CS663 Assignment-3

Saksham Rathi, Kavya Gupta, Shravan Srinivasa Raghavan
Department of Computer Science,
Indian Institute of Technology Bombay

Question 6

Solution

Let $F(\omega) = \mathcal{F}\{f(t)\}(\omega) = \int_{-\infty}^{\infty} e^{-j2\pi\omega t} f(t)dt$.

$$\begin{aligned}\mathcal{F}\{\mathcal{F}\{f(t)\}\}(\tau) &= \int_{-\infty}^{\infty} e^{-j2\pi\tau\omega} \left(\int_{-\infty}^{\infty} e^{-j2\pi\omega t} f(t)dt \right) d\omega \\ &= \int_{-\infty}^{\infty} e^{-j2\pi\tau\omega} F(w)d\omega \\ &= \int_{-\infty}^{\infty} e^{j2\pi(-\tau)\omega} F(w)d\omega\end{aligned}$$

Note that $f(\tau) = \int_{-\infty}^{\infty} e^{j2\pi\tau\omega} F(w)d\omega$ and $f(-\tau) = \int_{-\infty}^{\infty} e^{j2\pi(-\tau)\omega} F(w)d\omega$. Therefore,

$$\begin{aligned}\mathcal{F}\{\mathcal{F}\{f(t)\}\}(\tau) &= \int_{-\infty}^{\infty} e^{j2\pi(-\tau)\omega} F(w)d\omega = f(\tau) \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{f(t)\}\}(\tau) &= f(-\tau) \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{f(t)\}\}(t) &= f(-t) \qquad (1)\end{aligned}$$

The last step is possible because τ can be replaced by any variable and is essentially just a 'formal parameter'. Let $\mathcal{F}\{\mathcal{F}\{f(t)\}\}(t) = \mathbb{F}(t)$

From equation 1 we have,

$$\begin{aligned}\mathcal{F}\{\mathcal{F}\{f(t)\}\}(t) &= f(-t) \\ \Rightarrow \mathbb{F}(t) &= f(-t) \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{\mathbb{F}(t)\}\}(t) &= \mathcal{F}\{\mathcal{F}\{f(-t)\}\}(t) \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{\mathcal{F}\{\mathcal{F}\{f(t)\}\}\}\}(t) &= f(-(-t)) \text{ using eq1} \\ \Rightarrow \mathcal{F}\{\mathcal{F}\{\mathcal{F}\{\mathcal{F}\{f(t)\}\}\}\}(t) &= f(t) \qquad (2)\end{aligned}$$

and with equation 2 we are done.