Hamilton's Principle

PHYS 301: Analytical Mechanics

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Problem 1

A particle with mass m is subject to the potential energy $U = -A\left(e^{k_x x} + e^{k_y y}\right)$.

a. What is the Lagrangian for the particle?

b. What are the equations of motion for the particle?

c. Use *Mathematica* to solve the equations of motion for the trajectory.

d. Use *Mathematica* to plot the trajectory, investigating different values of k_x and k_y.

Problem 2

A particle of mass m rests on a smooth plane. The plane is raised to an inclination angle θ at a constant rate of ω ($\theta = 0$ at $t = 0$), causing the particle to move down the plane. Determine the motion of the particle and plot it in *Mathematica*.

Problem 3

A particle is constrained to move (without friction) on a circular wire rotating with constant angular speed ω about its vertical diameter.

a. What is the Lagrangian in terms of the single degree of freedom θ?

b. What is the equation of motion for the particle?

c. Find the equilibrium positions of the particle, and calculate the frequency of small oscillations about these positions.

d. Extra credit: Find and interpret physically a critical angular velocity $\omega = \omega_c$ that divides the particle's motion into two distinct types.

Problem 4

Follow the process of TM Example 7.9 on page 250 to find the equations of motion, force of constraint, and angular acceleration of a hollow sphere rolling down an inclined plane.

Problem 1

```
ClearAll["Global`*"]
```

$L = .5\,m\,v^2 + .5\,m\,w^2 + A\left(e^{k\,x} + e^{z\,y}\right);$

a) The Lagrangian for the particle: $L = .5\,m\,v^2 + .5\,m\,w^2 + A\left(e^{k\,x} + e^{z\,y}\right)$

v represents x-dot, w represents y-dot.
z= k_y, $k = k_x$.

D[L, x]

$A\,e^{k\,x}\,k$

D[L, v]

$1.\,m\,v$

D[L, y]

$A\,e^{y\,z}\,z$

D[L, w]

$1.\,m\,w$

b) The ELEoM for the particle are

$x''[t] - \frac{\left(A\,k\,e^{k\,x[t]}\right)}{m} == 0$ and $y''[t] - \frac{\left(A\,k\,e^{z\,y[t]}\right)}{m} == 0$

Solved for x[t] in the DSolve below. The solution is the same for y[t], only changing the k's to z's.

$\text{DSolve}\left[\left\{x''[t] - \frac{\left(A\,k\,e^{k\,x[t]}\right)}{m} == 0,\ x[0] == 0,\ x'[0] == 0\right\},\ x[t],\ t\right]$

$\left\{\left\{x[t] \to \frac{\text{Log}\left[1 + \text{Tan}\left[\frac{k\,t}{\sqrt{2}}\right]^2\right]}{k}\right\}\right\}$

c) The trajectory of the particle is as follows:

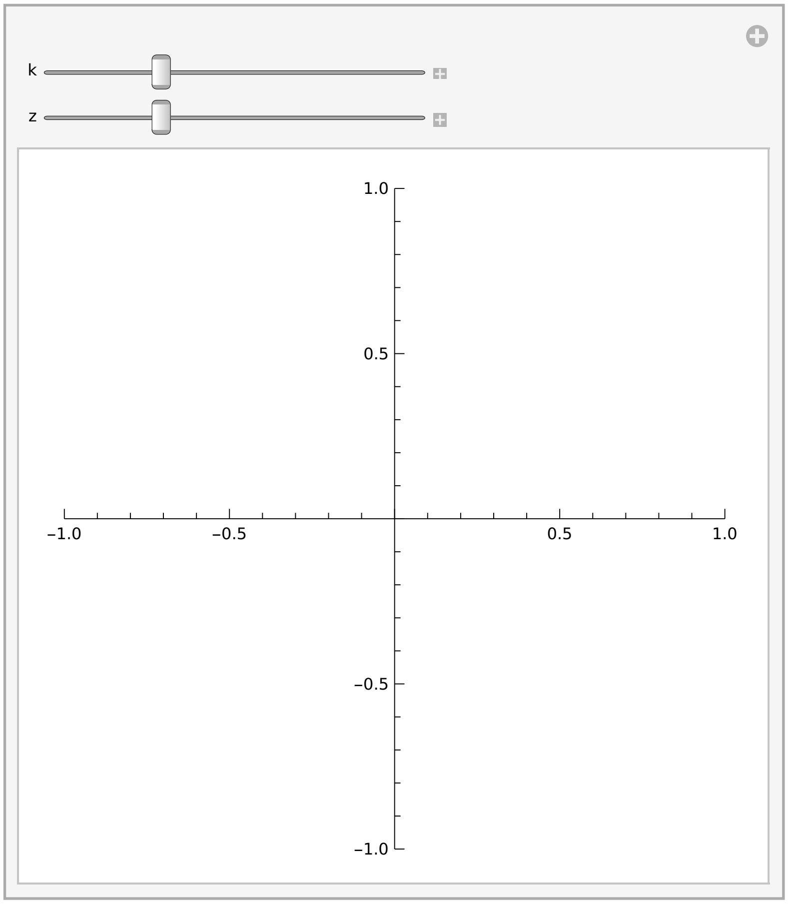
$x[t] = \frac{\text{Log}\left[1 + \text{Tan}\left[\frac{\sqrt{A}\,k\,t}{\sqrt{2}\,\sqrt{m}}\right]^2\right]}{k}$

$$y[t]=\frac{\text{Log}\left[1+\text{Tan}\left[\frac{\sqrt{A}\,z\,t}{\sqrt{2}\,\sqrt{m}}\right]^2\right]}{z}$$

d) Plotting: perfectly straight line when k=z

```
A = 1;
m = 1;
```

$$\text{Manipulate}\left[\text{ParametricPlot}\left[\left\{\frac{\text{Log}\left[1+\text{Tan}\left[\frac{\sqrt{A}\,k\,t}{\sqrt{2}\,\sqrt{m}}\right]^2\right]}{k},\ \frac{\text{Log}\left[1+\text{Tan}\left[\frac{\sqrt{A}\,z\,t}{\sqrt{2}\,\sqrt{m}}\right]^2\right]}{z}\right\},\ \{t,0,5\}\right],\ \{k,0,5\},\ \{z,0,5\}\right]$$



Power: Infinite expression $\frac{1}{0}$ encountered.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

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General: Further output of Power::infy will be suppressed during this calculation.

Infinity: Indeterminate expression 0. ComplexInfinity encountered.

General: Further output of Infinity::indet will be suppressed during this calculation.

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General: Further output of Infinity::indet will be suppressed during this calculation.

Problem 2

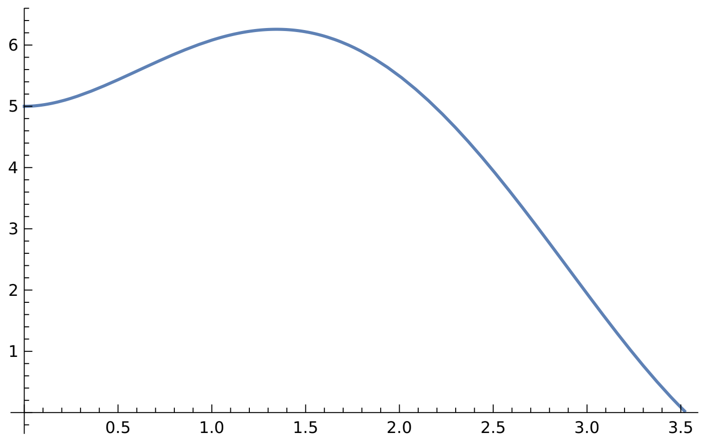
```
DSolve[{r''[t] == r ω^2 - g Sin[ω t], r[0] == l,  r'[0] == 0}, r[t], t]
```

DSolve: The function r appears with no arguments.

```
DSolve[{r''[t] == r ω^2 - g Sin[t ω], r[0] == l, r′[0] == 0}, r[t], t]
```

```
g = 9.8;
ω = 1;
l = 5;
```

$$\text{Plot}\left[\frac{1}{2}\left(l - \frac{g}{2\,\omega^2}\right)e^{\omega t} + \frac{1}{2}\left(l + \frac{g}{2\,\omega^2}\right)e^{-\omega t} + \frac{g}{2\,\omega^2}\,\text{Sin}[\omega\, t],\ \{t,\ 0,\ 3.52\}\right]$$



Problem 3

$$\text{D}\left[\frac{1}{2}\,m\,R^2\,\omega^2\left((\text{Sin}[\theta])^2\right) + m\,g\,R\,\text{Cos}[\theta],\ \theta\right]$$

$$-g\,m\,R\,\text{Sin}[\theta] + m\,R^2\,\omega^2\,\text{Cos}[\theta]\times\text{Sin}[\theta]$$

Problem 4

$$\text{Integrate}\left[\frac{2\,g\,\text{Sin}[\alpha]}{3\,R},\ t\right]$$

$$\frac{2\,g\,t\,\text{Sin}[\alpha]}{3\,R}$$

$$\textbf{Integrate}\left[\frac{2\,g\,t\,\text{Sin}[\alpha]}{3\,R},\ t\right]$$

$$\frac{g\,t^2\,\text{Sin}[\alpha]}{3\,R}$$

$$\textbf{Integrate}\left[\frac{2\,g\,\text{Sin}[\alpha]}{3},\ t\right]$$

$$\frac{2}{3}\,g\,t\,\text{Sin}[\alpha]$$

$$\textbf{Integrate}\left[\frac{2}{3}\,g\,t\,\text{Sin}[\alpha],\ t\right]$$

$$\frac{1}{3}\,g\,t^2\,\text{Sin}[\alpha]$$