

Question Paper for Midsem Examination

Course CS310

21 September, 2023

- The question paper carries 100 marks in total and consists of 7 questions.
- Partial answers carry partial marks. Hence it is advantageous to show working of your answers.
- This is a paper and pen examination. Answers must be written in an answersheet which must be submitted with role number clearly marked.
- Additionally, Answer to each question must ALSO be uploaded on SAFE as an image answer.
- The examination including uploading on SAFE must be completed in 120 minutes. Do not spend too much time on a single questions.
- Students may keep 3 printed or handwritten A4 size sheets with them for reference. Use of books, notebooks, laptops etc. is not allowed.
- Good Luck!

Q1 (16 marks) For each of the following statements please state whether it is true or false. Please provide a short justification in no more than 3 lines for your answer.

- A) A language L is co-finite if $\Sigma^* - L$ is finite. If L is co-finite then L is regular.
- B) If complement $\sim L$ is context-free then L must be context-free.
- C) If $L_1 \cap L_2$ is regular then L_1 and L_2 must be regular.
- D) If L_1 and L_2 are context-free then $(L_1 - L_2)$ is necessarily context-free.
- E) If L is context-free then L^* is necessarily context-free.
- F) $\{a^i b^j \mid i \neq j\}$ is context-free.
- G) $\{a^i b^j a^k b^l \mid i = j \wedge k = l\}$ is context-free.
- H) $\{a^i b^j a^k b^l \mid i = k \wedge j = l\}$ is context-free.

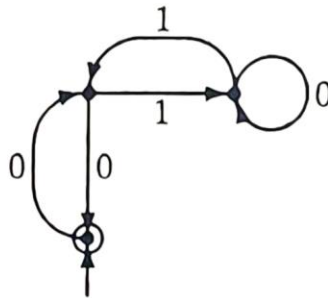
Question Q2 (8 marks) Let $\hat{\Delta} : 2^Q \times \Sigma^* \rightarrow 2^Q$ be the extended transition function for an NFA with transition function Δ . State whether each of the following statements is true or false.

- (A) If $P \subseteq R$ then $\hat{\Delta}(P, x) \subseteq \hat{\Delta}(R, x)$
- (B) If $\hat{\Delta}(X, x) = \emptyset$ then $X = \emptyset$.
- (C) $\hat{\Delta}((P \cup R), x) = \hat{\Delta}(P, x) \cup \hat{\Delta}(R, x)$.
- (D) $\hat{\Delta}((P \cap R), x) = \hat{\Delta}(P, x) \cap \hat{\Delta}(R, x)$

Question Q3 (9 marks) State whether the following pair of regular expressions are equivalent (i.e. they define the same language). Answer in yes or no.

- (A) $a^*(a^* + a)$ and a^* .
- (B) $(a + b)^*$ and $(a^*b)^*$.
- (C) $(a + b)^*$ and $(a^*b^*)^*$.

Question Q4 (6 marks) Which one of the following regular expressions matches the following NFA?



- (A) $\epsilon + 0(01^*1 + 00)^*01^*$
- (B) $\epsilon + 0(10^*1 + 00)^*0$
- (C) $\epsilon + 0(10^*1 + 10)^*1$
- (D) $\epsilon + 0(10^*1 + 10)^*10^*$

(Hint: think of eliminating the top left state in regexp construction).

2 Questions with Long Answers

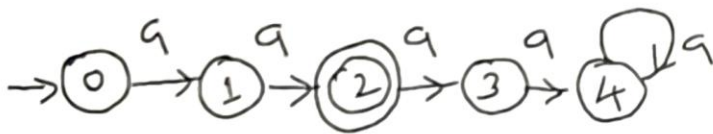
Question Q5 (8 marks) Give a DFA over alphabet $\{a, b\}$ accepting words which have at least an occurrence of factor aab (consecutively) but no occurrence of factor aaa anywhere.

Question Q6 (8 marks) Construct an NFA over alphabet $\Sigma = \{a_1, a_2, \dots, a_k\}$ for the language $\{w \cdot b \mid w \in \Sigma^* \wedge b \in \Sigma \wedge b \text{ occurs at least once in } w\}$. For full marks your NFA should have size $O(k)$. Intuitively answer how many states will the **minimal DFA** for this automaton have? Give a brief justification.

Question Q7 (8 marks) For the following ϵ -NFA construct DFA using the subset construction, retaining only the reachable states. Show steps of construction. Clearly label each state of DFA with the subset of ϵ -NFA states it represents. What language does the constructed automaton recognize (give the answer in regular expression form)? .

	ϵ	a	b
$\rightarrow p$	$\{q\}$	$\{r\}$	\emptyset
$q \ F$	\emptyset	\emptyset	$\{r\}$
r	$\{q\}$	\emptyset	\emptyset

Question Q8 (8 marks) Consider the DFA M_1 below.



Using the Hopcroft partition refinement algorithm for DFA minimization, determine which pairs of states are not equivalent. (Use the table data-structure introduced in class/Kozen Book). Clearly show in the table the phase in which pairs get separated. Draw the minimized DFA.