# CS754 Assignment-1

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**Declaration:** The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

## Question 3

### Solution

The P0 problem wants to find a solution $\theta$ to the following:

$$\min ||\theta||_0 \text{ such that } y = A\theta$$

Say there exist two distinct $s$-sparse solutions $\theta$ and $\theta'$ both satisfying $A\theta = A\theta' = y$.
Consider $\Delta = \theta - \theta'$. $\Delta$ is $2s$-sparse, since $\theta$ and $\theta'$ are $s$-sparse. Note that $A(\theta - \theta') = A\Delta = 0$
The RIC of a matrix $A$ is the smallest number $\delta_2 s$ such that the following is true for any $2s$-sparse vector x:

$$(1 - \delta_{2s})||x||_2^2 \leq ||Ax||_2^2 \leq (1 + \delta_{2s})||x||_2^2$$

Now, $\Delta$ is a $2s$-sparse vector in the null space of $A$. Hence, we have

$$(1 - \delta_{2s})||\Delta||_2^2 \leq ||A\Delta||_2^2 \leq (1 + \delta_{2s})||\Delta||_2^2$$

Now, we have $A\Delta = 0$, hence $||A\Delta||_2^2 = 0$. Thus,

$$(1 - \delta_{2s})||\Delta||_2^2 \leq 0$$

Hence, if $\delta_{2s} < 1$, then $||\Delta||_2^2$ has to be zero, which means $\theta = \theta'$, and only a unique solution exists.
Hence, for the P0 problem, $\delta_{2s} < 1$ must hold.