Vector Calculus III

PHYS 310 : Mathematical Methods in Physics

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Problem 1

a

Show that the force $\vec{F} = \frac{y\,\hat{x} + x\,\hat{y}}{\sqrt{1 - x^2\,y^2}}$ is conservative.

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D[y / Sqrt[(1 - x^2 y^2)], y]
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$$\frac{x^2\,y^2}{\left(1 - x^2\,y^2\right)^{3/2}} + \frac{1}{\sqrt{1 - x^2\,y^2}}$$

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D[x / Sqrt[(1 - x^2 y^2)], x]
```

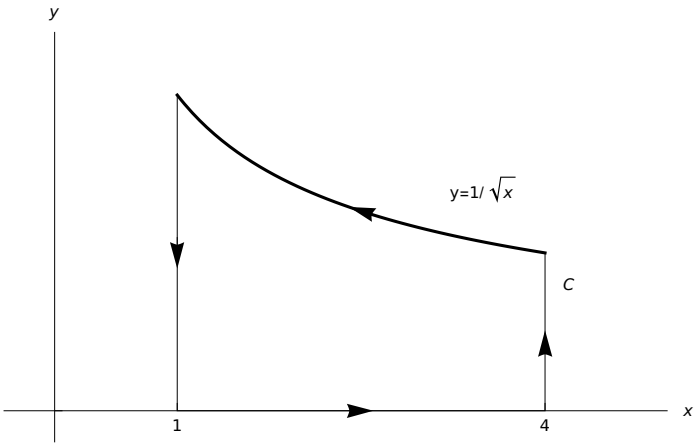
$$\frac{x^2\,y^2}{\left(1 - x^2\,y^2\right)^{3/2}} + \frac{1}{\sqrt{1 - x^2\,y^2}}$$

b

Find a scalar potential for the force.

Problem 2

Use Green's theorem to evaluate $\oint_C x\,y\,dx + x^2\,dy$, where the closed path C is as sketched.



$$\int_0^{\frac{1}{\sqrt{x}}} \int_1^4 x \, dx \, dy$$

$$\frac{15}{2\sqrt{x}}$$

Problem 3

Think before you try to do these problems!

a

Evaluate the integral below over the surface of a sphere with its center at the origin and a radius of 3.

$$\int \left(x \cos^2 y \, \hat{x} + x \, z \, \hat{y} + z \sin^2 y \, \hat{z} \right) \cdot d\vec{a}$$

$$\int_0^3 \int_0^{\pi} \int_0^{2\pi} r^2 \, \text{Sin}[\theta] \, d\phi \, d\theta \, dr$$

$$36\,\pi$$

b

Evaluate the integral below inside a sphere with its center at the origin and a radius of 5.

$$\int \vec{\nabla} \cdot (x^2 + y^2 + z^2) \left(x \, \hat{x} + y \, \hat{y} + z \, \hat{z} \right) dV$$

$$4\,\pi \left(5^5\right)$$

$$12\,500\,\pi$$

Problem 4

A point charge sitting at the origin produces a radial electric field $\vec{E} = k \frac{q}{r^2} \hat{r}$, where r is the usual coordinate in spherical coordinates. What is the electric flux $\left(\Phi_E \equiv \int \vec{E} \cdot d\vec{a}\right)$ through a circular disk of radius 1 in the $z = 2$ plane?

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kq ∫_0^.4636 ∫_0^2 π Sin[θ] dϕ dθ
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0.6632 kq
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