Downlink Beamforming with Imperfect Channel State Information at Transmitter

Sakthivel Velumani, Rijan Kusatha

***Abstract*—This paper studies the performance and results of sum-rate (SR) maximization algorithm proposed in [2]. We evaluated the algorithm for Multi User Multiple Input Single Output (MU-MISO) system with 2 and 4 users for different Channel State Information at Transmitter (CSIT) scenarios - perfect CSIT and imperfect CSIT. The objective of this paper is to evaluate the performance of algorithm that maximizes the SR under perfect CSIT and maximizes Ergodic sum-rate (ESR) under imperfect CSIT. Numerical simulations show the ESR over downlink SNR and also its dependency on CSIT quality.**

I. Introduction

THE use of multiple antennas at the transmitter and many single antenna receivers have proved to increase of overall throughput of a wireless communication system by exploiting spacial multiplexing capability. When CSI is available at the transmitter, the base station can transmit to multiple users simultaneously by passing the data through a transmit filter before sending it through the channel. This can be done through linear and non-linear transmission techniques. Since non-linear techniques are computationally complex, linear transmission techniques are used to find the transmit filter also called beamformer (also referred to as precoder in this paper). The simple transmit beamforming techniques such as Zero Forcing Beamforming (ZF-BF) and Minimum Mean Square Error (MMSE) beamforming do not maximize the SR but minimize the interference and MSE respectively and also require perfect CSIT.

The problem of finding an optimal beamformer to maximize the SR is non-convex because of decentralized receivers and coupled SR expression. [1] proposed an iterative algorithm to find a local SR-optimum by establishing a relationship between SR and Weighted sum-Minimum Mean Square Error (WMMSE) in the MU-MISO system. The relation between the KKT condition of SR problem and WMMSE problem was exploited by comparing the gradients of both cost functions w.r.t the beamformer. This approach uses a block-wise convex property. In each iteration three steps are involved such as finding MMSE receive filter, weights (for WMMSE) and finally the beamformer. This algorithm assumes a perfect CSIT exists and it finds a local-optimum of SR through WMMSE approach. More details of the algorithm are mentioned in III.

In a wireless communication system, the CSI at receiver and CSI at transmitter are not always guaranteed to be same. The CSIT might contain errors which depends on the feedback channel quality also we consider a fading channel so the channel estimate (imperfect CSIT) varies very often. For these reasons, ESR is considered instead of SR, in which the former is achieved over a long sequence of channel states. The maximization of ESR is done by designing a beamformer for a given channel estimate, such that the Averate SR (ASR) is maximized. This ASR is an average performance metric w.r.t channel state errors. A direct relation between SNR and channel state error variance is made such that that the error dacay with increased Signal to Noise Ratio (SNR) as $O(\text{SNR}^{-1})$ [2]. The ASR can be computed at the BS for a given instantaneous imperfect CSIT and with knowledge on probability distribution function of channel state errors. This stochastic problem is converted into a deterministic problem through Sample Average Approximation (SAA) method and the WMMSE approach (as mentioned earlier) is used to find the ASR-optimum. The algorithm that we evaluate for imperfect CSIT was proposed in [2]. More details are presented in IV.

In this paper we evaluate the performance and compare the results of both the algorithms for different channel and system scenarios. The structure of rest of the paper is as follows. Section II tells about the system model and channel considerations for the evaluation of the algorithm. Section III contains the problem formulation of cost function for perfect CSIT through Rate-WMMSE relation [1] [2], and flowchart of algorithm. Section IV contains details on SAA, problem formulation and flowchart algorithm for imperfect CSIT [2]. Finally the results are presented in section V and section VI concludes the paper.

II. System Model

Consider a MU-MIMO system with Base Station (BS) that has N_t antennas and single antenna users $\mathcal{K} \triangleq \{1, ..., K\}$ such that $K \leq N_t$. The received signal at kth user is given as

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k \tag{1}$$

where $\mathbf{h}_k \in \mathbb{C}^{N_t}$ is the channel vector between BS and the kth user, $\mathbf{x} \in \mathbb{C}^{N_t}$ is the transmit signal vector, and $n_k \sim \mathcal{CN}(0, \sigma_{n,k}^2)$ is the Additive White Gaussian Noise (AWGN) at the kth user. The transmitted signal has a power constraint defined by $\mathrm{E}\{\mathbf{x}^H \mathbf{x}\} \leq P_t$ and we assume equal noise variances across all users, i.e $\sigma_{n,k}^2 = \sigma_n^2 \ , \quad \forall k \in \mathcal{K}$. By definition the transmit SNR can be written as $\text{SNR} \triangleq P_t / \sigma_n^2$.

A. Channel State Information

Let us first describe about imperfect CSIT and then about perfect CSIT as the later is quite straight forward, trivial and a special case of the former. As mentioned earlier in section I we assume a fading channel model so the channel state is given by $\mathbf{H} \triangleq [\mathbf{h}_1, ..., \mathbf{h}_K]$ that varies according to an ergodic stationary process with probability density $f_{\mathrm{H}}(\mathbf{H})$. The receivers are assumed to have a perfect CSI while the BS has imperfect instantaneous channel state estimate (imperfect CSIT) given by $\widehat{\mathbf{H}} \triangleq [\widehat{\mathbf{h}}_1, ..., \widehat{\mathbf{h}}_K]$. Hence a joint distribution of $\{\mathbf{H}, \widehat{\mathbf{H}}\}$ characterizes a joint fading process which is also stationary and ergodic [2]. The error matrix for a given estimate is $\widetilde{\mathbf{H}} \triangleq [\widetilde{\mathbf{h}}_1, ..., \widetilde{\mathbf{h}}_K]$ and hence the overall channel relation can be written as $\mathbf{H} = \widehat{\mathbf{H}} + \widetilde{\mathbf{H}}$. The CSIT error is characterized by a conditional density of $f_{\mathbf{H}|\widehat{\mathbf{H}}}(\mathbf{H}|\widehat{\mathbf{H}})$. For each user, the marginal density of channel estimate writes as $f_{\mathbf{h}_{\boldsymbol{k}}|\widehat{\mathbf{h}}_{\boldsymbol{k}}}(\mathbf{h}_{\boldsymbol{k}}|\widehat{\mathbf{h}}_{\boldsymbol{k}})$. The covariance matrix of channel state estimate for kth user is $\mathbf{R}_{e,k}$ and assumed to be independent of $\widehat{\mathbf{h}}_{\boldsymbol{k}}$. The CSIT quality is allowed to change w.r.t SNR [2]. The CSIT quality is nothing but the channel state error variance $\sigma_{e,k}^2$ and it's relation with SNR is given by $\sigma_{e,k}^2 = P_t^{-\alpha}$ where α is a constant that can be thought as a feedback channel quality.

In case of perfect CSIT, the α is considered to be ∞ which make all the channel state error variances $\sigma_{e,1}^2, ..., \sigma_{e,K}^2$ to 0. So eventually $\widetilde{\mathbf{H}}$ disappears and supports the claim that CSIT is perfect $\mathbf{H} = \widehat{\mathbf{H}}$.

B. Precoder and Ergodic Sum Rate Problem

At a particular channel use, let us consider the symbol stream be $\mathbf{s} \triangleq [s_1, ..., s_K]$ for K users, and are mapped to the transmit antennas through a precoding matrix $\mathbf{P}_p \triangleq [\mathbf{p}_1, ..., \mathbf{p}_K]$, where $\mathbf{p}_k \in \mathbb{C}^{N_t}$ is the kth user's precoder. This yields a transmit signal model

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{p}_k s_k \tag{2}$$

Assuming the data symbol distribution as $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$, the transmit power constraint boils down to $\mathrm{E}\{\mathbf{x}^H\mathbf{x}\} = \mathrm{tr}(\mathbf{P}\mathbf{P}^H) \leq P_t$.

The average receive power at kth receiver for a given channel state and corresponding precoder is written as

$$T_k \triangleq \mathrm{E}\{|y_k|^2\} = \overbrace{|\mathbf{h}_k^H \mathbf{p}_k|^2}^{S_k} + \overbrace{\sum_{i \neq k} |\mathbf{h}_k^H \mathbf{p}_i|^2 + \sigma_n^2}^{I_k} \tag{3}$$

where S_k is the useful signal power and I_k is the interfering signal power. The instantaneous Signal to Interference plus Noise Ratios (SINRs) at the output of the kth receiver is

$$\gamma_k \triangleq S_k I_k^{-1} \tag{4}$$

and the information rate from the kth user point of view is written as

$$R_k = \log_2(1 + \gamma_k) \tag{5}$$

The precoder (or beamformer) is updated throughout the transmission according to the available channel state estimate. The challenge is to design an optimal precoder that maximize the rate for all users (SR). Since the channel fading process or the channel states are ergodic the ESR under perfect CSIT is written as

$$\max_{\mathrm{E_H}\left\{\mathrm{tr}\left(\mathbf{P}_{\mathrm{p}}\mathbf{P}_{\mathrm{p}}^H\right)\right\} \leq P_t} \mathrm{E_H}\left\{\sum_{k=1}^{K} R_k\right\} \tag{6}$$

In the above expression you can see that ESR problem has a long-term power constraint $\mathrm{E_H}\left\{\mathrm{tr}\left(\mathbf{P}_{\mathrm{p}}\mathbf{P}_{\mathrm{p}}^H\right)\right\} \leq P_t$. This imposes a inter-state power allocation which makes the problem intractable and complex. So the long-term power constraint is replaced with a short-term power constraint $\mathrm{tr}\left(\mathbf{P}_{\mathrm{p}}\mathbf{P}_{\mathrm{p}}^H\right) \leq P_t$ that allow us to design precoder $\mathbf{P}$ instantaneously and independently for each channel state. This may decrease the ESR but it improves the tractability of the problem. So now the problem (6) looks like

$$\max_{\mathrm{tr}\left(\mathbf{P}_{\mathrm{p}}\mathbf{P}_{\mathrm{p}}^H\right) \leq P_t} \sum_{k=1}^{K} R_k \tag{7}$$

C. Average Sum Rate Problem

Now let us consider the case of imperfect CSIT. The BS can still design the precoder using () assuming the estimates of channel state $\widehat{\mathbf{H}}$ to be perfect. But the resulting precoder cannot cope with MU interference and also leads to overestimation of rates. So the BS cannot predict the actual instantaneous rate but it can calculate the Average Rate (AR) for each user defined as $\bar{R}_k \triangleq \mathrm{E}_{\mathrm{H}|\widehat{\mathrm{H}}}\{R_k|\widehat{\mathbf{H}}\}\ \forall k \in \mathcal{K}$. In simple words, the AR is the expected rate over channel state error distribution for a given channel state estimate. Eventually the ER can be found by averaging ARs over all variations of $\widehat{\mathbf{H}}$. So the ESR is given by $\mathrm{E}_{\widehat{\mathbf{H}}}\left\{\sum_{k=1}^{K} \bar{R}_k\right\}$. Hence ESR boils down to ASR. By applying the same logic of short-term power constraint here, the problem would be to design optimal precoder to maximize ASR: $\sum_{k=1}^{K} \bar{R}_k$ for each $\widehat{\mathbf{H}}$ independently. This gives raise to ASR optimization problem given by

$$\mathcal{R}(P_t): \begin{cases} \max\limits_{\mathbf{P}_{\mathrm{p}}} & \sum\limits_{k=1}^{K} \bar{R}_k \\ \text{s.t.} & \mathrm{tr}\left(\mathbf{P}_{\mathrm{p}}\mathbf{P}_{\mathrm{p}}^H\right) \leq P_t \end{cases} \tag{8}$$

III. Algorithm with Perfect CSIT

The problem (8) is non-convex and very difficult to solve so we reformulate this into an equivalent Augmented WMMSE form written as

$$\mathcal{A}(P_t): \begin{cases} \min\limits_{\mathbf{P},\mathbf{u},\mathbf{g}} & \sum\limits_{k=1}^{K} \xi_k \\ \text{s.t.} & \mathrm{tr}\left(\mathbf{P}\mathbf{P}^H\right) \leq P_t \end{cases} \tag{9}$$

where ξ_k is the AWMMSE defined by $\xi_k = u_k \varepsilon_k^{\mathrm{MMSE}} - \log_2(u_k)$ where u_k is the weight, $\varepsilon_k^{\mathrm{MMSE}}$ is the MSE if MMSE equalizer is used at the receiver, $\mathbf{u}$ is the weight vector and $\mathbf{g}$ is the corresponding equalizer vector for all users. The Rate-AWMMSE relation is written as

$$\xi_k^{\mathrm{MMSE}} \triangleq \min_{u_k, g_k} \xi_k = 1 - R_k \tag{10}$$

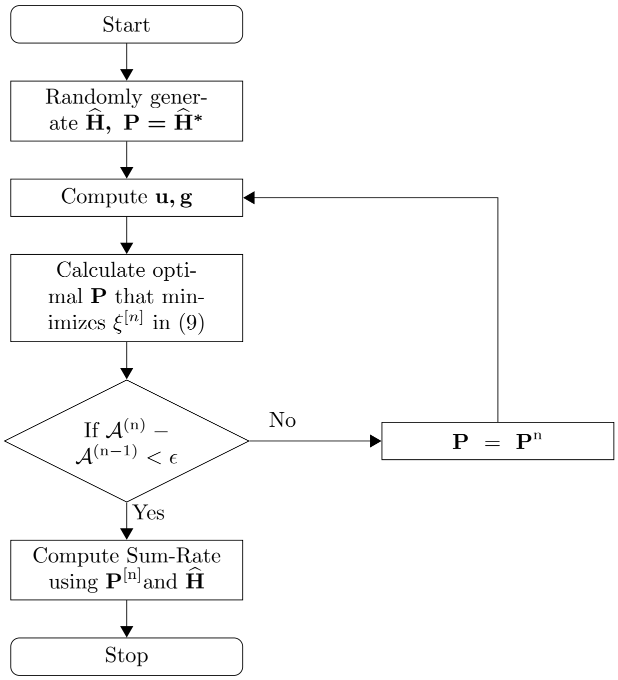

Fig. 1. Flowchart of the algorithm for perfect CSIT

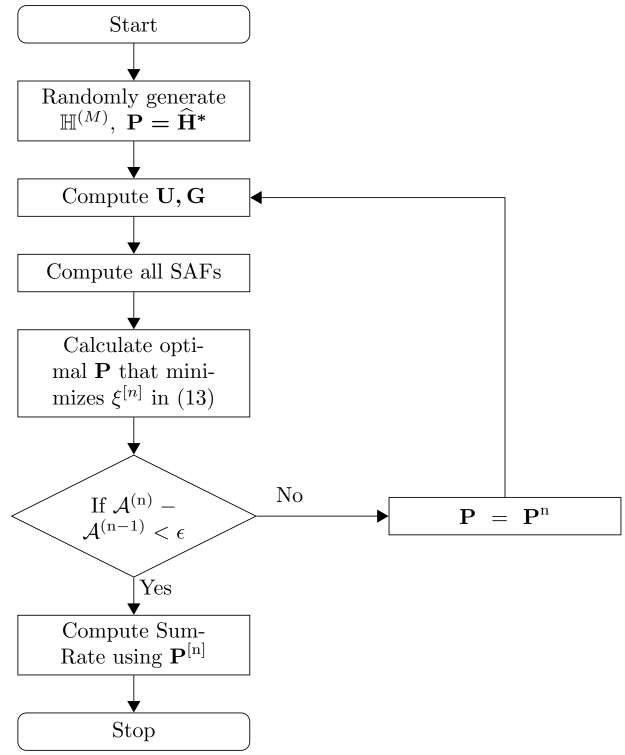

Fig. 2. Flowchart of the algorithm for imperfect CSIT

It can be seen the (9) is an extended version of (10) where all users are considered and optimal precoder is also determined w.r.t power constraints.

The problem (9) is non-convex in the joint set of optimization variables but it is convex in each variable of $\mathbf{P}$, $\mathbf{u}$ and $\mathbf{g}$ while fixing other two. So the problem is solved in an iterative manner since no closed form solution is available for all three optimization variables. In each iteration two steps are performed: 1) updating $\mathbf{u}$ and $\mathbf{g}$ for a given $\mathbf{P}$, 2) updating $\mathbf{P}$ for given $\mathbf{u}$ and $\mathbf{g}$. The steps of the algorithm are described in detail in Fig. 1.

IV. Algorithm with Imperfect CSIT

Under imperfect CSIT conditions the algorithm in III cannot be used directly as it leads to overestimation of rate performance and system would start to operate at undecodable rates. So to combat this, a technique called Sample Average Approximation is introduced into the same algorithm. For a given channel estimate $\widehat{\mathbf{H}}$ and index set $\mathcal{M} \triangleq \{1, ..., M\}$,

$$\mathbb{H}^{(M)} \triangleq \left\{\mathbf{H}^{(m)} = \widehat{\mathbf{H}} + \widetilde{\mathbf{H}}^{(m)} \mid \widehat{\mathbf{H}},\ m \in \mathcal{M}\right\} \tag{11}$$

be a sample of M i.i.d realizations, stated as a combination of channel estimate and possible channel error. The channel state error distribution depends on a constant factor α which defines the quality of feedback w.r.t downlink SNR. This sample is used to find the ARs that looks like

$$\mathcal{R}^{(M)}(P_t):\begin{cases}\max\limits_{\mathbf{P}} & \sum\limits_{k=1}^{K}\bar{R}_k^{(M)}\\ \text{s.t.} & \mathrm{tr}\big(\mathbf{P}\mathbf{P}^H\big)\le P_t\end{cases} \tag{12}$$

Refer [2] for more details on convergence of (12) to (8) when $M \to \infty$.

The Augmented WMMSE no becomes Averaged AWMMSE (shortly AWMSE) and the cost function can be written as

$$\mathcal{A}^{(M)}(P_t):\begin{cases}\min\limits_{\mathbf{P},\mathbf{U},\mathbf{G}} & \sum\limits_{k=1}^{K}\bar{\xi}_k^{(M)}\\ \text{s.t.} & \mathrm{tr}\big(\mathbf{P}\mathbf{P}^H\big)\le P_t\end{cases} \tag{13}$$

where $(\mathbf{U}, \mathbf{G})$ are weight and equalizer matrices of all users and all M channel state errors. To facilitate the calculation of precoder, some Sample Average Functions (SAFs) are used for the formulation of $\bar{\xi}_k^{(M)}$. Refer [2] for more details. The flowchat of algorithm as per its implementation is shown in Fig. 2.

V. Results

The simulation setup, results and findings for both CSIT scenarios are as follows

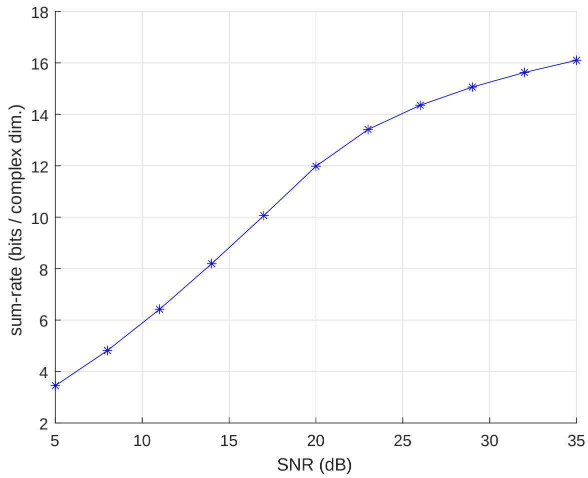

Fig. 3. Simulation Results of Imperfect CSIT with $N_t = 2,\ K = 2$ and average of 30 channel state estimates.

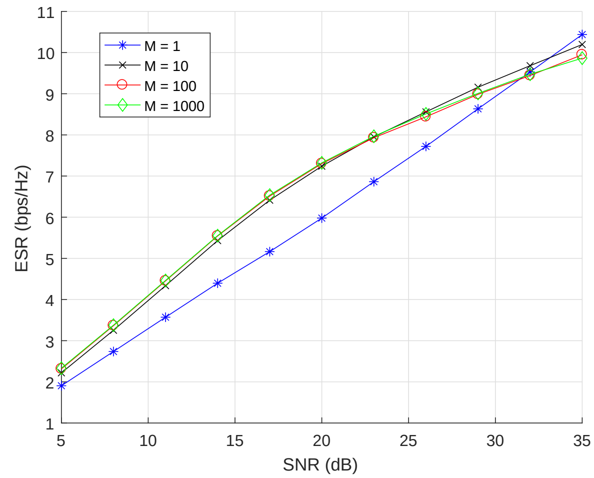

Fig. 5. Simulation Results of Imperfect CSIT with $N_t = 2,\ K = 2,\ M = [1, 10, 100, 1000],\ \alpha = 0.6$ and average of 100 channel state estimates.

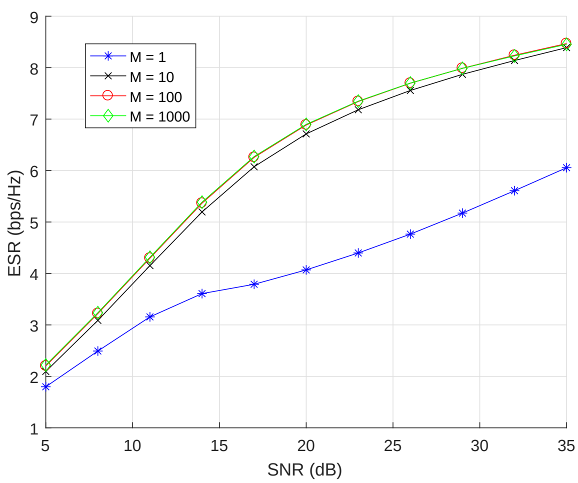

Fig. 4. Simulation Results of Imperfect CSIT with $N_t = 2,\ K = 2,\ M = [1, 10, 100, 1000],\ \alpha = 0.3$ and average of 100 channel state estimates.

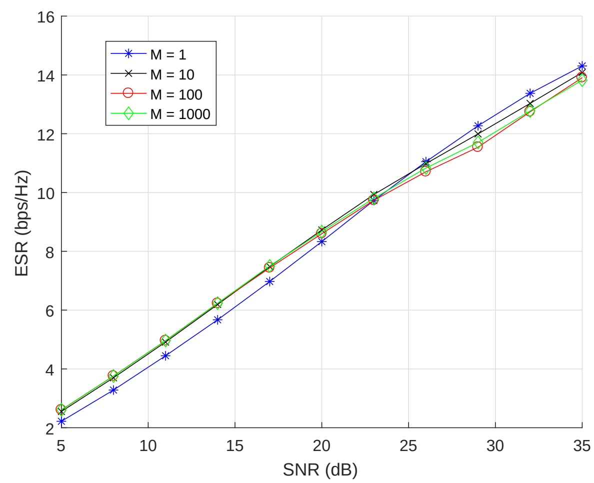

Fig. 6. Simulation Results of Imperfect CSIT with $N_t = 2,\ K = 2,\ M = [1, 10, 100, 1000],\ \alpha = 0.9$ and average of 100 channel state estimates.

A. Perfect CSIT

The no of transmit antennas at BS (N_t) is 2, no of receivers is 2 and no of iterations is 20. The sum-rate is calculated as per shown in Fig. 1. The same algorithm is repeated for 30 channel state realizations and sum-rates are averaged. The simulation result is seen in Fig. 3.

B. Imperfect CSIT

The no of transmit antennas at BS (N_t) is 2, no of receivers is 2 and no of iterations is 20. The no of samples M considered are $[1, 10, 100, 1000]$ and $\alpha = [0.3, 0.6, 0.9]$. The algorithm in Fig. 2. is repeated for 100 channel state estimate realizations and sum-rates are averaged. The simulation result is shown in Fig. 4, 5, 6.

VI. Conclusion

From Fig. 4, 5, 6. it is seen that as the sample size M of channel error increases the ESR increases logarithmicly for a given channel state estimate. It is also seen that as $\alpha \to 1$ the algorithm has no effect on M, in other words as the CSIT becomes perfect, the ESR does not depend on M. This effect can be seen well if both algorithms are compared as shown in Fig. 7.

References

[1] Søren Skovgaard Christensen, Rajiv Agarwal, Elisabeth de Carvalho, and John M. Cioffi *Weighted Sum-Rate Maximization using Weighted MMSE for MIMO-BC Beamforming Design*. IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 7, NO. 12, DECEMBER 2008

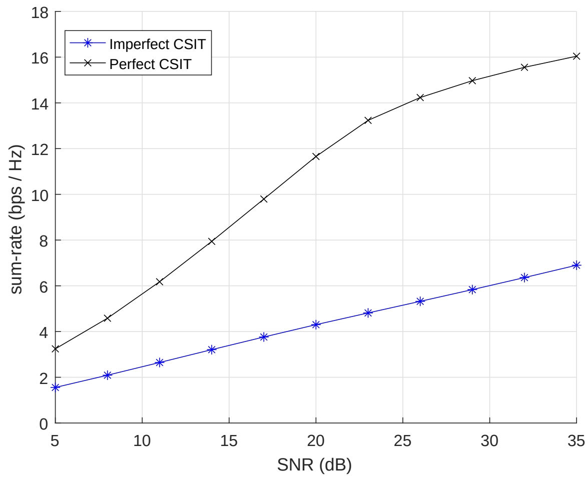

Fig. 7. Simulation Results of Imperfect CSIT compared with perfect CSIT for $N_t = 2,\ K = 2$, and average of 30 channel state estimates.

[2] Hamdi Joudeh and Bruno Clerckx *Sum-Rate Maximization for Linearly Precoded Downlink Multiuser MISO Systems With Partial CSIT: A Rate-Splitting Approach*. IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 64, NO. 11, NOVEMBER 2016.