Multiple Integrals II; Vector Calculus I

PHYS 310 : Mathematical Methods in Physics

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Problem 1

Do the following problem first in spherical coordinates, then in cylindrical coordinates.

Write a triple integral for the volume inside the cone $z^2 = x^2 + y^2$ and between the planes $z = 1$ and $z = 2$, then evaluate it.

```
V = ∫_0^(2 π) ∫_0^(π/4) ∫_Sec[θ]^(2 Sec[θ]) r^2 Sin[θ] dr dθ dϕ
```

$$\frac{7\,\pi}{3}$$

```
V = ∫_1^2 ∫_0^(2 π) ∫_0^z s ds dϕ dz
```

$$\frac{7\,\pi}{3}$$

Problem 2

A particle follows the trajectory $\vec{r}(t) = t^2\,\hat{x} - 2\,t\,\hat{y} + \left(t^2 + 2\,t\right)\hat{z}$, where lengths are in meters and times are in seconds. **a.** At what time does the particle pass through the point $(4, -4, 8)$? What is its **b.** velocity and **c.** acceleration vectors at this time?

```
r⃗(t) = (4)^2 - 2 (-4) + (8^2 + 2 (8))
```

```
104
```

```
v (t) = 2 (4) - 2 + (2 (8) + 2)
```

Set: Tag Times in t v is Protected.

```
24
```

```
a[t] = 2 + 2
```

Set: Tag Times in a t is Protected.

4

Problem 3

A particle travels in a counterclockwise horizontal circle of radius $\frac{1}{2}$m in the $z = 2$ m plane. Find the particle's **a.** angular velocity vector, **b.** angular momentum vector, and **c.** acceleration vector about the origin when the particle is above the $+x$-axis if its mass is 1 kg and its speed is 2 m/s. Do this problem in Cartesean coordinates (*Hint:* The general form for position of a particle traveling in a horizontal circle is $\vec{r} = r\cos(\omega t)\,\hat{x} + r\sin(\omega t)\,\hat{y} + z\,\hat{z}$, with appropriate values for r, ω, and z), and then in cylindrical coordinates ($\vec{r} = r\,\hat{s} + z\,\hat{z}$).

all by hand