Scott Kovacs

302 HW7

Problem 1

- J.J "discovered" electron by measuring charge-to-mass ratio of cathode rays:

a) passed beam through uniform $\vec{E}$ & $\vec{B}$ fields (perpendicular to each other and the beam) & adjusted $\vec{E}$ until he got 0 deflection. What was the speed of the particles in terms of E & B?

b) Turned off $\vec{E}$ & measured radius of curvature, R, of the beam deflected by only $\vec{B}$. In terms of E, B, & R what is the charge-to-mass ratio (q/m) of the particles?

a) To find the speed of the particles use the Lorentz Force Law:

$$\vec{F} = Q(\vec{E} + (v \times \vec{B}))$$

- If there is 0 deflection that means the force has to be 0.

$$0 = \cancel{Q}(\vec{E} + (\vec{v} \times \vec{B}))$$

$$0 = \vec{E} + (\vec{v} \times \vec{B})$$

$$\vec{E} = -(\vec{v} \times \vec{B})$$

* but since we are just concerned w/ speed which is a scalar, we don't need to worry about the - sign.

* $(\vec{v} \times \vec{B})$ is the same as vB since they are perpendicular.

$$\boxed{V = \frac{E}{B}}$$

b) W/ $\vec{E}$ off ($=0$) the magnetic force causing the curvature will be equivalent to the centripetal force on the particles.

- Centripetal force is given as: $F_c = \frac{mv^2}{R}$

- $F_B = F_c$: $\frac{mv^2}{R} = Q(\vec{v} \times \vec{B})$ * $\vec{v} \times \vec{B} = vB$ since they are perpendicular

- Solve for q/m:

$mv^2 = QvBR$

$$\boxed{\frac{Q}{m} = \frac{V}{BR}}$$

- if you want it in terms of the velocity from a): as $V = \frac{E}{B}$

$$\boxed{\frac{Q}{m} = \frac{E}{B^2 R}}$$

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302 HW7

Problem 2

a) phonograph carries a uniform density of "static electricity" σ.
It rotates at an angular velocity ω.
What is the surface current density $\vec{K}$ (including direction) a distance r from center?

b) A uniformly charged solid sphere radius R & total charge Q is centered @ origin & spinning @ constant angular rate ω about z-axis
Find current density $\vec{J}$ @ any point (r, θ, ϕ) w/in the sphere.

a) - Surface current density, $\vec{K}$, is defined as:

$$\vec{K} = \sigma \vec{v}$$

- where σ is surface charge & $\vec{v}$ is velocity

- The angular velocity ω is defined as:

$$\omega = \frac{\vec{v}}{r} \rightarrow \vec{v} = \omega r$$

- Plug in value for $\vec{v}$ into surface current density equation:

$$\boxed{\vec{K} = \sigma \omega r \ \hat{r}}$$

b) - Generally, volume charge density is given by:

$$\rho = \frac{Q}{V}$$

- where Q=charge V=volume

- w/ volume of a sphere $= \frac{4}{3}\pi R^3$, substitute in:

$$\rho = \frac{3Q}{4\pi R^3}$$

- Volume current density is:

$$\vec{J} = \rho \vec{v}$$

- where $\vec{v}$ is velocity & ρ= volume charge density

- the sphere has angular velocity ω about z axis, to get $\vec{v}$ from ω:

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{v} = \omega r \sin\theta \, \hat{\phi}$$

- substitute in $\vec{v}$ & ρ into $\vec{J} = \rho \vec{v}$:

$$\boxed{\vec{J} = \frac{3Q}{4\pi R^3}\left(\omega r \sin\theta \, \hat{\phi}\right)}$$

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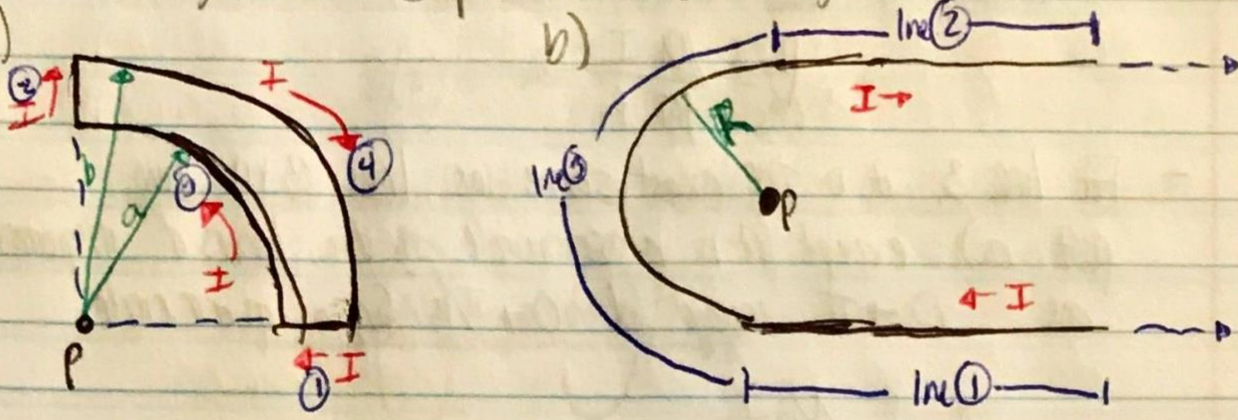
302 HW7

Problem 3

- Find magnetic field @ point P for each figure below.



a) Use the Biot-Savart Law to find the magnetic field

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\ell \times \hat{r}}{r}$$

- For lines 1 and 2 the magnetic field is 0 because the $d\ell$ segment and $\hat{r}$ are going to be collinear so their cross product $= 0$.
- For line 4, $\hat{r}$ will become $\hat{b}$ & r^2 will become b^2 since $\hat{r}$ is just the direction of the radius (which is b for line 4) & if r is b then magnitude of r^2 is b^2.

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\ell \times \hat{b}}{b^2}$$

- $d\ell \times \hat{b}$ will just be equal to $d\ell$ in magnitude since $d\ell$ & $\hat{b}$ are perpendicular.

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\ell}{b^2}$$

- $d\ell = r d\theta$ (or $b d\theta$ for line 4)

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{\cancel{b}}{b^{\cancel{2}}} d\theta = \frac{\mu_0 I}{4\pi} \int_0^{\pi/2} \frac{1}{b} d\theta$$

- limits of integration are 0 to $\pi/2$ since we're doing only one quarter of a circle.

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi b} \int_0^{\pi/2} d\theta \qquad \vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi b}\left(\frac{\pi}{2} - 0\right) \qquad \vec{B}(\vec{r}) = \frac{\mu_0 I}{8b}$$

- exact same for line 3 but a replaces r: $\vec{B}(\vec{r}) = \frac{\mu_0 I}{8a}$

- total $\vec{B}$ will $= \vec{B}_b - \vec{B}_a$: $B = \frac{\mu_0 I}{8b} - \frac{\mu_0 I}{8a}$

$$\boxed{B = \frac{\mu_0 I}{8}\left(\frac{1}{b} - \frac{1}{a}\right)}$$

b) - For lines 1 & 2 the magnetic field is just that of an infinitely long wire: (found in class)

$$B = \frac{\mu_0 I}{4\pi R}$$

- For line 3 it is the exact same as lines 3 + 4 from part a) except it is a semicircle so the limits of integration are $0 \to \pi$ instead of $0 \to \pi/2$ for a quarter circle.

$$B = \frac{\mu_0 I}{4R}$$

- Add all 3 together to get total B:

$$B_T = \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R}$$

$$B_T = \frac{\mu_0 I}{4R}\left(\frac{1}{\pi} + \frac{1}{\pi} + 1\right)$$

$$\boxed{B_T = \frac{\mu_0 I}{4R}\left(\frac{2}{\pi} + 1\right)}$$

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302 HW7

Problem 4

- Find the magnetic field @ point P on the axis of a tightly wound solenoid consisting of n turns per unit length wrapped around a cylindrical tube radius a & current I. Express in θ_1 & θ_2 (easiest)

- consider the turns to be essentially circular & use the result that the magnetic field strength distance z above the center of a circular loop radius R: $B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$

- What is the field on the axis of an infinite solenoid (in both directions)?

- B @ P: The magnetic field above a circular loop is:

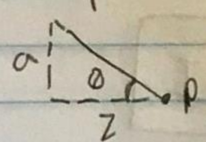
$B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$ - but our Radius is a; $R=a$ $\rightarrow B(z) = \frac{\mu_0 I}{2} \frac{a^2}{(a^2+z^2)^{3/2}}$

- to account for the "n turns per unit length" the 'n' must be included multiplying by I:

$B(z) = \frac{\mu_0 n I}{2} \frac{a^2}{(a^2+z^2)^{3/2}}$ - the thickness of the ring of wire also needs to be accounted for, which is done by integrating w/ respect to z:

$B(z) = \frac{\mu_0 n I}{2} \int \frac{a^2}{(a^2+z^2)^{3/2}} dz$ - to find dz, we first must find z:

- From the picture: $\tan\theta = \frac{a}{z} \rightarrow z\tan\theta = a \rightarrow z = \frac{a}{\tan\theta}$



$\frac{1}{\tan\theta} = \cot\theta \rightarrow z = a\cot\theta$

- differentiate $z = a\cot\theta$ to get dz: $\frac{d}{d\theta}\cot\theta = -\csc^2\theta\, d\theta$

$dz = -a\csc^2\theta\, d\theta$

$\csc\theta = \frac{1}{\sin\theta} \rightarrow \csc^2\theta = \frac{1}{\sin^2\theta} \rightarrow dz = \frac{-a}{\sin^2\theta} d\theta$

- we can plug in the value of $z = a\cot\theta$ into the integral
(but only focusing in on $(a^2+z^2)^{3/2}$ for now):

$(a^2 + (a\cot\theta)^2)^{3/2} = (a^2 + a^2\cot^2\theta)^{3/2} = (a^2(1+\cot^2\theta))^{3/2}$

$= (a^2)^{3/2}(1+\cot^2\theta)^{3/2} \rightarrow a^3(1+\cot^2\theta)^{3/2}$

- use $\csc^2\theta = \cot^2\theta + 1$:

$a^3(\csc^2\theta)^{3/2} = a^3\csc^3\theta$ - $\csc = \frac{1}{\sin}$, $\csc^3 = \frac{1}{\sin^3}$

$\frac{a^3}{\sin^3\theta} = (a^2+z^2)^{3/2}$

- Plug in $\frac{a^3}{\sin^3\theta} = (a^2+z^2)^{3/2}$ & $dz = \frac{-a}{\sin^2\theta}d\theta$ into the integral

$B = \frac{\mu_0 n I}{2}\int \frac{a^2}{\left(\frac{a^3}{\sin^3\theta}\right)} \frac{-a}{\sin^2\theta} d\theta$ - simplify

$-\frac{\mu_0 n I}{2}\int \frac{a^3\sin^3\theta}{a^3\sin^2\theta} d\theta = -\frac{\mu_0 n I}{2}\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$

- integrate:

$B = -\frac{\mu_0 n I}{2}(-\cos\theta)\Big|_{\theta_1}^{\theta_2}$ * bounds of θ_2 & θ_1 are from picture.

$$\boxed{B = \frac{\mu_0 n I}{2}(\cos\theta_2 - \cos\theta_1) \text{ @ point } P}$$

- Infinite Solenoid: if the solenoid is infinite, θ_2 will go to 0 and θ_1 will go to π (basically a straight line)

- plug in θ_2 & θ_1:

$B = \frac{\mu_0 n I}{2}(\cos 0 - \cos\pi)$

$= \frac{\mu_0 n I}{2}(1 - -1)$ $\boxed{B = \mu_0 n I}$

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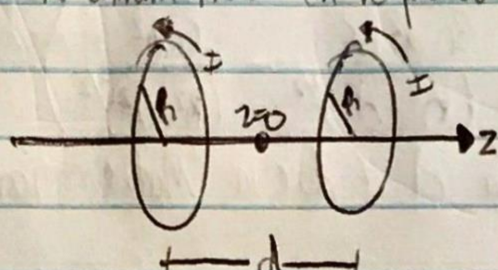
Problem 5

- The magnetic field on the axis of a circular loop:

$$B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$$

isn't uniform. A uniform field can be produced w/ 2 loops a distance d apart



a) Find B as a function of z & show that $\partial_z B = 0$ @ $z=0$

b) Find d such that $\partial_z^2 B = 0$ & find resulting magnetic field @ center.

a) B as a function of z:

- The equation for the magnetic field on the axis of a circular loop is already given, so all that needs to be done is adding the second loop in there. The only change is that instead of just z, one loop will have (z-d) and one loop will have (z+d).

$$\boxed{B(z) = \frac{\mu_0 I}{2}\left(\frac{R^2}{(R^2+(z+d)^2)^{3/2}} + \frac{R^2}{(R^2+(z-d)^2)^{3/2}}\right)}$$

- taking the partial derivative w/ respect to z yields: (MMA)

$$\partial_z B = \frac{\mu_0 I}{2}\left(\frac{-3R^2(z-d)}{((R^2+(z-d)^2)^{5/2}} - \frac{3R^2(z+d)}{((R^2+(z+d)^2)^{5/2}}\right)$$

- as $z \to 0$ (MMA)

$$\partial_z B = \frac{\mu_0 I}{2}\left(\frac{-3R^2(-d)}{(R^2+(-d)^2)^{5/2}} - \frac{3R^2(d)}{((R^2+(+d)^2)^{5/2}}\right) \to 0$$

$$= \frac{\mu_0 I}{2}\left(\frac{3R^2 d}{(R^2+d^2)^{5/2}} - \frac{3R^2 d}{(R^2+d^2)^{5/2}}\right)$$

$$\boxed{\partial_z B = 0}$$

b) d such that $\partial_z^2 B = 0$ & resulting B:

- Take $\partial_z^2 B$:

$$\partial_z^2 B = \frac{\mu_0 I}{2}\left(\frac{15R^2(z-d)^2}{((R^2+(z-d)^2))^{7/2}} - \frac{3R^2}{(R^2+(z-d)^2)^{5/2}} + \frac{15R^2(z+d)^2}{((R^2+(z+d)^2))^{7/2}} - \frac{3R^2}{((R^2+(z+d)^2)^{5/2}}\right)$$

- then apply $z \to 0$: (done in Mathematica)

$$\partial_z^2 B\,(z\to 0) = \frac{\mu_0 I}{2}\left(\frac{30d^2R^2}{(R^2+d^2)^{7/2}} - \frac{6R^2}{(R^2+d^2)^{5/2}}\right)$$

- Set = 0 and solve for d (done in Mathematica):

$$\boxed{d = \pm \frac{R}{2}}$$

- Plug $d = \frac{R}{2}$ (just + for simplicity) into the equation for B found in part a) after applying $z \to 0$:

(done in Mathematica)

$$B(z) = \frac{\mu_0 I}{2}\left(\frac{R^2}{((R^2+(z+d)^2)^{3/2}} + \frac{R^2}{((R^2+(z-d)^2)^{3/2}}\right)$$

$$B(z\to 0) = \frac{R^2 \mu_0 I}{(R^2+d^2)^{3/2}}$$

- plug in $d = R/2$:

$$B(z\to 0) = \frac{R^2 \mu_0 I}{(R^2+(\frac{R}{2})^2)^{3/2}}$$

– simplified in Mathematica

$$\boxed{B(z\to 0) = \frac{8\mu_0 I}{5\sqrt{5}\,R}}$$