Calculus Presentation

(Limits, Derivatives, Integrals, and Multivariate Gradients)

Introduction to Calculus

Calculus is the study of continuous change.
It has two main branches:

- Differential Calculus: Concerned with rates of change (slopes)
- Integral Calculus: Concerned with accumulation (areas under curves)

Applications:

- Physics (e.g., motion, force)
- Engineering (e.g., modeling systems)
- Economics (e.g., marginal cost analysis)
- Data science (e.g., optimization)

Limits

The limit of a function describes the behavior as the input approaches a certain value.

Notation:

- $\lim(x \to a) f(x) = L$

Example:

- $\lim(x \to 0) (\sin(x)/x) = 1$

Why it matters: Limits form the basis of derivatives and integrals.

Derivatives

A derivative tells us the rate of change or slope of a function at a point.

Notation:
- f'(x) or df/dx

Example:
- If f(x) = x^2, then f'(x) = 2x

Applications:
- Velocity (change of position)
- Economics (marginal cost)
- Machine learning (gradients in backpropagation)

Integrals

An integral represents the accumulation of quantities, like area under a curve.

Two types:

- Indefinite integrals (general form)
- Definite integrals (specific interval)

Example:

$\int x\, dx = x^2/2 + C$

Applications:

- Total distance from velocity
- Area under curves
- Volume calculations

Fundamental Theorem of Calculus

- Connects differentiation and integration.
- If F is an antiderivative of f, then:

$$\boxed{\int(a \text{ to } b)\ f(x)\ dx = F(b) - F(a)}$$

- This bridges the two main branches of calculus.

Multivariate Gradients

When functions have multiple variables, we use gradients.
The gradient vector points in the direction of steepest increase.

Notation:

- $\nabla f(x, y) = [\partial f/\partial x, \partial f/\partial y]$

Applications:

- Gradient Descent in Machine Learning
- Directional change in physics
- Optimization problems

Numerical Calculus using NumPy

Using NumPy:

1. Derivatives via finite differences:

$$df = (f(x+h) - f(x))/h$$

2. Integrals via trapezoidal rule:

np.trapz(y, x)

Use cases:

- Noisy data analysis
- Simulations
- Real-time processing

Summary

- Limits define behavior near a point
- Derivatives show change rates
- Integrals measure accumulated change
- Fundamental Theorem links the two
- Gradients extend to multiple variables
- NumPy helps with numerical methods in Python