CS754 Assignment-3

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Declaration: The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

Question 5

Solution

Given : Q_1 and Q_2 are observed particle images obtained by translating a zero shift particle image P_1 by $(\delta_{x_1}, \delta_{y_1})$ and $(\delta_{x_2}, \delta_{y_2})$ respectively. The common line for the particle images pass through the origins of the respective coordinate systems at angles θ_1 and θ_2 respectively.

0.1 Relationship between Observations and Shift Variables

In the coordinate system of P_1, the unit vector in the direction of the image is $(\cos\theta_1, \sin\theta_1)$. The shift can be projected onto this unit vector to get the shift components in the direction of the image. The shift components in the direction of the image are given by:

$$\delta_{s_1} = \delta_{x_1}\cos\theta_1 + \delta_{y_1}\sin\theta_1$$

Similarly, in the coordinate system of P_2, the unit vector in the direction of the image is $(\cos\theta_2, \sin\theta_2)$. The shift components in the direction of the image are given by:

$$\delta_{s_2} = \delta_{x_2}\cos\theta_2 + \delta_{y_2}\sin\theta_2$$

We can observe the following variables:

1. Relative Orientation between the two images: $\theta_2 - \theta_1$
2. Relative Shift between the two images: $|\delta_{s_2} - \delta_{s_1}|$

From the above observations, we can define the equation that relates the unknown shift variables $\delta_{x_1}, \delta_{y_1}, \delta_{x_2}, \delta_{y_2}$ to the observed variables $\theta_1, \theta_2, \delta_{s_1}, \delta_{s_2}$.

$$\delta_{x_1}\cos\theta_1 + \delta_{y_1}\sin\theta_1 - \delta_{x_2}\cos\theta_2 - \delta_{y_2}\sin\theta_2 = |\delta_{s_2} - \delta_{s_1}| \cdot \cos(\theta_2 - \theta_1)$$

0.2 Solving for the Shift Variables

To solve for the shifts, we need additional observations to furthur constrain the system. We can obtain additional observations as :

1. If there are identifiable features in the images, we can use their relative positions to obtain additional constraints.
2. Angles between such features in Q_1 and Q_2 can be used to resolve the ambiguity in the shift variables.

By using multiple equations of the form derived in the previous section, we can solve for the shift variables $\delta_{x_1}, \delta_{y_1}, \delta_{x_2}, \delta_{y_2}$.

0.3 Extension into N Projection Planes

The above method can be extended to N projection planes. The shift variables can be solved by using the following equation for each pair of projection planes i and j:

$$\delta_{x_i} \cos\theta_i + \delta_{y_i} \sin\theta_i - \delta_{x_j} \cos\theta_j - \delta_{y_j} \sin\theta_j = |\delta_{s_j} - \delta_{s_i}| \cdot \cos(\theta_j - \theta_i)$$

Thus, we get a total of $\binom{N}{2}$ equations to solve for the $2N$ shift variables. This creates a linear system of equations which can be solved to obtain the shift variables.