ISLAMIC UNIVERSITY OF TECHNOLOGY (IUT)
ORGANISATION OF ISLAMIC COOPERATION (OIC)
Department of Computer Science and Engineering (CSE)

MID SEMESTER EXAMINATION   SUMMER SEMESTER, 2022-2023
DURATION: 1 HOUR 30 MINUTES   FULL MARKS: 75

Math 4441: Probability and Statistics

Programmable calculators are not allowed. Do not write anything on the question paper.
Answer all **3 (three)** questions. Figures in the right margin indicate full marks of questions with
corresponding COs and POs in parentheses.

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1. a) A batch of 140 semiconductor chips is inspected by choosing a sample of five chips. Assume 10 of the chips do not conform to customer requirements.   `12.5 (CO1) (PO1)`

   i. How many different samples are possible?

   ii. How many samples of five contain exactly one nonconforming chip?

   iii. How many samples of five contain at least one nonconforming chip?

   b) Customers are often asked to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.   `12.5 (CO1) (PO1)`

   i. What is the probability that a product will attain a good review?

   ii. If a new design attains a good review, what is the probability that it will be a highly successful product?

2. a) Define Probability Mass Function (PMF) with necessary conditions. Consider the following function, $f(x)$:   `8 (CO1) (PO1)`

$$f(x) = \begin{cases} \dfrac{4}{5}\left(\dfrac{1}{5}\right)^x, & x = 0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$$

   Verify that $f(x)$ is a PMF and determine the following probabilities:

   i. $P(X = 4)$,

   ii. $P(X \leq 3)$,

   iii. $P(X > 2)$,

   iv. $P(X \geq 2)$.

   b) Find the Cumulative Distribution Function (CDF) of the distribution in Question 2.a and compute the following probabilities using the CDF: `7 (CO1) (PO1)`

   i. $P(X \leq 100)$,

   ii. $P(X > 150)$,

   iii. $P(80 < X < 120)$.

c) The Probability Density Function (PDF) for the diameter of a drilled hole in millimeters is: 10 (CO2) (PO2)

$$f(x) = \begin{cases} 10e^{-10(x-5)}, & x > 5 \\ 0, & \text{otherwise} \end{cases}$$

Although the target diameter is 5 millimeters, vibrations, tool wear, and other nuisances produce diameters larger than 5 millimeters.

   i. Determine the mean and variance of the diameter of the holes.

   ii. Determine the probability that a diameter exceeds 5.1 millimeters.

3.   a) Consider the following PMF, $f(x)$: 12 (CO1) (PO1)

$$f(x) = \begin{cases} \dfrac{2x + 1}{25}, & x = 0, 1, \dots, 4 \\ 0, & \text{otherwise} \end{cases}$$

Now, compute the following:

   i. $E(X)$,

   ii. $E(2X^3 - 3)$,

   iii. $Var(X)$,

   iv. $Var(3X^2 - 100)$.

b) The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline. 6 (CO2) (PO2)

   i. What is the probability that for exactly three calls the lines are occupied?

   ii. What is the probability that for at least one call the lines are not occupied?

   iii. What is the expected number of calls in which the lines are all occupied?

c) The number of content changes to a Web site follows a Poisson distribution with a mean of 0.25 per day. 7 (CO2) (PO2)

   i. What is the probability of two or more changes in a day?

   ii. What is the probability of no content changes in five days?

   iii. What is the probability of two or fewer changes in five days?