CS663 Assignment 2

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Question 4

The derivate operators for 1D images are defined as follows:

$$\frac{df}{dx} = f(x+1) - f(x)$$
$$\frac{d^2f}{dx^2} = f(x+1) + f(x-1) - 2f(x)$$

Let us consider a 1D image and it's derivatives. The size of the image is 15 pixels for simplicity.

5	5	5	4	3	2	1	1	1	1	1	6	6	6	6

(a) Image

0	0	-1	-1	-1	-1	-1	0	0	0	5	0	0	0	

(b) First derivatives

	0	-1	0	0	0	1	0	0	0	5	-5	0	0	

(c) Second derivatives

Let us take $\alpha = 0.1$. For convenience we will drop the captions and have all three rows of data directly one below the other in the same order as before. After one iteration we have,

5	5	4.9	4	3	2	1.1	1	1	1	1.5	5.5	6	6	6
0	-0.1	-0.9	-1	-1	-0.9	-0.1	0	0	0.5	4	0.5	0	0	
	-0.1	-0.8	-0.1	0	0.1	0.8	0.1	0	0.5	3.5	-3.5	-0.5	0	

Proceeding ahead we get,

5	4.99	4.82	3.99	3	2.01	1.18	1.01	1	1.05	1.85	5.15	5.95	6	6
-0.01	-0.17	-0.83	-0.99	-0.99	-0.83	-0.17	-0.01	0.05	0.80	3.3	0.80	0.05	0	
	-0.16	-0.66	-0.16	0	0.16	0.66	0.16	0.06	0.75	2.50	-2.5	-0.75	0	

Clearly, the intensities across edges are now closer than they were before the process ie the gradient across edges has reduced in magnitude.

Running a `MATLAB` script to do this resulted in the following images:

1. After a 100 iterations,

5	4.58	4.19	3.83	3.53	3.31	3.19	3.19	3.30	3.54	3.89	4.33	4.84	5.41	6

2. After a 1000 iterations,

5	5.07	5.13	5.19	5.27	5.33	5.40	5.47	5.54	5.62	5.69	5.77	5.85	5.92	6

3. After 10000 iterations,

5	5.07	5.14	5.21	5.29	5.36	5.43	5.50	5.57	5.64	5.71	5.79	5.86	5.93	6

Clearly, the intensities tend toward some local average value. This leads to blurring of the image. Earlier there were sharp intensitywe obtained the following results by running a `MATLAB` scriptn which repeated this process for about 1000 iterations:

5	5.07	5.14	5.21	5.29	5.36	5.43	5.50	5.57	5.64	5.71	5.79	5.86	5.93	6

gradients denoting clear edges. Therefore, adding the term $\alpha\frac{d^2I(x)}{dx^2}$ repeatedly causes blurring of edges resulting in a smooth image with intensities that are close in values throughout the image.

In the case of 2D images, adding the term $\alpha\nabla^2 I(x,y)$ to $I(x,y)$ achieves something similar. We can reason this as follows, clearly for every pixel (x,y), there are only four immediate neighbours $(x-1,y),(x+1,y),(x,y-1),(x,y+1)$ and therefore similar effects can be expected as the laplacian for 2D images takes the form

$$\nabla^2 I(x,y) = I(x-1,y) + I(x+1,y) + I(x,y-1) + I(x,y+1) - 4I(x,y)$$

So doing this for many iterations will result in every pixel tending towards some sort of average intensity of it's neighbourhood resulting in a distribution where most intensities are close by in value.

If, instead the term $\alpha\nabla^2 I(x,y)$ were subtracted however, we get an image which is quite different. This essentially increases the sharpness of the image and the range of intensities becomes wider resulting in an image where every edge is sharper ie higher gradients are found across edges.