CS & IT ENGINEERING

Theory of Computation

Push Down Automata

Lecture No. 8

By- DEVA Sir

TOPICS TO BE COVERED

01 Closure properties

02 practice

03

04

05

Closure properties for DCFLs [Remember closed operations]

closed operations → CPIFs

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $\overline{L}$
4. $L_1 - L_2$
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+
9. $L_1 \Delta L_2$
10. Subset(L)
11. prefix(L)
12. suffix(L)
13. Substring(L)
14. f(L)
15. h(L)
16. ε-free h(L)
17. $h^{-1}(L)$
18. L_1 / L_2
19. $L_1 \cup L_2 \cup L_3 \cup \cdots \cup L_k$
20. $L_1 \cap L_2 \cap L_3 \cap \cdots \cap L_k$
21. $L_1 - L_2 - L_3 - \cdots - L_k$
22. $L_1 \cdot L_2 \cdot L_3 \cdot \cdots L_k$
23. Finite Subset(L)
24. Finite Substitution(L)
25.
26.
27.
28.
29.
30.

25–30: Infinite ($\cup, \cap, -, \cdot, \subseteq, f$)

Closure properties for CFLs [Remember Not closed operations]

I C D S Q F $_{ID}$ I $_{All}$

1. $L_1 \cup L_2$
2. I $L_1 \cap L_2$
3. C $\overline{L}$
4. D $L_1 - L_2$
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+
9. S $L_1 \Delta L_2$
10. S Subset(L)
11. prefix(L)
12. suffix(L)
13. Substring(L)
14. f(L)
15. h(L)
16. ϵ-free h(L)
17. $h^{-1}(L)$
18. Q L_1 / L_2
19. $L_1 \cup L_2 \cup L_3 \cup \cdots \cup L_k$
20. F$_{ID}$ $L_1 \cap L_2 \cap L_3 \cap \cdots \cap L_k$
21. F$_{ID}$ $L_1 - L_2 - L_3 - \cdots - L_k$
22. $L_1 \cdot L_2 \cdot L_3 \cdot \cdots L_k$
23. Finite Subset(L)
24. Finite Substitution(L)

I$_{All}$ 25–30: Infinite ($\cup, \cap, -, \cdot, \subseteq, f$)

⑤ Concatenation

- → Not closed for DCFLs
- → Closed for CFLs

$DCFL_1 \cdot DCFL_2 \Rightarrow$ Need not be DCFL (Always CFL)

$CFL_1 \cdot CFL_2 \Rightarrow$ Always CFL

$L_1 = \{a^n b^n\}$ DCFL

$L_2 = \{a^n b^{2n}\}$ DCFL

$L_1 \cdot L_2 = \{a^{n_1} b^{n_1} a^{n_2} b^{2n_2}\}$ not DCFL

$n_1 = 0$

$\boxed{aaa}$ b

push

L_1 is CFL $\Rightarrow$ CFG_1 (S_1)

L_2 is CFL $\Rightarrow$ CFG_2 (S_2)

$\boxed{\text{Add } S \rightarrow S_1 S_2}$

$\Downarrow$

$S \rightarrow S_1 S_2$

$\boxed{CFG_1}$

$\boxed{CFG_2}$

I) $\{\underbrace{a^n b^n}\underbrace{a^k b^k}\} \longrightarrow$ DCFL

II) $\{\underbrace{a^n b^n}_{\text{may or may not}} a^k b^{2k}\} \longrightarrow$ CFL but not DCFL

III) $\{\underbrace{a^n b^n}_{\text{always comes}} a^k b^{2k} \mid n \geq 1\} \longrightarrow$ DCFL

(6) Reversal

→ not closed for DCFLs

→ closed for CFLs

I) $(\text{DCFL})^{Rev} \Longrightarrow$ need not be DCFL (Always CFL)

$L = \{a^n b^n a^k b^{2k} \mid n \geq 1, k \geq 0\}$ is DCFL

$L^{Rev} = \{b^{2k} a^k b^n a^n \mid n \geq 1, k \geq 0\}$ is not DCFL

$k=0$

bbbba

→ do we require to pop 1 b

→ do we require to pop 2 b's

L is CFL

$L = a^n b^n$

⇓

$S \to aSb \mid \varepsilon$

⇓ every rule, reverse it

$S \to bSa \mid \varepsilon$

⇓

$L = b^n a^n$ CFL

(7) Kleene star

- → Not closed for DCFLs
- → closed for CFLs

(8) Kleene plus

- → Not closed for DCFLs
- → closed for CFLs

$L = \{a\}$ CFL

⇓

$S \to a$ CFG

⇓ Add new start X and Add $X \to XS \mid \epsilon$

$X \to XS \mid \epsilon$

$S \to a$

⇓

$L = a^*$ is CFL

L is DCFL $\Rightarrow$ L^* need not be DCFL

$$L = \boxed{\{ca^nb^n\} \cup \{a^kb^{2k}\}} \Rightarrow L^* = \left(\{c\underline{a^n}\underline{b^n}\} \cup \{\underline{\underline{a^kb^{2k}}}\}\right)^*$$

$n \geq 0$ $\quad$ $k \geq 0$

is not DCFL

caabb

a

I) ca^2b^2

caabb

II) $\underbrace{\left(\{ca^0b^0\} \cup \_\right)}_{c} \underbrace{\left(\_ \cup a^2b^4\right)}_{aabbbb}$

(9) $L_1 \Delta L_2 = (L_1 - L_2) \cup (L_2 - L_1)$

$= (L_1 \cup L_2) - (L_1 \cap L_2)$

→ Not closed for DCFLs

" " " CFLs

(10) Subset of DCFL ____Need not be DCFL____

Subset of CFL ____Need not be CFL____

$L = (a+b)^*$ Reg (DCFL) (CFL)

- $\phi \subseteq L$ — Reg
- $a^* \subseteq L$ — Reg
- $a^n b^n \subseteq L$ — not reg, DCFL
- $ww^R \subseteq L$ — not reg, not DCFL, CFL
- $a^n b^n a^n \subseteq L$ — not CFL, not DCFL, not reg

Regular closures for DCFLs :

① $DCFL \cup Reg \Rightarrow$

*** ② $DCFL \cap Reg \Rightarrow$

③ $DCFL - Reg \Rightarrow$

④ $Reg - DCFL \Rightarrow$

$\}$ Always DCFL (need not be Regular)

$Reg - DCFL = Reg \cap \overline{DCFL}$

$= Reg \cap DCFL$

$= DCFL$

$$\boxed{\begin{aligned} DCFL - Reg &= DCFL \cap \overline{Reg} \\ &= DCFL \cap Reg \\ &= DCFL \end{aligned}}$$

Note: I) DCFL $\cap$ Reg $\Rightarrow$ Always DCFL

i) $\underset{\text{DCFL}}{a^n b^n} \cap \underset{\text{reg}}{a^* b^*} \Rightarrow \underset{\substack{\text{not reg} \\ \text{(DCFL)}}}{a^n b^n}$

ii) $\underset{\text{DCFL}}{\phi} \cap \underset{\text{reg}}{\phi} \Rightarrow \underset{\text{reg}}{\phi}$

II) Set of all DCFLs $\cap$ Set of all Regs $\Rightarrow$ Set of all regs

Regular closures for CFLs :

I) CFL $\cup$ Reg $\Rightarrow$
II) CFL $\cap$ Reg $\Rightarrow$
III) CFL $-$ Reg $\Rightarrow$

$\Big\}$ Always CFL

*** IV) Reg $-$ CFL $\Rightarrow$ Need not be CFL

$$\text{Reg} - \text{CFL} = \text{Reg} \cap \overline{\text{CFL}}$$
$$= \text{Reg} \cap \, ? \Rightarrow \text{need not be CFL}$$

I) DCFL $\boxed{\begin{array}{c}\cup \\ \cap \\ - \\ /\end{array}}$ Regular $\Longrightarrow$ DCFL

II) CFL $\boxed{\begin{array}{c}\cup \\ \cap \\ - \\ /\end{array}}$ Regular $\Longrightarrow$ CFL

① $L = \{a^n b^n\} \Rightarrow$ $\text{prefix}(L) = \{a^m b^n \mid m \geq n\}$

$\text{Suffix}(L) = \{a^m b^n \mid m \leq n\}$

$\text{Substring}(L) = \{a^* b^*\}$

② $L = \{a^n b^n\}$, $f(a) = \{0\}$, $f(b) = \{11\} \Rightarrow f(L) = 0^n 11^n = \{0^n 1^{2n}\}$

③ $L = \{ww^R \mid w \in \{a,b\}^*\} \Rightarrow$ $\text{prefix}(L) = (a+b)^*$

$\text{Suffix}(L) = (a+b)^*$

$\text{Substring}(L) = (a+b)^*$

prefix(DCFL) is DCFL

DCFL

$\Downarrow$

DPDA

$\Downarrow$ From initial to final path, \$ can come from any state

mod DPDA

$\Downarrow$

Pref(DCFL)

Note: Language L satisfies prefix property

iff

Every w in L is not prefix of any other string in L

Q1) $L = a^*$ Is it satisfy prefix property? $\Rightarrow$ No

$= \{\varepsilon, a, aa, \ldots\}$ $\Rightarrow$ Not satisfies prefix property

Q2) $L = \{a^n b^n \mid n \geq 1\} = \{ab, aabb, aaabbb, a^4b^4, \ldots\}$

$\hookrightarrow$ satisfies prefix property.

If any DCFL satisfies prefix property then we can make DPDA with empty stack.

$$\boxed{\text{DPDA} \cong \text{DPDA with final state}}$$

$\not\cong$ DPDA with Empty stack

$$\boxed{\text{DPDA with empty stack} < \text{DPDA}}$$

Note:

Set of all DCFLs

$\cong$

Set of all languages accepted by $\boxed{\text{DPDA}}$

$\cong$

Set of all languages accepted by $\boxed{\text{DPDA with final state}}$

$\cong$

Set of all languages generated by LR(1) CFGs
(LR(k))
$k \geq 1$

*** CFLs & DCFLs

$\boxed{\text{DPDA \& PDA}}$

closure properties

*** CFGs

$D^{\checkmark}_{CPIFs}$ $R^{\times}_{SI_{All}}$

$C^{\times}_{ICDSQF_{ID}I_{All}}$

a^+

DFA ✓

NFA ✓

DPDA ✓ with Final state

~~DPDA with empty stack~~

a ✓ doesn't satisfy prefix property

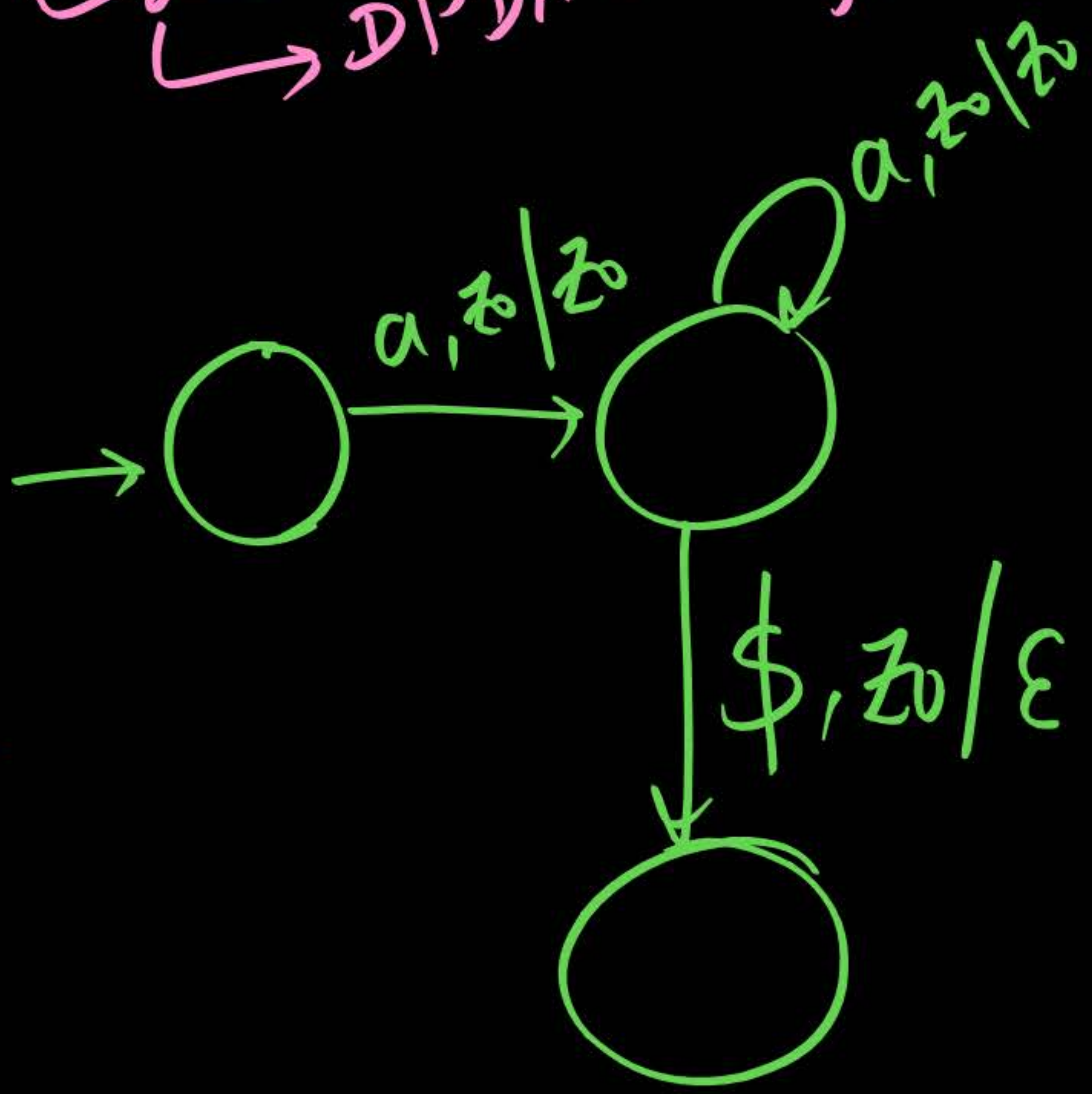
aa ✓

aaa ✓

No DPDA with empty stack

$a^+\$$ satisfy prefix property

→ DPDA with empty stack



PDA

With F.S. $\cong$ With E.S

DPDA

$\underbrace{\text{With F.S.}}_{\text{Same as DPDA}} \ncong \underbrace{\text{With E.S. (less powerful)}}$

Note:

CFG

$\cong$

CFL

$\cong$

PDA

$\cong$

PDA with F.S.

$\cong$

PDA with E.S.

DCFL

$\cong$

DPDA

$\cong$

DPDA with F.S.

$\cong$

LR(k) CFG $(k \geq 1)$

Inherently Ambiguous CFL :
(IAL)

IUL (Inherently unambiguous CFL)

1) a^* $\to$ Reg is not IAL — we can show unamb CFG

2) $a^n b^n$ $\to$ DCFL is not IAL

3) $\{ww^R \mid w \in \{a,b\}^*\}$ $\to$ CFL but not DCFL not IAL

*** 4) $\{a^m b^n c^k \mid m=n \text{ OR } n=k\}$ is IAL

$\to$ CFL but not DCFL

$\to$ There is no unamb CFG is the world

Ambiguous CFG

$\exists w$, >1 PT

IAL

L is IAL

iff

every CFG that generates L is Ambiguous

iff

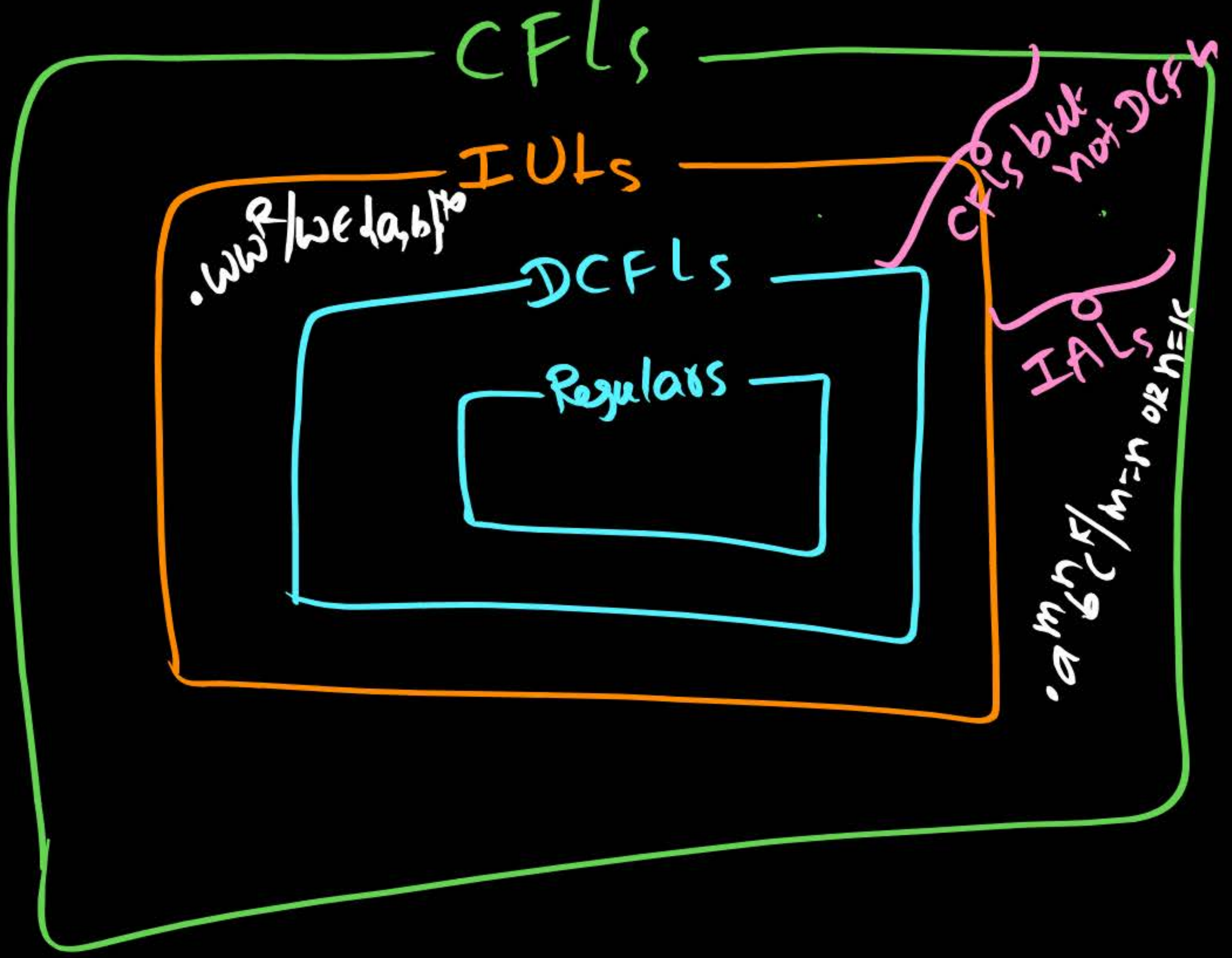
L has no unambiguous CFG

IUL

L is IUL

iff

Some unambiguous CFG exist for L



Every Reg is IUL

Every DCFL is IUL

CFL is either IUL or IAL

$\{WW^R \mid W \in \{a,b\}^*\}$

- → CFL but not DCFL
- → Unambiguous CFL

We can show unamb CFG.

$S \rightarrow aSa \mid bSb \mid \epsilon$

Unamb CFG

$\{a^m b^n c^k \mid m=n \text{ or } n=k\}$

- → CFL but not DCFL
- → Ambiguous CFL

There is no unambiguous CFG for this lang

If L is unambiguous CFL

$\Downarrow$

Show atleast one unambiguous CFG

$L = a^*$ is IUL

infinite CFGs exist

$\underbrace{S \to Sa \mid \varepsilon}$

atleast one Unamb CFG exist

If L is ambiguous CFL

⇓

We don't have unambiguous CFG

every CFG is amb CFG

$$L = \{ a^m b^n c^k \mid m = n \text{ or } n = k \}$$

There is no unamb CFG for this ✓

$$= \{ \underbrace{\underbrace{a^n b^n}_{X} \underbrace{c^*}_{Y}}_{A} \} \cup \{ \underbrace{\underbrace{a^*}_{P} \underbrace{b^n c^n}_{Q}}_{B} \}$$

$$
\boxed{
\begin{array}{ll}
S \to A \mid B & \\
A \to XY & B \to PQ \\
X \to aXb \mid \varepsilon & P \to aP \mid \varepsilon \\
Y \to cY \mid \varepsilon & Q \to bQc \mid \varepsilon
\end{array}
}
$$

1. $L = a^* b^* \longrightarrow$ Reg $\longrightarrow$ IUL

2. $L = a^n b^n \longrightarrow$ DCFL $\longrightarrow$ IUL

3. $L = \{a^m b^n c^k \mid m = n \text{ or } m = k\}$ $\to$ CFL but not DCFL $\to$ IAL

4. $L = \{a^n b^n c^*\} \longrightarrow$ DCFL $\longrightarrow$ IUL

5. $L = \{w \# w^R \mid w \in \{a, b\}^*\} \longrightarrow$ DCFL $\longrightarrow$ IUL

6. $L = \{w w^R \mid w \in (a+b)^*\} \longrightarrow$ CFL but not DCFL $\longrightarrow$ IUL

CFG

- Simplifications of CFG
  - Elimination null rules
  - " Unit rules
  - " Useless rules & symbols
- Normal Forms
  - CNF CFG
  - GNF CFG
- CYK Algorithms (Membership Algo)

Practice on CFLs

- class ✓
- DPPs & WTs ✓
- GATE PYQs ✓

Test Series

(After TOC complete)

Q

Which of the following are decidable? Last chapter

1. Whether the intersection of two regular languages is infinite
2. Whether a given context-free language is regular
3. Whether two push-down automata accept the same language
4. Whether a given grammar is context-free

[2008: 1 Marks]

(A) 1 and 2

(B) 1 and 4

(C) 2 and 3

(D) 2 and 4

Q

Consider the language

$L1 = \{0^i1^j \mid i \neq j\}$,

$L2 = \{0^i1^j \mid i = j\}$,

$L3 = \{0^i1^j \mid i = 2j + 1\}$,

$L4 = \{0^i1^j \mid i \neq 2j\}$.

DCFL → CFL

$0^{2j+1}1^j$

$0^{2j}1^j$

Which one of the following statements is true? [2010: 2 Marks]

(A) Only L2 is context free

(B) Only L2 and L3 are context free

(C) Only L1 and L2 are context free

(D) All are context free

Q

$0^+1^+ \rightarrow$ Reg

Consider the languages L1, L2 and L3 as given below:

$L1 = \{0^p1^q \mid p, q \in N\}$, $= 0^*1^*$ $\rightarrow$ Reg

$L2 = \{0^p1^q \mid p, q \in N \text{ and } p = q\}$ and $\rightarrow 0^n1^n \rightarrow$ DCFL

$L3 = \{0^p1^q0^r \mid p, q, r \in N \text{ and } p = q = r\}$ $\rightarrow 0^n1^n0^n \rightarrow$ CSL

Which of the following statements is NOT TRUE? [2011: 2 Marks]

(A) Push Down Automata (PDA) can be used to recognize L1 and L2. T

(B) L1 is a regular language. T

(C) All the three languages are context free. F

(D) Turing machines can be used to recognize all the languages. T

Q

Consider the following languages:

I. $\{a^m b^n c^p d^q \mid m + p = n + q, \text{ where } m, n, p, q \geq 0\}$

II. $\{a^m b^n c^p d^q \mid m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$

III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$

IV. $\{a^m b^n c^p d^q \mid mn = p + q, \text{ where } m, n, p, q \geq 0\}$

Which of the language above are context-free?

[2012: 2 Marks]

(A) I and IV only

(B) I and II only

(C) II and III only

(D) II and IV only

Q

Consider the following languages $0^*1^*0^*$

$L1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$

$L2 = \{0^p 1^q 0^r \mid p, q, r \geq 0, p \neq r\}$

$L_1 \cap L_2 = L2$

Which one of the following statements is FALSE?

[2013: 2 Marks]

(A) L2 is context-free

(B) $L1 \cap L2$ is context-free

(C) Complement of L2 is recursive

(D) Complement of L1 is context-free but not regular

L_1

L_2

Q

Consider the following languages over the alphabet $\Sigma = \{0, 1, c\}$:

$L_1 = \{0^n1^n \mid n \geq 0\}$

$L_2 = \{wcw^r \mid w \in \{0, 1\}^*\}$

$L_3 = \{ww^r \mid w \in \{0, 1\}^*\}$

Here, w^r is the reverse of the string w. Which of these languages are deterministic Context-free languages?

[2014-Set3: 2 Marks]

A None of the languages

B Only L_1

C Only L_1 and L_2

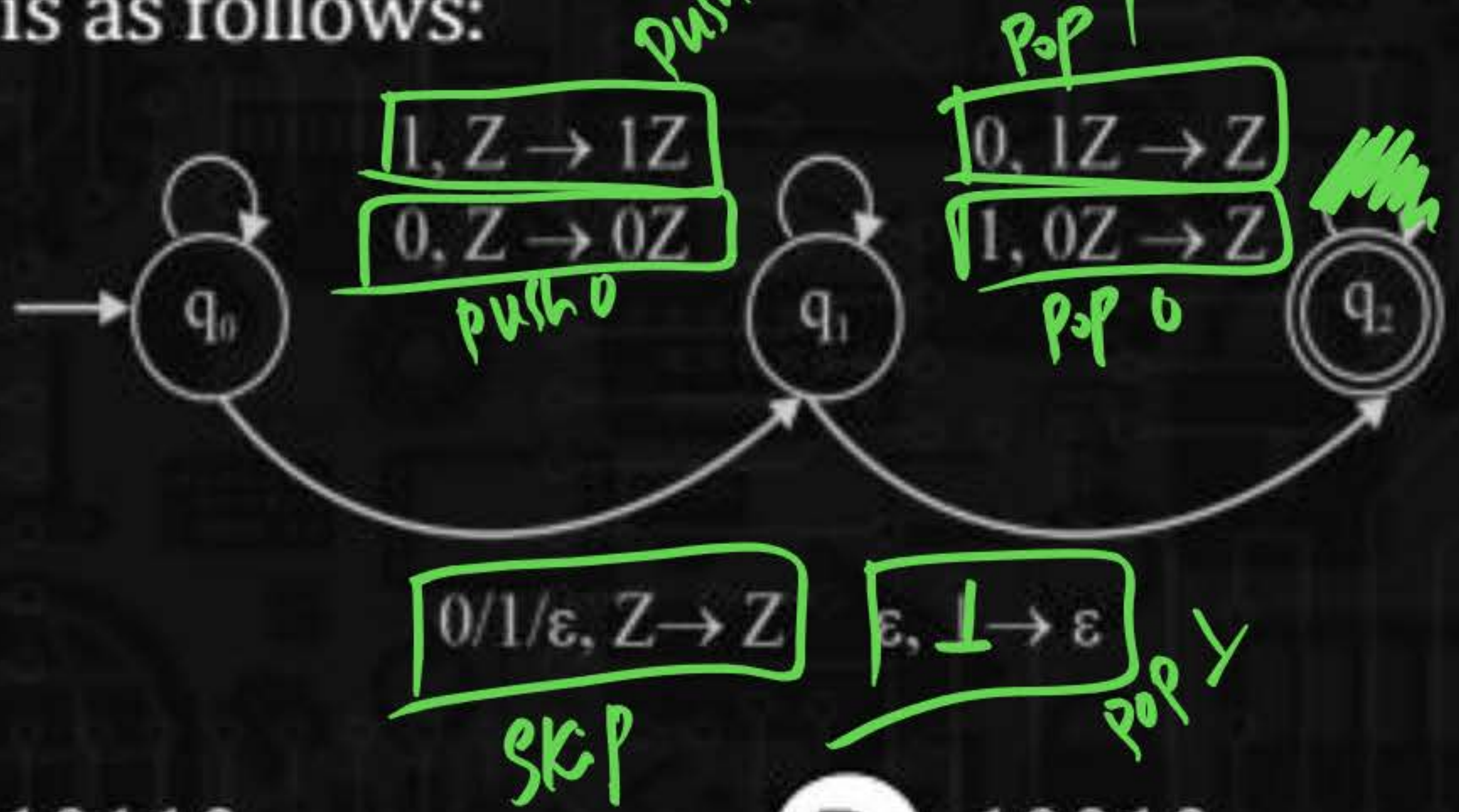
D All the three languages

Q ***

Consider the NPDA $\langle Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, \perp\}, \delta, q_0, \perp, F = \{q_2\} \rangle$, where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and F is the set of accepting states, The state transition is as follows:

$\perp = z$

$Z = \text{Any}$



Which of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

[2015-Set1: 2 Marks]

(A) 10110

(B) 10010

(C) 01010

(D) 01001

Practice on CFLs

Q

Which of the following languages are context-free?

$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$

$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$

$L_3 = \{a^m b^n \mid m = 2n + 1\}$

[2015(Set-3): 1 Marks]

(A) L_1 and L_2 only

(B) L_1 and L_3 only

(C) L_2 and L_3 only

(D) L_3 only

Practice on CFLs

Q

Consider the following context-free grammars:

$G_1: S \to aS \mid B,\ B \to b \mid bB$

$G_2: S \to aA \mid bB,\ A \to aA \mid B \mid \varepsilon,\ B \to bB \mid \varepsilon$

Which one of the following pairs of languages is generated by G_1 and G_2, respectively?

[2016(Set-1): 2 Marks]

(A) $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$.

(B) $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n \geq 0\}$.

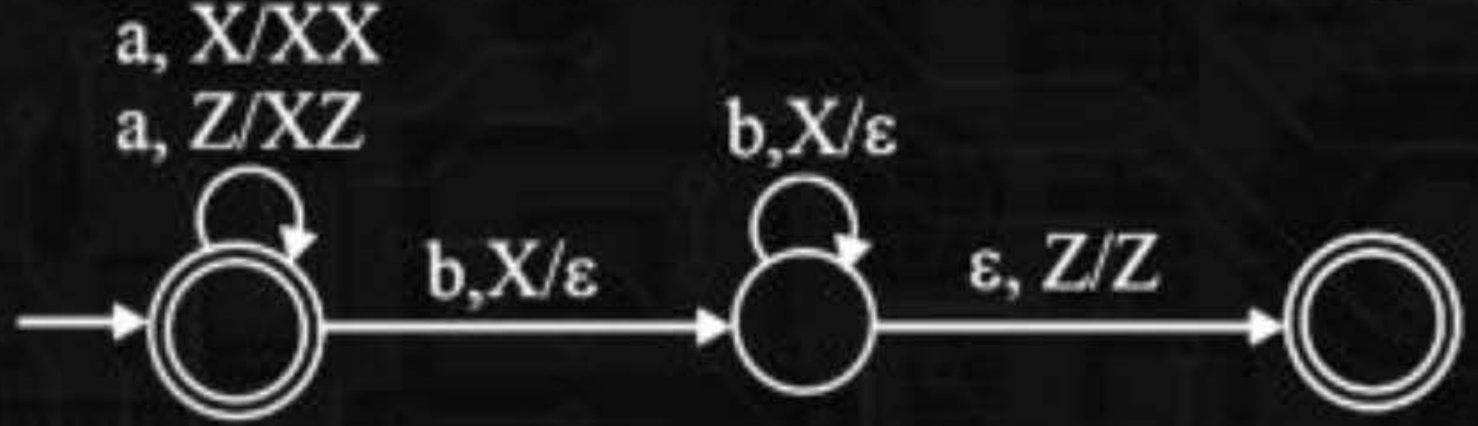
(C) $\{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ and } n > 0\}$.

(D) $\{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$ and $\{a^m b^n \mid m > 0 \text{ or } n > 0\}$.

Q

Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.

$Z = \perp = z_0$



Which one of the following is TRUE?

[2016(Set-1): 2 Marks]

(A) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata.

(B) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is not accepted by any deterministic PDA.

(C) L is not accepted by any Turing machine that halts on every input.

(D) $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and is deterministic context-free.

Q

Consider the following languages:

$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\}$

$L_2 = \{a^n b^n c^{2n} : n \geq 1\}$

Which one of the following is TRUE?

[2016(Set-2): 2 Marks]

(A) Both L_1 and L_2 are context-free.

(B) L_1 is context-free while L_2 is not context-free

(C) L_2 is context-free while L_1 is not context-free

(D) Neither L_1 nor L_2 is context-free

Q.

Language L_1 is defined by the grammar: $S_1 \rightarrow aS_1b|\varepsilon$
Language L_2 is defined by the grammar: $S_2 \rightarrow abS_2|\ \varepsilon$
Consider the following statements:
P: L_1 is regular
Q: L_2 is regular
Which one of the following is TRUE?

[2016(Set-2): 1 Marks]

(A) Both P and Q are true

(B) P is true and Q is false

(C) P is false and Q is true

(D) Both P and Q are false

Q

Consider the following context-free grammar over the alphabet $\Sigma = \{a, b, c\}$ with S as the start symbol

$S \rightarrow abScT \mid abcT$

$T \rightarrow bT \mid b$

Which one of the following represents the language generated by the above grammar?

[2017(Set-1): 1 Marks]

(A) $\{(ab)^n(cb)^n \mid n \geq 1\}$

(B) $\{(ab)^n cb^{m_1} cb^{m_2} \ldots cb^{m_n} \mid n, m_1, m_2, \ldots, m_n \geq 1\}$

(C) $\{(ab)^n(cb^m)^n \mid m, n \geq 1\}$

(D) $\{(ab)^n(cb^n)^m \mid m, n \geq 1\}$

Q

Consider the following language over the alphabet $\Sigma = \{a, b, c\}$.

Let $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ and

$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$.

Which of the following are context-free languages?

I. $L_1 \cup L_2$

II. $L_1 \cap L_2$

[2017(Set-1): 2 Marks]

(A) I only

(B) II only

(C) I and II

(D) Neither I nor II

Q

Consider the context-free grammars over the alphabet $\{a, b, c\}$ given below. S and T are non-terminals.

$$G_1: S \rightarrow aSb \mid T,\ T \rightarrow cT \mid \in$$

$$G_2: S \rightarrow bSa \mid T,\ T \rightarrow cT \mid \in$$

The language $L(G_1) \cap L(G_2)$ is [2017-Set1: 1 Mark]

(A) Finite

(B) Not finite but regular

(C) Context-free but not regular

(D) Recursive but not context-free

Q

Identify the language generated by the following grammar, where S is the start variable.

$$S \to XY$$
$$X \to aX \mid a$$
$$Y \to aYb \mid \epsilon$$

[2017(Set-2): 1 Marks]

(A) $\{a^m b^n \mid m \geq n, n > 0\}$

(B) $\{a^m b^n \mid m \geq n, n \geq 0\}$

(C) $\{a^m b^n \mid m > n, n \geq 0\}$

(D) $\{a^m b^n \mid m > n, n > 0\}$

Q

Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?

I. $L_1 \cup L_2$ is context-free

II. $\overline{L_1}$ is context-free

III. $L_1 - R$ is context-free

IV. $L_1 \cap L_2$ is context-free

[2017(Set-2): 1 Marks]

(A) I, II and IV only

(B) I and III only

(C) II and IV only

(D) I only

Q

Consider the following languages:

$L_1 = \{a^p \mid p \text{ is a prime number}\}$

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$

$L_4 = \{a^n b^n \mid n \geq 1\}$

Which of the following are CORRECT?

I. L_1 is context-free but not regular.

II. L_2 is not context-free.

III. L_3 is not context-free but recursive.

IV. L_4 is deterministic context-free.

[2017(Set-2): 2 Marks]

(A) I, II and IV only

(B) II and III only

(C) I and IV only

(D) III and IV only

Q

Which one of the following languages over $\Sigma = \{a, b\}$ is NOT context-free?

[2019: 2 Marks]

A $\{a^n b^i \mid i \in \{n, 3n, 5n\}, n \geq 0\}$

B $\{wa^n w^R b^n \mid w \in \{a, b\}^*, n \geq 0\}$

C $\{ww^R \mid w \in \{a, b\}^*\}$

D $\{wa^n b^n w^R \mid w \in \{a, b\}^*, n \geq 0\}$

Q

Consider the following languages:

$L_1 = \{wxyx \mid w, x, y \in (0+1)^+\}$

$L_2 = \{xy \mid x, y \in (a+b)^*, |x| = |y|, x \neq y\}$

Which of the following is TRUE

[2020: 2 Marks]

(A) L_1 is regular and L_2 is context-free.

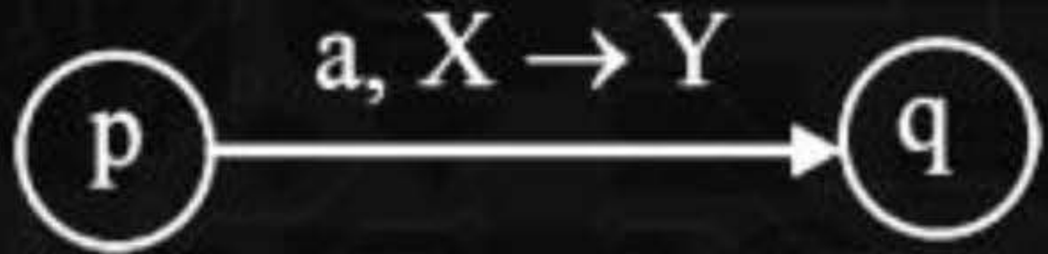
(B) L_1 is context-free but L_2 is not context-free.

(C) Neither L_1 nor L_2 is context-free.

(D) L_1 is context-free but not regular and L_2 context-free.

Q

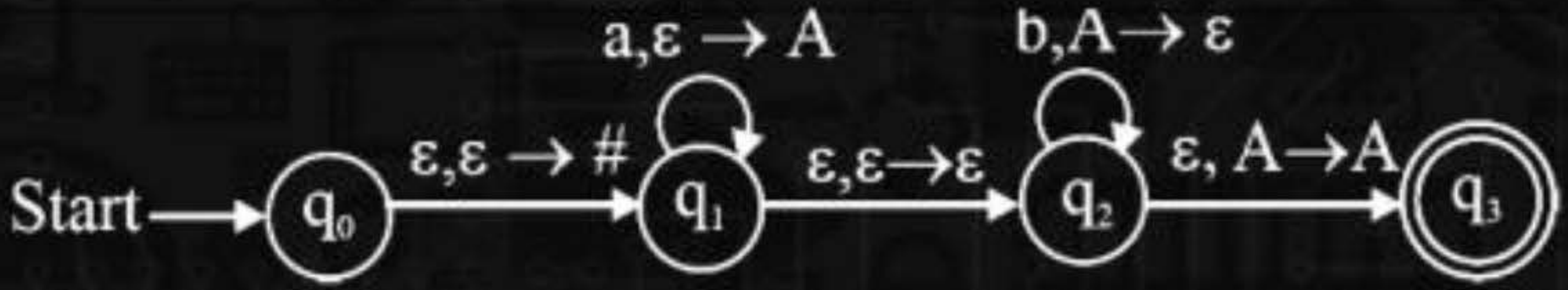
In a pushdown automaton $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$, a transition of the form,



Where $p, q, \in Q$, $a \in \Sigma \cup \{\epsilon\}$, and $X, Y \in \Gamma \cup \{\epsilon\}$ represents

$$(q, Y) \in \delta(p, a, X)$$

Consider the following pushdown automaton over the input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{\#, A\}$.



The number of strings of length 100 accepted by the above pushdown automaton is ______.

[2021(Set-1): 2 Marks]

Q

Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?

[2021(Set-1): 2 Marks]

(A) $L_1 \cdot L_2$

(B) $L_1 \cup L_2$

(C) $L_1 - L_2$

(D) $L_1 \cap L_2$

Q

For a string w, we define w^R to be the reverse of w. For example, if w = 01101 then w^R = 10110. Which of the following languages is/are context-free?

[2021(Set-2): 2 Marks]

A $\{wxw^Rx^R \mid w, x \in \{0, 1\}^*\}$

B $\{wxw^R \mid w, x \in \{0, 1\}^*\}$

C $\{ww^Rxx^R \mid w, x \in \{0, 1\}^*\}$

D $\{wxx^Rw^R \mid w, x \in \{0, 1\}^*\}$

Q

Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?

[2021(Set-2)MSQ: 1 Marks]

A) $L_1 \cap \overline{L_2}$

B) $\overline{\overline{L_1} \cup \overline{L_2}}$

C) $L_1 \cup (L_2 \cup \overline{L_2})$

D) $(L_1 \cap L_2) \cup (\overline{L}_1 \cap L_2)$

Q

Consider the following languages:

$L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$

$L_2 = \{wxw^R \mid w, x \in \{a, b\}^*\}, |w|, |x| > 0\}$

Note that w^R is the reversal of the string w. Which of the following is/are TRUE?

[2022: MSQ: 2 Marks]

(A) L_1 and L_2 are regular.

(B) L_1 and L_2 are context-free.

(C) L_1 is regular and L_2 is context-free.

(D) L_1 and L_2 are context-free but not regular.

Q

Consider the following languages:

$L_1 = \{ww \mid w \in \{a, b\}^*\}$

$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$

$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$

Which of the following statements is/are FALSE?

[2022: 2 Marks]

(A) L_1 is not context-free but L_2 and L_3 are deterministic context-free.

(B) Neither L_1 nor L_2 is context-free.

(C) L_2, L_3 and $L_2 \cap L_3$ all are context-free.

(D) Neither L_1 nor its complement is context-free

Summary

GATE PYQs practice

Next class

TM & Undecidability


THANK YOU GW
SOLDIERS !