CS & IT ENGINEERING

Theory of Computation

Turing Machine Recursively Enumerable 03

Lecture No. 3

By- DEVA Sir

01 Closure properties for Recursive languages

02 RELs

03 What is Decidable & undecidable?

04

05

Closure properties for Recursive Languages [Not closed: $\subseteq, f, h, I_{\infty}$]

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $\bar{L}$
4. $L_1 - L_2$
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+

~~9~~ Subset(L)

10. prefix(L)
11. suffix(L)
12. Substring(L)

~~13~~ $f(L)$

~~14~~ $h(L)$

15. ε-free $h(L)$
16. $h^{-1}(L)$

17. Finite $\cup$
18. Finite $\cap$
19. Finite Diff
20. Finite Concatenation
21. Finite Subset
22. $f_{finite}(L)$

~~23~~ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ Infinite $(\cup, \cap, -, \subseteq, f)$

$R^X_{Rec} \rightarrow SSH\, I_{ALL}$

Closure properties for RE Languages

[Not closed: $\bar{L}, -, \subseteq, F_D, I_{ALL}$

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. ~~$\bar{L}$~~
4. ~~$L_1 - L_2$~~
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+
9. ~~Subset(L)~~
10. prefix(L)
11. suffix(L)
12. Substring(L)
13. f(L)
14. h(L)
15. ε-free h(L)
16. $h^{-1}(L)$
17. Finite $\cup$
18. Finite $\cap$
19. ~~Finite Diff~~
20. Finite Concatenation
21. Finite Subset
22. $f_{finite}(L)$
23. ~~Infinite ($\cup, \cap, -, \subseteq, f$)~~
24. ~~Infinite ($\cup, \cap, -, \subseteq, f$)~~
25. ~~Infinite ($\cup, \cap, -, \subseteq, f$)~~
26. ~~Infinite ($\cup, \cap, -, \subseteq, f$)~~
27. ~~Infinite ($\cup, \cap, -, \subseteq, f$)~~
28. ~~Infinite ($\cup, \cap, -, \subseteq, f$)~~

$$REL^X_{CDSFI_{ALL}}$$

Note :

I) Subset & Infinite $(\cup, \cap, -, \cdot, \subseteq_f)$ are not closed for Reg languages
for DCFLs
for CFLs
for CSLs
for Recursives
for RELs

II) h^{-1} & Finite subset are closed:
for Regs / DCFLs / CFLs / CSLs / Recs / RELs

Note:

I) $R^X_{Regulars} \longrightarrow \subseteq, Infinite(\cup, \cap, -, \cdot, \subseteq, f)$

II) $D^{\checkmark}_{CFLs} \longrightarrow \bar{L}, Prefix, h^{-1}, finite\ subset$

III) $C^X_{FLs} \longrightarrow \cap, \bar{L}, -, \subseteq, /, F_{ID}, I_{ALL}$

IV) $R^X_{Recursives} \longrightarrow \subseteq, f, h, Inf(\cup, \cap, -, \cdot, \subseteq, f)$

V) $REL_S^X \longrightarrow \bar{L}, -, \subseteq, Finite\ Diff, I_{ALL}$

$R^X_{SI_{ALL}}$

$D^{\checkmark}_{CPIF_S}$

$C^X_{ICDSQF_{IDI_{ALL}}}$

$REC^X_{SSHI_{ALL}}$

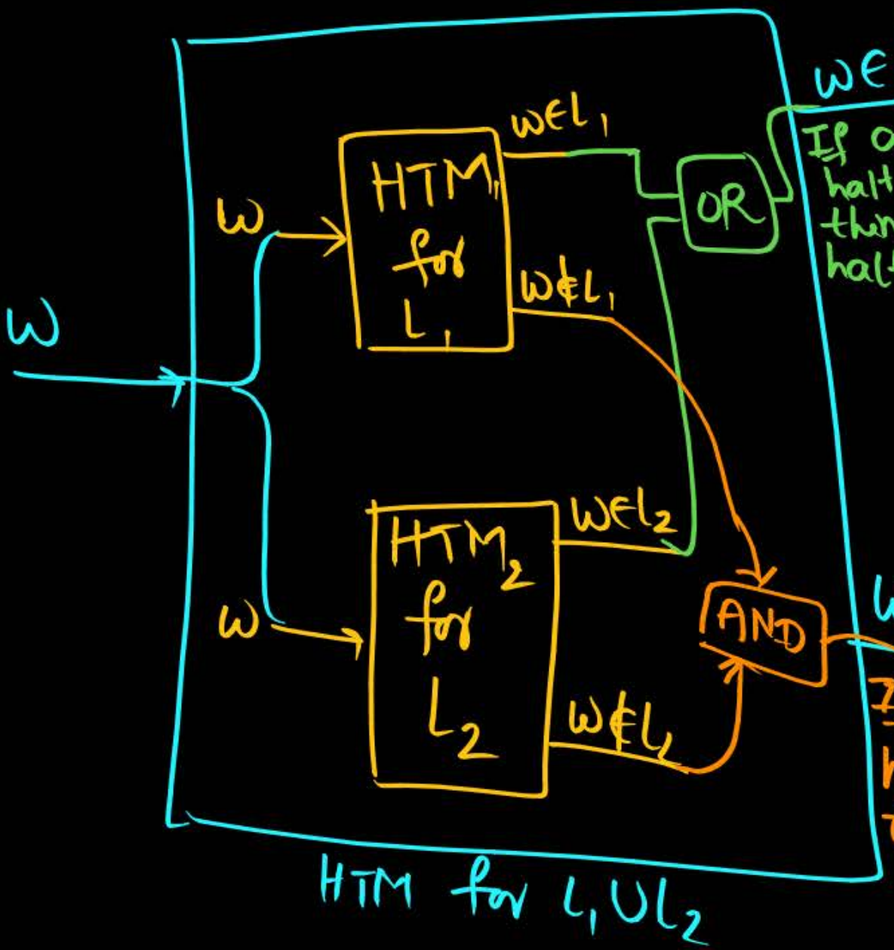
$REL^X_{CDSF_SI_{ALL}}$

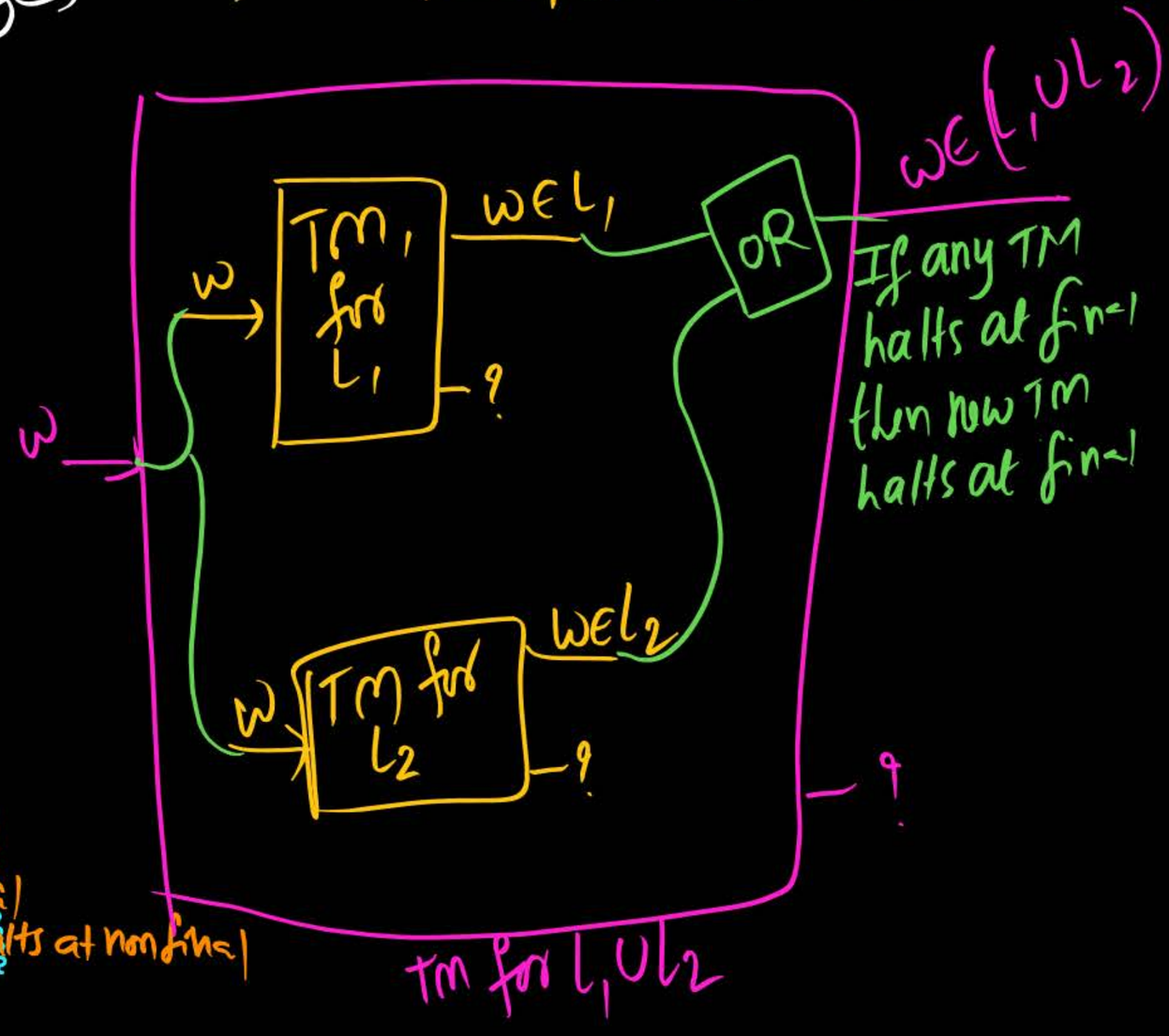
① Union

- → closed for Recursive languages (Decidable)
- → closed for RE languages (semi-decidable)

$Rec_1 \cup Rec_2 \Rightarrow Rec$

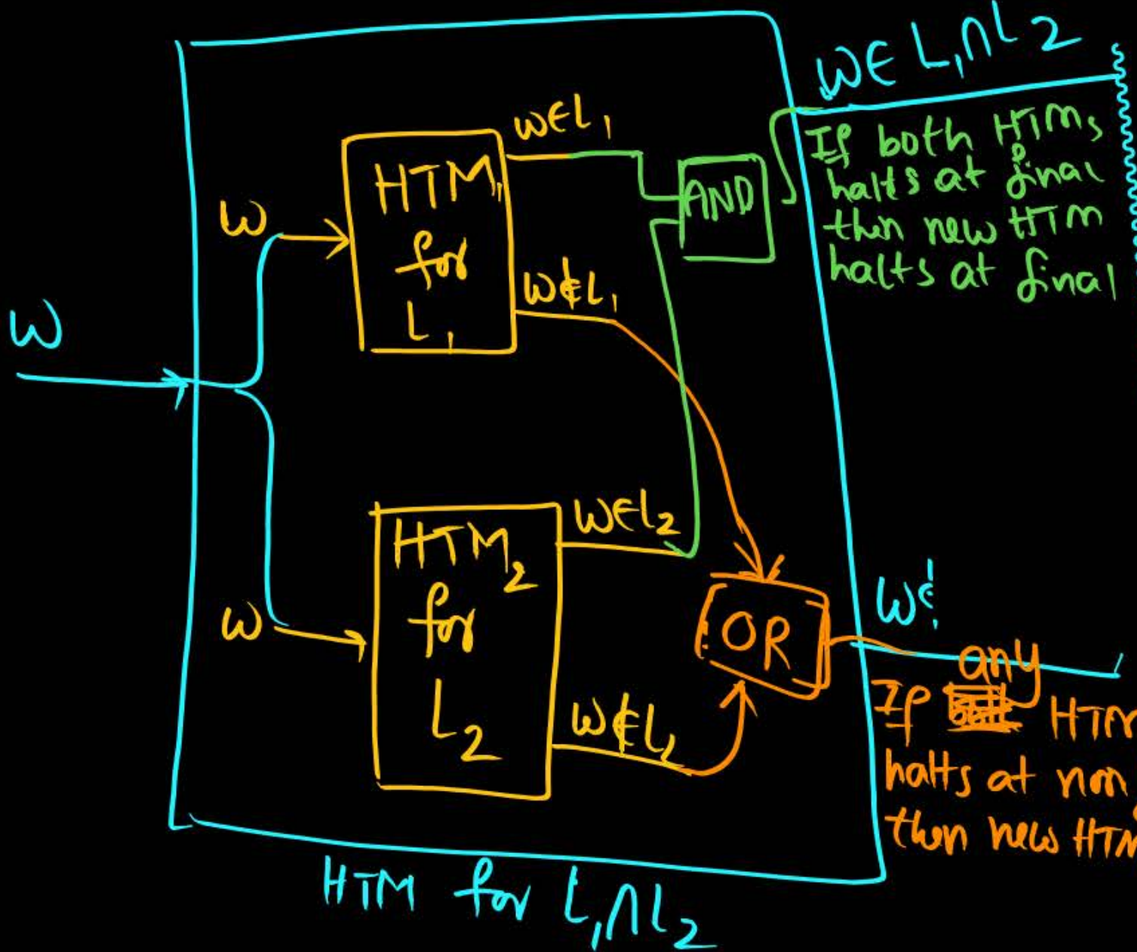
$REL_1 \cup REL_2 \Rightarrow REL$

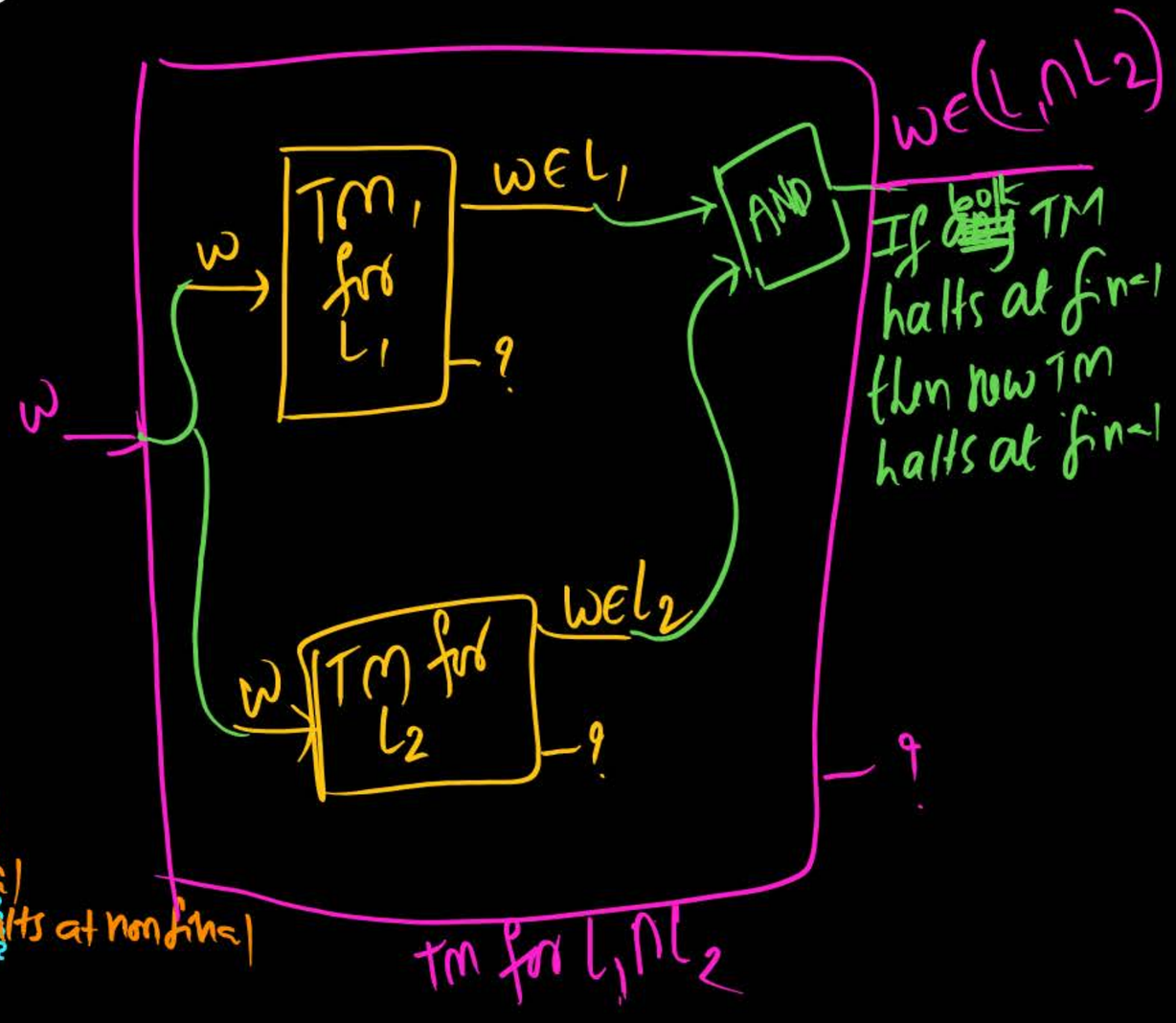




② Intersection

- closed for Recursive (Decidable) languages : $Rec_1 \cap Rec_2 \Rightarrow Rec$
- closed for RE (semi-decidable) languages : $REL_1 \cap REL_2 \Rightarrow REL$





A_1, A_2 are Recursives

B_1, B_2 are RELs

Given

I) $A_1 \cup A_2$ is Recursive

II) $B_1 \cup B_2$ is REL

Rec $\cup$ REL

III) $A_1 \cup B_1$ is REL

IV) $A_1 \cap A_2$ is Recursive

V) $B_1 \cap B_2$ is REL

Rec $\cap$ REL

VI) $A_1 \cap B_1$ is REL

$Reg \cap DCFL \Rightarrow DCFL$

$Reg \cap CFL \Rightarrow CFL$

$DCFL \cap CFL \Rightarrow CSL$

$CFL_1 \cap CFL_2 \Rightarrow CSL$

$CFL \cap CSL \Rightarrow CSL$

$CSL \cap Rec \Rightarrow Recursive$

$Rec \cap \overline{REL} \Rightarrow \overline{REL}$

$CFL \cap \overline{REL} \Rightarrow \overline{REL}$

Regular $\cap$ CFL $\Rightarrow$ Regular

a^*b^* $\quad$ a^nb^n $\quad$ a^nb^n

DCFL$_1$ $\cap$ DCFL$_2$ $\Rightarrow$ CSL (need not be DCFL) (need not be CFL)

CFL$_1$ $\cap$ CFL$_2$ $\Rightarrow$ CSL

DCFL $\cap$ CFL $\Rightarrow$ CSL

$a^nb^nc^* \cap a^*b^nc^n \Rightarrow a^nb^nc^n$

CSL

L is Recursive Language

iff

I) HTM exist (It does not mean every Tm is HTm)

II) Logic exist for both valid & Invalid

L is REL

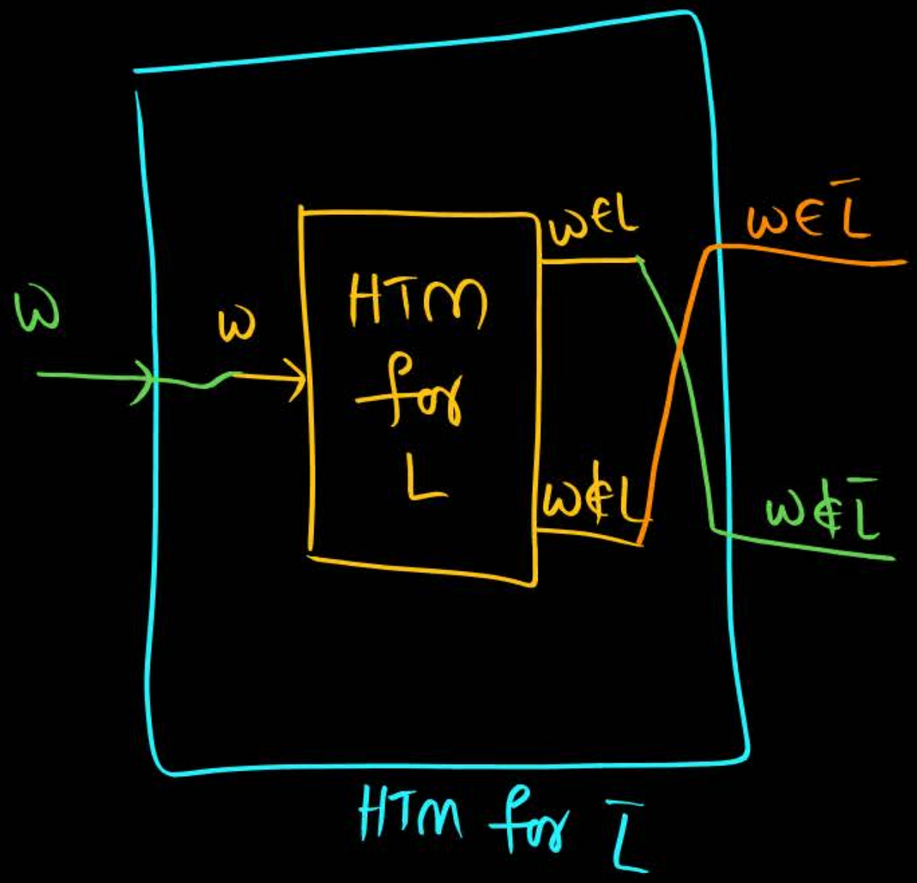
iff

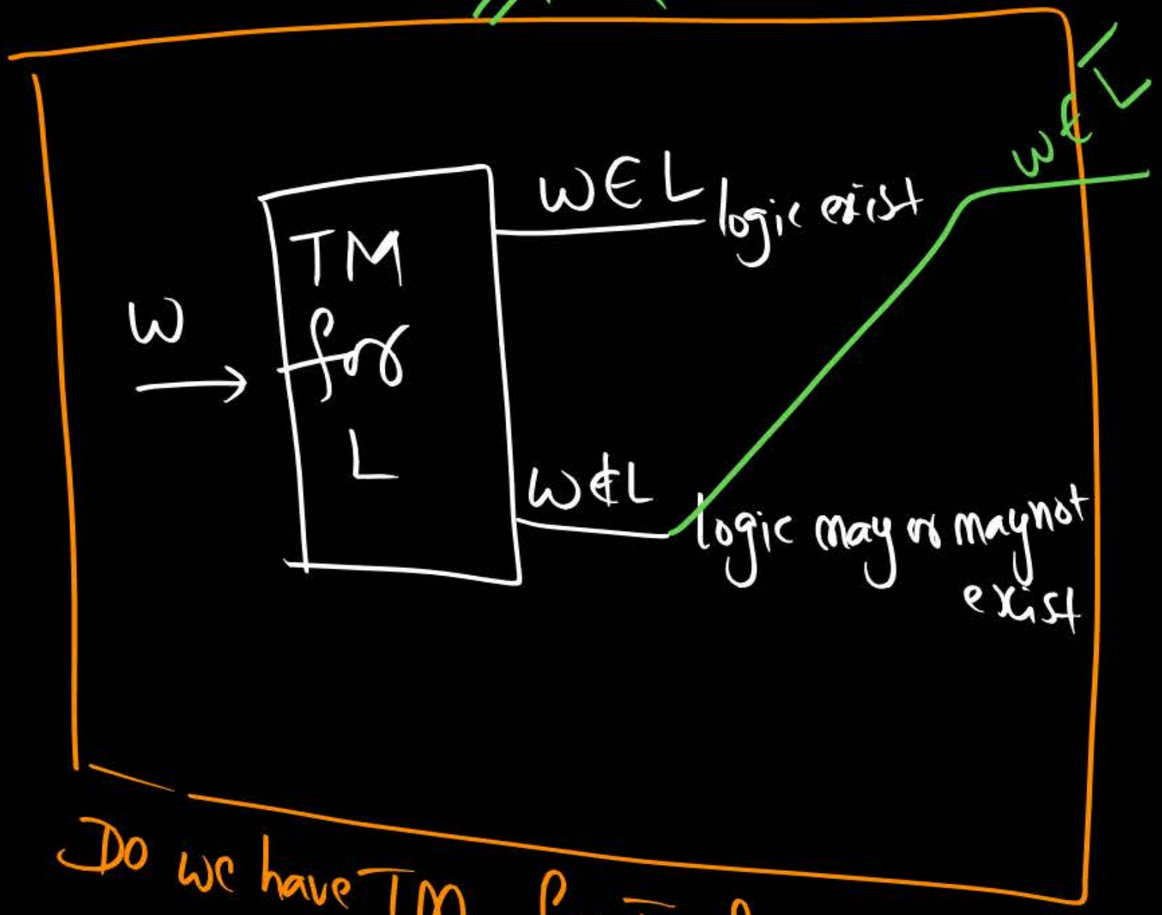
I) TM exist

II) Logic exist for valid
(logic may or may not exist for Invalid)

Complement

- closed for Recursive Languages (Decidable)
- Not closed for REL's





I) Recursive Language $\Rightarrow$ Recursive language

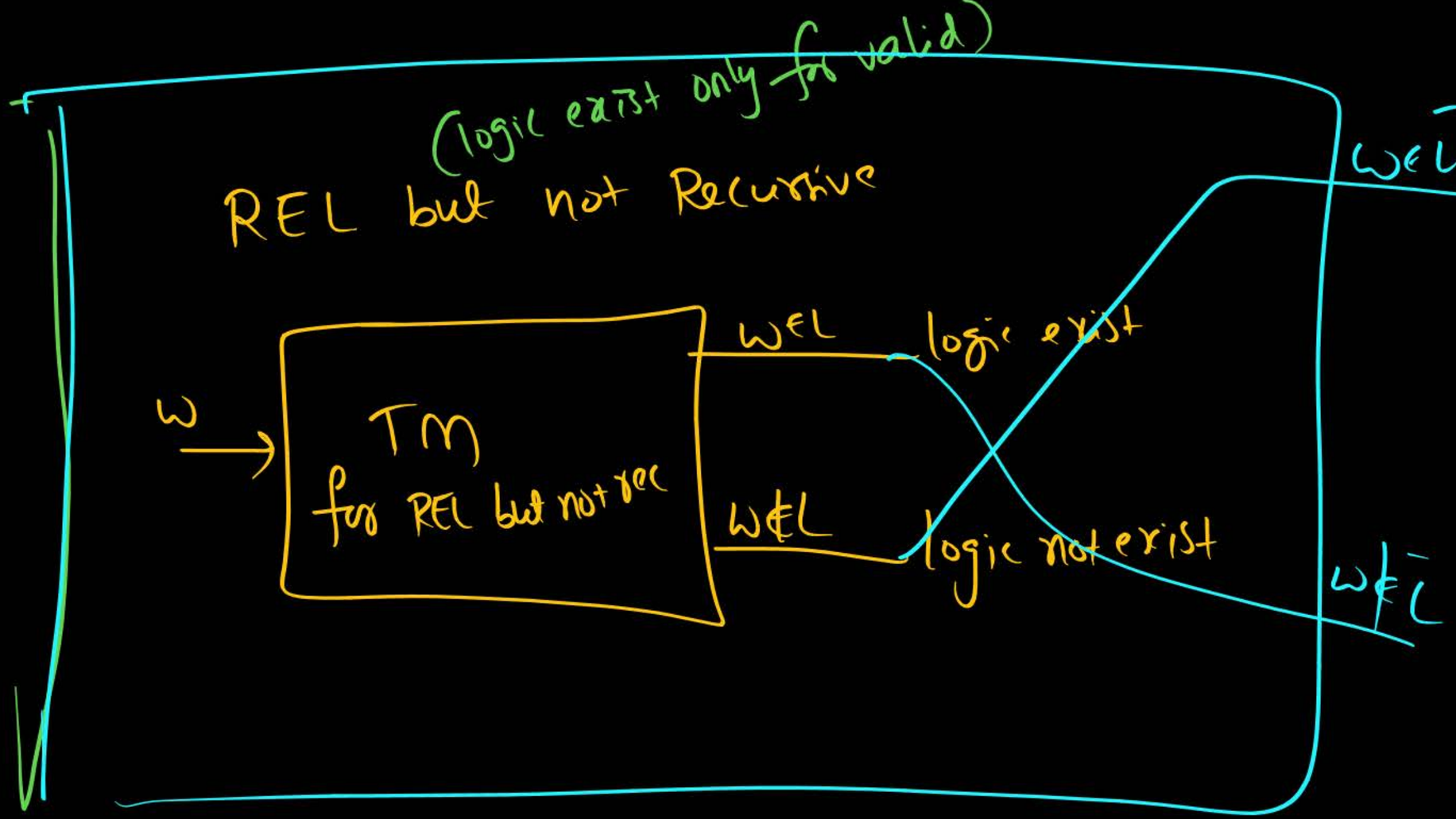
*** II) $\overline{REL}$ $\Rightarrow$ Never be "REL but not rec"

$\Rightarrow$ Need not be REL

$\Rightarrow$ Either Recursive Lang or "Not REL"

A_1 is REL

III) REL but not recursive $\Rightarrow$ Not REL



Rec ?

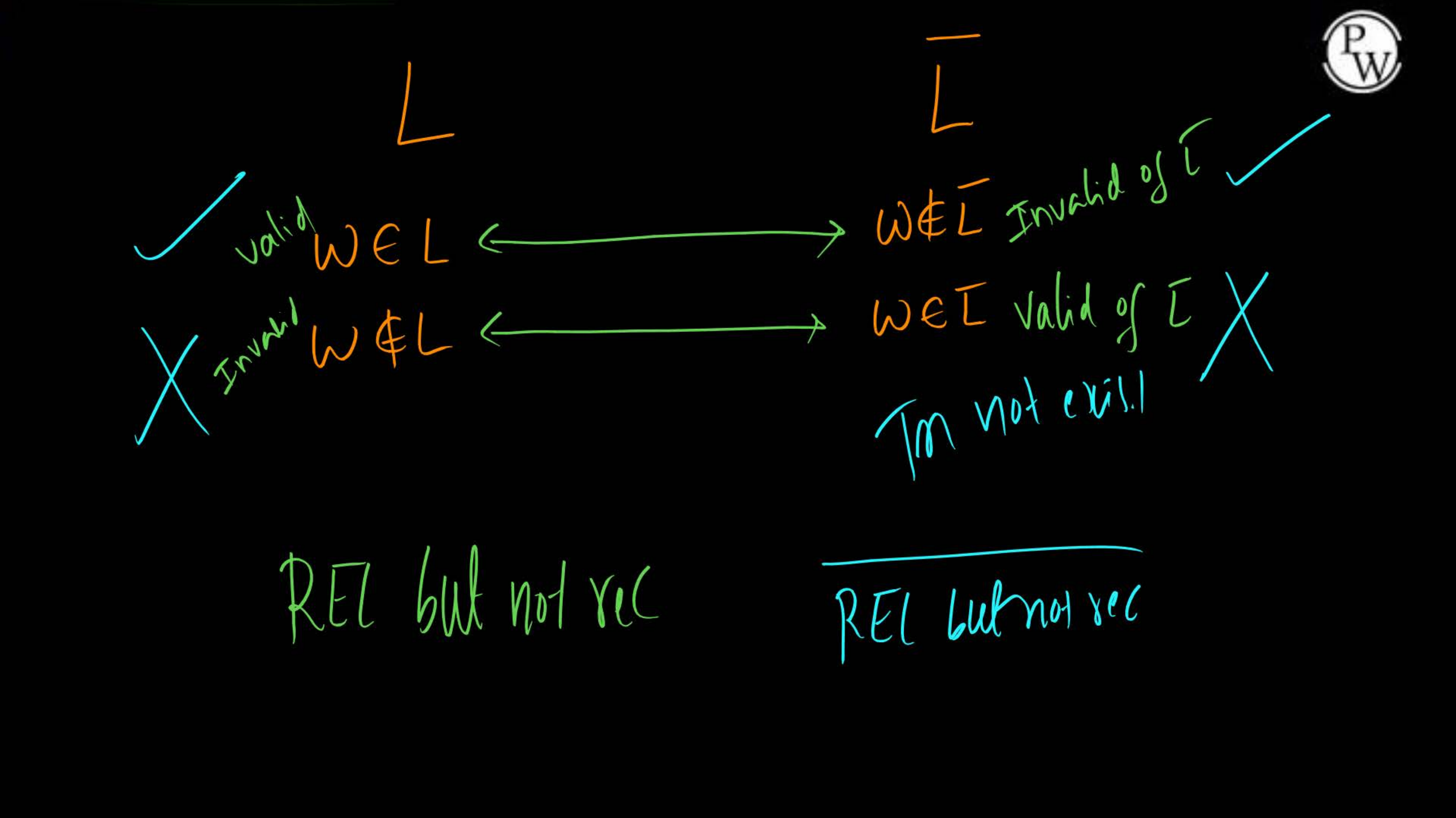
L is Rec $\Leftrightarrow$ $\bar{L}$ is Rec

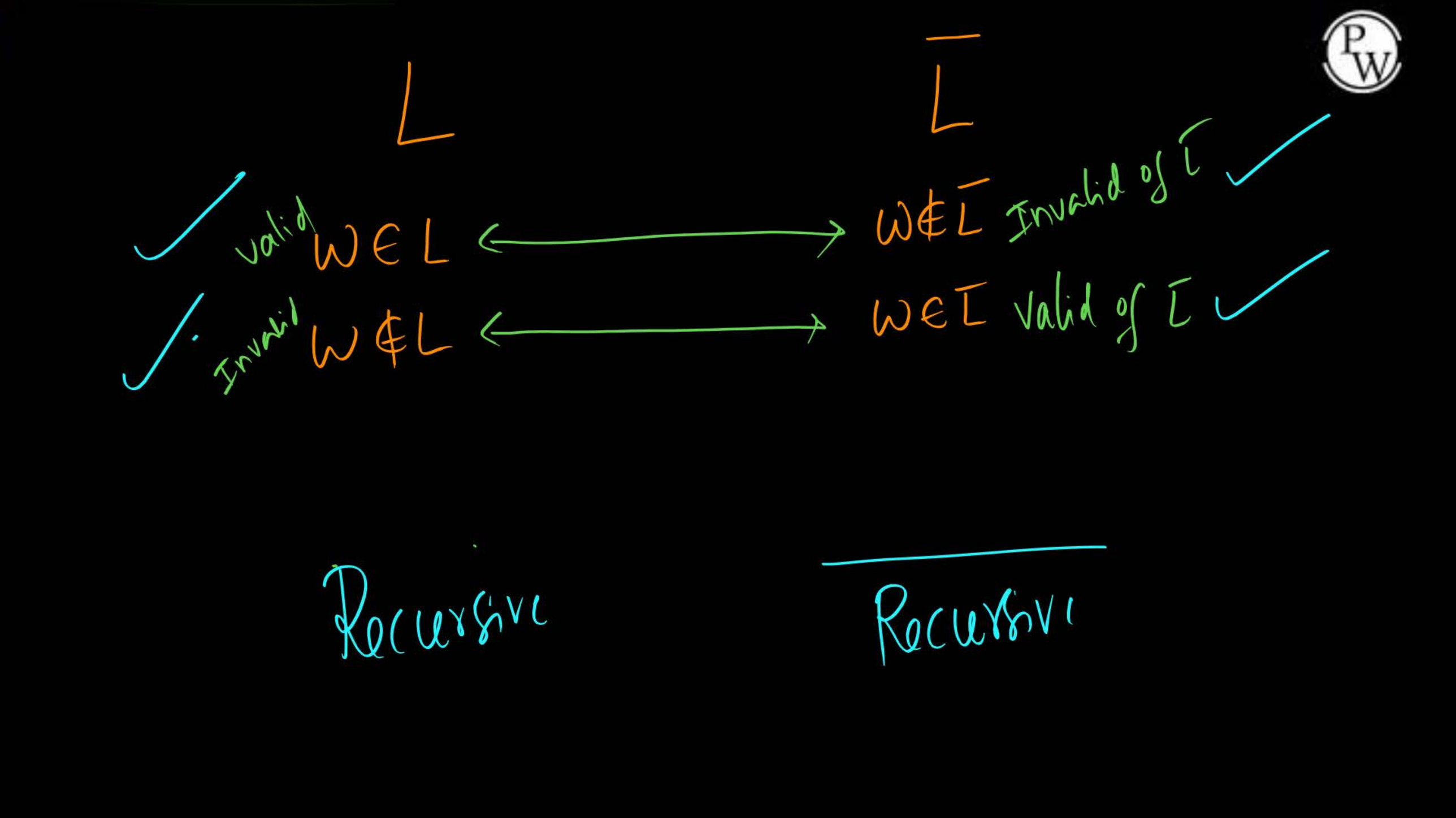
REL ?

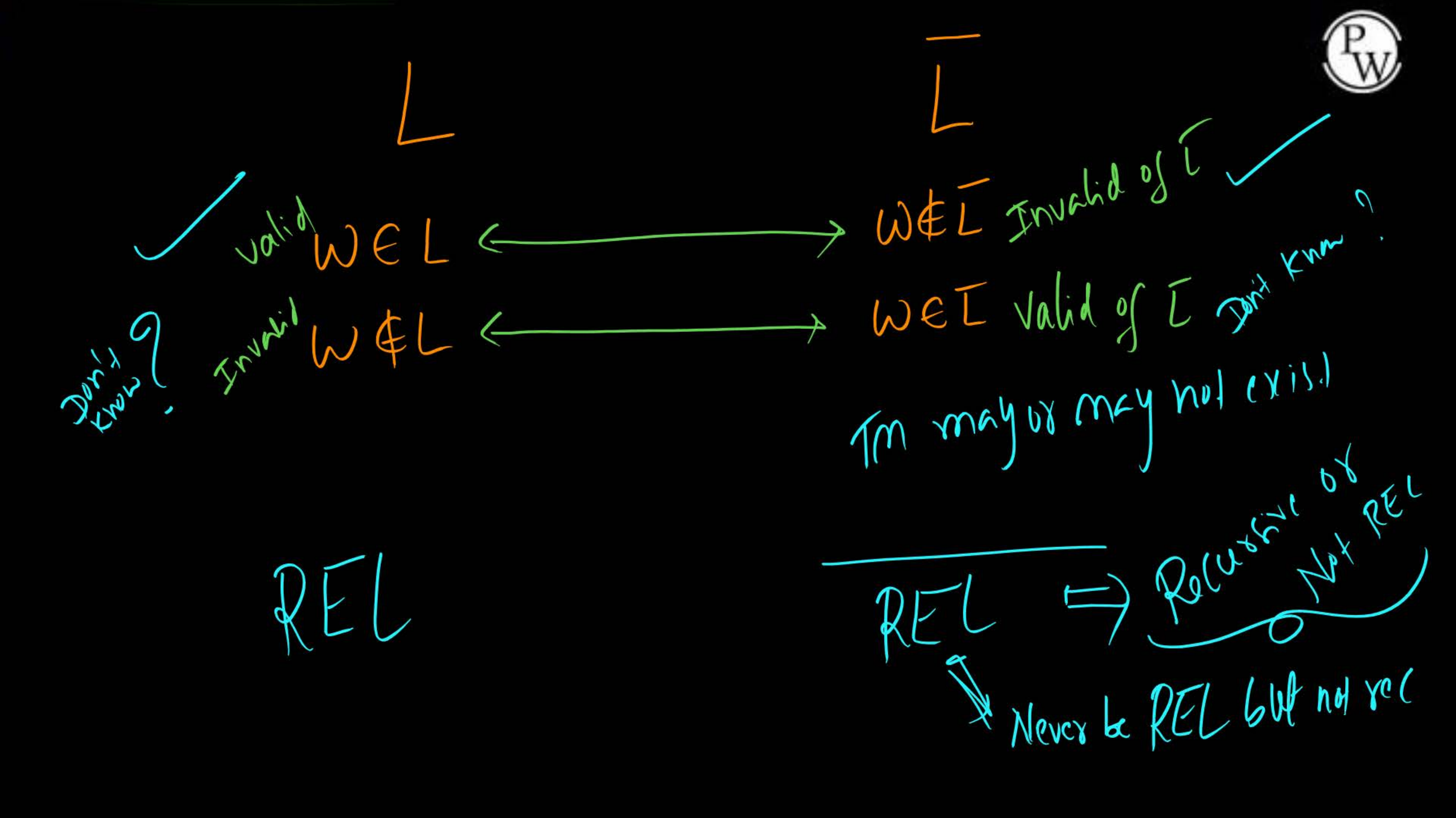
$\overline{\text{REL}}$ but not rec ?

Not REL ?

Undecidable ?


L
$\bar{L}$
valid
$w \in L$
$w \notin \bar{L}$ Invalid of $\bar{L}$
Invalid
$w \notin L$
$w \in \bar{L}$ valid of $\bar{L}$
TM not exist
REL but not rec
REL but not rec


L
$\bar{L}$
valid
$w \in L$
$w \notin \bar{L}$ Invalid of $\bar{L}$
Invalid
$w \notin L$
$w \in \bar{L}$ valid of $\bar{L}$
Recursive
Recursive


L
$\bar{L}$
Valid
$w \in L$
$w \notin \bar{L}$ Invalid of $\bar{L}$
Don't Know ?
Invalid
$w \notin L$
$w \in \bar{L}$ Valid of $\bar{L}$ Don't Know ?
TM may or may not exist
REL
REL ⇒ Recursive or Not REL
Never be REL but not rec

Note:

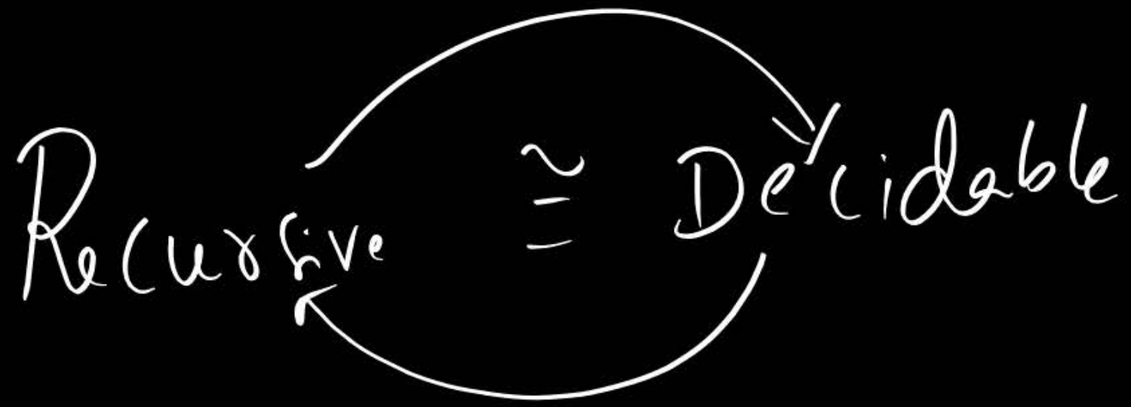
I) Recursive $\cup \cap - /$ Regular $\Rightarrow$ Recursive

Regular Closures

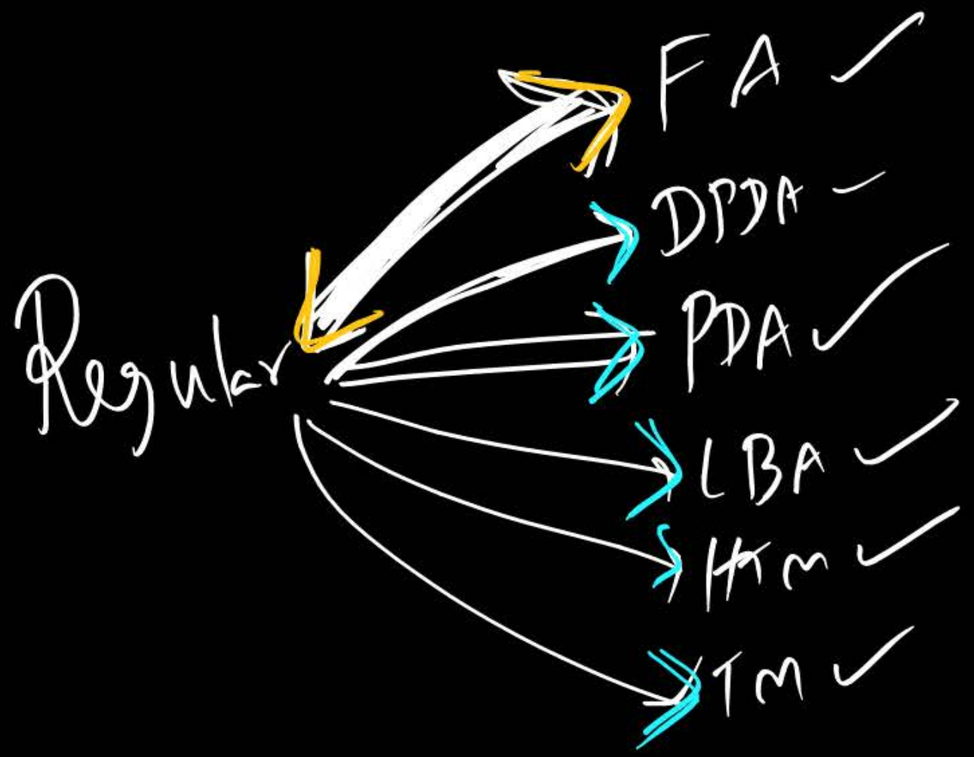
II) REL $\cup \cap - /$ Regular $\Rightarrow$ REL

I) Recursive is REL but REL need not be Recursive

II) Decidable is REL but REL need not be Decidable



Recursive $\xrightarrow{\text{is}}$ REL


Regular
FA
DPDA
PDA
LBA
TM

I) What is Recursive Language ?

L is Recursive Language

iff

L is Decidable Language

iff

L has HTM

iff

Logic exist for Valid of L and Invalid of L

iff

L is REL & $\bar{L}$ is REL

iff

L has TM & $\bar{L}$ has TM

valid of L

valid of $\bar{L}$

Invalid of L

L has HTM

L is Recursive language

iff

L is (Turing) Decidable

iff

L is Lexicographically Enumerable (Effectively)

iff

L has Algorithm

iff

L has Halting program

iff

L is computable

II) What is REL ?

L is REL

iff

L has TM

iff

Logic exist for valid members of L

iff

L is semi-decidable

iff

L is enumerable

iff

L has program

iff

L is Turing Recognizable

iff

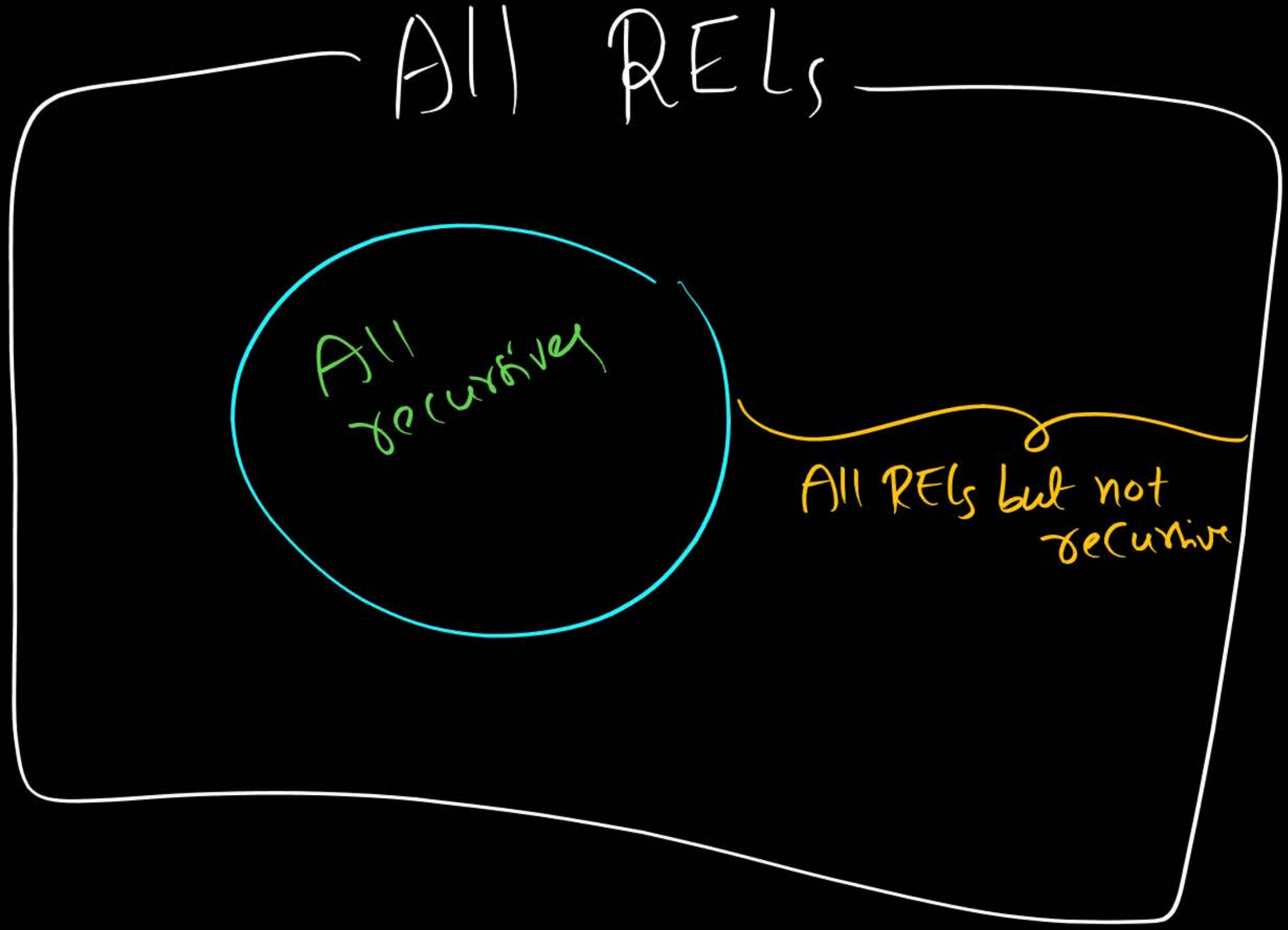
L is Turing Acceptable

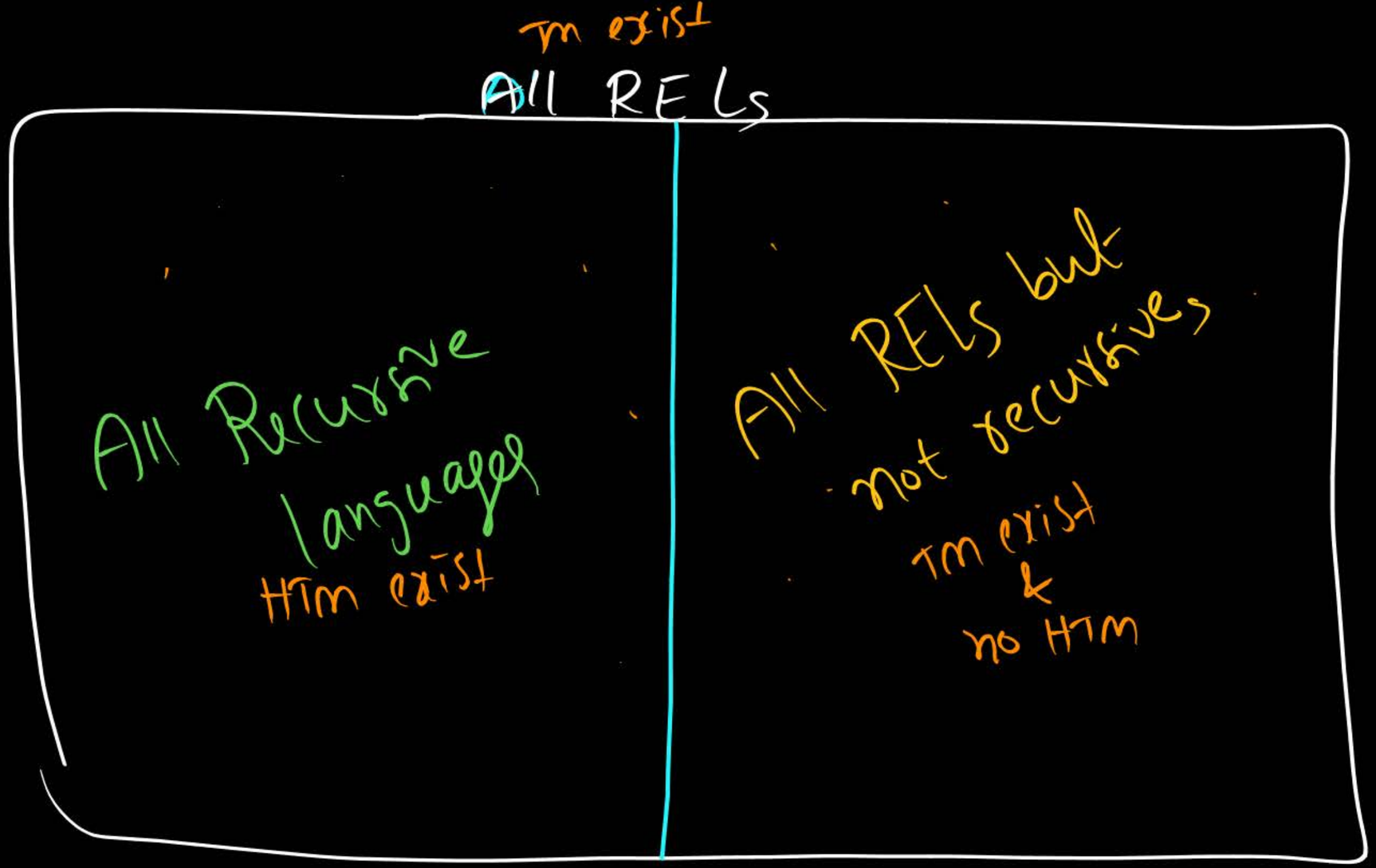
iff

L is Recognizable (Acceptable)

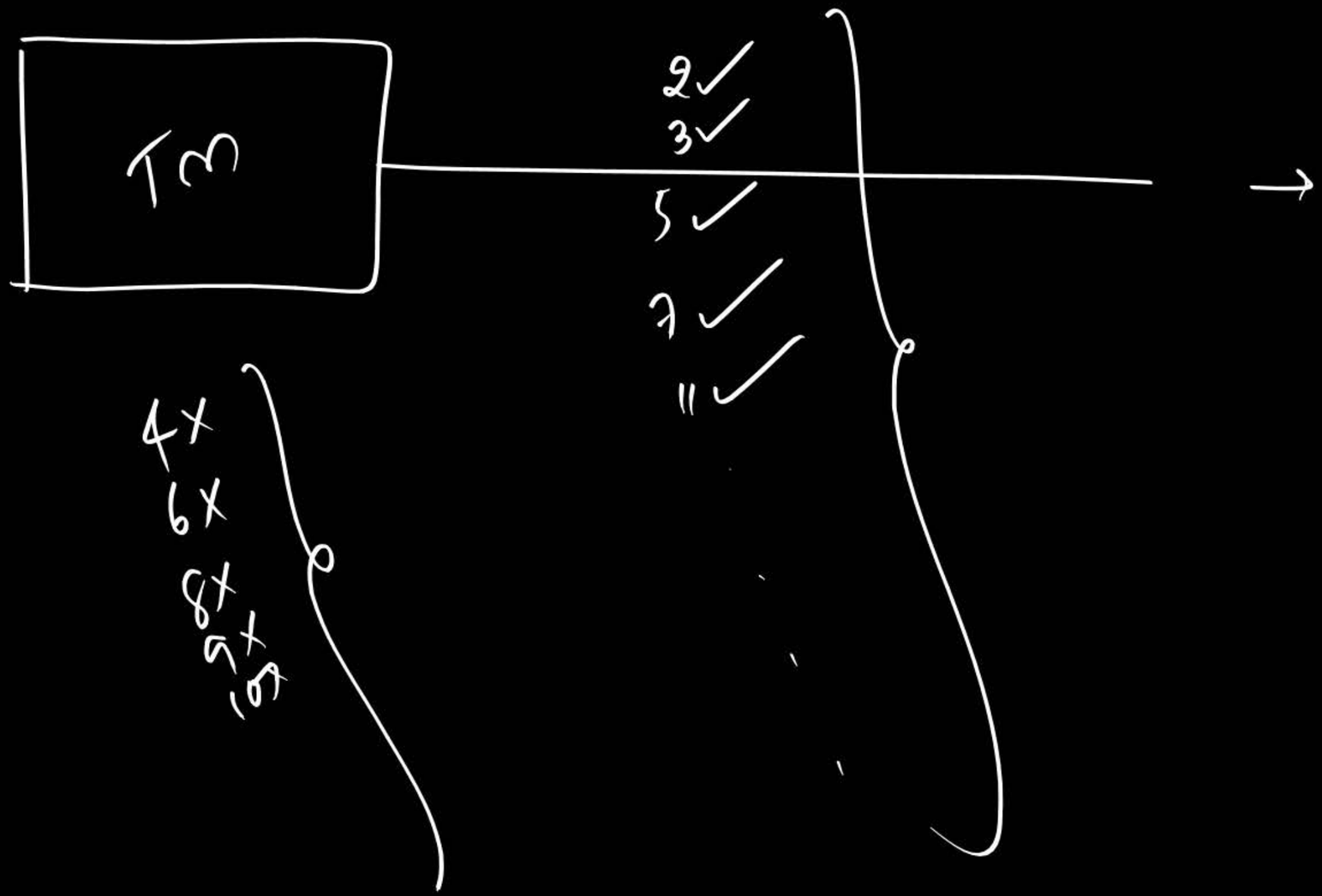
iff

L has unrestricted Grammar


All RELs
All recursively
All RELs but not recursive


TM exist
All RELs
All Recursive languages
HTM exist
All RELs but not recursives
TM exist
&
no HTM

Prime numbers



III) What is "REL but not Recursive" ?

L is REL but not recursive

$\equiv$ iff ⇕

L has TM but L has no TM

$\equiv$ iff ⇕

L has TM but $\bar{L}$ has no TM

$\equiv$ iff ⇕

L is Semidecidable (REL) & undecidable (not rec)

SDUD

iff

only valid of L has logic

iff

L is REL & $\bar{L}$ is not REL

REL but not recursive

SD & Undecidable

REL but not rec

(SDUD)

IV) What is Not REL ?

L is not REL

iff

L is not Semi decidable

iff

L has no TM

iff

valid strings of L has no logic

iff

L has no program

Not REL is Undecidable

but Undecidable is may be (REL but not RE) or (not REL)

VI) What is Undecidable ?

L is Undecidable

iff

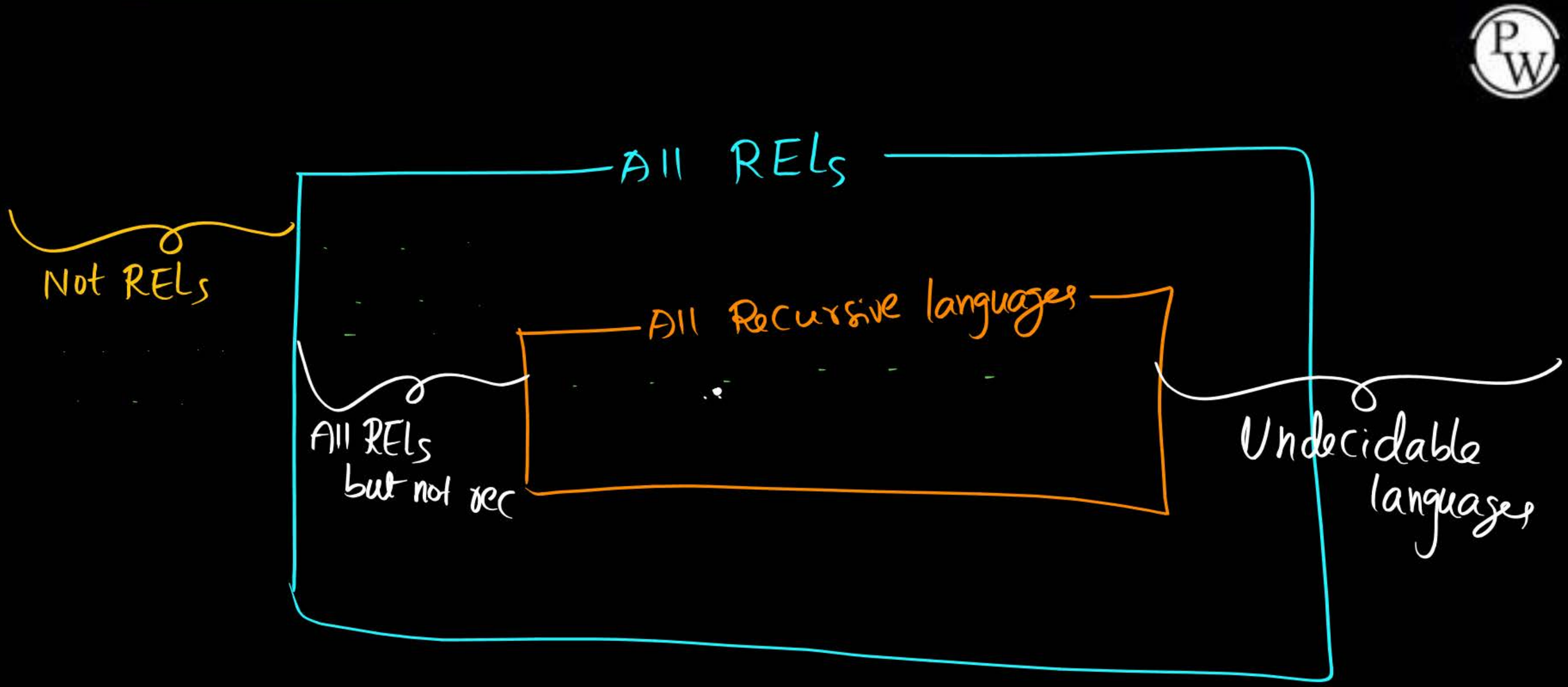
L is not decidable

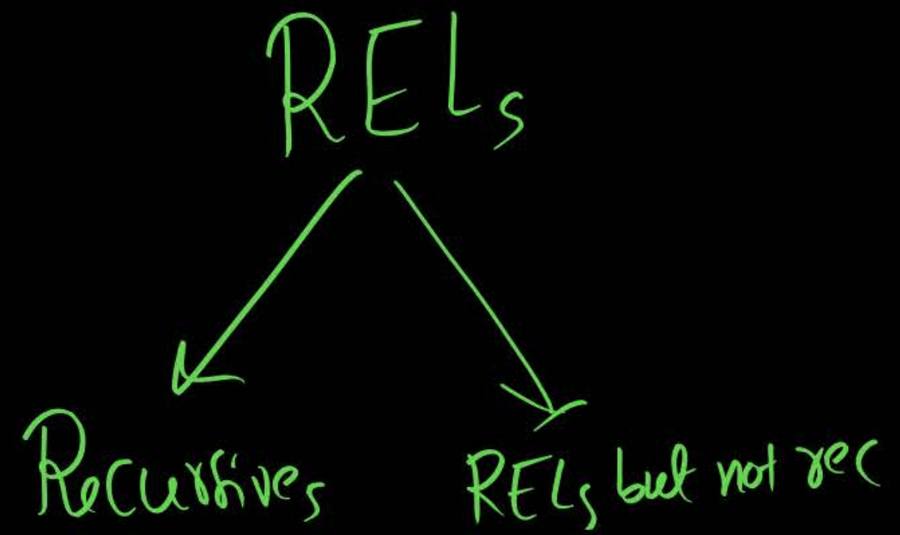
iff

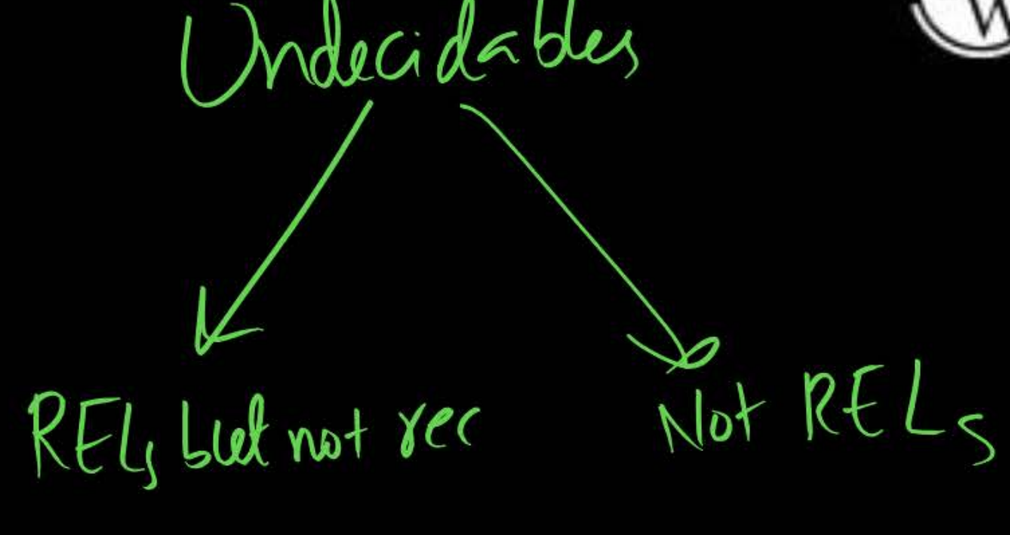
L is not recursive

iff

L has no TM


All RELs
Not RELs
All Recursive languages
All RELs but not rec
Undecidable languages





REL (SD)
- I) Recursive
- II) REL but not rec
- III) Not REL

} UD (not rec) (not Dec)

Summary

- Closure properties
- Definitions of D, SD, SDUD, NR, UD

Decidable / Recursive

Semidecidable (RE)

RE but not REC

Not RE

Undecidable

$\overline{D} \Rightarrow D$

$\overline{SD} \Rightarrow$ never be SDUD, either D or NR

$\overline{SUDUD} \Rightarrow NR$


THANK YOU
GW
SOLDIERS !