AI1110: Probability and Random Variable Assignment-1

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Question: 12.13.2.18

Problem Statement:
Two events A and B will be independent, if
(A) A and B are mutually exclusive
(B) $\Pr(A'B') = [1 - \Pr(A)][1 - \Pr(B)]$
(C) $\Pr(A) = \Pr(B)$
(D) $\Pr(A) + \Pr(B) = 1$
Answer: Option (B)
Solution:
(A) When tossing a coin, the event of getting a head and tail are mutually exclusive and let them be denoted by A and B respectively.

$$\Pr(A) = \Pr(B) = \frac{1}{2} \tag{1}$$
$$\implies \Pr(A) \times \Pr(B) = \frac{1}{4} \tag{2}$$
$$\Pr(AB) = 0 \neq \Pr(A) \times \Pr(B) \tag{3}$$

Hence A and B are not independent.
(B) $\Pr(A'B') = [1 - \Pr(A)][1 - \Pr(B)]$

$$\Pr(A'B') = [1 - \Pr(A)][1 - \Pr(B)] \tag{4}$$
$$\implies \Pr(A'B') = 1 - \Pr(A) - \Pr(B) + \Pr(A)\Pr(B) \tag{5}$$
$$\implies 1 - \Pr(A + B) = 1 - \Pr(A) - \Pr(B) + \Pr(A)\Pr(B) \tag{6}$$
$$\implies -[\Pr(A) + \Pr(B) - \Pr(AB)] = -\Pr(A) - \Pr(B) + \Pr(A)\Pr(B) \tag{7}$$
$$\implies -\Pr(A) - \Pr(B) + \Pr(AB) = -\Pr(A) - \Pr(B) + \Pr(A)\Pr(B) \tag{8}$$
$$\implies \Pr(AB) = \Pr(A) . \Pr(B) \tag{9}$$
$$\tag{10}$$

Hence it shows A and B are Independent events
(C) For the same counter example given for option A, $\Pr(A) = \Pr(B)$, But A and B are not independent events.
(D) For the same counter example given for option A, $\Pr(A) + \Pr(B) = 1$, But A and B are not independent events.

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