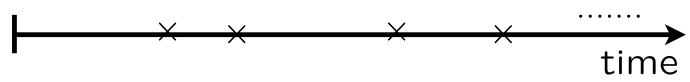
LECTURE 23: More on the Poisson process

- The sum of independent Poisson r.v.s
- Merging and splitting
- Random incidence

The sum of independent Poisson random variables

- Poisson process of rate $\lambda = 1$



- Consecutive intervals of length μ and ν

$$P(k,\tau) = \frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!}$$

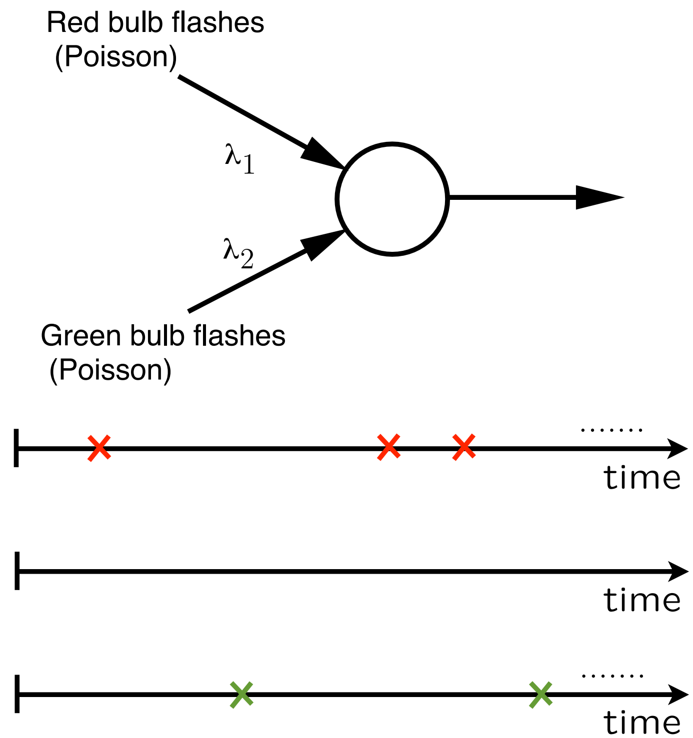
- Numbers of arrivals during these intervals: M and N

- M:
- N:
- Independent?
- $M + N$:

The sum of independent Poisson random variables,
with means/parameters μ and ν,
is Poisson with mean/parameter $\mu + \nu$

Merging of independent Poisson processes

		$1-\lambda_1\delta$	$\lambda_1\delta$	$O(\delta^2)$
		0	1	≥ 2
$1-\lambda_2\delta$	0			
$\lambda_2\delta$	1			
$O(\delta^2)$	≥ 2			

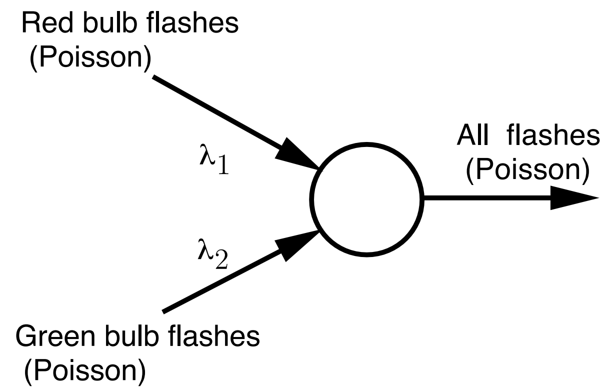


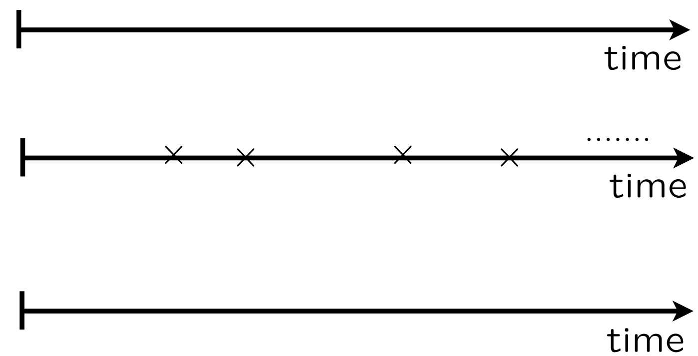
Merged process: Poisson($\lambda_1 + \lambda_2$)

Where is an arrival of the merged process coming from?

$\mathbf{P}(\text{Red} \mid \text{arrival at time } t) =$

		$1-\lambda_1\delta$	$\lambda_1\delta$	$O(\delta^2)$
		0	1	≥ 2
$1-\lambda_2\delta$	0	$1-(\lambda_1+\lambda_2)\delta$	$\lambda_1\delta$	
$\lambda_2\delta$	1	$\lambda_2\delta$	$O(\delta^2)$	
$O(\delta^2)$	≥ 2			





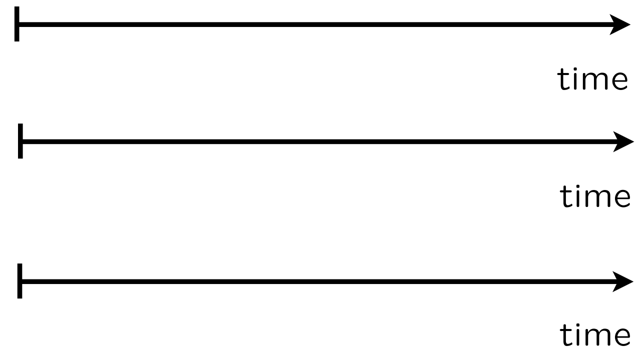
$\mathbf{P}(k\text{th arrival is Red}) =$

- Independence for different arrivals

$\mathbf{P}(4 \text{ out of first 10 arrivals are Red}) =$

The time the first (or the last) light bulb burns out

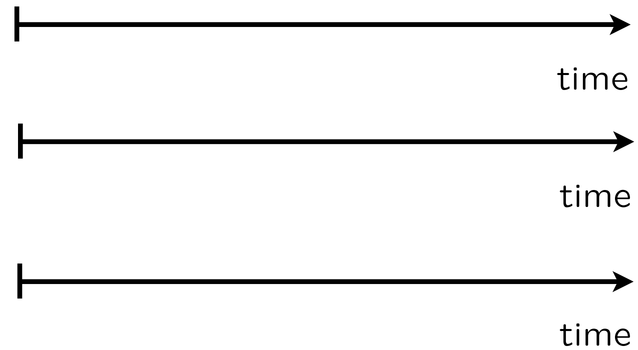
- Three lightbulbs
  - independent lifetimes X, Y, Z; exponential(λ)
- Find expected time until first burnout



- X, Y, Z: first arrivals in independent Poisson processes
- Merged process:
- $\min\{X, Y, Z\}$: 1st arrival in merged process

The time the first (or the last) light bulb burns out

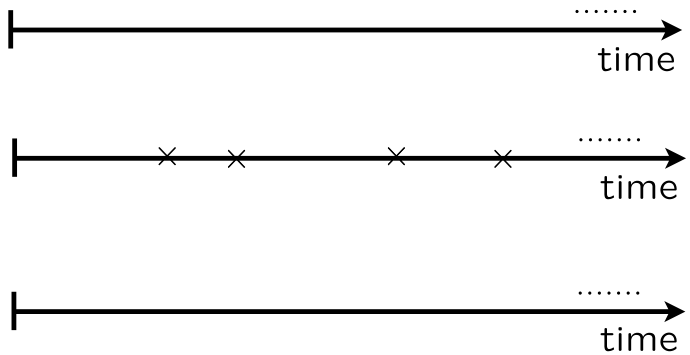
- Three lightbulbs
  - independent lifetimes X, Y, Z; exponential(λ)



- Find expected time until all burn out

Splitting of a Poisson process

- Split arrivals into two streams, using independent coin flips of a coin with bias q
  - assume that coin flips are independent from the original Poisson process



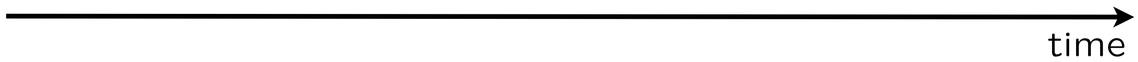
Resulting streams are Poisson, rates λq, $\lambda(1-q)$

- Are the two resulting streams independent?

  Surprisingly, yes!

“Random incidence” in the Poisson process

- Poisson process that has been running forever



- Believe that $\lambda = 4$/hour, so that $\mathbf{E}[T_k] =$

- Show up at some time and measure interarrival time

  – do it many times, average results, see something around 30 mins! Why?

“Random incidence” in the Poisson process — analysis

- Arrive at time t^*
- U: last arrival time
- V: next arrival time
- $V - U =$
- $\mathbf{E}[V - U] =$
- $V - U$: interarrival time you see, versus kth interarrival time

Random incidence “paradox” is not special to the Poisson process

- **Example:** interrarival times, i.i.d., equally likely to be 5 or 10 minutes

  expected value of kth interarrival time:

- you show up at a “random time”

  $\mathbf{P}$(arrive during a 5-minute interarrival interval) $=$

  expected length of interarrival interval during which you arrive $=$

- Calculation generalizes to “renewal processes:”
  i.i.d. interarrival times, from some general distribution

- “Sampling method” matters

Different sampling methods can give different results

- Average family size?
  - look at a “random” family (uniformly chosen)
  - look at a “random” person’s (uniformly chosen) family
- Average bus occupancy?
  - look at a “random” bus (uniformly chosen)
  - look at a “random” passenger’s bus
- Average class size?