CS 208 HW3 QUESTION 2

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22B1003

Question 2 :

1.

$$S \to AB \mid SS \mid 1S \mid 1$$
$$A \to 0S \mid 1B1 \mid \epsilon$$
$$B \to 1S \mid 1B$$

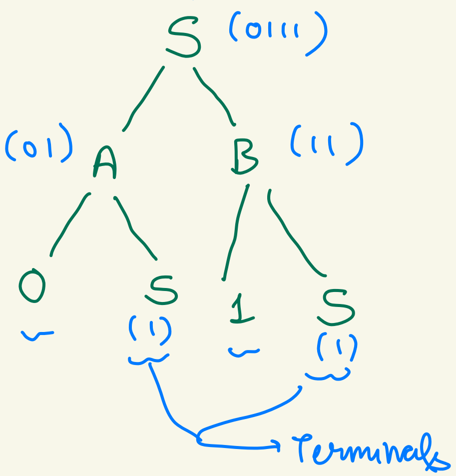
$\{S, A, B\}$ = set of non-terminals

S = start symbol

$\{0, 1\}$ = set of terminals

Consider $w_1 = 0111$

(atleast one occurrence of 0 and one occurrence of 1)

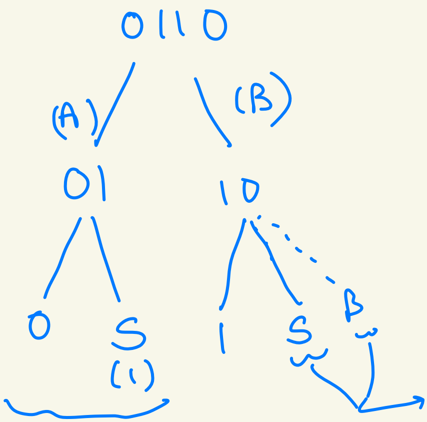


Derivation-Tree to support $w_1 \in L(G)$

Consider $w_2 = 0110$

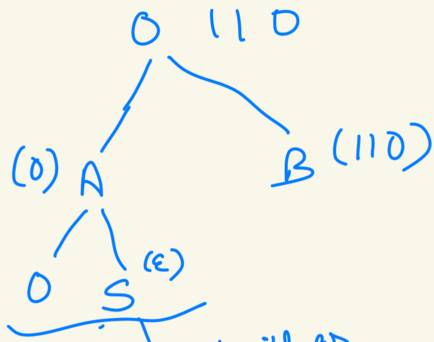
0 terminal is present in the expansion of A, hence the prefix of w_2 should belong to A. [S can also be broken into SS, but that will be equivalent to breaking in AB]

(i) $\underbrace{01}_{A}\underbrace{10}_{B}$ (ii) $\underbrace{0}_{A}\underbrace{110}_{B}$ [2 cases arise because B has to start with 1]



This is correct

Either of them should contain 0, but this is not possible



not possible as S does not contain ε

$\Rightarrow$ Since, we cannot form the tree for w_2 in both the cases $w_2 \notin L(G)$.

2. $A \to 0S \mid 1B1 \mid \varepsilon$

$S \to AB \mid SS \mid 1S \mid 1$

we can replace A here

$S \to 0SB \mid 1B1B \mid B \mid SS \mid 1S \mid 1$

$B \to 1S \mid 1B$

Now we have only two symbols

Seeing this, we can guess that B will be of the form of $1^n S$ $(n \geq 1)$

But, $1^n S$ is equivalent to $1^{n-1}(1S)$

This belongs to the production rule of S

so $1^n S \equiv 1^{n-1} S$

Now, we can move forward inductively

$1^n S \equiv 1^{n-1} S \equiv \ldots \equiv 1S$

So B can be reduced to:

$B \to 1S$

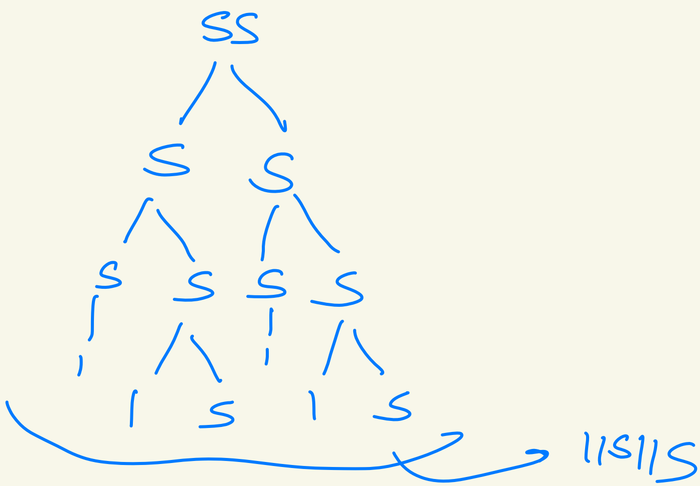
Now S can be modified as:

$S \to 0S1S \mid 11S11S \mid SS \mid 1S \mid 1$

All the steps are reversible, so we have found another CFG G' that uses only a single non-terminal symbol S. (and $L(G) = L(G')$)

3.

$S \rightarrow 0S1S \mid 11S11S \mid SS \mid 1S \mid 1$



thus $11S11S$ gets incorporated into SS

similarly $1S$ gets incorporated into SS

Thus, $S \rightarrow 0S1S \mid SS \mid 1$

string with $n_0 = 1 \rightarrow 0111$ (smallest string)

string with $n_0 = 2 \rightarrow 0(0111)11$ (smallest string)

Our PDA must have one state and one stack symbol.

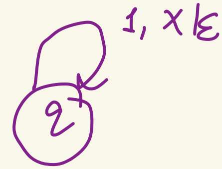
Initially, stack has X on it.

Now, string "1" is acceptable, so the transition $1, X/\varepsilon$ has to be present.

String "ε" is not acceptable, so $\varepsilon, X/\varepsilon$ not there.

similarly $0, X/\varepsilon$ transition is also not present.

PDA formed so far:

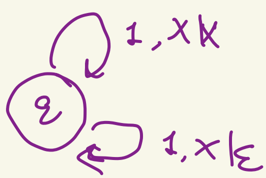


1^i is acceptable for every $i \geq 1$. (because of $S \to SS$)

So, if 11 has to be acceptable, then the stack transition should look like:

$$\underset{\text{Initial}}{X} \xrightarrow{\;1\;} \underset{\text{stack}}{\text{Some\_stack}} \xrightarrow{\;1\;} \underset{\text{stack}}{\text{Empty}}$$

Also, we can pop a single letter in one transition.
So, in the first transition, we can't add something.
If the first transition is $1, X|X$ then second has to be $1, X|\varepsilon$
If the first transition is $1, X|\varepsilon$ then second has to be $1, \varepsilon|\varepsilon$
So, one transition out of $1, X|X$ or $1, \varepsilon|\varepsilon$ has to be there. (Also both are equivalent, because $1^i \in L$ if $i \geq 1$)
So, PDA Constructed so far:



Now 0111 has to be present in L.

So, three transitions on seeing zero are possible:
(one out of these 3 has to be present):

(i) $0, X | X$

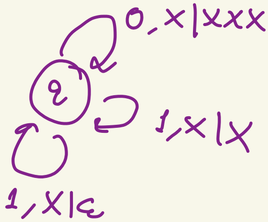
(ii) $0, X | XX$

(iii) $0, X | XXX$

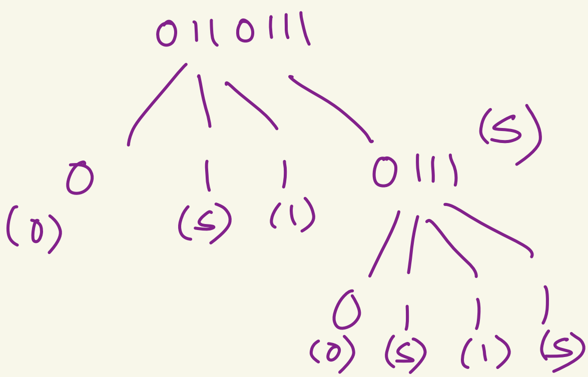
(We can't push 4 Xs as number of ones are 3)

01 not present, (i) is rejected

011 not present, (ii) is also rejected.



Consider the string 0110111



Thus $0110111 \in L$

This is also accepted by our PDA.

Due to the structure 0S1S, every suffix s:

$$2n_0(s) + 1 \le n_1(s)$$

So, on adding every 0, two ones have to be added
And for the last 0, three ones have to be added atleast.

Now, consider 0101111, this string is accepted by our PDA but this string is not in L (It does not fit the structure 0S1S)

Hence, we have achieved contradiction!

So, such PDA construction is not possible for this language.