CS754 Assignment-1

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Declaration: The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

Question 2

Solution

The restricted isometry constant (RIC) of a matrix A is defined as the smallest number δ_s such that the following is true for any s-sparse vector x:

$$(1-\delta_s)\|x\|^2 \leq \|Ax\|^2 \leq (1+\delta_s)\|x\|^2 \tag{1}$$

where $\|\cdot\|$ denotes the ℓ_2 norm.

We are given that $s < t$, and we need to compare δ_s and δ_t. Let x be an arbitrary s-sparse vector. Since, x contains atmost t zeroes, it follows that x is a t-sparse vector as well.

By definition of RIC, we have:

$$(1-\delta_s)\|x\|^2 \leq \|Ax\|^2 \leq (1+\delta_s)\|x\|^2 \tag{2}$$

Since x is t-sparse, we have:

$$(1-\delta_t)\|x\|^2 \leq \|Ax\|^2 \leq (1+\delta_t)\|x\|^2 \tag{3}$$

Now, by definition of RIC, we have that δ_s is the smallest number satisfying (2), and δ_t is the smallest number satisfying (3).

So, (2) is a special case of (3), and hence $\delta_s \leq \delta_t$.

So, both $(i)\delta_s < \delta_t$ and $(iii)\delta_s = \delta_t$ can be true.

We also need to show an example where equality holds. Consider A to be an orthonormal matrix, in that case:

$$\|Ax\|^2 = \|x\|^2 \tag{4}$$

So, $\delta_s = \delta_t = 0$ for any s and t. Hence, (iii) is true.