- The magnetic moment can be found w/:

$$m = IA$$

- For m_c:

$$m_c = I(\pi a^2)$$

- For m_s:

$$m_s = I(b^2)$$

- Plug into torque equation:

$$\tau = -\frac{\mu_0 I \pi a^2 I b^2}{4\pi r^3} \hat{x}$$

$$\boxed{\tau = -\frac{\mu_0 I^2 a^2 b^2}{4 r^3} \hat{x}}$$

- For the loops to be in equilibrium, the net torque has to be 0. To make that happen the square loop has to be oriented in the $-\hat{z}$ direction.

+ 9

Problem 2

- Toy has donut-shaped permanent magnets ($\vec{M}$ parallel to axis) which slide frictionlessly on a vertical rod. Treat the magnets as dipoles w/ mass m_d + dipole moment $\vec{m}$

a) If you put 2 back to back magnets on the rod, upper one will "float". At what height (z) does it float.

b) If you add a 3rd magnet what is the ratio of the 2 heights?

a) To find height the top magnet "floats":

* m_U is upper magnet moment
m_1 is lower magnet moment

- The magnetic field of a dipole is:

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

- for the upper magnet:

$$\vec{B} = \frac{\mu_0 m_U}{4\pi r^3}\left(2\cos 0\,\hat{r} + \sin 0\,\hat{\theta}\right)$$

* $\theta = 0°$ because they're collinear.

$$\vec{B}_U = \frac{\mu_0 m_U}{2\pi z^3}\hat{z}$$

* $r = z$ + $\hat{r}$ is $\hat{z}$

- To find the force by the field:

$$\vec{F} = \vec{\nabla}(\vec{m}_1 \cdot \vec{B}_U)$$

$$\vec{F} = \vec{\nabla}\left(\vec{m}_1 \cdot \frac{\mu_0 m_U \hat{z}}{2\pi z^3}\right)$$

$$\vec{F} = \vec{\nabla}\left(\frac{\mu_0 m_U m_1 \hat{z}}{2\pi z^3}\right)$$

* $\vec{m}_U = \vec{m}_1 = \vec{m}$

$$\vec{F} = \vec{\nabla}\left(\frac{\mu_0 m^2}{2\pi z^3}\hat{z}\right) \rightarrow \vec{F} = \frac{\partial}{\partial z}\left(\frac{\mu_0 m^2}{2\pi z^3}\hat{z}\right)$$

$$\vec{F} = \frac{3\mu_0 m^2}{2\pi z^4}$$

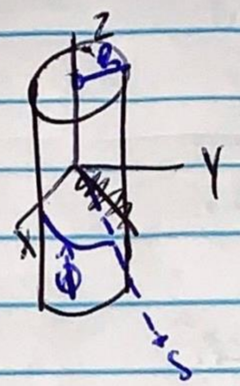
- The magnet floats when the magnetic force = gravitational force:

$$m_d g = \frac{3\mu_0 m^2}{2\pi z^4} \qquad z^4 = \frac{3\mu_0 m^2}{2\pi m g} \qquad \boxed{z = \sqrt[4]{\frac{3\mu_0 m^2}{2\pi m g}}}$$

Scott Kovos
302 HW9

Problem 3

- A long cylinder radius R carries a magnetization $\vec{M} = ks^2\hat{\phi}$ where k is a constant s is the distance from the axis. Find the magnetic field due to $\vec{M}$ for points inside & outside the cylinder.



- Bound volume current density is defined as:

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

- w/ $M_\phi = ks^2$
$M_s = 0$
$M_z = 0$

$$\vec{\nabla}\times\vec{M} = \frac{1}{s}\left[\frac{\partial M_z}{\partial\phi} - \frac{\partial M_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial M_s}{\partial z} - \frac{\partial M_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sM_\phi) - \frac{\partial M_s}{\partial\phi}\right]\hat{z}$$

- From $\nabla \times \vec{M}$ the $\hat{s}$ & $\hat{\phi}$ terms are 0 leaving only the $\hat{z}$ term: $\frac{\partial}{\partial s}(sks^2) = \frac{\partial}{\partial s}(ks^3) = 3ks^2\hat{z}$

$$\underline{\frac{1}{s}(3ks^2)\hat{z} = 3ks = J_b}$$

- Bound surface current density is defined as:

$\vec{K}_b = \vec{M}\times\hat{n}$ - $\hat{n} = \hat{s}$ direction for this case

$\vec{K}_b = ks^2\,\hat{\phi}\times\hat{s}$

$\vec{K}_b = -ks^2\hat{z}$ - s goes to R here b/c the surface is bound by the radius R

$$\underline{\vec{K}_b = -kR^2\hat{z}}$$

- To find the value of the current at the surface, use:

$$I = \int \vec{J}\cdot d\vec{a}$$

$$I_{surface} = \int_0^R 3ks\,(2\pi s\,ds)$$

$$I_{surface} = 6k\pi\int_0^R s^2 ds \rightarrow \underline{I_{surface} = 2\pi kR^3}$$

- The bound current is as follows:

$I_{bound} = \int \vec{K}_b \cdot d\vec{l}$

$I_{bound} = \int -KR^2 \, d\vec{l}$ $\quad$ $dl = 2\pi R$, circumference of the cylinder

$= -KR^2 \int d\vec{l}$

$= -KR^2 (2\pi R)$

$\underline{I_{bound} = -2\pi K R^3}$

- The total enclosed current is:

$I_{enc} = I_{surface} + I_{bound}$

$= 2\pi K R^3 - 2\pi K R^3$

$I_{enc} = 0$

So the field outside the cylinder is 0:

$$\boxed{B_{out} = 0}$$

- We can use $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ to find $\vec{B}_{inside}$:

$B(2\pi s) = \mu_0 (-2\pi K s^3)$

$\boxed{B_{inside} = \mu_0 K s^2}$ $\quad$ $\boxed{\text{or } \mu_0 \vec{M}}$

Scott Kovos

302 HW9

Problem 4

- An ∞ long cylinder radius R carries a "frozen in" magnetization parallel to the axis: $\vec{M} = ks\hat{z}$, k is constant s = distance from axis. There is 0 free current anywhere.
Find magnetic field inside & outside the cylinder using 2 different methods:
a) Locate all bound currents & calculate the field they produce.
b) Use Ampere's Law in the form of: $\oint \vec{H}\cdot d\vec{l} = I_{fenc}$ to find $\vec{H}$ then get $\vec{B}$ from: $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$

a) - To locate all the bound currents, begin with the volume bound current:

$\vec{J}_b = \vec{\nabla} \times \vec{M}$ w/ $\vec{M} = ks\hat{z}$

$\vec{J}_b = -\frac{\partial M_z}{\partial s}\hat{\phi}$ $\boxed{\vec{J}_b = -k\hat{\phi}}$

- Then surface bound current:

$\vec{K}_b = \vec{M} \times \hat{n}$ - $\hat{n}$ is in $\hat{s}$ direction. $\hat{\phi} \times \hat{s} = -\hat{z}$

$= ks\hat{\phi} \times \hat{n}$ - substitute R for s

$\vec{K}_b = -kR\hat{z}$

$\boxed{\text{Outside the cylinder } \vec{B} = 0}$

For inside the cylinder, need to find I_{enc}:

$I_1 = \oint \vec{J}_b \cdot d\vec{a}$ & $I_2 = \oint \vec{K}_b \cdot d\vec{l}$

$= \oint -k\hat{\phi}\cdot d\vec{a}$ $= \oint -kR\hat{z}\cdot d\vec{l}$

$= -kl(R-s)$ $= -kRl$

$I_{enc} = I_1 + I_2$

$I_{enc} = kls$ → $\oint B\cdot dl = \mu_0 I_{enc}$

$Bl = \mu_0 kls$

$\boxed{B_{outside} = \mu_0 ks\hat{z}}$

b) - Using $\oint \vec{H} \cdot d\vec{l} = I_{f\,enc}$

$\vec{H} = 0$ b/c there is no free current anywhere

$\vec{B}_{inside} = \mu_0 \vec{H} + \mu_0 \vec{M}$ (6.31)

- Substitute 0 for $\vec{H}$ and $K_s \hat{\phi}$ for $\vec{M}$

$$\boxed{\vec{B}_{inside} = \mu_0 K_s \hat{z}}$$

$\vec{B}_{outside}$, $\vec{M} = 0$

$$\boxed{\vec{B}_{outside} = 0}$$