一维热传导-对流方程的解析解

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耦合热传导对流方程

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} + W\frac{\partial T}{\partial z}$$

$k\frac{\partial^2 T}{\partial z^2}$ → 热传导项

$W\frac{\partial T}{\partial z}$ → 热对流项

$$W = -\frac{C_w}{C_g} w\theta$$

Gao. Z., X. Fan, and L. Bian. 2003:
An Analytical Solution to One-Dimensional Thermal Conduction-Convection in Soil. *Soil Science*, 168(2): 99-107.

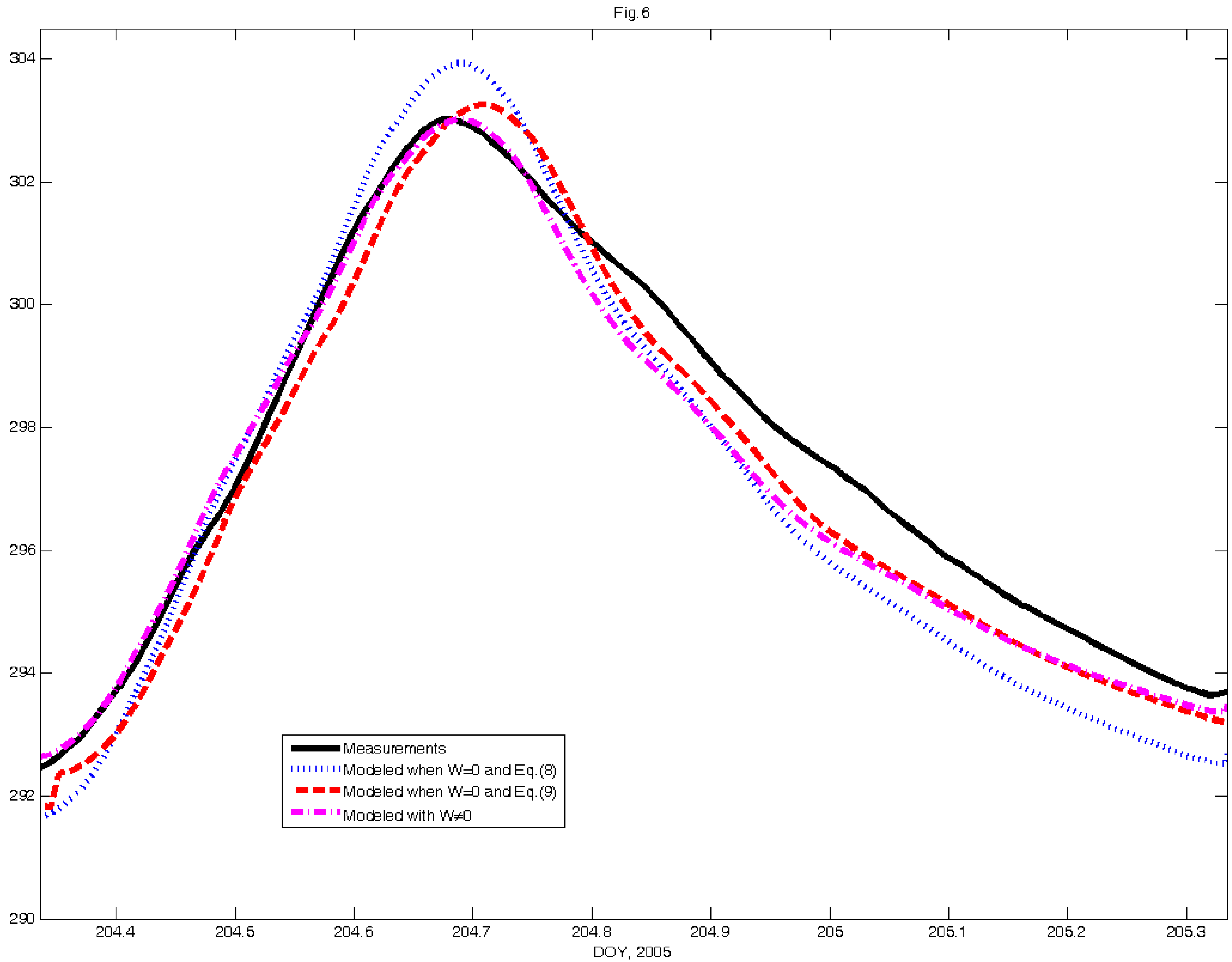
$$\frac{\partial T}{\partial t} = \frac{\lambda}{C_g}\frac{\partial^2 T}{\partial z^2} + \frac{1}{C_g}\frac{\partial \lambda}{\partial z}\frac{\partial T}{\partial z} \approx k\frac{\partial^2 T}{\partial z^2} + \frac{\partial k}{\partial z}\frac{\partial T}{\partial z}$$

$$\frac{\partial T}{\partial t} = -\frac{C_W}{C_g} w\theta \frac{\partial T}{\partial z}$$

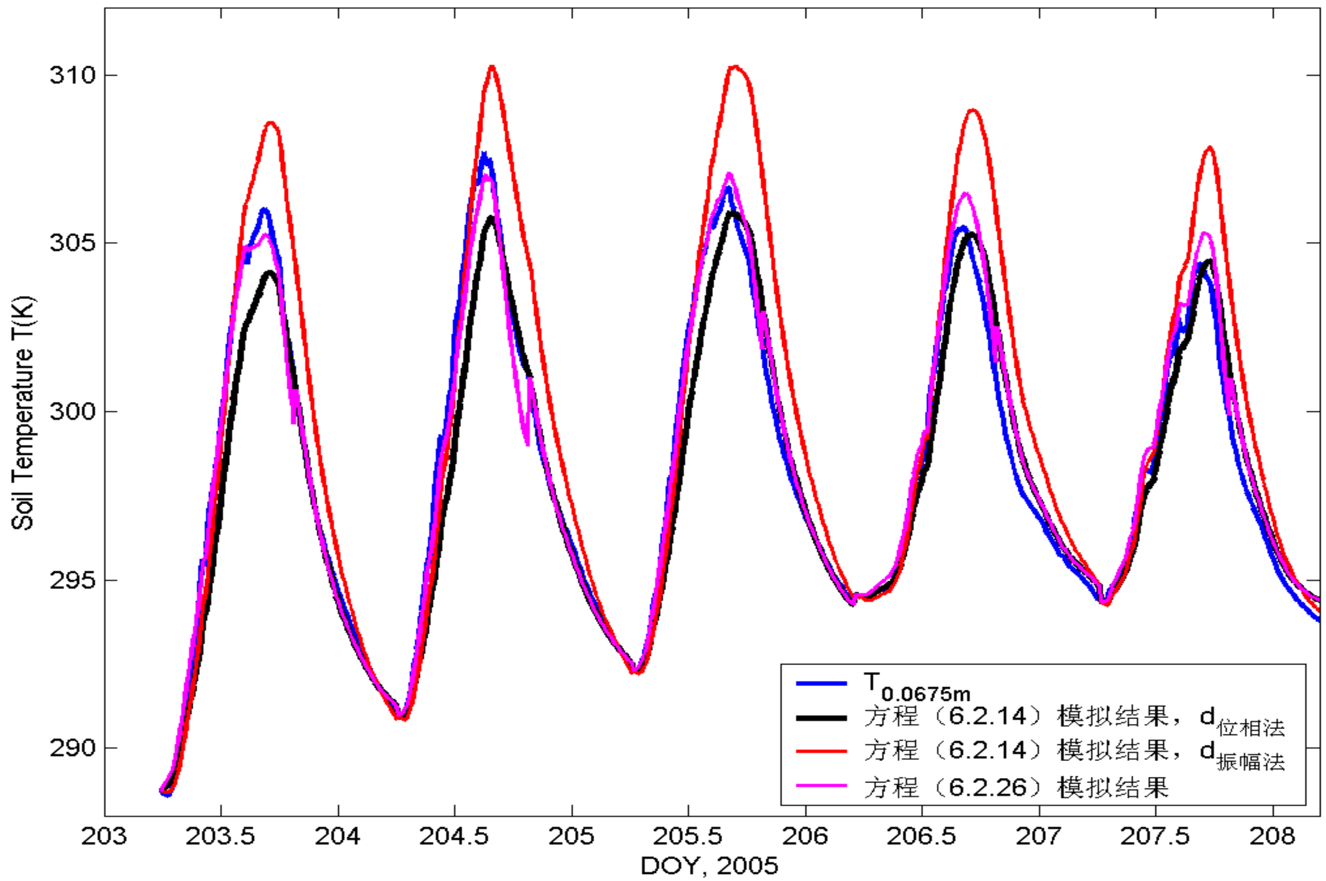
$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} + W\frac{\partial T}{\partial z}$$

$$\boxed{W \equiv \frac{\partial k}{\partial z} - \frac{C_W}{C_g} w\theta}$$

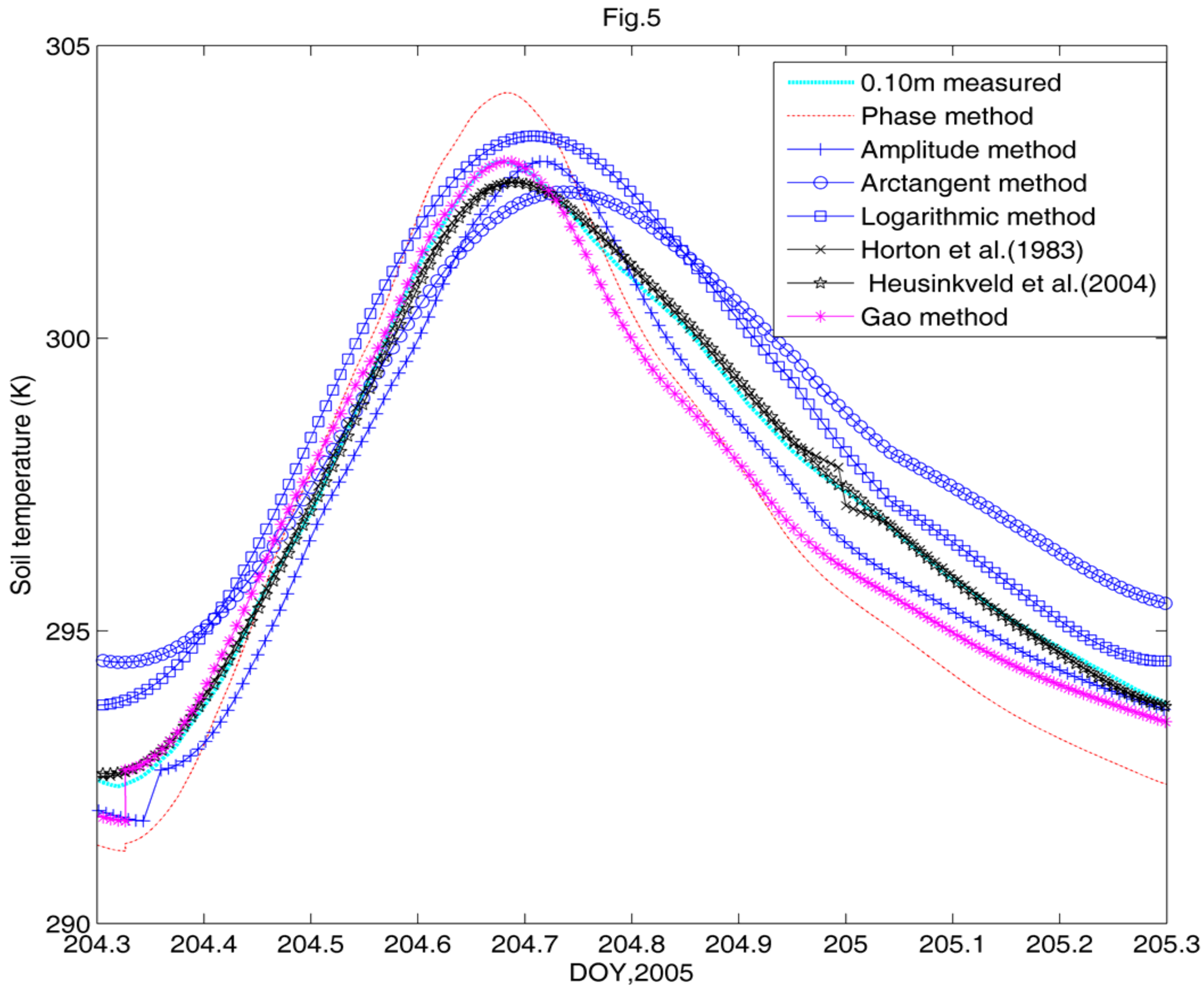
Gao, Z., D. H. Lenschow, R. Horton, M. Zhou, L. Wang, and J. Wen 2008, Comparison of Two Soil Temperature Algorithms for a Bare Ground Site on the Loess Plateau in China, *J. Geophys. Res.,* doi:10.1029/2008JD010285.



Gao, Z. (高志球), D. H. Lenschow, R. Horton, M. Zhou, L. Wang, and J. Wen 2008, Comparison of Two Soil Temperature Algorithms for a Bare Ground Site on the Loess Plateau in China, *J. Geophys. Res.*, doi:10.1029/2008JD010285.

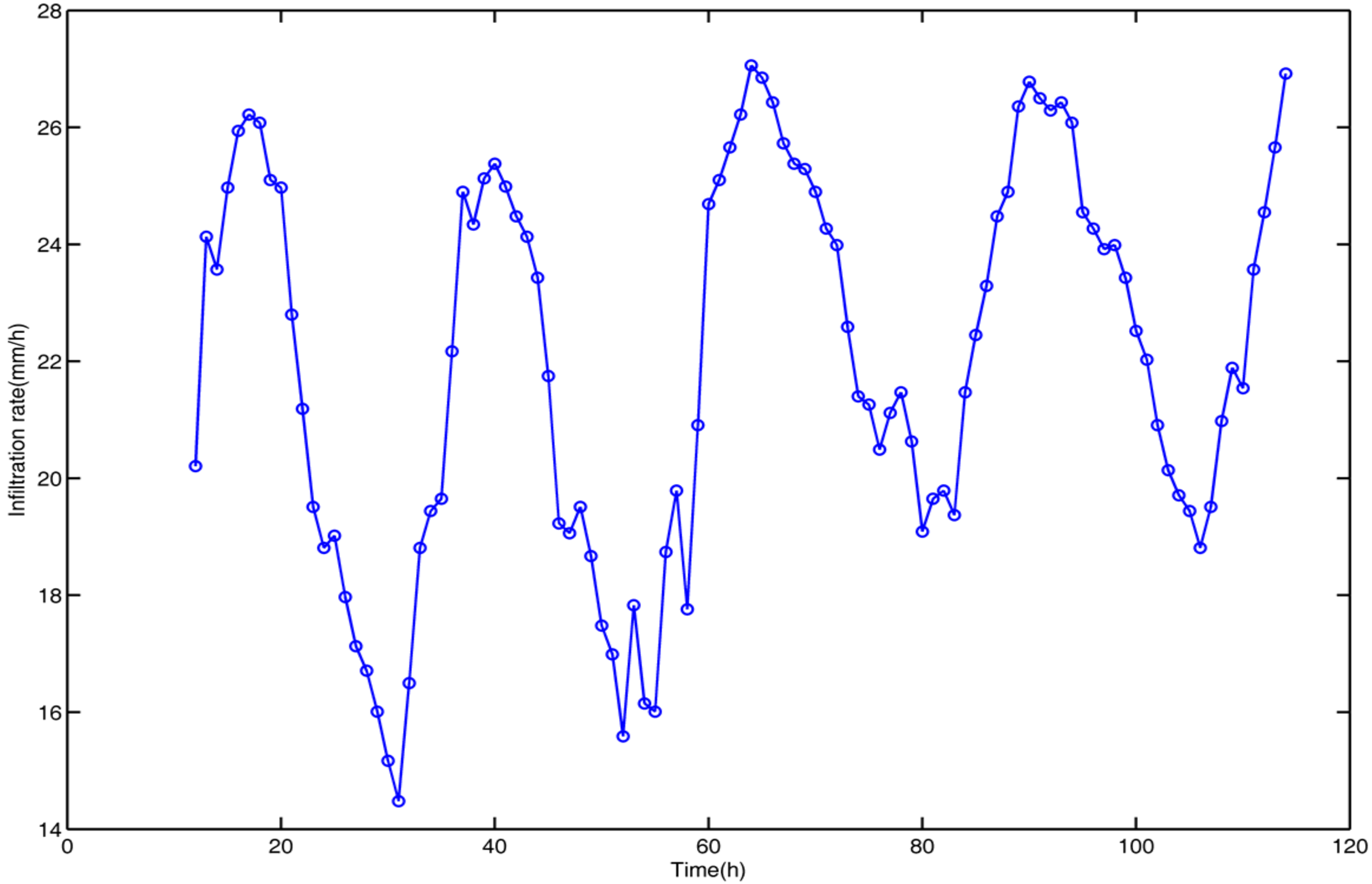


Gao Z., R. Horton, L. Wang, J. Wen, 2008: An Extension of the Force-Restore Method for Soil Temperature Prediction. *European Journal of Soil Sicence,* doi: 10.1111/j.1365-2389.2008.01060.x



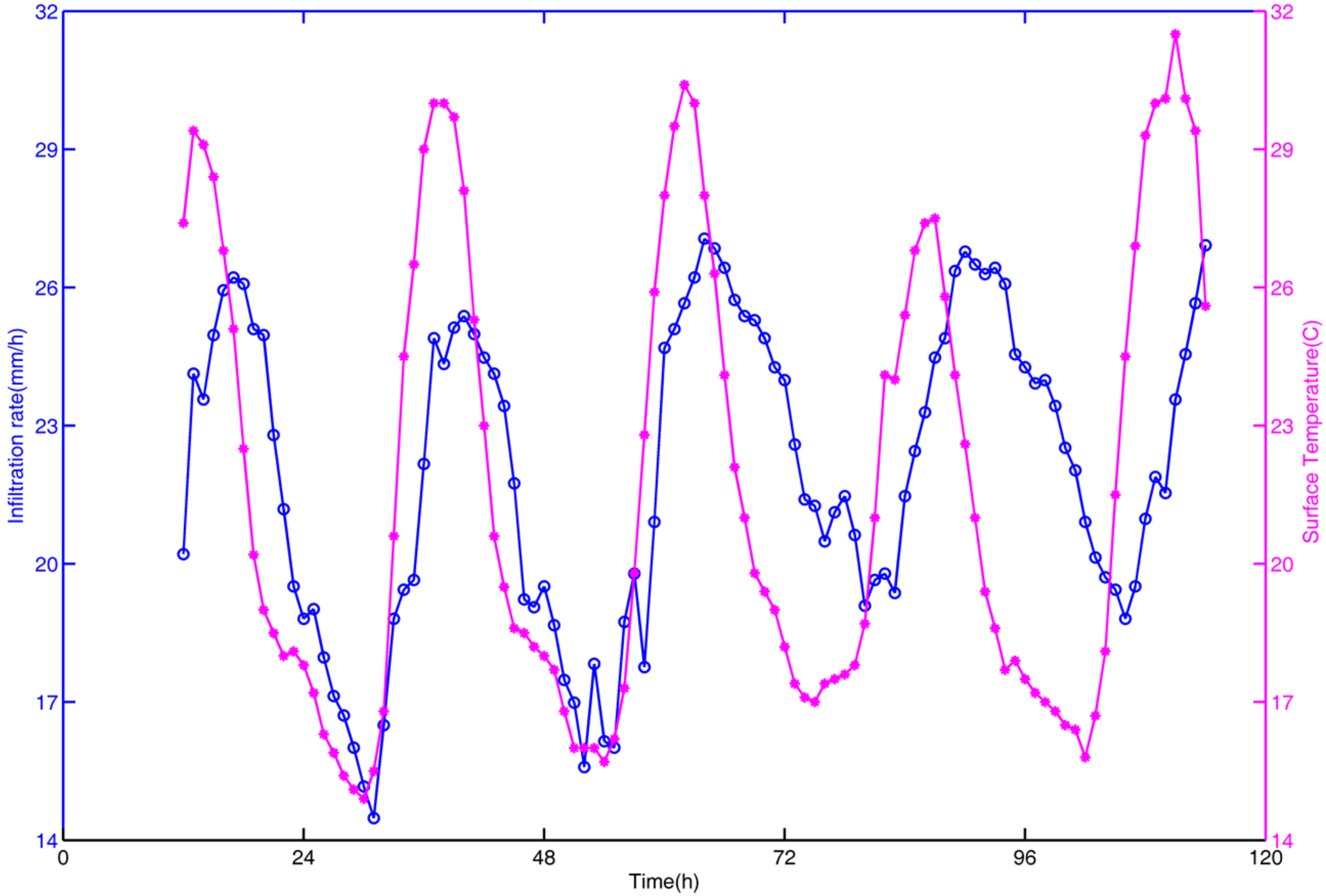
Wang, L., Z. Gao, R. Horton, Comparison of Six Methods to Determine the Surface Soil Thermal Diffusivity by using the data collected at the Bare Ground soil over the Loess Plateau in China.

土壤水通量密度



数据来源：Jaynes D. B., 1990: Temperature Variations Effect on Field-Measured Infiltration. *Soil Sci. Soc. Am. J.* 54: 305-312.

土壤水通量密度与土壤温度



Buckingham-Darcy通量定律：

$$W = -K(h)(\frac{\partial h}{\partial z} - 1)$$

考虑温度水分通量密度计算公式：

$$W(t) = -\frac{\eta_r}{\eta_t} K_r(h)(\frac{\partial h}{\partial z} - 1)$$

$$Kr(h) = Ksr\left|\frac{h_a}{h}\right|^n$$

当水流以常量下渗时，$\partial h/\partial = 0$ 此时水流只在重力梯度下移动

$$Kr(h) = Ksr$$

$$W(t) = -\frac{\eta_r}{\eta_t} K_{sr}$$

$$\frac{\eta_r}{\eta_T} = u_0 + u_1 T$$

$$T = T_0 + A\sin(\omega t)$$

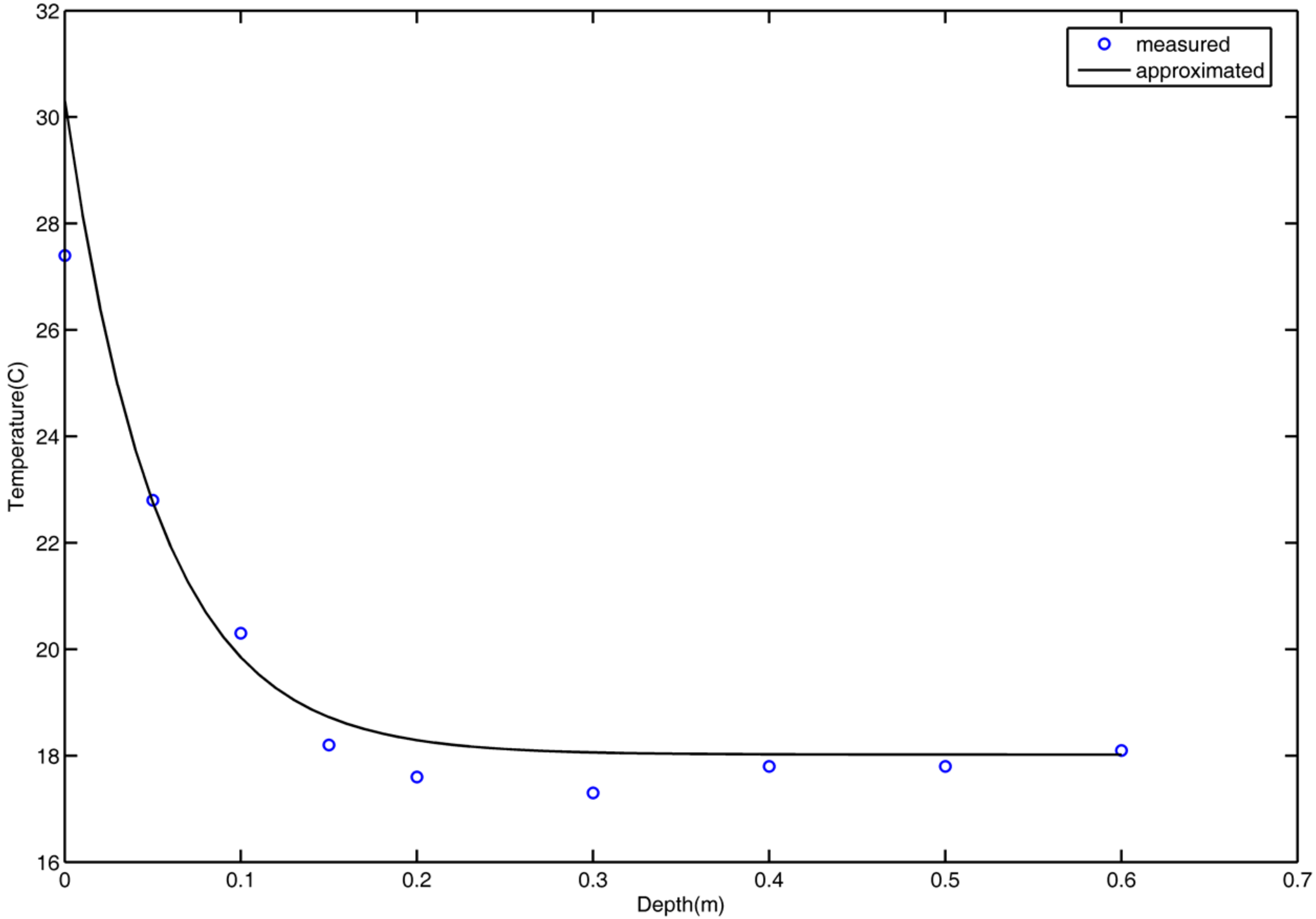
$$\boxed{W(t) = a + b\sin \omega t}$$

考虑土壤水分垂直运动日变化的土壤温度方程：

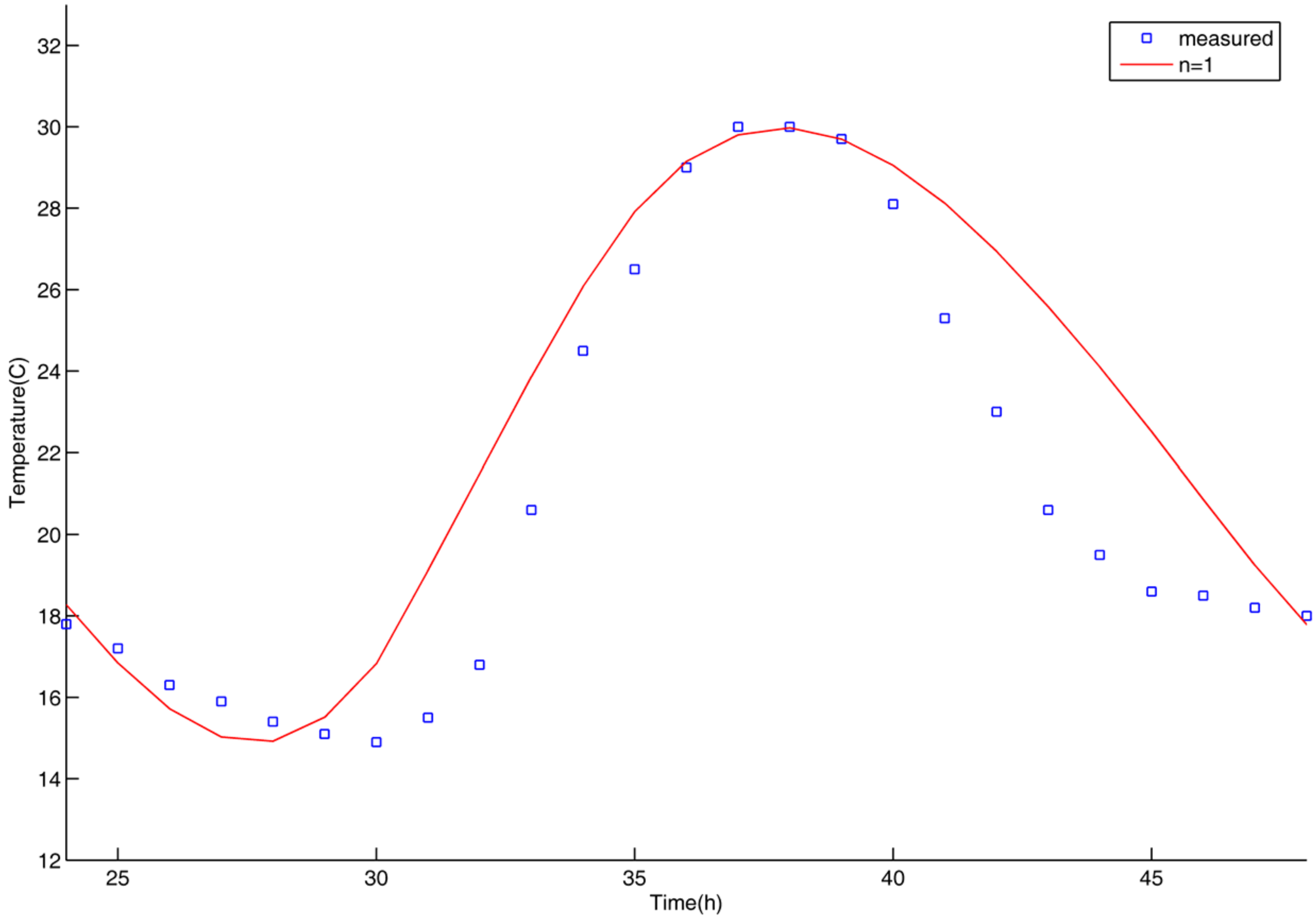
$$\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial z^2}-\left(a+b\sin\omega t\right)\frac{\partial T}{\partial z}$$

初始条件:

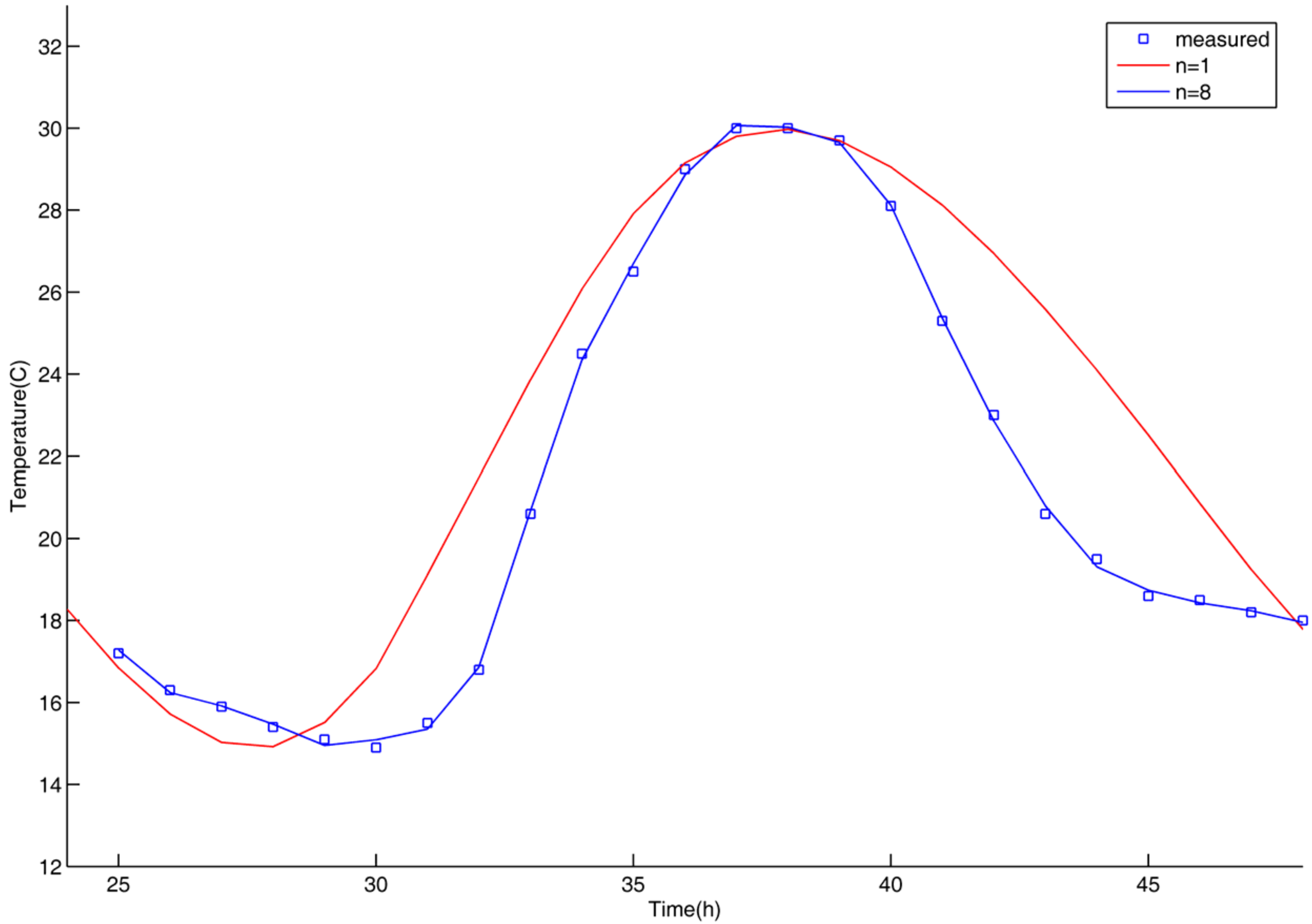
$$f(z) = T_1 + Be^{-kz}$$



边界条件： $T(0,t) = T_0 + A\sin(\omega t + \Phi)$



边界条件： $$T(0,t)=T_0+\sum_{i=1}^{n}A_i\sin(i\omega t+\Phi_i)$$ （$n=1,2,3\ldots n$）



$$\begin{cases} \dfrac{\partial T}{\partial t} = k\dfrac{\partial^2 T}{\partial z^2} - \boxed{\left(a+b\sin\omega t\right)}\dfrac{\partial T}{\partial z} \\ T(z,0)=f(z) \\ T(\infty,t)=T_1 \\ \boxed{T(0,t)=T_0+\sum_{i=1}^{n} A_i\sin(i\omega t+\Phi_i) \quad (n=1,2,3....n)} \end{cases}$$

$$T(z,t)=T_1+\exp\left(\frac{a_1 z}{2k}-\frac{a_1{}^2 t}{4k}\right)\frac{2}{\pi}\int_0^\infty V(\lambda,t)\sin(\lambda(x-\lambda_1(t))d\lambda$$

$$V(\lambda,t) = \exp(-kp^2 t + b_3\cos\omega t)(V_1+V_2+V_3+V_4+C)$$

$$V_1 = b_1\exp(b_2 t)\sum_{i=1}^{n}\frac{b_2 A_i\sin(i\omega t+\Phi_i) - i\omega A_i\cos(i\omega t+\Phi_i)}{b_2{}^2+(i\omega)^2}$$

$$V_2 = -b_1 b_3\sum_{i=1}^{n}[\cos\Phi_i V_2{}'(i) + \sin\Phi_i V_2{}''(i)]$$

$$V_2{}'(i) = \frac{1}{2}\frac{\exp(b_2 t)}{b_2{}^2+(i+1)^2\omega^2}[b_2 A_i\sin[(i+1)\omega t] - (i+1)\omega A_i\cos[(i+1)\omega t]]$$

$$+\frac{1}{2}\frac{\exp(b_2 t)}{b_2{}^2+(i-1)^2\omega^2}[b_2 A_i\cos[(i-1)\omega t] - (i-1)\omega A_i\sin[(i-1)\omega t]]$$

$$V_3 = \frac{b_4}{b_2}\exp(b_2 t)$$

$$V_4 = -b_4 b_3\exp(b_2\mathrm{t})\frac{b_2\cos(\omega t)+\omega\sin(\omega t)}{b_2{}^2+\omega^2}$$

$$C=\exp(-\frac{a_1 b}{2k\omega})V(\lambda,0)-[V_1(\lambda,0)+V_2(\lambda,0)+V_3(\lambda,0)+V_4(\lambda,0)]$$

谢谢！