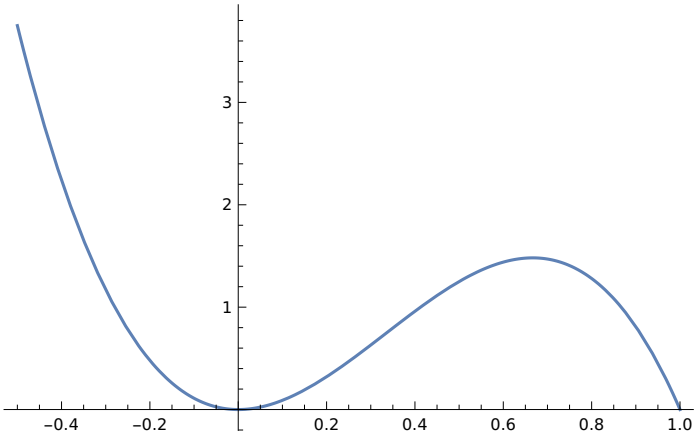
Problem 1

```
ClearAll["Global`*"]
U[x_] = (a x^2) - (b x^3)
```

$$a\,x^2 - b\,x^3$$

```
a = 10;
b = 10;
Plot[U[x], {x, -.5, 1.5 (2 a / 3 b)}]
```



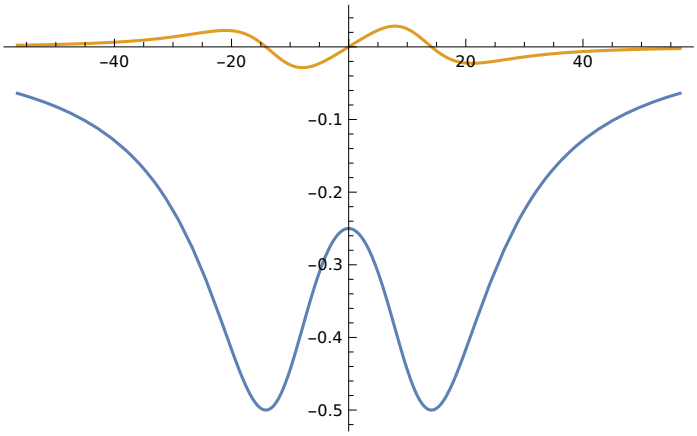
Problem 2

```
ClearAll["Global`*"]
```

```
a = 10;
U0 = 2;
U[x_] = -U0 ((a^2 (a^2 + x^2)) / (8 a^4 + x^4));
F[x_] = (2 a^2 (8 a^4 x - 2 a^2 x^3 - x^5) U0 /
    (8 a^4 + x^4)^2) \
  Limit[U[x], x → Infinity]
```

$$0$$

```
Plot[{U[x], F[x]}, {x, -4 * √2 a, 4 * √2 a}]
```




```
ClearAll["Global`*"]
```

$$U[x\_] = -U_0 \left(\frac{\left(a^2 \left(a^2 + x^2\right)\right)}{8\,a^4 + x^4} \right);$$

```
FullSimplify[-U'[x]]
```

$$\frac{2\,a^2 \left(8\,a^4\,x - 2\,a^2\,x^3 - x^5\right)}{\left(8\,a^4 + x^4\right)^2}$$

```
Solve[U'[x] == 0, x]
```

$$\{\{x \to 0\}, \{x \to -2\,i\,a\}, \{x \to 2\,i\,a\}, \{x \to -\sqrt{2}\,a\}, \{x \to \sqrt{2}\,a\}\}$$

```
FullSimplify[U''[x]]
```

$$-\frac{2\,a^2 \left(64\,a^8 - 48\,a^6\,x^2 - 96\,a^4\,x^4 + 10\,a^2\,x^6 + 3\,x^8\right)}{\left(8\,a^4 + x^4\right)^3}$$

```
U''[0]
```

$$-\frac{1}{4\,a^2}$$

```
U''[-2 i a]
```

$$\frac{1}{6\,a^2}$$

```
U''[2 ī a]
```

$$\frac{1}{6\,a^2}$$

```
U''[-√(2) a]
```

$$\frac{1}{3\,a^2}$$

```
U''[(√2) a]
```

$$\frac{1}{3\,a^2}$$

```
ω = √(U''[0]/m)
```

$$\frac{1}{2}\sqrt{-\frac{1}{a^2\,m}}$$

```
FullSimplify[α = ((7/72 y)/(1/2 m v^2 + 2/9 y))]
```

$$\frac{7\,y}{36\,m\,v^2+16\,y}$$

```
ClearAll["Global`*"]
U[x_] = -U0 ((a^2 (a^2 + x^2))/(8 a^4 + x^4));
```

Problem 3

```
ClearAll["Global`*"]
F[x_] = B (a^2/x^2 - 28 a^5/x^5 + 27 a^8/x^8);
U[x_] = (-∫(B (a^2/x^2 - 28 a^5/x^5 + 27 a^8/x^8) dx))
```

$$-a^2 B \left(-\frac{27 a^6}{7 x^7}+\frac{7 a^3}{x^4}-\frac{1}{x}\right)$$

```
Simplify[Solve[ F[x] == 0, x]]
```

$$\left\{\{x \to a\}, \{x \to 3 a\}, \left\{x \to -\frac{1}{2} i \left(-i+\sqrt{3}\right) a\right\},\right.$$
$$\left.\left\{x \to -\frac{3}{2} i \left(-i+\sqrt{3}\right) a\right\}, \left\{x \to \frac{1}{2} i \left(i+\sqrt{3}\right) a\right\}, \left\{x \to \frac{3}{2} i \left(i+\sqrt{3}\right) a\right\}\right\}$$

```
U[3 a/2]
```

$$-\frac{278 a B}{567}$$

```
U''[x]
```

$$-a^2 B \left(-\frac{216 a^6}{x^9}+\frac{140 a^3}{x^6}-\frac{2}{x^3}\right)$$

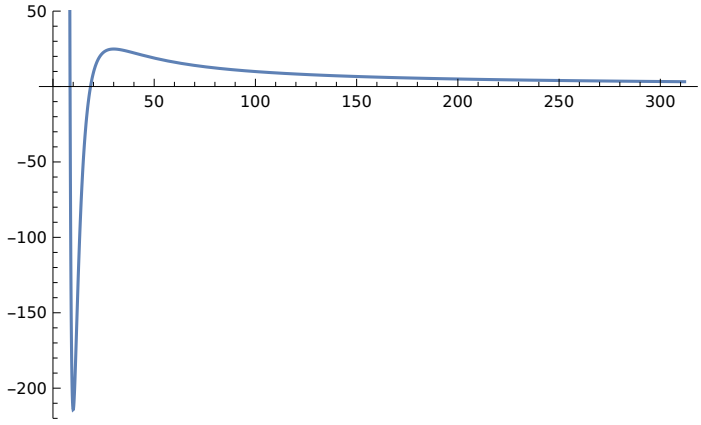
```
U''[a]
```

$$\frac{78 B}{a}$$

```
U''[3 a]
```

$$-\frac{26 B}{243 a}$$

```
a = 10;
B = 10;
Plot[U[x], {x, 4 (-26 B)/(243 a), 4 (78 B)/a}, PlotRange → {-220, 50}]
```



Problem 4

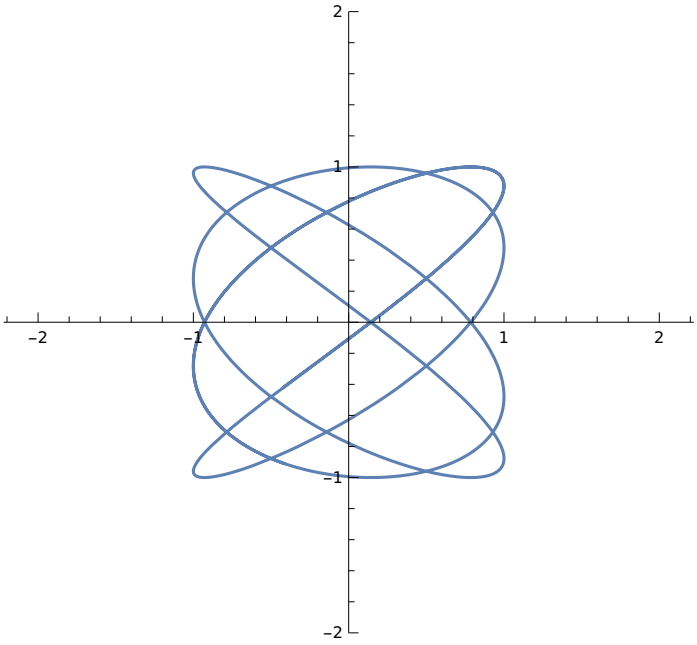
All work was done by hand.

Problem 5

```
ClearAll["Global`*"]
```

```
A = 1;
B = 1;
α = 2;
β = 2;
```

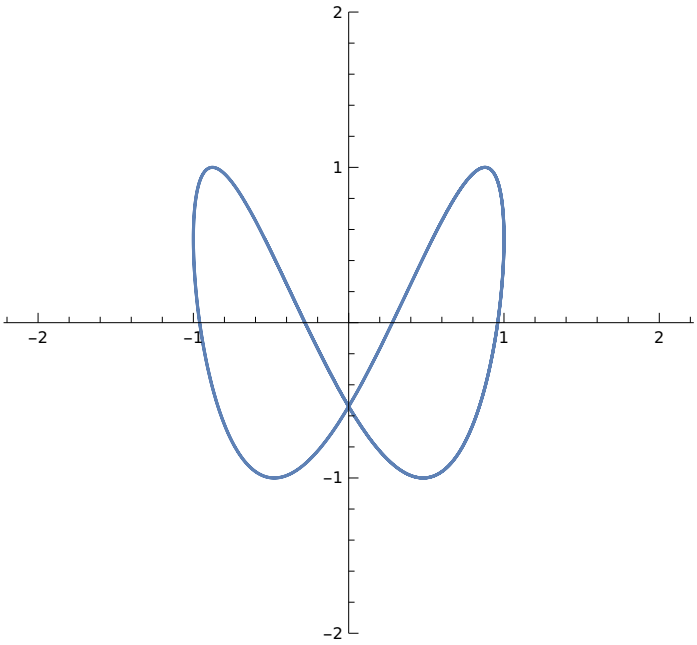
```
ParametricPlot[{A Cos[1 t - α], B Cos[.75 t - β]}, {t, 0, 10 π}, PlotRange → {-2, 2}]
```




```
ClearAll["Global`*"]

α = 1;
β = 1;
A = 1;
B = 1;
```

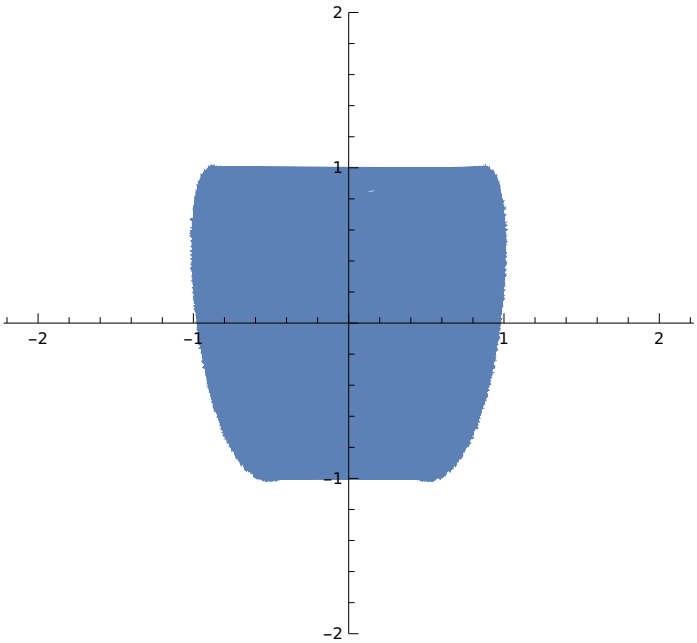
```
ParametricPlot[{A Cos[.5 t - α], B Cos[1 t - β]}, {t, 0, 10 π}, PlotRange → {-2, 2}]
```




```
ClearAll["Global`*"]
```

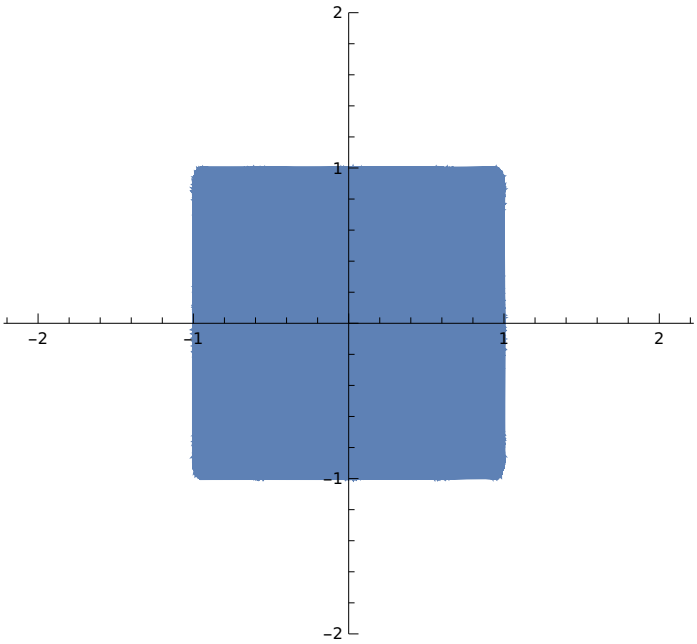
```
A = 1;
B = 1;
α = 4 π / 3;
β = π;
```

```
ParametricPlot[{A Cos[.5 t - α], B Cos[1 t - β]}, {t, 0, 10 000 000 π}, PlotRange → {-2, 2}]
```




```
ClearAll["Global`*"]
```

```
A = 1;
B = 1;
α = 3 π / 2;
β = π;
ParametricPlot[{A Cos[.6 t - α], B Cos[1 t - β]}, {t, 0, 1 100 000 π}, PlotRange → {-2, 2}]
```




```
ClearAll["Global`*"]
```

```
A = 1;
B = 1;
α = 1;
β = 1;
```

```
ParametricPlot[{A Cos[1 t - α], B Cos[(√2) t - β]}, {t, 0, 10 π}, PlotRange → {-2, 2}]
```