CS 726: Advanced Machine Learning, Spring 2025, Homework 1

Roll: _____

Name: _____
Mode: Credit/Audit/Sit-through _____

Write all your answers in the space provided. Do not spend time/space giving irrelevant details or details not asked for. Use the marks as a guideline for the amount of time you should spend on a question. You are only allowed to refer your hand-written notes, no one else's notes or textbook.

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1. Prove that if we factorize $\Pr(\mathbf{x})$ of a DAG as follows

$$\Pr(x_1 \dots x_n) = \prod_{i=1}^{n} f_i(x_i, x_{\pi_i})$$

where π_i denotes the indices corresponding to the parents of node i and $\sum_{x_i} f_i(x_i, x_{\pi_i}) = 1$ then $f_i(x_i, x_{\pi_i}) = \Pr(x_i \,|\, x_{\pi_i})$

..3

2. Let $P(x_1, \dots, x_4)$ be a distribution defined over binary variables as follows

$$P(x_1, \dots, x_4) = \frac{1}{Z} e^{x_1 \oplus x_2 \oplus x_3} e^{x_3 \oplus x_4} \tag{1}$$

where $\oplus$ denotes the XOR operation. XOR of two binary variables is 0 when both its arguments are the same and 1 otherwise. The value of the numerator for some of the entries have been filled in. You need to fill in the five missing entries.

x_1	x_2	x_3	x_4	$ZP(\mathbf{x})$
0	0	0	0	1
0	0	0	1	e
0	0	1	0	e^2
0	0	1	1	e
0	1	0	0	e
0	1	0	1	e^2
0	1	0	1	e
0	1	1	1	1
1	0	0	0	e
1	0	0	1	e^2
1	0	1	0	e
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

..1

(a) Calculate the value of Z

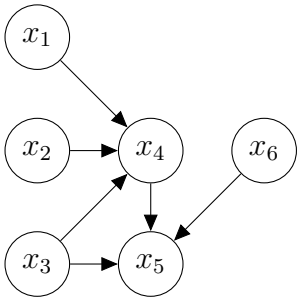
..1

(b) Draw a minimal Bayesian network representing the above distribution using the variable order x_1, x_2, x_3, x_4 to the right of the above table. ..2

(c) Write the CPD for $\Pr(x_1|\text{Pa}(x_1)), \Pr(x_2|\text{Pa}(x_2)), \Pr(x_3|\text{Pa}(x_3))$ in your Bayesian network above.

..3

3. Assume a distribution $P(x_1, x_2, \ldots, x_6)$ is represented by the Bayesian network H below:



(a) In the BN H, list all variables that are unconditionally independent of x_6

..1

(b) Using D separation rule on the above network to get answers to yes/no questions on conditional independencies, draw a fresh *minimal and correct* Bayesian network G using the variable order $x_6, x_5, x_4, x_3, x_2, x_1$.

..3

4. You are given the following statements about the conditional independence about a set of 6 variables A, B, C, D, E, F

   (a) C is independent of B given A

   (b) F is independent of $\{A, B\}$ given C

   (c) E is independent of $\{A, C, F\}$ given B

   (d) D is independent of $\{B, A, C\}$ given E, F

   Given these statements state which of the following are true. Give a brief justification for your answer. No marks without the right justification. [You may find it useful to first draw a suitable graphical model that is possible to draw given the above statements.]

   (1) Is F is independent of B given A? (2) Is C is independent of E?

..4

5. Draw a Bayesian network over five variables $x_1, \ldots, x_5$ assuming the variable order x_1, x_2, x_3, x_4, x_5. For this ordering, assume that the following set of local CIs hold in the distribution: $x_1 \perp\!\!\!\perp x_2$, $x_3 \perp\!\!\!\perp x_2 | x_1$, $x_4 \perp\!\!\!\perp x_1, x_3 | x_2$, $x_5 \perp\!\!\!\perp x_1, x_2 | x_3, x_4$

..2

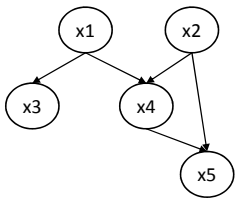
6. In the above Bayesian network, use only local CIs and the standard conditional probability axioms (2.7 to 2.10 from Chapter 2 of your textbook) to prove that $x_3 \perp\!\!\!\perp x_4$.

..5

7. Prove that in a BN where x_i, x_j are any two nodes with no edge between them, then $x_i \perp\!\!\!\perp x_j | Pa(x_i), Pa(x_j)$. [Use d-separation.]

..5

8. For the Bayesian network G below, perform the following operations



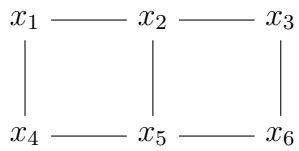
   (a) Convert it into a undirected graphical model H.

..2

(b) List two CIs that holds in G but do not hold in H.

..2

9. For the undirected graphical model H below, perform the following operations



(a) Convert it into a BN G using variable order $x_1, x_4, x_5, x_2, x_3, x_6$.

..2

(b) Choose a different variable order that leads to adding more edges in G than in the above ordering.

..2

(c) List two CIs that holds in H but do not hold in G.

..2

Total: 40