Problem 1 (continued)

b) using $\oint_S \vec{D} \cdot d\vec{a} = Q_{enc,free}$

- We know that there is no enclosed charge in $r<a$ and $r>b$
  So for both, $\vec{D}=0$

$$\boxed{\begin{array}{ll} \vec{D}=0 & r<a \\ \vec{D}=0 & r>b \end{array}}$$

- We already know $\vec{E}$ for $a<r<b$ so we can plug it into the equation: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ & we know $\vec{P} = \frac{k}{r}\hat{r}$

$$\vec{D} = \varepsilon_0 \left(\frac{-k}{r\varepsilon_0}\hat{r}\right) + \frac{k}{r}\hat{r} = 0 \qquad \boxed{\vec{D}=0 \text{ in } a<r<b}$$

- To get $\vec{E}$ from $\vec{D}=0$:

$$0 = \varepsilon_0 \vec{E} + \vec{P} \qquad \vec{E} = \frac{-\vec{P}}{\varepsilon_0}$$

$$\vec{E} = -\frac{\frac{k}{r}\hat{r}}{\varepsilon_0} \qquad \boxed{\vec{E} = -\frac{k}{r\varepsilon_0}\hat{r}}$$

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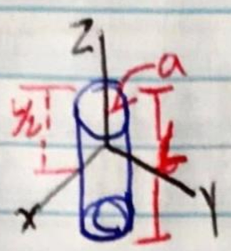
Problem 2

- Cylinder radius a, height L centered about z-axis has uniform polarization. $\vec{P} = P_0 \hat{z}$
  Find Electric field + Electric displacement everywhere on its axis



- There's a uniform polarization; so we know the volume polarization charge density $= 0$: $\rho_p = -\nabla \cdot \vec{P} = 0$
- σ_p @ $z = L/2 = P_0$ * b/c $\vec{P}$ is discontinuous @ upper + lower end
  σ_p @ $z = -L/2 = -P_0$ of cylinder. There is only $\vec{P}$ inside the cylinder
- We can basically treat this like 2 discs of charge, where the electric field for a disc of charge is: $\vec{E} = k\sigma 2\pi \left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$
  * $k = \frac{1}{4\pi\varepsilon_0}$ $\rightarrow$ $\vec{E} = \frac{\sigma 2\pi}{4\pi\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$
- Calculating $\vec{E}$ for each disc, top and bottom:

$$\vec{E}_{top} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{(z+\frac{L}{2})}{\sqrt{(z+\frac{L}{2})^2+a^2}}\right)$$

$$\vec{E}_{top} = \frac{P_0}{2\varepsilon_0}\left(1 - \frac{(z+\frac{L}{2})}{\sqrt{(z+\frac{L}{2})^2+a^2}}\right)$$

* z becomes $(z+\frac{L}{2})$ b/c we have to account that the cylinder is centered @ origin. $R \rightarrow a$ b/c a is the radius of our cylinder. + $\sigma_{top} = P_0$

- $\vec{E}_{bottom}$ is the same but $(z-\frac{L}{2})$ and $-P_0$

$$\vec{E}_{bottom} = \frac{-P_0}{2\varepsilon_0}\left(1 - \frac{(z-\frac{L}{2})}{\sqrt{(z-\frac{L}{2})^2+a^2}}\right)$$

- Add them together to get total $\vec{E}$:   * done in Mathematica

$$\boxed{\vec{E}_{total} = \frac{P}{2\varepsilon_0}\left(\frac{(z-\frac{L}{2})}{\sqrt{(z-\frac{L}{2})^2+a^2}} + \frac{(z+\frac{L}{2})}{\sqrt{(z+\frac{L}{2})^2+a^2}}\right)}$$

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Problem 2 (continued)

- To find $\vec{D}$, use equation $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$:

$\vec{D}_{total} = \varepsilon_0 (\vec{E}_{total}) + P_0$ *done in Mathematica w/c it was already there*

$$\boxed{\vec{D}_{total} = \frac{P_0}{2}\left(\frac{(z-\frac{L}{2})}{\sqrt{(z-\frac{L}{2})^2+a^2}} - \frac{(z+\frac{L}{2})}{\sqrt{(z+\frac{L}{2})^2+a^2}}\right)}$$

* $\vec{P}$ total is $P_{top} + P_{bottom} = P_0 + -P_0 = 0$
which is why the $\vec{P}$ term disappears.

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Problem 3

- Parallel Plate capacitor that are closely seperated in comparison to the size of the finite plates. Plates have a uniform charge density of $\pm\theta$

a) Find force of attraction b/w the plates assuming there's a linear dielectric ε_r b/w them.
b) What is the pressure on the surface of the plate?
c) If ε_r increases, how do a) + b) change?

a) To find the force of attraction, use the equation $F = q\vec{E}$
where q is the charge on one plate.
We can find q, the total charge on the plate by multiplying the charge density, θ by the area:
$q = \theta A$
- and the electric field inside a dielectric from a capacitor plate is: $E_{dielectric} = \frac{\theta}{\varepsilon}$ - where ε is the electric permeability of the dielectric

- From equation 4.34 in the book:
$$\varepsilon_r \equiv \frac{\varepsilon}{\varepsilon_0}, \quad \varepsilon = \varepsilon_r \varepsilon_0$$
So,
$$E_{dielectric} = \frac{\theta}{\varepsilon_r \varepsilon_0}$$
- Plug that into $\vec{F} = q\vec{E}$:
$$F = \theta A \left(\frac{\theta}{\varepsilon_r \varepsilon_0} \right) \qquad \boxed{F = \frac{\theta^2 A}{\varepsilon_r \varepsilon_0}}$$

b) Pressure is force over area so plug in $\vec{F}$ + divide by area:
$$P = \frac{\vec{F}}{A} \qquad P = \frac{\frac{\theta^2 A}{\varepsilon_r \varepsilon_0}}{A} \qquad \boxed{P = \frac{\theta^2}{\varepsilon_r \varepsilon_0}}$$

c) If ε_r increases:

a) $F = \dfrac{\sigma^2 A}{\varepsilon_r \varepsilon_0}$

b) $P = \dfrac{\sigma^2}{\varepsilon_r \varepsilon_0}$

If ε_r increases, the force will decrease.

if ε_r increases, pressure will decrease proportionally to the force

Scott [illegible]

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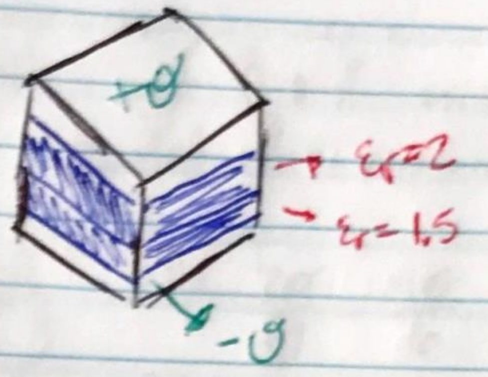
Problem 4



a) Find $\vec{D}$ in each slab:

The electric field in each slab is going to be the electric field - electric field from the polarization of the slabs which $= \dfrac{\sigma}{K\varepsilon_0}$

- The electric field inside the plates is twice the field due to each sheet (where the 2 comes from

E so we set:

$$E = \frac{\sigma}{2(K_1\varepsilon_0)} + \frac{\sigma}{2(K_2\varepsilon_0)} \qquad * \text{ w/ } K_1 = 2, \; K_2 = 1.5$$

$$E = \frac{\sigma}{2(2\varepsilon_0)} + \frac{\sigma}{2(1.5\varepsilon_0)} = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2} + \frac{1}{1.5}\right) = \frac{7\sigma}{12\varepsilon_0} = E$$

- The electric displacement $\vec{D}_{slab1} = K_1\varepsilon_0 \vec{E}$ and we can just substitute in what we know K_1 + E are:

$$\vec{D}_{slab1} = 2\varepsilon_0\left(\frac{7\sigma}{12\varepsilon_0}\right) \rightarrow \boxed{\vec{D}_{slab1} = \frac{7\sigma}{6}}$$

$$- \; \vec{D}_{slab2} = K_2\varepsilon_0 E = 1.5\varepsilon_0\left(\frac{7\sigma}{12\varepsilon_0}\right) \quad \boxed{\vec{D}_{slab2} = \frac{21\sigma}{24}}$$

) Find $\vec{E}$ in each slab:

- The electric field of a slab is $\vec{E} = \dfrac{\sigma}{\varepsilon}$ where $\varepsilon = \varepsilon_0\varepsilon_r$

$$\vec{E} = \frac{\sigma}{\varepsilon_0\varepsilon_r}$$

- substitute the known ε_r for each slab:

$$\boxed{\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}} \qquad \vec{E}_2 = \frac{\sigma}{1.5\varepsilon_0} \qquad \boxed{\vec{E}_2 = \frac{3\sigma}{2\varepsilon_0}}$$

Scott 1/d70s

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Problem 4 (continued)

c) Find $\vec{P}$ of each slab: w/ $\vec{D}$ & $\vec{E}$ we can use:

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ to find $\vec{P}$ in each slab:

$\vec{P} = \vec{D} - \varepsilon_0 \vec{E}$

$\vec{P}_1 = \frac{7\sigma}{6} - \varepsilon_0\left(\frac{\sigma}{2\varepsilon_0}\right)$ $\quad \vec{P}_1 = \frac{7\sigma}{6} - \frac{3\sigma}{6}$ $\quad \boxed{\vec{P}_1 = \frac{4\sigma}{6} = \frac{2\sigma}{3}}$

$\vec{P}_2 = \frac{21\sigma}{24} - \varepsilon_0\left(\frac{3\sigma}{2\varepsilon_0}\right)$ $\quad \vec{P}_2 = \frac{21\sigma}{24} - \frac{36\sigma}{24}$ $\quad \boxed{\vec{P}_2 = \frac{-15\sigma}{24}}$

d) The potential is $V = \vec{E}d$:

$V = (\vec{E}_1 + \vec{E}_2)d$ $\qquad * d = 2a$

$V = \left(\frac{\sigma}{2\varepsilon_0} + \frac{3\sigma}{2\varepsilon_0}\right)2a = \left(\frac{4\sigma}{2\varepsilon_0}\right)2a$ $\quad \boxed{\vec{V} = \frac{4\sigma a}{\varepsilon_0}}$

e) Location & amount of bound charge: $\sigma_b = \vec{P}\cdot\hat{n}$

$\vec{\sigma}_{b_1} = \vec{P}_1 \cdot \hat{n}$ $\quad$ * but $\vec{P}_1$ points down and $\hat{n}$ points up so change sign.

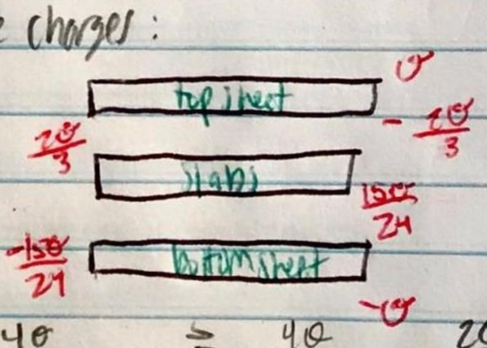
$\vec{\sigma}_{b_1} = -\vec{P}_1$ $\quad \boxed{\sigma_{b_1} = -\frac{2\sigma}{3} \text{ on top of slab 1}}$

$\sigma_{b2} = \vec{P}_2 \cdot \hat{n}$ $\quad$ * $\vec{P}_2$ points down & $\hat{n}$ points down so no change sign.

$\sigma_{b2} = \vec{P}_2$ $\quad \boxed{\sigma_{b2} = \frac{-15\sigma}{24} \text{ bottom of slab 2}}$

f) recalculate b) w/ bound & free charges:

a diagram is easiest to visualize:



Slab 1:

above: $\sigma - \frac{2\sigma}{3} = \frac{2\sigma}{3}$

below: $\frac{2\sigma}{3} - \frac{15\sigma}{24} + \frac{15\sigma}{24} - \sigma = \frac{-2\sigma}{3}$

total σ = top − bottom = $\frac{2\sigma}{3} + \frac{2\sigma}{3} = \frac{4\sigma}{3}$ $\qquad \vec{E}_1 = \frac{4\sigma}{6\varepsilon_0} = \frac{2\sigma}{3\varepsilon_0}$ $\qquad$ Somethings off

* get same value for $\vec{E}_2$, somethings off

Cott

Problem 5

- point charge, Q sitting @ center of a spherical, linear dielectric shell w/ permittivity ε & inner radius, a, outer radius, b.

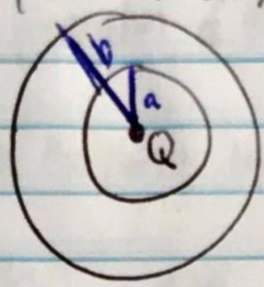
a) Use Gauss's Law to find $\vec{D}$ & $\vec{E}$ for all 3 regions: $r < a$, $a < r < b$, $r > b$

b) Find Polarization $\vec{P}$ & bound volume charge densities in the dielectric as well as the bound surface charge densities @ the two surfaces ($r = a$ & $r = b$)

c) What is the energy of this configuration?

Gauss' Law for $\vec{D}$

$\oint \vec{D} \cdot d\vec{a} = Q_{enc}$



a) To find the $\vec{D}$ for $r < a$

- substitute in the surface area of a sphere for $d\vec{a}$ and Q_{enc} is just Q, so: $\vec{D}(4\pi r^2) = Q$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2} \quad r < a}$$

For $a < r < b$

- Same goes for $a < r < b$, except r is $(b-a)$

$$\boxed{\vec{D} = \frac{Q}{4\pi (b-a)^2} \quad a < r < b}$$

For $r > b$

- Same for $r > b$, but r is $(r-b)$:

$$\boxed{\vec{D} = \frac{Q}{4\pi (r-b)^2} \quad r > b}$$