目录

2000 年美国大学生数学建模竞赛 MCM、ICM 试题 3

2000 MCM A: Air Traffic Control 3

2000 MCM B: Radio Channel Assignments 3

2000 ICM: Elephants: When is Enough, Enough? 5

2001 年美国大学生数学建模竞赛 MCM、ICM 试题 7

2001 MCM A: Choosing a Bicycle Wheel 7

2001 MCM B: Escaping a Hurricane's Wrath (An Ill Wind…) 8

2001 ICM: Our Waterways - An Uncertain Future 10

2002 年美国大学生数学建模竞赛 MCM、ICM 试题 14

2002 MCM A: Wind and Waterspray 14

2002 MCM B: Airline Overbooking 14

2002 ICM: Scrub Lizards 15

2003 年美国大学生数学建模竞赛 MCM、ICM 试题 19

2003 MCM A: The Stunt Person 19

2003 MCM B: Gamma Knife Treatment Planning 19

2003 ICM: Aviation Baggage Screening Strategies: To Screen or Not to Screen, that is the Question 20

2004 年美国大学生数学建模竞赛 MCM、ICM 试题 24

2004 MCM A: Are Fingerprints Unique? 24

2004 MCM B: A Faster QuickPass System 24

2004 ICM: To Be Secure or Not to Be? 24

2005 年美国大学生数学建模竞赛 MCM、ICM 试题 25

2005 MCM A: Flood Planning 25

2005 MCM B: Tollbooths 25

2005 ICM: Nonrenewable Resources 25

2006 年美国大学生数学建模竞赛 MCM、ICM 试题 27

2006 MCM A: Positioning and Moving Sprinkler Systems for Irrigation 27

2006 MCM B: Wheel Chair Access at Airports 27

2006 ICM: Trade-offs in the fight against HIV/AIDS 28
2007 年美国大学生数学建模竞赛 MCM、ICM 试题 32
2007 MCM A: Gerrymandering 32
2007 MCM B: The Airplane Seating Problem 32
2007 ICM: Organ Transplant: The Kidney Exchange Problem 33
2008 年美国大学生数学建模竞赛 MCM、ICM 试题 38
2008 MCM A: Take a Bath 38
2008 MCM B: Creating Sudoku Puzzles 38
2008 ICM: Finding the Good in Health Care Systems 38
2009 年美国大学生数学建模竞赛 MCM、ICM 试题 40
2009 MCM A: Designing a Traffic Circle 40
2009 MCM B: Energy and the Cell Phone 40
2009 ICM: Creating Food Systems: Re-Balancing Human-Influenced Ecosystems 41

2000 年美国大学生数学建模竞赛 MCM、ICM 试题

2000 MCM A: Air Traffic Control

To improve safety and reduce air traffic controller workload, the Federal Aviation Agency (FAA) is considering adding software to the air traffic control system that would automatically detect potential aircraft flight path conflicts and alert the controller. To that end, an analysit at the FAA has posed the following problems.

Requirement A: Given two airplanes flying in space, when should the air traffic controller consider the objects to be too close and to require intervention?

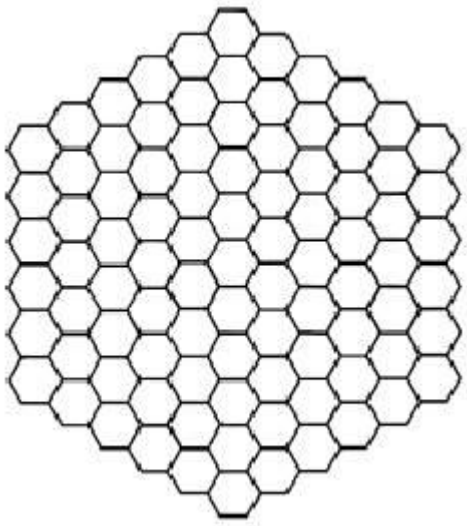
Requirement B: And airspace sector is the section of three-dimensional airspace that one air traffic controller controls. Given any airspace sector, how do we measure how complex it is from an air traffic workload perspective? To what extent is complexity determined by the number of aircraft simultaneously passing through that sector

1. at any one instant?
2. during any given interval of time?
3. during a particular time of day?

How does the number of potential conflicts arising during those periods affect complexity? Does the presence of additional software tools to automatically predict conflicts and alert the controller reduce or add to this complexity? In addition to the guidelines for your report, write a summary (no more than two pages) that the FAA analyst can present to Jane Garvey, the FAA Administrator, to defend your conclusions.

2000 MCM B: Radio Channel Assignments

We seek to model the assignment of radio channels to a symmetric network of transmitter locations over a large planar area, so as to avoid interference. One basic approach is to partition the region into regular hexagons in a grix (honeycomb-style), as shown in Figure 1, where a transmitter is located at the center of each hexagon.



An interval of the frequency spectrum is to be alloted for transmitter frequencies. The interval will be divided into regularly spaced channels, which we represent by integers 1,2,3, ⋯ . Each transmitter wil be assigned one positive integer channel. The same channel can be used at many locations, provided that interference from nearby transmitters is avoided.

Our goal is to minimize the width of the interval in the frequency spectrum that is needed to assugn channels subject to some constraints. This is achieved with the concept of a span. The span is the minimum, over all assignments satisfying the constraints, of the largest channel used at any location. It is not required that every channel smaller than the span be used in an assignment that attains the span.

Let s be the length of a side of one of the hexagons. We concentrate on the case that there are two levels of interference.

Requirement A: There are several contrainsts on the frequency assignments. First, no two transmitters within distance 4s of each other can be given the same channel. Second, due to spectral spreading, transmitters within distance 2s of each other must not be given the same or adjacent channels: Their channels must differ by at least 2. Under these contraints, what can we say about the span in Figure 1?

Requirement B: Repeat Requirement A, assuming the grid in the example spreads arbitrarily far in all directions.

Requirement C: Repeat Requirements A and B, except assume now more generally that channels for transmitters within distance 2s differ by at least some given integer k, while those at distance at most 4s must still differ by at least one. What cna we say about the span and about efficient strategies for designing assignments, as a function of k?

Requirement D: Consider generalizations of the problem, such as several levels of interference or irregular transmitter placements. What other factors may be important to consider?

Requirement E: Write an article (no more than 2 pages) for the local newspaper explaining your findings.

2000 ICM: Elephants: When is Enough, Enough?

"Ultimately, if a habitat is undesirably changed by elephants, then their removal should be considered -even by culling." National Geographic (Earth Almanac) – December 1999 A large National Park in South Africa contains approximately 11,000 elephants. Management policy requires a healthy environment that can maintain a stable herf of 11,000 elephants. Each year park rangers count the elephant population. During the past 20 years whole herds have been removed to keep the population as close to 11,000 as possible. The process involved shooting (for the most part) and occasionally relocating approximately 600 to 800 elephants per year.

Recently, there has been a public outcry against the shooting of these elephants. In addition, it is no longer feasible to relocate even a small population of elephants each year. A contraceptive dart, however, has been developed that can prevent a mature elephant cow from conceiving for a period of two years.

Here is some information about eh elephants in the Park:

- There is very little emigration of immigration of elephants.
- The gender ratio is very close to 1:1 and control measures have endeavored to maintain parity.
- The gender ratio of newborn calves is also about 1:1. Twins are born about 1.35% of the time.
- Cows first conceive between the ages of 10 and 12 and produce, on average, a calf every 3.5 years until they reach an age of about 60. Gestation is approximately 22 months.
- The contraceptive dart causes an elephant cow to come into oestrus every month (but not conceiving). Elephants usually have courtship only once in 3.5 years, so the monthly cycle can cause additional stress.
- A cow can be darted every year without additional detrimental effects. A mature elephant cow will not be able to conceive for 2 years after the last darting.
- Between 70% and 80% of newborn calves survive to age 1 year. Thereafter, the survival rate is uniform across all ages and is very high (over 95%), until about age 60; it is a good assumption that elephants die before reading age 70.

- There is no hunting and negligible poaching in the Park.

The park management has a rough data file of the approximate ages and gender of the elephants they have transported out of the region during the past 2 years. This data is available on website: icm2000data.xls. Unfortunately no data is available for the elephants that have been shot or remain in the Park.

Your overall task is to develop and use models to investigate how the contraceptive dart might be used for population control. Specifically:

Task 1: Develop and use a model to speculate about the likely survival rate for elephants aged 2 to 60. Also speculate about the current age structure of the elephant population.

Task 2: Estimate how many cows would need to be darted each year to keep the population fixed at approximately 11,000 elephants. Show how the uncertainty in the data at your disposal affects your estimate. Comment on any changes in the age structure of the population and how this might affect tourists. (You may want to look ahead about 30-60 years.)

Task 3: If it were feasible to relocate between 50 and 300 elephants per year, how would this reduce the number of elephants to be darted? Comment on the trade-off between darting and relocation.

Task 4: Some opponents of darting argue that if there were a sudden loss of a large number of elephants (due to disease or uncontrolled poaching), even if darting stopped immediately, the ability of the population to grow again would be seriously impeded. Investigate and respond to this concer.

Task 5: The management in the Park is skeptical about modeling. In particular, they argue that a lack of complete data makes a mockery of any attempt to use models to guide their decision. In addition to your technical report, include a carefully crafted report (3-page maximum) written explicitly for the park management that responds to their concerns and provides advice. Also suggest ways to increase the park managers confidence in your model and your conclusions.

Task 6: If your model works, other elephant parks in Africa would be interested in using it. Prepare a darting plan for parks of various sizes (300-25,000 elephants), with slightly different survival rates and transportation possibilities.

2001 年美国大学生数学建模竞赛 MCM、ICM 试题

2001 MCM A: Choosing a Bicycle Wheel

Cyclists have different types of wheels they can use on their bicycles. The two basic types of wheels are those constructed using wire spokes and those constructed of a solid disk (see Figure 1) The spoked wheels are lighter, but the solid wheels are more aerodynamic. A solid wheel is never used on the front for a road race but can be used on the rear of the bike.

Professional cyclists look at a racecourse and make an educated guess as to what kind of wheels should be used. The decision is based on the number and steepness of the hills, the weather, wind speed, the competition, and other considerations. The director sportif of your favorite team would like to have a better system in place and has asked your team for information to help determine what kind of wheel should be used for a given course.

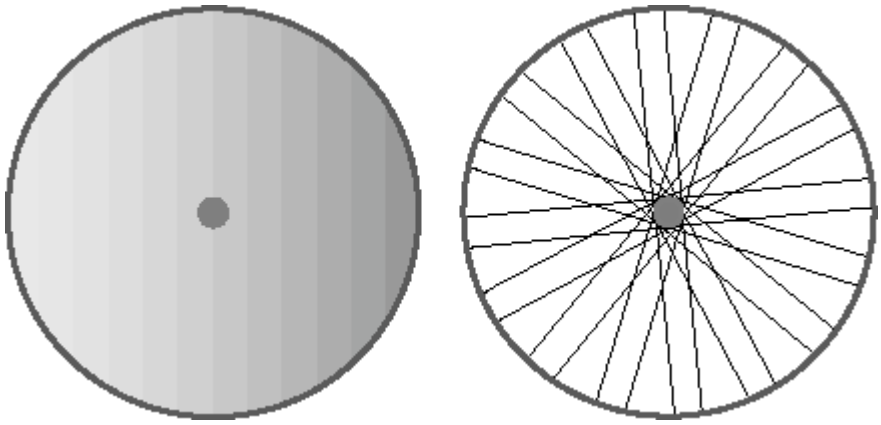


Figure 1: A solid wheel is shown on the left and a spoked wheel is shown on the right.

The director sportif needs specific information to help make a decision and has asked your team to accomplish the tasks listed below. For each of the tasks assume that the same spoked wheel will always be used on the front but there is a choice of wheels for the rear.

- Task 1. Provide a table giving the wind speed at which the power required for a solid rear wheel is less than for a spoked rear wheel. The table should include the wind speeds for different road grades starting from zero percent to ten percent in one percent increments. (Road grade is defined to be the ratio of the total rise of a hill divided by the length of the road. If the hill is viewed as a triangle, the grade is the sine of the angle at the bottom of the hill.) A rider starts at the bottom of the hill at a speed of 45 kph, and the deceleration of the rider is proportional

to the road grade. A rider will lose about 8 kph for a five percent grade over 100 meters.

- Task 2. Provide an example of how the table could be used for a specific time trial course.
- Task 3. Determine if the table is an adequate means for deciding on the wheel configuration and offer other suggestions as to how to make this decision.

2001 MCM B: Escaping a Hurricane's Wrath (An Ill Wind...)

Evacuating the coast of South Carolina ahead of the predicted landfall of Hurricane Floyd in 1999 led to a monumental traffic jam. Traffic slowed to a standstill on Interstate I-26, which is the principal route going inland from Charleston to the relatively safe haven of Columbia in the center of the state. What is normally an easy two-hour drive took up to 18 hours to complete. Many cars simply ran out of gas along the way. Fortunately, Floyd turned north and spared the state this time, but the public outcry is forcing state officials to find ways to avoid a repeat of this traffic nightmare.

The principal proposal put forth to deal with this problem is the reversal of traffic on I-26, so that both sides, including the coastal-bound lanes, have traffic headed inland from Charleston to Columbia. Plans to carry this out have been prepared (and posted on the Web) by the South Carolina Emergency Preparedness Division. Traffic reversal on principal roads leading inland from Myrtle Beach and Hilton Head is also planned.

A simplified map of South Carolina is shown. Charleston has approximately 500,000 people, Myrtle Beach has about 200,000 people, and another 250,000 people are spread out along the rest of the coastal strip. (More accurate data, if sought, are widely available.)

The interstates have two lanes of traffic in each direction except in the metropolitan areas where they have three. Columbia, another metro area of around 500,000 people, does not have sufficient hotel space to accommodate the evacuees (including some coming from farther north by other routes), so some traffic continues outbound on I-26 towards Spartanburg; on I-77 north to Charlotte; and on I-20 east to Atlanta. In 1999, traffic leaving Columbia going northwest was moving only very slowly. Construct a model for the problem to investigate what strategies may reduce the congestion observed in 1999. Here are the questions that need to be addressed:

1. Under what conditions does the plan for turning the two coastal-bound lanes of I-26 into two lanes of Columbia-bound traffic, essentially turning the entire I-26 into one-way traffic, significantly improve evacuation traffic flow?

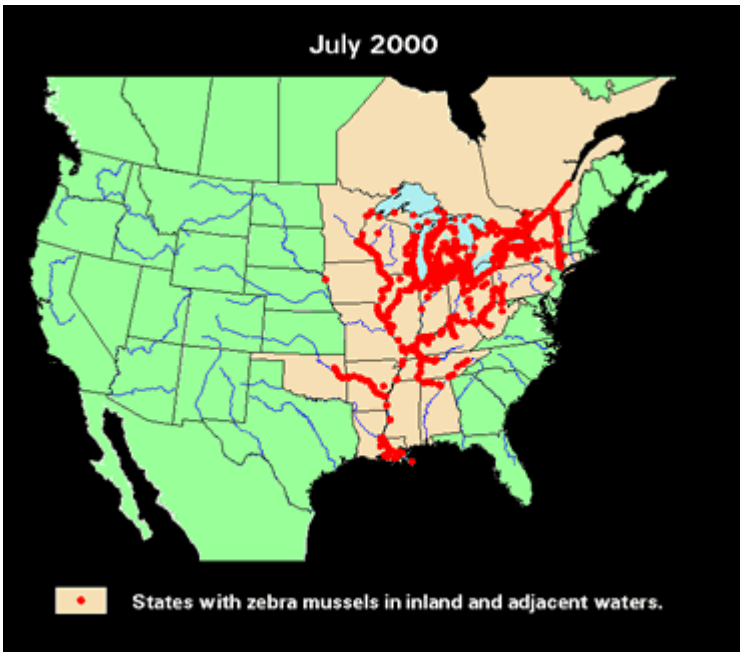
2. In 1999, the simultaneous evacuation of the state's entire coastal region was ordered. Would the evacuation traffic flow improve under an alternative strategy that staggers the evacuation, perhaps county-by-county over some time period consistent with the pattern of how hurricanes affect the coast?
3. Several smaller highways besides I-26 extend inland from the coast. Under what conditions would it improve evacuation flow to turn around traffic on these?
4. What effect would it have on evacuation flow to establish more temporary shelters in Columbia, to reduce the traffic leaving Columbia?
5. In 1999, many families leaving the coast brought along their boats, campers, and motor homes. Many drove all of their cars. Under what conditions should there be restrictions on vehicle types or numbers of vehicles brought in order to guarantee timely evacuation?
6. It has been suggested that in 1999 some of the coastal residents of Georgia and Florida, who were fleeing the earlier predicted landfalls of Hurricane Floyd to the south, came up I-95 and compounded the traffic problems. How big an impact can they have on the evacuation traffic flow? Clearly identify what measures of performance are used to compare strategies. Required: Prepare a short newspaper article, not to exceed two pages, explaining the results and conclusions of your study to the public.

Clearly identify what measures of performance are used to compare strategies.

Required: Prepare a short newspaper article, not to exceed two pages, explaining the results and conclusions of your study to the public.

South Carolina

2001 ICM: Our Waterways - An Uncertain Future



Zebra mussels, Dreissena polymorpha, are small, fingernail-sized, freshwater mollusks unintentionally introduced to North America via ballast water from a transoceanic vessel. Since their introduction in the mid 1980s, they have spread through all of the Great Lakes and to an increasing number of inland waterways in the United States and Canada. Zebra mussels colonize on various surfaces,

such as docks, boat hulls, commercial fishing nets, water intake pipes and valves, native mollusks and other zebra mussels. Their only known predators, some diving ducks, freshwater drum, carp, and sturgeon, are not numerous enough to have a significant effect on them. Zebra mussels have significantly impacted the Great Lakes ecosystem and economy. Many communities are trying to control or eliminate these aquatic pests. SOURCE: Great Lakes Sea Grant Network http://www.sgnis.org/.

Researchers are attempting to identify the environmental variables related to the zebra mussel infestation in North American waterways. The relevant factors that may limit or prevent the spread of the zebra mussel are uncertain. You will have access to some reference data to include listings of several chemicals and substances in the water system that may affect the spread of the zebra mussel throughout waterways. Additionally, you can assume individual zebra mussels grow at a rate of 15 millimeters per year with a life span between 4 - 6 years. The typical mussel can filter 1 liter of water each day.

Requirement A: Discuss environmental factors that could influence the spread of zebra mussels.

Requirement B: Utilizing the chemical data provided at: http://www.comap/undergraduate/contests/icm/imagesdata/LakeAChem1.xls, and the mussel population data provided at: http://www.comap/undergraduate/contests/icm/imagesdata/LakeAPopulation 1.xls model the population growth of zebra mussels in Lake A. Be sure to review the Information about the collection of the zebra mussel data.

Requirement C: Utilizing additional data on Lake A from another scientist provided at : http://www.comap/undergraduate/contests/icm/imagesdata/LakeAChem2.xls and additional mussel population data provided at: http://www.comap/undergraduate/contests/icm/imagesdata/LakeAPopulation 2.xls corroborate the reasonableness of your model from Requirement B. As a result of this additional data, adjust your earlier model. Analyze the performance of your model. Discuss the sensitivity of your model.

Requirement D: Utilizing the Chemical data from two lakes (Lake B and Lake C) in the United States provided at http://www.comap/undergraduate/contests/icm/imagesdata/LakeB.xls and http://www.comap/undergraduate/contests/icm/imagesdata/LakeC.xls determine if these lakes are vulnerable to the spread of zebra mussels. Discuss your prediction.

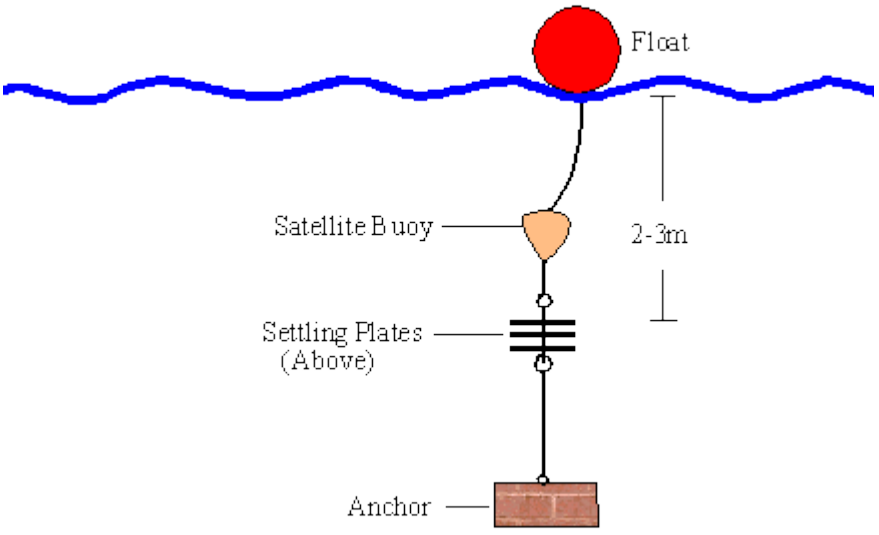
Requirement E: The community in the vicinity of Lake B (in requirement D) is considering specific policies for the de-icing of roadways near the lake during

the winter season. Provide guidance to the local government officials regarding a policy on "de-icing agents." In your guidance include predictions on the long-term impact of de-icing on the zebra mussel population.

Requirement F: It has been recommended by a local community in the United States to introduce round goby fish. Zebra mussels are not often eaten by native fish species so they represent a dead end ecologically. However, round gobies greater than 100 mm feed almost exclusively on zebra mussels. Ironically, because of habitat destruction, the goby is endangered in its native habitat of the Black and Caspian Seas in Russia. In addition to your technical report, include a carefully crafted report (3-page maximum) written explicitly for the local community leaders that responds to their recommendation to introduce the round goby. Also suggest ways to help reduce the growth of the mussel within and among waterways.

Information about the collection of the zebra mussel data

The developmental state of the Zebra mussel is categorized by three stages: veligers (larvae), settling juveniles, and adults. Veligers (microscopic zebra mussel larvae) are free-swimming, suspended in the water for one to three weeks, after which they begin searching for a hard surface to attach to and begin their adult life. Looking for zebra mussel veligers is difficult because they are not easily visible by the naked eye. Settled juvenile zebra mussels can be felt on smooth surfaces like boats and motors. An advanced zebra mussel infestation can cover a surface, even forming thick mats sometimes reaching very high densities. The density of juveniles was determined along the lake using three 15×15 cm settling plates. The top plate remained in the water for the entire sampling season (S - seasonal) to estimate seasonal accumulation. The middle and bottom plates are collected after specific periods (A – alternating) of time denoted by "Lake Days" in the data files.



The settling plates are placed under the microscope and all juveniles on the undersides of the plate are counted and densities are reported as juveniles/m^2.

2002 年美国大学生数学建模竞赛 MCM、ICM 试题

2002 MCM A: Wind and Waterspray

An ornamental fountain in a large open plaza surrounded by buildings squirts water high into the air. On gusty days, the wind blows spray from the fountain onto passersby. The water-flow from the fountain is controlled by a mechanism linked to an anemometer (which measures wind speed and direction) located on top of an adjacent building. The objective of this control is to provide passersby with an acceptable balance between an attractive spectacle and a soaking: The harder the wind blows, the lower the water volume and height to which the water is squirted, hence the less spray falls outside the pool area.

Your task is to devise an algorithm which uses data provided by the anemometer to adjust the water-flow from the fountain as the wind conditions change.

2002 MCM B: Airline Overbooking

You're all packed and ready to go on a trip to visit your best friend in New York City. After you check in at the ticket counter, the airline clerk announces that your flight has been overbooked. Passengers need to check in immediately to determine if they still have a seat.

Historically, airlines know that only a certain percentage of passengers who have made reservations on a particular flight will actually take that flight. Consequently, most airlines overbook-that is, they take more reservations than the capacity of the aircraft. Occasionally, more passengers will want to take a flight than the capacity of the plane leading to one or more passengers being bumped and thus unable to take the flight for which they had reservations.

Airlines deal with bumped passengers in various ways. Some are given nothing, some are booked on later flights on other airlines, and some are given some kind of cash or airline ticket incentive.

Consider the overbooking issue in light of the current situation: Less flights by airlines from point A to point B Heightened security at and around airports Passengers' fear Loss of billions of dollars in revenue by airlines to date

Build a mathematical model that examines the effects that different overbooking schemes have on the revenue received by an airline company in order to find an optimal overbooking strategy, i.e., the number of people by which an airline should overbook a particular flight so that the company's revenue is maximized. Insure that your model reflects the issues above, and

consider alternatives for handling "bumped" passengers. Additionally, write a short memorandum to the airline's CEO summarizing your findings and analysis.

2002 ICM: Scrub Lizards

The Florida scrub lizard is a small, gray or gray-brown lizard that lives throughout upland sandy areas in the Central and Atlantic coast regions of Florida. The Florida Committee on Rare and Endangered Plants classified the scrub lizard as endangered.

You will find a fact sheet on the Florida Scrub Lizard at http://www.comap.com/undergraduate/contests/mcm/contests/2002/problems/icm2002data/scrublizard.pdf

The long-term survival of the Florida scrub lizard is dependent upon preservation of the proper spatial configuration and size of scrub habitat patches.

Task 1: Discuss factors that may contribute to the loss of appropriate habitat for scrub lizards in Florida. What recommendations would you make to the state of Florida to preserve these habitats and discuss obstacles to the implementation of your recommendations?

Task 2: Utilize the data provided in Table 1 to estimate the value for Fa (the average fecundity of adult lizards); Sj (the survivorship of juvenile lizards-between birth and the first reproductive season); and Sa (the average adult survivorship).

Table 1

Summary data for a cohort of scrub lizards captured and followed for 4 consecutive years. Hatchling lizards (age 0) do not produce eggs during the summer they are born. Average clutch size for all other females is proportional to body size according to the function y = 0.21*(SVL)-7.5, where y is the clutch size and SVL is the snout-to-vent length in mm.

Year	Age	Total Number Living	Number of Living Females	Avg. Female Size (mm)
1	0	972	495	30.3
2	1	180	92	45.8
3	2	20	11	55.8
4	3	2	2	56.0

Task 3: It has been conjectured that the parameters Fa , Sj , and Sa , are related to the size and amount of open sandy area of a scrub patch. Utilize the data provided in Table 2 to develop functions that estimate Fa, Sj , and Sa for different patches. In addition, develop a function that estimates C, the carrying capacity of scrub lizards for a given patch.

Table 2

Summary data for 8 scrub patches including vital rate data for scrub lizards. Annual female fecundity (Fa), juvenile survivorship (Sj), and adult survivorship (Sa) are presented for each patch along with patch size and the amount of open sandy habitat.

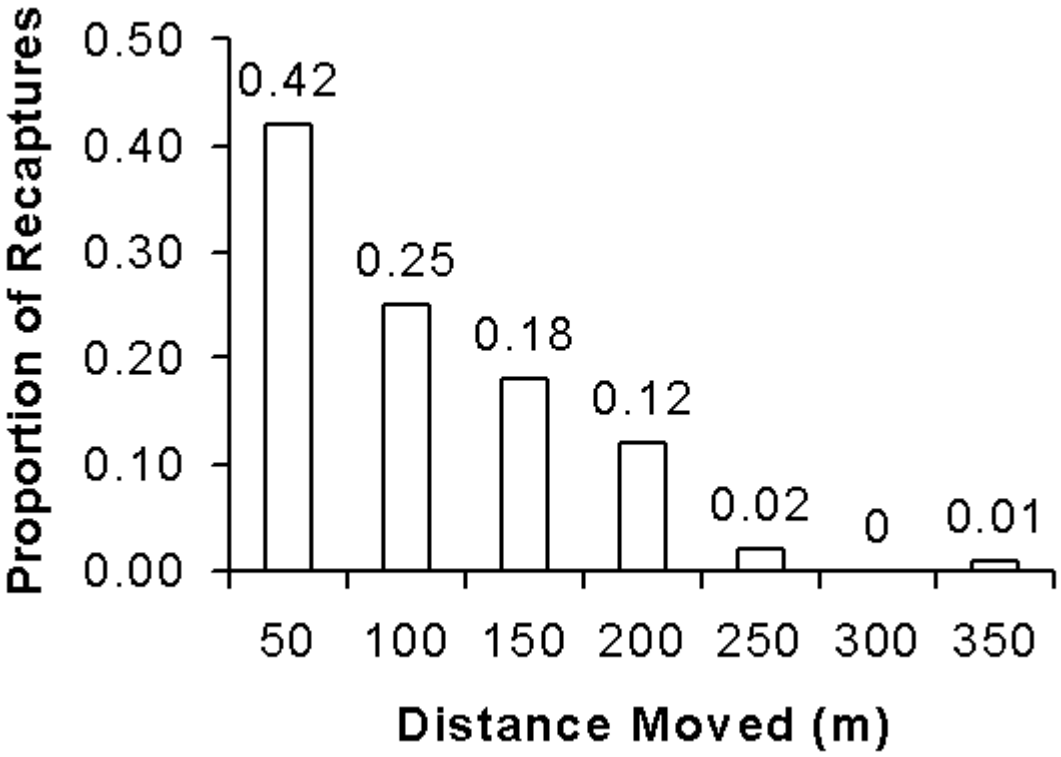
Patch	Patch Size (ha)	Sandy Habitat (ha)	Fa	Sj	Sa	Density (lizards/ha)
a	11.31	4.80	5.6	0.12	0.06	58
b	35.54	11.31	6.6	0.16	0.10	60
c	141.76	51.55	9.5	0.17	0.13	75
d	14.65	7.55	4.8	0.15	0.09	55
e	63.24	20.12	9.7	0.17	0.11	80
f	132.35	54.14	9.9	0.18	0.14	82
g	8.46	1.67	5.5	0.11	0.05	40
h	278.26	84.32	11.0	0.19	0.15	115

Task 4: There are many animal studies that indicate that food, space, shelter, or even reproductive partners may be limited within a habitat patch causing individuals to migrate between patches. There is no conclusive evidence on why scrub lizards migrate. However, about 10 percent of juvenile lizards do migrate between patches and this immigration can influence the size of the population within a patch. Adult lizards apparently do not migrate. Utilizing the data provided in the histogram below estimate the probability of lizards surviving the migration between any two patches i and patch j.

Table 3

Histogram

Migration data for juvenile lizards marked, released, and recaptured up to 6 months later. Surveys for recapture were conducted up to 750m from release sites.



Task 5: Develop a model to estimate the overall population size of scrub lizards for the landscape given in Table 3. Also, determine which patches are suitable for occupation by scrub lizards and which patches would not support a viable population.

Patch size and amount of open sandy habitat for a landscape of 29 patches located on the Avon Park Air Force Range. See:

http://www.comap.com/undergraduate/contests/icm/2002problem/map.jpg

for a map of the landscape.

Patch Identification	Patch Size (ha)	Sandy Habitat (ha)
1	13.66	5.38
2	32.74	11.91
3	1.39	0.23
4	2.28	0.76
5	7.03	3.62
6	14.47	4.38
7	2.52	1.99
8	5.87	2.49
9	22.27	8.44

10	19.25	7.58
11	11.31	4.80
12	74.35	19.15
13	21.57	7.52
14	15.50	2.82
15	35.54	11.31
16	2.93	1.15
17	47.21	10.73
18	1.67	0.13
19	9.80	2.23
20	39.31	7.15
21	2.23	0.78
22	3.73	1.02
23	8.46	1.67
24	3.89	1.89
25	1.33	1.11
26	0.85	0.79
27	8.75	5.30
28	9.77	6.22
29	13.45	4.69

TASK 6: It has been determined from aerial photographs that vegetation density increases by about 6% a year within the Florida scrub areas. Please make a recommendation on a policy for controlled burning.

2003 年美国大学生数学建模竞赛 MCM、ICM 试题

2003 MCM A: The Stunt Person

An exciting action scene in a movie is going to be filmed, and you are the stunt coordinator! A stunt person on a motorcycle will jump over an elephant and land in a pile of cardboard boxes to cushion their fall. You need to protect the stunt person, and also use relatively few cardboard boxes (lower cost, not seen by camera, etc.).

Your job is to:

- determine what size boxes to use
- determine how many boxes to use
- determine how the boxes will be stacked
- determine if any modifications to the boxes would help
- generalize to different combined weights (stunt person & motorcycle) and different jump heights

Note that, in "Tomorrow Never Dies", the James Bond character on a motorcycle jumps over a helicopter.

2003 MCM B: Gamma Knife Treatment Planning

Stereotactic radiosurgery delivers a single high dose of ionizing radiation to a radiographically well-defined, small intracranial 3D brain tumor without delivering any significant fraction of the prescribed dose to the surrounding brain tissue. Three modalities are commonly used in this area; they are the gamma knife unit, heavy charged particle beams, and external high-energy photon beams from linear accelerators.

The gamma knife unit delivers a single high dose of ionizing radiation emanating from 201 cobalt-60 unit sources through a heavy helmet. All 201 beams simultaneously intersect at the isocenter, resulting in a spherical (approximately) dose distribution at the effective dose levels. Irradiating the isocenter to deliver dose is termed a "shot." Shots can be represented as different spheres. Four interchangeable outer collimator helmets with beam channel diameters of 4, 8, 14, and 18 mm are available for irradiating different size volumes. For a target volume larger than one shot, multiple shots can be used to cover the entire target. In practice, most target volumes are treated with 1 to 15 shots. The target volume is a bounded, three-dimensional digital image that usually consists of millions of points.

The goal of radiosurgery is to deplete tumor cells while preserving normal structures. Since there are physical limitations and biological uncertainties involved in this therapy process, a treatment plan needs to account for all those limitations and uncertainties. In general, an optimal treatment plan is designed to meet the following requirements.

1. Minimize the dose gradient across the target volume.
2. Match specified isodose contours to the target volumes.
3. Match specified dose-volume constraints of the target and critical organ.
4. Minimize the integral dose to the entire volume of normal tissues or organs.
5. Constrain dose to specified normal tissue points below tolerance doses.
6. Minimize the maximum dose to critical volumes.

In gamma unit treatment planning, we have the following constraints:

1. Prohibit shots from protruding outside the target.
2. Prohibit shots from overlapping (to avoid hot spots).
3. Cover the target volume with effective dosage as much as possible. But at least 90% of the target volume must be covered by shots.
4. Use as few shots as possible.

Your tasks are to formulate the optimal treatment planning for a gamma knife unit as a sphere-packing problem, and propose an algorithm to find a solution. While designing your algorithm, you must keep in mind that your algorithm must be reasonably efficient.

2003 ICM: Aviation Baggage Screening Strategies: To Screen or Not to Screen, that is the Question

You are an analysis team in the Office of Security Operations for the Transportation Security Administration (TSA), responsible for the Midwest Region of the United States. New laws will soon mandate 100% screening of all checked bags at the 429 passenger airports throughout the nation by explosive detection systems (EDSs; see Figure 1). EDSs use computed tomography (CT) technology to scan checked bags, similar to how CAT scans are used in hospitals. Using multiple x-rays of each bag, EDSs create three-dimensional images of a bag’s content, showing the density of each item. This information is utilized to determine whether an explosive device is present. Experimentation with EDSs indicate that each device is operational about 92% of the time and each device can examine between 160 and 210 bags per hour.

The TSA has been actively purchasing EDSs and deploying them at airports throughout the nation. Given that these devices cost nearly $1 million each, weigh as much as eight tons, and cost several thousand dollars to install in an

airport, determining the correct number of devices to deploy at each airport and how to best use them (once operational) are important problems.

Currently, manufacturers are not able to produce the expected number of EDSs required to meet the federal mandate of 100% screening of checked luggage. Because of the limited number of EDS machines available, the Director of Airport Security for the Midwest Region (Mr. Sheldon) is not surprised that the TSA is requesting a detailed analysis on the estimated number of EDSs required at all airports. In addition, given the limited space and funds available for each airport, Mr. Sheldon believes that at some point a detailed analysis of emerging technologies will be needed. Promising technologies with more modest space and labor costs will emerge in the coming decade (e.g. x-ray diffraction; neutron-based detection; quadropole resonance; millimeter wave imaging; and microwave imaging).

Task 1: You have been tasked by your Director, Mr. Sheldon, to develop a model to determine the number of EDSs required at two of the largest facilities in the region, Airports A & B, which are described in the Technical Information Sheet (TIS) – Appendix A. Carefully describe the assumptions that you make in designing the model, then use your model to recommend the number of EDSs required using the data provided in Table 1 of the TIS.

Task 2: Prepare a short (one page) position paper to accompany your model that describes the security-related objectives of the airlines and the constraints that the airlines must work within for the sets of flights described in Table 1 of the TIS.

Task 3: Since security screening takes time and might delay passengers, the airport managers at Airports A & B request that you develop a model that can help the airlines determine how to schedule the departure of different types of flights within the peak hour. Carefully describe all the assumptions that you make in designing the model and use your model to produce a schedule for the two airports with the data provided in Table 1.

Task 4: Based on your analysis, what can you recommend to Mr. Sheldon and the airlines about checked baggage screening for the flights during the peak hours at your two airports?

Task 5: Mr. Sheldon realizes that your work may have national impact and requests that you write a memo explaining how your models can be adapted to determine the number of EDSs and airline scheduling for all 193 airports in the Midwest Region. He will send the memo along with the models and the analysis to the Director of the Office of Security Operations (his boss) at the TSA and to all security directors of other airports in the region for their comment and possible implementation.

Additional security measures associated with higher risks may require that up to 20% of the passengers will need to have all their checked bags screened through both an EDS and an explosive trace detection (ETD) machine, even though an EDS is 98.5% accurate in identifying explosive devices in checked bags. ETD machines use mass spectrometry technology to detect minute particles of explosive compounds. Each ETD machine costs $45,000 to purchase, however, the labor cost to operate the ETD machine is approximately 10 times that of the EDS. ETD can process 40 to 50 bags per hour; they are operational 98% of the time, and they are 99.7% accurate in identifying explosive materials on checked bags. At this time, ETD machines have not been federally certified, but Mr. Sheldon believes that they will soon be an integral part of national airport security systems.

Task 6: Modify your EDS models to incorporate the use of ETD machines and determine how many ETD machines are needed for Airports A & B and if the schedules need to be changed. Since this information may affect national level decisions, write a memo to the Director of Homeland Security and the Director of TSA with a technical analysis of this enhanced screening policy. Is the cost of such a policy justified in light of the value that it provides? Should the ETDs replace any of the EDS devices?

Task 7: The Director of Homeland Security must also decide how to best fund future scientific research programs. Use your EDS/ETD model to examine the possible effect of changes in the device technology, cost, accuracy, speed, and operational reliability. Include recommendations for the science, technology, engineering, and mathematics (STEM) research areas that will have the biggest impact on security system performance. Add your recommendation to the memo prepared in Task 7.

Appendix A Technical Information Sheet (TIS)

Table 1 Peak Hour Flight Departures for Airports A and B Note: On average, 2% of flights are cancelled each day

Flight Type	Number of Seats on Each Flight	Airport A Number of Flights of Each Type	Airport B Number of Flights of Each Type
1	34	10	8
2	46	4	6
3	85	3	7
4	128	3	5
5	142	19	9
6	194	5	10
7	215	1	2

8	350	1	1

Although all the flights in Table 1 depart during a peak hour, their actual departure times are set by the airline when designing their flight schedule. A flight cannot depart until all its checked bags are screened using an EDS. The airline has the flexibility to schedule their flights during the peak hour to avoid undesirable flight delays due to unscreened bags.

Historical data indicates that flights with 85 or fewer seats typically fly with between 70% and 100% of their seats occupied. Flights with between 128 and 215 seats typically fly with between 60% and 100% of their seats occupied. Flights with 350 seats typically fly with between 50% and 100% of their seats occupied. Passengers typically arrive for their flight between forty-five minutes and two hours prior to their scheduled departure time. For flights other than "shuttle" service, airlines claim that 20% of the passengers do not check any luggage, 20% check one bag, and the remaining passengers check two bags.

Preliminary estimates indicate that it will cost $100,000 to modify existing infrastructure (reinforced flooring, etc.) to install each EDS at Airport A and $80,000 to install each EDS at Airport B.

2004 年美国大学生数学建模竞赛 MCM、ICM 试题

2004 MCM A: Are Fingerprints Unique?

It is a commonplace belief that the thumbprint of every human who has ever lived is different. Develop and analyze a model that will allow you to assess the probability that this is true. Compare the odds (that you found in this problem) of misidentification by fingerprint evidence against the odds of misidentification by DNA evidence.

2004 MCM B: A Faster QuickPass System

“QuickPass” systems are increasingly appearing to reduce people's time waiting in line, whether it is at tollbooths, amusement parks, or elsewhere. Consider the design of a QuickPass system for an amusement park. The amusement park has experimented by offering QuickPasses for several popular rides as a test. The idea is that for certain popular rides you can go to a kiosk near that ride and insert your daily park entrance ticket, and out will come a slip that states that you can return to that ride at a specific time later. For example, you insert your daily park entrance ticket at 1:15 pm, and the QuickPass states that you can come back between 3:30 and 4:30 pm when you can use your slip to enter a second, and presumably much shorter, line that will get you to the ride faster. To prevent people from obtaining QuickPasses for several rides at once, the QuickPass machines allow you to have only one active QuickPass at a time. You have been hired as one of several competing consultants to improve the operation of QuickPass. Customers have been complaining about some anomalies in the test system. For example, customers observed that in one instance QuickPasses were being offered for a return time as long as 4 hours later. A short time later on the same ride, the QuickPasses were given for times only an hour or so later. In some instances, the lines for people with Quickpasses are nearly as long and slow as the regular lines.

The problem then is to propose and test schemes for issuing QuickPasses in order to increase people's enjoyment of the amusement park. Part of the problem is to determine what criteria to use in evaluating alternative schemes. Include in your report a non-technical summary for amusement park executives who must choose between alternatives from competing consultants.

2004 ICM: To Be Secure or Not to Be?

Click the title to view a PDF version of this problem: To Be Secure or Not to Be?

2005 年美国大学生数学建模竞赛 MCM、ICM 试题

2005 MCM A: Flood Planning

Lake Murray in central South Carolina is formed by a large earthen dam, which was completed in 1930 for power production. Model the flooding downstream in the event there is a catastrophic earthquake that breaches the dam.

Two particular questions:

Rawls Creek is a year-round stream that flows into the Saluda River a short distance downriver from the dam. How much flooding will occur in Rawls Creek from a dam failure, and how far back will it extend?

Could the flood be so massive downstream that water would reach up to the S.C. State Capitol Building, which is on a hill overlooking the Congaree River?

2005 MCM B: Tollbooths

Heavily-traveled toll roads such as the Garden State Parkway, Interstate 95, and so forth, are multi-lane divided highways that are interrupted at intervals by toll plazas. Because collecting tolls is usually unpopular, it is desirable to minimize motorist annoyance by limiting the amount of traffic disruption caused by the toll plazas. Commonly, a much larger number of tollbooths is provided than the number of travel lanes entering the toll plaza. Upon entering the toll plaza, the flow of vehicles fans out to the larger number of tollbooths, and when leaving the toll plaza, the flow of vehicles is required to squeeze back down to a number of travel lanes equal to the number of travel lanes before the toll plaza. Consequently, when traffic is heavy, congestion increases upon departure from the toll plaza. When traffic is very heavy, congestion also builds at the entry to the toll plaza because of the time required for each vehicle to pay the toll.

Make a model to help you determine the optimal number of tollbooths to deploy in a barrier-toll plaza. Explicitly consider the scenario where there is exactly one tollbooth per incoming travel lane. Under what conditions is this more or less effective than the current practice? Note that the definition of “optimal” is up to you to determine.

2005 ICM: Nonrenewable Resources

Select a vital nonrenewable or exhaustible resource (water, mineral, energy, food, etc.) for which your team can find appropriate world-wide historic data on its endowment, discovery, annual consumption, and price.

The modeling tasks are:

1. Using the endowment, discoveries, and consumption data, model the depletion or degradation of the commodity over a long horizon using resource modeling principles.
2. Adjust the model to account for future economic, demographic, political and environmental factors. Be sure to reveal the details of your model, provide visualizations of the model's output, and explain limitations of the model.
3. Create a fair, practical "harvesting/management" policy that may include economic incentives or disincentives, which sustain the usage over a long period of time while avoiding severe disruption of consumption, degradation or rapid exhaustion of the resource.
4. Develop a "security" policy that protects the resource against theft, misuse, disruption, and unnecessary degradation or destruction of the resource. Other issues that may need to be addressed are political and security management alternatives associated with these policies.
5. Develop policies to control any short- or long-term "environmental effects" of the harvesting. Be sure to include issues such as pollutants, increased susceptibility to natural disasters, waste handling and storage, and other factors you deem appropriate.
6. Compare this resource with any other alternatives for its purpose. What new science or technologies could be developed to mitigate the use and potential exhaustion of this resource? Develop a research policy to advance these new areas.

2006 年美国大学生数学建模竞赛 MCM、ICM 试题

2006 MCM A: Positioning and Moving Sprinkler Systems for Irrigation

There are a wide variety of techniques available for irrigating a field. The technologies range from advanced drip systems to periodic flooding. One of the systems that is used on smaller ranches is the use of "hand move" irrigation systems. Lightweight aluminum pipes with sprinkler heads are put in place across fields, and they are moved by hand at periodic intervals to insure that the whole field receives an adequate amount of water. This type of irrigation system is cheaper and easier to maintain than other systems. It is also flexible, allowing for use on a wide variety of fields and crops. The disadvantage is that it requires a great deal of time and effort to move and set up the equipment at regular intervals.

Given that this type of irrigation system is to be used, how can it be configured to minimize the amount of time required to irrigate a field that is 80 meters by 30 meters? For this task you are asked to find an algorithm to determine how to irrigate the rectangular field that minimizes the amount of time required by a rancher to maintain the irrigation system. One pipe set is used in the field. You should determine the number of sprinklers and the spacing between sprinklers, and you should find a schedule to move the pipes, including where to move them.

A pipe set consists of a number of pipes that can be connected together in a straight line. Each pipe has a 10 cm inner diameter with rotating spray nozzles that have a 0.6 cm inner diameter. When put together the resulting pipe is 20 meters long. At the water source, the pressure is 420 Kilo- Pascal's and has a flow rate of 150 liters per minute. No part of the field should receive more than 0.75 cm per hour of water, and each part of the field should receive at least 2 centimeters of water every 4 days. The total amount of water should be applied as uniformly as possible.

2006 MCM B: Wheel Chair Access at Airports

One of the frustrations with air travel is the need to fly through multiple airports, and each stop generally requires each traveler to change to a different airplane. This can be especially difficult for people who are not able to easily walk to a different flight's waiting area. One of the ways that an airline can make the transition easier is to provide a wheel chair and an escort to those people who ask for help. It is generally known well in advance which passengers require help, but it is not uncommon to receive notice when a passenger first registers at the airport. In rare instances an airline may not receive notice from a passenger until just prior to landing.

Airlines are under constant pressure to keep their costs down. Wheel chairs wear out and are expensive and require maintenance. There is also a cost for making the escorts available. Moreover, wheel chairs and their escorts must be constantly moved around the airport so that they are available to people when their flight lands. In some large airports the time required to move across the airport is nontrivial. The wheel chairs must be stored somewhere, but space is expensive and severely limited in an airport terminal. Also, wheel chairs left in high traffic areas represent a liability risk as people try to move around them. Finally, one of the biggest costs is the cost of holding a plane if someone must wait for an escort and becomes late for their flight. The latter cost is especially troubling because it can affect the airline's average flight delay which can lead to fewer ticket sales as potential customers may choose to avoid an airline.

Epsilon Airlines has decided to ask a third party to help them obtain a detailed analysis of the issues and costs of keeping and maintaining wheel chairs and escorts available for passengers. The airline needs to find a way to schedule the movement of wheel chairs throughout each day in a cost effective way. They also need to find and define the costs for budget planning in both the short and long term.

Epsilon Airlines has asked your consultant group to put together a bid to help them solve their problem. Your bid should include an overview and analysis of the situation to help them decide if you fully understand their problem. They require a detailed description of an algorithm that you would like to implement which can determine where the escorts and wheel chairs should be and how they should move throughout each day. The goal is to keep the total costs as low as possible. Your bid is one of many that the airline will consider. You must make a strong case as to why your solution is the best and show that it will be able to handle a wide range of airports under a variety of circumstances.

Your bid should also include examples of how the algorithm would work for a large (at least 4 concourses), a medium (at least two concourses), and a small airport (one concourse) under high and low traffic loads. You should determine all potential costs and balance their respective weights. Finally, as populations begin to include a higher percentage of older people who have more time to travel but may require more aid, your report should include projections of potential costs and needs in the future with recommendations to meet future needs.

2006 ICM: Trade-offs in the fight against HIV/AIDS

As the HIV/AIDS pandemic enters its 25th year, both the number of infections and number of deaths due to the disease continue to rise. Despite an enormous amount of effort, our global society remains uncertain on how to most effectively allocate resources to fight this epidemic.

You are a team of analysts advising the United Nations (UN) on how to manage the available resources for addressing HIV/AIDS. Your job is to model several scenarios of interest and to use your models to recommend the allocation of financial resources. The narrative below provides some background information, and outlines specific tasks.

Task #1: For each of the continents (Africa, Asia, Europe, North America, Australia, and South America), choose the country you believe to be most critical in terms of HIV/AIDS. Build a model to approximate the expected rate of change in the number of HIV/AIDS infections for these countries from 2006 to 2050, in the absence of any additional interventions. Fully explain your model and the assumptions that underlie your model. In addition, explain how you selected the countries to model.

There are a number of interventions that HIV/AIDS funding could be directed towards – including prevention interventions (voluntary counseling and testing, condom social marketing, school-based AIDS education, medicines to prevent mother-to-child transmission, etc.) and care interventions (treating other untreated sexually transmitted diseases, treating opportunistic infections, etc.). You should focus on only two potential interventions: provision of antiretroviral (ARV) drug therapies, and provision of a hypothetical HIV/AIDS preventative vaccine.

Task #2: First, estimate the level of financial resources from foreign aid donors that you realistically expect to be available to address HIV/AIDS, by year, from 2006 to 2050, for the countries you selected in Task #1. Then use the model you developed in Task #1 and these estimates of financial resources to estimate the expected rate of change in the number of HIV/AIDS infections for your selected countries from 2006 to 2050 under realistic assumptions for the following three scenarios: (1) Antiretroviral (ARV) drug therapy (2) A preventative HIV/AIDS vaccine (3) Both ARV provision and a preventative HIV/AIDS vaccine Assume in these scenarios that there is no risk of emergence of drug-resistant strains of HIV (you will examine this issue in Task #3).

Be sure to carefully describe the assumptions that underlie your model.

You can choose whether these scenarios should be implemented for all of the countries you selected in Task #1, or for certain subsets of countries based on income cut-offs, disease burden, etc. Available for use if you wish is a spreadsheet of country-level income data.

ARV drug therapies can have tremendous benefits in terms of prolonging the lives of individuals infected with HIV/AIDS. ARVs are keeping a high proportion of HIV/AIDS-infected individuals in rich countries alive, and policy makers and international institutions are facing tremendous political pressure to increase

access to ARVs for individuals in poor countries. Health budgets in low-income countries are very limited, and it seems unlikely that poor countries will be able to successfully expand these programs to the majority of their populations using their own resources. Appendix 1 presents country-specific data from UNAIDS on current access to ARVs for a number of countries

The efficacy of ARVs depends in large part on adherence to the treatment regimen and to proper monitoring. The most favorable conditions for ARVs are structured programs with extensive counseling and physician care, as well as regular testing to monitor for disease progression and the onset of opportunistic infections. Non-adherence or inadequate treatment carries with it two very serious consequences. First, the treatment may not be effective for the individual undergoing treatment. Second, partial or inadequate treatments are thought to directly lead to the emergence of drug-resistant strains of HIV.

The price of the drugs initially used to treat patients has come down to several hundred dollars a year per patient, but delivering them and providing the necessary accompanying medical care and further treatment is the key administrative and financial challenge. It is estimated that purchasing and delivering antiretrovirals using the clinically-recommended approach (DOTS, or directly observed short course treatments) which is intended to minimize the emergence of drug-resistant strains would cost less than $1,100 per person per year. (Adams, Gregor et al. [2001]. "Consensus Statement on Antiretroviral Treatment for AIDS in Poor Countries,")

For a preventative HIV vaccine, make assumptions you feel are reasonable about the following (in addition to other factors you may choose to include in your model):

1. The year in which an HIV/AIDS preventative vaccine might be available
2. How quickly vaccination rates might reach the following steady-state levels of vaccination:

- If you wish to immunize new cohorts (infants), assume the steady-state level for new cohorts of the country-by-country immunization rates for the third dose of the diphtheria-pertussis-tetanus vaccine (DTP3), as reported by the WHO (2002)
- If you wish to immunize adults (any group over age 5), assume the steady-state level for older cohorts is the second dose of the tetanus toxoid (TT2) rate, as reported by the WHO (2002)

1. The efficacy and duration of protection of the vaccine
2. Whether there would be epidemiological externalities from vaccination
3. Assume the vaccine is a three-dose vaccine, and can be added to the standard package of vaccines delivered under the WHO's Expanded

Programme on Immunization (EPI) at an incremental cost of addition of $0.75

Task #3: Re-formulate the three models developed in Task #2, taking into consideration the following assumptions about the development of ARV-resistant disease strains.

Current estimates suggest that patients falling below 90-95 percent adherence to ARV treatment are at a "substantial risk" of producing drug resistant strains. Use as an assumption for your analysis that a person receiving ARV treatment with adherence below 90 percent has a 5 percent chance of producing a strain of HIV/AIDS which is resistant to standard first-line drug treatments.

Second- and third-line ARV drug therapies are available, but assume for your analysis that these drugs are prohibitively expensive to implement in countries outside of Europe, Japan, and the United States

Task #4: Write a white paper to the United Nations providing your team's recommendations on the following:

1. Your recommendations for allocation of the resources available for HIV/AIDS among ARV provision and a preventative HIV vaccine
2. Your argument for how to weigh the importance of HIV/AIDS as an international concern relative to other foreign policy priorities
3. Your recommendations for how to coordinate donor involvement for HIV/AIDS

For (1): assume that between now and 2010 the available financial resources could be allocated so as the speed the development of a preventative HIV vaccine – through directly-financing vaccine research and development (R&D), or through other mechanisms. Any gains from such spending would move the date of development you assumed in Task #2 to some earlier date.

Here is a zip file which contains a PDF of the problem and all associated data (10) files. (File Size 3MB)

2007 年美国大学生数学建模竞赛 MCM、ICM 试题

2007 MCM A: Gerrymandering

Gerrymandering The United States Constitution provides that the House of Representatives shall be composed of some number (currently 435) of individuals who are elected from each state in proportion to the state's population relative to that of the country as a whole. While this provides a way of determining how many representatives each state will have, it says nothing about how the district represented by a particular representative shall be determined geographically. This oversight has led to egregious (at least some people think so, usually not the incumbent) district shapes that look "unnatural" by some standards.

Hence the following question: Suppose you were given the opportunity to draw congressional districts for a state. How would you do so as a purely "baseline" exercise to create the "simplest" shapes for all the districts in a state? The rules include only that each district in the state must contain the same population. The definition of "simple" is up to you; but you need to make a convincing argument to voters in the state that your solution is fair. As an application of your method, draw geographically simple congressional districts for the state of New York.

2007 MCM B: The Airplane Seating Problem

Airlines are free to seat passengers waiting to board an aircraft in any order whatsoever. It has become customary to seat passengers with special needs first, followed by first-class passengers (who sit at the front of the plane). Then coach and business-class passengers are seated by groups of rows, beginning with the row at the back of the plane and proceeding forward.

Apart from consideration of the passengers' wait time, from the airline's point of view, time is money, and boarding time is best minimized. The plane makes money for the airline only when it is in motion, and long boarding times limit the number of trips that a plane can make in a day.

The development of larger planes, such as the Airbus A380 (800 passengers), accentuate the problem of minimizing boarding (and deboarding) time.

Devise and compare procedures for boarding and deboarding planes with varying numbers of passengers: small (85 – 210), midsize (210 – 330), and large (450 – 800).

Prepare an executive summary, not to exceed two single-spaced pages, in which you set out your conclusions to an audience of airline executives, gate agents, and flight crews.

An article appeared in the NY Times Nov 14, 2006 addressing procedures currently being followed and the importance to the airline of finding better solutions. The article can be seen at:
http://travel2.nytimes.com/2006/11/14/business/14boarding.html

2007 ICM: Organ Transplant: The Kidney Exchange Problem

Transplant Network: Despite the continuing and dramatic advances in medicine and health technology, the demand for organs for transplantation drastically exceeds the number of donors. To help this situation, US Congress passed the National Organ Transplant Act in 1984, establishing the Organ Procurement and Transplantation Network (OPTN) to match organ donors to patients with organ needs. Even with all this organizational technology and service in place, there are nearly 94,000 transplant candidates in the US waiting for an organ transplant and this number is predicted to exceed 100,000 very soon. The average wait time exceeds three years—double that in some areas, such as large cities. Organs for transplant are obtained either from a cadaver queue or from living donors. The keys for the effective use of the cadaver queue are cooperation and good communication throughout the network. The good news is that the system is functioning and more and more donors (alive and deceased) are identified and used each year with record numbers of transplants taking place every month. The bad news is that the candidate list grows longer and longer. Some people think that the current system with both regional and national aspects is headed for collapse with consequential failures for some of the neediest patients. Moreover, fundamental questions remain: Can this network be improved and how do we improve the effectiveness of a complex network like OPTN? Different countries have different processes and policies, which of these work best? What is the future status of the current system?

Task 1: For a beginning reference, read the OPTN Website (http://www.optn.org/) with its policy descriptions and data banks (http://www.optn.org/data and http://www.optn.org/latestData/viewDataReports.asp). Build a mathematical model for the generic US transplant network(s). This model must be able to give insight into the following: Where are the potential bottlenecks for efficient organ matching? If more resources were available for improving the efficiency of the donor-matching process, where and how could they be used? Would this network function better if it was divided into smaller networks (for instance at the state level)? And finally, can you make the system more effective by saving

and prolonging more lives? If so, suggest policy changes and modify your model to reflect these improvements.

Task 2: Investigate the transplantation policies used in a country other than the US. By modifying your model from Task 1, determine if the US policy be would improved by implementing the procedures used in the other country. As members of an expert analysis team (knowledge of public health issues and network science) hired by Congress to perform a study of these questions, write a one-page report to Congress addressing the questions and issues of Task 1 and the information and possible improvements you have discovered from your research of the different country' s policies. Be sure to reference how you used your models from Task 1 to help address the issues.

Focusing on Kidney Exchange: Kidneys filter blood, remove waste, make hormones, and produce urine. Kidney failure can be caused by many different diseases and conditions. People with end-stage kidney disease face death, dialysis (at over $60,000/yr), or the hope for a kidney transplant. A transplant can come from the cadavers of an individual who agreed to donate organs after death or from a live donor. In the US, about 68,000 patients are waiting for a kidney from a deceased donor, while each year only 10,000 are transplanted from cadavers and 6,000 from living individuals (usually relatives of the patients). Hence the median wait for a matching kidney is three years—unfortunately, some needy patients do not survive long enough to receive a kidney.

There are many issues involved in kidney transplantation—the overall physical and mental health of the recipient, the financial situation of the recipient (insurance for transplant and post-operation medication), and donor availability (is there a living donor willing to provide a kidney). The transplanted kidney must be of a compatible ABO blood type. The 5-year survival of the transplant is enhanced by minimizing the number of mismatches on six HLA markers in the blood. At least 2,000 would-be-donor/recipient pairs are thwarted each year because of blood-type incompatibility or poor HLA match. Other sources indicate that over 6,000 people on the current waiting list have a willing but incompatible donor. This is a significant loss to the donor population and worthy of consideration when making new policies and procedures.

An idea that originated in Korea is that of a kidney exchange system, which can take place either with a living donor or with the cadaver queue. One exchange is paired-kidney donation, where each of two patients has a willing donor who is incompatible, but each donor is compatible with the other patient; each donor donates to the other patient, usually in the same hospital on the same day. Another idea is list paired donation, in which a willing donor, on behalf of a particular patient, donates to another person waiting for a cadaver kidney; in return, the patient of the donor-patient pair receives higher priority for a

compatible kidney from the cadaver queue. Yet a third idea is to expand the paired-kidney donation to 3-way, 4-way, or a circle (n-paired) in which each donor gives to the next patient around the circle. On November 20, 2006, 12 surgeons performed the firstever 5-way kidney swap at Johns Hopkins Medical Facility. None of the intended donor-recipient transplants were possible because of incompatibilities between the donor and the originally intended recipient. At any given time, there are many patient-donor pairs (perhaps as many as 6,000) with varying blood types and HLA markers. Meanwhile, the cadaver queue receives kidneys daily and is emptied daily as the assignments are made and the transplants performed.

Task 3: Devise a procedure to maximize the number and quality of exchanges, taking into account the medical and psychological dynamics of the situation. Justify in what way your procedure achieves a maximum. Estimate how many more annual transplants your procedure will generate, and the resulting effect on the waiting list.

Strategies: Patients can face agonizing choices. For example, suppose a barely compatible—in terms of HLA mismatches—kidney becomes available from the cadaver queue. Should they take it or wait for a better match from the cadaver queue or from an exchange? In particular, a cadaver kidney has a shorter half-life than a live donor kidney.

Task 4: Devise a strategy for a patient to decide whether to take an offered kidney, or to even participate in a kidney exchange. Consider the risks, alternatives, and probabilities in your analysis.

Ethical Concerns: Transplantation is a controversial issue with both technical and political issues that involve balancing what is best for society with what is best for the individual. Criteria have been developed very carefully to try to ensure that people on the waiting list are treated fairly, and several of the policies try to address the ethical concerns of who should go on to the list or who should come off. Criteria involved for getting on or coming off the list can include diagnosis of a malignant disease, HIV infection or AIDS, severe cardiovascular disease, a history of non-compliance with prior treatment, or poorly controlled psychosis. Criteria used in determining placement priority include: time on the waiting list, the quality of the match between donor and recipient, and the physical distance between the donor and the recipient. As a result of recent changes in policy, children under 18 years of age receive priority on the waiting list and often receive a transplant within weeks or months of being placed on the list. The United Network for Organ Sharing Website recently (Oct 27, 2006) showed the age of waiting patients as:

Under 18: 748 18 to 34: 8,033 35 to 49: 20,553 50 to 64: 28,530 65 and over: 10,628

One ethical issue of continual concern is the amount of emphasis and priority on age to increase overall living time saved through donations. From a statistical standpoint, since age appears to be the most important factor in predicting length of survival, some believe kidneys are being squandered on older recipients.

Political issues: Regionalization of the transplant system has produced political ramifications (e.g., someone may desperately need a kidney and is quite high on the queue, but his or her deceased neighbor's kidney still can go to an alcoholic drug dealer 500 miles away in a big city). Doctors living in small communities, who want to do a good job in transplants, need continuing experience by doing a minimum number of transplants per year. However, the kidneys from these small communities frequently go to the hospitals in the big city and, therefore, the local doctors cannot maintain their proficiency. This raises the question, should transplants be performed only in a few large centers, by a few expert and experienced surgeons? Would that be a fair system and would it add or detract from system efficiency?

Many other ethical and political issues are being debated. Some of the current policies can be found at http://www.unos.org/policiesandbylaws/policies.asp?resources=true For example, recent laws have been passed in the US that forbid the selling or mandating the donation of organs, yet there are many agencies advocating for donors to receive financial compensation for their organ. The state of Illinois has a new policy that assumes everyone desires to be an organ donor (presumed consent) and people must opt out if they do not. The Department of Health and Human Services Advisory Committee on Organ Transplantation is expected to recommend that all states adopt policies of presumed consent for organ donation. The final decision on new national policies rests with the Health Resources and Services Administration within the US Department of Health and Human Services.

Task 5: Based on your analysis, do you recommend any changes to these criteria and policies? Discuss the ethical dimensions of your recommended exchange procedure and your recommended patient strategy (Tasks 3 and 4). Rank order the criteria you would use for priority and placement, as above, with rationale as to why you placed each where you did. Would you consider allowing people to sell organs for transplantation? Write a onepage paper to the Director of the US Health Resources and Services Administration with your recommendations.

Task 6: From the potential donor’s perspective, the risks in volunteering involve assessing the probability of success for the recipient, the probability of survival of the donor, the probability of future health problems for the donor, the probability of future health risks (such as failure of the one remaining

kidney), and the postoperative pain and recovery. How do these risks and others affect the decision of the donor? How do perceived risks and personal issues (phobias, irrational fears, misinformation, previous experiences with surgery, level of altruism, and level of trust) influence the decision to donate? If entering a list paired network rather than a direct transplant to the relative or friend, does the size n of the n-paired network have any effect on the decision of the potential donor? Can your models be modified to reflect and analyze any of these issues? Finally, suggest ways to develop and recruit more altruistic donors.

PDF version of ICM 2007

2008 年美国大学生数学建模竞赛 MCM、ICM 试题

2008 MCM A: Take a Bath

Consider the effects on land from the melting of the north polar ice cap due to the predicted increase in global temperatures. Specifically, model the effects on the coast of Florida every ten years for the next 50 years due to the melting, with particular attention given to large metropolitan areas. Propose appropriate responses to deal with this. A careful discussion of the data used is an important part of the answer.

2008 MCM B: Creating Sudoku Puzzles

Develop an algorithm to construct Sudoku puzzles of varying difficulty. Develop metrics to define a difficulty level. The algorithm and metrics should be extensible to a varying number of difficulty levels. You should illustrate the algorithm with at least 4 difficulty levels. Your algorithm should guarantee a unique solution. Analyze the complexity of your algorithm. Your objective should be to minimize the complexity of the algorithm and meet the above requirements.

2008 ICM: Finding the Good in Health Care Systems

Nations have systems for providing health care for their residents. Issues that are often of concern to people and are often in the news include which systems are better and whether current systems can be improved. Aspects of these systems vary widely between nations: how they are funded; whether services are delivered through public, private, or non-profit organizations; whether public insurance is universal for all residents; who is eligible for assistance; what care is covered; whether the latest medical procedures are available; and how much is required as user fees. Other factors that are often debated in determining the quality of care include: coverage for complementary care (glasses, dental, prostheses, prescription drugs, etc); which diseases are the most critical in affecting overall health; percentage of GDP spent on health care; percentage of health care costs that goes toward labor/administrative/malpractice insurance; ratio of public to private spending on health care; per capita spending on health care; growth of per capita spending on health care; number of participating physicians; per capita sick days; fairness of care in terms of age, race, gender, socio-economic class; and many more. Adding to the complications are health-related factors such as personal exercise, food availability, climate, occupations of citizens, and smoking habits.

The World Health Organization (WHO), an agency of the United Nations, is a source of data on health factors. The annual World Health Report (http://www.who.int/whr/en/index.html) assesses global health factors and World Health Statistics (http://en.wikipedia.org/wiki/World_Health_Organisation) provides health statistics for the countries in the UN. The production and dissemination of health statistics is a major function of WHO. To many people, these data and the associated analyses are considered unbiased and very valuable to the world community. There are many other sources of reliable health data available.

Part I: Describe several different outcomes (metrics) that could be used to evaluate the effectiveness of a country's health care system, such as average life expectancy of its residents. What metric would you use to make comparisons between existing and potential systems? Can you combine your metrics to make them even more useful in measuring quality?

Part II: Identify current sources of data that provide the raw data needed to compute the metrics you have identified above. You may need to modify your list of metrics based on the availability of data. Explain why you have selected those data and demonstrate how they can be used to assess and compare the relative effectiveness of health care systems as they exist in different countries.

Part III: Choose at least three of the most important and viable metrics for comparing health care systems. Justify why these are the most useful for this purpose. Can any of these help measure the historical change in an existing health care system? Are they measurable and can the data be easily collected?

Part IV: Use your three (or more) metrics to compare the United States health care system with one other country that is considered to have good health care using the most recent year for which you have data. Which country has the better health care system? Is your answer definitive?

Part V: Using your metrics, compare the United States and one other country which is considered to have poor health care using the most recent year for which you have data. Which country has the better health care system?

Part VI: Pick a country's (US or other) health care system and restructure it to improve the system based on your metrics. Build predictive models to test various changes to determine if the changes will improve the overall quality of the system. Suggest major change(s) that can improve the system.

PDF version

2009 年美国大学生数学建模竞赛 MCM、ICM 试题

2009 MCM A: Designing a Traffic Circle

Many cities and communities have traffic circles—from large ones with many lanes in the circle (such as at the Arc de Triomphe in Paris and the Victory Monument in Bangkok) to small ones with one or two lanes in the circle. Some of these traffic circles position a stop sign or a yield sign on every incoming road that gives priority to traffic already in the circle; some position a yield sign in the circle at each incoming road to give priority to incoming traffic; and some position a traffic light on each incoming road (with no right turn allowed on a red light). Other designs may also be possible.

The goal of this problem is to use a model to determine how best to control traffic flow in, around, and out of a circle. State clearly the objective(s) you use in your model for making the optimal choice as well as the factors that affect this choice. Include a Technical Summary of not more than two double-spaced pages that explains to a Traffic Engineer how to use your model to help choose the appropriate flow-control method for any specific traffic circle. That is, summarize the conditions under which each type of traffic-control method should be used. When traffic lights are recommended, explain a method for determining how many seconds each light should remain green (which may vary according to the time of day and other factors). Illustrate how your model works with specific examples.

2009 MCM B: Energy and the Cell Phone

This question involves the "energy" consequences of the cell phone revolution. Cell phone usage is mushrooming, and many people are using cell phones and giving up their landline telephones. What is the consequence of this in terms of electricity use? Every cell phone comes with a battery and a recharger.

Requirement 1

Consider the current US, a country of about 300 million people. Estimate from available data the number H of households, with m members each, that in the past were serviced by landlines. Now, suppose that all the landlines are replaced by cell phones; that is, each of the m members of the household has a cell phone. Model the consequences of this change for electricity utilization in the current US, both during the transition and during the steady state. The analysis should take into account the need for charging the batteries of the cell phones, as well as the fact that cell phones do not last as long as landline phones (for example, the cell phones get lost and break).

Requirement 2

Consider a second "Pseudo US"—a country of about 300 million people with about the same economic status as the current US. However, this emerging country has neither landlines nor cell phones. What is the optimal way of providing phone service to this country from an energy perspective? Of course, cell phones have many social consequences and uses that landline phones do not allow. A discussion of the broad and hidden consequences of having only landlines, only cell phones, or a mixture of the two is welcomed.

Requirement 3

Cell phones periodically need to be recharged. However, many people always keep their recharger plugged in. Additionally, many people charge their phones every night, whether they need to be recharged or not. Model the energy costs of this wasteful practice for a Pseudo US based upon your answer to Requirement 2. Assume that the Pseudo US supplies electricity from oil. Interpret your results in terms of barrels of oil.

Requirement 4

Estimates vary on the amount of energy that is used by various recharger types (TV, DVR, computer peripherals, and so forth) when left plugged in but not charging the device. Use accurate data to model the energy wasted by the current US in terms of barrels of oil per day.

Requirement 5

Now consider population and economic growth over the next 50 years. How might a typical Pseudo US grow? For each 10 years for the next 50 years, predict the energy needs for providing phone service based upon your analysis in the first three requirements. Again, assume electricity is provided from oil. Interpret your predictions in term of barrels of oil.

2009 ICM: Creating Food Systems: Re-Balancing Human-Influenced Ecosystems

Creating Food Systems: Re-Balancing Human-Influenced Ecosystems(PDF version)