Frequency of a Stretchy String

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I. INTRODUCTION

The frequency of a stretchy spring fascinates most people before they ever hear of the term frequency. It's the classic case of plucking a rubber band and hearing a sound, then pulling it tighter to hear a higher note. A more notable example are string instruments such as the guitar. A guitarist needs to know how tight to make the strings on his instrument to give him the desired notes so he's not missing chord after chord in his/her song.

This experiment will provide both an equation relating frequency to tension with no other dependence, as well as a graph to show how tension affects the frequency of a string.

II. THEORY

In order to explore the relationship between the tension of a string and its' frequency mathematically, their relationship must be derived from the concepts of waves, tension, and Hooke's Law. The goal is to derive an equation with a direct relationship of tension to frequency, with no other dependence. To begin, we have the relationship between the frequency and velocity of a wave:

$$f = \frac{V_{wave}}{2L'} \tag{1}$$

From Equation 1, we can plug in the equation for the velocity of a wave on a string:

$$v = \sqrt{\frac{T}{\frac{m}{L'}}} \tag{2}$$

This produces an equation for the frequency of a wave on a string with components of tension,T, mass, m, and initial string length, L':

$$f = \frac{\sqrt{\frac{T}{\frac{m}{L'}}}}{2L'} \tag{3}$$

The next addition to the equation is the combination of Equation 4, Hooke's Law, with the standard definition of $\triangle$L, Equation 5:

$$F = k\triangle L \tag{4}$$

$$\triangle L = L' - L_{eq} \tag{5}$$

For the experiment, the force in the Hooke's Law equation will be replaced with T, the tension in the string, as it is the only force that will be applied to the string. With the variable change from F to T and the combination of Equation 4 and Equation 5 we are left with:

$$T = k(L' - L_{eq}) \tag{6}$$

Solving Equation 6 for L' in order to plug the result into Equation 3 produces:

$$L' = \frac{T}{k} + L_{eq} \tag{7}$$

Equation 7 is plugged in to Equation 3 which produces the desired equation for the frequency of a string solely dependent on the tension of the string:

$$f = \frac{\sqrt{\frac{T}{\frac{m}{\frac{T}{k}+L_{eq}}}}}{2(\frac{T}{k} + L_{eq})} \tag{8}$$

The relationship is direct, but due to the complexity of the equation, it is hard to predict the behavior of the function. Equation 8 was simplified in Mathematica to produce an equation that is easier to analyze:

$$f = \frac{1}{2}\sqrt{\frac{T}{(\frac{T}{k} + L_{eq})m}} \tag{9}$$

A final substitution must be made to remove mass from Equation 9. The mass will be replaced by the linear mass density, $m = \mu_0 L_{eq}$ resulting in the final equation:

$$f = \sqrt{\frac{T}{2(\frac{T}{k} + L_{eq})\mu_0 L_{eq}}} \tag{10}$$

Equation 10 will be used as a measure of fitness to data that will be collected during the experiment. That data will be subject to a Fourier Transformation to find the frequency of a string at the measured tension. A graph of frequency vs. tension will be produced and should look like that of a logarithmic function where initially as tension increases, frequency increases greatly, but as the tension gets higher and higher, the change in frequency gets smaller and smaller until a point where it essentially does not change.

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III. EXPERIMENT

To gather the data, a Vernier Dual-Range Force Sensor will be used to measure the tension of the string and when plucked will produce a force vs. time graph that will be subjeect to a Fourier Tranform to produce the frequency of the string. A graph of frequency vs. tension will be produced and Equation 10 will be used as a measure of fitness.

The first step is to calculate the force sensor. The device will be set up vertically on the metal stand with a spring attached. The length of the unstretched spring will be measured with the 0 end of the meter stick at the tip of the spring attached to the force sensor to the end of the spring. A light mass is hung from the spring and after a few seconds allowing LoggerPro to settle on a reading, the force is to be recorded. Repeat the above steps for a heavier mass. These measurements will be used to calculate the coefficients for scale and intercept in the calibration equation.

To measure the unstretched length of the string, lay one end of the string at the 0 end of the meter stick. Pull the string to straighten it, but be careful to not pull hard enough to stretch the string as to change its length. This length is L_0 which will be used in the calculation of the linear mass density of the string, μ_0.

To find the mass of the string, obtain your scale and place the string on the scale. If the scale is digital, wait until the scale has a unchanging reading and record that mass. If you are using a triple beam balance scale, adjust the weight on the balance until the beam is no longer bouncing up and down and is lined up with the mark on the edge of the beam indicating that the beam is balanced. Record this mass, which will be used with L_0 to calculate μ_0.

Set up the force sensor horizontally to begin measuring the initial (unstretched) length of the string, L_{eq} when attached to the force sensor. Repeat the process for measuring unstretched string length that is described above. Then, pull the string tight to stretch the string and attach it to the other metal base to keep the tension and distance steady throughout the measurement process for the stretched string length,L'. Put the 0 end of the meter stick at the metal base where the string is attached and measure the distance to the other metal base where the string is attached. The LoggerPro should have a steady reading for force at this point, if so, record that force. If not, wait until the reading is steady. This force which will be used with L_{eq} and L' in Equation 6 to find the spring constant, k, for the string.

Without altering the set up, pluck the string while LoggerPro is set to collect data. The tension has already been recorded as the force that was used in the Hooke's Law equation. Once the string stops vibrating, stop collecting data. LoggerPro will have a table of force and time data. Save this data, as it will be used in the Fourier Transformation to find the frequency of the string. Stretch the string even further while attached to the metal bases. Wait for a steady force reading in LoggerPro and record that force, then pluck the string. Repeat the process above until ten trials have been completed.