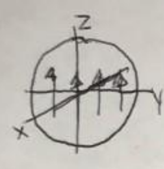
Scott Kobos Exam 3

Problem 1

- 3 objects carry a frozen-in magnetization as shown. Re-draw the object and its magnetization and sketch & label the bound currents that arise and write a sentence or two explaining how you determined them.

a) Sphere w/ uniform magnetization along z-axis.



Magnetization: ■
Bound Volume: ■
Bound Surface: ■

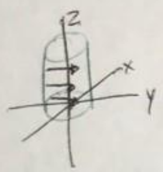
- part a) is basically Example 6.1 from the book.
- there is no curl, so the can be no bound volume current.

$\vec{\nabla} \times \vec{M} = 0$, $J_b = 0$.

- $K_b = \sin\theta\,\hat{\phi}$, which would just be lines around the surface.

* Math for $J_b = 0$ on seperate page

b)



Magnetization: ■
Bound Volume: ■
Bound Surface: ■

- The magnetization is only in the y-direction in cylindrical coordinates, $\hat{y} = \sin\phi\,\hat{s} + \cos\phi\,\hat{\phi}$

in cylindrical, $\hat{n}$ is $\hat{s}$

$\vec{M} = \hat{y}$

$$J_b = \vec{\nabla} \times (\sin\phi\,\hat{s} + \cos\phi\,\hat{\phi})$$

$$J_b = \cancel{(0-0)}\hat{s} + \cancel{(0-0)}\hat{\phi} + \frac{1}{s}\left(\frac{\partial}{\partial s}(s\cos\phi) - \frac{\partial}{\partial \phi}(\sin\phi)\right)\hat{z}$$

$$J_b = \frac{1}{s}(\cos\phi - \cos\phi)$$

$J_b = 0$, No bound volume current

$$K_b = \vec{M} \times \hat{n}$$

$$K_b = (\sin\phi\,\hat{s} + \cos\phi\,\hat{\phi}) \times \hat{s}$$

$$\underline{K_b = \cos\phi\,\hat{z}}$$

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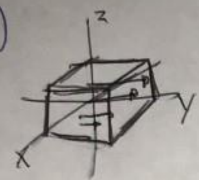
Problem 1 (continued)

c) 

Magnetization:
Bound Volume:
Bound Surface:

- The magnetization is in the y-direction, but dependent on z,

$$\vec{M} = M(z)\,\hat{y}$$

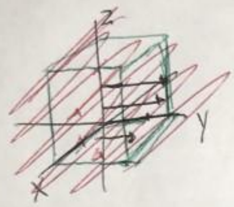
$$J_b = \vec{\nabla} \times \vec{M}$$

$$J_b = \vec{\nabla} \times M(z)\,\hat{y}$$

$$J_b = \left(-\frac{\partial M_y}{\partial z}\right)\hat{x} + \left(\frac{\partial M_y}{\partial x}\right)\hat{z}$$

$\ast$ $\frac{\partial M_y}{\partial x}\hat{z}$ must be 0 b/c the magnetization is symmetric (constant) along the x-axis

$$J_b = \frac{-\partial M_y}{\partial z}\,\hat{x}$$



~~the bound volume~~

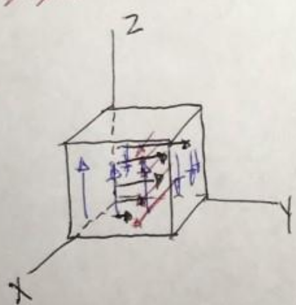
$$K_b = \vec{M} \times \hat{n}$$

$$K_b = M(z)\,\hat{y} \times \langle \hat{x}, \hat{y}, \hat{z} \rangle$$

$$K_b = \langle M_z, 0, M_z \rangle$$

$\ast$ $\hat{n}$ could be in $\hat{x}$, $\hat{y}$, or $\hat{z}$ depending on what surface you're looking at.

$\ast$ $\hat{y}$ portion would be 0
$\hat{y} \times \hat{y} = 0$



- My drawing is tough, I can't draw it the way I'm seeing it in my head.

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Scott Kuros Exam 3

Problem 1 (continued)

* math for why $J_b = 0$ for part a)

M is only in $\hat{z}$-direction, $\hat{z}$ in spherical coordinates is $\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$$J_b = \vec{\nabla} \times \vec{M}$$

$$= \underbrace{\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta V_\phi) - \frac{\partial V_\theta}{\partial\phi}\right)}_{0}\hat{r} + \underbrace{\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{\partial}{\partial r}(rV_\phi)\right)}_{0}\hat{\theta} + \frac{1}{r}\underbrace{\left(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial\theta}\right)}\hat{\phi}$$

b/c there's no V_ϕ & V_θ doesn't have ϕ in it

b/c there's no V_ϕ and V_r doesn't have ϕ in it.

$$\frac{1}{r}\left(\frac{\partial}{\partial r}(r\cdot -\sin\theta) - \frac{\partial}{\partial\theta}\cos\theta\right)\hat{\phi}$$

$$\frac{1}{r}\left(\frac{\partial}{\partial r}(-r\sin\theta) - -\sin\theta\right)\hat{\phi}$$

$$\frac{1}{r}(-\sin\theta - -\sin\theta)\hat{\phi}$$

$$\frac{1}{r}(0)\hat{\phi}$$

$$0$$

$$\underline{J_b = 0}$$

Scott Kobos Exam 3

Problem 2

- Paramagnetism and diamagnetism are the two temporary forms of magnetism that occur in materials. Briefly describe the physical cause at the atomic level in each case.

- For a material to be paramagnetic, it must have unpaired electrons that result in a net magnetic moment and attracts the material to a magnetic field. Also has a positive magnetic susceptibility, χ_m

- For a material to be diamagnetic, it must have paired electrons that result in no net magnetic moment and repels the material from a magnetic field. Also has a negative magnetic susceptibility, χ_m.

Scott Kurus Exam 3

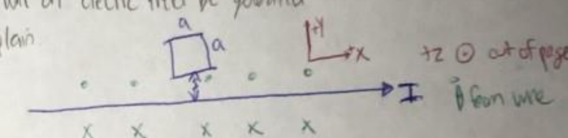
Problem 3

- Square loop of wire w/ sides a lies on a table a distance s from a very long straight wire which carries a current I as shown. Recall the $\vec{B}$ of a long thin wire is $\frac{\mu_0 I}{2\pi s}$.

Consider the following scenarios and a) identify whether a current is generated & in what direction. Justify w/ a sentence or two b) identify whether there is an induced magnetic field from the loop and its direction from the loop.

1. The loop is pulled away (upwards) from the wire at speed v.
2. The loop is pulled to the right at speed v
3. The loop is held stationary but the current increases over time @ $I(t) = I_0 + C\Delta t$ where C is a positive constant

c) For which of the scenarios in part a) will an electric field be generated even if the loop is not present? Explain



① Loop pulled up @ speed v:

a) Current: Yes, a current is generated by pulling the loop up at speed v. The current is in the counter clockwise direction.

b) magnetic field: yes, a magnetic field is induced from the loop and is in the $+\hat{z}$ (out of page) direction

- $\vec{B}$ decreases as it gets further away from the wire, decreasing the flux. The current flows through the loop in an effort to keep the flux constant. The direction of the $\vec{B}$ from the wire is $+\hat{z}$ due to RHR. The induced $\vec{B}$ is in the same direction as the initial $\vec{B}$ in this case b/c $\vec{B}$ is decreasing. This makes the current counterclockwise by Right Hand Rule.

② Loop pulled right @ speed v:

a) current: Yes, current is generated in the loop when the loop is pulled to the right @ speed v. This is essentially Faraday's Experiment 1 but in his experiment, $\vec{B}$ was into the page so I_{ind} was clockwise. In this case, $\vec{B}$ is out of the page, so I_{ind} is counterclockwise

b) magnetic field: There is no induced magnetic field in this scenario. $\vec{B}$ is dependent on distance from the wire, not along the wire. The $\vec{B}$ is constant, no ΔB, no $B_{induced}$.

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Problem 3 (continued)

3) Stationary loop, $I(t) = I_0 + c\Delta t$

a) Current: Yes, a current is generated when the loop is still but the current increases over time. It is in the clockwise direction. This example is Faradays Experiment 3

Flux increases, so a current is generated.

b) magnetic field: yes a magnetic field is induced. Current increases, so $B_{initial}$ increases, ΔB is in same direction as $B_{initial}$, & $\vec{B}_{induced}$ opposes. $\vec{B}_{induced}$ is in $-\hat{z}$ direction (into page) since it opposes the original field that is in the $+\hat{z}$ direction.

Current direction is CW by RHR from $\vec{B}_{induced}$

c) An electric field will be generated even if the loop is not present in Scenario 3. The increasing current creates a changing magnetic field and a changing magnetic field induces an electric field.

Scott Kovas Exam 3

Problem 4

- a thick wire radius a carries a free current I_f distributed along the wire's width such that $J_f = 6ks^2 \hat{z}$

a) Determine magnitude and direction of $\vec{H}$ for an arbitrary point inside the wire.

b) If the wire has $\vec{M} = -\frac{1}{6} ks^3 \hat{\phi}$, what is the magnitude and direction of $\vec{B}$ for an arbitrary point inside the wire.

c) Bonus: is the wire paramagnetic or diamagnetic?

a) Magnitude & direction of $\vec{H}$:

$$\vec{\nabla} \times \vec{H} = J_f \qquad J_f = 6ks^2 \hat{z}$$

- so $\vec{\nabla} \times \vec{H} = 6ks^2 \hat{z}$, which means $\vec{\nabla} \times \vec{H}$ will only have the z-component of the curl, $\frac{1}{s}\left(\frac{\partial}{\partial s}(sH_\phi)\right)\hat{z}$.

- $\frac{1}{s}\left(\frac{\partial}{\partial s}(sH_\phi)\right)\hat{z} = 6ks^2 \hat{z}$

$$\cancel{\int \frac{\partial}{\partial s}}(sH_\phi) = \int 6ks^3 \, ds$$

$$sH_\phi = 6k\left(\frac{1}{4}s^4\right)$$

$$sH_\phi = \frac{6ks^4}{4} \rightarrow \boxed{H_\phi = \frac{3ks^3}{2} \hat{\phi}}$$

b) $\vec{M} = -\frac{1}{6}ks^3 \hat{\phi}$ find magnitude & direction of $\vec{B}$:

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \rightarrow \vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$\vec{B} = \mu_0\left(\frac{3ks^3}{2}\hat{\phi} + -\frac{1ks^3}{6}\hat{\phi}\right)$$

$$\vec{B} = \mu_0\left(\frac{9ks^3}{6}\hat{\phi} - \frac{1ks^3}{6}\hat{\phi}\right)$$

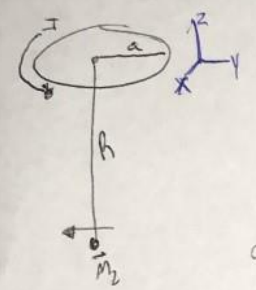
$$\vec{B} = \frac{8}{6}ks^3\mu_0 \hat{\phi} \qquad \boxed{\vec{B} = \frac{4ks^3\mu_0}{3}\hat{\phi}}$$

c) The wire is paramagnetic because $\vec{B}$ is parallel to $\vec{M}$.

Scott Kobos Exam3

Problem 5

- Consider the circular loop shown, which has a current I & radius a. Suppose another magnetic dipole, m_2, is placed a distance R below the loop as shown. Clearly mark axes: Determine:
  a) Force on m_2 from the loop
  b) Torque on m_2 from the loop
  c) The equilibrium position of the loop if it is free to rotate



- The magnetic field for a loop at a point R distance away on its z-axis is as follows:

$$B_2 = \frac{\mu_0 I}{2} \frac{a^2}{(a^2+R^2)^{3/2}} \; -\hat{z} \text{ in our case}$$

a) Force: $\vec{F} = \nabla(\vec{m}_2 \cdot \vec{B}_2)$

$\vec{m}_2 \cdot \vec{B}_2 = 0$ because they are orthogonal

So $\boxed{\vec{F} = 0}$

b) Torque: $\tau = \vec{m}_2 \times \vec{B}_2$ $\qquad \vec{m}_2 = m_2 \hat{y}$, $\vec{B}_2 = B \cdot -\hat{z}$

$\tau = m_2 \hat{y} \times B \, -\hat{z}$

$\tau = m_2 B \, \hat{x}$

$\tau = m_2 \left(\frac{\mu_0 I}{2} \frac{a^2}{(a^2+R^2)^{3/2}} \right) \hat{x}$

$$\boxed{\tau = \frac{\mu_0 I m_2}{2} \frac{a^2}{(a^2+R^2)^{3/2}} \hat{x}}$$

c) The equilibrium position will be when the loop is parallel (or antiparallel) to $\vec{m}_2$