CS663 Assignment 2

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Question 5

Let us first consider the case of zero-mean Gaussian Filter. We will choose a filter of size $2a+1$ and it is already given that the ramp image has infinite extent. The 1D image has intensities of the form:

$$I(x) = cx + d$$

The result of applying the Gaussian filter on I will be:

$$J(x) = \frac{\sum_{i=-a}^{a} G(i) I(x+i)}{\sum_{i=-a}^{a} G(i)}$$

where $G(i) = e^{-\frac{i^2}{2\sigma^2}}$ (the constant has been skipped because we are normalizing the weights after applying the filter).

So,

$$J(x) = \frac{\sum_{i=-a}^{a} e^{-\frac{i^2}{2\sigma^2}} (c(x+i)+d)}{\sum_{i=-a}^{a} e^{-\frac{i^2}{2\sigma^2}}}$$

We can break the numerator sum as follows:

$$\sum_{i=-a}^{a} e^{-\frac{i^2}{2\sigma^2}} (c(x+i)+d) = (cx+d) \sum_{i=-a}^{a} e^{-\frac{i^2}{2\sigma^2}} + c \sum_{i=-a}^{0} i \times e^{-\frac{i^2}{2\sigma^2}} + c \sum_{i=0}^{a} i \times e^{-\frac{i^2}{2\sigma^2}}$$

The second and third terms on the right hand side, will cancel each other, so here is the final expression of the filtered image:

$$J(x) = \frac{(cx+d) \sum_{i=-a}^{a} e^{-\frac{i^2}{2\sigma^2}}}{\sum_{i=-a}^{a} e^{-\frac{i^2}{2\sigma^2}}} = cx + d = I(x)$$

Therefore, we have deduced that the filtered image $J(x)$ is essentially the same as the original unfiltered image.

Let us move to the case of bilateral filter. Again, we will choose a window of size $2a+1$.

$$J(x) = \frac{\sum_{i=-a}^{a} G_{\sigma_s}(|(x+i)-x|) G_{\sigma_r}(|I(x+i)-I(x)|) I(x+i)}{\sum_{i=-a}^{a} G_{\sigma_s}(|(x+i)-x|) G_{\sigma_r}(|I(x+i)-I(x)|)}$$

$$J(x) = \frac{\sum_{i=-a}^{a} G_{\sigma_s}(|i|) G_{\sigma_r}(|ci|) I(x+i)}{\sum_{i=-a}^{a} G_{\sigma_s}(|i|) G_{\sigma_r}(|ci|)}$$

$$J(x) = \frac{\sum_{i=-a}^{a} e^{-\frac{i^2}{\sigma_s^2}} e^{-\frac{(ci)^2}{\sigma_s^2}} (c(x+i)+d)}{\sum_{i=-a}^{a} e^{-\frac{i^2}{\sigma_s^2}} e^{-\frac{(ci)^2}{\sigma_s^2}}}$$

Constants from the Gaussian PDF have been removed because of normalization in the denominator.

Similar to the previous case, we will break the numerator sum as follows:

$$\sum_{i=-a}^{a} e^{-\frac{i^2}{\sigma_s^2}} e^{-\frac{(ci)^2}{\sigma_s^2}} (c(x+i)+d) = (cx+d) \sum_{i=-a}^{a} e^{-\frac{i^2}{\sigma_s^2}} e^{-\frac{(ci)^2}{\sigma_s^2}} + c \sum_{i=-a}^{0} i \times e^{-\frac{i^2}{\sigma_s^2}} e^{-\frac{(ci)^2}{\sigma_s^2}} + c \sum_{i=0}^{a} i \times e^{-\frac{i^2}{\sigma_s^2}} e^{-\frac{(ci)^2}{\sigma_s^2}}$$

The second and the third terms from the sum will cancel each other, so here is the final expression of the filtered image:

$$J(x) = \frac{(cx+d)\sum_{i=-a}^{a} e^{-\frac{i^2}{\sigma_s^2}} e^{-\frac{(ci)^2}{\sigma_s^2}}}{\sum_{i=-a}^{a} e^{-\frac{i^2}{\sigma_s^2}} e^{-\frac{(ci)^2}{\sigma_s^2}}} = cx + d = I(x)$$

Therefore, we have deduced that the output image $J(x)$ (after applying bilateral filter) is essentially the same as the original unfiltered image.