CS663 Assignment-3

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Question 5

Solution

The Discrete Fourier Transform (DFT) of a signal $f(x, y)$ of size $M \times N$ and the Inverse Discrete Fourier Transform (IDFT) are respectively given by

$$F(u,v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$
$$f(x,y) = \frac{1}{\sqrt{MN}} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

Let $f(x, y)$ be a real function and $F(u, v)$ be it's DFT. By linearity of the conjugate operator and the fact that $f(t)$ is real, we can show that $F^*(u, v) = F(-u, -v)$

$$\begin{aligned}
F^*(u,v) &= \left(\frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}\right)^* \\
&= \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(f(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}\right)^* \\
&= \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f^*(x,y) \left(e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}\right)^* \\
&= \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)} \\
&= \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{j2\pi\left(\frac{-(-u)x}{M}+\frac{-(-v)y}{N}\right)} \\
&= \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{(-u)x}{M}+\frac{(-v)y}{N}\right)} \\
&= F(-u,-v) \\
\Rightarrow F^*(u,v) &= F(-u,-v)
\end{aligned}$$

Let $f(x, y)$ be real and even. We already showed that if f is real then $F^*(u, v) = F(-u, -v)$. Now if we show that $F(u, v) = F^*(u, v)$ then we are done because F would then be both real and even.

From the previous calculations we have

$$
\begin{aligned}
F^*(u,v) &= \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)} \\
&= \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(-x,-y) e^{j2\pi\left(\frac{(-u)(-x)}{M}+\frac{(-v)(-y)}{N}\right)} \\
&= \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(-x,-y) e^{-j2\pi\left(\frac{u(-x)}{M}+\frac{v(-y)}{N}\right)} \\
&= \frac{1}{\sqrt{MN}} \sum_{x=-(M-1)}^{0} \sum_{y=-(N-1)}^{0} f(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}
\end{aligned}
$$