Oscillations II

PHYS 301: Analytical Mechanics

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Problem 1

You're building a bathroom scale and you want the platform deflection to be one inch when a 200-lb person stands on it. You want the motion to be critically damped. What should the spring constant k and the damping constant b be? What type of motion will occur if someone lighter than 200 lbs steps onto the scale?

Problem 2

Find the motion of an object subject to a linear restoring force and a linear retarding force when it starts from equilibrium at initial velocity v_0. Consider underdamped, critically damped, and over-damped cases, and use *Mathematica* to plot x-vs-t graphs for each case.

Problem 3

An undamped driven harmonic oscillator satisfies the equation of motion $m\left(d^2 x / d t^2 + \omega_0^2 x\right) = F(t)$. The driving force $F(t) = F_0 \sin(\omega t)$ is switched on at $t = 0$.

a. Find $x(t)$ for $t > 0$ for the initial conditions $x = 0$ and $\dot{x} = 0$ at $t = 0$.

b. Find $x(t)$ for $\omega = \omega_0$ by taking the limit $\omega \to \omega_0$ in your result for part a. Sketch your result for $x(t)$.

Problem 4

A force $F_0 e^{-\alpha t}$ acts on a damped harmonic oscillator. Find a particular solution of the equation of motion by starting from the guess that there should be a solution with the same time dependence as the applied force. *Extra credit:* If the oscillator starts from rest at equilibrium, find the motion of the oscillator and plot it in *Mathematica*.

Problem 5

Use *Mathematica* to plot the response function ($x(t)$) for a driving force $F(t) = 3\cos(t) + 2\cos(2t)$ with initial conditions $x(0) = \dot{x}(0) = 0$. Use values $b = 0.05$/s, $k = 10$kg/m, and $m = 1$kg.