Problem 3

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ClearAll["Global`*"]
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$$x[t\_] = \frac{\left(-F_0\left((\text{Sin}[\omega t])^2\right)\right)}{m\,\omega\left(\omega\,\text{Sin}[\omega\,t] - \omega_0\,\text{Cos}[\omega\,t]\right)} + A\,\text{Sin}[\omega_0\,t] + B\,\text{Cos}[\omega_0\,t]$$

$$B\,\text{Cos}[t\,\omega_0] + A\,\text{Sin}[t\,\omega_0] - \frac{\text{Sin}[\omega t]^2\,F_0}{m\,\omega\left(\omega\,\text{Sin}[t\,\omega] - \text{Cos}[t\,\omega]\,\omega_0\right)}$$

$$\text{Solve}\left[B\,\text{Cos}[t\,\omega_0] + A\,\text{Sin}[t\,\omega_0] - \frac{\text{Sin}[\omega t]^2\,F_0}{m\,\omega\left(\omega\,\text{Sin}[t\,\omega] - \text{Cos}[t\,\omega]\,\omega_0\right)} == 0,\ B\right] /.\ t \to 0$$

$$\left\{\left\{B \to -\frac{\text{Sin}[\omega t]^2\,F_0}{m\,\omega\,\omega_0}\right\}\right\}$$

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x'[t]
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$$A\,\text{Cos}[t\,\omega_0]\,\omega_0 - B\,\text{Sin}[t\,\omega_0]\,\omega_0 + \frac{\text{Sin}[\omega t]^2\,F_0\left(\omega^2\,\text{Cos}[t\,\omega] + \omega\,\text{Sin}[t\,\omega]\,\omega_0\right)}{m\,\omega\left(\omega\,\text{Sin}[t\,\omega] - \text{Cos}[t\,\omega]\,\omega_0\right)^2}$$

$$B = -\frac{\text{Sin}[\omega t]^2\,F_0}{m\,\omega\,\omega_0};$$

$$\text{Solve}\left[A\,\text{Cos}[t\,\omega_0]\,\omega_0 - B\,\text{Sin}[t\,\omega_0]\,\omega_0 + \frac{\text{Sin}[\omega t]^2\,F_0\left(\omega^2\,\text{Cos}[t\,\omega] + \omega\,\text{Sin}[t\,\omega]\,\omega_0\right)}{m\,\omega\left(\omega\,\text{Sin}[t\,\omega] - \text{Cos}[t\,\omega]\,\omega_0\right)^2} == 0,\ A\right] /.\ t \to 0$$

$$\left\{\left\{A \to -\frac{\omega\,\text{Sin}[\omega t]^2\,F_0}{m\,\omega_0^3}\right\}\right\}$$

$$A = -\frac{\omega\,\text{Sin}[\omega t]^2\,F_0}{m\,\omega_0^3};$$

$$y[t\_] = \frac{\left(-F_0\,((\text{Sin}[\omega\,t])^2)\right)}{m\,\omega\left(\omega\,\text{Sin}[\omega\,t] - \omega_0\,\text{Cos}[\omega\,t]\right)} + A\,\text{Sin}[\omega_0\,t] + B\,\text{Cos}[\omega_0\,t]$$

$$-\frac{\omega\,\text{Sin}[\omega t]^2\,\text{Sin}[t\,\omega_0]\,F_0}{m\,\omega_0^3} - \frac{\text{Cos}[t\,\omega_0]\,\text{Sin}[\omega t]^2\,F_0}{m\,\omega\,\omega_0} - \frac{\text{Sin}[\omega t]^2\,F_0}{m\,\omega\left(\omega\,\text{Sin}[t\,\omega] - \text{Cos}[t\,\omega]\,\omega_0\right)}$$

The solution x(t) is shown above as y[t_].

$$\text{FullSimplify}\left[-\frac{\omega\,\text{Sin}[\omega t]^2\,\text{Sin}[t\,\omega_0]\,F_0}{m\,\omega_0^3} - \frac{\text{Cos}[t\,\omega_0]\,\text{Sin}[\omega t]^2\,F_0}{m\,\omega\,\omega_0} - \frac{\text{Sin}[\omega t]^2\,F_0}{m\,\omega\left(\omega\,\text{Sin}[t\,\omega] - \text{Cos}[t\,\omega]\,\omega_0\right)}\right]$$

$$\frac{\text{Sin}[\omega t]^2\,F_0\left(\frac{1}{-\omega\,\text{Sin}[t\,\omega]+\text{Cos}[t\,\omega]\,\omega_0} - \frac{\omega^2\,\text{Sin}[t\,\omega_0]+\text{Cos}[t\,\omega_0]\,\omega_0^2}{\omega_0^3}\right)}{m\,\omega}$$

$$H = D\left[F\,\text{Sin}[\omega t]^2\left(\frac{1}{-\omega\,\text{Sin}[t\,\omega] + \text{Cos}[t\,\omega]\,\omega_0} - \frac{\omega^2\,\text{Sin}[t\,\omega_0] + \text{Cos}[t\,\omega_0]\,\omega_0^2}{\omega_0^3}\right), \omega\right]$$

$$F\left(-2\,\omega\,\text{Sin}[t] - \frac{-t\,\omega\,\text{Cos}[t\,\omega] - \text{Sin}[t\,\omega] - t\,\text{Sin}[t\,\omega]}{\left(\text{Cos}[t\,\omega] - \omega\,\text{Sin}[t\,\omega]\right)^2}\right)\text{Sin}[\omega t]^2$$

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ClearAll["Global`*"]
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$$\text{Limit}\left[\frac{F_0\left(-2\,\omega\,\text{Sin}[t] - \frac{-t\,\omega\,\text{Cos}[t\,\omega]-\text{Sin}[t\,\omega]-t\,\text{Sin}[t\,\omega]}{\left(\text{Cos}[t\,\omega]-\omega\,\text{Sin}[t\,\omega]\right)^2}\right)\text{Sin}[\omega t]^2}{m}, \omega \to \omega_0\right]$$

$$\frac{\left(-2\,\text{Sin}[t] - \frac{-t\,\text{Cos}[t]-\text{Sin}[t]-t\,\text{Sin}[t]}{(\text{Cos}[t]-\text{Sin}[t])^2}\right)\text{Sin}[\omega t]^2}{m}$$

```
m = 1;
ω = 1;
Plot[(-2 Sin[t] - (-t Cos[t] - Sin[t] - t Sin[t])/(Cos[t] - Sin[t])^2) Sin[ω t]^2/m, {t, 0, 5}]
```