# CS663 Assignment 2

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# Question 4

The derivate operators for 1D images are defined as follows:

$$\frac{df}{dx} = f(x+1) - f(x)$$
$$\frac{d^2 f}{dx^2} = f(x+1) + f(x-1) - 2f(x)$$

Let us consider a 1D image and it's derivatives. The size of the image is 15 pixels for simplicity.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 5 | 5 | 4 | 3 | 2 | 1 | 1 | 1 | 1 | 1 | 6 | 6 | 6 | 6 |

(a) Image

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | -1 | -1 | -1 | -1 | -1 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | |

(b) First derivatives

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | -1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 5 | -5 | 0 | 0 | |

(c) Second derivatives

Let us take $\alpha = 0.1$. For convenience we will drop the captions and have all three rows of data directly one below the other in the same order as before. After one iteration we have,

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 5 | 4.9 | 4 | 3 | 2 | 1.1 | 1 | 1 | 1 | 1.5 | 5.5 | 6 | 6 | 6 |
| 0 | -0.1 | -0.9 | -1 | -1 | -0.9 | -0.1 | 0 | 0 | 0.5 | 4 | 0.5 | 0 | 0 | |
| | -0.1 | -0.8 | -0.1 | 0 | 0.1 | 0.8 | 0.1 | 0 | 0.5 | 3.5 | -3.5 | -0.5 | 0 | |

Proceeding ahead we get,

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 4.99 | 4.82 | 3.99 | 3 | 2.01 | 1.18 | 1.01 | 1 | 1.05 | 1.85 | 5.15 | 5.95 | 6 | 6 |
| -0.01 | -0.17 | -0.83 | -0.99 | -0.99 | -0.83 | -0.17 | -0.01 | 0.05 | 0.80 | 3.3 | 0.80 | 0.05 | 0 | |
| | -0.16 | -0.66 | -0.16 | 0 | 0.16 | 0.66 | 0.16 | 0.06 | 0.75 | 2.50 | -2.5 | -0.75 | 0 | |

Clearly, the intensities across edges are now closer than they were before the process ie the gradient across edges has reduced in magnitude.

Running a `MATLAB` script to do this resulted in the following images:

1. After a 100 iterations,

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 4.58 | 4.19 | 3.83 | 3.53 | 3.31 | 3.19 | 3.19 | 3.30 | 3.54 | 3.89 | 4.33 | 4.84 | 5.41 | 6 |

2. After a 1000 iterations,

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 5.07 | 5.13 | 5.19 | 5.27 | 5.33 | 5.40 | 5.47 | 5.54 | 5.62 | 5.69 | 5.77 | 5.85 | 5.92 | 6 |

3. After 10000 iterations,

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 5.07 | 5.14 | 5.21 | 5.29 | 5.36 | 5.43 | 5.50 | 5.57 | 5.64 | 5.71 | 5.79 | 5.86 | 5.93 | 6 |

## Explanation

Clearly, the intensities tend toward some local average value. This leads to blurring of the image. Earlier there were sharp intensity gradients denoting clear edges. Therefore, adding the term $\alpha\frac{d^2I(x)}{dx^2}$ repeatedly causes blurring of edges resulting in a smooth image with intensities that are close in values throughout the image. This can be justified as follows- In the case of a ramp like situation where the intensities start increasing owing to an edge, the laplacian is negative at the start and positive at the end. So adding it results in reducing the intentisities at the start of a ramp and increases it at the end of a ramp making it flatter. The reverse works when subtracting. This makes the ramp sharper by decreasing the intensities at the start and increasing it at the already high end. Thus the range of intensities becomes wider resulting in an image where every edge is sharper ie higher gradients are found across edges.

In the case of 2D images, adding the term $\alpha\nabla^2 I(x,y)$ to $I(x,y)$ achieves something similar. We can reason this as follows, clearly for every pixel $(x,y)$, there are only four immediate neighbours $(x-1,y), (x+1,y), (x,y-1), (x,y+1)$ and therefore similar effects can be expected as the laplacian for 2D images takes the form

$$\nabla^2 I(x,y) = I(x-1,y) + I(x+1,y) + I(x,y-1) + I(x,y+1) - 4I(x,y)$$

So doing this for many iterations will result in every pixel tending towards some sort of average intensity of it's neighbourhood resulting in a distribution where most intensities are close by in value.