CS663 Assignment-3

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Question 6

Solution

Let $F(\omega) = \mathcal{F}\{f(t)\}(\omega) = \int\limits_{-\infty}^{\infty} e^{-j2\pi\omega t} f(t)dt$.

$$\begin{aligned}
\mathcal{F}\{\mathcal{F}\{f(t)\}\}(\tau) &= \int_{-\infty}^{\infty} e^{-j2\pi\tau\omega} \left(\int_{-\infty}^{\infty} e^{-j2\pi\omega t} f(t)dt \right) d\omega \\
&= \int_{-\infty}^{\infty} e^{-j2\pi\tau\omega} F(w)d\omega \\
&= \int_{-\infty}^{\infty} e^{j2\pi(-\tau)\omega} F(w)d\omega
\end{aligned}$$

Note that $f(\tau) = \int_{-\infty}^{\infty} e^{j2\pi\tau\omega} F(w)d\omega$ and $f(-\tau) = \int_{-\infty}^{\infty} e^{j2\pi(-\tau)\omega} F(w)d\omega$. Therefore,

$$\begin{aligned}
\mathcal{F}\{\mathcal{F}\{f(t)\}\}(\tau) &= \int_{-\infty}^{\infty} e^{j2\pi(-\tau)\omega} F(w)d\omega = f(-\tau) \\
\Rightarrow \mathcal{F}\{\mathcal{F}\{f(t)\}\}(\tau) &= f(-\tau) \\
\Rightarrow \mathcal{F}\{\mathcal{F}\{f(t)\}\}(t) &= f(-t) \qquad (1)
\end{aligned}$$

The last step is possible because τ can be replaced by any variable and is essentially just a 'formal parameter'. Let $\mathcal{F}\{\mathcal{F}\{f(t)\}\}(t) = \mathbb{F}(t)$

From equation1 we have,

$$\begin{aligned}
\mathcal{F}\{\mathcal{F}\{f(t)\}\}(t) &= f(-t) \\
\Rightarrow \mathbb{F}(t) &= f(-t) \\
\Rightarrow \mathcal{F}\{\mathcal{F}\{\mathbb{F}(t)\}\}(t) &= \mathcal{F}\{\mathcal{F}\{f(-t)\}\}(t) \\
\Rightarrow \mathcal{F}\{\mathcal{F}\{\mathcal{F}\{\mathcal{F}\{f(t)\}\}\}\}(t) &= f(-(-t)) \text{ using eq1} \\
\Rightarrow \mathcal{F}\{\mathcal{F}\{\mathcal{F}\{\mathcal{F}\{f(t)\}\}\}\}(t) &= f(t) \qquad (2)
\end{aligned}$$

and with equation2 we are done.

In case of functions like *rect* and *sinc* which are Fourier transform pairs, the *rect* function is sufficiently simple to evaluate numerically whereas the *sinc* function requires a lot more approximations to evaluate the value at a single point. So the identity in equation1 can be used to calculate the Fourier transform of *sinc* which is just $rect(-t)$ (obtained as follows $\mathcal{F}\{sinc\}(t) = \mathcal{F}\{\mathcal{F}\{rect\}\}(t) = rect(-t)$). Similarly one can find other Fourier transform pairs where one function is analytically much simpler than the other and use it's values directly to calculate the Fourier transform of the more complex function.

When dealing with symmetric or anti-symmetric functions, knowing that applying the Fourier transform twice results in a time reversal can help quickly determine the Fourier transform of a function without explicitly computing it.

For even functions, this property tells us that the function is unchanged after applying the Fourier transform twice. This can be used to identify symmetries and simplify calculations. Similarly, for odd functions, the property tells us that the function changes sign after applying the Fourier transform twice. This can be used to identify anti-symmetries and simplify calculations.