CS754 Assignment-3

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Declaration: The work submitted is our own, and we have adhered to the principles of academic honesty while completing and submitting this work. We have not referred to any unauthorized sources, and we have not used generative AI tools for the work submitted here.

Question 4

Solution

The Radon Transform of a 2D image $f(x,y)$ at an angle θ is defined as:

$$\mathcal{R}_\theta f(\rho) = \int_{-\infty}^{\infty} f(x,y)\delta(\rho - (xcos\theta + ysin\theta))dxdy \tag{1}$$

Since, $g(x,y)$ is another 2D image which is a version of $f(x,y)$ shifted by (x_0, y_0), we can write it as follows:

$$g(x,y) = f(x - x_0, y - y_0)$$

The Radon transform of $g(x,y)$ at an angle θ is:

$$\mathcal{R}_\theta g(\rho) = \int_{-\infty}^{\infty} g(x,y)\delta(\rho - (xcos\theta + ysin\theta))dxdy \tag{2}$$

$$\mathcal{R}_\theta g(\rho) = \int_{-\infty}^{\infty} f(x - x_0, y - y_0)\delta(\rho - (xcos\theta + ysin\theta))dxdy \tag{3}$$

Let $x' = x - x_0$ and $y' = y - y_0$

$$\mathcal{R}_\theta g(\rho) = \int_{-\infty}^{\infty} f(x', y')\delta(\rho - ((x' + x_0)cos\theta + (y' + y_0)sin\theta))dxdy \tag{4}$$

$$\mathcal{R}_\theta g(\rho) = \int_{-\infty}^{\infty} f(x', y')\delta((\rho - x_0cos\theta - y_0sin\theta) - (x'cos\theta + y'sin\theta))dxdy \tag{5}$$

One can easily notice the similarity of this equation with Eq:1 with a modified $\rho' = (\rho - x_0cos\theta - y_0sin\theta)$.

Therefore,

$$\mathcal{R}_\theta g(\rho) = \mathcal{R}_\theta f(\rho') = \mathcal{R}_\theta f(\rho - x_0cos\theta - y_0sin\theta) = \mathcal{R}_\theta f(\rho - (x_0, y_0) \cdot (cos\theta, sin\theta)) \tag{6}$$