CS663 Assignment 3

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Question 2

The correlation of two continuous 2D signals in the continuous domain is represented by the equation:

$$h(x,y) = (f \otimes g)(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)g(x+r,y+s)drds \tag{1}$$

We need to derive the 2D fourier transform of $h(x,y)$. We know that the 2D fourier transform of a function $p(x,y)$ is given by:

$$P(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y)e^{-j2\pi(ux+vy)}dxdy \tag{2}$$

Taking the fourier transform of $h(x,y)$, we get:

$$H(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)e^{-j2\pi(ux+vy)}dxdy \tag{3}$$

Substituting the value of $h(x,y)$ from equation (1) into equation (3), we get:

$$H(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)g(x+r,y+s)drds\right) e^{-j2\pi(ux+vy)}dxdy \tag{4}$$

Rearranging the terms, we get:

$$H(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x+r,y+s)e^{-j2\pi(ux+vy)}dxdy\right) dudv \tag{5}$$

Consider $x' = x + r$ and $y' = y + v$. Thus, $x = x' - r$ and $y = y' - s$.

$$H(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')e^{-j2\pi(u(x'-r)+v(y'-s))}dx'dy'\right) dudv \tag{6}$$

$$H(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')e^{-j2\pi(ux'+vy')}e^{j2\pi(ur+vs)}dx'dy'\right) dudv \tag{7}$$

$$H(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')e^{-j2\pi(ux'+vy')}dx'dy'\right) e^{j2\pi(ur+vs)}dudv \tag{8}$$

$$H(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)G(u,v)e^{j2\pi(ur+vs)}dudv \tag{9}$$

We know that the fourier transform of $f(u,v)$ is given by:

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)e^{-j2\pi(ux+vy)}dxdy \tag{10}$$

Thus, the fourier transform of $f(u,v)$ is given by:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)e^{-j2\pi(ux+vy)}dxdy = F(u,v) \tag{11}$$

Similarly, the fourier transform of $g(x', y')$ is given by:

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y')e^{-j2\pi(ux'+vy')}dx'dy' \tag{12}$$

Substituting the value of $G(u, v)$ from equation (11) into equation (10), we get:

$$H(u,v) = F(u,v)e^{j2\pi(ur+vs)} \tag{13}$$

We know that the fourier transform of $g(x + u, y + v)$ is given by:

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x+u,y+v)e^{-j2\pi(ux+vy)}dxdy \tag{14}$$

Substituting the value of $G(u, v)$ from equation (6) into equation (5), we get:

$$H(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)G(u,v)dudv \tag{15}$$

Thus, the 2D fourier transform of $h(x, y)$ is given by:

$$H(u,v) = F(u,v)G(u,v) \tag{16}$$

The correlation of two continuous 2D signals in the discrete domain is represented by the equation:

$$h(x,y) = (f \otimes g)(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f(m,n)g(x+m,y+n) \tag{17}$$