CS754 Assignment-4

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Declaration: The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

Question 3

Solution

Here is the approach we can use:

1. The matrix M is of size $n_1 \times n_2$ with an unknown rank r. The measurement matrix Y is of size $m \times n_2$ obtained by taking dot products of the columns of M with m different random vectors drawn from a zero-mean Gaussian distribution. This means that the measurements are of the form:
$$Y = A(M)$$
where A is the linear operator that takes the dot product with the random vectors.

2. From lecture slides, we have that random Gaussian Matrices satisfy the RIP condition. We can even use Theorem 2 from lecture slides, which states that if a matrix $A \in \mathbb{R}^{m \times n_1 n_2}$ such that $\delta_{5r}(A) < 0.1$ for $r \geq 1$ and $M \in \mathbb{R}^{n_1 \times n_2}$ has rank at most r, then there exists a unique solution to the following optimization problem:
$$M^* = min_M ||M||_*$$
subject to $Y = Avec(M)$ (which is the same as $Y = A(M)$).

3. Our matrix satisfies the RIP condition, so we can use the optimization problem to recover the matrix M.

4. There are various algorithms to solve the optimization problem such as a simple gradient descent on the loss function $||Y - A(M)||_2^2 + \lambda ||M||$ will also work.

5. Now, since we have got our matrix M (which is unique from the above therem), we will perform SVD on it to get the rank r matrix. We can use the Singular Value Decomposition (SVD) to decompose the matrix M into three matrices:
$$M = U \Sigma V^T$$
where U is an orthogonal matrix of size $n_1 \times n_1$, Σ is a diagonal matrix of size $n_1 \times n_2$ with the singular values on the diagonal, and V is an orthogonal matrix of size $n_2 \times n_2$. The rank of the matrix M is equal to the number of non-zero singular values in Σ (we need to zero out the small singular values). Hence, we have got the rank of the matrix M.