Newtonian Mechanics—Single Particle; Oscillations

PHYS 301: Analytical Mechanics

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Problem 1

An object of mass m is subjected to a potential energy $U(x) = a\,x^2 - b\,x^3$.

a. Use *Mathematica* or some other computer program to plot the potential energy for positive values of a and b.

b. Find the force.

c. The particle starts at the origin $x = 0$ with initial speed v_0. Show that, if v_0 is less than a certain critical velocity v_c, the particle will remain confined to a region near the origin. Find v_c.

Problem 2

An elementary particle moves in a potential well given by

$$U[x] = -U_0 \frac{a^2 \left(a^2 + x^2\right)}{8\,a^4 + x^4}.$$

a. Use *Mathematica* or some other computer program to plot the potential energy and the force for positive values of U_0 and a.

b. Discuss the motions which may occur. Locate all equilibrium points and determine the frequency of small oscillations about any that are stable. (You probably want to use *Mathematica* or some other computer program.)

c. The particle starts a great distance away from the potential well with velocity v_0 toward the well. As it passes the point $x = a$, it suffers a collision with another particle, during which it loses a fraction α of its kinetic energy. How large must α be in order that the particle thereafter remains trapped in the well? How large must α be in order that the particle be trapped in one side of the well? Find the turning points of the new motion if $\alpha = 1$.

Problem 3

A particle of mass m is subject to a force given by

$$F = B\left(\frac{a^2}{x^2} - 28\,\frac{a^5}{x^5} + 27\,\frac{a^8}{x^8}\right).$$

The particle moves only about the positive *x*-axis.

a. Find and plot the potential energy. (*B* and *a* are each positive.)

b. Locate all equilibrium points and determine the frequency of small oscillations about any which are stable. Describe the types of motion which may occur.

c. A particle starts at $x = 3\,a/2$ with a velocity $v = -v_0$, where v_0 is positive. What is the smallest value of v_0 for which the particle may eventually escape to a very large distance? Describe the motion in that case. What is the maximum velocity the particle will have? What velocity will it have when it is very far from its starting point? Report all three values as decimal coefficients of $\sqrt{B\,a/m}$.

Problem 4

A simple harmonic oscillator consists of a 100-g mass attached to a spring whose force constant is 10 N/m. The mass is displaced 3 cm and released from rest. Calculate the normal frequency, the period, the total energy, and the maximum speed.

Problem 5

For the general case of two-dimensional oscillations (equation 3.27 in the book), plot the path of the oscillator in the following cases: (*Hint:* In *Mathematica*, use **ParametricPlot[]**, plugging in values for all quantities except time. Make the upper limit on time pretty large so that you can see as much of the motion as possible.)

a. $A = B$, $\alpha = \beta$, $4\,\omega_y = 3\,\omega_x$

b. $A = B$, $\alpha = \beta$, $\omega_y = 2\,\omega_x$

c. $A = B$, $\alpha = \beta + \pi/3$, $\omega_y = 2\,\omega_x$

d. $A = B$, $\alpha = \beta + \pi/2$, $5\,\omega_y = 3\,\omega_x$

e. $A = B$, $\alpha = \beta$, $\omega_y = \sqrt{2}\,\omega_x$