

CS & IT ENGINEERING

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COMPUTER NETWORKS

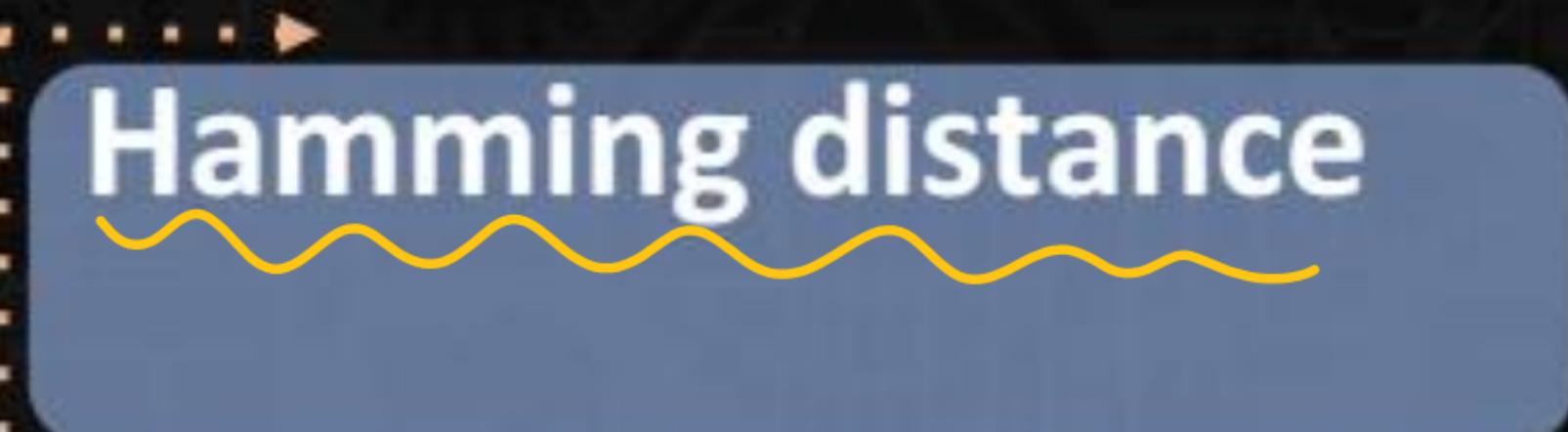


Error Control
Lecture No-2



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TOPICS TO
BE
COVERED



Hamming distance

X-OR → ⊕ → Mod 2 sum or Mod 2 Addition

Hamming Distance

$$\begin{array}{r} 0 \\ + \\ \hline 1 \end{array} \quad \begin{array}{r} 1 \\ + \\ \hline 0 \end{array} \quad \begin{array}{r} 0 \\ + \\ \hline 0 \end{array} \quad \begin{array}{r} 1 \\ + \\ \hline 0 \end{array} \quad \begin{array}{r} 1 \\ + \\ \hline 1 \end{array} \quad \begin{array}{r} 1 \\ + \\ \hline 1 \end{array}$$

Hamming Distance :

Hamming distance between two Binary string of same size is the number of differences between corresponding bits.

Hamming distance between two Binary string is denoted by $d(x, y)$

- ① $d(\underline{000}, \underline{011}) = 2$ (Hamming distance)
- ② $d(\underline{100}, \underline{011}) = 3$ (Hamming distance)
- ③ $d(\underline{10101}, \underline{11110}) = 3$ (Hamming distance)

Hamming distance can easily be found if we apply XOR operation (\oplus) on the two words and count the number of 1's in the result.

$$\begin{array}{r}
 10101 \\
 \text{EX-OR} \\
 11110 \\
 \hline
 01011 \rightarrow \text{No. of } 1\text{'s} = 3 \\
 \hline
 \end{array}$$

Hamming distance

Hamming Distance :

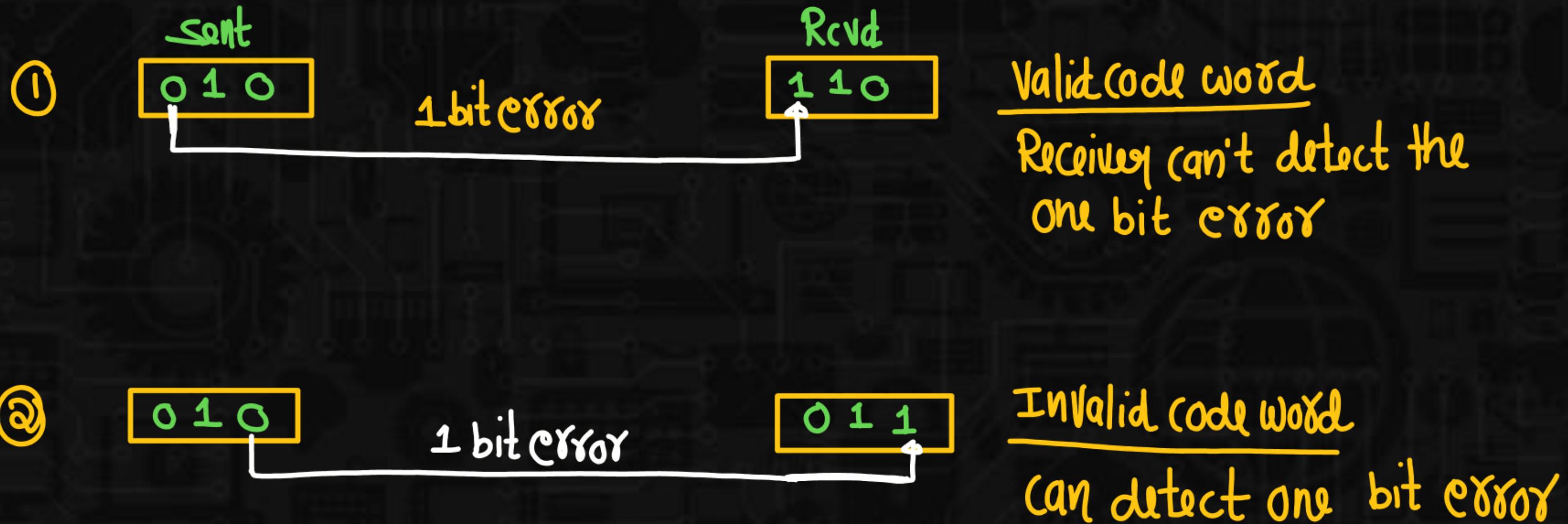
In a set of codewords, the minimum Hamming distance is the smallest Hamming distance between all possible pairs of code words.

Ex-1 Valid code word

0	1	0	(a)
1	0	1	(b)
1	1	0	(c)
0	0	1	(d)

$$\left. \begin{array}{l} d(a,b) = 3 \\ d(a,c) = 1 \\ d(a,d) = 2 \\ d(b,c) = 2 \\ d(b,d) = 1 \\ d(c,d) = 3 \end{array} \right\} \text{minimum Hamming distance} = 1$$

Minimum Hamming distance for Error detection :



Note: All one bit errors can't be detected

min
 $H \cdot D = 1$



Can't detect all
one bit errors

Ex2 :

Valid code word

0 0 0 (a)

0 1 1 (b)

1 0 1 (c)

1 1 0 (d)



$$d(a,b) = 2$$

$$d(a,c) = 2$$

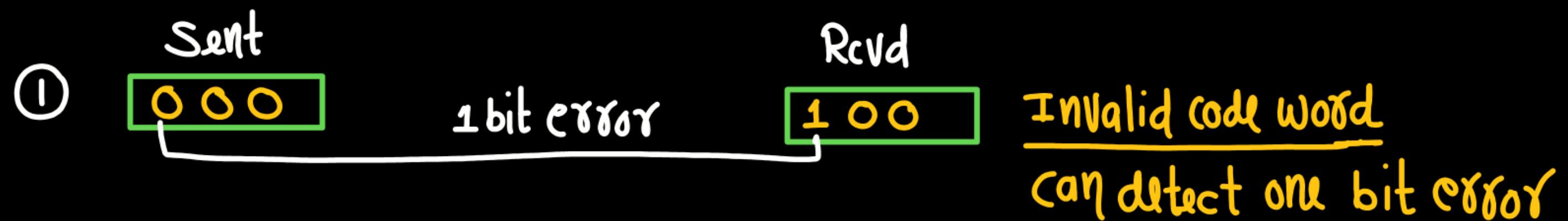
$$d(a,d) = 2$$

$$d(b,c) = 2$$

$$d(b,d) = 2$$

$$d(c,d) = 2$$


minimum Hamming
distance = 2



(a) **0 0 0** $\xrightarrow{1 \text{ bit error}}$ **1 0 0**
0 1 0
0 0 1 } Invalid code word

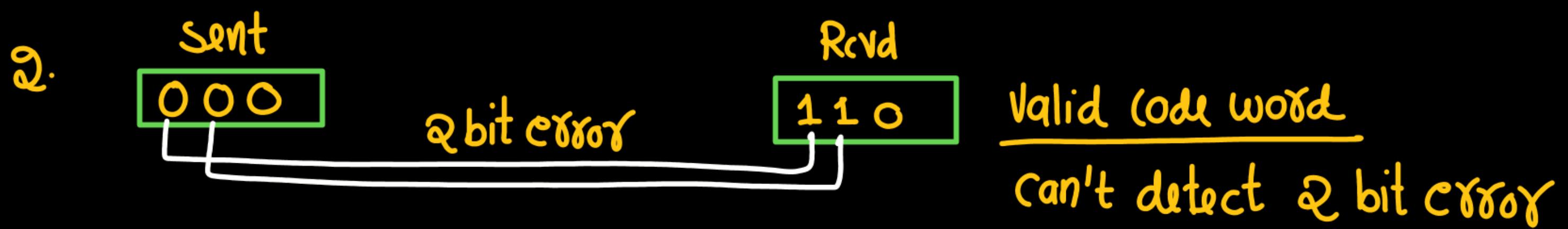
(b) **0 1 1** $\xrightarrow{1 \text{ bit Error}}$ **1 1 1**
0 0 1
0 1 0 } Invalid code word

(c) $101 \xrightarrow{1\text{ bit error}} \begin{matrix} 001 \\ 111 \\ 100 \end{matrix} \left. \right\} \text{InValid Code word}$

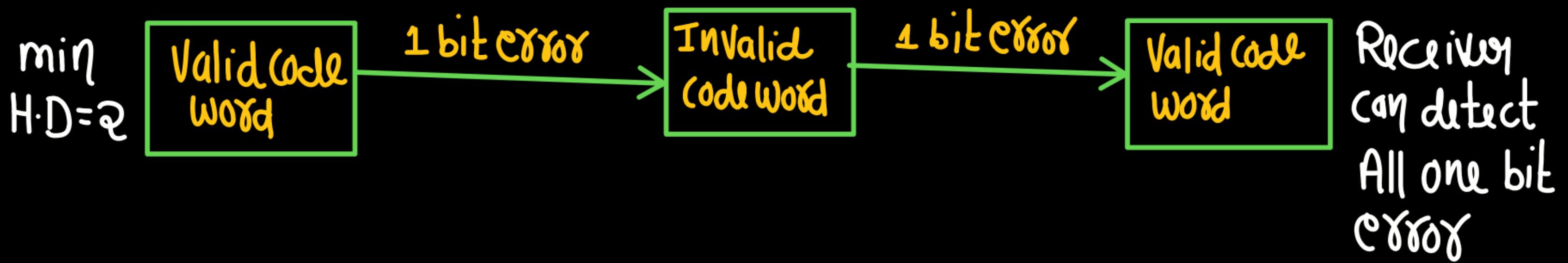
P
W

(d) $110 \xrightarrow{1\text{ bit error}} \begin{matrix} 010 \\ 100 \\ 111 \end{matrix} \left. \right\} \text{InValid Code word}$

Note: All one bit error can be detected



Note: All 2 bit error can't be detected



Ex3 :

Valid code word

0 0 0 0 0 (a)

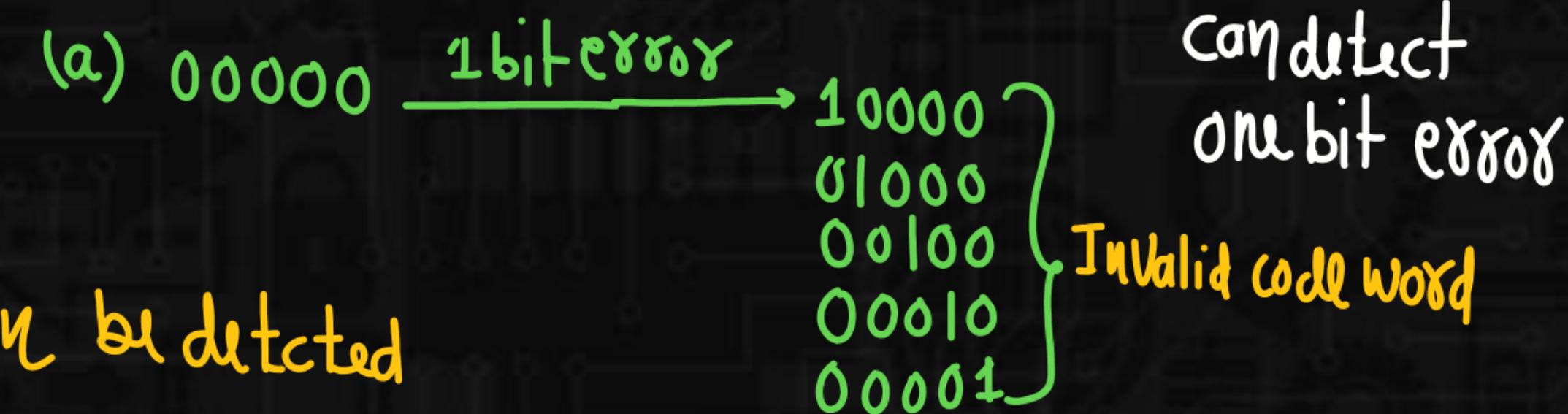
0 1 0 1 1 (b)

1 0 1 0 1 (c)

1 1 1 1 0 (d)

$$\begin{aligned} d(a,b) &= 3 \\ d(a,c) &= 3 \\ d(a,d) &= 4 \\ d(b,c) &= 4 \\ d(b,d) &= 3 \\ d(c,d) &= 3 \end{aligned}$$

minimum Hamming distance = 3



Note: All bit errors can be detected

2.

00000

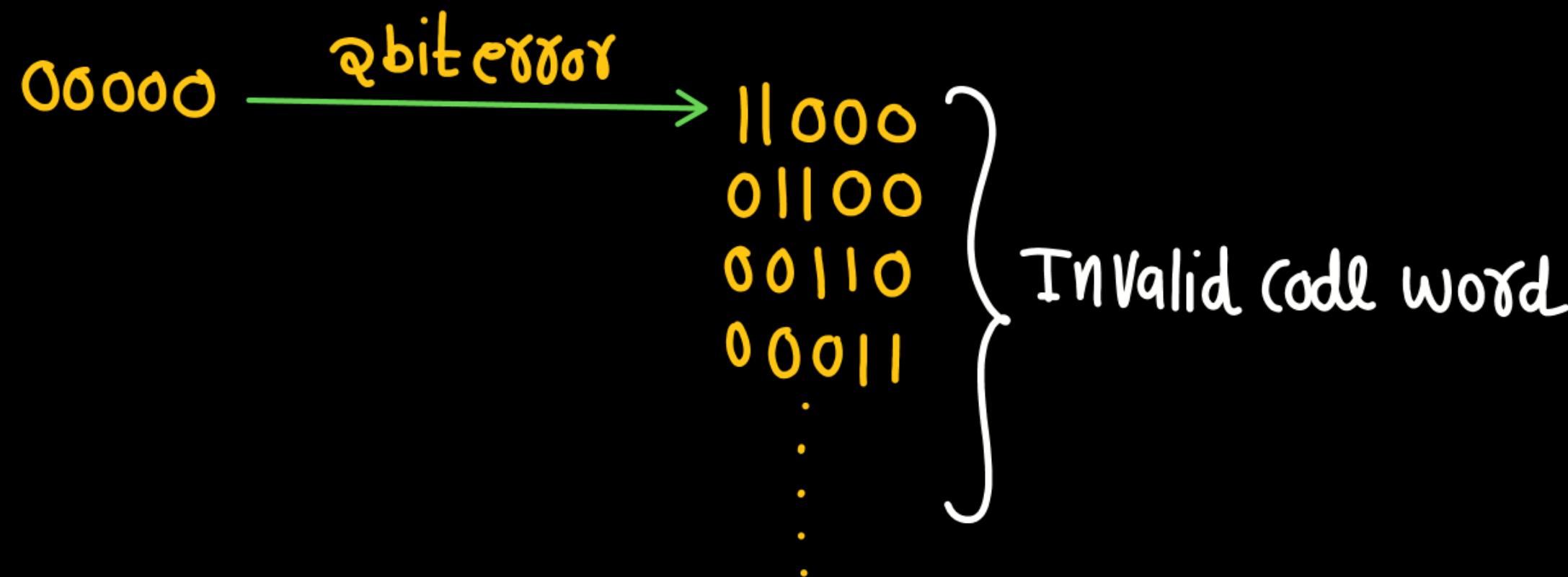
2 bit error

11000

Invalid code word

can detect 2 bit error

P
W



Note: All 2 bit error detected

3.



Invalid code word

can detect 3 bit error

P
W



Valid code word

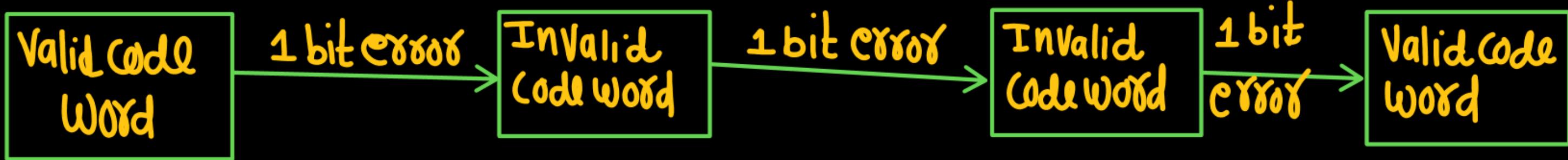
can't detect 3 bit error

Note: All 3 bit errors can't be detected

Note

GF minimum Hamming distance = 3

- All one bit error detected
- All two bit error detected
- All three bit error can't be detected

min.
H.D=3

can't detect
All 3 bit
Errors

P
W

① If minimum Hamming distance = d
then we can detect upto $(d-1)$ bit errors

② To detect ' d ' bit errors minimum
Hamming distance required = $d+1$

Linear Block codes :

- A Linear block code is a code in which the XOR (\oplus) of two valid code words create another valid code word.
- Today all most all error detecting codes are linear block codes: Non Liner block codes are difficult to implement.
- It is simple to find the minimum Hamming distance for linear block code the minimum Hamming distance is the number of 1's in a Non zero valid code word with the smallest Number of 1's.

Ex1 :

Valid code word

(a) 0 0 0

(b) 0 1 1
(c) 1 0 1
(d) 1 1 0

Non zero Valid Code word

 $\text{XOR } (a, b) = 011 \text{ (valid code word)}$ $\text{XOR } (a, c) = 101 \text{ (valid code word)}$ $\text{XOR } (a, d) = 110 \text{ (valid code word)}$ $\text{XOR } (b, c) = 110 \text{ (valid code word)}$ $\text{XOR } (b, d) = 101 \text{ (valid code word)}$ $\text{XOR } (c, d) = 011 \text{ (valid code word)}$

So above code word is Liner block code.

Min Hamming distance = 2 (min. no. of 1's in the non zero code word)

Assume it is Linear Block code

x 000
Nonzero Code word { 001 }
 011 } minimum No. of 1's = 1
 111
 110 }
 }
 } minimum Hamming distance = 1

