CS & IT ENGINEERING

Theory of Computation

Finite Automata

Lecture No. 13

By- DEVA Sir

TOPICS TO BE COVERED

01 NFA $\Leftrightarrow$ DFA

Algorithms

02 i) NFA without ϵ moves $\Rightarrow$ DFA

03 ii) NFA with ϵ $\Rightarrow$ NFA without ϵ

04 iii) NFA with ϵ $\Rightarrow$ DFA

05

Algo: $\underbrace{\text{NFA without } \epsilon \text{ moves} \Longrightarrow \text{DFA}}_{\text{(Subset construction)}}$

$$\begin{array}{ccc} & \text{DFA} & \\ & 2^n \text{ states} & \\ \text{NFA} \xrightarrow{\text{Algo}} \text{DFA} & & \text{DFA} \xrightarrow{\text{Algo}} \text{Min DFA} \\ \text{NFA} & \Longrightarrow & \text{Min DFA} \end{array}$$

n states

How many states?

$\boxed{\text{Atmost } 2^n}$

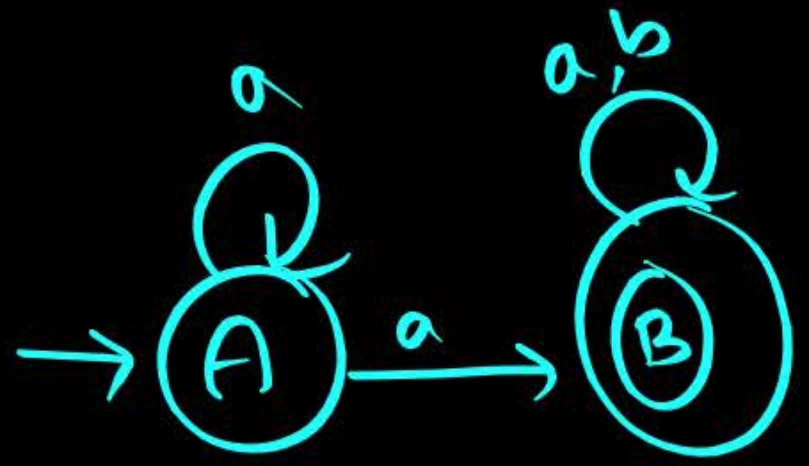
$\leq 2^n$

Given NFA $\Longrightarrow$ DFA

$$(Q, \Sigma, \delta_{NFA}, q_0, F) \xrightarrow{\text{subset construction}} (2^Q, \Sigma, \delta_{DFA}, \{q_0\}, F')$$

$\Downarrow$

Set of states of DFA = Set of all subsets of Q



NFA

2 states

$Q = \{A, B\}$

Initial state in DFA

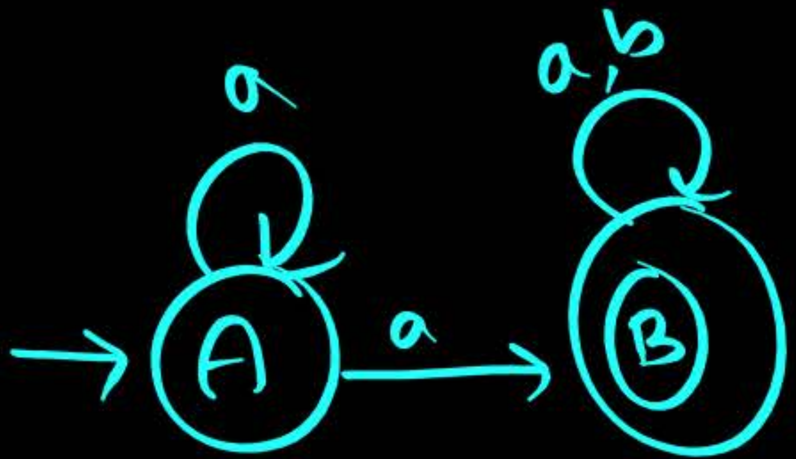
$$2^Q = \{\phi, \{A\}, \{B\}, \{A, B\}\}$$

Subset of Q

$\boxed{\text{State in DFA}}$

Given NFA $\Longrightarrow$ DFA

$$(Q, \Sigma, \delta_{NFA}, q_0, F) \xrightarrow{\text{subset construction}} (2^Q, \Sigma, \delta_{DFA}, \{q_0\}, F')$$



NFA

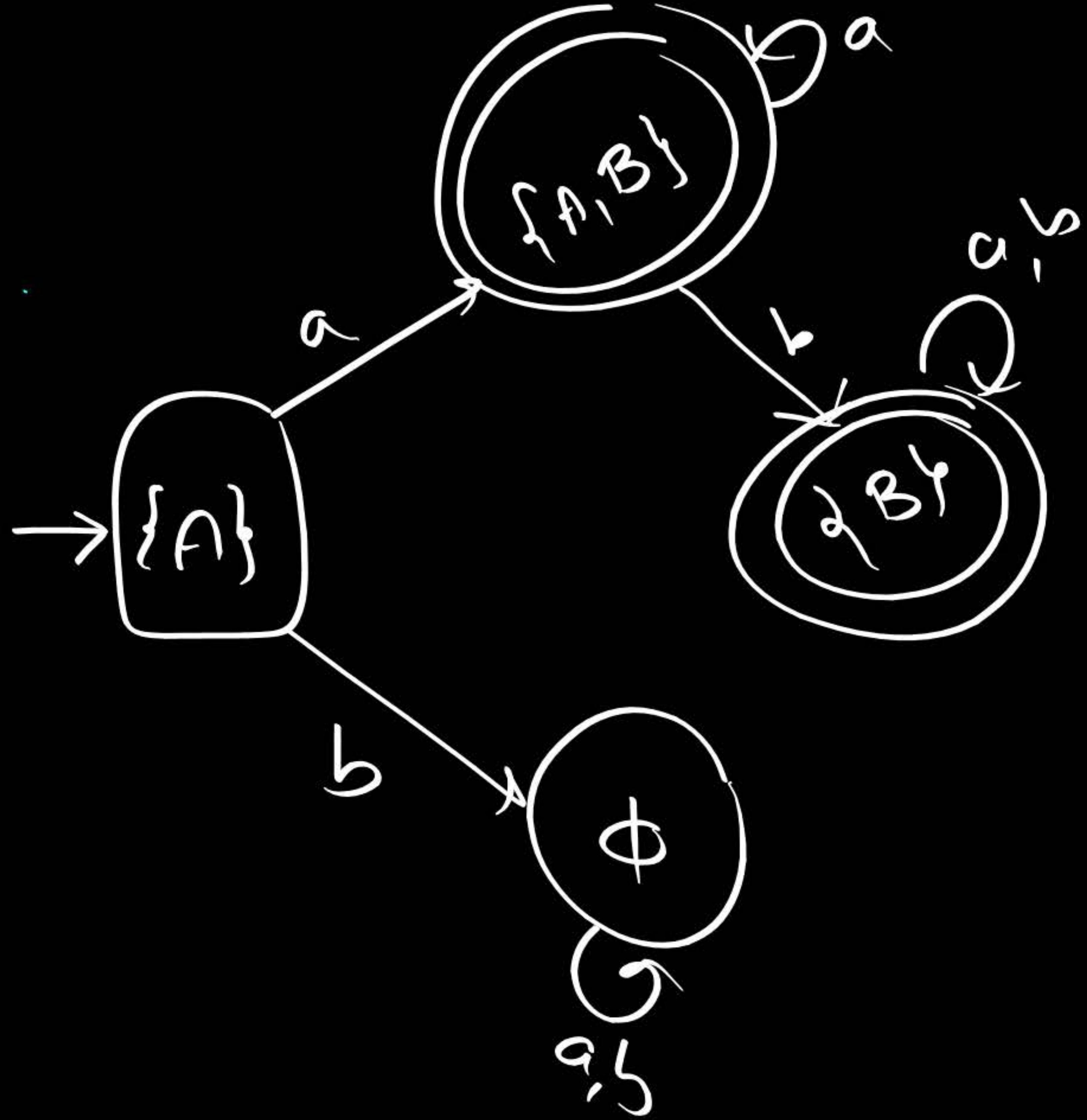
2 states

$Q = \{A, B\}$

NFA	a	b
$\rightarrow A$	$\{A, B\}$	ϕ
$*B$	$\{B\}$	$\{B\}$

$\Longrightarrow$

DFA	a	b
ϕ	ϕ	ϕ
$\rightarrow \{A\}$	$\{A, B\}$	ϕ
$*\{B\}$	$\{B\}$	$\{B\}$
$*\{A, B\}$	$\{A, B\}$	$\{B\}$



DFA

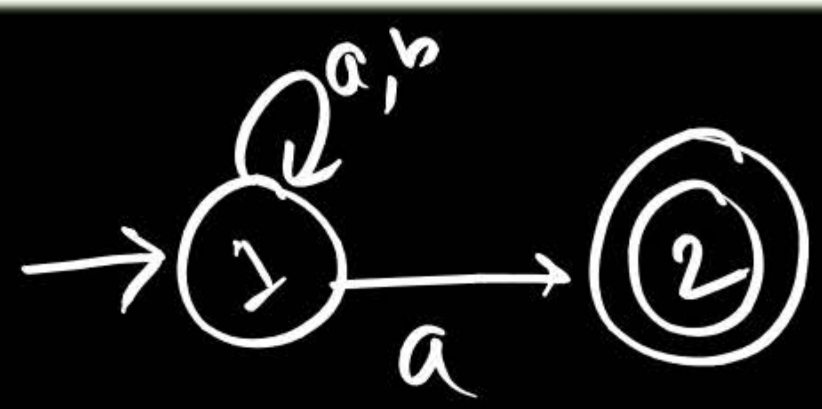
$$\left(2^{Q}, \Sigma, \delta_{DFA}, \{q_0\}, F'\right)$$

DFA	a	b
ϕ	ϕ	ϕ
$\rightarrow\{A\}$	$\{A,B\}$	ϕ
$*\{B\}$	$\{B\}$	$\{B\}$
$*\{A,B\}$	$\{A,B\}$	$\{B\}$

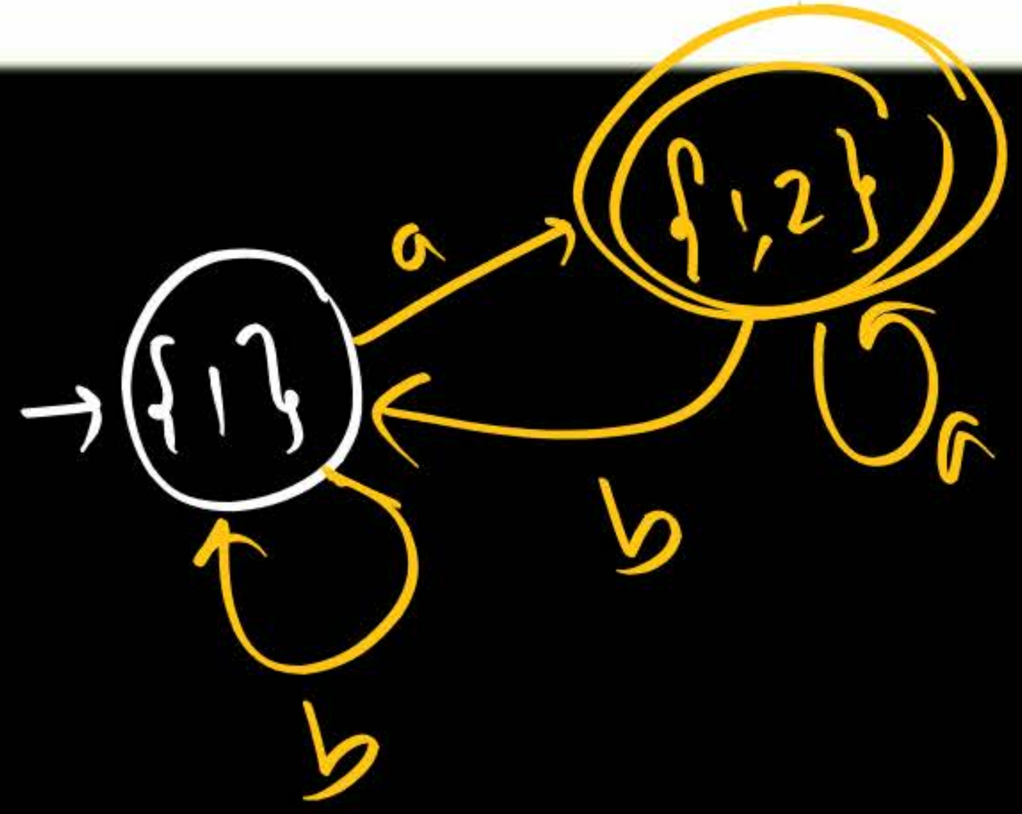
$$\delta_{DFA}(\underbrace{\{p,q\}}_{\text{state of DFA}}, i) = \delta_{NFA}(p,i) \cup \delta_{NF}(q,i)$$

$$= X \cup Y$$

NFA	i
p	X
q	Y



$\Longrightarrow$

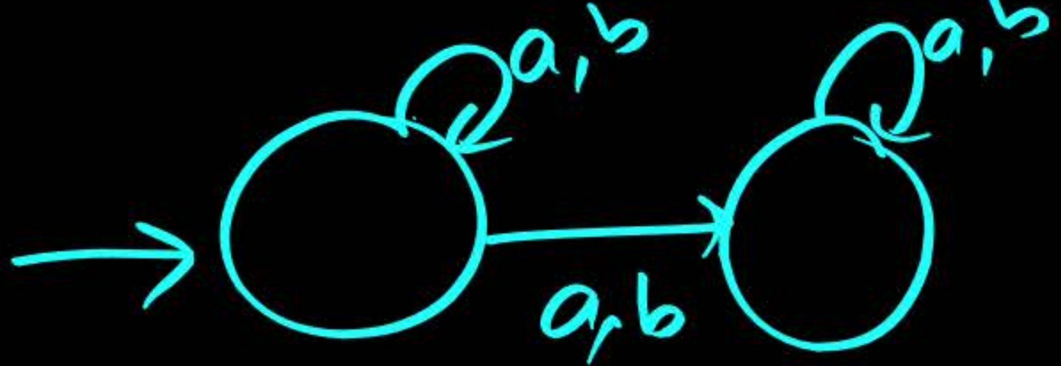


NFA

NFA	a	b
→ 1	{1,2}	{1}
* 2	ϕ	ϕ

	a	b
→ {1}	{1,2}	{1}
* {1,2}	{1,2}	{1}

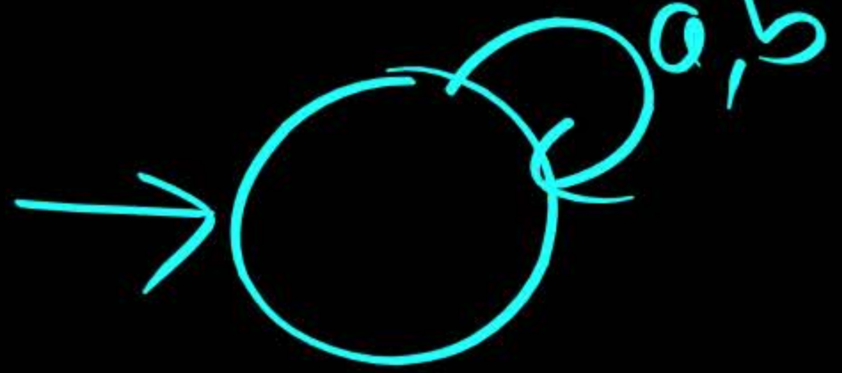
Q1)



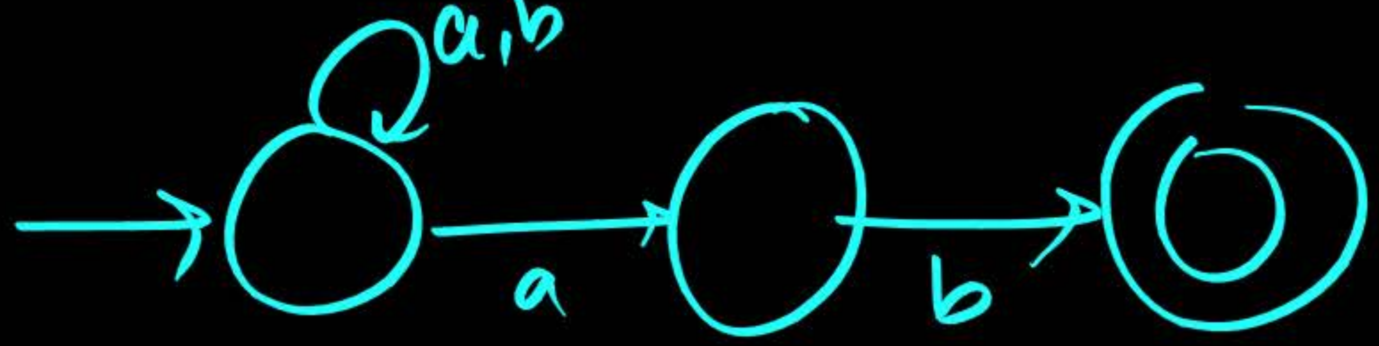
$\Rightarrow$ No. of states in equivalent Min DFA = 1

$\Downarrow$

$L = \phi \Longrightarrow$



Q2)



$L = (a+b)^* ab$

min = ab

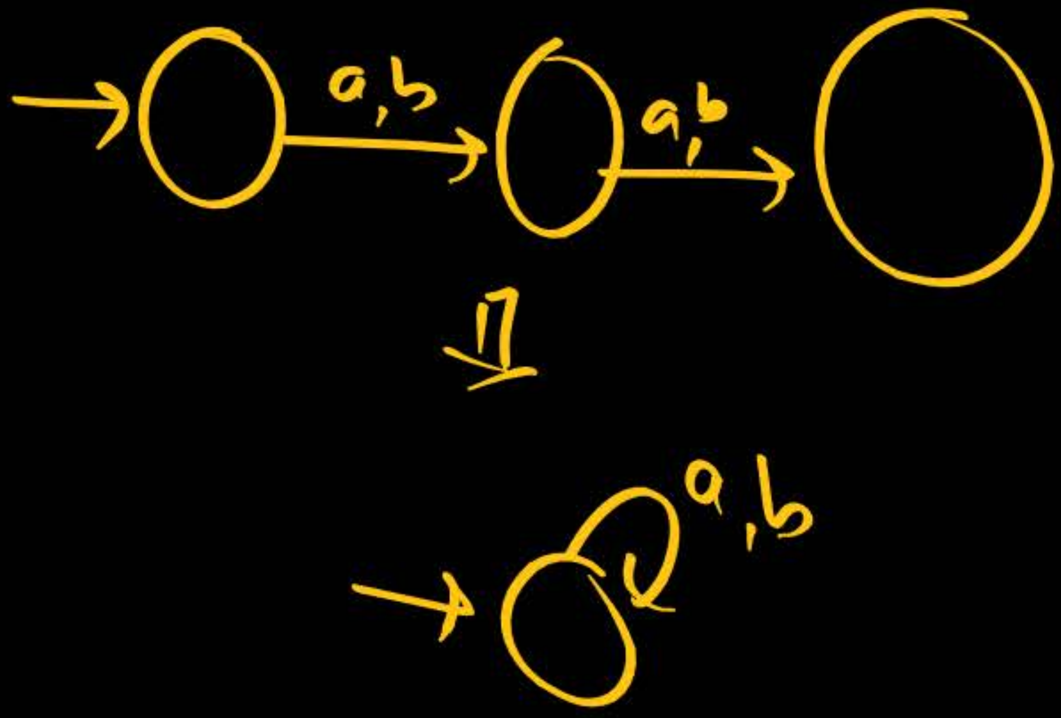
$|ab| + 1 = 2 + 1 = 3$ states

$n(\text{Min DFA}) = 3$

I) NFA $\Longrightarrow$ Min DFA

If n states present in NFA then

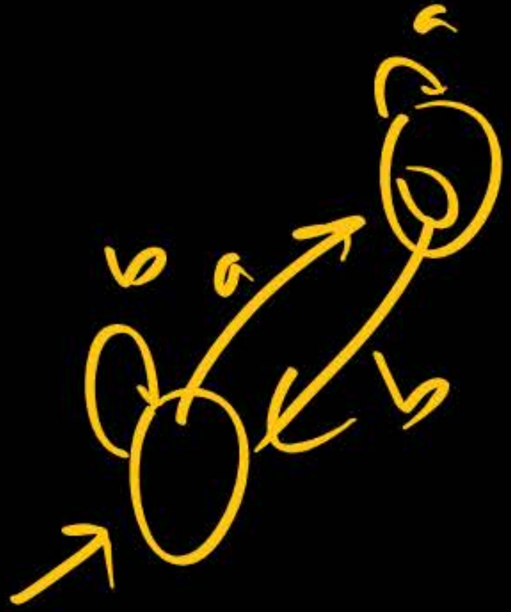
$$1 \le \boxed{\text{No. of states in Min DFA}} \le \underbrace{2^n}_{\text{always Important}}$$



II) Min NFA $\Longrightarrow$ Min DFA

If n states present in NFA then

$$n \le \boxed{\text{No. of states in Min DFA}} \le \underbrace{2^n}_{\text{always Important}}$$

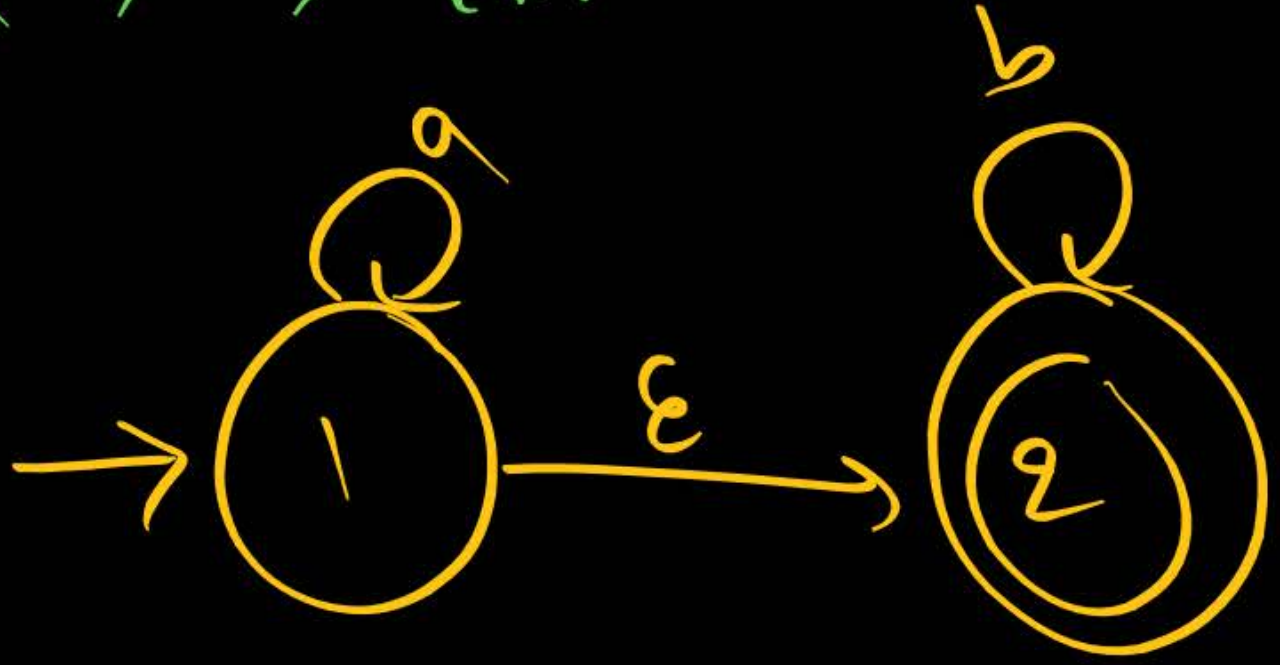


Algo:

NFA $\Longrightarrow$ NFA

with ε moves — without ε moves

$(Q, \Sigma, \delta_{\varepsilon\text{-NFA}}, q_0, F)$ — $(Q, \Sigma, \delta_{NFA}, q_0, F')$



$\varepsilon\text{-clo}(1) = \{1, 2\}$

$\varepsilon\text{-clo}(2) = \{2\}$

without ε moves

① $\delta_{NFA}(1,a) = ?$

② $\delta_{NFA}(1,b) = ?$

③ $\delta_{NFA}(2,a) = ?$

④ $\delta_{NFA}(2,b) = ?$

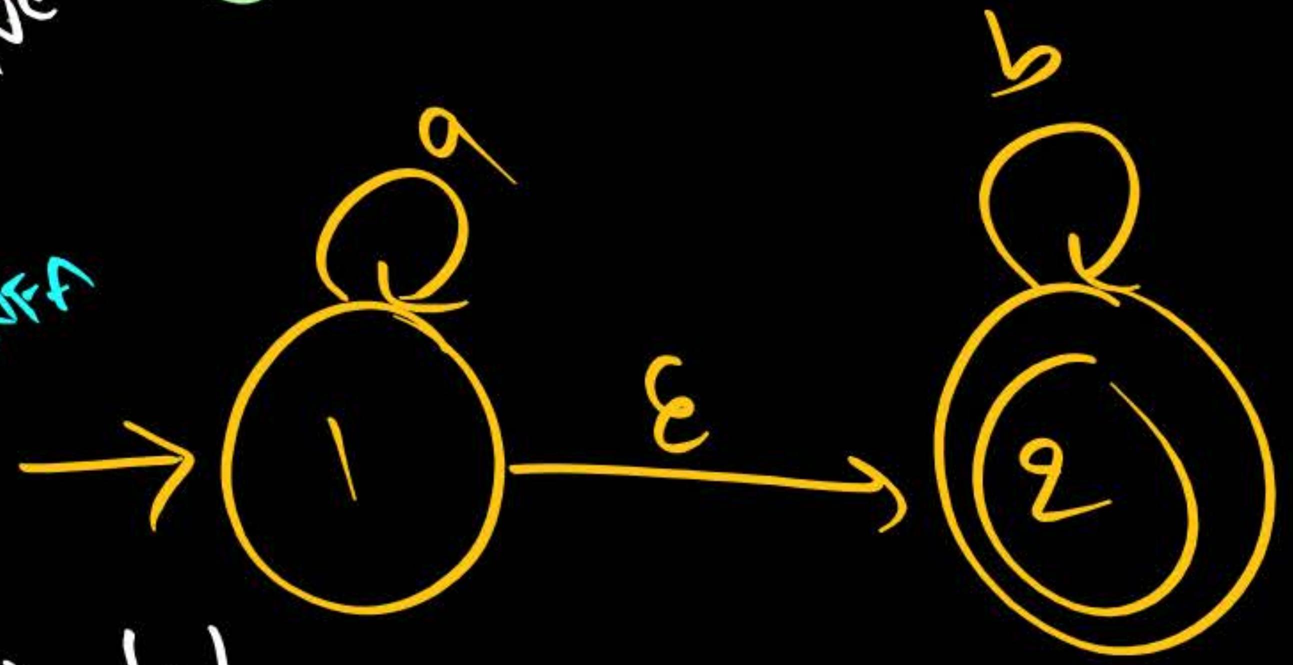
$$\delta_{NFA}(1,a) = \varepsilon\text{-clo}\Big(\underbrace{\delta\big(\underbrace{\varepsilon\text{-clo}(1)}_{1st}, a\big)}_{2nd}\Big)$$

$$= \varepsilon\text{-clo}\left[\delta(\{1,2\}, a)\right]$$

$$= \varepsilon\text{-clo}\left[\underbrace{\delta(1,a)}_{\{1\}} \cup \underbrace{\delta(2,a)}_{\phi}\right] = \varepsilon\text{-clo}(1) = \{1,2\}$$

Given

$\delta_{\varepsilon\text{-NFA}}$



$\delta(1,a) = \{1\}$

$\delta(1,b) = \phi$

$\delta(2,a) = \phi$

$\delta(2,b) = \{2\}$

$\varepsilon\text{-clo}(1) = \{1, 2\}$

$\varepsilon\text{-clo}(2) = \{2\}$

NFA without ε-moves

① $\delta_{NFA}(1,a) = ?$

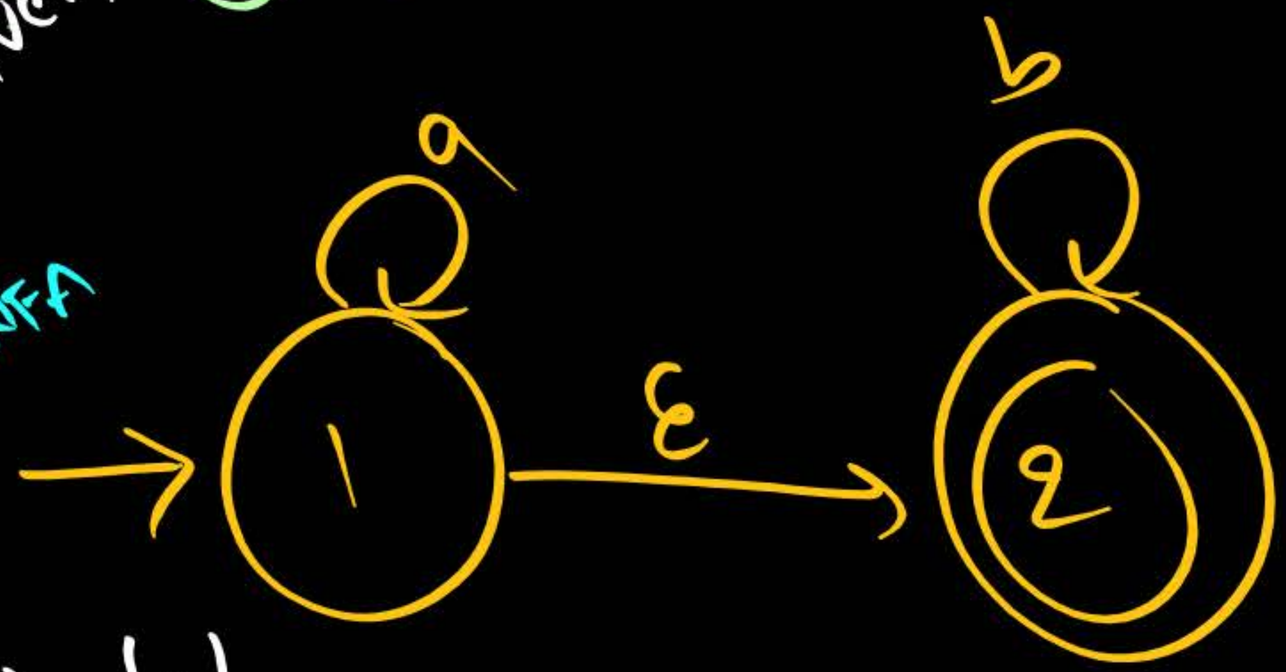
② $\delta_{NFA}(1,b) = ?$

③ $\delta_{NFA}(2,a) = ? = \phi$

④ $\delta_{NFA}(2,b) = ? = \{2\}$

Given

$\delta_{\varepsilon\text{-}NFA}$



$\delta(1,a) = \{1\}$

$\delta(1,b) = \phi$

$\delta(2,a) = \phi$

$\delta(2,b) = \{2\}$

$\varepsilon\text{-clo}(1) = \{1, 2\}$

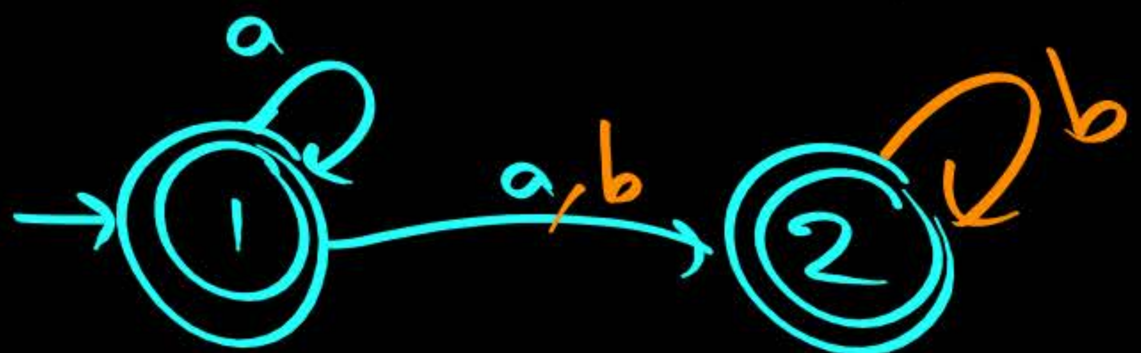
$\varepsilon\text{-clo}(2) = \{2\}$

$$\delta_{NFA}(1,b) = \varepsilon\text{-clo}\Big(\underbrace{\delta\big(\underbrace{\varepsilon\text{-clo}(1)}_{1st}, b\big)}_{2nd}\Big)$$

$$= \varepsilon\text{-clo}\left[\delta(\{1,2\}, b)\right]$$

$$= \varepsilon\text{-clo}\left[\underbrace{\delta(1,b)}_{\phi} \cup \underbrace{\delta(2,b)}_{\{2\}}\right] = \varepsilon\text{-clo}(2)$$

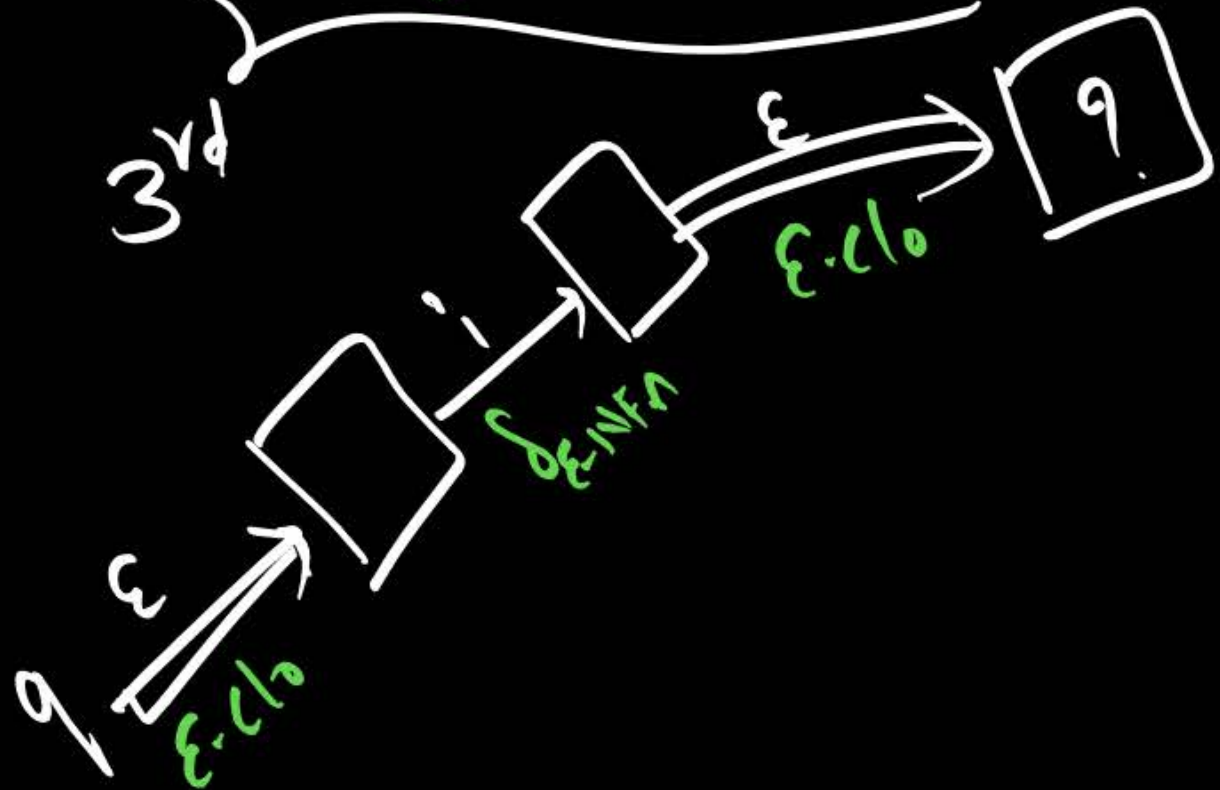
$$= \{2\}$$



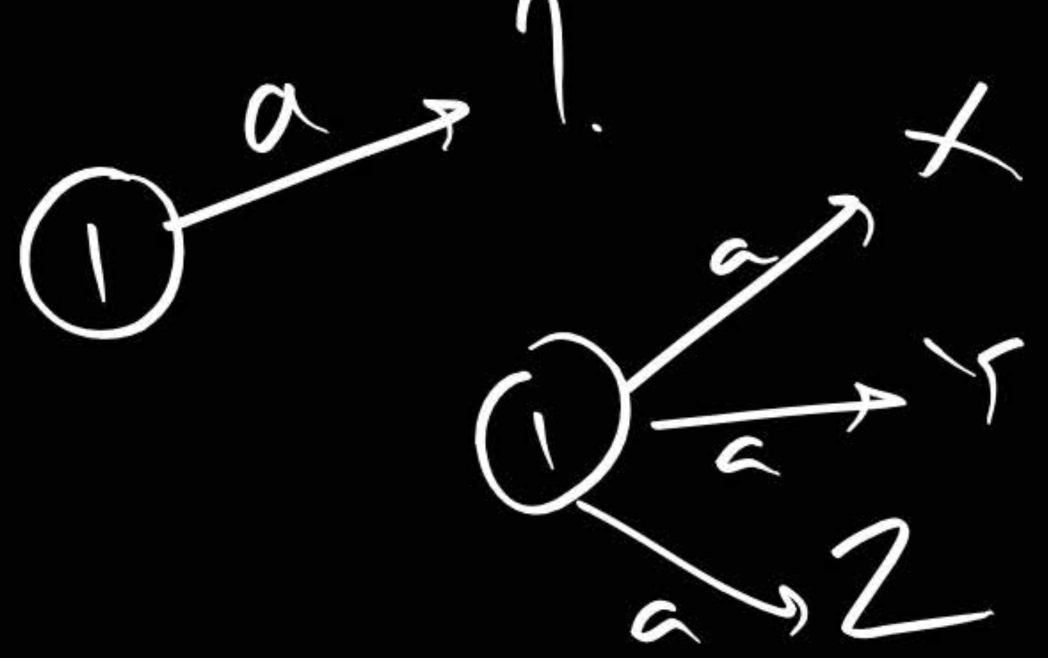
NFA without ε-moves

$$\delta_{NFA}(q, i) = \underbrace{\varepsilon\text{-closure}\Big(\underbrace{\delta_{\varepsilon\text{-NFA}}\big(\underbrace{\varepsilon\text{-closure}(q)}_{1^{st}}, i\big)}_{2^{nd}}\Big)}_{3^{rd}}$$

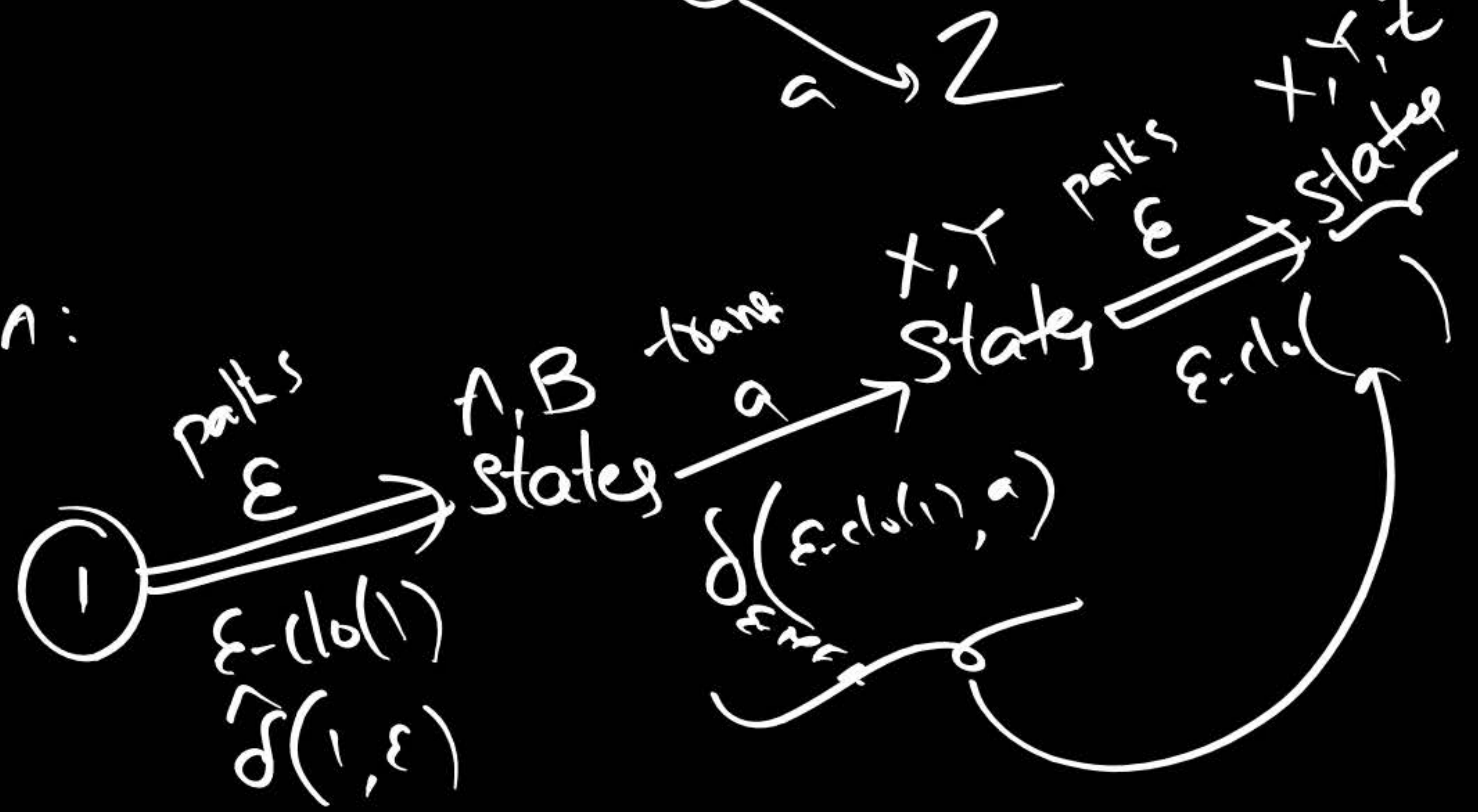
$$q \xrightarrow{i}$$



NFA:



Given ε-NFA:



I) Initial of NFA is same as initial of ε-NFA

II) If ε-closure(q) contains final of ε-NFA then make q as final in NFA

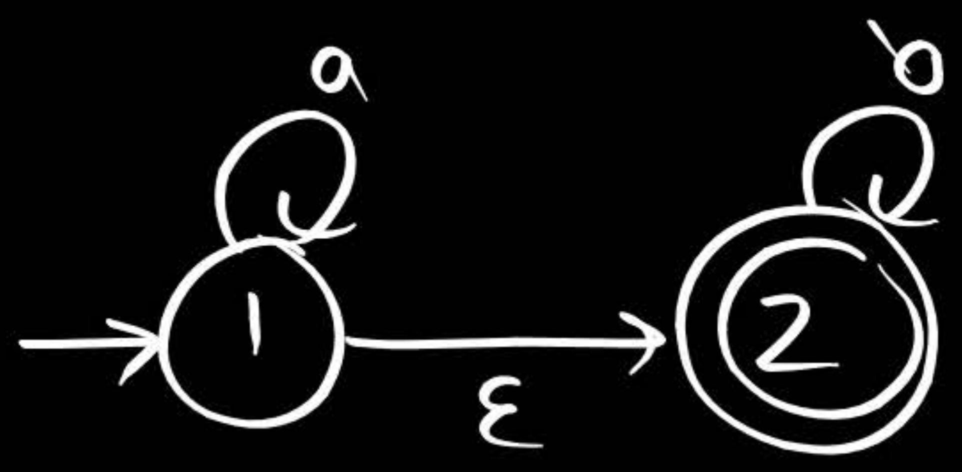
III) $\delta_{NFA}(q, i) = \underbrace{\varepsilon\text{-clo}\left(\underbrace{\delta_{\varepsilon\text{-NFA}}\left[\underbrace{\varepsilon\text{-clo}(q)}_{1st}, i\right]}_{2nd}\right)}_{3rd}$

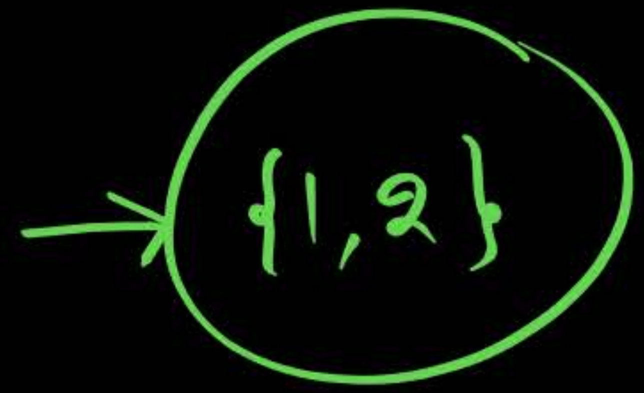
$\forall q \in Q$

$\forall i \in \Sigma$

Algo: NFA with ε moves $\xRightarrow{\text{Subset const.}}$ DFA

$$(Q, \Sigma, \delta_{\varepsilon\text{-NFA}}, q_0, F) \Longrightarrow (2^Q, \Sigma, \delta_{DFA}, \varepsilon\text{-clo}(q_0), F')$$





$\delta(1, a) = \{1\}$

$\delta(1, b) = \phi$

$\delta(2, a) = \phi$

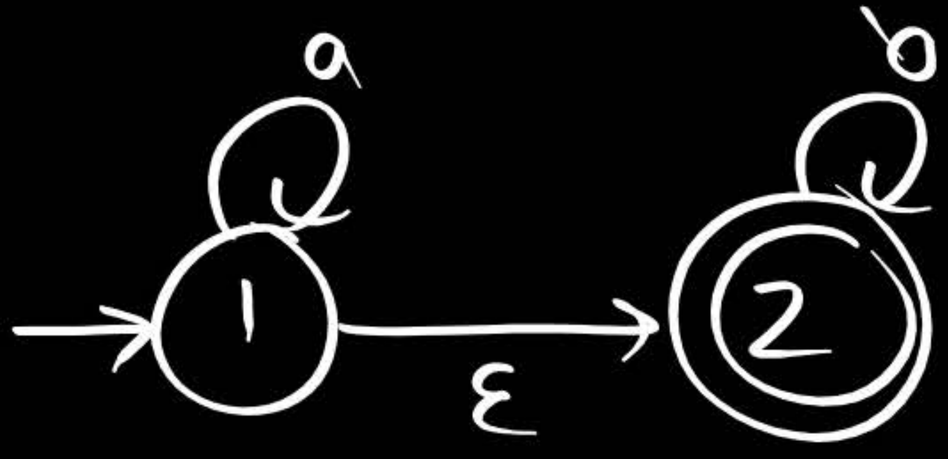
$\delta(2, b) = \{2\}$

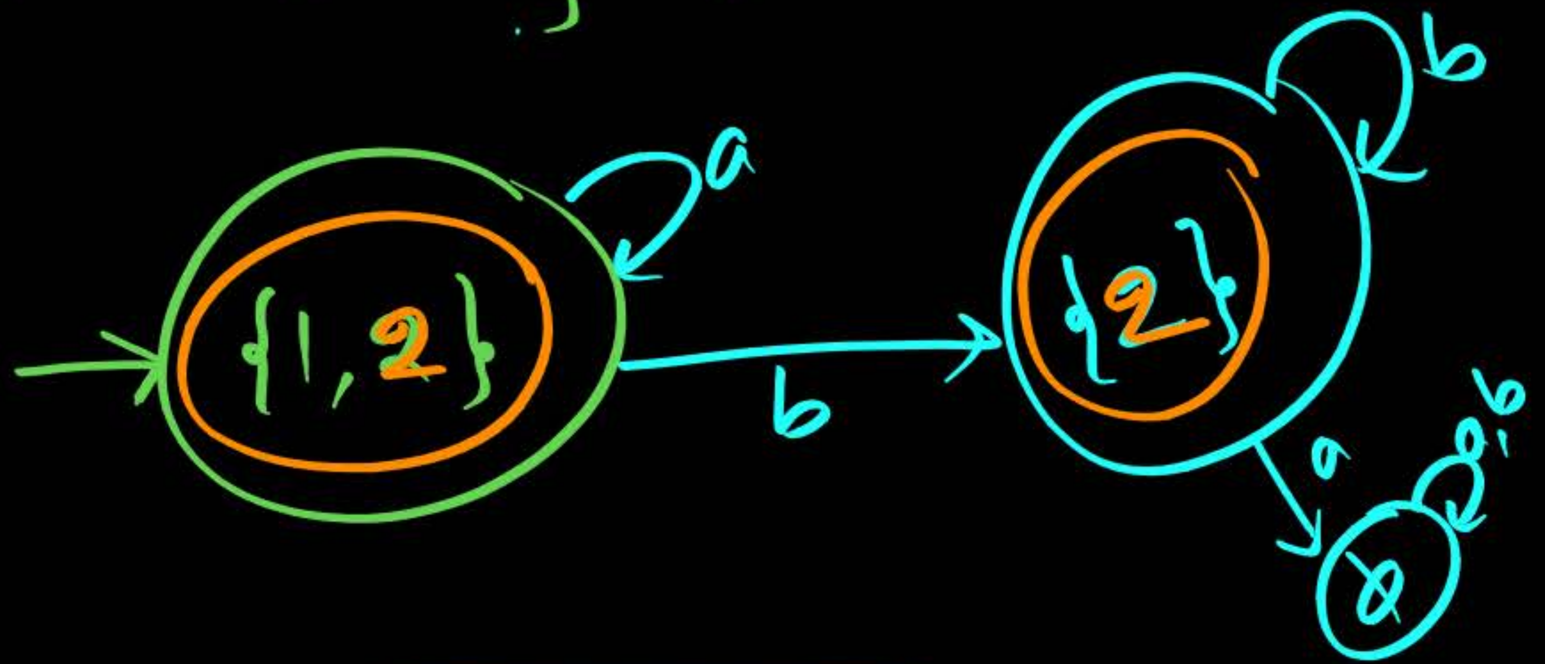
$\varepsilon\text{-clo}(1) = \{1, 2\}$

$\varepsilon\text{-clo}(2) = \{2\}$

Algo : NFA with ε moves $\Longrightarrow$ DFA

$$\delta_{DFA}(\{1,2\},a) = \varepsilon\text{-clo}\left[\delta_{\varepsilon\text{-NFA}}(\{1,2\},a)\right]$$
$$= \varepsilon\text{-clo}\left[\{1\} \cup \{\}\right] = \varepsilon\text{-clo}(1) = \{1,2\}$$





$\delta(1,a) = \{1\}$

$\delta(1,b) = \phi$

$\delta(2,a) = \phi$

$\delta(2,b) = \{2\}$

$\varepsilon\text{-clo}(1) = \{1,2\}$

$\varepsilon\text{-clo}(2) = \{2\}$

$$\delta_{DFA}(\{1,2\},b) = \varepsilon\text{-clo}\left[\delta_{\varepsilon\text{-NFA}}(\{1,2\},b)\right]$$
$$= \varepsilon\text{-clo}(2) = \{2\}$$

I) Initial state of DFA = ε-clo(Initial of ε-NFA)

II) $$\forall i \in \Sigma \quad \delta_{DFA}(\underbrace{\{p,q\}}_{\substack{\text{I} \\ \text{subset} \\ \text{of } Q}}, i) = \underbrace{\varepsilon\text{-clo}\left[\underbrace{\delta_{\varepsilon\text{-NFA}}(\{p,q\}, i)}_{1^{st}}\right]}_{2^{nd}}$$

Every new subset

III) Final of DFA:

If any subset of Q contain final of ε-NFA

Note:

NFA
$\boxed{\text{NFA with } \varepsilon\text{-moves or NFA without } \varepsilon\text{-moves}}$ $\Rightarrow$ No. of states in min DFA is atmost 2^n

$\Downarrow$

n States

$\Rightarrow$ Max no. of states in min DFA is 2^n

RegExp $\cong$ FA:

I) FA $\Longrightarrow$ RegExp [Using State Elimination]

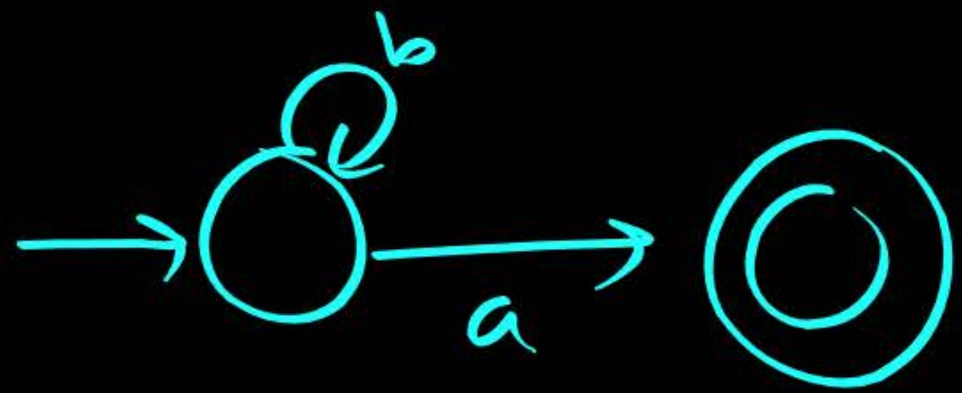
II) RegExp $\Longrightarrow$ FA [Induction Method]

I) FA ⇒ RegExp

1.

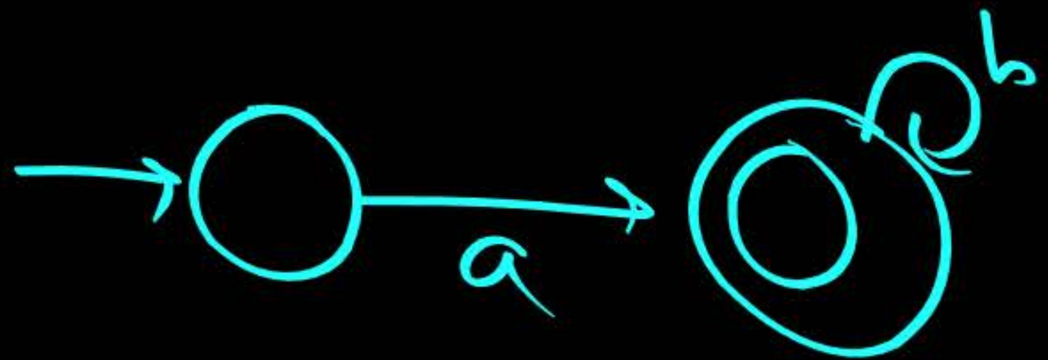
$L_1 = a$

2.



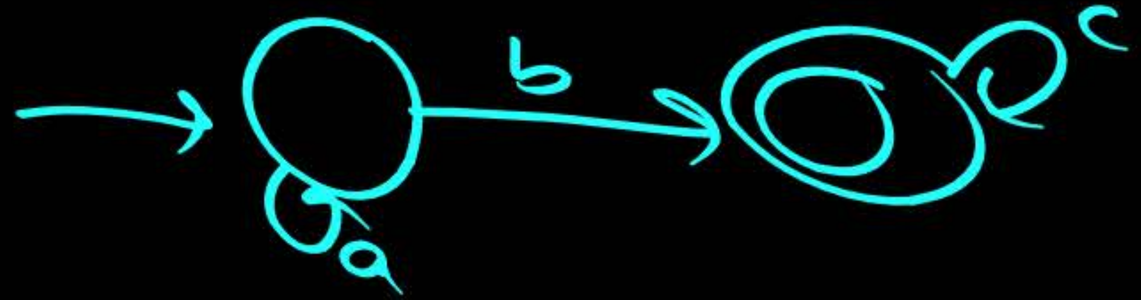
$L_2 = b^* a$

3.

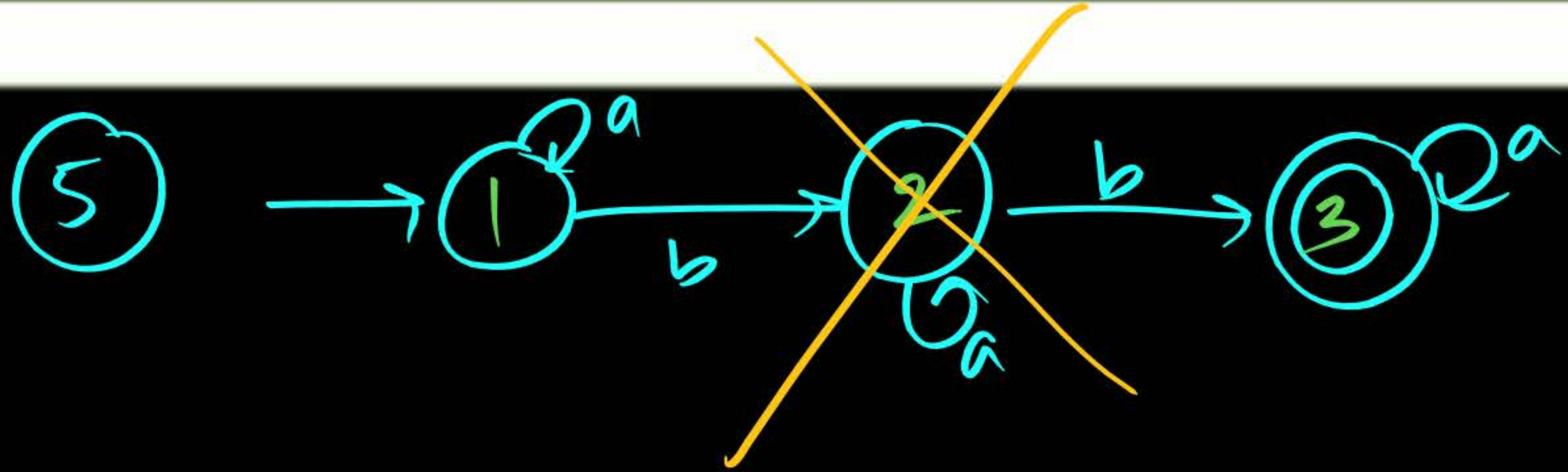


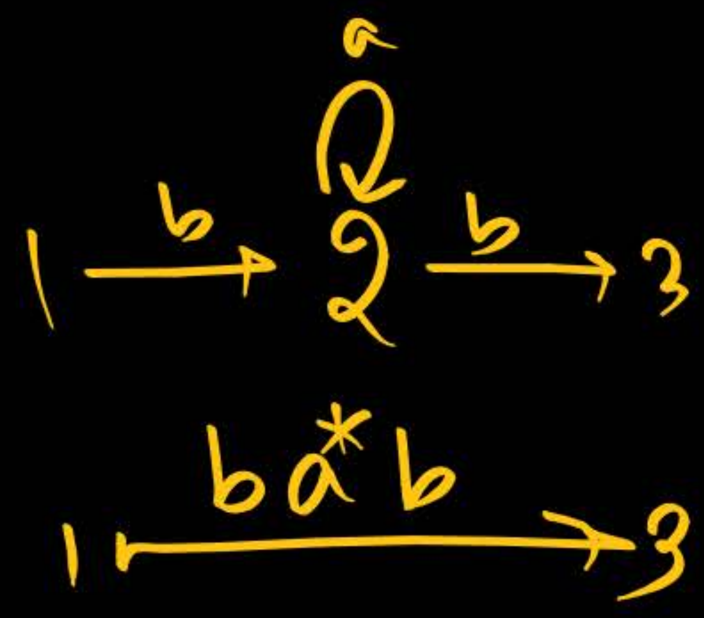
$L_3 = a b^*$

4.

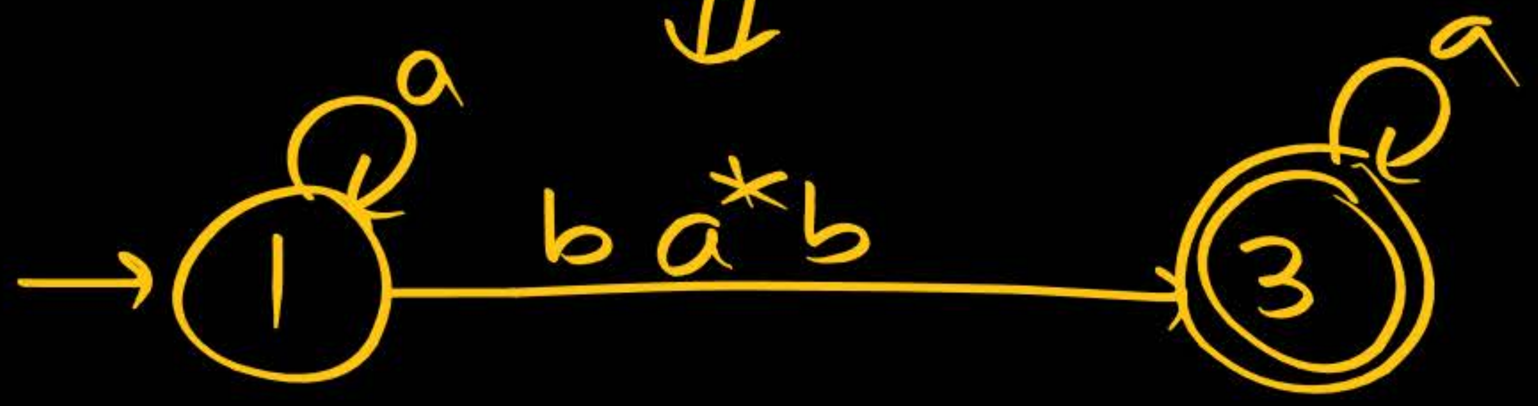


$L_4 = a^* b c^*$



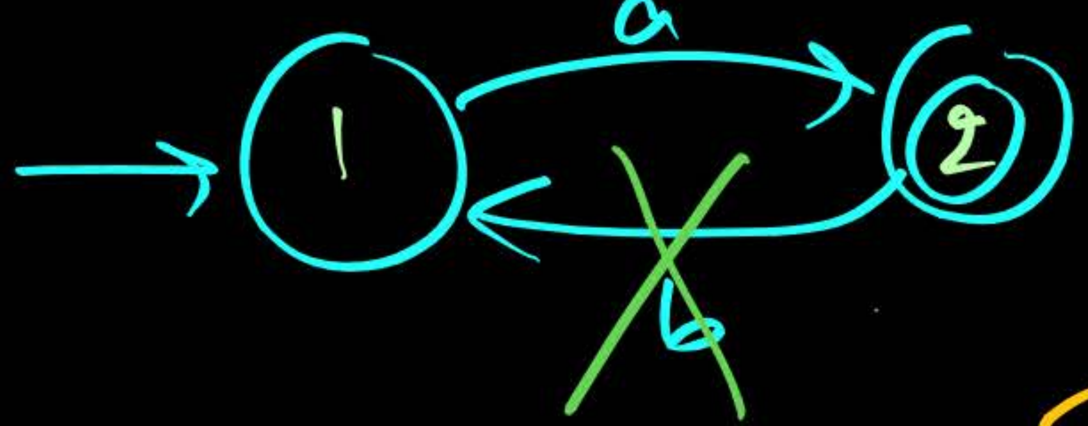


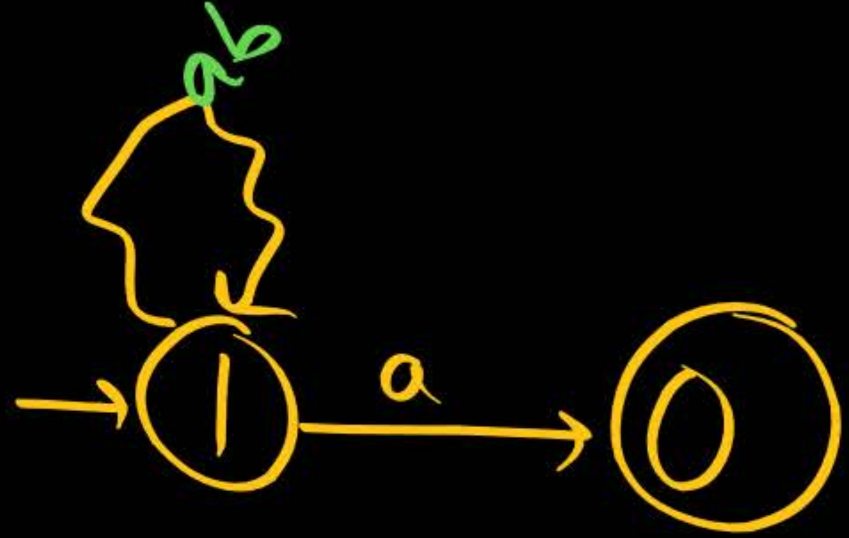
⇓



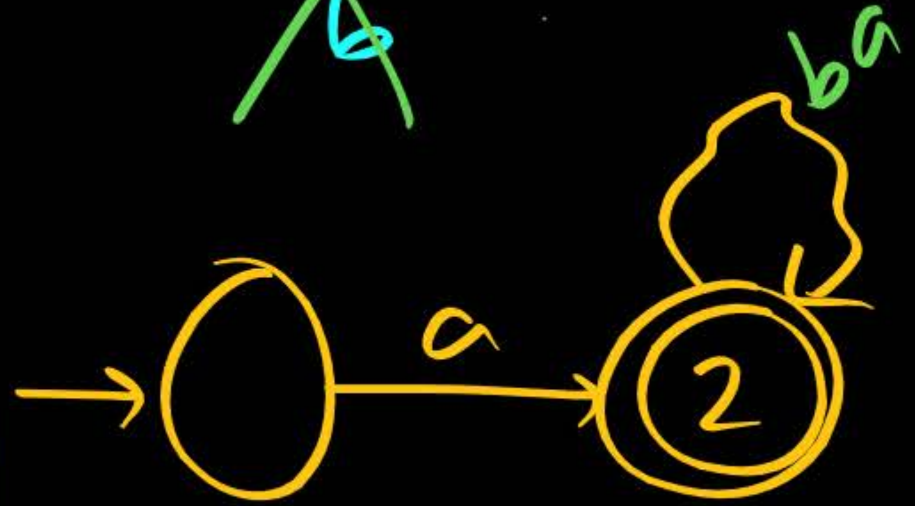
$$L = a^* b a^* b a^*$$

6

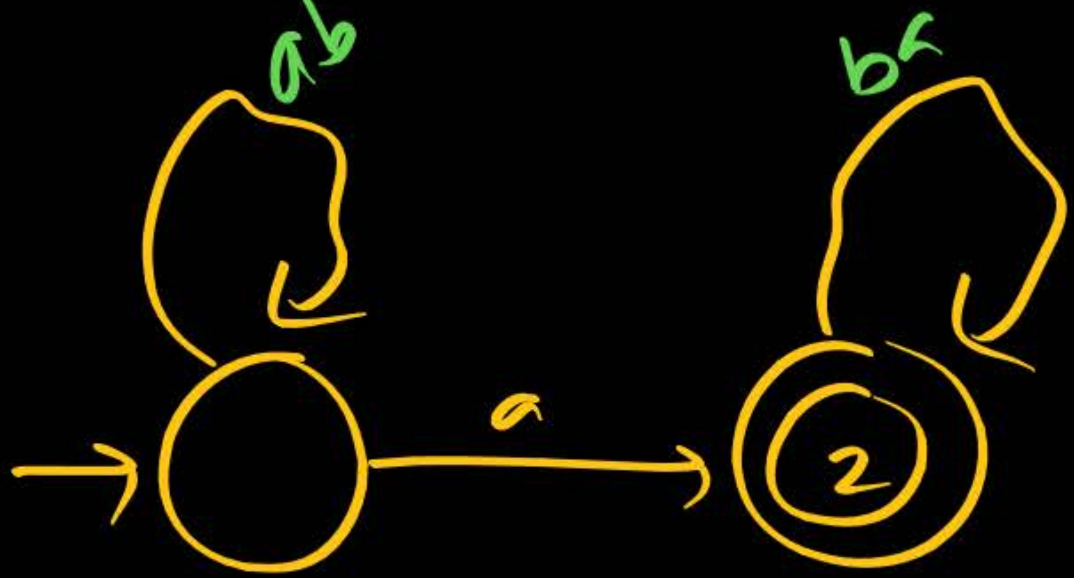




$= (ab)^* a$

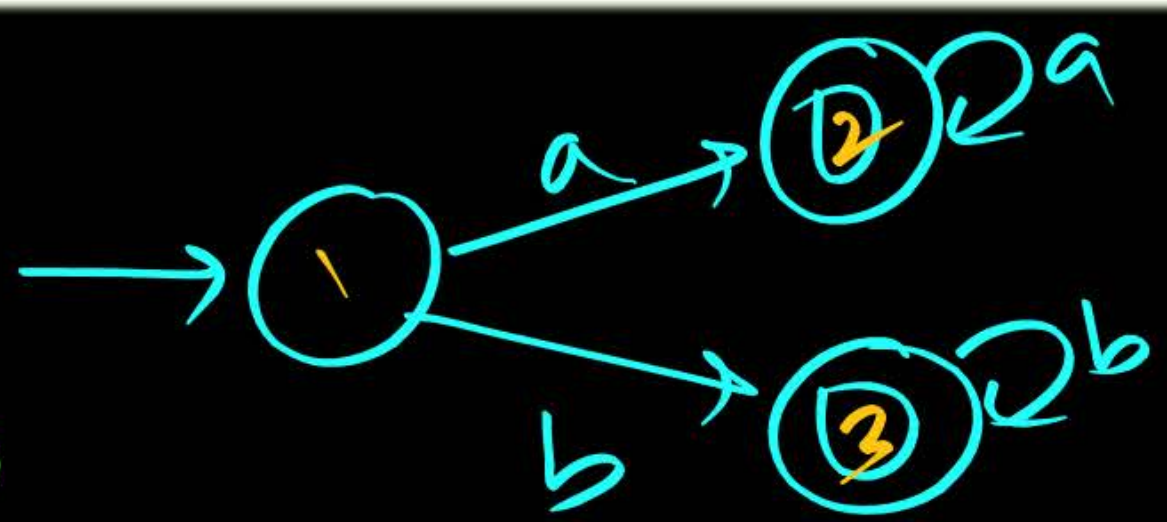


$= a(ba)^*$



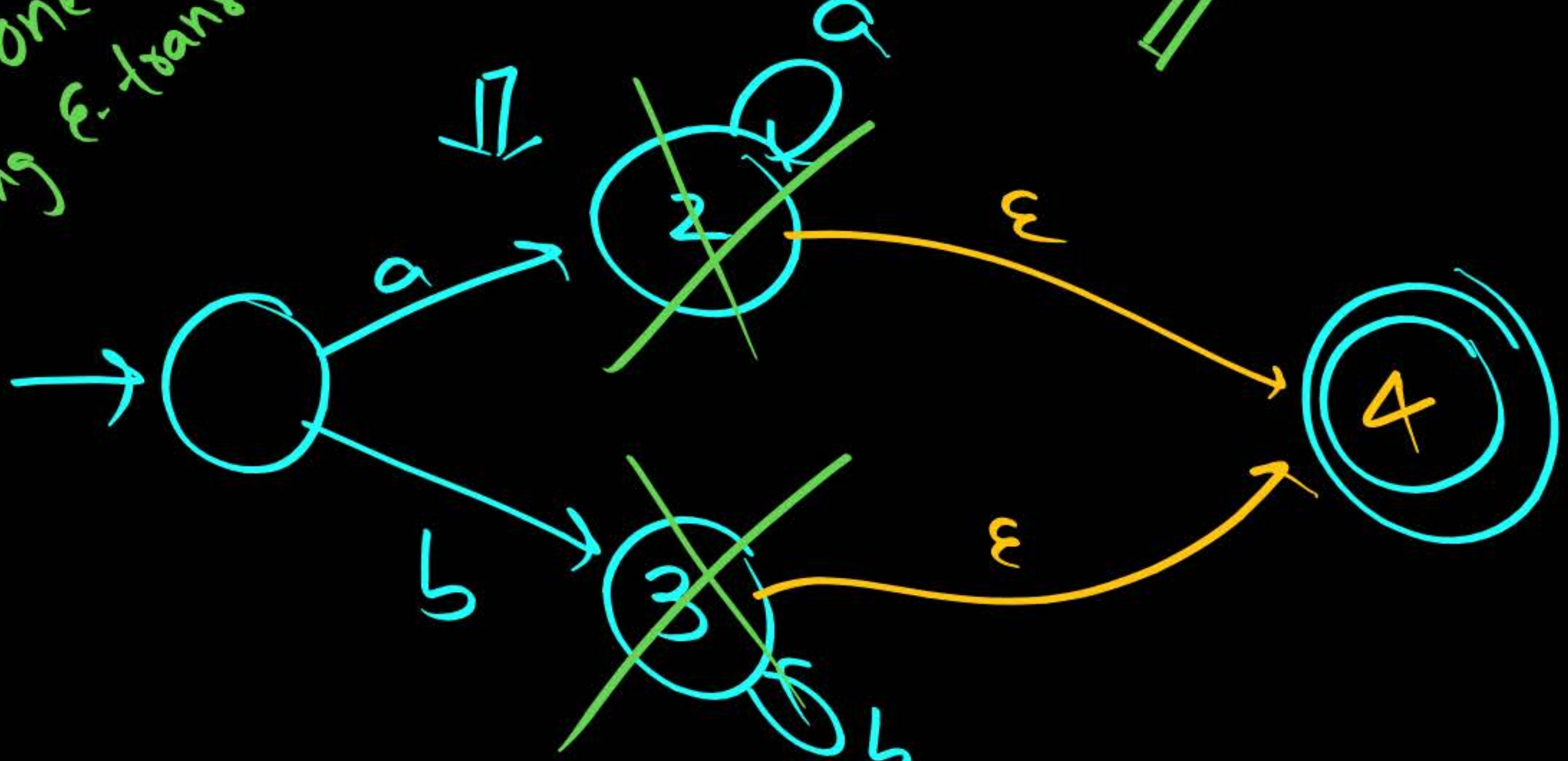
$= (ab)^* a (ba)^*$

7

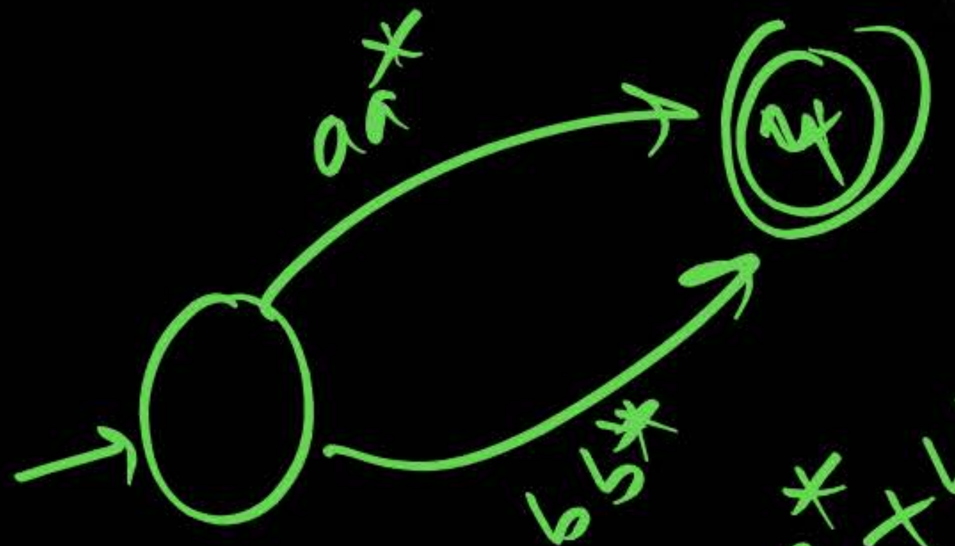


If more finals
⇓
make one final
using ε-transitions

⇓

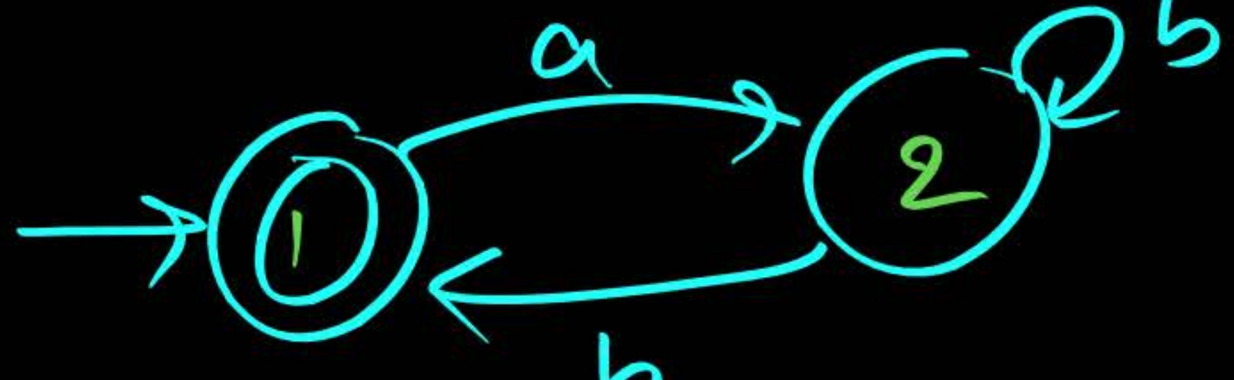


⇓

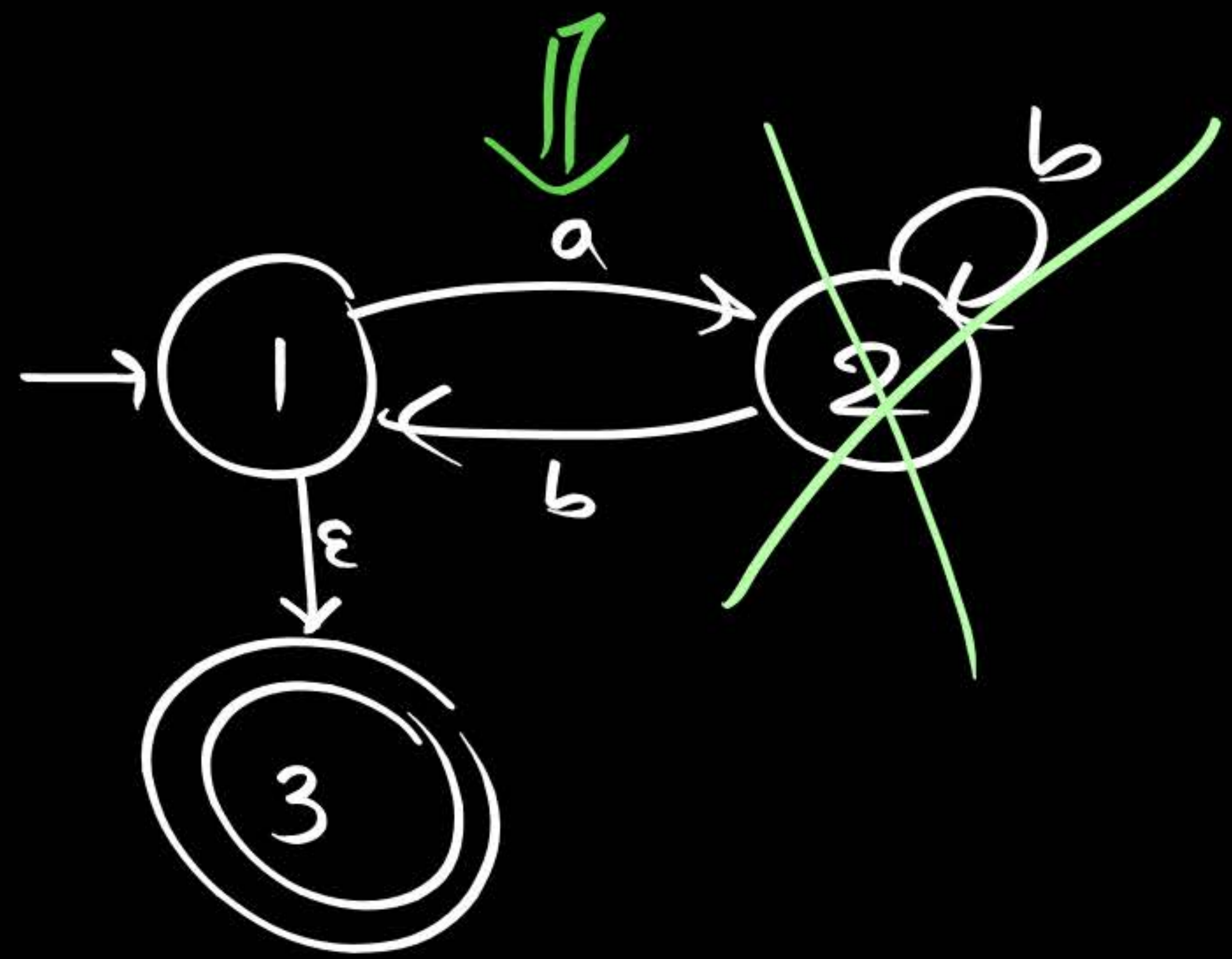


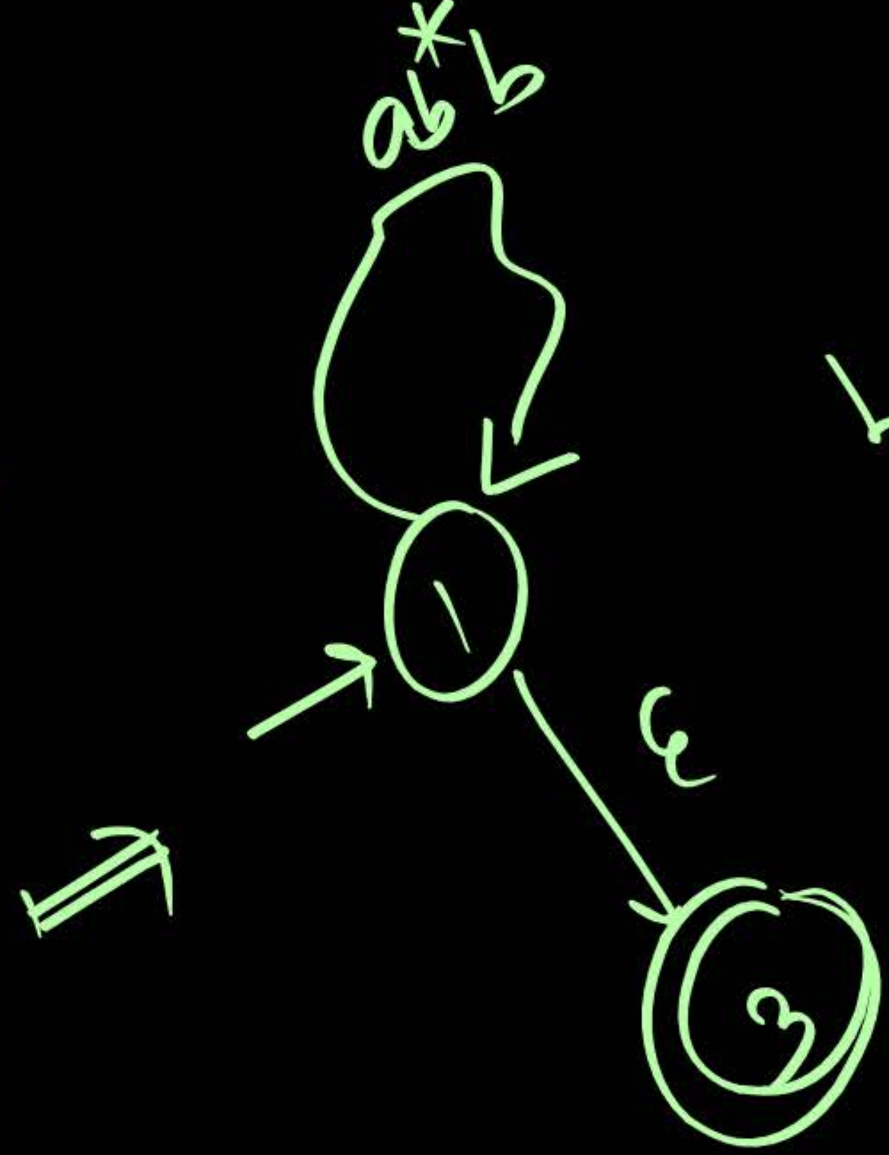
$L = aa^* + bb^*$
$= a^+ + b^+$

8



Separate initial & Final

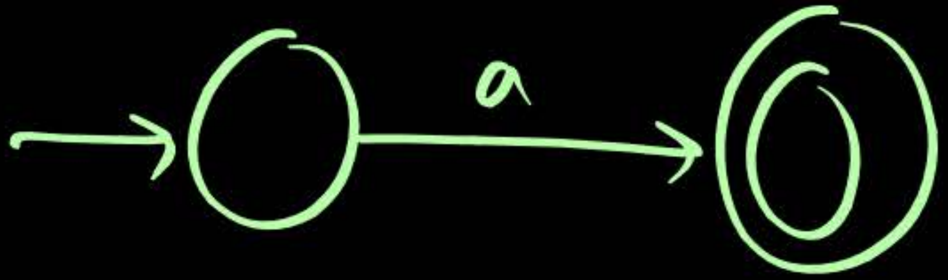




$L = (ab^*b)^*$

II) RegExp $\Longrightarrow$ FA

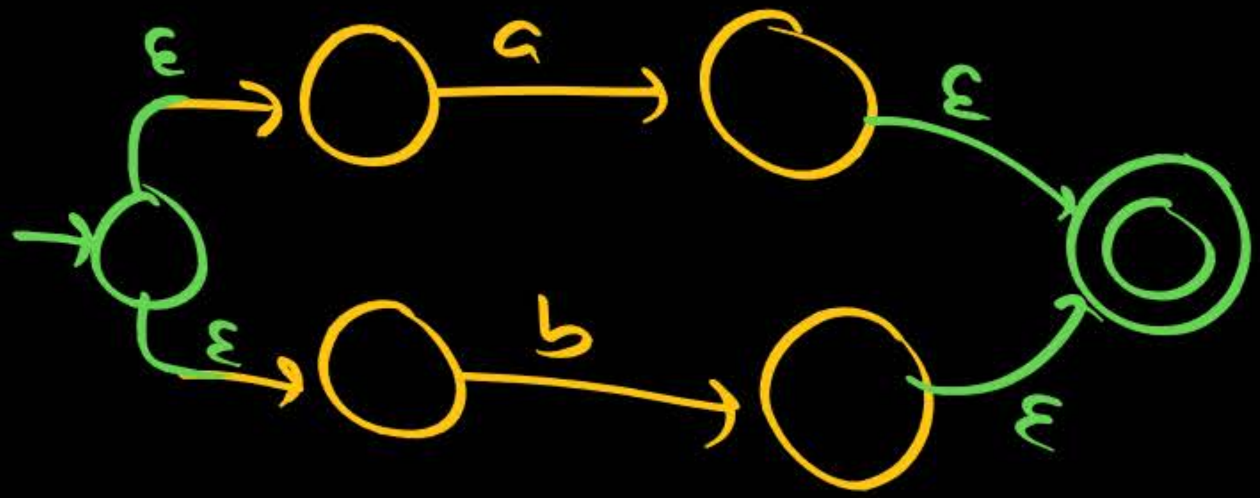
1) $R = a$



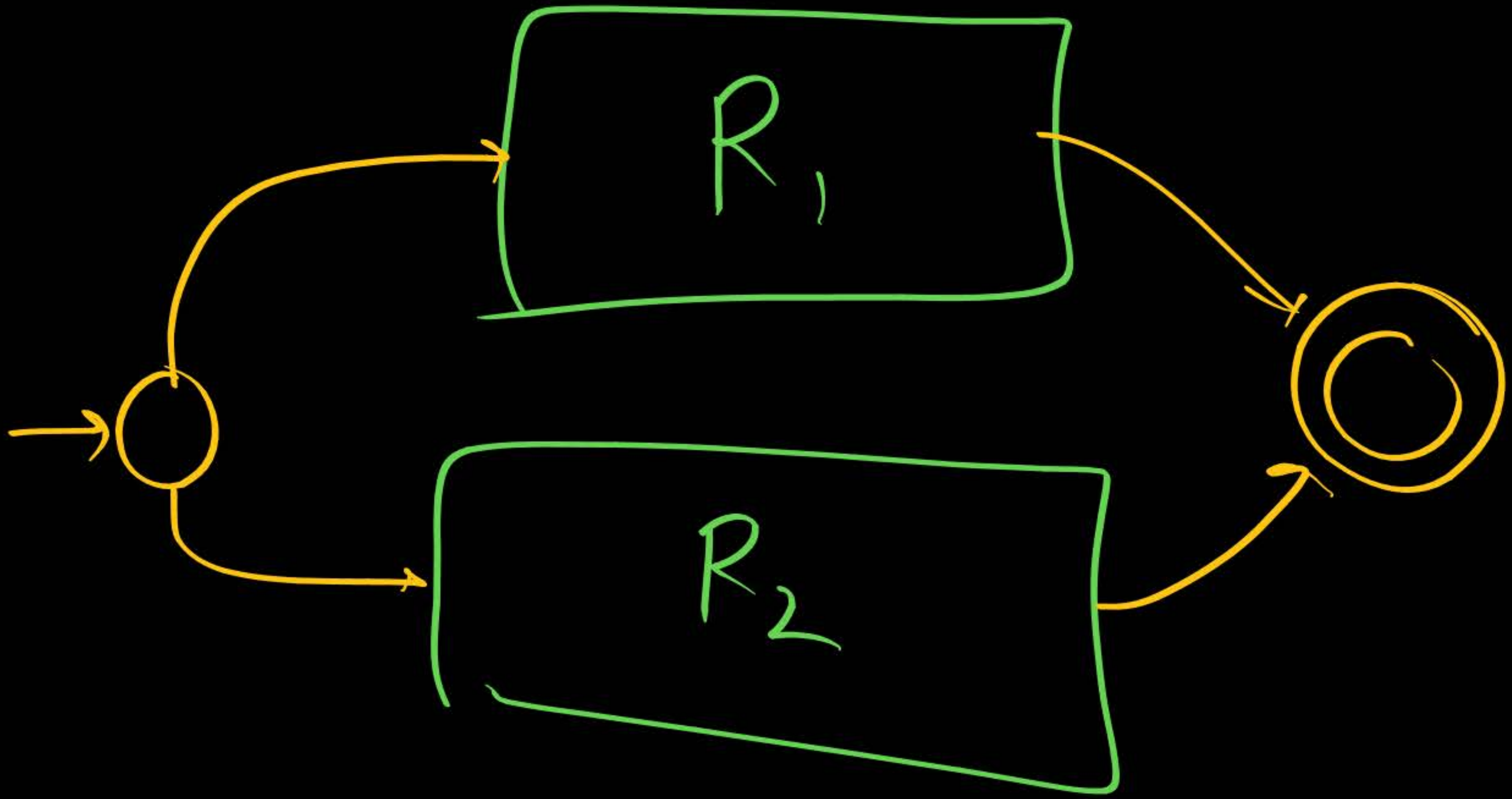
2) $R = b$

3) $R = a + b$

$= R_1 + R_2$

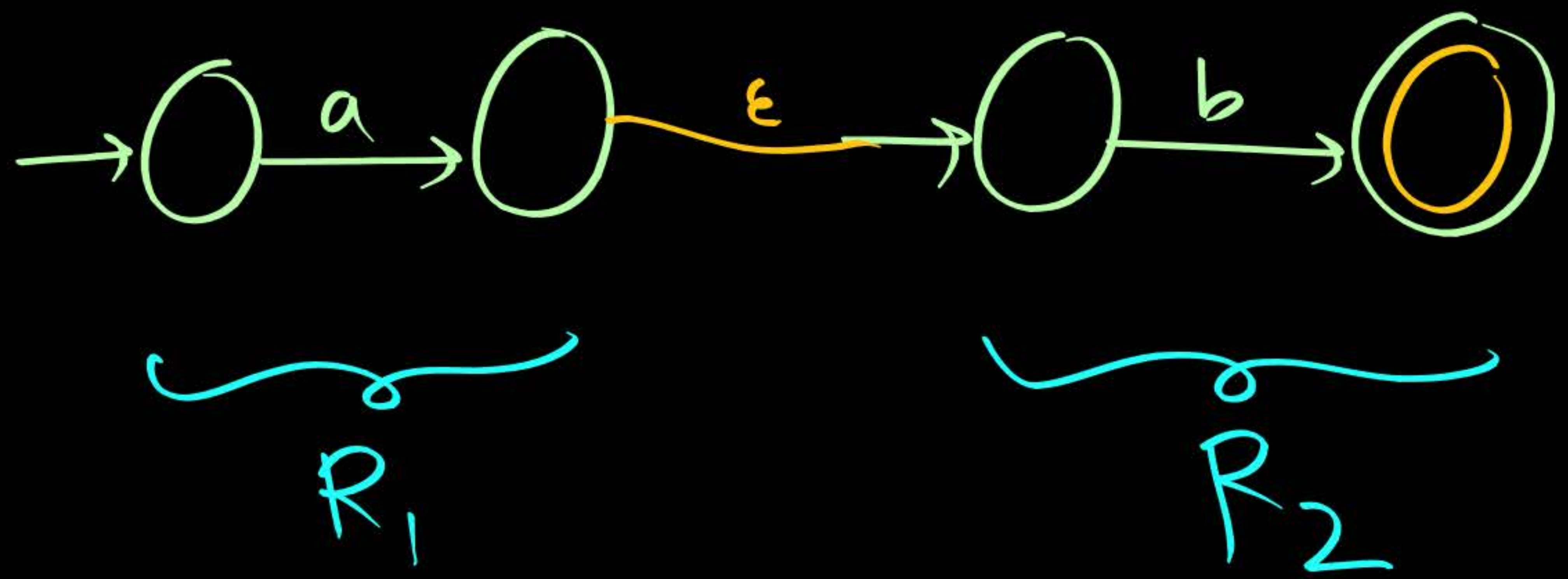


$R_1 + R_2$

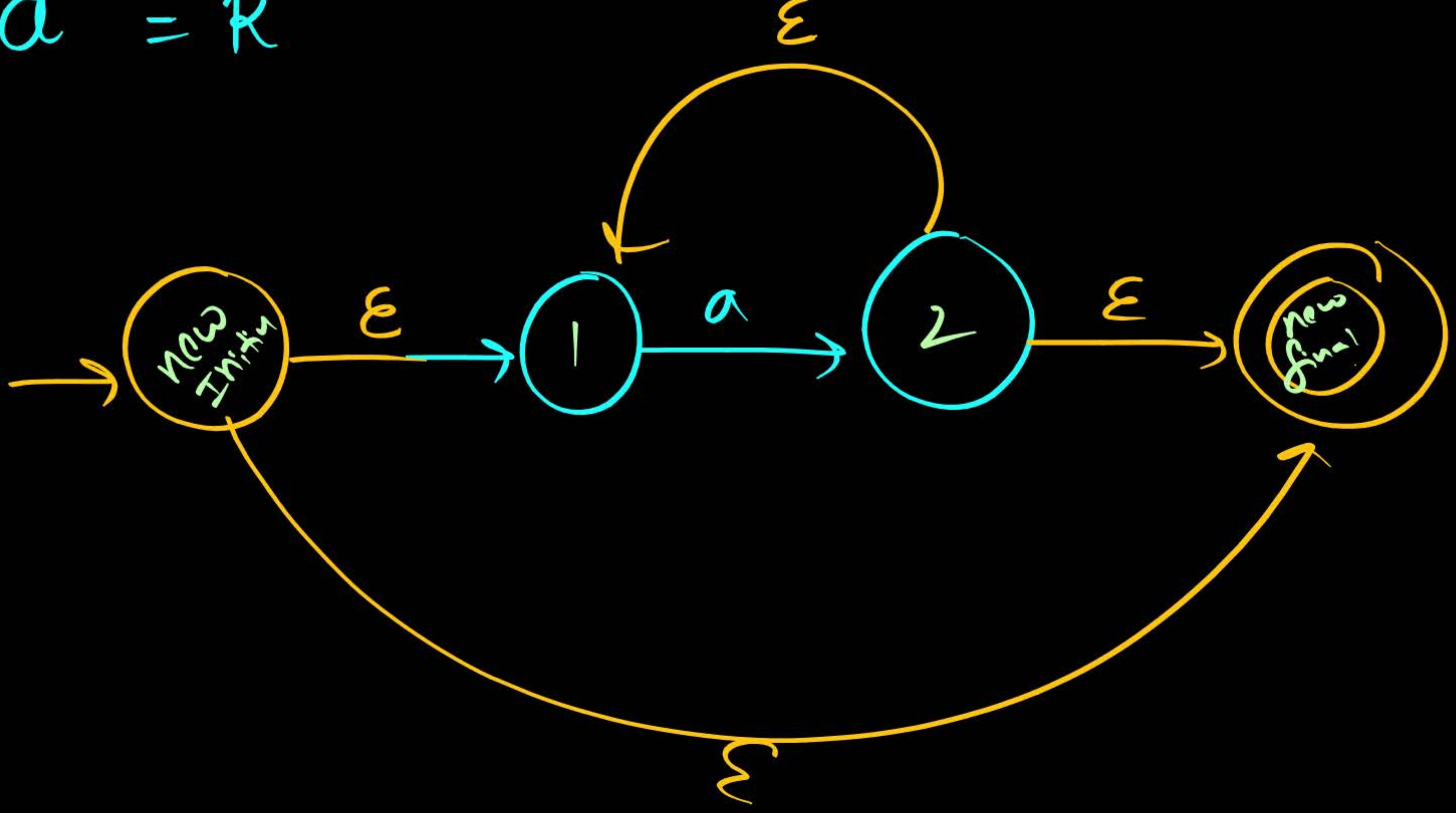


4) $R = a \cdot b$

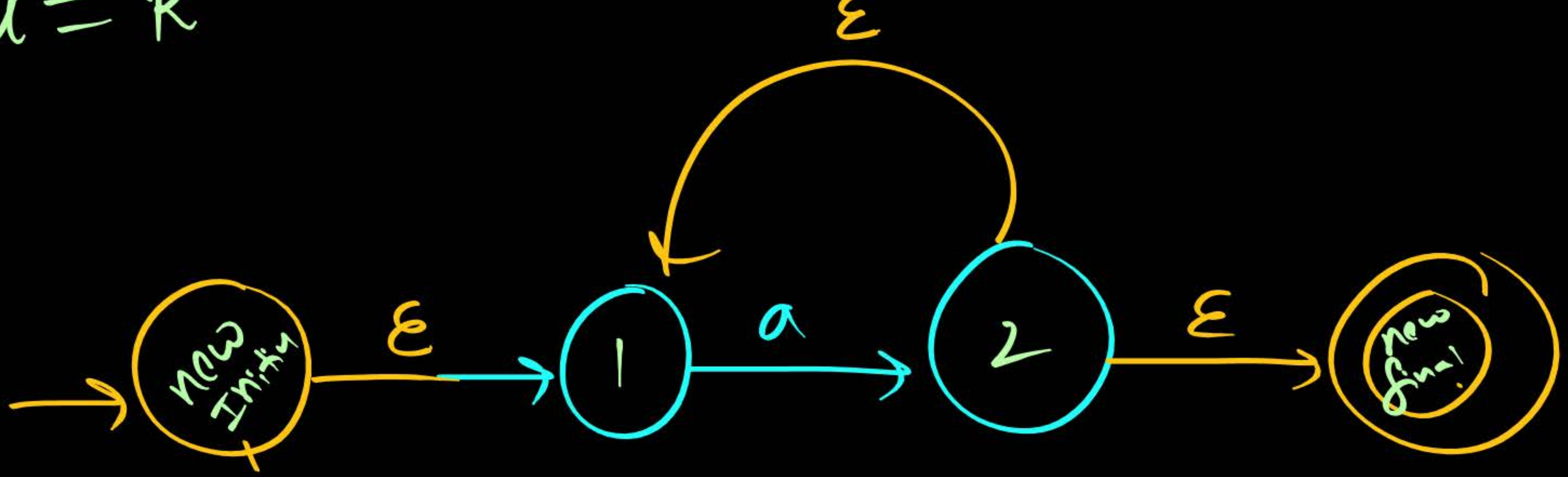
$R_1 \cdot R_2$



5) $a^* = R^*$



6) $a^+ = R^+$



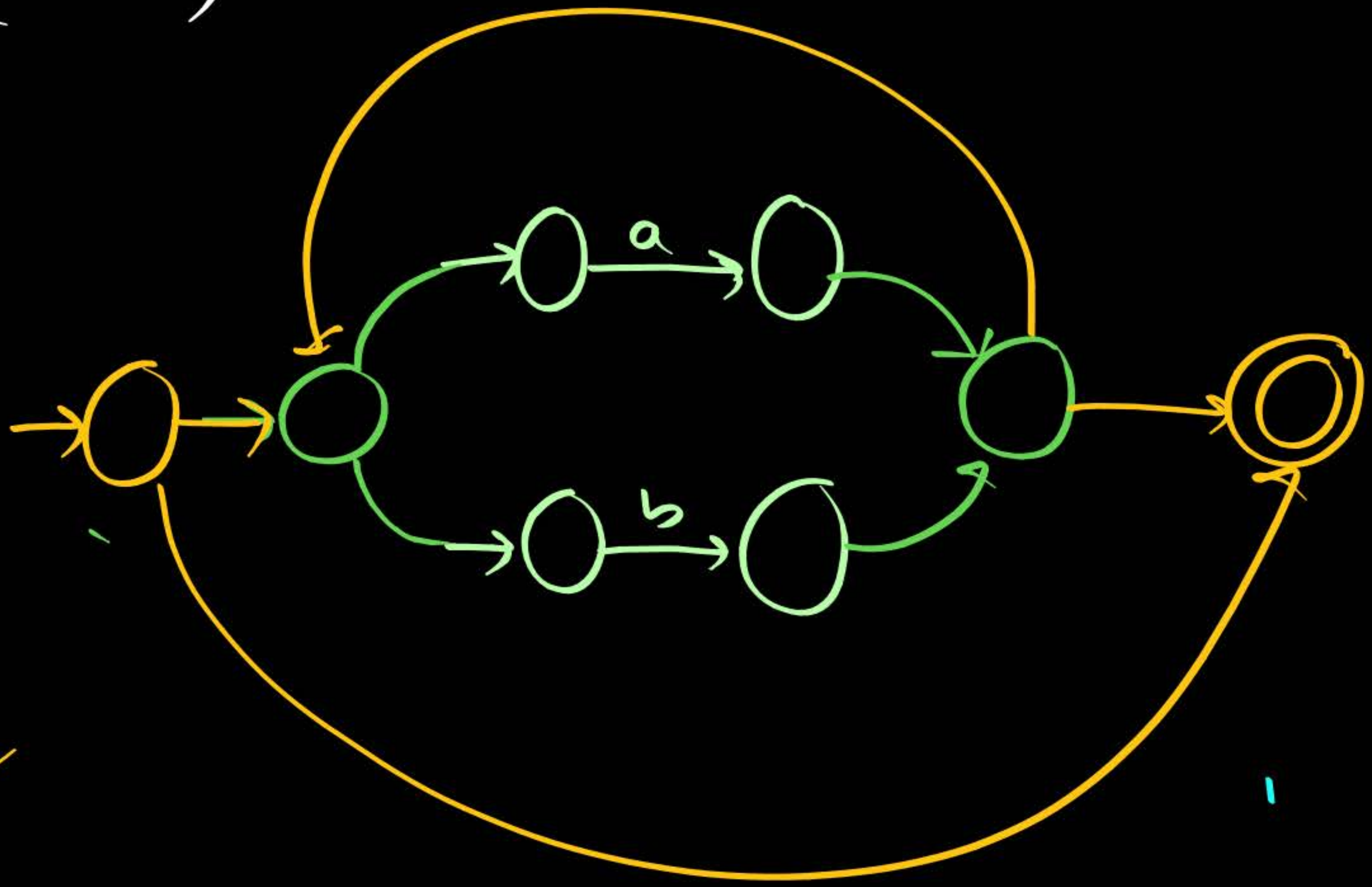
7) $(a+b)^*$

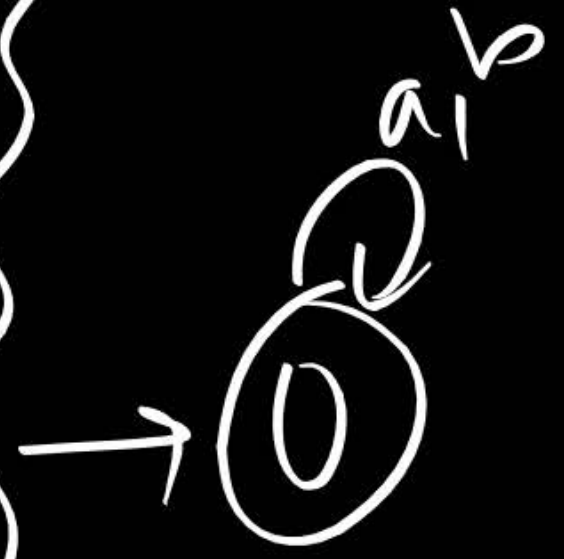
a ✓

b ✓

a+b ✓

$(a+b)^*$ ✓





Directly min DFA

8) $[(ab)^* + ab^*b].aaa = R$

a ✓

b ✓

ab ✓

$(ab)^*$ ✓

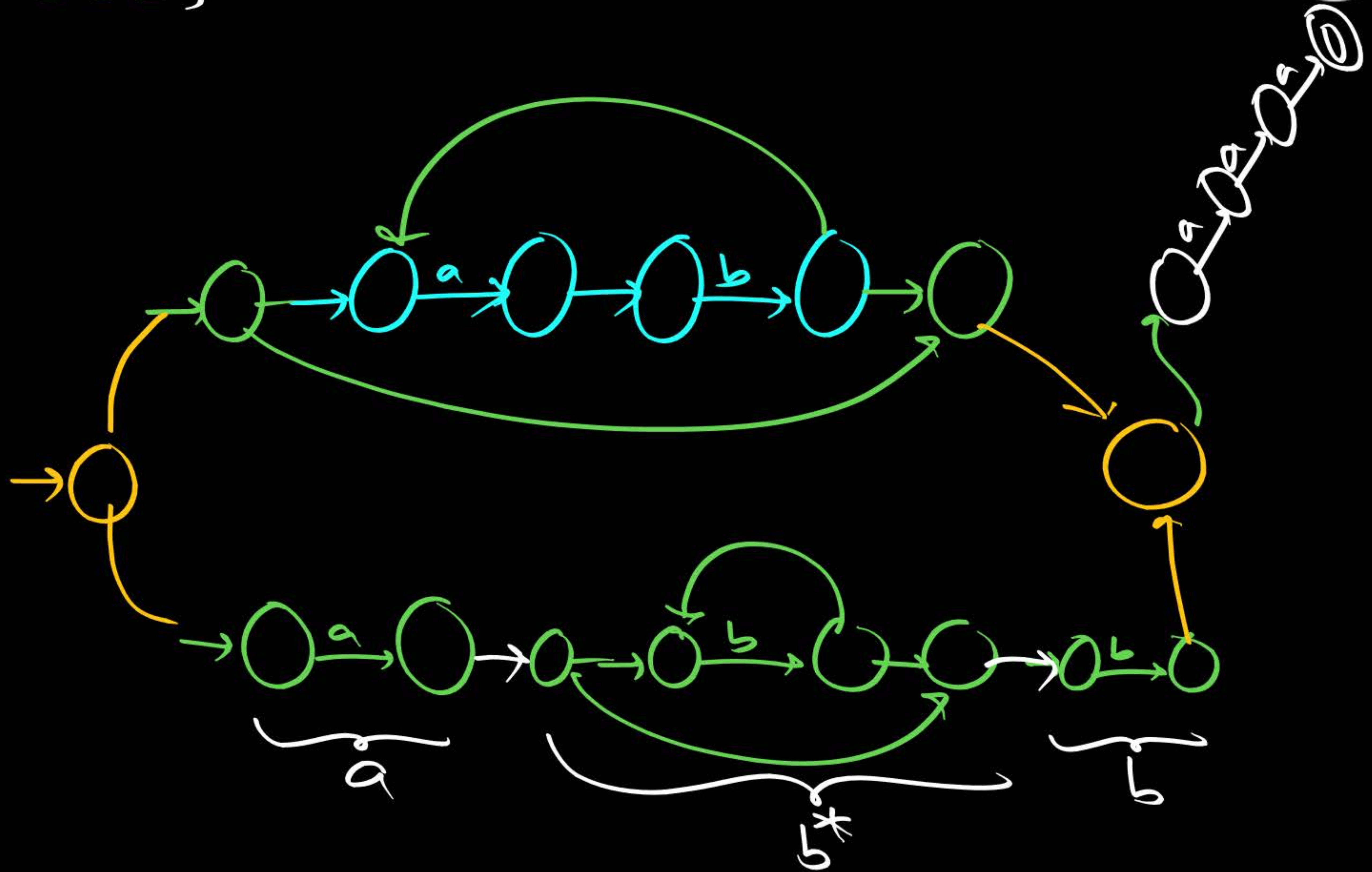
b^* ✓

ab^*b ✓

$(ab)^* + ab^*b$ ✓

aaa ✓

R ✓



$$\begin{tikzcd} \text{Reg Exp} \arrow[r, Rightarrow] & \text{FA} \arrow[l, bend left] \end{tikzcd}$$

$$\boxed{\text{RegExp} \cong \text{FA}}$$

Summary

DFA

$\cong$

NFA without ε moves

$\cong$

NFA with ε moves

$\cong$

RegExp


THANK YOU
GW
SOLDIERS !