Solution $\boxed{6.1}$ ←

In order to find out heavy bottle we need to distinguish different bottles from each other. The best way to do is to take different count of pills from each bottle so that we can get unique extra weight amount for the bottle with heavy pill.

So if we take i pills from i th bottle

total no. of pills =

$$1 + 2 + 3 + \cdots + 20 = \frac{n(n+1)}{2} = \frac{20 \times 21}{2}$$

$$= 210 \text{ pills}$$

extra weight = actual weight − 210 grams

Bottle number ⇒ $\dfrac{\text{extra weight}}{0.1 \text{ gram.}}$

extra weight per pill.

Solution $\boxed{6.2}$:-

$\boxed{\text{Game 1}}$: one shot to make the hoop.
Let this probability be 'p'.

$\boxed{\text{Game 2}}$: 3 shots, 2 of these three should be made.
So probability of winning = $P(2 \text{ out of } 3)$
$+$
$P(3 \text{ out of } 3)$

$$P(3 \text{ out of } 3) = p \times p \times p = p^3$$

$$P(2 \text{ out of } 3) = P(1 \text{ success}, 2 \text{ success}, 3 \text{ fail}) + P(1 \text{ fail}, 2 \text{ success}, 3 \text{ success}) + P(1 \text{ success}, 2 \text{ fail}, 3 \text{ success}).$$

$$= p \times p \times (1-p) + (1-p)\, p \times p + p\,(1-p)\,p$$

$$= 3p^2(1-p) = 3p^2 - 3p^3$$

So total winning probability for Game 2 $\Rightarrow$

$$p^3 + 3p^2 - 3p^3 \Rightarrow 3p^2 - 2p^3$$

So we can choose $\boxed{\text{Game 1}}$ if

$$p > 3p^2 - 2p^3$$

$$1 > 3p - 2p^2$$

$$2p^2 - 3p + 1 > 0 \qquad (2p-1)(p-1) > 0$$

we know p is between 0 & 1. So terms should be (-)ve $\quad 2p - 1 < 0 \quad \Rightarrow \quad p < 0.5$

So if $p < 0.5$ we should choose Game 1

otherwise Game 2.

Solution $\boxed{6.3}$:-

We have removed two diagonally opposite corners of the chessboard, which are of same color (for a $n \times n$ chessboard).

So now we have 30 squares of one color & 32 squares of other color.

Each domino will cover two squares (one white and other black) i.e. with 31 dominos we can cover only

$$\left.\begin{array}{l}\text{31 white squares} \\ \text{31 black squares}\end{array}\right\} \text{which is not possible in our case.}$$

Solution $\boxed{6.4}$:-

Probability of any ant in moving towards one direction $= 1/2$.

P (all moving in one direction) $= 1/2 \times \frac{1}{2} \times 1/2 = \left(\frac{1}{2}\right)^3$

P (all moving in other direction) $= 1/2 \times 1/2 \times 1/2 = (1/2)^3$

P (moving in same direction) $= (1/2)^3 + (1/2)^3 = 2 \times \frac{1}{8}$

$= 1/4$.

P (of collision) $= 1 - P$ (moving in same direction)

$= 1 - \frac{1}{4} = 3/4$.

For n ants on an n-vertex polygon :-

P (all moving in same direction) $= 2\,(1/2)^n = \left(\frac{1}{2}\right)^{n-1}$

P (of collision) $= 1 - \left(\frac{1}{2}\right)^{n-1}$

Solution $\boxed{6.5}$:-

Step 1 :- Fill 5 quart completely & let 3 quart be empty.

Step 2 :- Pour content of 5 into 3 till 3 is full, remaining content in 5 quart will be 2 quart.

Step 3 :- Empty 3 quart completely & Pour remaining content from 5 quart (ie 2 quart) into 3 quart.

Step 4 :- Refill 5 quart completely and use it to fill up 3 quart (which is having 2 quart already) and then we will be left with $\boxed{\text{4 quarts}}$ in 5 quart jar.

Solution $\boxed{6.6}$:-

Case when only 1 person has blue eyes -

If he is unable to see anyone with blue eyes, he is intelligent enought to know that he will be the one as atleast one person is having blue eyes so he leaves on day 1.

If two people have blue eyes, both can see one guy with blue eyes and the both will expect that guy to leave on day 1, if the other guy does not leave on day 1 that means both of them are with blue eyes and will depart on day 2.

Similarly if N people have blue eyes it will take N days to leave the island.
And all N will leave on Nth night together.

Solution $\boxed{6.7}$ -

Probability of any child being a girl or boy is 0.5

So be there any rule or giving birth or limit on

count, roughly half of the new borns should be

girls and half of them should be boys.

Solution $\boxed{6.9}$:-

For 100 Lockers in a hallway, a door (n) is toggled for each and every factor of (n)

eg. door no. 6 - will be toggled at - 1, 2, 3, 6

open Close open Close

So ultimately door 6 will be closed.

So we can see that a door will be left open if there are odd factors (number of factors) for any door (n).

And no. of factors are odd for <u>perfect squares</u>

So as there are 10 perfect squares between <u>1</u> to 100 so (10) doors will be open by the end of this complete experiment.

Solution $\boxed{6.10}$:-

We can make use of binary representation for each bottle among 1000 bottles and this can be done by using $\boxed{\text{10 bits}}$ for each id.

$2^{10} = 1024$.

So we can use 10 strips as 10 bit locations and add a drop from a bottle if i th bit of any bottle id is set. The drop is added to i th strip.

It will take 7 days to get the result and positive result strips will actually reveal the id of poisoned bottle.