CS 208

HW4 - Q3

SAKSHAM RATHI
22B1003

3b) $\mathcal{F} = \{L_i \mid L_i \subseteq \Sigma^*, i \in \mathbb{N}\}$

$L' =$ infinite intersection of all languages in $\mathcal{F} = \{w \mid \forall i \in \mathbb{N}, w \in L_i\}$

= Co-recursively enumerable.

This is equivalent to proving that the complement of L' is recursively enumerable

$L' = L_1 \cap L_2 \cap \dots$

$\overline{L'} = \overline{L_1} \cup \overline{L_2} \cup \dots$

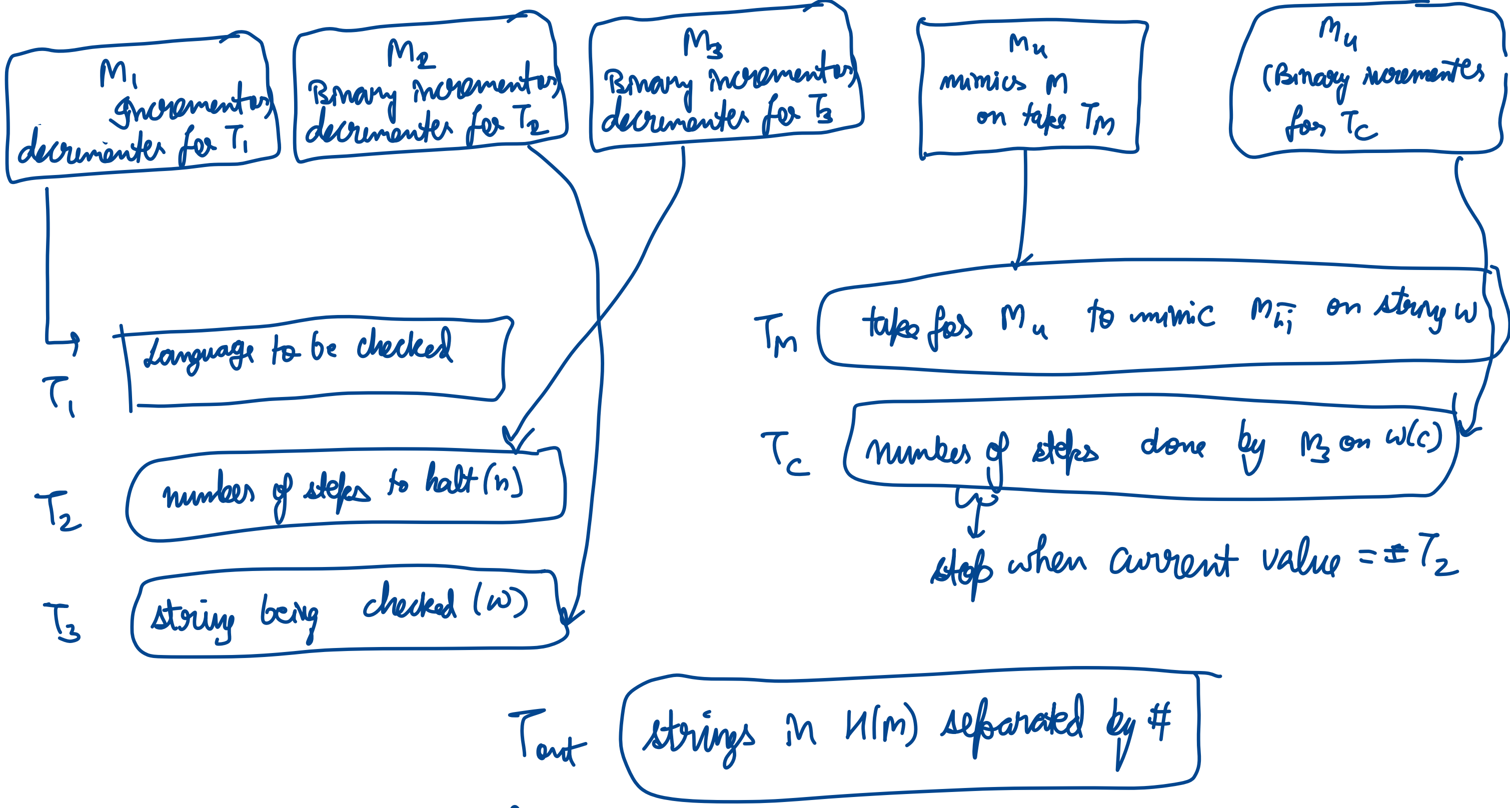
Now, we need to enumerate $\overline{L'}$ using a turing machine. (Hence, we will prove that $\overline{L'}$ is RE.)

The enumeration which we saw in class was based on two dimensions. Here will have three dimensions.

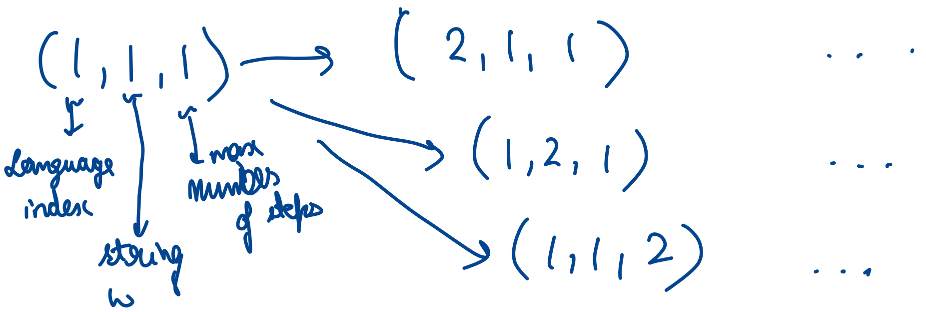
(i) which language to choose $(\overline{L_i})$
(ii) encoding of the string (w)
(iii) numbers of steps to halt (n)

Every string $w \in \overline{L'}$ will belong to some $\overline{L_i}$. Also, it will have a finite number of steps n after which $M_{\overline{L_i}}$ halts since $w \in \overline{L_i} = H(M_{\overline{L_i}})$

Here is how our turing machine will look like:



So, the checking will go on like the following:



(Basically a bijection for N^3 to N)

All words in the language are written on the output tape T_{out} and since M does not halt for all the strings which are not in the language, they will not be written on T_{out}. Hence, M enumerates exactly all the strings in L'.

$\Rightarrow L' = RE$

$\Rightarrow \bar{L} =$ Co-recursively enumerable

Hence proved!