We noticed later that $\mathbf{A}(\mathbf{A}^T\mathbf{A})^{+}\mathbf{A}^T$ is a projector on the range($\mathbf{A}$). This significantly simplifies some of the algebra in the paper and the algorithm in Appendix B, which can be rewritten as follows.

01 **function** $[\mathbf{E}_2] = \textbf{ienkf\_cycle}(\mathbf{E}_1^{(0)}, \mathbf{y}_2, \mathbf{R}_2, \mathcal{M}_{12}, \mathcal{H}_2)$
02 $\quad \mathbf{x}_1^{(0)} = \mathbf{E}_1^{(0)}\,\mathbf{1}/m$
03 $\quad \mathbf{A}_1^{(0)} = \mathbf{E}_1^{(0)} - \mathbf{x}_1^{(0)}\mathbf{1}^{\mathrm{T}}$
04 $\quad \mathbf{E}_1 = \mathbf{E}_1^{(0)}, \quad \mathbf{T} = \mathbf{I}, \quad \mathbf{w} = \mathbf{0}$
05 $\quad$ **repeat**
06 $\qquad$ <u>IEnKF:</u> $\mathbf{E}_1 = \mathbf{x}_1\mathbf{1}^{\mathrm{T}} + \mathbf{A}_1^{(0)}\mathbf{T}$
06 $\qquad$ <u>IEKF:</u> $\mathbf{E}_1 = \mathbf{x}_1\mathbf{1}^{\mathrm{T}} + \varepsilon_1\mathbf{A}_1^{(0)}$
07 $\qquad \mathbf{E}_2 = \mathcal{M}_{12}(\mathbf{E}_1)$
08 $\qquad \mathbf{Hx} = \mathcal{H}_2(\mathbf{E}_2)\,\mathbf{1}/m$
09 $\qquad$ <u>IEnKF:</u> $\mathbf{HA} = [\mathcal{H}_2(\mathbf{E}_2) - \mathbf{Hx}\,\mathbf{1}^{\mathrm{T}}]\,\mathbf{T}^{-1}$
09 $\qquad$ <u>IEKF:</u> $\mathbf{HA} = [\mathcal{H}_2(\mathbf{E}_2) - \mathbf{Hx}\,\mathbf{1}^{\mathrm{T}}]\,/\,\varepsilon_1$
10 $\qquad \nabla J = (\mathbf{HA})^{\mathrm{T}}\mathbf{R}_2^{-1}(\mathbf{y}_2 - \mathbf{Hx})/(m-1) - \mathbf{w}$
11 $\qquad \mathbf{M} = \mathbf{I} + (\mathbf{HA})^{\mathrm{T}}\mathbf{R}_2^{-1}\mathbf{HA}/(m-1)$
12 $\qquad \Delta\mathbf{w} = \mathbf{M}^{-1}\nabla J$
13 $\qquad \mathbf{w} = \mathbf{w} + \Delta\mathbf{w}$
14 $\qquad \mathbf{T} = \mathbf{M}^{-1/2}$
15 $\qquad \mathbf{x}_1 = \mathbf{x}_0^{(0)} + \mathbf{A}_1^{(0)}\mathbf{w}$
16 $\quad$ **until** $\|\Delta\mathbf{w}\| < \varepsilon_2$
17 $\quad \mathbf{E}_2 = \mathbf{E}_2\,\mathbf{1}\mathbf{1}^{\mathrm{T}}/m + (1+\delta)\,\mathbf{E}_2\,(\mathbf{I} - \mathbf{1}\mathbf{1}^{\mathrm{T}}/m)$

The simplified algorithm was presented in later papers Bocquet and Sakov (2014); Sakov et al. (2018).

References

Bocquet, M. and P. Sakov, 2014: An iterative ensemble Kalman smoother. *Q. J. R. Meteorol. Soc.*, **140**, 1521–1535.

Sakov, P., J.-M. Haussaire, and M. Bocquet, 2018: An iterative ensemble Kalman filter in the presence of additive model error. *Q. J. R. Meteorol. Soc.*, **144**, 1297–1309.