- There are 12 unknown parameters $A, B, C, D, E, F, a, b, c, d, e$ and f, therefore we would need at least $12/2 = 6$ control points.
- Let the pairs of physically corresponding control points be $\{(x_{11}, y_{11}), (x_{21}, y_{21})\}$, $\{(x_{12}, y_{12}), (x_{22}, y_{22})\}$, $\{(x_{13}, y_{13}), (x_{23}, y_{23})\}$, $\{(x_{14}, y_{14}), (x_{24}, y_{24})\}$. $\{(x_{15}, y_{15}), (x_{25}, y_{25})\}$ and $\{(x_{16}, y_{16}), (x_{26}, y_{26})\}$.

We are given that for any pair of corresponding points $\{(x_1, y_1), (x_2, y_2)\}$ the motion model is

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} a & b & c & d & e & f \\ A & B & C & D & E & F \end{pmatrix} \cdot \begin{pmatrix} x_1^2 \\ y_1^2 \\ x_1 y_1 \\ x_1 \\ y_1 \\ 1 \end{pmatrix} \tag{1}$$

For each pair of control points that we have taken, there would be an equation similar to 1. The general format would look like this

$$\begin{pmatrix} x_{2k} \\ y_{2k} \end{pmatrix} = \begin{pmatrix} a & b & c & d & e & f \\ A & B & C & D & E & F \end{pmatrix} \cdot \begin{pmatrix} x_{1k}^2 \\ y_{1k}^2 \\ x_{1k} y_{1k} \\ x_{1k} \\ y_{1k} \\ 1 \end{pmatrix} \tag{2}$$

where $k \in \{1, 2, 3, 4, 5, 6\}$.

Combining all 6 eqations we get,

$$\begin{pmatrix} x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \end{pmatrix} = \begin{pmatrix} a & b & c & d & e & f \\ A & B & C & D & E & F \end{pmatrix} \cdot \begin{pmatrix} x_{11}^2 & x_{12}^2 & x_{13}^2 & x_{14}^2 & x_{15}^2 & x_{16}^2 \\ y_{11}^2 & y_{12}^2 & y_{13}^2 & y_{14}^2 & y_{15}^2 & y_{16}^2 \\ x_{11} y_{11} & x_{12} y_{12} & x_{13} y_{13} & x_{14} y_{14} & x_{15} y_{15} & x_{16} y_{16} \\ x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{3}$$

Let $X_2 = \begin{pmatrix} x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \end{pmatrix}$, $X_1 = \begin{pmatrix} x_{11}^2 & x_{12}^2 & x_{13}^2 & x_{14}^2 & x_{15}^2 & x_{16}^2 \\ y_{11}^2 & y_{12}^2 & y_{13}^2 & y_{14}^2 & y_{15}^2 & y_{16}^2 \\ x_{11} y_{11} & x_{12} y_{12} & x_{13} y_{13} & x_{14} y_{14} & x_{15} y_{15} & x_{16} y_{16} \\ x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} \\ y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

and $\mathcal{M} = \begin{pmatrix} a & b & c & d & e & f \\ A & B & C & D & E & F \end{pmatrix}$. Equation 3 can be written compactly as

$$X_2 = \mathcal{M} \cdot X_1$$

$$\Rightarrow \boxed{\mathcal{M} = X_2 \cdot X_1^{-1}} \tag{4}$$

Therefore selecting control points such X_1 is invertible and solving equation 4 will provide the matrix $\mathcal{M}$ whose entries are the constants $A, B, C, D, E, F, a, b, c, d, e$ and f.