# CS754 Assignment-3

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**Declaration:** The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

## Question 2

**Solution**

## 1 Methodology

### 1.1 Problem Formulation

The coded snapshot $E_u$ is obtained as:

$$E_u = \sum_{t=1}^{T} C_t \odot F_t + \eta$$

where:

1. $C_t$ is a binary code modulating frame $F_t$,
2. $\eta$ is additive Gaussian noise with $\sigma = 2$.

**Objective:** Given $E_u$ and $\{C_t\}_{t=1}^{T}$, recover $\{F_t\}_{t=1}^{T}$.

### 1.2 Linear System Representation

The problem is reformulated as $\mathbf{A}\mathbf{x} = \mathbf{b}$, where:

1. $\mathbf{x}$: Vectorized form of all frames $\{F_t\}$,
2. $\mathbf{b}$: Vectorized coded snapshot $E_u$,
3. $\mathbf{A}$: Block-row matrix where each block is $\text{diag}(\text{vec}(C_t))$.

$$A = [diag(C_1)\ \ diag(C_2) \cdots diag(C_T)] \tag{1}$$

For **patch-wise processing** (8×8 patches):

1. $\mathbf{x}$: Sparse coefficients of patches in 2D-DCT basis,

2. **A**: Constructed as:

$$\mathbf{A} = \left[\text{diag}(\text{vec}(C_1))\mathbf{\Psi} \quad \cdots \quad \text{diag}(\text{vec}(C_T))\mathbf{\Psi}\right]$$

   where $\mathbf{\Psi}$ is the 2D-DCT basis.

3. **b**: Vectorised patch vector

## 1.3 Reconstruction via OMP

1. **Patch Extraction:**
   (a) Divide $E_u$ into overlapping $8\times8$ patches.
   (b) For each patch, solve:

$$\mathbf{A}_{\text{patch}}\mathbf{x}_{\text{patch}} = \mathbf{b}_{\text{patch}}$$

   using OMP (sparsity constraint: $k = 10$).

2. **Sparse Recovery:**
   (a) OMP iteratively selects the most correlated basis vectors and solves a least-squares problem.

3. **Frame Reconstruction:**
   (a) Reconstruct each patch via $\mathbf{\Psi x}$.
   (b) Average overlapping regions to suppress artifacts.

## 1.4 Results

To quantify the quality of the reconstruction we computed the RRMSE (Normalised Root Mean Squared Error). A trend that was observed was that the the RRMSE for frames increases on increasing the values of T. This can be explained by the fact that the total information available to us in the long exposure frame remains that same but we are inferring more number of frames from that leading to an increase in the error and decrease in reconstruction quality.

| **Image Identification** | **RMSE** |
|---|---|
| Cars T = 3 | 0.7915 |
| Cars T = 5 | 0.8056 |
| Cars T = 7 | 0.8107 |
| Flame T = 5 | 1.1015 |

Table 1: Mean RMSE Values for Frame Reconstruction

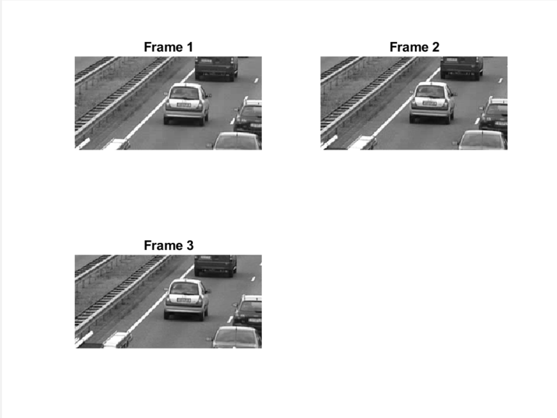


Figure 1: Original Frames for T = 3

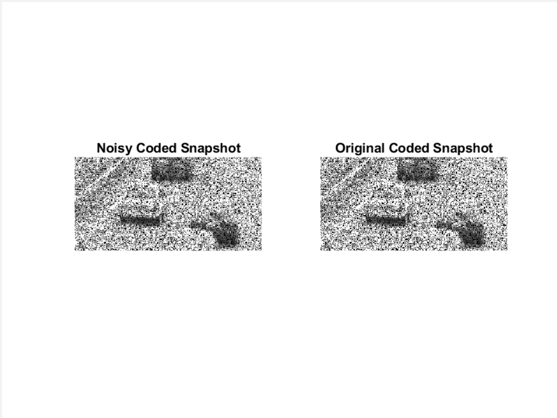


Figure 2: Noisy coded snapshot for T = 3

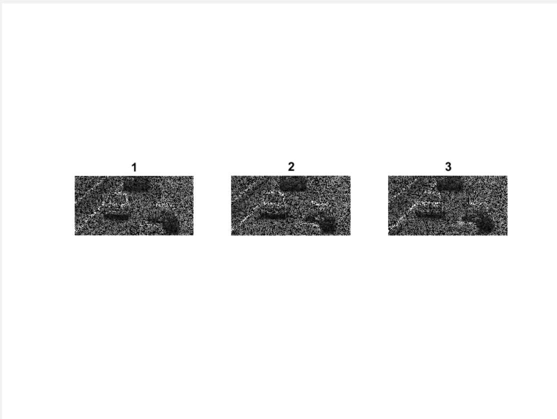


Figure 3: Reconstructed Frames for T = 3

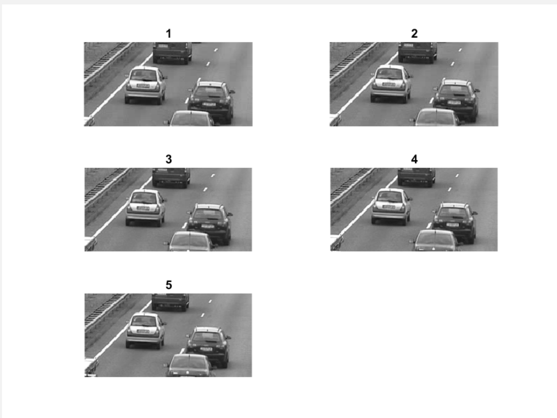


Figure 4: Original Frames for T = 5

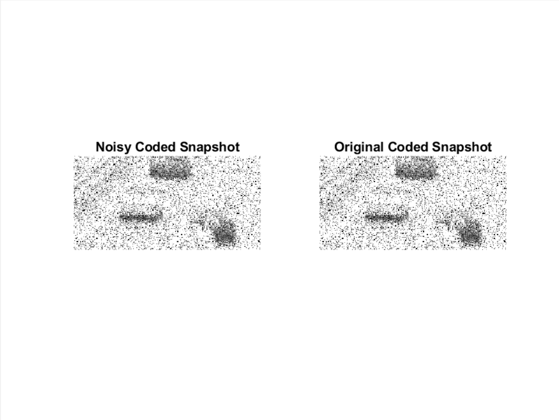


Figure 5: Noisy coded snapshot for T = 5

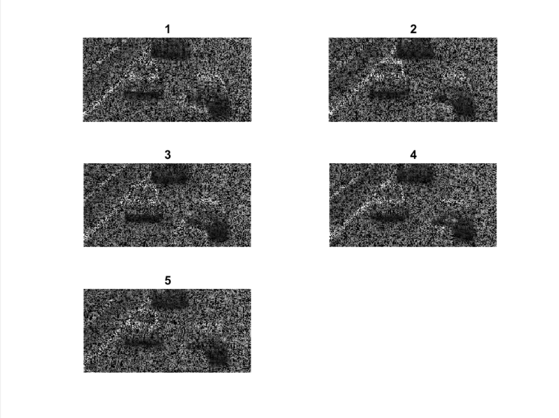


Figure 6: Reconstructed Frames for T = 5

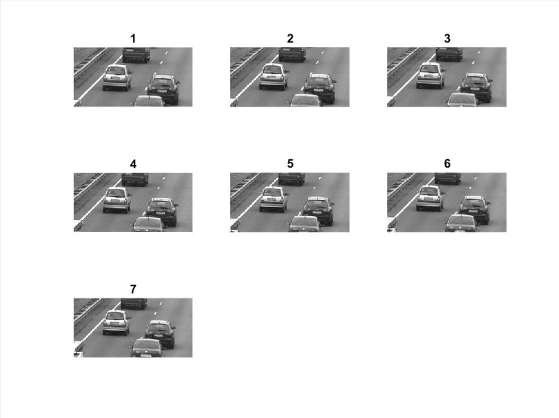


Figure 7: Original Frames for T = 7

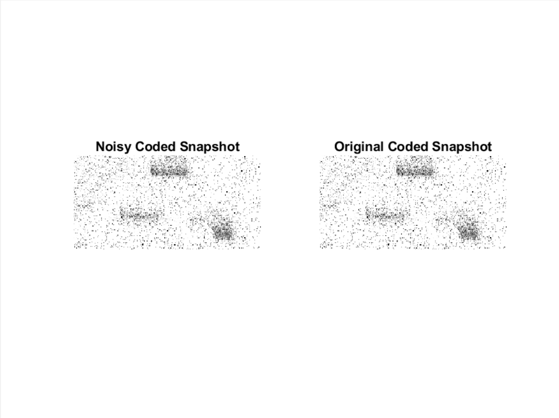


Figure 8: Noisy coded snapshot for T = 7

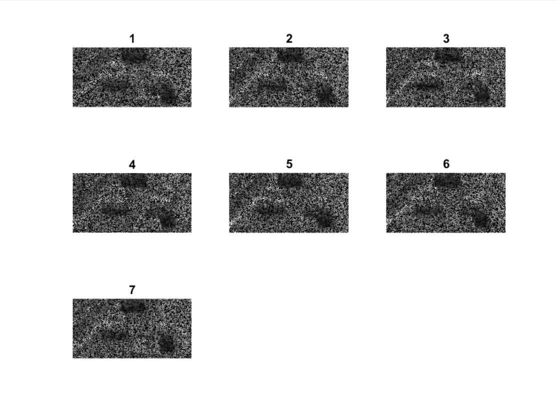


Figure 9: Reconstructed Frames for T = 7

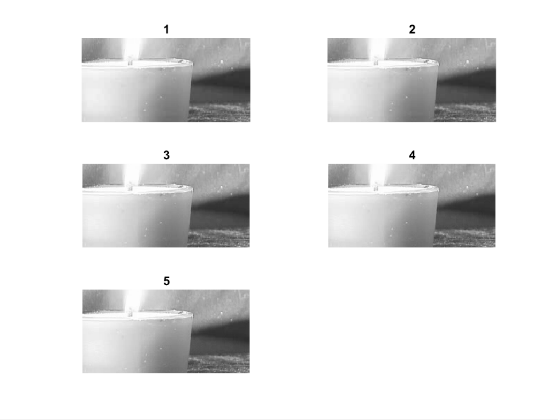


Figure 10: Original Frames for Flame Video

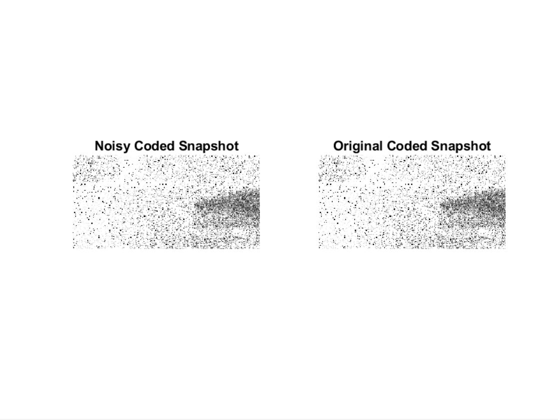


Figure 11: Noisy coded snapshot for Flames Video

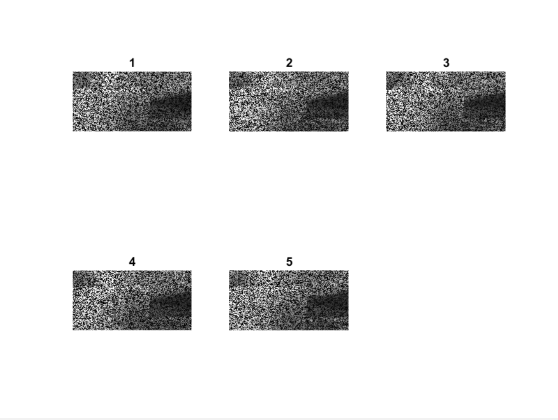


Figure 12: Reconstructed Frames for Flames Video