# CS754 Assignment-5

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**Declaration:** The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

## Question 2

**Solution**

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### 1 Primary Objective

**Robust PCA**: The objective is to separate the observed matrix $M$ into a sufficiently low-rank component $L$ and a sparse matrix $S$ with unknown support. This hence allows us to separate a given image into a background, and a foreground component.

**L1-norm PCA**: The objective is to find a collection of orthogonal directions, that is, a matrix $Q$, thereby obtaining an outlier restraining embedding of the data, with lower dimension, hence, improving upon standard PCA which is sensitive to outliers.

### 2 Optimisation Problem

**Robust PCA**: The optimisation problem to be solved is convex. Consider a matrix $M$ of size $n_1 \times n_2$. The solution to be optimisation problem

$$\min_{L,S} \|L\|_* + \frac{1}{\sqrt{\max(n_1, n_2)}} \|S\|_1 \text{ s.t. } L + S = M$$

yields exact estimates of $L$ and $S$ with very high probability.

**L1-norm PCA**: The optimisation problem to be solved, is

$$\max_{Q \in \mathbb{R}^{D \times K}} \|X^\top Q\|_1 \text{ s.t. } Q^\top Q = I_K$$

where $X \in \mathbb{R}^{D \times N}$, which is $N$ $D$-dimensional data points. Unlike RPCA, this optimisation problem is non convex, since $Q^\top Q = I_K$ is not a convex space.

# 3 Algorithm Used

**Robust PCA**: The technique used to solve the RPCA optimisation problem is Augmented Lagrangian method (ALM). It uses an Alternating Minimisation Algorithm which alternates between optimising $L, S$ and $Y$.

**L1-norm PCA**: The exact solution can be obtained in exponential time, using the binary nuclear-norm maximisation problem. An approximate solution can be found efficiently using the L1-BF algorithm