

CS663 Assignment-3

Saksham Rathi, Kavya Gupta, Shravan Srinivasa Raghavan

Department of Computer Science,
Indian Institute of Technology Bombay

Question 6

Solution

Let $F(\omega) = \mathcal{F}\{f(t)\}(\omega) = \int_{-\infty}^{\infty} e^{-j2\pi\omega t} f(t) dt$.

$$\begin{aligned}\mathcal{F}\{\mathcal{F}\{f(t)\}\}(\tau) &= \int_{-\infty}^{\infty} e^{-j2\pi\tau\omega} \left(\int_{-\infty}^{\infty} e^{-j2\pi\omega t} f(t) dt \right) d\omega \\ &= \int_{-\infty}^{\infty} e^{-j2\pi\tau\omega} F(\omega) d\omega \\ &= \int_{-\infty}^{\infty} e^{j2\pi(-\tau)\omega} F(\omega) d\omega\end{aligned}$$

Note that $f(\tau) = \int_{-\infty}^{\infty} e^{j2\pi\tau\omega} F(\omega) d\omega$ and $f(-\tau) = \int_{-\infty}^{\infty} e^{j2\pi(-\tau)\omega} F(\omega) d\omega$. Therefore,

$$\begin{aligned}\mathcal{F}\{\mathcal{F}\{f(t)\}\}(\tau) &= \int_{-\infty}^{\infty} e^{j2\pi(-\tau)\omega} F(\omega) d\omega = f(\tau) \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{f(t)\}\}(\tau) &= f(-\tau) \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{f(t)\}\}(t) &= f(-t)\end{aligned}\tag{1}$$

The last step is possible because τ can be replaced by any variable and is essentially just a ‘formal parameter’. Let $\mathcal{F}\{\mathcal{F}\{f(t)\}\}(t) = \mathbb{F}(t)$

From equation1 we have,

$$\begin{aligned}\mathcal{F}\{\mathcal{F}\{f(t)\}\}(t) &= f(-t) \\ \Rightarrow \mathbb{F}(t) &= f(-t) \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{\mathbb{F}(t)\}\}(t) &= \mathcal{F}\{\mathcal{F}\{f(-t)\}\}(t) \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{\mathcal{F}\{\mathcal{F}\{f(t)\}\}\}\}(t) &= f(-(-t)) \text{ using eq1} \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{\mathcal{F}\{\mathcal{F}\{f(t)\}\}\}\}(t) &= f(t)\end{aligned}\tag{2}$$

and with equation2 we are done.

In case of functions like *rect* and *sinc* which are Fourier transform pairs, the *rect* function is sufficiently simple to evaluate numerically whereas the *sinc* function requires a lot more approximations to evaluate the value at a single point. So the identity in equation1 can be used to calculate the Fourier transform of *sinc* which is just *rect*(-t) (obtained as follows $\mathcal{F}\{sinc\}(t) = \mathcal{F}\{\mathcal{F}\{rect\}\}(t) = rect(-t)$). Similarly one can find other Fourier transform pairs where one function is analytically much simpler than the other and use its values directly to calculate the Fourier transform of the more complex function.