Problem 1

Done by hand.

Problem 2

```
ClearAll["Global`*"]
```

```
P[t_] = e^(-β t) (A e^(√(β^2 - ((ω0)^2)) t) + B e^(-√(β^2 - ((ω0)^2)) t))
```

$$e^{-t\,\beta}\left(B\,e^{-t\,\sqrt{\beta^2-\omega_0^2}}+A\,e^{t\,\sqrt{\beta^2-\omega_0^2}}\right)$$

```
Solve[0 == e^(-β t) (A e^(√(β^2 - ((ω0)^2)) t) + B e^(√(β^2 - ((ω0)^2)) t)), A] /. t → 0
```

{{A → -B}}

```
Simplify[P'[t]] /. -B → A
```

$$e^{-t\left(\beta+\sqrt{\beta^2-\omega_0^2}\right)}\left(A\,e^{2\,t\,\sqrt{\beta^2-\omega_0^2}}\left(-\beta+\sqrt{\beta^2-\omega_0^2}\right)+A\left(\beta+\sqrt{\beta^2-\omega_0^2}\right)\right)$$

```
Simplify[Solve[v0 == e^(-t (β + √(β^2 - ω0^2))) (e^(2 t √(β^2 - ω0^2)) A (-β + √(β^2 - ω0^2)) + A (β + √(β^2 - ω0^2))), A]] /. t → 0
```

$$\left\{\left\{A \to \frac{v_0}{2\sqrt{\beta^2-\omega_0^2}}\right\}\right\}$$

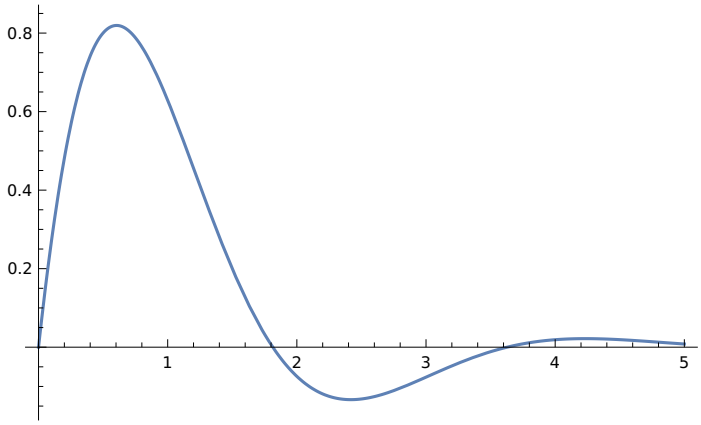
```
A = v0 / (2 √(β^2 - ω0^2));
```

```
B = -v0 / (2 √(β^2 - ω0^2));
```

Plotting the Underdamped Case

```
v0 = 3;
β = 1;
ω0 = 2;
ω1 = √((ω0)^2 - β^2);
G = A e^(-β t) e^(i ω1 t) + B e^(-β t) e^(-i ω1 t);
```

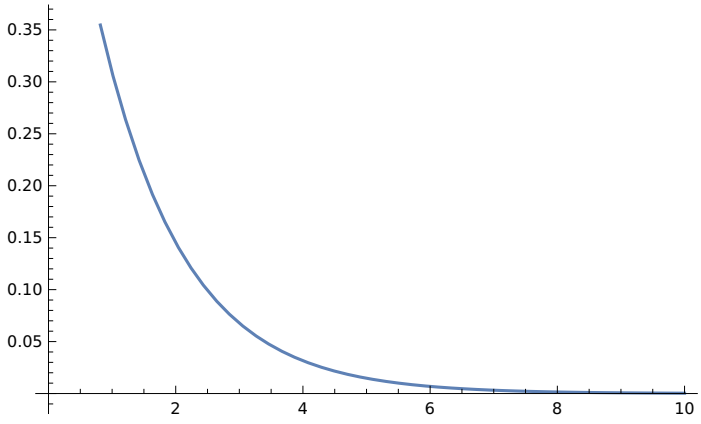
```
Plot[G, {t, 0, 5}]
```



Plotting the Over Damped Case

```
β = 3;
ω0 = 2;
ω2 = √(β^2 - (ω0)^2);
v0 = 3;
J = A e^(-β t) e^(ω2 t) + B e^(-β) e^(-ω2 t);
```

```
Plot[J, {t, 0, 10}]
```



Plotting the Critically Damped Case

```
ClearAll["Global`*"]
```

```
Solve[(A + B t) e^(-β t) == 0, A] /. t → 0
```

{{A → 0}}

```
F[t_] = (A + B t) e^(-β t);
```

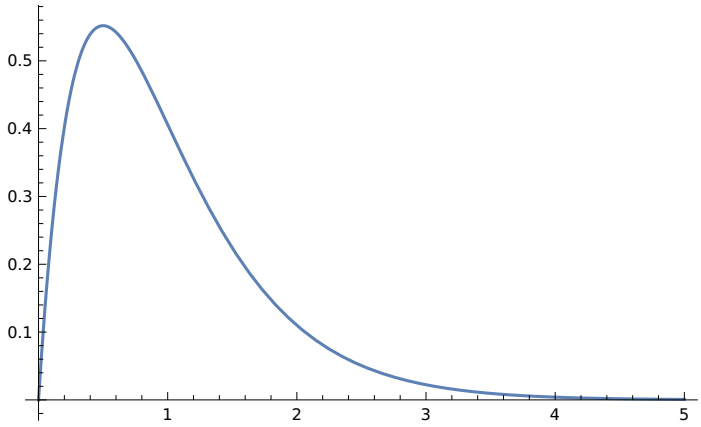
```
F'[t]
```

$B e^{-t\beta} - e^{-t\beta} (A + B t) \beta$

```
A = 0;
Solve[B e^(-t β) - e^(-t β) (A + B t) β == v, B] /. t → 0
```

{{B → v}}

```
B = v0;
v0 = 3;
β = 2;
H = (A + B t) e^(-β t);
Plot[H , {t, 0, 5}]
```



Problem 4

```
ClearAll["Global`*"]
```

```
Simplify[A α^2 e^(-α t) + 2 β (-A e^(-α t)) + ω0^2 A e^(-α t) == F0 e^(-α t)]
```

$$e^{-t\,\alpha}\left(A\left(4+\alpha^2-2\,\beta\right)-F_0\right)==0$$

```
Solve[e^(-t α) (A (4 + α^2 - 2 β) - F0) == 0, A]
```

$$\left\{\left\{A\to\frac{F_0}{4+\alpha^2-2\,\beta}\right\}\right\}$$

Problem 5

```
s = NDSolve[{x''[t] + 2 β x'[t] + (ω)^2 x[t] == 3 Cos[t] + 2 Cos[2 t], x[0] == 0, x'[0] == 0}, x, {t, 0, 200}]
```

$$\left\{\left\{x\to \texttt{InterpolatingFunction}\left[\right]\right\}\right\}$$

Domain: {{0., 200.}}
Output: scalar

```
b = .05;
k = 10;
m = 1;
ω = √10;
```

```
Plot[Evaluate[ x[t] /. s], {t, 0, 200}]
```