CS726 Scribe Notes

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1 Marginal Probability

Here is the pseudocode for sharing messages between the maximal cliques of the graph.

Firstly, we show how to calculate the Z value for the given graph.

Here is the pseudocode for computing the marginal probabilities in the graphical model using message passing.

Algorithm 1 Computation of Partition Function Z

Require: Graphical Model with maximal cliques and potentials
Ensure: Partition function Z

Construct the junction tree JT from maximal cliques
Initialize adjacency list JT_{adj} from JT
Select a root clique C_{root}
Initialize depth map with C_{root} at depth 0
function DFS($node, parent, depth$)
    for each child in $JT_{adj}[node]$ **do**
        if child $\neq$ parent **then**
            Update depth map
            Call DFS on child with depth $+1$
        end if
    end for
end function
Perform DFS from C_{root}
function SENDMESSAGE(C_{from}, C_{to})
    Compute separator set $S = C_{from} \cap C_{to}$
    Initialize message vector M of size $2^{|S|}$
    Modify clique potential based on incoming messages
    for each state assignment in C_{from} **do**
        Compute corresponding separator index
        Aggregate message value
    end for
    Store message $M(C_{from} \rightarrow C_{to})$
end function
Initialize messages dictionary
Initialize clique potentials
for each clique from deepest to root **do**
    Send messages to parent cliques
end for
for each clique from root to leaves **do**
    Send messages to child cliques
end for
Compute partition function Z using root clique potential and received messages
return Z

Algorithm 2 Computation of Marginal Probabilities

Require: Graphical Model with maximal cliques, clique potentials, and messages
Ensure: Marginal probabilities for each variable

Initialize adjacency list for junction tree
Retrieve partition function Z using previously computed values
Initialize marginal probability list M with zeros
for each variable X_i in the graphical model **do**
    Find a maximal clique C containing X_i
    Extract the potential function for clique C
    for each neighboring clique C' of C **do**
        Compute separator set $S = C \cap C'$
        Retrieve message $M(C' \to C)$
        for each assignment in C **do**
            Identify corresponding index in S
            Multiply message values with clique potential
        end for
    end for
    Compute marginal probability for X_i
    Normalize values using Z
end for
return M