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ORIGINAL ARTICLE

Ranking Nodes in Signed Social Networks

Moshen Shahriari · Mahdi Jalili

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Abstract Social networks are inevitable part of modern life. A class of social networks is those with both positive (friendship or trust) and negative (enmity or distrust) links. Ranking nodes in signed networks remains a hot topic in computer science. In this manuscript, we review different ranking algorithms to rank the nodes in signed networks, and apply them to the sign prediction problem. Ranking scores are used to obtain *reputation* and *optimism*, which are used as features in the sign prediction problem. Reputation of a node shows patterns of voting towards the node and its optimism demonstrates how optimistic a node thinks about others. To assess the performance of different ranking algorithms, we apply them on three signed networks including Epinions, Slashdot and Wikipedia. In this paper, we introduce three novel ranking algorithms for signed networks and compare their ability in predicting signs of edges with already existing ones. We use logistic regression as the predictor and the reputation and optimism values for the trustee and trustor as features (that are obtained based on different ranking algorithms). We find that ranking algorithms resulting in correlated ranking scores, leads to almost the same prediction accuracy. Furthermore, our analysis identifies a number of ranking algorithms that result in higher prediction accuracy compared to others.

Keywords Social networks · Signed networks · Ranking algorithms · Sign prediction · Link prediction · Classification

M. Shahriari · M. Jalili (✉)
Department of Computer Engineering, Sharif University of Technology, Azadi Avenue, Tehran, Iran
e-mail: mjalili@sharif.edu

1 Introduction

Online social networks get ever increasing importance nowadays and have become an inevitable element of modern societies. A social network, in its simplest form, consists of a group of individuals (or organizations) with a number of connections in between. These connections can be due to family ties, friendship, working relations or mutual interests. The connections in a network can be directed or undirected, weighted or unweighted. An elegant way of analyzing social networks is to use various tools available in graph theory and to study the static and dynamic behavior of the networks. There has been tremendous progress in network during the last decade, which is mainly due to the availability of data over the Internet and sophisticated processing power (Babaei, Mirzasoleiman, Safari, & Jalili, 2012; Boccaletti, Latora, Moreno, Chavez, & Hwang, 2006; Hasan & Zaki, 2011; Newman, 2003).

Vast majority of social networks have only one type of edge; however, some networks might have two (or more) types of connections. A class of networks consists of links with both positive and negative signs (Cartwright & Harary, 1956; J. Leskovec, Huttenlocher, & Kleinberg, 2010a, 2010b; Symeonidis, Tiakas, & Manolopoulos, 2010). Positive links denote friendship, trust or voting in favor, while negative links may show enmity, distrust or voting against. Some examples of such networks are Epinions (an online review website in which the users can say their positive/negative views toward each other), Slashdot (a network of technology news where the users can rate them) and Wikipedia (an encyclopedia where the users can vote for selecting managers) (Burke & Kraut, 2008; Guha, Kumar, Raghavan, & Tomkins, 2004; Kunegis, Lommatzsch, & Bauckhage, 2009). Trust networks are another example of such networks with links having positive or negative sign. In online social networks, trust is an important aspect that has attracted much attention in recent years (T. DuBois, J. Golbeck, & Srinivasan, 2011; Ziegler & Lausen, 2005; Korovaiko & Thomo, 2013). In a peer-to-peer system, for instance, a malicious user may disseminate viruses to others. Therefore, discerning malicious users from good ones is an important task in these environments (Kamvar, Schlosser, & Garcia-Molina, 2003; Zhang & Wang, 2012). When a person shares a link with us in a social network, we might be interested in knowing how much trustable is this person, and thus, how much trustable are the information coming from this node. Therefore, we need to have a criterion to evaluate the amount of trustworthiness of a person (Mishra & Bhattacharya, 2011).

To find out how much important (or trustable) is a node in a social network, one should rank the nodes using a ranking algorithm (Farahat, Lofaro, Miller, Rae, & Ward, 2006; Jiao, Yan, Zhao, & Fan, 2009). There has been much interest in devising efficient algorithms for ranking networks with only positive connections such as WWW. Indeed, most of the web search engines are based on such ranking algorithms to obtain the rank of the webpages in WWW (Easley & Kleinberg, 2010). The simplest ranking algorithm one may consider, is the in-degree of nodes where the more incoming links a node receives, the higher its ranking in the network (Easley & Kleinberg, 2010). Most of the algorithms proposed for ranking the nodes in networks are recursive in nature; they start from an initial rank vector for the nodes and by performing a number of repetitions they end up with the final rank values (Kleinberg, 1999; Lempel & Moran, 2000; Page, Brin, Motwani, & Winograd, 1999; Sahand, 2010). Although many scholarly works have been carried out on ranking the nodes in networks with only one type of edges (i.e., positive edges), only several of them deal with ranking in signed networks. Ranking the nodes in signed networks could be linked to their trustworthiness and reliability. This means nodes with high ranking scores are more trustable (or reliable) than those with lower ranking scores. In this paper, we review different ranking algorithms to rank the nodes in signed networks, and apply them for the sign prediction problem. The core contribution of our work is to apply the ranking algorithms for sign prediction. We also extend two existing ranking algorithms, namely Hyperlink-Induced Topic Search (HITS) and PageRank (that have been introduced for networks with only positive links) for signed networks.

Sign prediction problem is a special case of link prediction problem (Ilham Esslimani, Armelle Brun, 2011; Liaghat, Rasekh, & Mahdavi, 2013) where it is assumed that an edge exists but its sign is unknown. The unknown sign is predicted based on the information obtained from the structure of the network (Chiang, Natarajan, Tewari, & Dhillon, 2011; J. Leskovec, Huttenlocher, & Kleinberg, 2010c; Symeonidis et al., 2010). In this work, first, the nodes are ranked and a ranking score is obtained for each node. Then, two quantities, namely *reputation* and *optimism*, are calculated for the nodes based on their ranking scores. Optimism of a node shows how much optimistic a node thinks about the environment and its reputation represents pattern of voting towards that node. We use logistic regression with cross-validation to perform the prediction task and evaluate the performance by prediction accuracy. Results indicate that some algorithms outperform others in that they result in higher prediction accuracy when predicting the sign of the links. We also calculate correlations between the ranking scores obtained through different algorithms (including those proposed in this work) to find the most similar algorithms. Algorithms with similar rankings scores likely result in close prediction accuracy. The contribution of this manuscript can be summarized as follows:

- Three new ranking algorithms are proposed for networks with positive and negative links.
- Different ranking algorithms are reviewed and investigated in sign prediction problem. The ranking and sign prediction problem are linked via the normalized optimism and reputation features. It is found that some ranking algorithms result in higher prediction accuracy than others.

2 Ranking algorithms for signed networks

Let us consider a directed graph $G(V,E)$ in which V represents the set of nodes and E shows the set of relations between these nodes. Each link has a label that can be positive or negative; E^+ is the set of positive links and E^- is the one for negative links. Positive links show friendship (or trust) between two nodes, while possible enmity (or distrust) between any pair of nodes is designated by negative links. Alternatively, the network can be represented by adjacency matrix $A = (a_{ij})$ where $a_{ij} = 1$ when there is a positive link from node i to node j, $a_{ij} = -1$ when the link is negative, and $a_{ij} = 0$ otherwise. We can also denote the adjacency matrix of the subgraph with positive links by A^+ and the one with negative links with A^- (the entries of A^+ and A^- corresponding to the links are 1). Having positive or negative relationships between the nodes, the ranking problem can be formally stated as follows: How we can compute the rank of each node in a trustworthy manner from the structural information of the network. Ranking the nodes in networks with only positive links has been heavily studied in the last decade, for example, see (Estrada & Rodríguez-Velázquez, 2005; Farahat et al., 2006; Ilyas & Radha, 2011; Kleinberg, 1999; Langville & Meyer, 2004; Lempel & Moran, 2000). In-degree of a node might be the simplest measure of its centrality (Barabási, 2009; Barabasi & Albert, 1999). More sophisticated algorithms have been proposed to find the ranking of the nodes. However, only several algorithms exist for ranking the nodes in networks with positive or negative links that are reviewed in the following.

2.1 Existing ranking algorithms for signed networks

2.1.1 *Prestige*

Similar to in-degree for non-signed networks, prestige is the simplest algorithm for ranking the nodes in signed networks (Zolfaghar & Aghaie, 2010). Computational complexity of Prestige for graph with N nodes is $O(N)$. Prestige considers only positive and negative incoming links. If a node receives many positive incoming links, it should have high prestige value. On the other hands, nodes with many negative incoming links will have small values of prestige. Let us denote the number of positive incoming links to node i by $|IN_i^{(+)}|$ and those with negative sign by $|IN_i^{(-)}|$. The prestige of node i (Pr_i) is calculated as

$$Pr_i = \frac{|IN_i^{(+)}| - |IN_i^{(-)}|}{|IN_i^{(+)}| + |IN_i^{(-)}|}. \tag{1}$$

Indeed, prestige takes into account the normalized influence of positive and negative incoming links. Nodes with high prestige are likely to be trusted in the future, i.e., receive more positive (trust) links, while those with small prestige (or negative prestige) are less likely to have such positive connections.

2.1.2 *Exponential ranking*

This algorithm starts with the problem of discrete choice theory which wants to select the node with the highest reputation value (Traag, Nesterov, & Dooren, 2010). Thus, it comes to a probabilistic formulation for updating the rank vector. The algorithm is based on social balance in the sense that the enemy of a node's enemy needs not to be necessarily its friend. Indeed, it assumes a node with a negative reputation to be partially trustworthy, i.e., if this node points negatively to another node, we should not take it as positive, and we should only trust its judgments less than nodes with higher reputation. In fact, the algorithm somehow tries to deal with negative reputations. In exponential ranking algorithm, the rank value is a global trust value which is obtained from local trust values. The column rank vector a can be calculated as

$$a = A^T P \tag{2}$$

where A is the adjacency matrix and P is a positive definite column probability vector with $\|P\|_1 = 1$, which is computed in an iterative formula as

$$P(t+1) = \frac{\exp\left(\frac{1}{x} A^T P(t)\right)}{\left\|\exp\left(\frac{1}{x} A^T P(t)\right)\right\|_1} \tag{3}$$

where x is the amount of noise imposed in selecting the highest reputable judge according to discrete choice theory (V.A. Traag, 2010). For some initial condition $P(0)$, it is possible to prove that the probability vector converges to a unique vector P^* independent of the initial value. Indeed,

$$P^* = \lim_{t \to \infty} P(t) \tag{4}$$

and the final rank vector can be calculated as

$$a^* = A^T P^*. \tag{5}$$

The idea behind this ranking algorithm is that a node's rank depends on the rank of the nodes linking to it. The algorithms is iterated for T time steps and its computational complexity is $O(TN)$.

2.1.3 *PageRank*

PageRank—adopted from Larry Page from Google—is one of the most widely used ranking algorithms (Page et al., 1999). The idea behind PageRank is to surf the graph and to follow the directed edges with probability of α or to teleport to a new node with probability of $1-\alpha$. Let us denote the rank of node i (obtained through PageRank algorithm) by PR_i. This rank value is computed (in an iterative manner) as

$$PR_i(t+1) = \alpha \sum_{j \in IN_i} \frac{PR_j(t)}{|OUT_j|} + (1-\alpha)\frac{1}{N} \tag{6}$$

where α is a forgetting factor indicating that we go from node j to node i with probability α or we start a new walk from random node to node i with probability $1-\alpha$. N is the number of nodes in the network.

PageRank algorithm is one of the first ranking methods used in signed networks (Kunegis et al., 2009). The original PageRank was proposed for networks with only positive connections. Therefore, in this work negative connections are removed from the network and the algorithms is applied on the subgraphs with only positive connections. Computational complexity of this algorithm is $O(TN)$.

2.1.4 *PageTrust*

PageTrust is a modified version of PageRank to suit it for networks with signed links (Cristobald & Dooren, 2008). The difference between these two algorithms is that in PageTrust, nodes receiving negative connections are visited less as compared to those with positive sign in the process of random walk (Cristobald & Dooren, 2008). Let us consider $P = (P_{ij})$ as distrust matrix in which P_{ij} $(i \neq j)$ is the proportion of walkers having a negative (distrust) link from node i to node j. Denoting the ranking value of node i (obtained through PageTrust algorithm) by PT_i, the following update rule is used to compute the ranking scores (C. & Dooren, 2008)

$$PT_i(t+1) = (1 - Q_{ii}(t)) \left[\alpha \sum_{j,(j,i)\in G^+} \frac{PT_j(t)}{|OUT_j^{(+)}|} + (1-\alpha)\frac{1}{N} \right] \tag{7}$$

where α is forgetting factor and Q is a matrix calculated as

$$Q(t+1) = T(t)P(t). \tag{8}$$

In the above equation, $T(t)$ is the transition matrix at time t, i.e., the row-normalized version of A^+. In each iteration step, we correct the distrust matrix P considering the links with negative sign. The correction rule for updating P is as follows (C. & Dooren, 2008)

$$P_{ij}(t+1) = \begin{cases} 1 & \text{if } (i \neq j); (i,j) \in G^- \\ 0 & \text{if } (i = j); (i,j) \in G^- \\ Q_{ij}(t+1) & \text{otherwise} \end{cases} \tag{9}$$

Furthermore, the initial values are set as $P(0) = Q(0) = A^-$ (C. & Dooren, 2008), where $\bar{A}$ is the adjacency matrix of the subgraph with only negative links. Computational complexity of the algorithm is in $O(TNN_n)$ where N_n is the number of nodes receiving negative links (C. & Dooren, 2008).

2.1.5 Bias and Deserve

This algorithm has been proposed for ranking nodes in signed networks, taking into account their bias and deserve (Mishra & Bhattacharya, 2011). Bias and deserve terms are defined as follows (Mishra & Bhattacharya, 2011). Bias (or trustworthiness) of a link reflects the expected weight of an outgoing connection. Deserve (or prestige) of a link reflects the expected weight of an incoming connection from an unbiased node. Indeed, bias is related to outgoing links a node generates towards others and deserve pertains to incoming links a node receives from others. If propensity of a particular node towards other nodes is high, it is referred to as a biased node. Not only opinions produced by an optimistic or biased user should not be considered significantly, but also a node receiving positive signs from highly biased nodes has a lower prestige as compared to the one receiving such links from unbiased nodes (Mishra & Bhattacharya, 2011). Let $DES_i(t)$ and $BIAS_i(t)$ denote, respectively, the value of deserve and bias for node i at time t. The iterative formula for computing DES is as (Mishra & Bhattacharya, 2011)

$$DES_i(t+1) = \frac{1}{|IN_i|} \sum_{k \in IN_i} [a_{ki}(1 - X_{ki}(t))] \tag{10}$$

where $|IN_i|$ shows the number of nodes in the set IN_i. a_{ki} is the corresponding entry from the adjacency matrix A, i.e., it is equal to 1 for positive links and to -1 for negative links. $X_{ki}(t)$ indicates the influence that bias of node k has on its outgoing link to node i (at time t) and is computed as (A. Mishra & Bhattacharya, 2011)

$$X_{ki}(t) = \max\{0, BIAS_k \times a_{ki}\}. \tag{11}$$

Similarly, the values of bias vector is updated based on the following recursive rule

$$BIAS_i(t+1) = \frac{1}{2|OUT_i|} \sum_{k \in OUT_i} [a_{ik} - DES_k(k)]. \tag{12}$$

The above formulations for bias and deserve enable one to study how trust is propagated through the network. Similar to other recursive algorithms, this algorithm also starts with some initial values for $BIAS$ and DES vectors and runs for a sufficient number of steps until the stopping conditions are met. It has been shown that regardless of initial values, the algorithm converges to unique vector of $BIAS$ and DES (Mishra & Bhattacharya, 2011). The algorithm is similar to $HITS$, in which DES values are corresponding to authority values obtained through $HITS$ and $BIAS$ values are corresponding to hub values in $HITS$. Therefore, the DES vector is taken into account as the vector indicating the rankings of the nodes in the network. Computational complexity of Bias and Deserve is $O(TN)$.

2.2 Novel Ranking Algorithms

In this work, we introduce three novel ranking algorithms for signed networks by extending HITS (Kleinberg, 1999) and PageRank (Page et al., 1999). HITS and PageRank have been originally proposed for networks with only positive links and cannot be directly used for signed networks. They might not result in satisfactory performance when applied to signed networks. We modify the original HITS and Rage-Rank to better fit them for signed networks. To assess whether this modification is indeed effective, we apply the ranking results to the sign prediction problem; the ranking algorithms resulting in higher sign prediction accuracy could be the winners (at least for this specific application).

2.2.1 Hyperlink-Induced Topic Search (HITS)

HITS was originally proposed to extract useful information by analyzing the link structure in WWW (Kleinberg, 1999). Our approach in using HITS for ranking the nodes in signed networks is, first, to divide the network into two subgraphs (one with only positive links and another with only negative connections), and then, to run it separately on these two subgraphs. Let us denote the subgraph with only positive links by $G^{+}(V,E^{+})$ and the one with negative connections by $G^{-}(V,E^{-})$. HITS algorithm runs separately on these subgraphs and the update equations are as

$$\begin{cases} h_i^{(+)} = \sum\limits_{j \in IN_i^{(+)}} a_j^{(+)} \,; a_i^{(+)} = \sum\limits_{j \in OUT_i^{(+)}} h_j^{(+)} \\ h_i^{(-)} = \sum\limits_{j \in IN_i^{(-)}} a_j^{(-)} \,; a_i^{(-)} = \sum\limits_{j \in OUT_i^{(-)}} h_j^{(-)} \end{cases} \tag{13}$$

where h_i indicates the hub vector and a_i the authority vector for the nodes. '+' sign above hub and authority vectors indicates the values in G^{+} and "$-$" in G^{-}. The sum in the above update equations (for node i) is carried out to the authority and hub sets of node i. $OUT_i^{(+)}$ shows the set of nodes who receive positive links from node i and $OUT_i^{(-)}$ indicates those receiving negative links from node i. Similarly, $IN_i^{(+)}$ and $IN_i^{(-)}$ indicate the set of nodes pointing to node i with positive and negative links, respectively. One can rewrite the above equations as

$$\begin{cases} a^{(+)}(t+1) = A^{+T}A^{+}a^{(+)}(t); a^{(-)}(t+1) = A^{-T}A^{-}a^{(-)}(t) \\ h^{(+)}(t+1) = A^{+}A^{+T}h^{(+)}(t); h^{(-)}(t+1) = A^{-}A^{-T}h^{(-)}(t) \end{cases} \tag{14}$$

where $a^{(+)}(t)$ and $h^{(+)}(t)$ show authority and hub vectors in G^{+} at time t. A^{+} and A^{-} are the adjacency matrices of G^{+} and G^{-}. The algorithm starts with some random initial values for the hub and authority vectors (often taken as 1 for all of them), and converges to the final values provided that it runs for sufficient steps. This algorithm is indeed the original HITS and has guaranteed convergence. Finally, the authority value for node i is obtained as:

$$a_i = a_i^{(+)} - a_i^{(-)} \tag{15}$$

where a_i shows the ranking value for node i that can be used to compute features for sign prediction in the network.

2.2.2 Modified HITS

We propose another modification to the original HITS algorithm to better suit it for signed networks. In this modified HITS algorithm the network is not divided into subgraphs with positive and negative links, and the hub and authority values are obtained by considering the incoming and outgoing links with corresponding signs. More precisely, the authority and hub values for node i (a_i and h_i, respectively) are computed as

$$\begin{cases} h_i(t+1) = \dfrac{\sum_{j \in IN_i^{(+)}} a_j(t) - \sum_{j \in IN_i^{(-)}} a_j(t)}{\sum_{j \in IN_i^{(+)}} a_j(t) + \sum_{j \in IN_i^{(-)}} a_j(t)} \\ a_i(t+1) = \dfrac{\sum_{j \in OUT_i^{(+)}} h_j(t) - \sum_{j \in OUT_i^{(-)}} h_j(t)}{\sum_{j \in OUT_i^{(+)}} h_j(t) + \sum_{j \in OUT_i^{(-)}} h_j(t)} \end{cases} \tag{16}$$

The algorithm starts from some initial conditions and ends up with the final values for the authority and hub vectors provided that it is repeated for sufficient number of steps or when a stopping criterion is met. The authority vector indicates the rankings of the nodes in the network. Unlike the previous method, in this algorithm there is no need to separate the positive and negative graphs. Indeed, in each of the iteration steps, positive and negative hub/authority sets influence the final hubness and authorityness of the nodes. Our simulations on the data show that the algorithm converges with a computational complexity similar to the original HITS algorithm. Although we are unable to provide the convergence proof of this modified HITS algorithm, our simulations show that the algorithm converges provided that sufficient number of iterations is performed. In practice, we consider a stopping criterion $(a(t + 1) - a(t)) < \varepsilon$ with sufficiently small ε.

2.2.3 Modified PageRank

As for HITS, we extend PageRank algorithm for networks with both positive and negative links. In this ranking, we apply PageRank separately on G^{+} (subgraph with positive links) and G^{-} (subgraph with negative links). The update rules are as

$$\begin{aligned} PR_i^{+}(t+1) &= \alpha \sum_{j \in IN_i} \frac{PR_j^{+}(t)}{|OUT_j^{(+)}|} + (1-\alpha)\frac{1}{N} \\ PR_i^{-}(t+1) &= \alpha \sum_{j \in IN_i} \frac{PR_j^{-}(t)}{|OUT_j^{(-)}|} + (1-\alpha)\frac{1}{N} \end{aligned} \tag{17}$$

where $|OUT_j^{(+)}|$ and $|OUT_j^{(-)}|$ are the number of positive and negative outgoing links from node j, respectively. The algorithms starts with some initial condition for PR^{+} and PR^{-} vectors and after enough iterations it converges to the final rank vectors for G^{+} and G^{-}. The final rank vector (PR) is calculated by

$$PR = PR^{+} - PR^{-}. \tag{18}$$

This algorithm is indeed a standard PageRank algorithm with the same computational complexity and guaranteed convergence.

3 Sign Prediction Problem

To study the effect of ranking on sign prediction, we use rankings scores obtained through these algorithms to compute features for sign prediction problem. (J. Leskovec et al., 2010a, 2010c; Jure Leskovec, Huttenlocher, & Kleinberg, 2010; Shahriari, Sichani, Gharibshah, & Jalili, 2012; Symeonidis & Tiakas, 2013); some algorithms resulting in better prediction accuracy could be interpreted as better models in sign prediction. The sign prediction problem in networks with positive and negative links can be formally defined as follows. Consider a signed social network G where the sign of all links are given except for the link from node i to node j that is denoted by $s(i,j)$. We would like to know how reliably we can predict $s(i,j)$ based on the information obtained from the rest of the network (such as the sign of other links). Indeed, the sign prediction problem tries to find out to what extent the evolution of a network can be predicted using its structural features.

In this paper, we investigate ability of different ranking algorithms in the sign prediction problem. The values resulted from the ranking algorithms are taken into account as structural information on the networks to be used for predicting the sign of the links. We propose measures obtained for the nodes – called reputation (*Rep*) and optimism (*Opt*) – as features for sign prediction. These metrics are computed taking into account the ranking scores for the nodes. Let us denote the ranking score for node i by R_i that is obtained through the ranking algorithms described in the previous section. The reputation of node i (Rep_i) based on this ranking score is defined as

$$\mathrm{Rep}_i = \frac{\sum_{j \in IN_i^{(+)}} R_j - \sum_{j \in IN_i^{(-)}} R_j}{\sum_{j \in IN_i^{(+)}} R_j + \sum_{j \in IN_i^{(-)}} R_j} \tag{19}$$

and its optimism (Opt_i) is calculated as

$$Opt_i = \frac{\sum_{j \in OUT_i^{(+)}} R_j - \sum_{j \in OUT_i^{(-)}} R_j}{\sum_{j \in OUT_i^{(+)}} R_j + \sum_{j \in OUT_i^{(-)}} R_j} \tag{20}$$

Reputation of a node shows how much a node is popular in the network. Acceptability and social significance of the users can be measured by this factor; however, not only pattern of voting towards node i is important in its reputation, but also ranking of nodes voted towards this node is crucial. Therefore, pattern of voting towards node i as well as the ranking score of nodes voted for node i are combined to compute its reputation. Optimism somehow quantifies the pattern of votes the nodes make in the network. Some users might be more optimistic than others, meaning their votes are more likely to be positive. Similarly, Opt_i combines voting pattern of node i towards other nodes along with their ranking scores. To predict the sign of the link from node i to node j, we consider four features to use in the classification system. These features include Rep_i, Opt_i, Rep_j, and Opt_j. Indeed, the reputation and optimism values for the trustor and trustee are considered as classification features and the proposed measures for optimism and reputation are somehow normalized incoming and outgoing ranking scores.

We use logistic regression as the classification tool for the problem of sign prediction. Functions of the form $f: X \to Y$ or $P(Y|X)$ can be learnt using logistic regression approach (Mitchell, 1997). In this function X is the set of discrete or continuous values and Y is the set of discrete values. In the problem of sign prediction in networks with positive and negative links, Y is a Boolean variable indicating the classes for positive and negative signs. In the case where Y can have Boolean variables, the parametric model for logistic regression can be stated as

$$\begin{cases} P(Y=1|X) = \dfrac{1}{1+\exp(w_0 + \sum_{i=1}^{n} w_i x_i)} \\ P(Y=0|X) = \dfrac{\exp(w_0 + \sum_{i=1}^{n} w_i x_i)}{1+\exp(w_0 + \sum_{i=1}^{n} w_i x_i)} \end{cases} \tag{21}$$

where $(x_1,\ldots,x_n)$ is the feature vector and w_i's are the estimated coefficients based on the training data. This classifier has been previously used for the problem of sign prediction (Leskovec et al., 2010a). Each ranking algorithm yields a different set of features resulting in different prediction accuracy. We use the measure of accuracy for comparing the performance of different ranking algorithms. Accuracy of prediction is obtained as

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN} \times 100 \tag{22}$$

where TP, TN, FP, FN are, respectively, the rates of true positive, true negative, false positive and false negative in the classifier. Prediction accuracy largely depends on the goodness of the features used for classification, i.e., the values of *Rep* and *Opt* as obtained through different ranking algorithms.

4 Results

In this section, we provide the experimental results of the algorithms on a number of real-world signed networks from social media.

4.1 Data description

We apply the algorithms on three social networks with positive or negative connections including Epinions, Slashdot and Wikipedia (Burke & Kraut, 2008; Guha et al., 2004; Kunegis et al., 2009). It is worth mentioning that a link can only have one of positive or negative signs and both of them cannot be present at the same time in a single edge. Some information about characteristics of these social networks is given in the following (these datasets are freely available to download at http://snap.stanford.edu/data/).

Epinions: Epinions is an online review website that users say publicly their views toward each other. Topics in this website range from consumer commodities like cars and toasters to multimedia things like music and movies. This is a signed network with both trust (positive) and distrust (negative) relationship among the nodes. In the network people state their opinion about each other by $+1$ and -1 signs. Epinions dataset can be considered as a directed graph containing 131828 nodes and 841372 edges with about 85 % of positive sign.

Slashdot: This dataset represents the social network of the technology news website which contains 82144 nodes and 549202 edges and was founded in 1997. There exist stories that are published by editors or submitted by users. Users are allowed to comment on these stories and can rate each other positively or negatively. Friend and foe relationships correspond to positive and negative endorsements. In addition to friend and foe relationship, Slashdot also uses a concept of fan and freak. A user can be the fan of his friend and the freak of his enemies. And finally, there is a trolling property which is disseminating of false and disruptive information to deceive other users. The high number of troll users can justify the existence of foe relationship. Therefore, we need ranking to determine such malicious users.

Wikipedia: Wikipedia is a famous encyclopedia that has been created by volunteer people around the world. This online glossary has over 6.8 million enrolled users helping in creating 2.3 million English articles (merely in English language until 2008). The system is managed by some administrators for its maintenance. The maintenance process comprises administrative issues such as deleting copyright violation, protecting vandalized pages, blocking malicious users and editing the front page. These managers are elected via an adminship election where users can vote positively, negatively or neutrally to elect their preferred candidate. Finally, the zero votes are omitted and only positive and negative votes are left for processing.

Table 1 summarize the topological information on these networks including the number of nodes, the number of edges and the portion of links with positive or negative signs.

Table 1 Datasets statistics (http://snap.stanford.edu/)

	Node	Edges	+ Edges (%)	− Edges (%)
Epinions	119217	841200	85.0	15.0
Slashdot	82144	549202	77.4	22.6
Wikipedia	7118	103747	78.7	21.2

4.2 Sign prediction

We run the ranking algorithms (include Prestige, PageRank, PageTrust, Exponential, Bias and Deserve (BAD), HITS, modified HITS and modified PageRank) on Epinions, Slashdot and Wikipedia networks. Moreover, *Rep* and *Opt* vectors, as described by Eqs. (19) and (20), are obtained based on the ranking scores resulting from 8 ranking algorithms on above networks. Therefore, for each of the networks, we have 8 prediction tasks each with four features, i.e., *Rep* and *Opt* vectors for the trustees and trustors. We use logistic regressors for our classification tasks. We use tenfold cross-validation, and each time, 80 % of randomly selected edges are used for training and 20 % for test. In other words, all edges in the test set (20 % of the edges) are considered to have hidden signs.

In all three datasets, the links with positive sign are more represented than those with negative sign. This may bias the training phase and random guessing the sign results in a prediction accuracy of at least 75 %. Thus, we apply the classifier not only to the original version of the datasets but also to balanced versions where the number of links with positive sign is equal to the number of links with negative sign (J. Leskovec et al., 2010c; Shahriari et al., 2012). To this end, all links with negative sign are preserved in the networks and the equal numbers of remaining ones with positive signs are randomly chosen. Indeed, optimism and reputation are obtained considering all information available for the networks and the only difference between the balanced and original is in their training/test sets. For each dataset, random selection of links with positive sign is performed 10 times and the prediction accuracy is averaged over them. This will overcome the bias caused by the unbalanced number of links with positive and negative signs in the datasets. It is expecting that the prediction accuracy in the balanced datasets to be less as compared to the original versions.

Figures 1, 2, 3 show the prediction accuracy for both original and balanced versions of Epinions, Slashdot and Wikipedia datasets, respectively. The predictor based on the features obtained from modified HITS algorithm outperforms other algorithms for the balanced version of all three datasets. The modified HITS, shows a prediction accuracy of 93.78, 89.21, and 85.97 for the balanced version of Epinions, Slashdot, and Wikipedia datasets, respectively. This is followed by HITS, BAD, and Prestige for balanced Slashdot, and Wikipedia networks. In all balanced datasets PageRank has the worst performance.

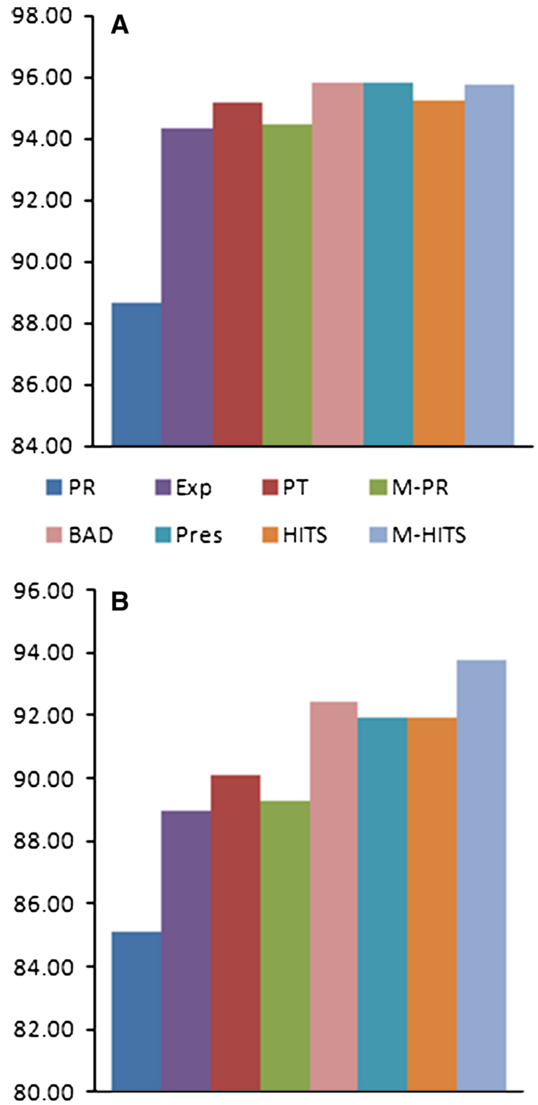

Fig. 1 Prediction accuracy of Rep (Eq. (20)) and Opt (Eq. (21)) based on different ranking algorithms including Prestige (Pres), HITS, modified HITS (M-HITS), PageRank (PR), modified PageRank (M-PR), PageTrust (PT), Exponential (Exp) and Bias and Deserve (BAD). The networks are **a** original Epinions dataset and **b** balanced version of Epinions dataset where the number of links with positive sign is equal to those with negative sign

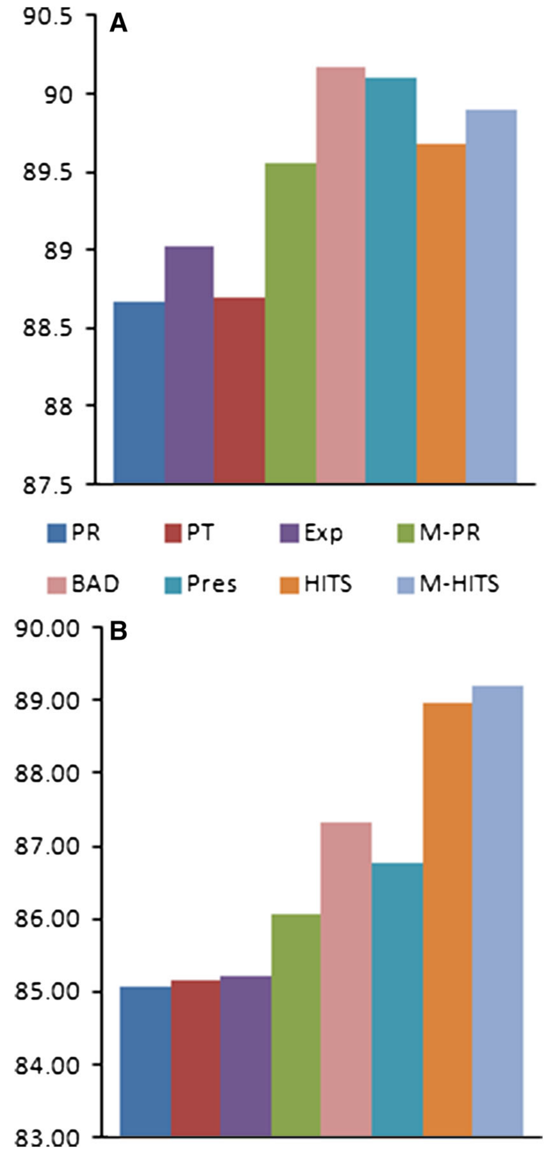

Fig. 2 Prediction accuracy for **a** original Slashdot dataset and **b** balanced version of Slashdot dataset. Other designations are as Fig. 1

The order of prediction accuracy for the original version of datasets is somewhat different as compared to the balanced versions. The top four performers are HITS, modified HITS, Prestige and BAD. Moreover, Exponential ranking prediction accuracy is lower than BAD, Prestige and modified HITS. The regressor based on BAD demonstrates an accuracy of 95.84 for Epinions dataset and is the top performer. The second rank is for the classifier based on Prestige with an accuracy of 95.82 followed by those based on modified HITS (prediction accuracy of 95.76) and HITS (prediction accuracy of 95.23). For this dataset, PageRank shows significantly worse performance as compared to others. Therefore, in this dataset, when computational complexity is not a determining factor, any of the algorithms could be chosen except PageRank that results in poor prediction accuracy. However, if this factor is important, one may choose Prestige and Exponential that have lower computational complexity than others.

For Slashdot dataset, the top classifier is the one based on the values obtained through BAD with prediction accuracy of 90.17. This is followed by the classifier based on Prestige ranking algorithm with accuracy of 90.11, the ones based on modified HITS with accuracy of 89.90 and HITS with accuracy of 89.68. For this dataset, the classification results based on PageRank and PageTrust ranking algorithms have the poorest performance among others.

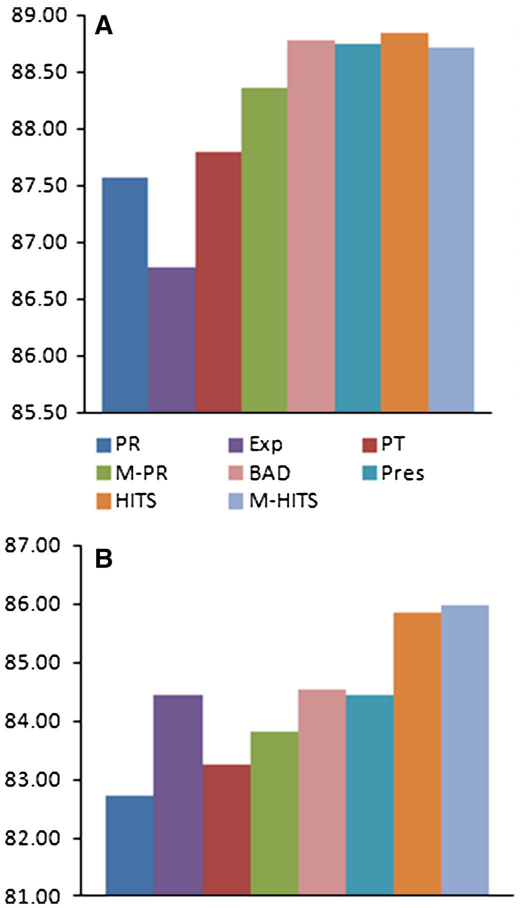

Fig. 3 Prediction accuracy for **a** original Wikipedia dataset and **b** balanced version of Wikipedia dataset. Other designations are as Fig. 1

The top performer for Wikipedia dataset is HITS with prediction accuracy of 88.84, followed by BAD (with accuracy of 88.77), Prestige (with accuracy of 88.75) and modified HITS (with accuracy of 88.72). Exponential algorithm shows the worst performance in Wikipedia dataset followed by PageRank and PageTrust.

4.3 Correlation of ranking algorithms

We compute correlation between different ranking algorithms. Tables 2, 3, 4 show the Spearman correlation coefficients between different ranking algorithms in Epinions, Slashdot and Wikipedia datasets, respectively. The ranking scores are highly correlated in Epinions dataset (with 67.85 % of the correlation coefficients higher than 0.7), followed by Wikipedia (with 53.57 % of the correlation coefficients higher than 0.7) and Slashdot (with 21.42 % of the correlation coefficients higher than 0.7). The rankings obtained by PageTrust show the least correlation with other ranking scores and none of the correlation values exceed 0.41. Among others, Prestige shows high correlations with many of the ranking scores. Prestige that takes into account only the incoming links with negative and positive sign is the simplest algorithm to compute. These results show that not only different ranking algorithms are somehow correlated with the simplest one (Prestige), but also it results in satisfactory prediction accuracy, and thus, for large signed networks where complex algorithms cannot be performed, one might rank the nodes based on their Prestige values.

In Epinions dataset, the highest correlations are between the pair Prestige-BAD followed by Prestige-modified HITS, Prestige-Exponential, BAD-Exponential, modified HITS-BAD and modified HITS-Exponential. This indicates that the results of these four algorithms (Prestige, Exponential, modified HITS, and BAD) are tightly connected. The correlation between Prestige and BAD is also high in Slashdot dataset. The order of highest correlation in Wikipedia dataset is as follows: Prestige-BAD, BAD-exponential, Prestige-Exponential, modified HITS-Prestige, modified HITS-Exponential and modified HITS-BAD. Similar to Epinions dataset, Prestige, Exponential, modified HITS, and BAD are the four ranking algorithms resulting in very similar ranking orders in Wikipedia. Although these algorithms perform similarly in ranking the nodes, they might behave differently when they are used for making an analysis on the network. Finally, the results of Sect. 4.2 and 4.3 indicate that the algorithms resulting in close prediction accuracy also show high correlation in their ranking scores, but the other way around is not always the case.

5 Discussion and future works

A class of social networks are those with both positive (or trust) and negative (or distrust) links. Ranking the nodes in such networks is important. For example, one may want to identify the most reputable, trustable or malicious users. Ranking scores might also be used for identifying the structure of the network, e.g., predicting the sign of the links. There are two main challenges in signed networks. One is to propose a ranking algorithm that best describes nodes' trustworthiness and the other one is to predict signs of hidden edges.

In this manuscript, we consider a number of ranking algorithms for signed networks. Standard PageRank and HITS are considered and are properly modified to fit for signed networks. We also consider available algorithms including Prestige (based on incoming links with positive and negative signs), PageTrust (a modified version of PageRank), Exponential, and Bias and Deserve. These algorithms are applied to three real signed networks including Epinions, Slashdot and Wikipedia datasets. The

Table 2 Spearman correlation coefficient between different ranking scores in Epinions dataset (bold numbers show those correlations higher than 0.7). The ranking algorithms are Prestige (Pres), HITS, modified HITS (M-HITS), PageRank (PR), modified PageRank (M-PR), PageTrust (PT), Exponential (Exp) and Bias and Deserve (BAD) ranking algorithms

	Pres	M-HITS	HITS	Exp	PT	PR	M-PR	BAD
Pres	–	**0.891**	**0.834**	**0.869**	0.189	**0.736**	**0.830**	**0.960**
M-HITS	**0.891**	–	**0.764**	**0.853**	0.259	**0.751**	**0.786**	**0.854**
HITS	**0.834**	**0.764**	–	**0.839**	0.234	0.605	0.675	**0.819**
Exp	**0.869**	**0.853**	**0.839**	–	0.403	**0.810**	**0.852**	**0.860**
PT	0.189	0.259	0.234	0.403	–	0.374	0.368	0.213
PR	**0.736**	**0.751**	0.605	**0.810**	0.374	–	**0.838**	**0.720**
M-PR	**0.830**	**0.786**	0.675	**0.852**	0.368	**0.838**	–	**0.819**
BAD	**0.960**	**0.854**	**0.819**	**0.860**	0.213	**0.720**	**0.819**	–

Table 3 Spearman correlation coefficient between different ranking scores in Slashdot dataset. Other designations are as Table 2

	Pres	M-HITS	HITS	Exp	PT	PR	M-PR	BAD
Pres	–	0.679	**0.736**	0.652	0.020	0.473	0.639	**0.933**
M-HITS	0.679	–	0.424	0.624	0.138	0.562	0.577	0.626
HITS	**0.736**	0.424	–	0.605	0.054	0.267	0.440	**0.714**
Exp	0.652	0.624	0.605	–	0.344	**0.734**	**0.749**	0.635
PT	0.020	0.138	0.054	0.344	–	0.400	0.330	0.025
PR	0.473	0.562	0.267	**0.734**	0.400	–	**0.807**	0.429
M-PR	0.639	0.577	0.440	**0.749**	0.330	**0.807**	–	0.594
BAD	**0.933**	0.626	**0.714**	0.635	0.025	0.429	0.594	–

Table 4 Spearman correlation coefficient between different ranking scores in Wikipedia dataset. Other designations are as Table 2

	Pres	M-HITS	HITS	Exp	PT	PR	M-PR	BAD
Pres	–	**0.881**	**0.780**	**0.983**	0.392	0.562	**0.841**	**0.992**
M-HITS	**0.881**	–	**0.743**	**0.869**	0.372	0.539	**0.761**	**0.880**
HITS	**0.780**	**0.743**	–	**0.769**	0.273	0.271	**0.770**	**0.786**
Exp	**0.983**	**0.869**	**0.769**	–	0.393	0.553	**0.838**	**0.986**
PT	0.392	0.372	0.273	0.393	–	0.407	0.339	0.389
PR	0.562	0.539	0.271	0.553	0.407	–	0.432	0.552
M-PR	**0.841**	**0.761**	**0.770**	**0.838**	0.339	0.432	–	**0.842**
BAD	**0.992**	**0.880**	**0.786**	**0.986**	0.389	0.552	**0.842**	–

scores resulted from these ranking algorithms are used to obtain the features to use in the sign predictor (logistic regression in this work). The predictions are performed on original networks as well as on balanced version of the networks where the numbers of links with negative and positive signs are equal. We find that one of the algorithms proposed in this work (the modified HITS) shows the best performance in terms of prediction accuracy in the balanced version of all three networks. However, for the original networks, no single algorithm results in the best prediction accuracy and four of them (HITS, modified HITS, Prestige and Bias and Deserve) has higher accuracies than others.

One should consider characteristics of ties (e.g., their sign) in a social network before choosing a ranking algorithm. In other words, many of existing ranking algorithms that have been developed for simple networks (those with only positive edges) cannot be applied to signed networks; and thus, specific ranking algorithms should be used for such networks. Each algorithm implemented in this paper except PageRank can be selected if relationships in the social network have properties of trust and distrust and computation complexity is not an issue. But, if computational complexity is an issue, Prestige and exponential ranking are preferred over others. In comparing the performance of the ranking algorithms in different datasets, one can have a closer look at the properties of these datasets. In Wikipedia dataset, users vote on each other based on reputation of the trustee node. Users with reputation often receive more positive votes. While, in Slashdot and Epinions datasets, the votes represent the users' taste and preferences.

Signed networks are important type of networks with many applications. There has been fewer works on these networks than those with only positive links. Community detection methods are among the topics that deserve further attention in signed networks. Most of community detection algorithms try to minimize a fitness function in which it maximizes intra-community densities and simultaneously minimizes inter-community densities. However, community detection in signed graphs has a different perspective and not only the density of the inter- and intra-community links is important, but also their sign is important (maximizing the number of positive links within communities and negative links between them). This could be a future direction of research in this field. Also, community detection could improve the performance of sign prediction and we leave it for a future work.

6 Conclusion

In this work, we investigated ranking nodes in signed networks. We considered a number of existing ranking algorithms and introduced three novel algorithms as modifications to well-known HITS and PageRank. Different ranking algorithms were applied on signed networks inlcluding Epinions, Slashdot and Wikipedia. We also investigated the effect of ranking on sign prediction problem. To this end, we proposed two measures for the nodes based on their ranking scores. These measures were then used in the classification task. By applying the algorithms on three real datasets of signed social networks, we could identify algorithms in which produce higher prediction accuracy in comparison to others. Sign prediction problem is only one of the possible applications of ranking algorithms. Machine learning framework for prediction of the missing signs needs to have a number of node-specific features, and the ranking scores resulting from the algorithms is a possible class of such features. The results obtained in this manuscript showed that these features perform much better than previously proposed ones. Therefore, our work not only introduces excellent features for application in the sign prediction problem, but also provides a platform (i.e., sign prediction) to compare different ranking algorithms.

In this paper, we used only three available benchmarks that have been frequently used in analyzing signed networks; this can be one of the limitations of our results. Unavailability of datasets was the main reason for choosing only three datasets. To have better comparisons, the findings of this manuscript should be verified in more diverse datasets.

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