

# PHY 107

## 1D motion

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# OUTLINE

- ▶ Motion
- ▶ Position and Displacement
- ▶ Velocity
- ▶ Acceleration
- ▶ Concept of instantaneous and average quantities
- ▶ Constant Acceleration Equations
- ▶ Free Fall
- ▶ Graphical Integration

# Motion

## **Kinematics: Classification and comparison of motions**

Assumptions:

1. Motion is along a straight line (horizontal, vertical, slanted)
2. No interest in force
3. The object is treated as a particle

# Displacement

It means change in position.

Displacement is a vector quantity (magnitude and direction)

1. A particle moves from  $x=5\text{m}$  to  $x=7\text{m} \rightarrow \text{Displacement} = 2\text{m}$
2. A particle is at position  $x=2\text{m}$ . It then moves to  $x=100\text{m}$  and finally comes to position  $x=2\text{m}$

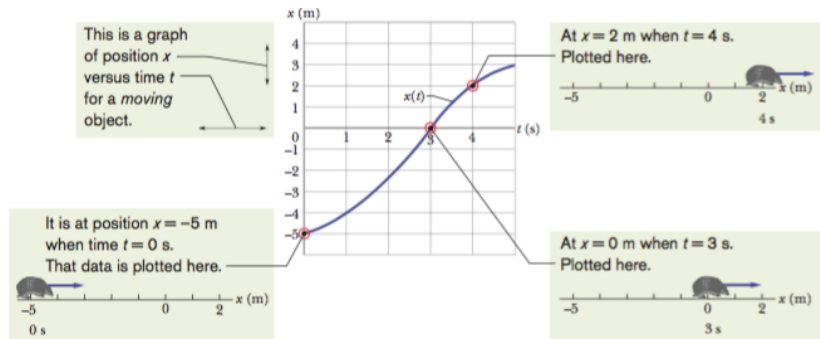
Displacement =  $x_2 - x_1 = 0\text{ m}$

What about distance?

# Average Velocity

It is the ratio of the displacement  $\Delta x$  that occurs in a time interval  $\Delta t$  to the time interval  $\Delta t$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

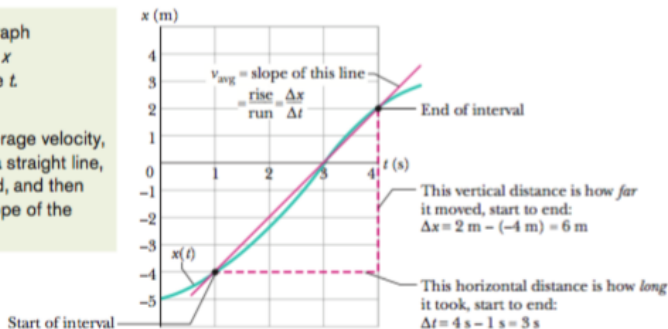


**Fig. 2-3** The graph of  $x(t)$  for a moving armadillo. The path associated with the graph is also shown, at three times.

# Average Velocity

This is a graph of position  $x$  versus time  $t$ .

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.



$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{6 \text{ m}}{3 \text{ s}} = 2 \text{ m/s}$$

# Instantaneous Velocity

The velocity at any instant is obtained from average velocity by shrinking the time interval  $\Delta t$  closer to zero (but NOT equal to zero)

$$v_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Calculus comes into play!

# Acceleration

Acceleration is the change in velocity in a given amount of time

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

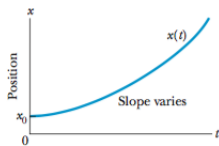
Instantaneous acceleration:  $a = \frac{dv}{dt} = \frac{d}{dt} \frac{dx}{dt} = \frac{d^2x}{dt^2}$

**Example** A particle's position on the x-axis is given by  $x = 4 - 27t + t^3$  with x in meters and t in seconds

1. Find  $v(t)$  and  $a(t)$
2. Is there ever a time when  $v=0$ ?
3. Describe the particle's motion for  $t \geq 0$

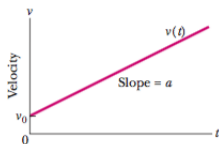


# Constant acceleration



(a)

Slopes of the position graph are plotted on the velocity graph.



(b)

Slope of the velocity graph is plotted on the acceleration graph.



(c)

**Table 2-1**

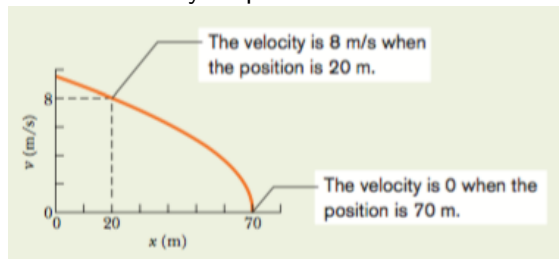
**Equations for Motion with Constant Acceleration<sup>a</sup>**

| Equation Number | Equation                           | Missing Quantity |
|-----------------|------------------------------------|------------------|
| 2-11            | $v = v_0 + at$                     | $x - x_0$        |
| 2-15            | $x - x_0 = v_0t + \frac{1}{2}at^2$ | $v$              |
| 2-16            | $v^2 = v_0^2 + 2a(x - x_0)$        | $t$              |
| 2-17            | $x - x_0 = \frac{1}{2}(v_0 + v)t$  | $a$              |
| 2-18            | $x - x_0 = vt - \frac{1}{2}at^2$   | $v_0$            |

<sup>a</sup>Make sure that the acceleration is indeed constant before using the equations in this table.

# Constant acceleration

**EXAMPLE** The plot below shows a particle's velocity versus its position as it moves along an  $x$  axis with constant acceleration. Find its velocity at position  $x=0$ .



1.  $v^2 = v_0^2 + 2a(x - x_0)$
2. Use two points on the curve to find acceleration.
3. Use the computed  $a$  to find  $v(x = 0)$

# Free Fall Acceleration

Toss an object up or down

Eliminate effects of air on the flight

Free fall acceleration: constant downward acceleration of the object

Mass, density or shape have no impact on this acceleration.



**Fig. 2-10** A feather and an apple free fall in vacuum at the same magnitude of acceleration  $g$ . The acceleration increases the distance between successive images. In the absence of air, the feather and apple fall together. (Jim Sugar/Corbis Images)

The directions of motion are along the  $y$  axis with the positive direction of  $y$  upward.

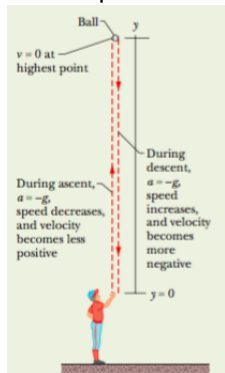
The free-fall acceleration near Earth's surface:

$$a = -g = -9.8\text{m/s}^2$$

# Free fall acceleration

**EXAMPLE** A pitcher tosses a baseball up along a  $y$  axis, with an initial speed of  $12\text{ m/s}$ .

- How long does the ball take to reach its maximum height?
- What is the ball's max height above its release point?
- How long does the ball take to reach a point  $5.0\text{ m}$  above its release point?



# Free fall acceleration

$$(a) \ t = \frac{v-v_0}{a} \text{ Ans: } 1.2 \text{ s}$$

$$(b) \ y = \frac{v^2-v_0^2}{2a} \text{ Ans: } 7.3 \text{ m}$$

$$(c) \ y = v_0 t - 0.5gt^2$$

$$5 = 12t - 0.5(9.8)t^2$$

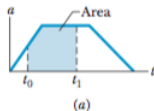
$$4.9t^2 - 12t + 5 = 0$$

$$t = 0.53 \text{ s and } t = 1.9 \text{ s}$$

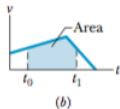
# Graphical Integration in Motion Analysis

$$a = \frac{dv}{dt}: \text{Fundamental theorem of Calculus: } v_1 - v_0 = \int_{t_0}^{t_1} a \, dt$$

$$v = \frac{dx}{dt}: \text{Fundamental theorem of Calculus: } x_1 - x_0 = \int_{t_0}^{t_1} v \, dt$$



This area gives the change in velocity.



This area gives the change in position.

# Reference

[1] Fundamentals of Physics by Halliday and Resnik