Scott Vobos

302 HW8

Problem 1

a) determine magnetic field in all 4 regions:
① $s<a$ ② $a<s<b$ ③ $b<s<c$ ④ $s>c$

b) What should the relationship b/w I_1 & I_2 be in order to produce no magnetic field outside the cable? Rewrite the field in the other 3 regions under these circumstances in terms of just I_1.

* current density $\vec{J}$ is uniformly distributed through wire & shell.

a)

① $s<a$

- To find $\vec{B}$ inside the wire, we start w/ an Amperian loop inside the wire. See picture to the right →
- To begin, we need to find the enclosed current in the wire to put into Ampere's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$
- To find I_{enc} we start with the current density:

$\vec{J} = I_1/A$ — substitute πr^2 in for A:

$\vec{J} = I_1/\pi r^2$ * substitute a in for r.

$\vec{J} = I_1/\pi a^2$

- The expression for I_{enc} is as follows: $I_{enc} = J(A) = J(\pi r^2)$
Plug in J.

$$I_{enc} = \frac{I_1}{\pi a^2}(\pi r^2) = \frac{I_1 r^2}{a^2}$$

- Plug I_{enc} into Ampere's Law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0\left(\frac{I_1 r^2}{a^2}\right)$$

$$B = \frac{\mu_0 I_1 r^2}{2\pi r a^2} \rightarrow \boxed{B(s<a) = \frac{\mu_0 I_1 r}{2\pi a^2}}$$

② $a<s<b$

- Same as above but I_{enc} is just the total I_1 from within the wire: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_1$

$$\boxed{B(a<s<b) = \frac{\mu_0 I_1}{2\pi r}}$$

③ $b<s<c$

- For this scenario, draw an Amperian Loop inside the shell:

- For I_{enc}, it will be all of $+I_1$ that is in the wire, plus the part of I_2 that the loop encloses. Use the same method w/ current density as part a) ①.

$J = I/A$ $\quad A = \pi r^2$, use b for r

$J = I/\pi c^2$

$I_{enc} = J(A) = \frac{I_2}{\pi c^2}(\pi(r-b)^2) \rightarrow I_{enc} = \frac{I_2(r-b)^2}{c^2}$

- So $I_{enc} = I_1 + \frac{-I_2(r-b)^2}{c^2}$ * I_2 is opposite direction so it is −

- Plug that into Ampere's Law:

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$B\,2\pi r = \mu_0\left(I_1 - \frac{I_2(r-b)^2}{c^2}\right) \rightarrow \boxed{B = \frac{\mu_0\left(I_1 - \frac{I_2(r-b)^2}{c^2}\right)}{2\pi r}}$

④ $s>c$

- Same as part a) ② but loop is around whole wire & shell so

$I_{enc} = I_1 + I_2$ * I_2 is down so it's −

$I_{enc} = I_1 - I_2$

- Plug into Ampere's Law:

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$\boxed{B = \frac{\mu_0(I_1 - I_2)}{2\pi r}}$

Problem 1

b) To produce a magnetic field of 0 outside the coaxial cable, $|I_1|$ and $|I_2|$ must be equal.

w/ $|I_1|=|I_2|$, part a) ④ becomes:

$$B=\frac{\mu_0(I_1-I_1)}{2\pi r} \rightarrow \frac{\mu_0(0)}{2\pi r}=0$$

- rewriting all other scenarios w/ $I_1=I_2$ (which I will just call 'I')

① $B=\dfrac{\mu_0 I r}{2\pi a^2}$

② $B=\dfrac{\mu_0 I}{2\pi r}$

③ $B=\dfrac{\mu_0\left(I-\frac{I(r-b)^2}{c^2}\right)}{2\pi r}$

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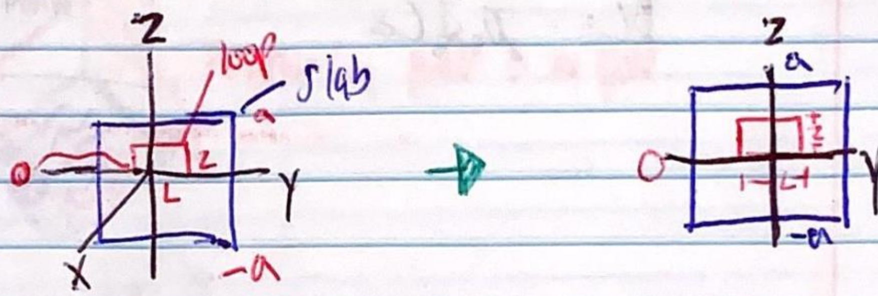
Problem 2

- thick slab from a to $-a$ in z (∞ in x & y) carries a uniform volume current of $\vec{J} = J_0 \hat{x}$
- Find $\vec{B}$ as a function of z inside and outside the slab.
- To chose your Amperian loop, make it a rectangle inside the slab w/ length L & height z:



- To find the magnetic field, you can use Ampere's Law:
  $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$
  but first need to find I_{enc} for inside the slab, above, and below the slab.
- The total enclosed charge is: $I_{enc} = \oint J \cdot da$
  but w/ a uniform volume current it is just:
  $I_{enc} = J_0$ (Area of Loop)
  $\underline{I_{enc} = J_0 (Lz)}$
- with I_{enc}, we can now use Ampere's Law:
  $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (J_0 L z)$
  $BL = \mu_0 J_0 L z$

$\boxed{\vec{B}_{inside} = \mu_0 J_0 z \hat{y} \quad \text{(direction of } -\hat{y} \text{ by RHR)}}$

- The process repeats for above and below the slab:

$I_{enc} = J_0 L z$ — but for above the slap the charge is only enclosed up to a so $\underline{I_{enc\,above} = J_0 L a}$
— same for below but $-a$ $\underline{I_{enc\,below} = -J_0 L a}$

- Above the slab Ampere's Law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 J_0 L a$$

$$BL = \mu_0 J_0 L a$$

$$\boxed{\vec{B}_{above} = -\mu_0 J_0 a \,\hat{y}}$$

(direction by RHR)

- Below the slab Ampere's Law:

$$\oint \vec{B}\cdot d\vec{l} = -\mu_0 J_0 L a$$

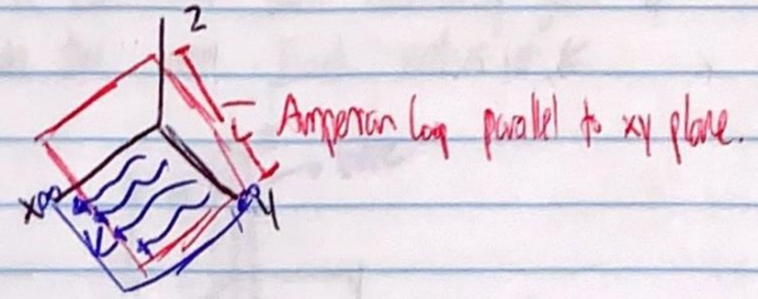
$$BL = -\mu_0 J_0 L a$$

$$\boxed{\vec{B}_{below} = \mu_0 J_0 a \,\hat{y}}$$

(RHR)

Problem 3

- a) Use Ampere's Law to find the magnetic field from an infinite uniform surface current $\vec{K} = K_0 \hat{x}$ flowing over the xy plane.
- b) Redo part a) by first calculating the magnetic vector potential & then deriving the magnetic field from it.



a)

- Ampere's Law is as follows: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

- Plugging in our uniform surface current to find I_{enc}:

$$I_{enc} = \oint \vec{K} \cdot d\vec{l} = K_0 L \quad \text{(since our current is uniform over the surface)}$$

- Back to Ampere's law w/ the new I_{enc}:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 K_0 L$$
$$2BL = \mu_0 K_0 L \rightarrow B = \frac{\mu_0 K_0}{2}$$

* The '2' in $2BL$ comes from the top & bottom of the sheet $(1+1)$.

- For top and bottom:

$$\boxed{B_{top} = -\frac{\mu_0 K_0}{2} \hat{y} \qquad B_{bottom} = \frac{\mu_0 K_0}{2} \hat{y}}$$

(direction from RHR)

b)

- For a surface current, the magnetic vector potential is defined as:

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{K}}{r} \, da'$$

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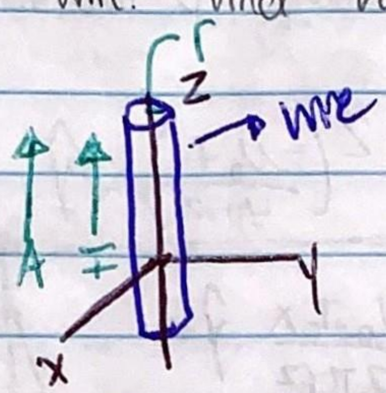
Problem 4

- Round wire radius R carries current I uniformly distributed over the cross-section of the wire. Let the axis of the wire be the z axis w/ $\hat{z}$ direction of current. Show that a vector potential of the form:

$$\vec{A} = k(x^2+y^2)\hat{z}$$

where k is a constant will correctly give $\vec{B}$ @ all points in the wire. Find value of k.



- The magnetic field from the magnetic potential can be found using the equation: $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

- Plug in $\vec{A}$

$$\vec{B} = \vec{\nabla} \times k(x^2+y^2)\hat{z}$$

$$\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & k(x^2+y^2) \end{bmatrix}$$

$$\vec{B} = 2ky\,\hat{x} - 2kx\,\hat{y}$$

- From Ampere's Law the magnetic field from a current carrying wire is:

$$\vec{B} = \frac{\mu_0 I(r)}{2\pi r}\hat{\theta}$$

- For the unit vector $\hat{\theta}$ the value is:

$$\hat{\theta} = \frac{-y}{r}\hat{x} + \frac{x}{r}\hat{y}$$

- Plug $\hat{\theta}$ into $\vec{B}$:

$$\vec{B} = -\frac{\mu_0 I y}{2\pi r^2}\hat{x} + \frac{\mu_0 I x}{2\pi r^2}\hat{y}$$

- Compare to $\vec{B}$ above:

$$2ky\,\hat{x} - 2kx\,\hat{y} = -\frac{\mu_0 I y}{2\pi r^2}\hat{x} + \frac{\mu_0 I x}{2\pi r^2}\hat{y}$$

- If K is plugged back into $\vec{B}$ found from $\vec{B} = \vec{\nabla} \times \vec{A}$:

$$\vec{B} = 2Ky\,\hat{x} - 2Kx\,\hat{y}$$

$$\vec{B} = 2\left(\frac{-\mu_0 I}{4\pi r^2}\right)y\,\hat{x} - 2\left(\frac{-\mu_0 I}{4\pi r^2}\right)x\,\hat{y}$$

$$\boxed{\vec{B} = -\frac{\mu_0 I y}{2\pi r^2}\,\hat{x} + \frac{\mu_0 I x}{2\pi r^2}\,\hat{y}}$$

which matches $\vec{B}$ from Amperes Law.

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Problem 5

- Circular loop of wire, radius R, lies in xy plane centered @ origin & carries current I running counterclockwise as viewed from the $+z$ axis.

a) What is the dipole moment?

b) What is the (approximate) magnetic field far from the origin?

c) Show that, for points on the z-axis your answer is consistent w/ the exact field:

$$B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}} \quad \text{when } z \gg R$$

a) - The magnetic dipole moment is:

$$\vec{m} = I \int d\vec{a} = IA$$

- The area of our loop is that of a circle, πR^2

so m is:

$$\boxed{\vec{m} = I\pi R^2 \hat{z}} \quad \text{(direction by RHR)}$$

b) The magnetic field of a dipole is:

$$\vec{B}_{dip}(r) = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

Plugging in m from part a):

$$\vec{B}_{dip}(r) = \frac{\mu_0 I \pi R^2}{4\pi r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

$$\boxed{\vec{B}_{dip}(r) = \frac{\mu_0 I R^2}{4 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$$

c) - When $z \gg R$ above the loop, $\theta = 0°$ & $r = z$ + $\hat{r} = \hat{z}$ * r is distance away from reference point to field.

plug them in: $\vec{B}_{dip}(r) = \frac{\mu_0 I R^2}{4 z^3} 2\hat{z}$ * $\cos 0 = 1$ $\sin 0 = 0$

- for below, $\theta = 180°$ & $r = z$ + $\hat{r} = -\hat{z}$ * $\cos 180 = -1$ $\sin 180 = 0$

$$\vec{B}_{dip}(r) = -\frac{\mu_0 I R^2}{4 z^3} 2\hat{z}$$

- Simplify ar B:

$$\bar{B}_{dip}(r) = \frac{\mu_0 I R^2}{2z^3}\hat{z}$$

and compare to:

$$B(z) = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$$

as $z \gg R$:

$$B(z) = \frac{\mu_0 I R^2}{2}\left(\frac{1}{(R^2+z^2)^{3/2}}\right) \quad R \to 0$$

$$B(z) = \frac{\mu_0 I R^2}{2}\left(\frac{1}{(0+z^2)^{3/2}}\right)$$

$$\underline{B_z = \frac{\mu_0 I R^2}{2z^3}} \quad \checkmark$$