# Central-Force Motion

## PHYS 301: Analytical Mechanics

Scott Kobos

### Problem 1

**A particle of mass $m$ moves under the influence of a central force whose potential is $U(r) = K r^4$, where $K$ is a positive constant.**
**a. For what energy and angular momentum will the orbit be a circle of radius $a$ about the origin?**

**b. What is the period of this circular motion, in terms of $a$, $K$, and $m$?**

### Problem 2

**The Yukawa potential, describing the attractive force between a neutron and a proton, is described by the potential $U(r) = -k\, e^{-\alpha r} \big/ r$, where $k$ is a positive constant.**

**a. Find the force, and compare it with an inverse square law of force.**

**b. Discuss the types of motion which can occur if a particle of mass $m$ moves under such a force. Strategy: Try to find equilibrium points and show why it's impossible to do, at the very least without some really advanced math. Then try a different tactic: let $x \equiv \alpha r$ and express the potential in terms of $x$ and dimensionless parameter $b \equiv \frac{\alpha l^2}{2 m k}$. Plot $V(x)$, letting $k\,\alpha = 1$ Joule, and vary $b$ from 0.001 to 1 (perhaps using the Manipulate function in *Mathematica*). What happens at around $b = 0.42$? Describe the motion of particles for $b < 0.42$, and for $b > 0.42$.**

### Problem 3

***Sputnik 1* had a perigee 227 km <u>above Earth's surface</u>, at which point its speed was 28,710 km/hr. Find its apogee distance from Earth's surface and its period of revolution.**

### Problem 4

**A satellite is moving in circular orbit of radius $R$ about Earth at speed $v$. To what should the velocity be increased in order for the satellite to be in an elliptical orbit with $r_{\min} = R$ and $r_{\max} = 2R$?**

### Problem 5

**Use a graphing program such as *Mathematica* to plot the orbit of an object under the influence of an inverse-square-law central force with energy *E*, allowing *E* to vary (using Manipulate in *Mathematica*, for example).**

# Problem 1

$$\mathrm{D}\left[k\, r^4 + \frac{l^2}{2\, m\, r^2},\ r\right]$$

$$-\frac{l^2}{m\, r^3} + 4\, k\, r^3$$

$$\mathrm{FullSimplify}\left[\mathrm{Solve}\left[-\frac{l^2}{m\, r^3} + 4\, k\, r^3 == 0,\ l\right]\right]$$

$$\{\{l \to -2\sqrt{k}\sqrt{m}\, r^3\}, \{l \to 2\sqrt{k}\sqrt{m}\, r^3\}\}$$

$$\left(2\sqrt{k}\sqrt{m}\, r^3\right)^2$$

$$4\, k\, m\, r^6$$

$$\frac{1}{2}\, \frac{(4\, k\, m\, a^6)}{m\, a^2}$$

$$2\, a^4\, k$$

$$2\, \frac{\pi}{\frac{2\, a\sqrt{k\, m}}{m}}$$

$$\frac{m\, \pi}{a\sqrt{k\, m}}$$

# Problem 3

```
ClearAll["Global`*"]
```

```
L = 4.399 * 10^12;
m1 = 83.6;
m2 = 5.972 * 10^24;
G = 6.674 * 10^-11;
e = -2 391 593 755;
```

$$\text{Solve}\left[0 == .5\ m1 \left(\frac{L}{m1\ r2}\right)^2 - \frac{(G\ m1\ m2)}{r2} - e,\ r2\right]$$

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

$$\{\{r2 \to 6.59942 \times 10^6\}, \{r2 \to 7.33295 \times 10^6\}\}$$

# Problem 4

```
ClearAll["Global`*"]
```

$$\frac{\left(\frac{G\,M\,m}{r} + \frac{G\,M\,m}{2\,r}\right)}{m}$$

$$\frac{3\,G\,M}{2\,r}$$