Integration Rules

Basic Differentiation Rules

The rules for you to note/recall:

- The constant rule
$$\int a \, \mathrm{d}x = ax + C$$
- The polynomial rule
$$\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1} + C = \frac{1}{n+1} x^{n+1} + C$$
- The scalar multiple rule
$$\int kf(x) \, \mathrm{d}x = k \int f(x) \, \mathrm{d}x$$
- The sum rule
$$\int f(x) \pm g(x) \, \mathrm{d}x = \int f(x) \, \mathrm{d}x \pm \int g(x) \, \mathrm{d}x$$
- The reciprocal rule
$$\int \frac{1}{x} \, \mathrm{d}x = \ln |x| + C$$
- The exponential rules
$$\int e^x \, \mathrm{d}x = e^x + C$$
- The trigonometric rules
$$\begin{aligned} \int \cos x \, \mathrm{d}x &= \sin x + C \\ \int \sin x \, \mathrm{d}x &= -\cos x + C \\ \int \sec^2 x \, \mathrm{d}x &= \tan x + C \end{aligned}$$

Other Integration Rules

- **Integration by Substitution**
If the function $u = g(x)$ has a continuous derivative and f is continuous then
$$\int f(g(x))g'(x) \, \mathrm{d}x = \int f(u) \, \mathrm{d}u \, .$$
Look for one part of the function that is the derivative of the other part.

For example: $\int \frac{(\ln x)^2 - 4\ln x}{x}\,\mathrm{d}x$.

Solution:

Let $u = \ln x$, thus

$$\begin{aligned} \frac{du}{dx} &= \frac{1}{x}, \\ dx &= x\,du. \end{aligned}$$

$$\begin{aligned} \int \frac{(\ln x)^2 - 4\ln x}{x} dx &= \int \frac{(u^2 - 4u)}{x} x\,du \\ &= \int u^2 - 4u\,du \\ &= \frac{u^3}{3} - 2u^2 + C \\ &= \frac{(\ln x)^3}{3} - 2(\ln x)^2 + C. \end{aligned}$$

- **Integration by Parts**

  If u and v are functions of x and have a continuous derivative, then

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x.$$

  Guidelines for integration by parts:

  - Try letting u be the portion of the integrand whose derivative is a simpler function than u.
  - Try letting $\frac{\mathrm{d}v}{\mathrm{d}x}$ be a trigonometric function or an exponential function.

  For example: $\int xe^x\,\mathrm{d}x$.

  Solution:

  Let

$$\begin{aligned} u = x &\rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \\ \frac{\mathrm{d}v}{\mathrm{d}x} = e^x &\rightarrow v = e^x \end{aligned}$$

  Integrating by parts gives:

$$\begin{aligned} \int xe^x\,\mathrm{d}x &= \int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x \\ &= uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x \\ &= xe^x - \int e^x\,\mathrm{d}x \\ &= xe^x - e^x + C \\ &= e^x(x-1) + C. \end{aligned}$$