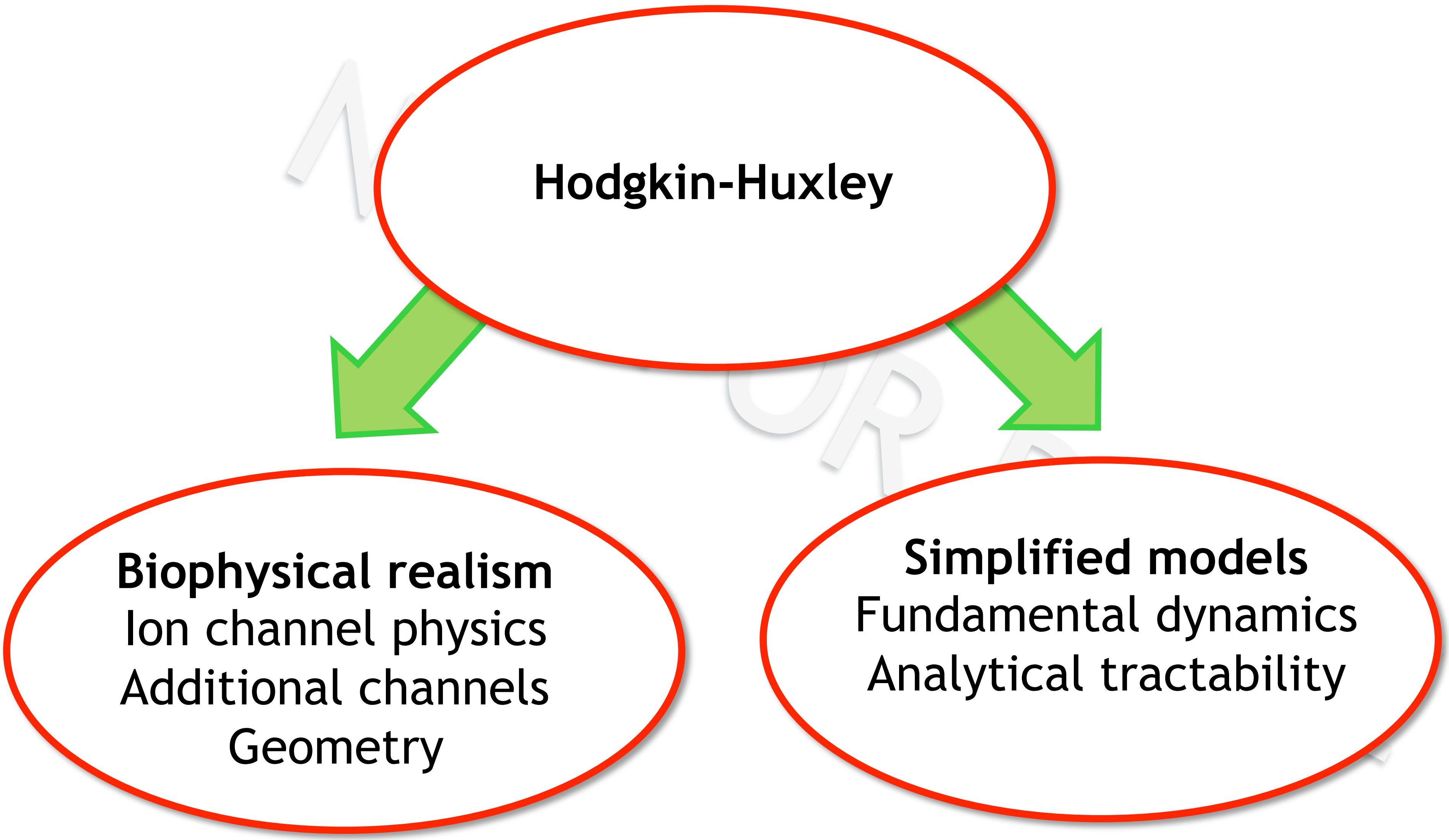
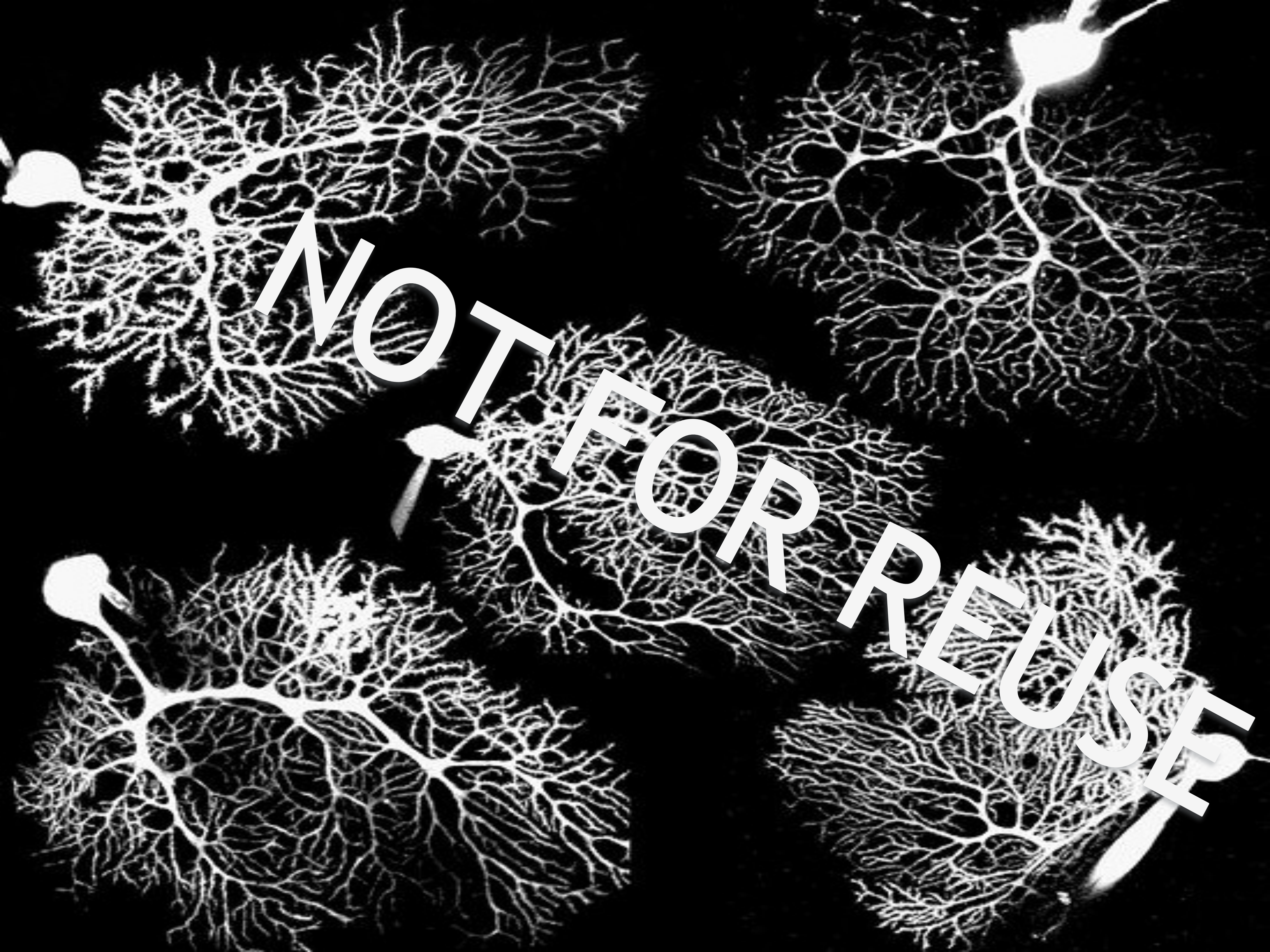
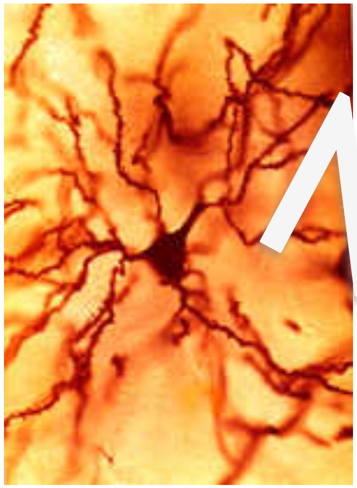
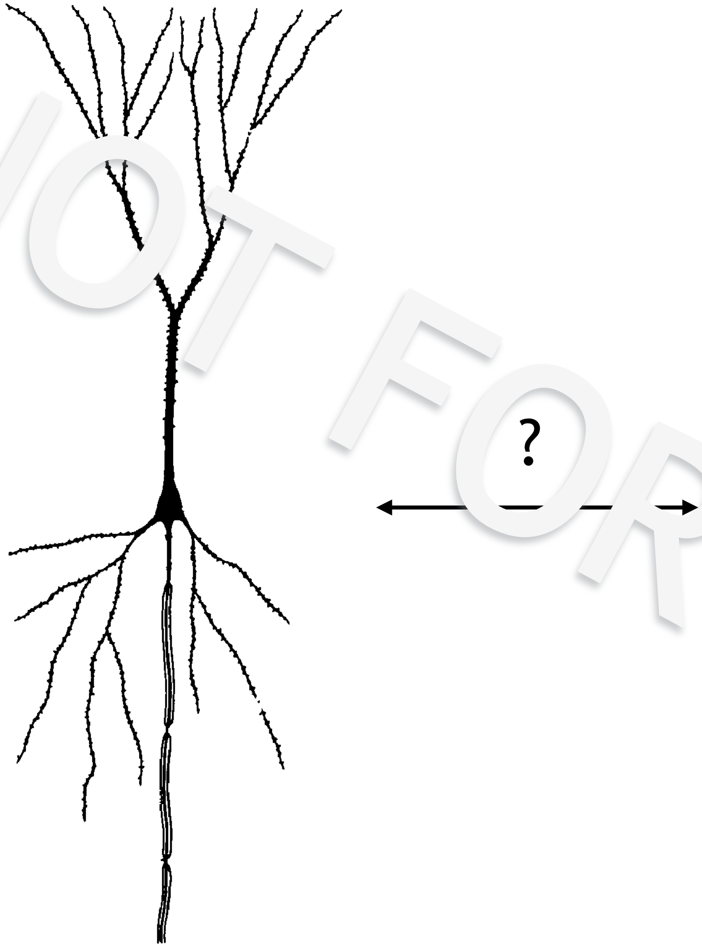
Where to from here?

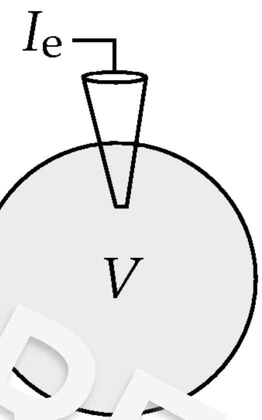




Neurons have complicated spatial structures



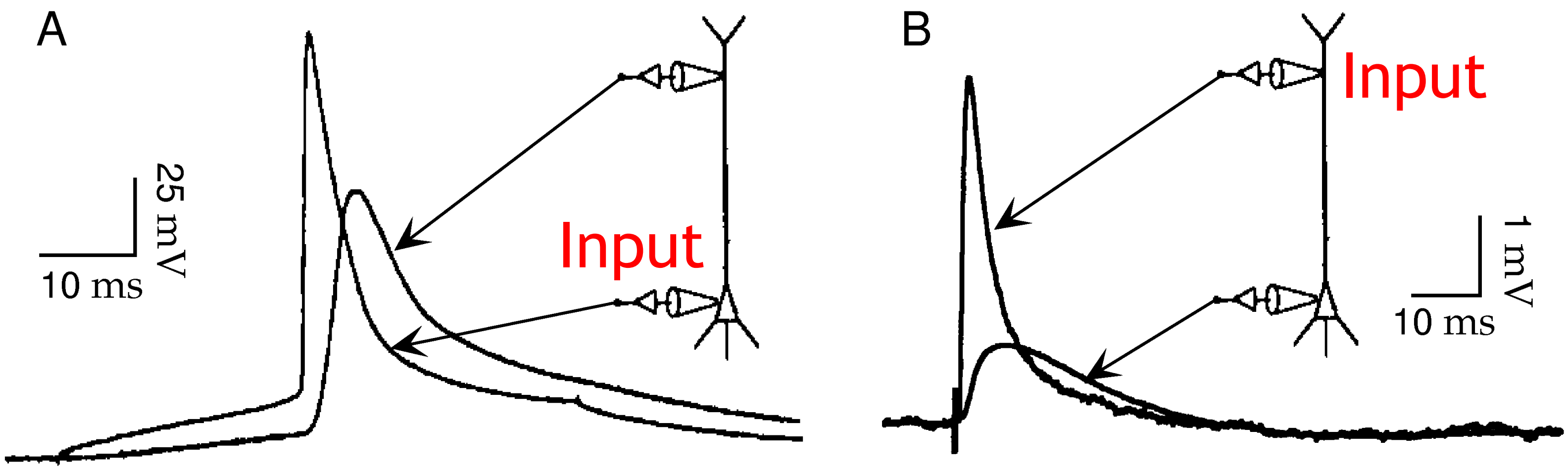




Real Neurons

Our Model

Geometry matters!



Inject current at the cell body and record effect in a dendrite

Inject current in a dendrite and record effect at the cell body

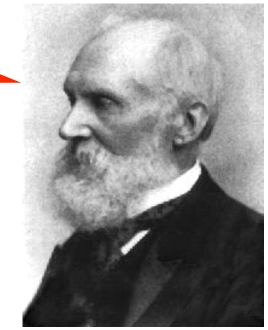
The thinner the dendrite the larger the voltage change but generally the further away the the more that input gets filtered and attenuated.

Voltage decays with distance in passive membranes (How?)

Dayan and Abbott

Linear cables

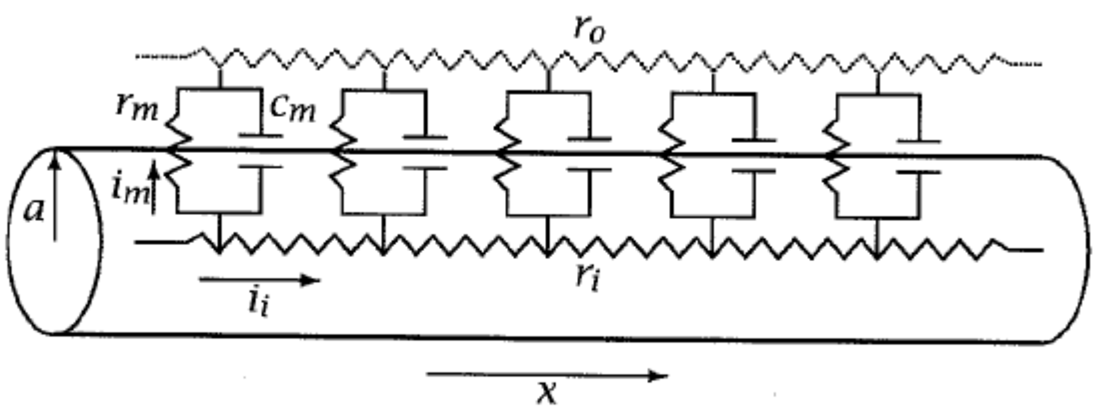
This problem sounds familiar!



Lord Kelvin
(1824-1907)
Developed cable theory for undersea cables

Voltage *V* is a function of *both x and t*

space and time
PDE



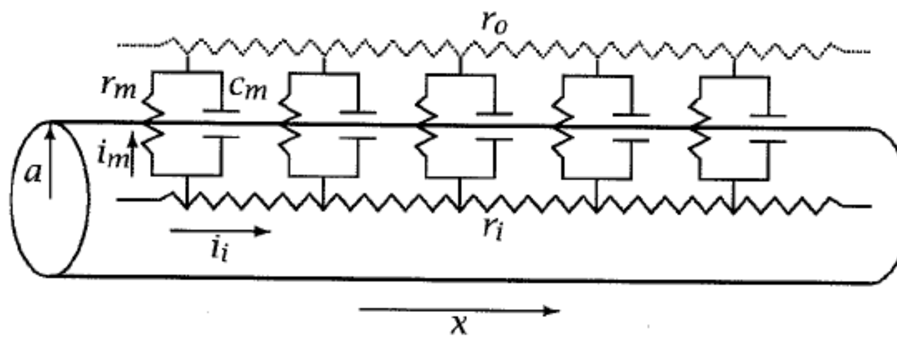
r_m and r_i are the membrane and axial resistances, i.e. the resistances of a thin slice of the cylinder

Johnston and Wu

The cable equation

Before we had:

$$i_m = i_C + i_{\text{ionic}} = c_m \frac{\partial V_m}{\partial t} + \frac{V_m}{r_m}$$



Now we also have to consider i_i, the current down the cable, due to voltage changes in *x.*
That current works against internal resistance, r_i

$$\boxed{\frac{1}{r_i}\frac{\partial^2 V_m(x,t)}{\partial x^2} = c_m \frac{\partial V}{\partial t} + \frac{V_m}{r_m}.}$$

or

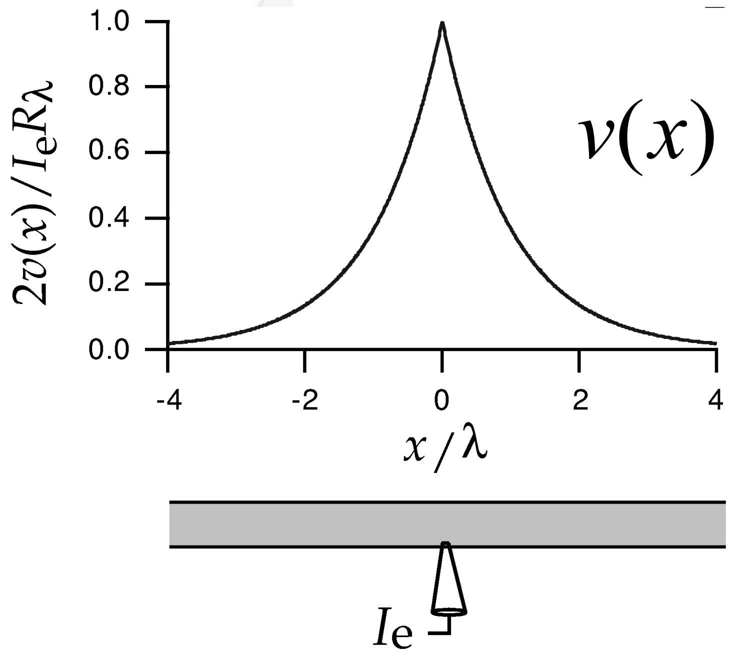
$$\boxed{\lambda^2 \frac{\partial^2 V_m}{\partial x^2} = \tau_m \frac{\partial V_m}{\partial t} + V_m}$$

where

$\tau_m = r_m c_m$ Time constant

$\lambda = \sqrt{\frac{r_m}{r_i}}$ Space constant

How does voltage decay in space?



$$v(x) \propto e^{\left(-\frac{|x|}{\lambda}\right)}$$

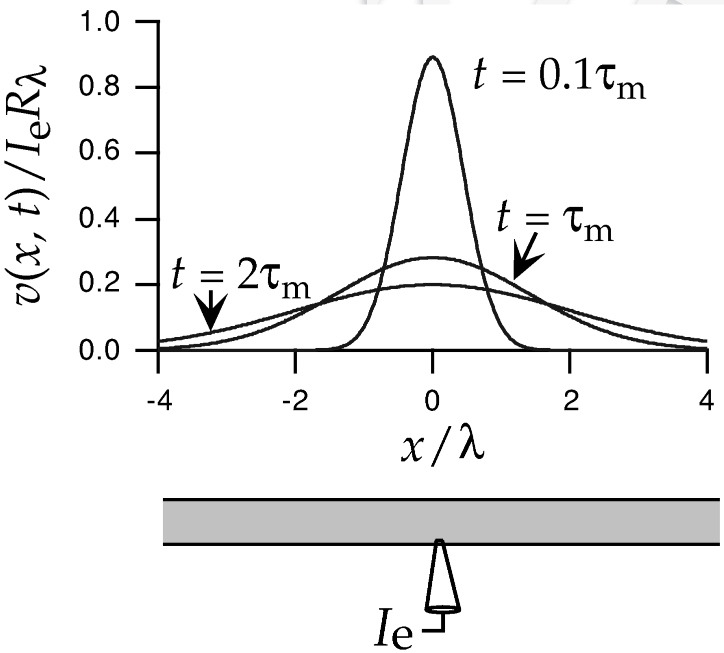
Potential decays exponentially from x = 0

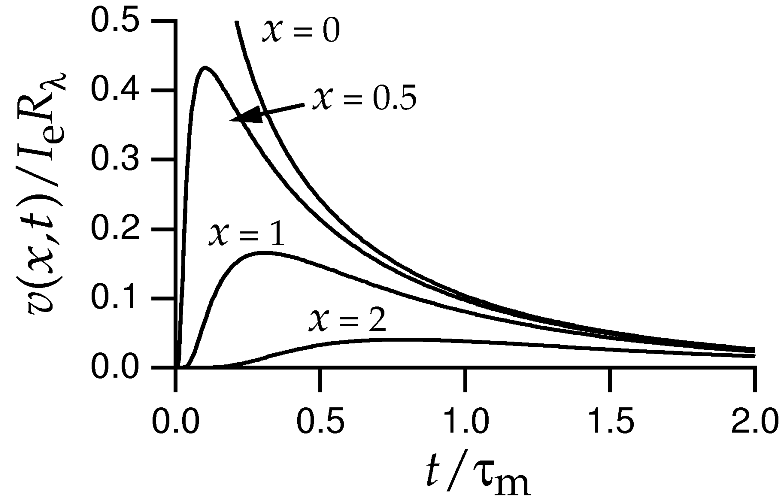
Infinite Cable,
Constant current at x = 0

Dayan and Abbott

How does voltage decay over space and time?

Infinite Cable,
Current pulse at t = 0, x = 0





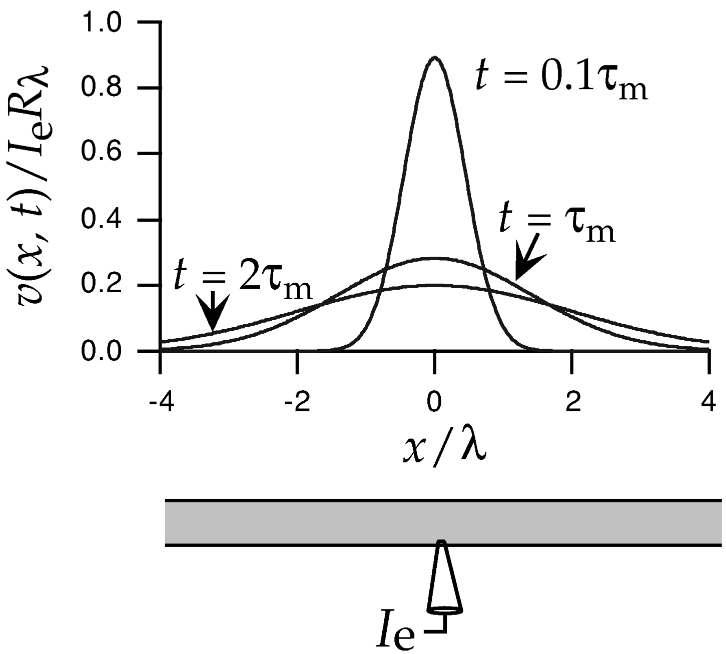
Potential peaks later (and at lower values) for points further away from input

General solution: filter and impulse response

$$V(x,t) \propto \sqrt{\frac{\tau}{4\pi\lambda^2 t}}\, e^{-\frac{t}{\tau} - \frac{\tau x^2}{4\lambda^2 t}}$$

Exponential decay

Diffusive spread



This is called a Green's function, or a filter, like we saw in Lecture 2: the full solution for an arbitrary input is found by convolving the input with this filter.

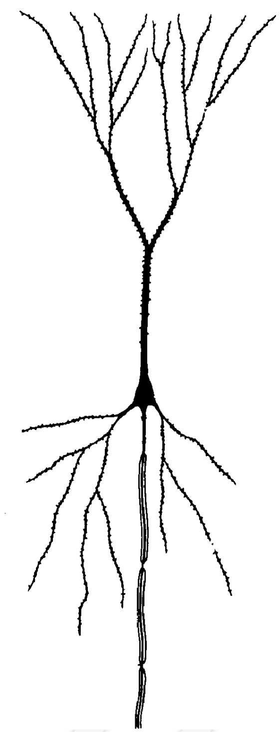
OK: now what?

1. The geometry can be extremely complicated

Cable Equation

$$\frac{1}{r_i}\frac{\partial^2 V_m(x,t)}{\partial x^2} = c_m \frac{\partial V}{\partial t} + \frac{V_m}{r_m} + i_e$$

?

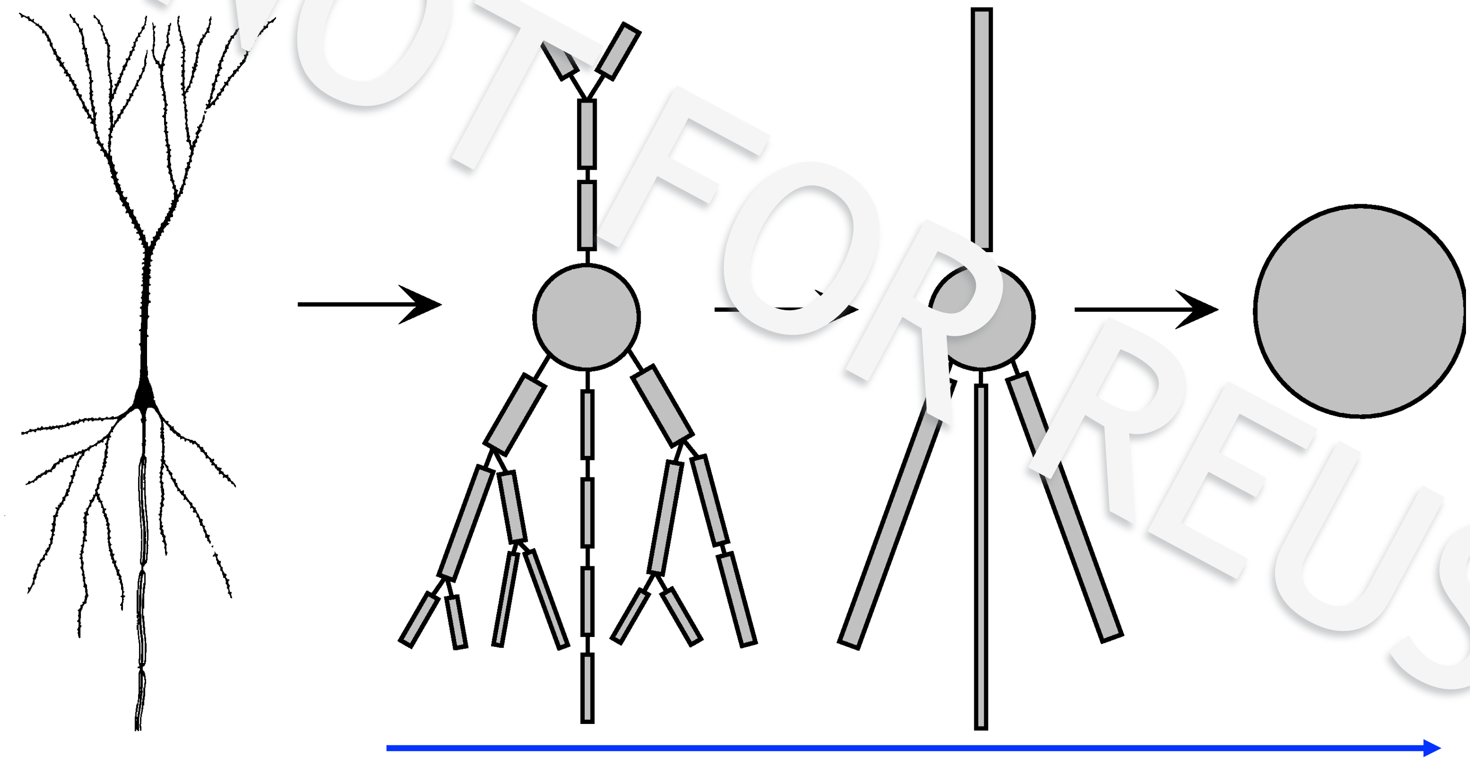


2. And, um, ion channels?

→ Quickly becomes intractable to solve analytically for realistic neurons

Solution: Divide and Conquer

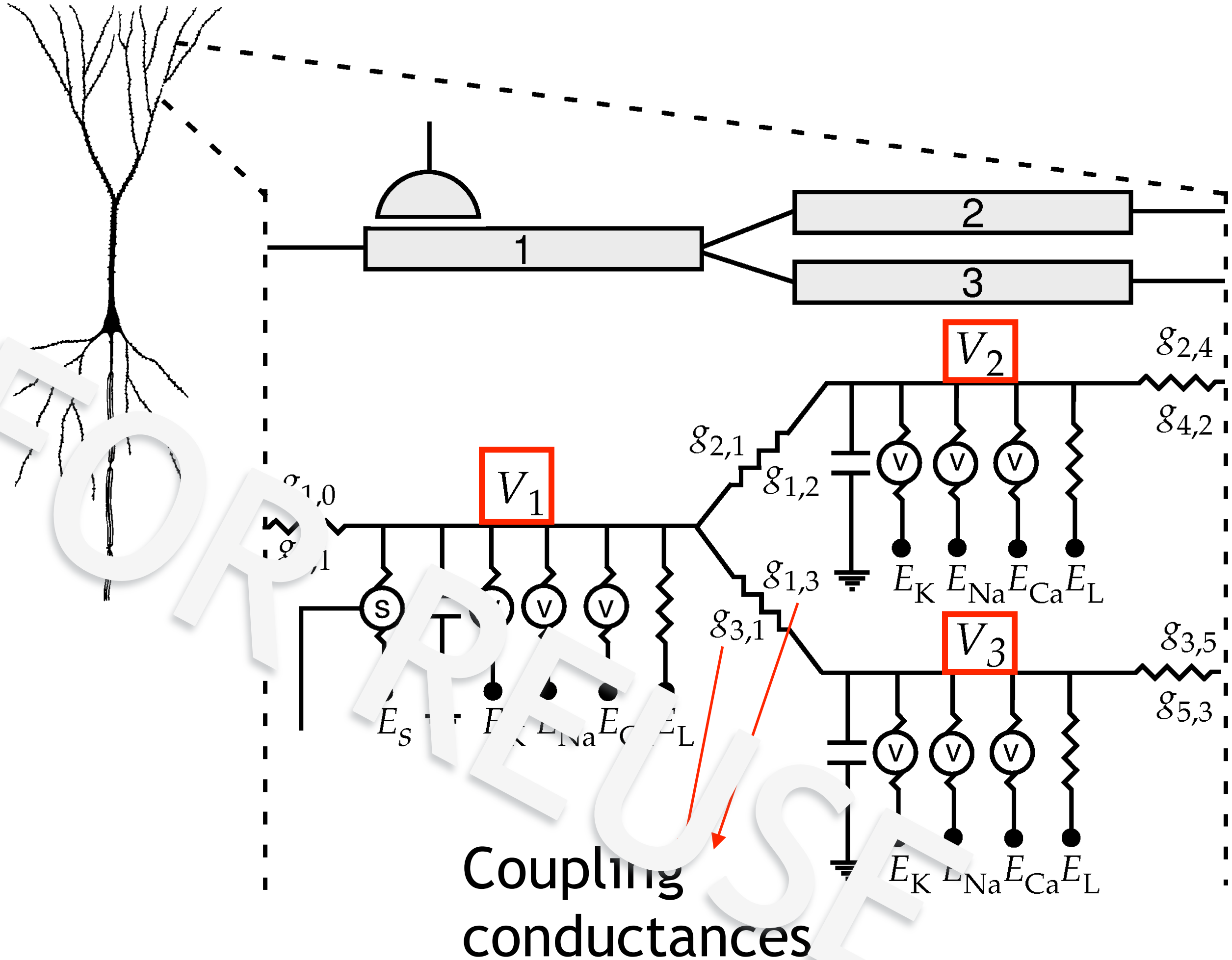
Compartmental models



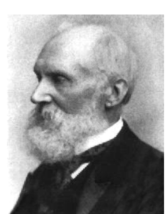
Decreasing number of "compartments"

Each compartment = one dV/dt equation
(usually no dependence on x)

The gory details

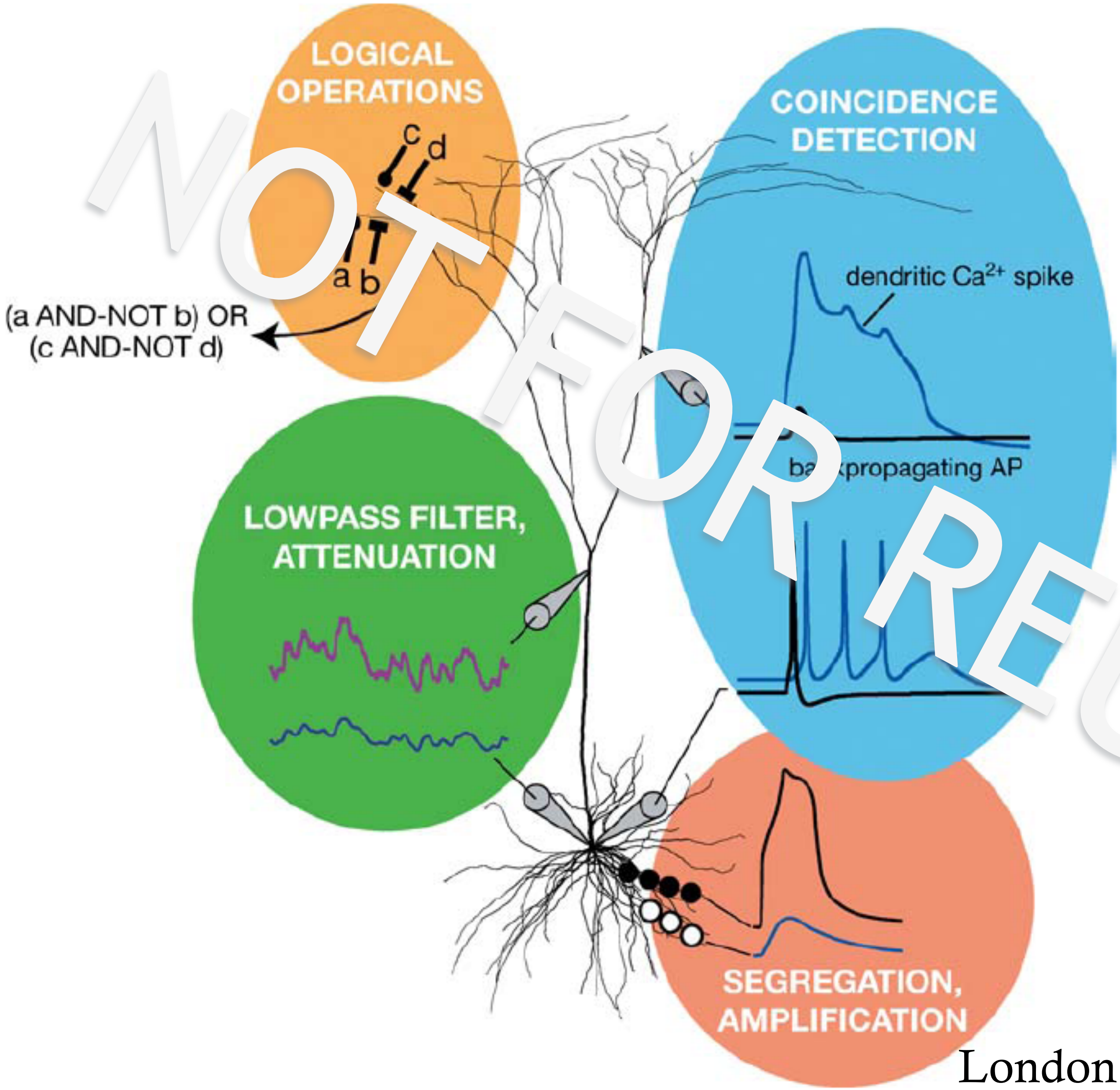


And now you see why Genesis and NEURON were developed—thank you!



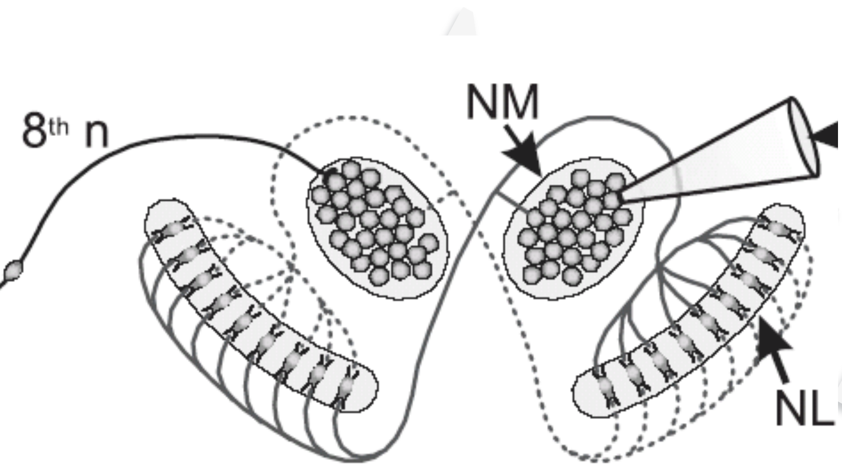
Dayan and Abbott

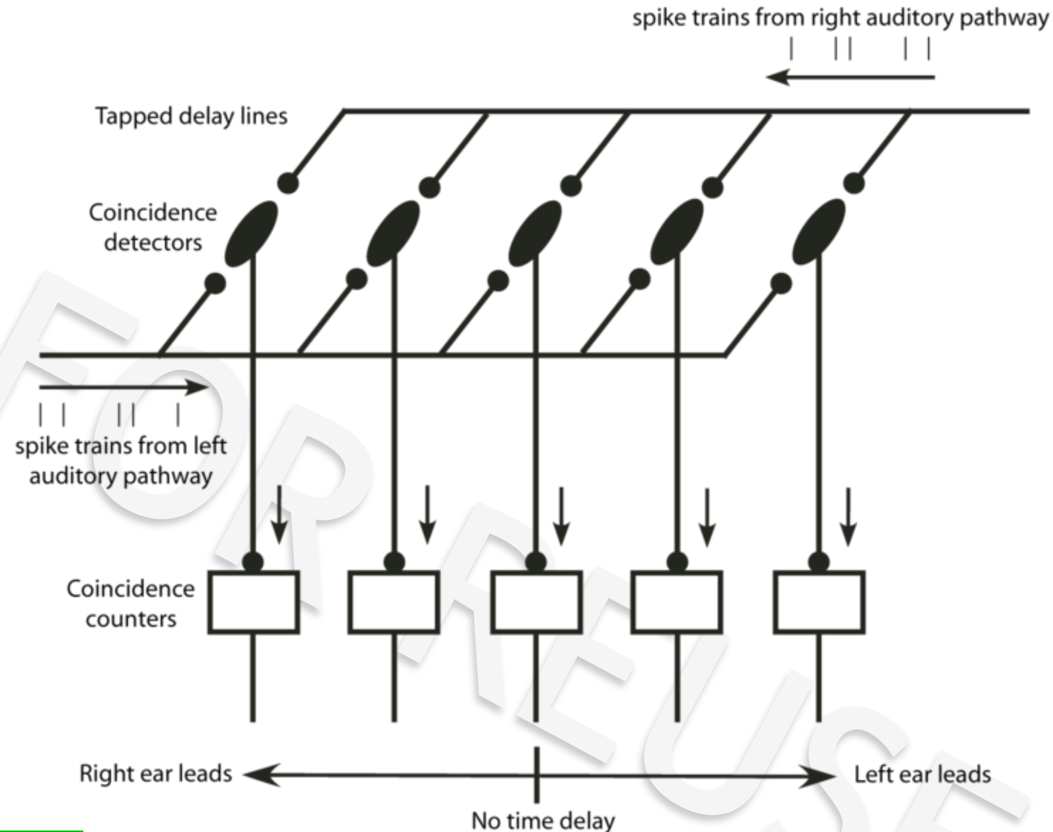
What do dendrites add to neuronal computation?



London and Hausser, 2005

Delay lines in sound localization



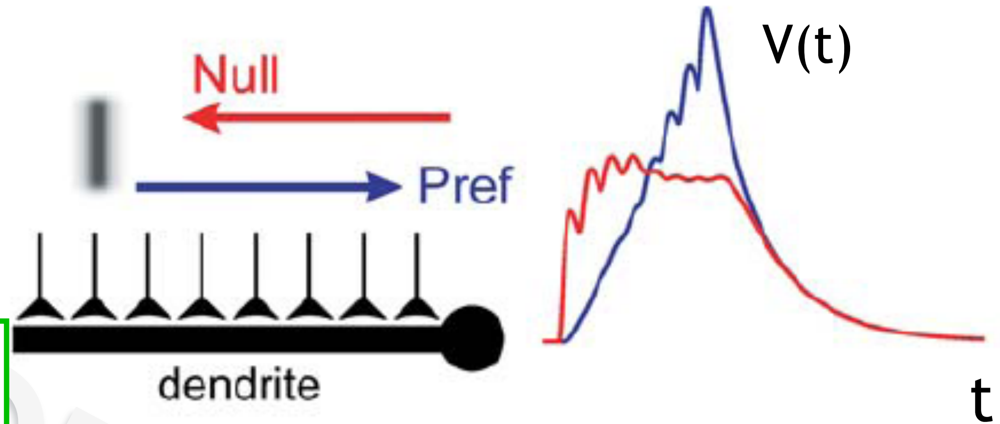


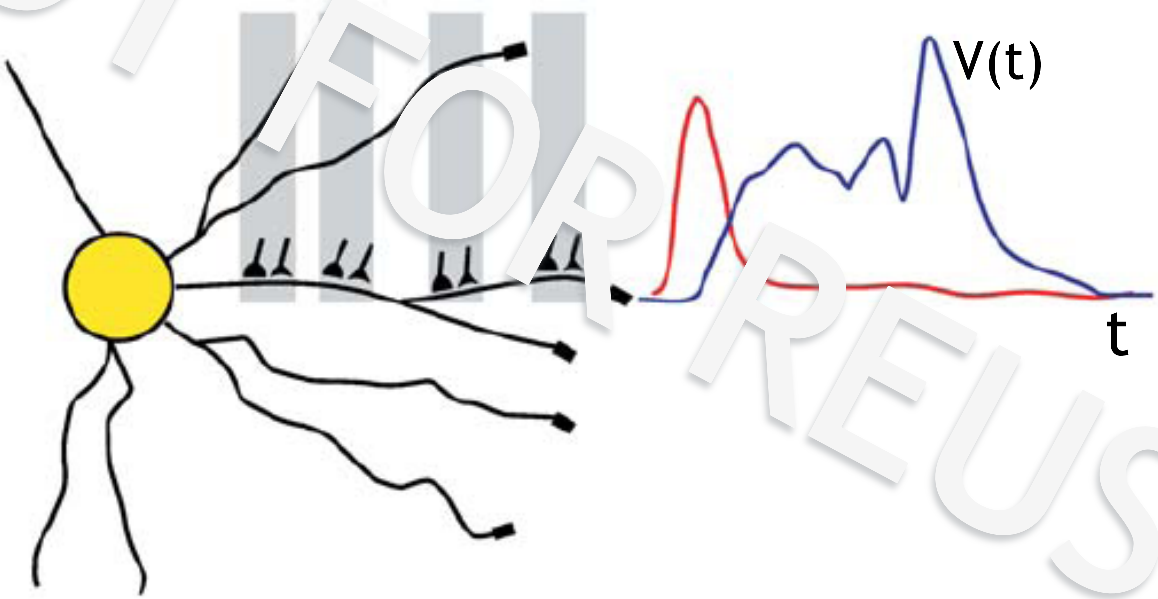
A sound that comes from your left enters your left ear first. The signal from your left ear then has more time to travel down the delay line leading to your coincidence detecting neurons. This signal gets further along the delay line before it meets the signal from your right ear, which began slightly after the signal from your left ear. Thus, the two signals meet at a coincidence detecting neuron that is closer to the right ear, and your brain infers that the sound came from your left.

Spain; Scholarpedia

Direction selectivity

和input的序列顺序有关，远端先激活，就会越来越大叠加，近端先激活，就不会这样。





Rall; London and Hausser, 2005

Enthusiastically recommended references

- **Johnson and Wu, *Foundations of Cellular Neurophysiology*, Chap 4**

The classic textbook of biophysics and neurophysiology: lots of problems to work through. Good for HH, ion channels, cable theory.

- **Koch, *Biophysics of Computation***

Insightful compendium of ion channel contributions to neuronal computation

- **Izhikevich*, Dynamical Systems in Neuroscience***

An excellent primer on dynamical systems theory, applied to neuronal models

- **Magee, *Dendritic integration of excitatory synaptic input*,**

Nature Reviews Neuroscience, 2000
Review of interesting issues in dendritic integration

- **London and Hausser, *Dendritic Computation*,**

Annual Reviews in Neuroscience, 2005
Review of the possible computational space of dendritic processing