

CS663 Assignment 3

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Question 4

We have a 201×201 image, where:

- All pixels are black (value 0).
- The central column (column index 101) has all pixels with a value of 255.

Let $f(x, y)$ be the pixel value at position (x, y) :

- For $x = 100$, $f(x, y) = 255$ for all y . (Assuming that our image indices are from 0 to 200.)
- For all other x , $f(x, y) = 0$ for all y .

Given a 2D image $f(x, y)$, its 2D DFT is defined as:

$$F(u, v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})} \quad (1)$$

We are asked to find the 2D DFT of the given image.

$$F(u, v) = \frac{1}{\sqrt{201 \times 201}} \sum_{x=0}^{200} \sum_{y=0}^{200} f(x, y) e^{-j2\pi(\frac{ux}{201} + \frac{vy}{201})} \quad (2)$$

$$F(u, v) = \frac{1}{\sqrt{201 \times 201}} \sum_{y=0}^{200} 255 \times e^{-j2\pi(\frac{u \times 100}{201})} e^{-j2\pi(\frac{vy}{201})} \quad (3)$$

$$F(u, v) = \frac{255 \times e^{-j2\pi(\frac{u \times 100}{201})}}{\sqrt{201 \times 201}} \sum_{y=0}^{200} e^{-j2\pi(\frac{vy}{201})} \quad (4)$$

For $v = 0$, the sum is 201. For all other v :

$$F(u, v) = \frac{255 \times e^{-j2\pi(\frac{u \times 100}{201})}}{\sqrt{201 \times 201}} \frac{1 - e^{-j2\pi(\frac{201v}{201})}}{1 - e^{-j2\pi(\frac{v}{201})}} \quad (5)$$

$$F(u, v) = \frac{255 \times e^{-j2\pi(\frac{u \times 100}{201})}}{\sqrt{201 \times 201}} \frac{1 - 1}{1 - e^{-j2\pi(\frac{v}{201})}} = 0 \quad (6)$$

So, the 2D DFT of the given image is:

$$F(u, v) = \frac{255 \times e^{-j2\pi(\frac{u \times 100}{201})}}{\sqrt{201 \times 201}} \times 201 \times \delta(v) \quad (7)$$

where $\delta(v)$ is the Kronecker delta function (discrete image).

```
1  F = fft2(image);
2  F_shifted = fftshift(F);
3  magnitude = abs(F_shifted);
4  log_magnitude = log(1 + magnitude); % Logarithm for better
   visibility
5  figure;
6  imagesc(log_magnitude);
7  title('Log_Magnitude_of_Fourier_Transform');
8  xlabel('Frequency_u');
9  ylabel('Frequency_v');
```

Listing 1: MATLAB code for Fourier Transform

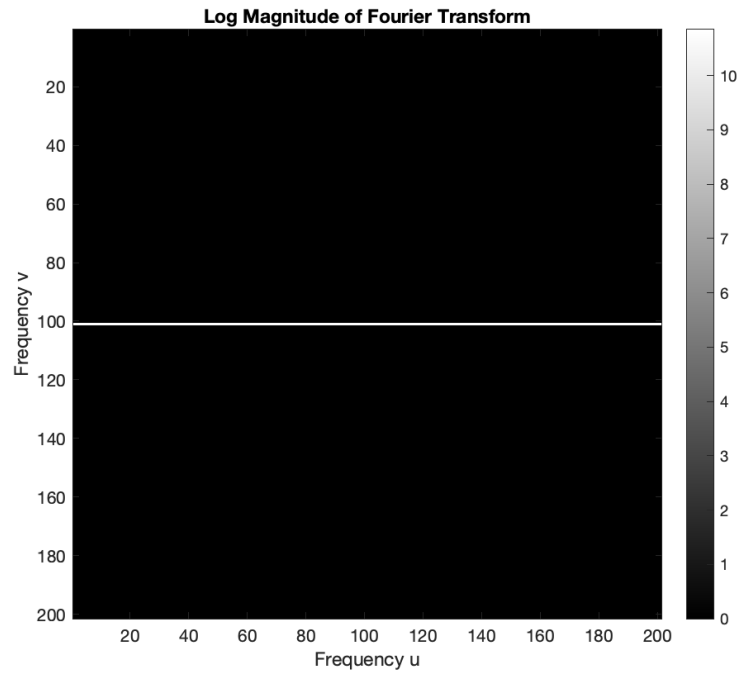


Figure 1: Log Magnitude of Fourier Transform

The MATLAB code for computing the 2D DFT of the given image is shown above.