Final Project: Vehicle Steering using MPC

Multivariable Control Theory, MAE 6780

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Abstract

In autonomous vehicles, such as self driving cars, control is essential for applications like collision avoidance and navigation. Control theoretic approaches such as Model Predictive Control (MPC), have been shown effective at capturing the behavious of human drivers for a given vehicle model, trajectory and speed profile [2]. MPC is chosen because of its capability of systematically taking into account nonlinearities, future predictions and operating constraints during the control design stage. MPC is demonstrated to be effective for the chosen vehicle model, in achieving a specified reference trajectory without violating designer specified constraints.

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1 Introduction and Theory

1.1 Vehicle Model

The modeling of vehicle dynamics has been extensively studied in the past decades and a wide spectrum of vehicle models have appeared in literature. Figure 1 shows a sketch of a vehicle body fixed frame of reference and the forces acting on the vehicles center of gravity. x, y, z are the vehicle's longitudinal, lateral and verticle axes respectively, $\dot{x}$, $\dot{y}$ and $\dot{\psi}$ are the longitudinal, lateral velocities and yaw rate. F_x, F_y and M_z are the longitudinal, lateral forces acting on the vehicle CoG and the rotating moment about z axes.

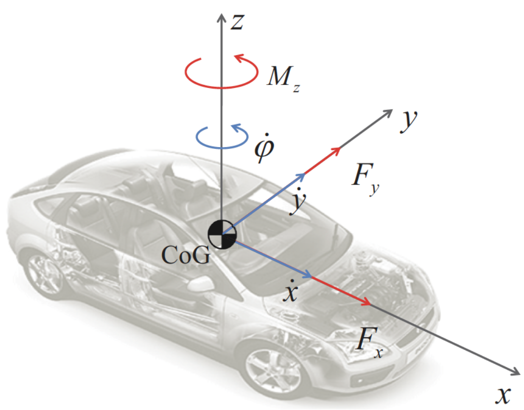


Figure 1: Vehicle body fixed frame of reference

The rigid body dynamic equations [1] are :

$$m\ddot{x} = m\dot{y}\dot{\psi} + F_x \tag{1.1}$$

$$m\ddot{y} = -m\dot{x}\dot{\psi} + F_y \tag{1.2}$$

$$I_z\ddot{\psi} = M_z \tag{1.3}$$

where m is the mass of the vehicle and I_z is the vehicle's moment of inertia about the z-axis. In this project, a linear bicycle model is chosen, which makes the following assumptions and simplifications:

i) Only the front steering angle can be controlled. Moreover, the front left and front right wheel steering angles are equal.

ii) At the vehicle front and rear axels, the left and right wheels are lumped together as shown in Figure 2.

iii) A linear tire model is used in which $F_y = K\alpha$, where F_y is the lateral force on the tire, K is the respective cornering stiffness and α is the slip angle.

iv) The vehicle travels at a constant longitudinal velocity, i.e. $\ddot{x} = 0$.

v) $v_y << v_x$ i.e small slide slip angle

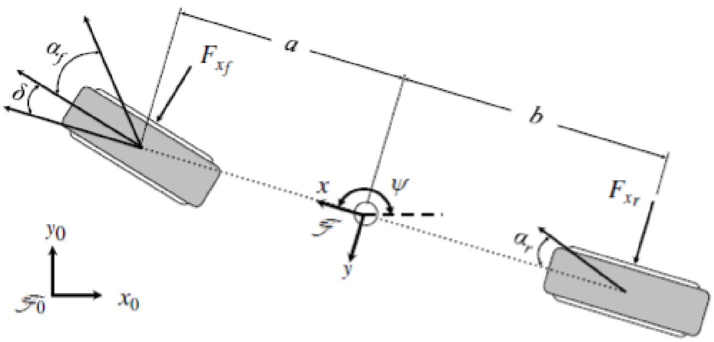


Figure 2: Linear bicycle model

Using the above assumptions and simplifications the dynamics equations become the following:

$$m(\ddot{y} + \dot{\psi}\dot{x}) = F_{yr} + F_{yf} \tag{1.4}$$

$$I_z\ddot{\psi} = aF_{yf} - bF_{yr} \tag{1.5}$$

where the front and rear lateral forces are modelled as:

$$F_{yf} = K_f\alpha_f \tag{1.6}$$

$$F_{yr} = K_r\alpha_r \tag{1.7}$$

and the front and rear slip angles can be written as:

$$\alpha_f = \delta - \frac{a\ddot{\psi} + \dot{y}}{V} \tag{1.8}$$

$$\alpha_r = \frac{b\dot{\psi} + \dot{y}}{V} \tag{1.9}$$

The description of the parameters and symbols used in the above equations are summarized in Table 1

m	mass of the vehicle
F_{yf}	front lateral force
F_{yr}	rear lateral force
a	distance of front tire from center
b	distance of rear tire from center
α_f	front slip angle
α_r	rear slip angle
K_f	front tire cornering stiffness
K_r	rear tire cornering stiffness
δ	steering angle
ψ	yaw

Table 1: Model parameters and variables

1.2 State Space Model

The coordinate system used in this project for a straight, single lane vehicle is shown in Figure 3. The state vector, output and control input is defined as:

$$\mathbf{x} = \begin{bmatrix} Y & V_y & \psi & \dot{\psi} \end{bmatrix}^\top \tag{1.10}$$

$$\mathbf{y} = \begin{bmatrix} Y & \psi \end{bmatrix}^\top \tag{1.11}$$

$$\mathbf{u} = \begin{bmatrix} \delta & \nu \end{bmatrix}^\top \tag{1.12}$$

where ν is the yaw rate disturbance describer later in this report. The state space model can be described as:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{1.13}$$

$$\mathbf{y} = C\mathbf{x} \tag{1.14}$$

where,

$$A = \begin{bmatrix} 0 & 1 & V & 0 \\ 0 & -\frac{2K_f + 2K_r}{mV} & 0 & -V - -\frac{2K_f a - 2K_r b}{mV} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2K_f a - 2K_r b}{I_z V} & 0 & -\frac{2K_f a^2 + 2K_r b^2}{I_z V} \end{bmatrix} \tag{1.15}$$

$$B = \begin{bmatrix} 0 & 0 \\ \frac{2K_f}{m} & 0 \\ 0 & 0 \\ \frac{2K_f a}{I_z} & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{1.16}$$

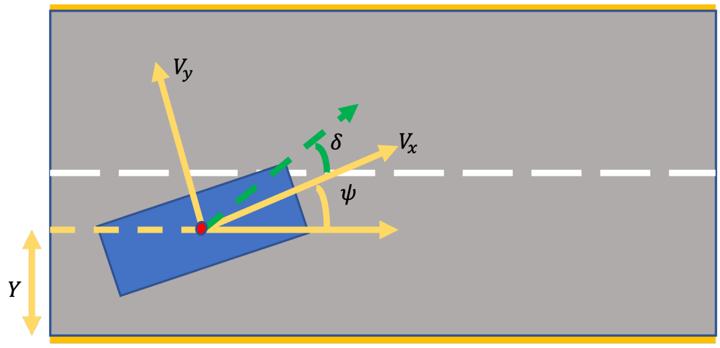


Figure 3: Coordinate system and state variables

1.3 Disturbance model

Unmeasured disturbances like cross-winds add an undesirable yawing effect on the vehicle. A wind disturbance model is used to generate realistic noise for the vehicle which depends on resultant velocity between the vehicle and the wind (V_{res}), wheelbase (W), relative angle between the vehicle and the cross-wind (β) and the moment of inertia (I_z). The yaw acceleration noise is given by equation 1.17, as described by Zhang et al. [3]

$$\nu = \frac{C_{YM}\rho V_{res}^2 AW}{2I_z} \tag{1.17}$$

1.4 Model Predictive Control

Model predictive control (MPC) is based on iterative, finite-horizon optimization of a plant model. An online calculation is used to explore state trajectories and find a cost minimizing strategy. Only the first step of the control strategy is implemented and then the calculations are repeated. The cost function used in this project is defined as:

$$J = \sum_{i=1}^{N} w_{x_i}(r_i - x_i)^2 + \sum_{i=1}^{N} w_{u_i}(\Delta u_i)^2 \tag{1.18}$$

where x_i is the state variable, r_i is the reference state, u_i is the manipulated variable (in our case, the steering angle), w_x is the weighting coefficient matrix for states, and w_u is the weighting coefficient matrix for change in the manipulated variable. In model predictive control, J is minimized using the control input u without violating the constraints for the states or the control input. In this project, the constraints supplied were as follows:

$$-\pi/2 \leq \delta \leq \pi/2 \tag{1.19}$$

$$-\pi/6(s^{-1}) \leq \dot{\delta} \leq \pi/6(s^{-1}) \tag{1.20}$$

$$-\pi/4 \leq \psi \leq \pi/4 \tag{1.21}$$

2 Procedure

2.1 Simulink Model

To tune the controller and simulate the plant response, we created a Simulink model of the plant and the controller as shown in Figure 4. The plant model is specified in state space form (labeled as Bicycle model), a signal generator block is used to provide the reference values for the lateral position (Y) and the yaw angle (ψ). These reference values are sent as input into the MPC block along with the measured output from the plant block. The output of the MPC block (steering angle) is fed into the plant as input, along with the yaw rate noise, which is generated in a separate block. Scope blocks are used to record and display the ouputs.

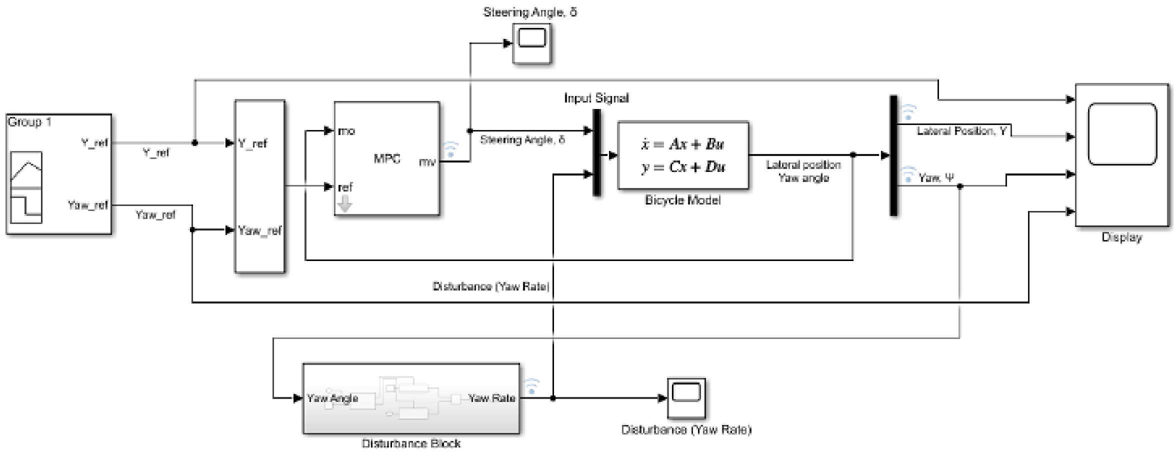


Figure 4: Simulink Model

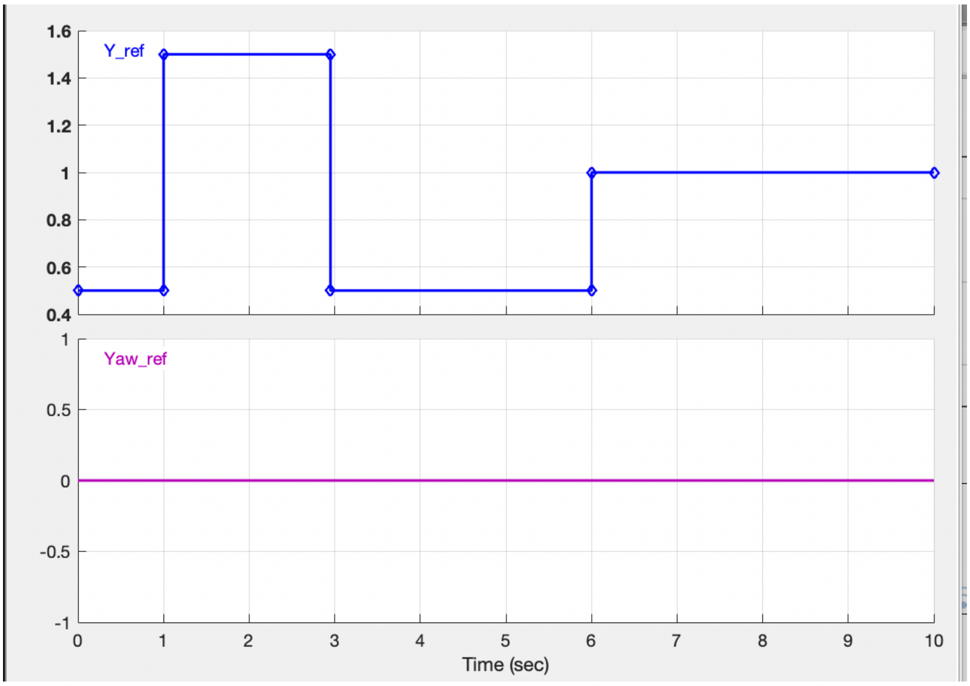


Figure 5: Reference signal

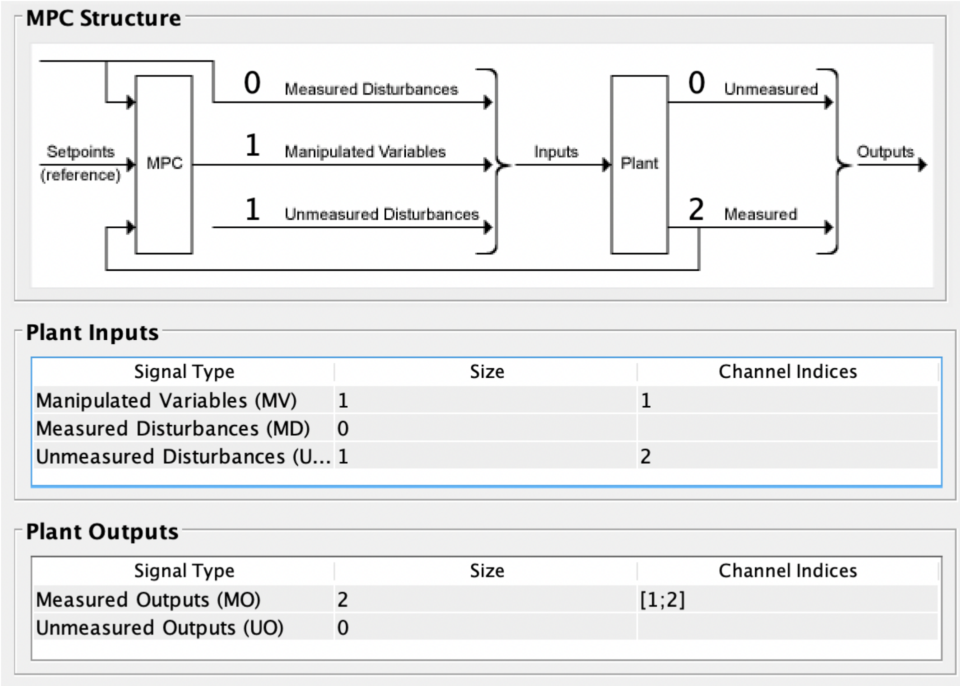
2.2 Generating reference values

Reference values were generated keeping in mind a simple example of a car wanting to change lanes on a one-way road to avoid obstacles as shown in Figure 5. In this example, the reference signal expects the car to stay in the right lane for 1 second, shift to the left lane for 2 seconds, switch back to the right lane for 3 seconds, and then move to the center of the road till the end of the simulation.

2.3 MPC Tuning

There are several parameters that dictate the performance of the MPC block in the simulation. The main MPC structure is shown in Figure 6 where the plant has two measured outputs, two inputs out of which one is a manipulated variable (steering angle) which is an output of the MPC block and one is an unmeasured disturbance (yaw rate disturbance due to wind).

The horizon parameters determine how many future time-steps the MPC will evaluate the trajectory and also how many control steps it will optimise. These are summarised in Figure 7.



Signal Type	Size	Channel Indices
Manipulated Variables (MV)	1	1
Measured Disturbances (MD)	0	
Unmeasured Disturbances (U...	1	2

Signal Type	Size	Channel Indices
Measured Outputs (MO)	2	[1;2]
Unmeasured Outputs (UO)	0	

Figure 6: MPC structure

The weight matrices for the control input and the outputs are summarised in Figure 8.

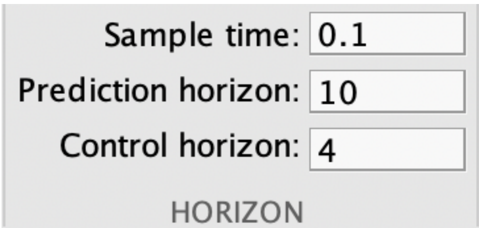
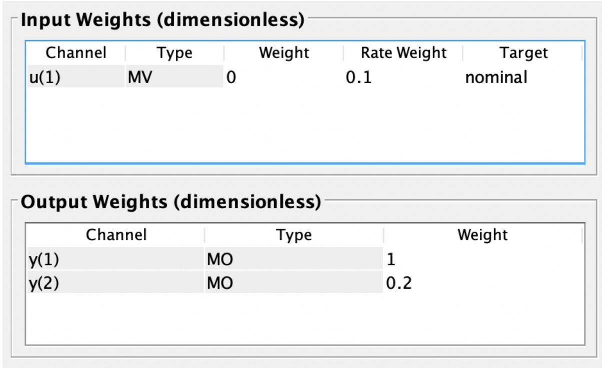


Figure 7: Horizon parameters

There is only one control input, hence the cost function penalises the change in steering angle. Out of the two measured outputs, error in lateral position (Y) has a higher weight than error in yaw angle (ψ). This is because the lateral position of the vehicle on the road is more important to us than the path it took to get there as long as its within reasonable constraints (1.21).

Constraints were specified for the respective states and control input as defined in equations 1.19 - 1.21 as shown in Figure 9



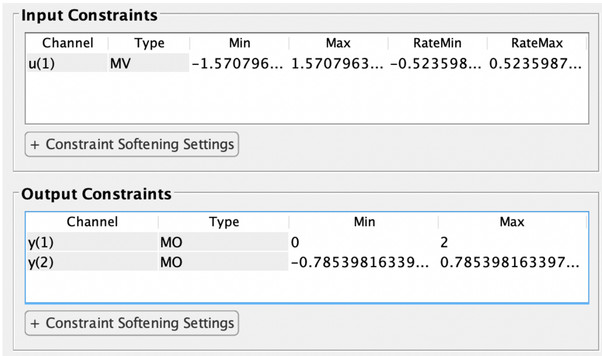
Input Weights (dimensionless)

Channel	Type	Weight	Rate Weight	Target
u(1)	MV	0	0.1	nominal

Output Weights (dimensionless)

Channel	Type	Weight
y(1)	MO	1
y(2)	MO	0.2

Figure 8: Weights



Input Constraints

Channel	Type	Min	Max	RateMin	RateMax
u(1)	MV	-1.570796...	1.5707963...	-0.523598...	0.5235987...

+ Constraint Softening Settings

Output Constraints

Channel	Type	Min	Max
y(1)	MO	0	2
y(2)	MO	-0.78539816339...	0.785398163397...

+ Constraint Softening Settings

Figure 9: Constraints

2.4 Simulation

To generate a video simulation of an imaginary car following the prescribed trajectory, a rectangular block was plotted at each time step the center of which was at $[x_0, Y(t)]$. The corners of the rectangular block were rotated by $\psi(t)$ using the following rotation given in equation 2.1 to show vehicle's relative position w.r.t the road.

$$R(\psi) = \begin{bmatrix} cos(\psi) & -sin(\psi) \\ sin(\psi) & cos(\psi) \end{bmatrix} \tag{2.1}$$

3 Results

The results obtained for the reference signal mentioned above are summarised below in Figure 10, 11a and 11b. We can observe that the vehicle was able to follow the reference trajectory with reasonably low lag time. Since the weight for penalising error in yaw angle was kept low, the the vehicle approaches the reference lateral position by

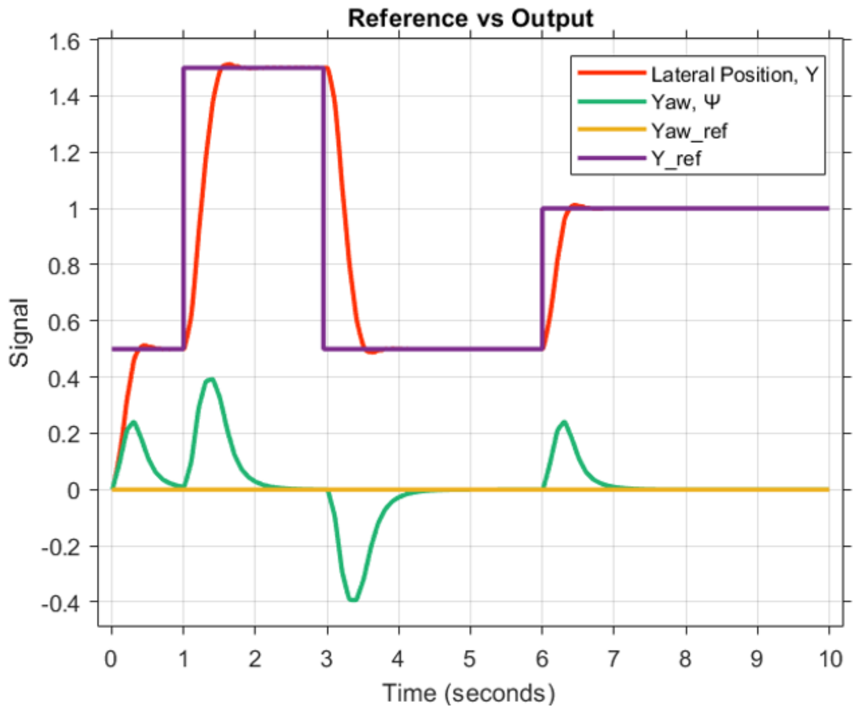
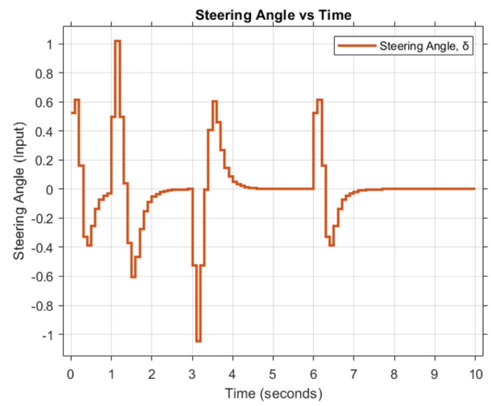


Figure 10: Reference vs actual trajectory

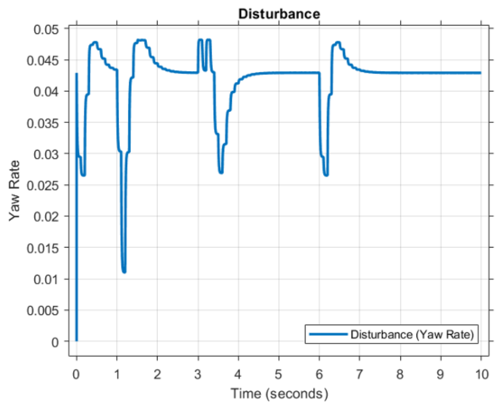
temporarily deviating from the yaw angle reference value. Some snapshots of the video simulation of the motion of the vehicle are included in Figure 12. We can see that these results are consistent with our expectations from the controller.

4 Conclusions

This project presents a control framework for lane keeping and obstacle avoidance, for the linear bicycle model of a vehicle. Simulations for a manually specified reference values were done and the designed MPC block was successful in controlling the plant to make it follow the specified reference trajectory whilst satisfying the specified constraints. Even with the added unmeasured yaw acceleration disturbance due to wind, the controller was able to steady the vehicle and follow the reference trajectory. Future work on this project could include making the plant model more realistic by using the four-wheel model or a non-linear bicycle model as described in [1]. This model can also be extended to include curved roads with more complex trajectories and objectives. The effect of measurement noise/state estimation errors on the controller effectiveness also needs to be studied

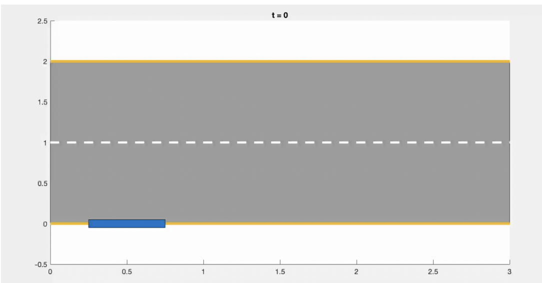


(a) Control input

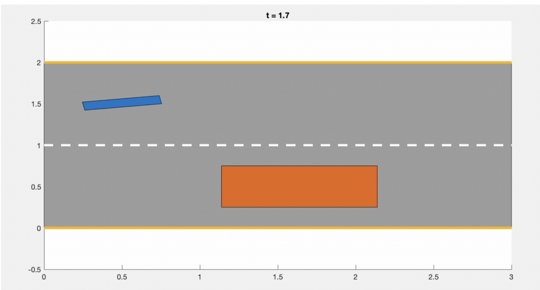


(b) Yaw acceleration disturbance

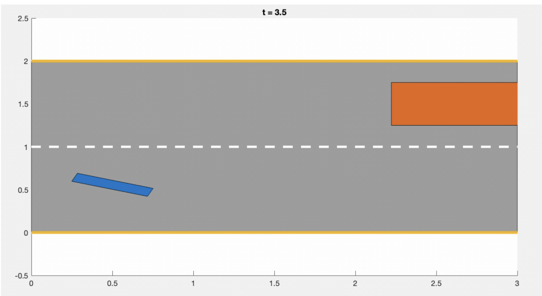
Figure 11: Control Input and Disturbance



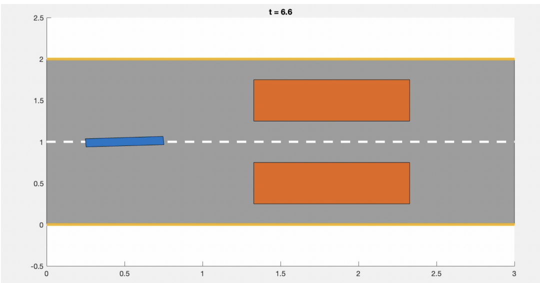
(a) $t = 0$



(b) $t = 1.7s$



(c) $t = 3.5s$



(d) $t = 6.6s$

Figure 12: Snapshots of video simulation

References

[1] Y. Gao, F. Borrelli, J. K. Hedrick, and L. E. Ghaoui. Model predictive control for autonomous and semiautonomous vehicles, 2014.

[2] H. Wei, W. Ross, S. Varisco, P. Krief, and S. Ferrari. *Modeling of Human Driver Behavior via Receding Horizon and Artificial Neural Network Controllers.* 2013.

[3] Q. Zhang, C. Su, Y. Zhou, C. Zhang, J. Ding, and Y. Wang. Numerical investigation on handling stability of a heavy tractor semi-trailer under crosswind. *Applied Sciences (Switzerland)*, 10, 6 2020.