Multiple Integrals I

PHYS 310 : Mathematical Methods in Physics

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Problem 1

A cube with side length 2 has a mass density proportional to the square of the distance from the center, i.e. $\rho(x, y, z) = \rho\, r^2$, where $r \equiv \sqrt{x^2 + y^2 + z^2}$. Find the total mass of the cube in terms of ρ.

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} p\left(x^2 + y^2 + z^2\right) dz\, dy\, dx$$

$$8\, p$$

Problem 2

A partially silvered mirror covers the square areas with vertices at $(\pm 1, \pm 1)$. The fraction of incident light which it reflects at (x, y) is $(x - y)^4 / 16$. Assuming a uniform intensity of incident light, find the fraction of the light reflected.

$$\int_{-1}^{1}\int_{-1}^{1} (x - y)^4 / 16\, dy\, dx$$

$$\frac{4}{15}$$

Problem 3

A rod of length l has linear mass density proportional to distance from the center of the rod. Find the moment of inertia about its center in terms of its length and total mass.

$$2 \int_{0}^{L/2} x^3\, \rho\, dx$$

$$\frac{L^4 \rho}{32}$$

$$2 \int_0^{L/2} \rho\, x\, dx$$

$$\frac{L^2 \rho}{4}$$

Problem 4

A right circular cylinder (i.e., a “normal” cylinder) has radius 3 m and height 4 m. Using integrals, find numerical values for its

a. volume,

b. mass if its density is $\rho(\vec{r}) = \frac{1}{20} \cos^2 \phi$ in appropriate units, and

c. moment of inertia about the cylinder’s axis.

$$\int_0^3 \int_0^4 \int_0^{2\pi} s\, d\Phi\, dz\, ds$$

$$36\,\pi$$

$$\int_0^3 \int_0^4 \int_0^{2\pi} s\, \frac{1}{20}\, \mathrm{Cos}\,[\phi]^2\, d\phi\, dz\, ds$$

$$\frac{9\,\pi}{10}$$

$$\int_0^3 \int_0^4 \int_0^{2\pi} s\, \frac{1}{20}\, \mathrm{Cos}[\phi]^2 \left(.5\, s^2\right) d\phi\, dz\, ds$$

6.36173

Syntax: Incomplete expression; more input is needed .