Design & Analysis of Algorithm

UNIT-I

Basics of Algorithm

→ What is an algorithm?

An algorithm is a step by step procedure or set of rules to solve a problem or perform a task.

→ History of Algorithm :-

ⓐ Origin – Developed by Al-Khwarizmi in 9th Century

ⓑ Early use – To solve a simple calculation

ⓒ Modern Era –

→ Components of an algorithm

ⓐ Input ⓑ Process ⓒ Output

ⓓ Control structures ⓔ Termination

→ Characterstics of a good algorithm

ⓐ Correctness ⓑ Efficiency ⓒ Clarity

ⓓ Finiteness ⓔ Scalability

→ Application of algorithms

ⓐ finance ⓑ Mathematics ⓒ daily life

ⓓ Artificial Intelligence ⓔ Computer Science.

→ Why do we need Algorithm.

(a) Problem solving (b) Automation
(c) Efficiency (d) Consistancy (e) Enovation

Summation or Sigma notation

It is significant in algorithm analysis. when analysing algorithms specialy loop like structure. It represents total comulational cost over a series of steps

$$\text{Sigma sign} \rightarrow \sum_{i=0}^{n} x_i \leftarrow \text{elements}$$

stopping point (upper limit of summation) → n

index of summation → i

$i = 0$ ← Starting point (lower limit of Summation)

Summation formulas

(i) $\sum_{i=1}^{n} 1 = n$

(ii) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ (Sum of natural number)

(iii) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ (Sum of square of natural no.)

(iv) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$ (sum of cube ")

(v) $\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$ (Sum of fourth " ")

(vi) $\sum_{i=1}^{n} 2i = n(n+1)$ (Sum of even)

(vii) $\sum_{i=1}^{n} = (2i+1) = n^2$ (Sum of Odd)

~~NPR~~ and NCR — Counting

NCR – indicates combination it is used when order does not matter

$$\boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

→ ${}^nP_r$ – It denote permutation It is used when order does matter

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Notes

${}^nC_r$ focuses on ~~loops~~ groups

${}^nP_r$ focuses on arrangements

Q. We have four players A, B, C, D we need to choose a Captain and vice Captain

(1) Selection Order does not matter we only Select two without asigning roles

$${}^nC_r = \frac{4!}{2!(4-2)!}$$

$= 4! = 4\times3\times2\times1$
$= \underline{24}$

$= 2! = 2\times1 = 2$

$=(4-2)! = 2! = 2$

$= \underline{6}$

A, B, C, D

(A, B) (A, C), (A, D)

AB is the same as BA because we are just Picking two players.

^nP_r arrangement order matters we asign Captain and vice captain roles so that Order matters.

A(Captain), B(vice) $\rightarrow$ (A, B)
B (Captain), A (vice) $\rightarrow$ (B, A)

$$^nP_r = \frac{n!}{(n-r)!} = \frac{4!}{(4-2)!}$$

$$= \frac{4\times 3\times 2\times 1}{(\cancel{4}\ 24 - 2\times 1)} = \underline{12}$$

A, C $\rightarrow$ (A, C)
C, A $\rightarrow$ (C, A)
A, D $\rightarrow$ (A, D)
D, A $\rightarrow$ (D, A)
B, C $\rightarrow$ (B, C)
C, B $\rightarrow$ (C, B)
B, D $\rightarrow$ (B, D)

D, B $\rightarrow$ (D, B)
C, D $\rightarrow$ (C, D)
D, C $\rightarrow$ (D, C)

A, B not equal B, A because A as a Captain and B as vice captain is different from B as a Captain and A as a vice captain

Conclusion –

^nC_r with 6 cases it denotes just selection and Order does not matter

^nP_r with 12 cases it denotes roles are asign and Order matters

asign
hat

Q.1) Solve it sigma

$$\sum_{i=1}^{5} i = 1+2+3+4+5 = \frac{5(5+1)}{2} = \frac{30}{2}$$

ii) Solve it sigma m^2 $m=4$, $i=1$

$$\sum_{m=1}^{4} m^2 = 1^2+2^2+3^2+4^2$$
$$= \underline{\underline{30}}$$

Q. Finding Σ when using a constant instead of i

(a) when constant $= 1$

$$\sum_{i=1}^{n} 1 = 1+1+1 \ldots\ldots + 1 \text{ (times)}$$

ex $\rightarrow$ $$\sum_{i=1}^{10} 1 = 10$$

(b) when constant $= c$ (any fixed number)

$$\sum_{i=1}^{n} c = c+c+c \ldots\ldots + c \text{ (n times)} = c \times n$$

or

$$\sum_{i=1}^{3} 4 = 3 \times 4 = 12$$

Sigma with varying lower bound

(a) lower bound is not 1

$$\boxed{\sum_{i=2}^{n} i = \sum_{i=1}^{n} i - \sum_{i=1}^{1} i}$$

ex $\rightarrow$ $$\sum_{i=2}^{5} i = \sum_{i=1}^{5} i - \sum_{i=1}^{1} i$$

$$= \frac{n(n+1)}{2} = 15 - 1 = 14$$

(11) Constant term with varying lower bound

$$\boxed{\sum_{i=a}^{n=b} c = c \times (b-a+1)}$$

ex $\rightarrow b=6 \quad a=3 \quad c=4$

$$= 4 \times (6-3+1)$$

$$\sum_{3}^{6} 4 = 4 \times (6-3+1)$$

$$= 16 \cancel{(6-3+1)}$$

$$= \underline{16}$$

```
int b = 6, a = 3, C = 4; res = 0
for (int i = 0; i < (b - a + 1); i++) {
        res += C;
    }
    Printf("Res: %d\n", res);
```

Counting

The rules of Sum and Product :-

Q. The rules of

If a sequence of task $T_1, T_2 \ldots T_n$ can be done $w_1, w_2 \ldots w_n$ ways respectively the condition is that no task can be perform simultaneously then the no. of ways ~~loop 1~~ do one of these task is $w_1 + w_2 + \ldots w_n$

If we consider two Task a and b which are disjoint $A \cap B = \phi$ then mathematically $|A \cup B| = |A| + |B|$

The rules of Product

Similarly $A \cap B = \phi$

$|A * B| = |A| * |B|$

Q. A boy lives at X and wants to go to school at Z from his home X has ~~to~~ first reach Y and then Y to Z. He may go ~~to~~ X to Y either ~~he~~ 3 bus roots ~~to~~ ~~or~~ or 2 train from there he can either choose 4 bus roots or 5 train roots to reach Z. How many ways for where to go from X to Z

Soln

from X to Y he can go in ~~3 bus to~~ 3+2 = 5 ways (rule of sum)
There-fore he can go Y to Z 4+5 = 9 ways (rule of sum) hence from X to Z he can go in 5 ~~to~~ 9 = 45 ways (rule of product)

X Y Z
3,2 4,5

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Q. Solve it

$$\frac{1}{n}\sum_{x=1}^{n}(x^2+x+1)$$

$$= \frac{1}{n}\left[\sum_{x=1}^{n}x^2 + \sum_{x=1}^{n}+x + \sum_{x=1}^{n}1\right]$$

$$= \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n\right]$$

$$= \frac{1}{n}\left[\frac{n(n+1)(2n+1)+3n(n+1)+6n}{6}\right]$$

$$= \frac{n}{n}\left[\frac{(n+1)(2n+1)+3(n+1)+6}{6}\right]$$

$$= \frac{2n^2+n+2n+1+3n+3+6}{6}$$

$$= \frac{2n^2+6n+10}{6} = \frac{\cancel{2}(n^2+3n+5)}{\cancel{6}}$$

$$= \frac{n^2+3n+5}{3} \quad \underline{\text{Ans}}$$

Set Theory – Sets in mathematics are simply a collection of disting object forming a group. A set can have any group of items such as collection of numbers all days.

$A = \{1, 2, 3, 4, 5\}$

$B = \{SUN, MON, TUE\}$

Representation of sets in set theory.

There are different set notation use for the representation of sets in set theory such as

(i) Semantic form (iii) Set builder form
(ii) Roster form

Example :-

(i) Sematic form – A set of first five even natural number.

(ii) $\{2, 4, 6, 8, 10\}$ (iii) $\{x \in N / x \leq 10$ and n is even$\}$

The statement says all the element of set a are even numbers that are less than or equal to 10.

Note

Sometimes a colon (:) is used as the place of vertical bar.

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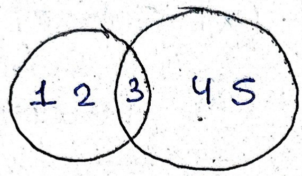
Representation of set using venn diagram

Its is factorial representation of set in which each set is represented by a circle. The element of a set are present inside the circle. Sometimes a rectangle in closes in circle which are known as universal set



Set A = {1, 2, 3}
Set B = {3, 4, 5}

Common Element of set A and set B is 3

Set Symbol –

Symbol	meaning
{ }	Symbol of set
U	Universal set
$n(X)$	Cardinal no. of set
$b \in A$	b is an element of set A
$a \in B$	a is not an element of set B
ϕ	Empty set or null set
$A \cup B$	Set A union set B
$A \cap B$	Set A intersection Set B
$A \subseteq B$	Set A is a subset of Set B
$B \subseteq A$	Set B is the super set of set A

Types of sets

(i) Singleton set
(ii) Finite set
(iii) Infinite set
(iv) Empty or null set
(v) Equal set
(vi) Unequal set
(vii) Equivalent set
(viii) Overlapping set
(ix) Disjoint set
(x) Subset and superset set
(xi) Universal set
(xii) Power set

(i) $S = \{1\}$ only one element
(ii) $S = \{1, 2, 3, 4, 5\}$ fixed element
(iii) $S = \{1, 2, 3, 4 \ldots\}$ unlimited element
(iv) $S = \{\ \}$ no element
(v) $A = \{1, 2, 3\}$ $B = \{1, 2, 3\}$ $A = B$
(vi) $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4\}$ $A \neq B$
(vii) $A = \{1, 2, 3\}$ $B = \{a, b, c\}$ Same no. of element
(viii) ~~A = {1, 2, 3~~ ~~B = {a, b, c}~~
$A = \{1, 2, 3\}$ $B = \{1, 4, 5\}$ at least one common
(9) $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$ no common element
(10) $A = \{2, 3, 5, 7\}$
$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Subset – A is subset of B
Superset – B is a superset of A

(11) $U = \{0, 1, 2, 3 \ldots 9\}$
all possible elements
(12) $A = \{1, 2, 3\}$ $P(A) = \{\varnothing\}, \{1\}, \{1, 2\} \{1, 3\}$ 2^n
all possible subset $\{2, 3\}$ $\{1, 2, 3\}$

Examples -

(I) Used in base cases of recursion

(II) Used in shorting algorithm where input size is.

(III) Used in theory of computation (Infinite loops)

(IV) Used in edge cases in searching in graph traversal

(V) Used in checking data inconsisting

(VI) Used in finding missing element

(VII) Used in checking memory allocation

(VIII) Used in intersaction operation in data bases

(IX) Used in graph colouring that ensures no two ~~abs~~ adcent nodes share a color.

(X) Used in pattern matching and search algorithms

(XI) Used in complement operations (finding missing elements)

(XII) Used in backtracking algorithm

Sigma Notation in Sets

(1) With finite sets

$$\sum_{i=1}^{n} s_i \quad \text{If } S = \{2, 4, 6, 8\}$$

Soln

$$\sum_{i=1}^{4} s_i = 2+4+6+8 = 20$$

(ii) for counting elements in sets

Sigma can count occurances in a universal set U $S = \{1,2,3,4,5,6\}$

Ex → $$f(s) = \begin{cases} 1, & \text{if } s \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{i=1}^{6} f(s) = 0+1+0+1+0+1 = 3$$

(iii) for Powersets

$A = \{1,2,3\}$

$B = \{\phi\}, \{1\}, \{1,2\}, \{1,2,3\}$

all possible subset

ex → $$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$

$$\sum_{i=0}^{3} \binom{3}{0} = 2^3 = 8$$

(iv) ~~Union~~ for set operation

Union and Intersaction

$A = \{1,2,3\}$ $B = \{3,4,5\}$

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= \sum_{i=1}^{6} |A| + |B| - |A \cap B|$$
$$= 3+3-1 = 5$$

(v) for weighted samations in set

$X = \{(2,0.3), (4,0.5), (6,0.2)\}$

Soln

$$\sum_{i=1}^{3} x_i w_i = (2 \times 0.3) + (4 \times 0.5) + (6 \times 0.2)$$
$$= \underline{3.8}$$

a Universal

Properties of Sets

(I) Commutative (II) Associative (III) distributive
(IV) Identity (V) Complements (VI) Idempotent

If $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 3, 4, 5\}$ $C = \{1, 2, 3, 4, 5\}$

(I) $A \cup B = B \cup A$
$A \cap B = B \cap A$

(II) $(A \cap B) \cap C = A \cap (B \cap C)$
$(A \cup B) \cup C = A \cup (B \cup C)$

(III) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(IV) $A = \{1, 2, 3\}$ $U = \{1, 2, 3, 4, 5\}$
$A \cup \emptyset = A$
$A \cap U = A$

(V) $A \cup A' = U$
$A \cap A' = \emptyset$

(6) $A \cup A = A$
$A \cap A = A$

(VI) Same set repeated gives the same set

Conclusion –
(I) Order doesn't matter
(II) grouping doesn't matter
(III) Union & intersection distributive
(IV) Empty & universal set act as a identity
(V) a set and its complement form a whole

Probability –

It defines the likely of occurrance of the implem event. It can define as the ratio of the no. of favorable outcomes of an to the total no. of outcomes of events.

$$\text{Probability (Event)} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{x}{n}$$

$P(E) = 0$ If and only if E is an impossible event

$P(E) = 1$ if and only if E is certain event.

$0 \le P(E) \le 1$

Q. In a coin there is probability $\frac{1}{2}$. Both side of a coin as equal probability that is 0.5

$P(H) = \frac{1}{2}$ $P(T) = \frac{1}{2}$

(a) tossing two point we have total 4 outcome
2^2 (H, H), (H, T), (T, H) (T, T)

(b) Rolling one dice
Sample space {1, 2, 3, 4, 5, 6}
P(Even) = 2, 4, 6 P(Prime) = 2, 3, 5
P(odd) = 3, 1, 5

(c) Rolling two dice
Total no. of outcome rolling two dice is 36

$= \frac{18}{36} = \frac{1}{2} = 0.5$

(d) Probability of drawing cards
~~bb~~ black - 26
Hearts - 13
face ~~phase~~ - 12
Card numbered - 4
Red card numbered - 4

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Basic Probability rules

(a) Additional rules for mutually exclusive events $P(A \cup B) = P(A) + P(B)$

ex - Probability of rolling a two (2) or a five (5) on a dice

$P(A \cup B) = P(2) + P(5)$

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} + \frac{1}{3}$

(b) Mo

Ex -

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(c) Pro

for a

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(d)

Ex

= (

(e)

Both sides
is 0.5

4 Outcomes

(b) Multiplication rule – (for independent events)

$P(A \cap B) = P(A) \times P(B)$

Ex – Crossing two coin probability of getting two heads

$P(HH) = P(H) \times P(H)$

$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(c) Probability distribution

$$\sum_{i=1}^{n} P(X = x_i) = 1$$

For a discreate random variable x without comes $x_1, x_2, x_3 \ldots\ldots x_n$

Example –

A dice has 6 outcomes the ~~some~~ sum of Probability

$$\sum_{i=1}^{6} \frac{1}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

(d) Expected mean value

$$E(x) = \sum_{i=1}^{n} x_i P(X = x_i)$$

Ex – A weighted dice has probabilities $P(1) = 0.1$, $P(2) = 0.2$, $P(3) = 0.3$, $P(4) = 0.2$, $P(5) = 0.1$, $P(6) = 0.1$

$= (1 \times 0.1) + (2 \times 0.2) + (3 \times 0.3) + (4 \times 0.2) + (5 \times 0.1) + (6 \times 0.1)$

$= 3.3$

(e) Variance –

$$Var(x) = \sum_{i=1} x_i - E(x))^2 \; P(x - x_i)$$

$= (1 - 3.3)^2 + (2 - 3.3)^2 + (3 - 3.3)^2 + (4 - 3.3)^2 + (5 - 3.3) + (6 - 3.3)^2$

$= 5.29 + 1.69 + 0.09 + 0.49 + 2.89 + 7.29$

$= 17.74$

Relation — A relation is a collection b/w elements of two sets if A and B are sets then a relation R is a subset of there cartesian product [A X B]

Example — $A = \{1,2\}$, $B = \{3,4\}$

$R = A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

What is Recurrance relation ?

A recurrence relation defines a sequence where each term depends on previous term

Example - fibonacci series.

$f(n) = f(n-1) + f(n-2)$

where $f(0) = 0$, $f(1) = 1$

Relation b/w recurrence relations and relation

It defines a relationship within a set (sequence of numbers)

To connects a term in a sequence to its previous terms

Example — factorial relation

$$\boxed{n! = n(n+1)!}$$

Relation and Function :-

A function is a special type of relation when each input (x in A) have exactly one output (y in B)

ex — $f(x) = x+3$

$f(1) = 1+3 = 4$

$f(2) = 2+3 = 5$

$f(3) = 3+3 = 6$

$R = \{(1,4), (2,5), (3,6)\}$

Growth of function –

A describes how a function be hours as input (n) increases. It is used to analyzed the efficiency of algorithms interms of algorithm of efficiency time and space complexity

ex – $f(n) = n^2$, $g(n) = n$

- $f(n)$ growth faster than $g(n)$
- for n the difference is small
- for large n $f(n)$ becomes must larger than $g(n)$

$n = 1, 0.2, 2$

$f(1) = (1)^2 = 1$

$f(0.2) = (0.2)^2 = 0.04$

$f(2) = (2)^2 = 4$

Growth analysis using sigma

i) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ It increases quadraticolly

ii) $\sum_{i=1}^{n} 1 = n$ constant linear $O(n)$

Domain & Range with function

The domain of the functions its the set of all Possible input values (x) for which the function is defined

$f(x) = x^2$

Any real no. $x \to (-\infty, \infty)$

$f(x) = \frac{1}{x}$

undefined where $x = 0$ $(-\infty, 0) \cup (0, \infty)$

Range – The range of the function is the set of the all possible output (y) when we input value from the domain.

(i) $f(x) = x^2$

Since x^2 is always positive then the output is $x \to [0, \infty)$

(ii) $y \neq f(x) = \frac{1}{x}$ any real number except $y = 0$ so range $(-\infty, 0) \cup (0, \infty)$

(iii) $f(x) = \sqrt{x}$

domain $x \geq 0$ [Square root is undefined for negative numbers)

range $y \geq 0$ output is always positive

Domain and range using flipkart, hub and delivary location

(i) Flipcart hub :- (Input) Domain

(ii) Delivary location (Output) Range

(iii) A function maps hub to delivary location based certain rules.

(a) Delivary time ~~location~~ function -

$T(d) = 2d + 1$

$T(d)$ = delivery time in hours

d = ~~delivery~~ distance from hub in km

Domain (input) = Distance from hub $d \geq 0$

Range (output) = Delivary time $T(d) \geq 1$ hours

If location is 0km away the retusning time is 1 hours is

range

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(ii) If location is 5km away the returning time is 11 hours

$T(d) = 2d + 1$

$= 2 \times 5 + 1 = 11h$

(b) Delivery charges function

$C(d) = 10 + 5d$

$C(d) =$ delivery charge in rupees

$d =$ distance in km

domain (input) :- Distance cannot be -ve

$d \geq 0$

Range (output) :- minimum 4 ~~stop~~ Charge is

$C(d) \geq 10$

(i) If ~~min~~ location ~~location~~ is 0km away the ~~returning~~ delivery charge ~~time~~ is ~~10 h~~

$C(d) = 10 + 5 \times 0$

$= 10$

(ii) If location ~~Charges~~ is 5km away the ~~returning~~ delivery charge ~~time~~ is ~~11 h~~.

$T(d) = 10 + 5 \times 5$

$= 10 + 25$

$= 35$

(c) Number of deliveries per hours

$D(h) = 500h$

$D(h) =$ total delivery

$h =$ No. of flipcart hubs

Domain (input) : atleast 1 hub is needed $h \geq 1$

Range (output) : Each hub delivers atleast 500 parcel $d(h) \geq 5$

(i) If ~~location~~ min is 1 hub

$D(h) = 500h$

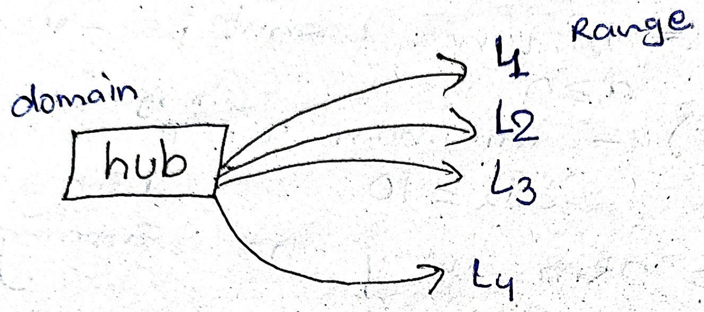
$= 500 \times 1 = 500$

(ii) If ~~location~~ max is 5 hub

$D(h) = 500h$

$= 500 \times 5$

$= 2500$



Analysis of algorithm

(i) Performance — how much time or memory or disk is used when a program is run. This depends on the machine, compiler etc as well as the code we write

(ii) Complexity — how do the resource requirements of a program or algorithm scale that is what happens as the size of the problem being solved ~~whit~~ by the code gets larger

$$\left(1 + \frac{1}{n}\right)^n$$

Problem
↓
Algorithm
↓
input → Program → output

Types of Algorithm analysis

(I) Best case :- Defines the input to which algo takes minimum time it calculate lower count.

(II) Worst case :- It takes maximum time on give it calculates upper count

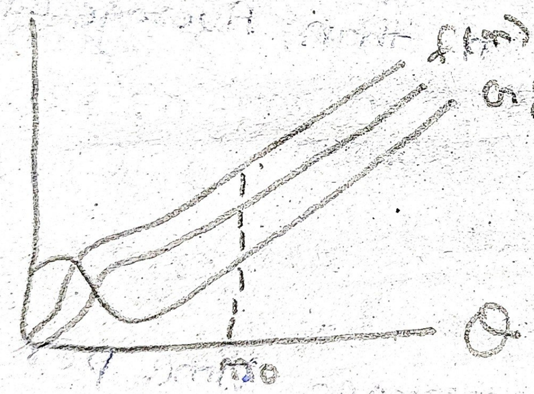
(III) Average case :- It takes all random inputs & calculate the compleation time.

Asympotatic Notation

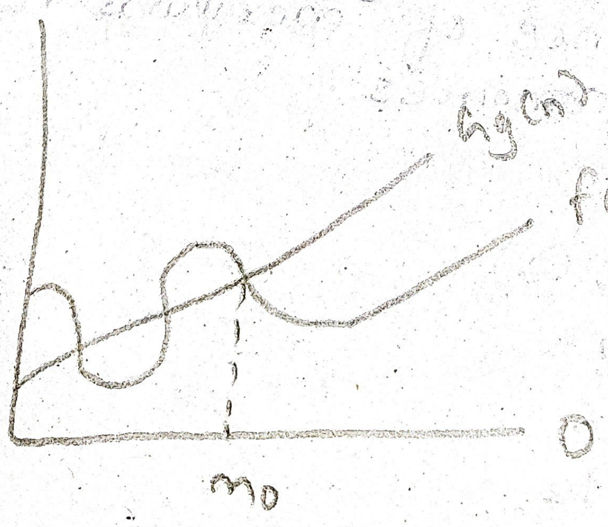
Big (O) maximum amount

Omega (Ω) minimum

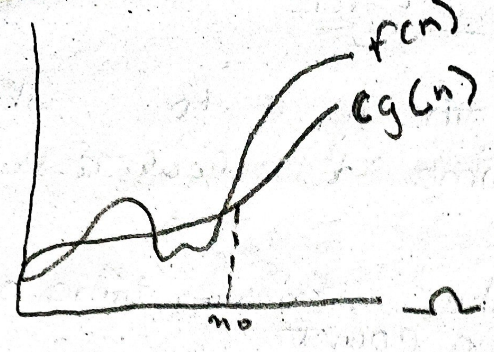
Theta (θ) Average amount



[Average case]



[Worst case]



[Best case]

i.e.g $f(x) = x^2 + x + 1$

- Best case – $x = 0$
  $f(0) = 0^2 + 0 + 1 = 1$
- worst case – $x = n$
  $f(n) = n^2 + n + 1$
- Average case – $x = \frac{1}{n} \sum_{x=1}^{n} (x^2 + x + 1)$

Over the range from 1 to n, than Average case Complexity.

$$\frac{1}{n} \sum_{x=1}^{n} (x^2 + x + 1)$$

Amortized Analysis :–

Is used to find the average time per operation over a sequence of operations it focus on long term performances.

Suppose $f(x) = x^2 + x$
operation for Small (x)

$f(1) = 1^2 + 1 = 2$
$f(2) = 2^2 + 2 = 6$
$f(3) = 3^2 + 3 = 12$

When x is small [1, 2, 3] the lower order term matters more in calculation

- When x is larger [10, 100, 1000] the higher order term dominates and lower terms become insignificant.

$f(10) = x^2 + x$ (Asympotation)
$= 10^2 + 10$
$= 110$

$f(100) = (100)^2 + 100$
$= 10,100$

$f(1000) = (1000)^2 + 1000$
$= 1001,000$

(i) At $x = 2$, $f(x) = 6$ both x^2 and x are important

(ii) At, $x = 100$ $f(100) = 10,000$, x^2 dominates and x is negligable.
Which mean it x value is lower such as (1, 2, 3) compair to [10, 100, 1000]

Q. Difference between

Amortized analysis	Asymoptative
(i) Sequence of operation	(i) individual Operation
(ii) Average performance	(ii) Worst-case Performance
(iii) For algorithms with expensive operation	(iv) For describing limiting behaviour

(i) Big - O Notation (upper bound)

$$f(x) \le c_1 g(x) \text{ for all } x \ge x_0$$

let $f(x) = x^2 + x$ and $g(x) = x^2$

$$x^2 + x \le 2x^2 \text{ for } x \ge 1$$

(ii) Big - Ω Notation (lower bound)

$$f(x) \ge C_2 \, g(x) \text{ for all } x \ge x_0$$

let $f(x) = x^2 + x$ and $g(x) = x^2$

$$x^2 + x \ge 1x^2$$

(iii) Big - Θ Notation (Tight Bound)

$$C_2 \, g(x) \le f(x) \le c_1 \, g(x) \text{ for } x \ge x_0$$

$$1x^2 \le x^2 + x \le 2x^2$$

$f(x) = x^2 + x$ $\quad g(x) = x^2$ $\quad f(x) \le c_1 g(x)$

$f(1) = (1)^2 + 1 = 2$ $\quad g(1) = 1$ $\quad 2 \le 2(1)$

$f(2) = (2)^2 + 2 = 6$ $\quad g(2) = (2)^2 = 4$ $\quad 6 \le 2(4)$

$f(3) = (3)^2 + 3 = 12$ $\quad g(3) = (3)^2 = 9$ $\quad 12 \le 2(9)$

$f(4) = (4)^2 + 4 = 20$ $\quad g(4) = (4)^2 = 16$ $\quad 20 \le 2(16)$

$f(5) = (5)^2 + 5 = 30$ $\quad g(5) = (5)^2 = 25$ $\quad 30 \le 2(25)$

for $f(x) = x^2 + x$ upper bound is at most
Big-O x^2 lower bound as at least x^2
and Tight bound exactly x^2

(I) for x like 1, 2, 3, 4 x is no disable
(II) for large like 10, 100, 1000 dominates

Explain divide complex and combine approach

Divide - break the problem in smaller sum problem

wrong

Combine - Marge the problem to get the final solution

Example -

Binary search, Marge short

Power calculation of x^n

for even :- $x^n = (x^{n/2})(x^{n/2})$

for odd - $x^n = (x^{(n-1)/2})(x^{(n-1)/2})$

~~Pow~~