Motion in a Noninertial Reference Frame

PHYS 301: Analytical Mechanics

Scott Kobos

Problem 1

Calculate the numerical values of the accelerations experienced by objects near the surface of Earth due to:
a. Earth’s gravity,
b. Earth's rotation about its own axis, and
c. Earth's revolution around the Sun.

Problem 2

A bucket of water is set spinning about its symmetry axis. Two forces act on a small piece of water on the surface as the bucket spins: gravity and a pressure gradient within the water. Use these forces to determine the shape of the water in the bucket. (*Hints:* Draw a free-body diagram. You’re looking to define the direction of the pressure gradient force, which defines the shape of the water’s surface. Your answer will be an equation in cylindrical coordinates for *z* in terms of *s*.)

```
Clear[w]
Clear[g]
```

```
Integrate[(w^2 s)/g, s]
```

$$\frac{s^2 w^2}{2 g}$$

Problem 3

If a particle is projected vertically upward to a height h above a point on Earth's surface at a northern latitude λ, show that it strikes the ground at a point $\frac{4}{3}\,\omega\cos\lambda\,\sqrt{8\,h^3/g}$ to the west. (Neglect air resistance, and consider only small vertical heights.) The latitude for Conway, SC is $\lambda = 30\,°50'\,17''$.

Use precise values of g and ω_{Earth} to find what speed a projectile would have to be launched at for the deviation to be one meter.

```
Integrate[-2 w Cos[λ] (v - g t), t]
```

$$2\left(\frac{g\,t^2}{2} - t\,v\right) w\,\mathrm{Cos}[\lambda]$$

```
Integrate[2 ((g t^2)/2 - t v) w Cos[λ], t]
```

$$\left(\frac{g\,t^3}{3} - t^2\,v\right) w\,\mathrm{Cos}[\lambda]$$

```
Solve[0 == v t - 1/2 g t^2, t]
```

$$\left\{\{t \to 0\}, \left\{t \to \frac{2\,v}{g}\right\}\right\}$$

```
Clear[v]
t = (2 v)/g;
y = ((g t^3)/3 - t^2 v) w Cos[λ]
```

$$-\frac{4\,v^3\,w\,\mathrm{Cos}[\lambda]}{3\,g^2}$$

```
FullSimplify[(√(2 g h))^3]
```

$$2\sqrt{2}\,(g\,h)^{3/2}$$

```
(9.780327 (1 + .0053024 ((Sin[30 Degree])^2) - .0000058 ((Sin[2 * 30 Degree])^2))) + ((-3.086 * 10^-6) (50))
```

9.79309

```
Solve[1 == (4 v^3 (7.3 * 10^-5) Cos[30 Degree])/(3 (9.793094967104894)^2), v]
```

$$\{\{v \to -52.1978 - 90.4093\,i\}, \{v \to -52.1978 + 90.4093\,i\}, \{v \to 104.396\}\}$$

Problem 4

A British warship fires a projectile due south near the Falkland Islands during World War I at latitude 50°S. If the shells are fired at 37° elevation with a speed of 800 m/s, by how much do the shells miss their target and in what direction? Ignore air resistance.

```
Clear[t]
```

```
Integrate[2 ((-w v Cos[θ] × Sin[ϕ]) - ((v Sin[θ] - g t) (-w Cos[ϕ]))), t]
```

$$-g\, t^2\, w \,\mathrm{Cos}[\phi] + 2\, t\, v\, w\, \mathrm{Sin}[\theta - \phi]$$

```
Integrate[-g t^2 w Cos[ϕ] + 2 t v w Sin[θ - ϕ], t]
```

$$-\frac{1}{3}\, g\, t^3\, w\, \mathrm{Cos}[\phi] + t^2\, v\, w\, \mathrm{Sin}[\theta - \phi]$$

```
ϕ = 50 Degree;
θ = 37 Degree;
v = 800;
g = 9.8;
w = 7.3 * 10^-5;
```

$$\mathrm{FullSimplify}\left[-\frac{1}{3}\, g \left(\frac{(2\, v\, \mathrm{Sin}[\theta])}{g}\right)^3 w\, \mathrm{Cos}[\phi] + \left(\frac{(2\, v\, \mathrm{Sin}[\theta])}{g}\right)^2 v\, w\, \mathrm{Sin}[\theta - \phi]\right]$$

-272.228

Problem 5

Consult TM example 10.2 in the book. Reproduce TM figure 10-4, but use **Manipulate[]** to control the initial speed v_0 and initial angle θ. Find which initial velocity will make the puck motionless in the fixed system and show it on your figure. (*Hint:* What shape does this make in the rotating system?) What interesting thing happens for $v_0 = 0.512$ and $\theta = \pi/4$ (as measured from the $+x$-axis)?

DSolve equation 10.27 in book man. par. plot t0, 100

```
Clear["Global`*"]
```

```
ω = {0, 0, 1};
R = 1;
r = {-.5 R, 0, 0};
v = {r Cos[θ], r Sin[θ], 0};
```

```
a = Cross[-ω , Cross[ω, {rx, ry, 0}]] - 2 (Cross[ ω, {vx, vy, 0}])
{rx + 2 vy, ry - 2 vx, 0}
```

```
Simplify[DSolve[ {x''[t] == x[t] + 2 y'[t], y''[t] == y[t] - 2 x'[t],
    x[0] == -.5, y[0] == 0, x'[0] == v, y'[0] == q}, {x[t], y[t]}, t]]
{{x[t] → e^(-i t)
     (-0.25 + e^(2 i t) (-0.25 + t ((0. + 0.25 i) - (0. + 0.5 i) q + 0.5 v)) + t ((0. - 0.25 i) + (0. + 0.5 i) q + 0.5 v)),
  y[t] → e^(-i t) ((0. + 0.25 i) + e^(2 i t) ((0. - 0.25 i) + t (-0.25 + 0.5 q + (0. + 0.5 i) v)) +
       t (-0.25 + 0.5 q - (0. + 0.5 i) v))}}
```

```
x[t_] = Re[e^(-i t)
    (-0.25 + e^(2 i t) (-0.25 + t ((0. + 0.25 i) - (0. + 0.5 i) q + 0.5 v)) + t ((0. - 0.25 i) + (0. + 0.5 i) q + 0.5 v))]
Re[e^(-i t) (-0.25 + e^(2 i t) (-0.25 + t ((0. + 0.25 i) - (0. + 0.5 i) q + 0.5 v)) + t ((0. - 0.25 i) + (0. + 0.5 i) q + 0.5 v))]
```

```
y[t_] = Re[e^(-i t)
    ((0. + 0.25 i) + e^(2 i t) ((0. - 0.25 i) + t (-0.25 + 0.5 q + (0. + 0.5 i) v)) + t (-0.25 + 0.5 q - (0. + 0.5 i) v))]
Re[e^(-i t) ((0. + 0.25 i) + e^(2 i t) ((0. - 0.25 i) + t (-0.25 + 0.5 q + (0. + 0.5 i) v)) + t (-0.25 + 0.5 q - (0. + 0.5 i) v))]
```

```
Manipulate[ v = i Cos[θ]; q = i Sin[θ];
 ParametricPlot[{x[t], y[t]}, {t, 0, 10}, PlotRange → {{-5, 5}, {-5, 5}}], {i, 0, 5}, {θ, 0, 2 π}]
```