ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)

ORGANISATION OF ISLAMIC COOPERATION (OIC)

Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION **SUMMER SEMESTER, 2021-2022**

DURATION: 1 HOUR 30 MINUTES **FULL MARKS: 100**

Math 4241: Integral Calculus and Differential Equations

Programmable calculators are not allowed. Do not write anything on the question paper.

Answer **all 3 (three)** questions. Figures in the right margin indicate full marks of questions whereas corresponding CO and PO are written within parentheses.

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1. a) i. State the Mean Value Theorem for Integrals. Verify this theorem for $f(x) = x^2 + x$ on the interval [-12, 0]. 5 (CO1) (PO1)

ii. Evaluate $\int_0^3 f(x)dx$ if $f(x) = \begin{cases} x^2 & ; x < 2 \\ 3x - 2 & ; x \geq 2 \end{cases}$ 5 (CO1) (PO1)

iii. Define antiderivative of a function. Use antiderivative method to find the area under the graph of $y = x^2$ over the interval $[0, 1]$. 4 (CO2) (PO1)

b) Write down the statement of Fundamental Theorem of Calculus. Hence, evaluate the following integrals using the theorem: 2+12 (CO2) (PO1)

i. $\int_1^e x^2 \ln x dx$;

ii. $\int \frac{\sqrt{x^2-9}}{x} dx$;

iii. $\int \frac{dx}{x^2-4x+5}$;

iv. $\int \tan^{-1} x dx$.

c) What do you mean by net signed area? Find the net signed area between the graph of $f(x) = x - 1$ and the interval $[0, 2]$. Here, x_k^* chosen to be the left endpoint of each subinterval. 5 (CO2) (PO1)

2. a) i. Derive the formula for the volume of a right pyramid whose altitude is h and base is a square with sides of length a. 5 + 5 (CO2) (PO1)

ii. Find the area of the region that is enclosed between the curves $y^2 = 4x$ and $y = 2x - 4$.

b) i. Find the volume of the solid generated when the region between the graphs of the equation $f(x) = \sqrt{25 - x^2}$ and $g(x) = 3$ is revolved about the x-axis. 9 + 9 (CO2) (PO1)

ii. Use cylindrical shells to find the volume of the solid that is generated when the region that is enclosed by $y = x^3$, $y = 1$, and $x = 0$ is revolved about the line $y = 1$.

c) Use the concept of sigma notation with x_k^* as the right endpoint of each subinterval to find the area between the graph of $f(x) = x^2$ and the interval $[0, 1]$. 5 (CO2) (PO1)

3. a) i. Find the reduction formula for $\int \operatorname{cosec}^n x dx$. 4 + 6 (CO2) (PO1)

ii. If $I_n = \int_0^{\pi/4} \tan^n \theta d\theta$, then show that

$$n\left(I_{n+1} + I_{n-1}\right) = 1$$

Now, from this relation, find the value of I_8.

b) i. Define gamma and beta function. Show that $\Gamma(n+1) = n\Gamma(n) = n!$. 3 × 5 (CO2) (PO1)

ii. Show that $\int_0^{\pi/2} \sin^5 \theta \cos^4 \theta d\theta = \frac{8}{315}$.

iii. Prove that $\int_0^{\pi/2} \sin^p \theta d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{1}{2}\right)$, hence evaluate $\Gamma\left(\frac{1}{2}\right)$.

c) i. Find the arc length of the curve $y = \ln\left|\frac{e^x-1}{e^x+1}\right|$ from $x = 1$ to $x = 3$. 5 + 4 (CO2) (PO1)

ii. Find the area of the region that is enclosed between the curves $y^2 = 4x$ and $y = 2x - 4$.