CS & IT ENGINEERING

COMPUTER NETWORKS

Error Control

Lecture No-2

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TOPICS TO BE COVERED

Hamming distance

X-OR $\rightarrow \oplus \rightarrow$ Mod 2 sum or Mod 2 Addition

Hamming Distance

$0 \oplus 1 = 1$ $\quad$ $1 \oplus 0 = 1$ $\quad$ $0 \oplus 0 = 0$ $\quad$ $1 \oplus 1 = 0$ $\quad$ $1 \oplus 0 \oplus 1 = 0$ $\quad$ $1 \oplus 1 \oplus 1 = 1$

Hamming Distance :

Hamming distance between two Binary string of same size is the number of differences between corresponding bits.

Hamming distance between two Binary string is denoted by d(x, y)

sent Rcvd

① d(000, 011) = 2 (Hamming distance)

② d(100, 011) = 3 (Hamming distance)

③ d(10101, 11110) = 3 (Hamming distance)

10101
EX-OR
11110
01011 → No. OF 1's = 3 → Hamming distance

Hamming distance can easily be found if we apply XOR operation (⊕) on the two words and count the number of 1's in the result.

minimum

Hamming Distance :

In a set of codewords, the minimum Hamming distance is the smallest Hamming distance between all possible pairs of code words.

ex-1 Valid code word

0	1	0	(a)
1	0	1	(b)
1	1	0	(c)
0	0	1	(d)

$$\left.\begin{aligned} d(a,b) &= 3 \\ d(a,c) &= 1 \\ d(a,d) &= 2 \\ d(b,c) &= 2 \\ d(b,d) &= 1 \\ d(c,d) &= 3 \end{aligned}\right\}$$

minimum Hamming distance = 1

Minimum Hamming distance for Error detection :

① Sent $\boxed{0\,1\,0}$ —— 1 bit error ——→ Rcvd $\boxed{1\,1\,0}$

<u>Valid code word</u>

Receiver can't detect the one bit error

② $\boxed{0\,1\,0}$ —— 1 bit error ——→ $\boxed{0\,1\,1}$

<u>Invalid code word</u>

can detect one bit error

Note: All one bit error can't be detected min $H \cdot D = 1$

$\boxed{\text{Valid code word}} \xrightarrow{\text{1 bit error}} \boxed{\text{Valid code word}}$

Can't detect all one bit error

Ex2 :

Valid code word

000 (a)

011 (b)

101 (c)

110 (d)

$d(a,b) = 2$

$d(a,c) = 2$

$d(a,d) = 2$

$d(b,c) = 2$

$d(b,d) = 2$

$d(c,d) = 2$

minimum Hamming distance = 2

① Sent $\boxed{0\,0\,0}$ $\xrightarrow{\text{1 bit error}}$ Rcvd $\boxed{1\,0\,0}$

Invalid code word

can detect one bit error

(a) $000 \xrightarrow{\text{1 bit error}} \left.\begin{array}{c} 100 \\ 010 \\ 001 \end{array}\right\}$ Invalid code word

(b) $011 \xrightarrow{\text{1 bit Error}} \left.\begin{array}{c} 111 \\ 001 \\ 010 \end{array}\right\}$ Invalid code word

(c) $101 \xrightarrow{\text{1 bit error}} \left.\begin{matrix} 001 \\ 111 \\ 100 \end{matrix}\right\}$ Invalid code word

(d) $110 \xrightarrow{\text{1 bit error}} \left.\begin{matrix} 010 \\ 100 \\ 111 \end{matrix}\right\}$ Invalid code word

Note: All one bit error can be detected

2. Sent: $\boxed{000}$ — 2 bit error → Rcvd: $\boxed{110}$

Valid code word

can't detect 2 bit error

Note: All 2 bit error can't be detected

min H.D = 2

$\boxed{\text{Valid code word}} \xrightarrow{\text{1 bit error}} \boxed{\text{Invalid code word}} \xrightarrow{\text{1 bit error}} \boxed{\text{Valid code word}}$

Receiver can detect All one bit error

Ex3 :

Valid code word

0 0 0 0 0 (a)

0 1 0 1 1 (b)

1 0 1 0 1 (c)

1 1 1 1 0 (d)

$d(a,b) = 3$

$d(a,c) = 3$

$d(a,d) = 4$

$d(b,c) = 4$

$d(b,d) = 3$

$d(c,d) = 3$

minimum Hamming distance $= 3$

① Sent $\boxed{00000}$ $\xrightarrow{\text{1 bit error}}$ Rcvd $\boxed{10000}$ Invalid code word

can detect one bit error

(a) 00000 $\xrightarrow{\text{1 bit error}}$

10000
01000
00100
00010
00001

Invalid code word

Note: All bit error can be detected

2. $\boxed{0\,0\,0\,0\,0}$ 2 bit error $\boxed{1\,1\,0\,0\,0}$ Invalid code word

can detect 2 bit error

$$00000 \xrightarrow{\text{2 bit error}} \left.\begin{matrix} 11000 \\ 01100 \\ 00110 \\ 00011 \\ \vdots \\ \vdots \end{matrix}\right\} \text{Invalid code word}$$

Note: All 2 bit error detected

3:

$\boxed{00000}$ 3bit error $\boxed{11100}$ Invalid code word

can detect 3 bit error

Sent $\boxed{00000}$ 3bit error Rcvd $\boxed{01011}$ Valid code word

can't detect 3 bit error

Note: All 3 bit error can't be detected

Note If minimum Hamming distance = 3

- All one bit error detected
- All two bit error detected
- All three bit error can't be detected

min. H.D = 3

$$\boxed{\text{Valid code word}} \xrightarrow{\text{1 bit error}} \boxed{\text{Invalid code word}} \xrightarrow{\text{1 bit error}} \boxed{\text{Invalid code word}} \xrightarrow{\text{1 bit error}} \boxed{\text{Valid code word}}$$

can't detect All 3 bit error

1. If minimum Hamming distance = d
   then we can detect upto (d-1) bit error

2. To detect 'd' bit error minimum
   Hamming distance required = d+1

Linear Block codes :

- A Linear block code is a code in which the XOR ($\oplus$) of two valid code words create another valid code word.
- Today all most all error detecting codes are linear block codes: Non Liner block codes are difficult to implement.
- It is simple to find the minimum Hamming distance for linear block code the minimum Hamming distance is the number of 1's in a Non zero valid code word with the smallest Number of 1's

Ex1 :

Valid code word

(a) 0 0 0

(b) 0 1 1

(c) 1 0 1

(d) 1 1 0

Non zero Valid code word

min H·D=2

XOR (a, b) = 011 (valid code word)

XOR (a, c) = 101 (valid code word)

XOR (a, d) = 110 (valid code word)

XOR (b, c) = 110 (valid code word)

XOR (b, d) = 101 (valid code word)

XOR (c, d) = 011 (valid code word)

So above code word is Liner block code.

Min Hamming distance = 2 (min. no. of 1's in the non zero code word)

Assume it is Linear Block code

× 000

$$\text{Non zero Code word}\left\{\begin{matrix} 001 \\ 011 \\ 111 \\ 110 \end{matrix}\right\}\text{minimum No. of 1's} = 1$$

$\downarrow$

minimum Hamming distance = 1


THANK YOU
GW
SOLDIERS !