CS663 Assignment 2

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Contents

Question 1

The original image $I(x, y)$ is corrupted by additive gaussian noise $N(x, y)$. The final image can be expressed as:

$$I'(x, y) = I(x, y) + N(x, y)$$

We need to express the PDF of the final image in terms of the PDF of the original image and the noise. Let us consider, random variables A, B and:

$$C = A + B$$

Let us try to express the CDF of C in terms of A and B:

$$F_C(c) = \mathbb{P}(C \leq c) = \mathbb{P}(A + B \leq c) = \int_{-\infty}^{\infty} \mathbb{P}(B \leq c - a) f_A(a) da = \int_{-\infty}^{\infty} F_B(c - a) f_A(a) da$$

The right hand side inequality comes from the definition of CDF and PDF of random variables, where we want the sum to be less than c, so $B \leq c - a$, where a is a value taken by the distrubution A, with probability $f_A(a)$. And then we perform the integral across all such values of a. Now:

$$f_C(c) = \frac{d}{dc} F_C(c) = \frac{d}{dc} (\int_{-\infty}^{\infty} F_B(c - a) f_A(a) da) = \int_{-\infty}^{\infty} \frac{d}{dc} (F_B(c - a)) f_A(a) da$$

where the last inequality comes from the fact that $f_A(a)$ does not depend on C.

$$f_C(c) = \int_{-\infty}^{\infty} \frac{d}{dc} (F_B(c - a)) f_A(a) da = \int_{-\infty}^{\infty} f_B(c - a) f_A(a) da$$