Program	B. Tech. (SoCS/SoAE)	Semester	II
Course	Advanced Engineering Mathematics II	Course Code	MATH1065
Session	January-May 2025	Unit I	Numerical Methods and Optimization

1. (i) Compute the real root of the equation $x^3 - 5x + 3 = 0$, starting with $x_0 = 1$ and $x_1 = 2$ using the bisection method. Perform ***two*** iterations.
   (ii) Compute a real root of the equation $e^x = 3x$, using the bisection method correct up to ***three*** decimal places.
   (iii) Use the bisection method to find out the positive square root of 28, correct up to ***three*** decimal places.
2. (i) Compute the real root of the equation $x^3 - 3x - 5 = 0$, using Newton-Raphson method correct up to ***three*** decimal places.
   (ii) Find a positive real root of the equation $3x - \cos x = 1$ by Newton-Raphson method, correct up to ***three*** decimal places.
   (iii) The bacteria concentration in a reservoir varies as $C = 4e^{-2t} + e^{-0.1t}$. Using Newton-Raphson method, calculate the time required for the bacteria concentration to be 0.5.
3. (i) Solve the following system of linear equations:
$$2x + 4y + z = 3; \quad 3x + 2y - 2z = -2; \quad x - y + z = 6;$$
   using Gauss's elimination method.
   (ii) Use Gauss-Seidel iterative method to solve the following system of simultaneous equations:
$$9x + 4y + z = -17; \quad x - 2y - 6z = 14; \quad x + 6y = 4;$$
   Perform four iterations.
4. Prove the following relations, where the symbols have their usual meanings:
   (i) $\Delta(1+\Delta)^{-1/2} \equiv \nabla(1-\nabla)^{-1/2}$.
   (ii) $D \equiv \frac{1}{h}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \cdots\right]$.
   (iii) $\nabla - \Delta \equiv -\nabla\Delta$.
5. The following table gives the population of a town during the last six census. Estimate the population in 1913 by Newton's forward difference interpolation formula.

$Years\ (x)$	1911	1921	1931	1941	1951	1961
$Population\ (y)(in\ thousands)$	12	15	20	27	39	52

6. Evaluate $f(3.8)$ from the following table using Newton's backward difference interpolation formula.

x	0	1	2	3	4
$f(x)$	1	1.5	2.2	3.1	4.6

7. The viscosity of a certain kind of oil is experimentally measured at different temperatures as shown in the following table.

Temperature in Celsius	110	130	160	190
Viscosity in Pascal-second	10.8	8.1	5.5	4.8

Find the viscosity of the oil at 140^0C, by Lagrange's method of interpolation.

8. Establish a cubic polynomial of the curve $y = f(x)$ passing through the points $(0, 18)$, $(1, 10)$, $(3, -18)$, $(6, 90)$ using Lagrange's interpolation formula. Also, find the slope of the curve at $x = 2$.

9. Using Newton's divided difference formula, calculate the value of $f(6)$ from the following data:

x	1	2	7	8
$f(x)$	1	5	5	4

10. Develop the divided difference table from the data given below and obtain the interpolation polynomial $f(x)$ using Newton's divided difference formula. Also, find $f''(3.5)$.

x	0	2	3	4
$f(x)$	7	11	28	63

11. Compute $\int_{0.6}^{2} y dx$, where $y(x)$ is given by the following table:

x	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
y	1.23	1.58	2.03	4.32	6.25	8.36	10.23	12.45

12. A curve $y = f(x)$ is drawn to pass through the points given in the following table:

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Find the area bounded by the curve $y = f(x)$, the straight lines $x = 1, x = 4$ and the x-axis using Simpson's $\left(\frac{1}{3}\right)^{\text{rd}}$ rule.

13. The velocities of a car running on a straight road at intervals of 2 minutes are given below:

$Time\ (minutes)$	0	2	4	6	8	10	12
$Vel.\ (km/hr.)$	0	22	30	27	18	7	0

Apply Simpson's $\left(\frac{3}{8}\right)$ rule to find the distance covered by the car.

14. Using Picard's method of successive approximations, obtain a solution up to third approximation of the equation $\frac{dy}{dx} = 1 + xy$, such that $y = 0$ when $x = 0$.

15. Use Euler's method to obtain an approximate value of $y(0.1)$ for the equation

$$\frac{dy}{dx} = x + y + xy, y(0) = 1. \ [\textit{Choose step size, } h = 0.025]$$

16. Estimate $y(1)$ if $2yy' = x^2$ and $y(0) = 2$ using the Runge-Kutta method of fourth-order by taking $h = 0.5$. Also, compare the result with the exact value.

17. Examine the convexity of the set
$$S = \{(x_1, x_2, x_3): 2x_1 - x_2 + x_3 \leq 4\} \subset R^3.$$

18. By applying the Simplex method, solve the LPP:
$$Max. z = 25x_1 + 20x_2$$
subject to the constraints
$$16x_1 + 12x_2 \leq 100$$
$$8x_1 + 16x_2 \leq 80$$
and $$x_1, x_2 \geq 0.$$