Universality of Hooke's Law in Tension Springs

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I. INTRODUCTION

Springs are a common component in mechanical systems and perform a wide variety of functions. Tension springs are often used to regulate flow through gates and compression springs can be used to keep interfaces in contact through motion. Given the ubiquity and range of uses, it is valuable to experimentally verify the basic principle governing springs: Hooke's Law. In this paper, we will determine the degree to which Hooke's law is a valid description of two different tension springs.

II. THEORY

An ideal spring is one which obeys Hooke's Law

$$F = -k\Delta x \tag{1}$$

where a net force F will stretch the spring by a distance Δx. If a spring is hung vertically and an object with a mass m is attached to the bottom, the gravitational force ($F^G = mg$) will put the spring in tension. Once in equilibrium, Newton's Second Law states that

$$mg = k\Delta x. \tag{2}$$

Assuming the equilibrium position of the spring is the origin ($\Delta x = x$) and solving for the final position of the spring, we get

$$x = \frac{g}{k}m. \tag{3}$$

Thus, for an ideal spring, the spring's stretch should be proportional to the mass used to keep it in tension.

III. EXPERIMENT

To evaluate this model, two springs will be tested. For each spring, an array of weights with varying mass will be hung and the stretch of the spring will be determined using an ultrasonic position sensor. The relationship between the hung mass and spring stretch is expected to be linear following Equation 3.

The spring will be hung in the vertical orientation from a lab stand. Placing it on a threaded connector will keep it from sliding off. A mass holder will be hung from the bottom of the spring and the ultrasonic position sensor will be placed on the surface directly underneath.

Before any measurements are performed, the sensor needs to be calibrated for each spring. The zero-mass position is found by lifting the mass holder by the stem until the spring coils are closed. The length of the spring's stretch is the difference between measured positions and the zero-mass position.

Once calibrated, the mass on the hanger can be adjusted and the hanger slowly lowered until it is in equilibrium. For each hung mass value, the ultrasonic position sensor is used to record at the position twenty times. Means and standard error are calculated and reported.

The ranges of hung mass values are chosen so that the smallest produces clearly visible stretching of the spring coils and the largest is less than 1-kg.

IV. ANALYSIS

The data from each spring, thick and thin, is compared to Hooke's Law using Equation 3 to determine fitness, using the model:

$$x = c_1 m. \tag{4}$$

With parameter c_1 defined as:

$$c_1 = \frac{g}{k}. \tag{5}$$

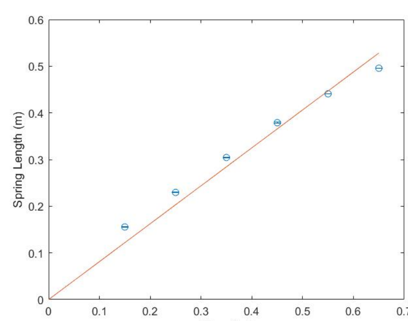

FIG. 1. Thin spring comparison of mass vs. length with fitness.

In Figure 1 the stretch of a spring as a function of the mass hung from it shows the fitness of the linear model to the data. The mass was hung from the spring and released above the position sensor, allowing the position sensor to track the oscillation of the spring, allowing for the calculation of the mean position and standard error. While the fit is not perfect, by visual inspection it can be seen that the data is linear in form, but strays from the line of fit due to error that could arise from the balance of the lab stand, the possibility of an imperfect release of the mass above the position sensor, or other sources of error. The same process is used to produce the data for the thick spring as follows:

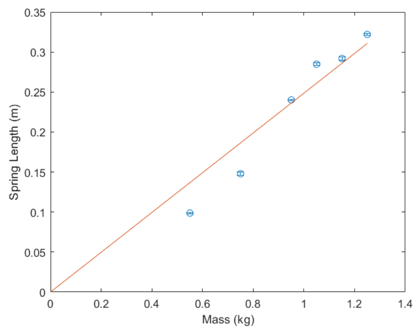

FIG. 2. Thick spring comparison of mass vs. length with fitness.

From Figure 2 we can see that the data does not accurately fit the linear model presented by Equation 3. This is due to the ability of the thick spring to resist stretching at light masses due to its' higher spring constant. To properly fit the data to the thick spring, an additional coefficient, c_2, must be added to account for the threshold mass to initiate stretch in the thick spring:

$$x = c_2 + c_1 m \tag{6}$$

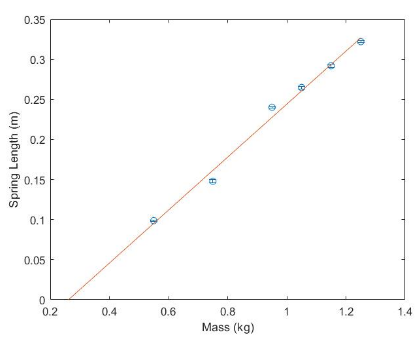

FIG. 3. Thick spring comparison of mass vs. length with fitness to updated model.

With the inclusion of the extra coefficient to account for the stronger spring constant of the thick spring and its' resistance to stretching at low masses, the updated model fits much better.

The values of the coefficients in the models for each spring were found to be the following

Spring	$c_1 \frac{m^2}{Ns^2}$	c_2
Thick	$-.1 \pm .01$	$.3 \pm .02$
Thin	$.8 \pm .02$	N/A

TABLE I. Summary of coefficient values

Using the values of the coefficients, the spring constants, k, of the springs were calculated

Spring	k (N/m)
Thick	39.4 ±.01
Thin	12.1 ±.01

TABLE II. Spring constant values

V. CONCLUSION

The Hooke's Law model is found to be applicable only to springs with small spring constants where the initial mass needed to stretch the spring is negligible. If the spring has a large enough spring constant, an additional coefficient must be added to accurately fit the data from that spring.