Special Theory of Relativity I

PHYS 301: Analytical Mechanics

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Problem 1

Alex is riding in a car that has LEDs at each end. The car passes Jordan, who is standing on the ground 25 m from the car's path, with velocity 0.8 *c*. Flashes from the two LEDs reach Jordan simultaneously, and when he observes them, he observes one to be 3.75 m to his left, and the other to be 3.75 m to his right. According to Alex, how far apart are the LEDs, and what was the delay between their flashes?

Problem 2

The star Deneb goes supernova. Ten years later and 100 light-years away, as measured by astronomers in the galaxy, the star Mimosa explodes as well. An alien spacecraft passing through measures the distance between the two explosions as 180 light-years. What do the aliens measure as the time between explosions?

```
Clear[v]
```

```
Solve[180 == 1/(Sqrt[1 - v^2/1]) (100 - 10 v), v]
```

$$\left\{\left\{v \to -\frac{4}{5}\right\}, \left\{v \to \frac{56}{65}\right\}\right\}$$

$$v = \frac{1}{\left(\sqrt{\left(1 - \frac{\left(\left(\frac{4}{5} c\right)\right)^2}{c^2}\right)}\right)}$$

$$\frac{5}{3}$$

```
Simplify[t = 5/3 (10 - ((-4/5) (100))/1^2)]
```

```
150
```

Problem 3

Inside a linear accelerator, a beam of electrons moves at speed u_e and a beam of protons moves at speed u_p. Find formulas for the speed of the electrons relative to the protons if:

a. the beams are moving away from the origin along the same axis, and

b. the beams are moving away from the origin along perpendicular axes.

c. Apply both of your formulas for the case where $u_e = u_p = 0.99\,c$.

```
u = (.99 c + .99 c)/(1 - (.99 c) (-.99 c)/c^2)
```

```
0.999949 c
```

```
.99 c/Sin[45 Degree]
```

```
1.40007 c
```

Problem 4

Train 1 is 90 m long and travels westward at a speed of 0.6 c. Train 2 is 60 m long and travels eastward at a speed of 0.85 c.

a. How long does Train 2 appear to a passenger on Train 1?

```
Clear[u]
```

```
Clear[c]
```

```
u = (.85 c + .6 c)/((1 - (.85 c (-.6 c))/c^2))
```

```
0.960265 c
```

$$\gamma = \frac{1}{\left(\sqrt{\left(1 - \frac{\left(0.9602649006622517\, c\right)^2}{c^2}\right)}\right)}$$

```
3.58307
```

$$l = \frac{60}{3.58307}$$

```
16.7454
```

b. How long does Train 1 appear to a passenger on Train 2?

$$x = \frac{90}{3.58307}$$

```
25.1181
```

Problem 5

At what speed would one have to move so that light from one end of the visible spectrum is Doppler-shifted to the opposite end of the spectrum?

$$\text{Solve}\left[A == \left(\frac{\sqrt{\left(1 + \frac{v}{c}\right)}}{\sqrt{\left(1 - \frac{v}{c}\right)}}\right) B,\ v\right]$$

$$\left\{\left\{v \to \frac{(A^2 - B^2)\, c}{A^2 + B^2}\right\}\right\}$$

$$A = 7.985 * 10^{14};$$
$$B = 4.054 * 10^{14};$$
$$c = 3 * 10^{8};$$
$$v = \frac{(A^2 - B^2)\, c}{A^2 + B^2}$$

$$1.77038 \times 10^{8}$$