CS663 Assignment 3

Saksham Rathi, Kavya Gupta, Shravan Srinivasa Raghavan

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Question 4

We have a 201×201 image, where:

- All pixels are black (value 0).
- The central column (column index 101) has all pixels with a value of 255.

Let $f(x, y)$ be the pixel value at position (x, y):

- For $x = 100$, $f(x, y) = 255$ for all y. (Assuming that our image indices are from 0 to 200.)
- For all other x, $f(x, y) = 0$ for all y.

Given a 2D image $f(x, y)$, its 2D DFT is defined as:

$$F(u, v) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} \tag{1}$$

We are asked to find the 2D DFT of the given image.

$$F(u, v) = \frac{1}{\sqrt{201 \times 201}} \sum_{x=0}^{200} \sum_{y=0}^{200} f(x, y) e^{-j2\pi\left(\frac{ux}{201} + \frac{vy}{201}\right)} \tag{2}$$

$$F(u, v) = \frac{1}{\sqrt{201 \times 201}} \sum_{y=0}^{200} 255 \times e^{-j2\pi\left(\frac{u \times 100}{201}\right)} e^{-j2\pi\left(\frac{vy}{201}\right)} \tag{3}$$

$$F(u, v) = \frac{255 \times e^{-j2\pi\left(\frac{u \times 100}{201}\right)}}{\sqrt{201 \times 201}} \sum_{y=0}^{200} e^{-j2\pi\left(\frac{vy}{201}\right)} \tag{4}$$

For $v = 0$, the sum is 201. For all other v:

$$F(u, v) = \frac{255 \times e^{-j2\pi\left(\frac{u \times 100}{201}\right)}}{\sqrt{201 \times 201}} \frac{1 - e^{-j2\pi\left(\frac{201v}{201}\right)}}{1 - e^{-j2\pi\left(\frac{v}{201}\right)}} \tag{5}$$

$$F(u, v) = \frac{255 \times e^{-j2\pi\left(\frac{u \times 100}{201}\right)}}{\sqrt{201 \times 201}} \frac{1 - 1}{1 - e^{-j2\pi\left(\frac{v}{201}\right)}} = 0 \tag{6}$$

So, the 2D DFT of the given image is:

$$F(u, v) = \frac{255 \times e^{-j2\pi\left(\frac{u \times 100}{201}\right)}}{\sqrt{201 \times 201}} \times 201 \times \delta(v) \tag{7}$$

where $\delta(v)$ is the Kronecker delta function (discrete image).

```
F = fft2(image);
F_shifted = fftshift(F);
magnitude = abs(F_shifted);
log_magnitude = log(1 + magnitude); % Logarithm for better
    visibility
figure;
imagesc(log_magnitude);
title('Log Magnitude of Fourier Transform');
xlabel('Frequency u');
ylabel('Frequency v');
```

Listing 1: MATLAB code for Fourier Transform

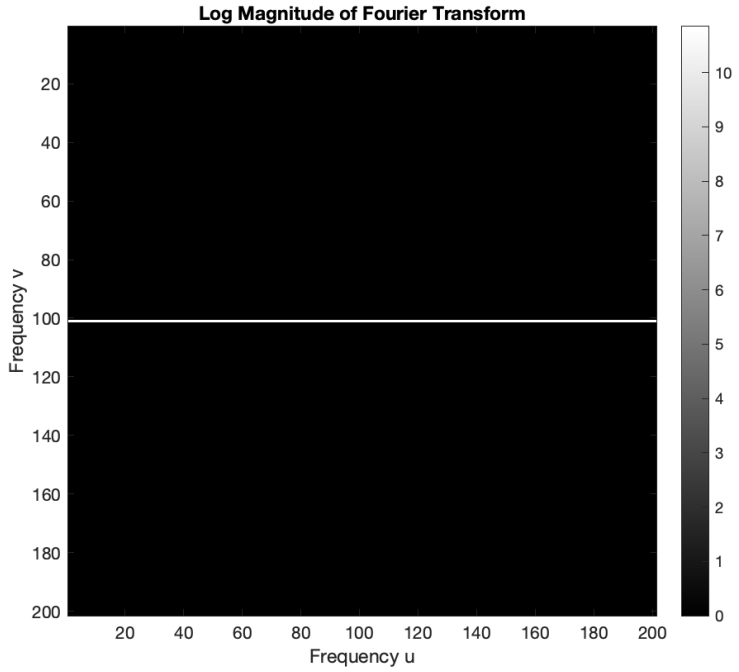

Figure 1: Log Magnitude of Fourier Transform

The MATLAB code for computing the 2D DFT of the given image is shown above.

The vertical line is at $u = 100$ because of the fftshift we are doing. (If we comment out that line, the peak occurs at $u = 0$, which is what we have got from the mathematical derivation.)