

CS663 Assignment-5

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Question 2

Solution

We have the following two equations:

$$g_1 = f_1 + h_2 * f_2 \quad (1)$$

$$g_2 = f_2 + h_1 * f_1 \quad (2)$$

where, f_1 and f_2 are the outside scene and reflection images respectively, g_1 and g_2 are the observed images and h_1 and h_2 are the blurring kernels.

It is difficult to solve equations involving convolution operation directly. Since, we know that fourier transforms can transform convolution operation into multiplication, we will take fourier transform of the above equations.

Taking fourier transform of equation (1) and (2) we get:

$$G_1(\mu) = F_1(\mu) + H_2(\mu)F_2(\mu) \quad (3)$$

$$G_2(\mu) = F_2(\mu) + H_1(\mu)F_1(\mu) \quad (4)$$

Multiplying equation (3) by $H_1(\mu)$ and equation (4) by $H_2(\mu)$ we get:

$$H_1(\mu)G_1(\mu) = H_1(\mu)F_1(\mu) + H_1(\mu)H_2(\mu)F_2(\mu) \quad (5)$$

$$H_2(\mu)G_2(\mu) = H_2(\mu)F_2(\mu) + H_2(\mu)H_1(\mu)F_1(\mu) \quad (6)$$

Subtracting equation (5) from equation (4) we get:

$$G_2(\mu) - H_1(\mu)G_1(\mu) = F_2(\mu)(1 - H_1(\mu)H_2(\mu)) \quad (7)$$

Similarly, subtracting equation (6) from equation (3) we get:

$$G_1(\mu) - H_2(\mu)G_2(\mu) = F_1(\mu)(1 - H_1(\mu)H_2(\mu)) \quad (8)$$

From equation (7) and (8) we can solve for $F_1(\mu)$ and $F_2(\mu)$ as:

$$F_1(\mu) = \frac{G_1(\mu) - H_2(\mu)G_2(\mu)}{1 - H_1(\mu)H_2(\mu)} \quad (9)$$

$$F_2(\mu) = \frac{G_2(\mu) - H_1(\mu)G_1(\mu)}{1 - H_1(\mu)H_2(\mu)} \quad (10)$$

Now, we can take inverse fourier transform of $F_1(\mu)$ and $F_2(\mu)$ to get the outside scene and reflection images respectively.

The solution is well defined for all values of μ except where $H_1(\mu)H_2(\mu) = 1$. These are blurring kernels (they are defocussing the images). So, essentially they are removing the high frequency components from the images. Therefore, these are low pass filters.

Also, these blurring kernels integrate to 1:

$$\int_{-\infty}^{\infty} h_1(x)dx = 1 \quad (11)$$

(Because they can't increase or decrease the intensity levels of the images, they can only redistribute the intensity levels.)

Also,

$$H_1(0) = \int_{-\infty}^{\infty} h_1(x) e^{-j0x} dx = 1 \quad (12)$$

Similarly, $H_2(0) = 1$.

For all other values of μ , $|H_1(\mu)|$ and $|H_2(\mu)|$ are ≤ 1 . So, $H_1(\mu)H_2(\mu) \leq 1$ (because these are blurring (low pass) filters).

(These fourier transforms can also be defined over $[a, b]$ instead of $(-\infty, \infty)$.)

Therefore our solution is undefined for low frequencies where $H_1(\mu)H_2(\mu) = 1$ (such as $\mu = 0$).

Let us also consider the case where there is an image noise η_1 and η_2 in the observed images g_1 and g_2 respectively.

The equations will now be:

$$g_1 = f_1 + h_2 * f_2 + \eta_1 \quad (13)$$

$$g_2 = f_2 + h_1 * f_1 + \eta_2 \quad (14)$$

We will incur losses (by solving similarly as above): $\frac{N_1(\mu) - H_2(\mu)N_2(\mu)}{1 - H_1(\mu)H_2(\mu)}$ and $\frac{N_2(\mu) - H_1(\mu)N_1(\mu)}{1 - H_1(\mu)H_2(\mu)}$.

The noise errors will get amplified for lower frequencies where $H_1(\mu)H_2(\mu) \approx 1$. This is a bit different from the case of inverse filter.