Measure, Plot, and Fit

Group 4
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Overview

Your default procedure whenever you plan or measure data should be to take multiple measurements, plot your results, then fit your data using your model. Today you will record two data sets, plot and fit and assess the results.

You are going to experimentally record the relationship between applied force and the stretch of a spring. Thrilling, I know, except you will do it efficiently and accurately. Use the flowcharts below to set-up, calibrate, and record data. You will repeat the procedure for two springs.

Experiment Plan

An ideal spring is one which obeys Hooke's Law

$$F = -k\Delta x \tag{1}$$

where a net force F will stretch the spring by a distance Δx. If our spring is hung vertically and a mass is attached to the bottom, we expect, via Newton's Second Law,

$$mg = k\Delta x. \tag{2}$$

Assuming the equilibrium position of the spring is the origin ($\Delta x = x$) and solving for the final position of the spring, we get

$$x = \frac{g}{m}k. \tag{3}$$

Thus, for an ideal spring, the spring's stretch should be proportional to the mass hung from it.

The experiment will hang a variety of masses from a vertically oriented spring and measure the change in the position of the end of the mass holder.

Experiment Details

The set-up and procedure can be found on the next page. Some additional details are included here.

- You will follow the set-up and procedure for each of the two types of springs, the thin one and the thick one. This means that you will produce one graph for each spring.
- For each spring, perform measurements at six different masses. Make sure that the spring is visibly stretched for every measurement. Note: The two springs will use a different series of masses.
- When using the motion sensor, you will record LOTS of measurements in quick succession. You will find an Excel sheet included with this assignment that will calculate the mean and standard error of a bunch of numbers. Copy the measurements from LoggerPro and paste them into the Excel file to quickly record the mean and standard error for the position of the mass. You will then want to copy these values into a separate spreadsheet or directly into MatLab to be graphed.

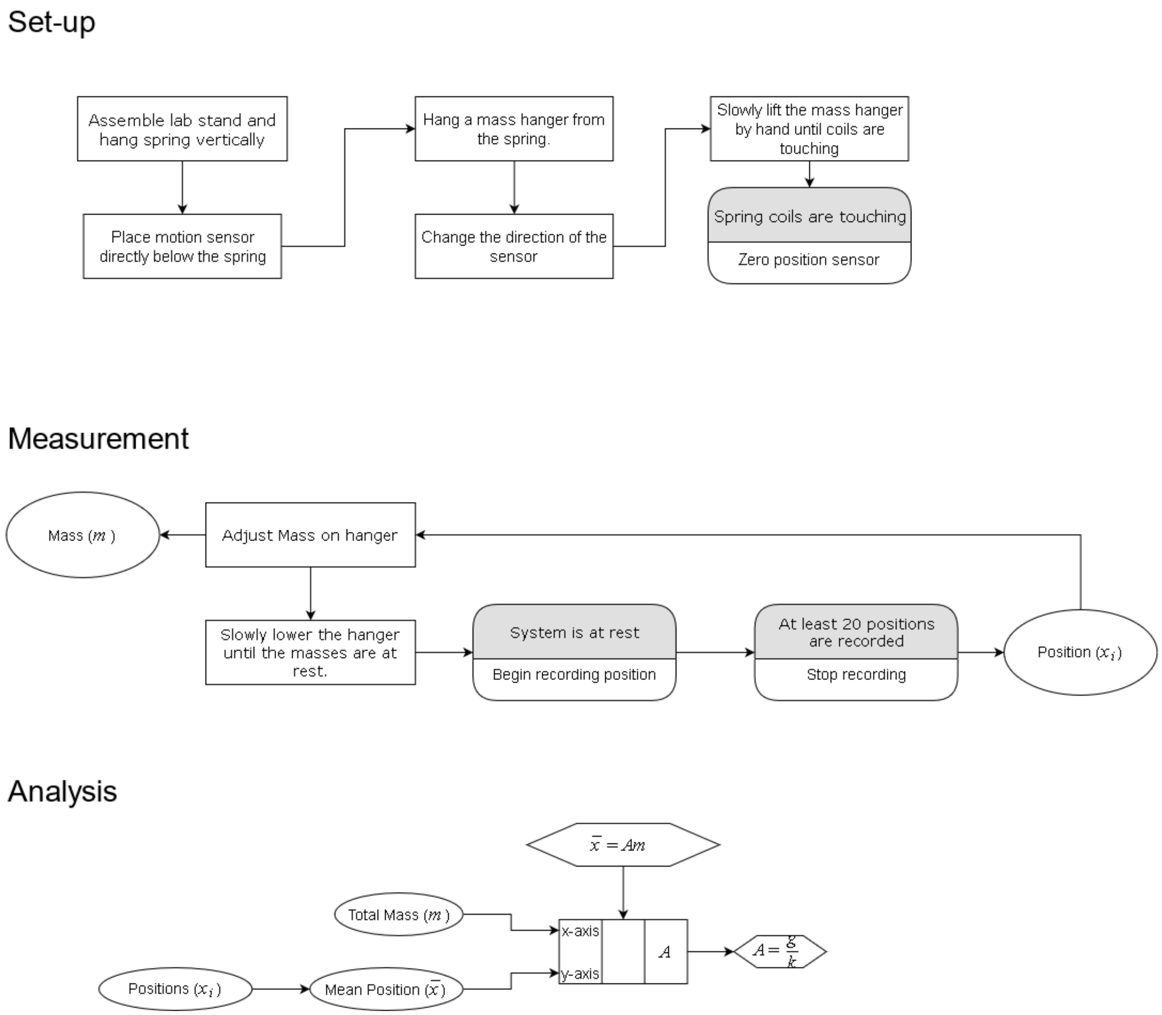

Figure 1: Flowcharts illustrating the setup, procedure, and analysis for measuring the spring constant of a single spring.

Fitting

Using Equation 3 as a template, determine your fit function using the minimum number of coefficients to reproduce the equation (Hint, you only need one). Write your fit function below. Refer to last class's instructions for an example of what this should look like.

$$y = c_1 x \tag{4}$$

Likewise, for each of your coefficients, write the equation describing how they relate to your model function:

$$c_1 = \frac{g}{k} \tag{5}$$

Apply this fit equation to each of your sets of measurements and include the graphs below.

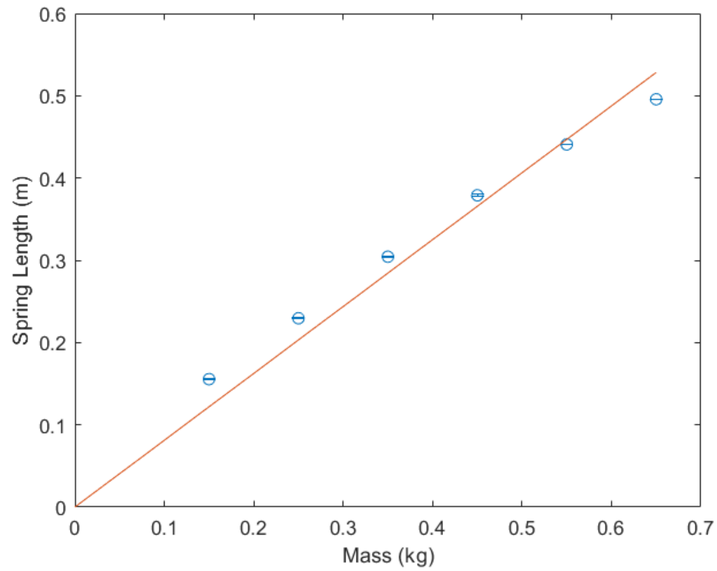

Figure 2: This graph shows the fit of the relationship between stretched length of the thin spring and the mass hung from the spring.

Write a paragraph discussing the quality of the fit for each spring. In it, include the value for the spring constant k for each spring.

For the thin spring in Figure 2, the linear model serves us well for fitting the data. While it doesn't cross through each point, it is pretty close to lining up well with the data points. From this model, we can calculate the value of the spring constant for the thing spring from Equation 5 by dividing the value for gravity the slope of the line, c_1. For the thin spring, the spring constant was found to be $12.0586 N/m$. The same can be done for the thick spring, whose spring constant was calculated to be $39.4208 N/m$. For the thick spring in Figure 3, the model isn't quite as well-fitting as it was for the thin spring. Crosses through two data points, but is further away from the first two points. This is expected for the thick spring, as a refitting is performed later in the assignment.

Making a New Model

Where the model falls short, we can propose an experimentally motivated model. Looking at the data for the stiff spring, the model CLEARLY falls short. Propose a change to the model that drastically improves the fit and write the fit function here.

$$y = c_1 + c_2 x \tag{6}$$

Use your new fit function from Equation 6 to fit your data. Include a graph and a table of coefficients below.

From your results, answer the following questions:

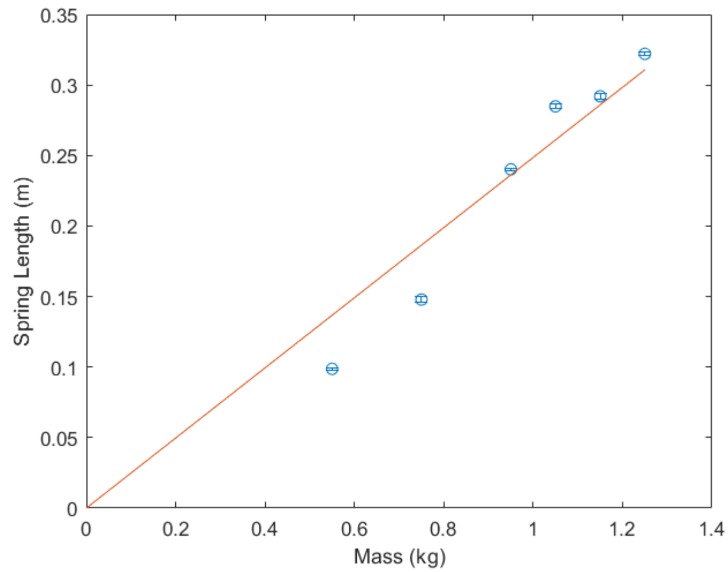

Figure 3: This graph shows the fit of the relationship between stretched length of the thick spring and the mass hung from the spring.

Coeff	Value
c_1	-0.087 $\pm .017 kg$
c_2	0.331 $\pm .017 m/kg$

Table 1: Coefficients using Equation 6 to fit data for the stiff spring.

1. How did you need to modify the original model to produce a good fit?

   The original model was modified by adding a coefficient in front of the original term of Equation 4 to shift the intercept.

2. What, physically, does this modification mean?

   Physically, this parameter means that we are accounting for the difference in strength of the springs. For the thin spring, it was weak enough to stretch with just about any mass suspended from it, but the thick spring was much stronger and needed a couple hundred grams suspended from it to even start to stretch. The addition of the coefficient takes that into account.

3. Based on your modifications, is the stiff spring sometimes, always, or never an ideal spring?

   Based on these modifications, the thick spring is sometimes an ideal spring, once it is loaded enough to being to stretch, but is not an ideal spring while it is loaded with insufficient weight to stretch it.

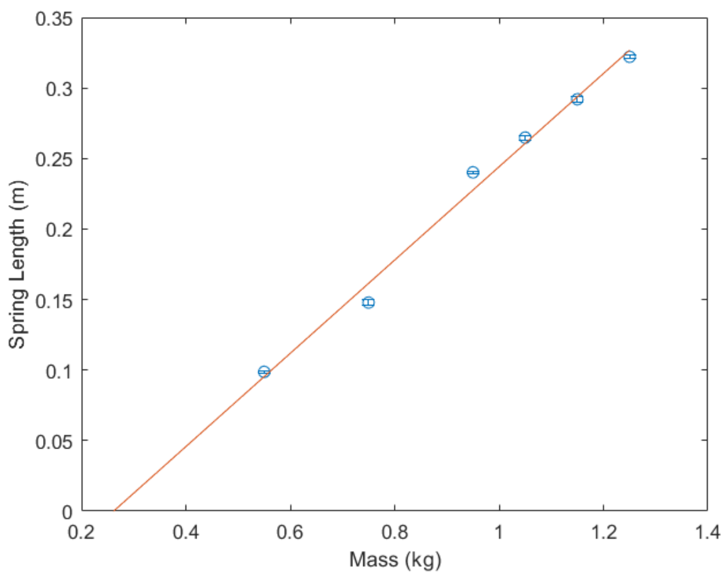

Figure 4: This graph shows the revised fit of the relationship between stretched length of the thick spring and the mass hung from the spring.