


CS & IT ENGINEERING

Theory of Computation

Regular Languages

Lecture No.- 03



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Topics to be Covered



Topic

Basics of TOC



- operations on strings
- Chomsky Hierarchy
- Language Automata Grammar



TOPIC:



operations on strings:

1) Concatenation(\cdot)

$$w_1 = ab$$

$$w_2 = baa$$

$$w_1 \cdot w_2 = w_1 w_2 = abbaa$$

$$w_2 w_1 = baab$$

$$\left. \begin{array}{l} |w_1| = k \\ |w_2| = n \end{array} \right\} \Rightarrow |w_1 w_2| = |w_2 w_1| = k + n$$

$$\Sigma = \{0, 1\}$$

Σ
 0
 1
 00
 01
 10
 11

$$\Sigma = \{\underbrace{\text{gate}}_0, \underbrace{\text{exam}}_1\} = \{0, 1\}$$

Σ
 gate
 exam
 gate gate
 gate exam
 exam gate
 exam exam



TOPIC:



2) prefix of a string:

$$\Sigma = \{a, b\}$$

$w = \overrightarrow{abbb}$

\hookrightarrow prefixes of $w \Rightarrow$

- ϵ
- a
- ab
- abb

} beginning sequence of w

$\Sigma = \{a, b, c\}$



prefixes = ϵ

a

ab

abc

Prefixes of $(w) = \{u \mid \underline{u}v = w\}$

= Set of all beginning sequences of w

abc

$w = u$

v

$= \epsilon \cdot abc$

$= a \cdot bc$

$= ab \cdot c$

$= \underline{abc} \cdot \epsilon$

Prefixes

Suffixes

abc

$$u \cdot v = w$$

$$\epsilon \cdot abc = abc$$

$$a \cdot bc = abc$$

$$ab \cdot c = abc$$

$$abc \cdot \epsilon = abc$$

empty string

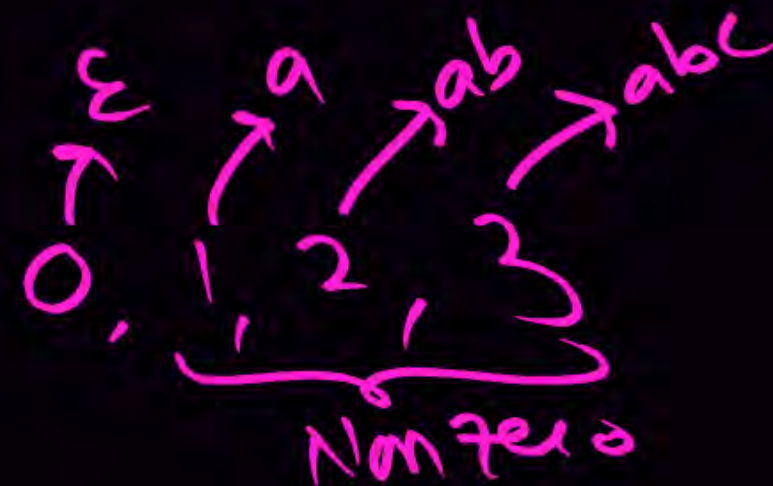
non empty strings

ϵ, a, ab, abc

prefixes

4 prefixes

Lengths:





TOPIC:



Let w be a string.

Q1) No. of prefixes of w = ? $= |w| + 1$

Q2) No. of non empty prefixes of w = ? $= |w|$

Q3) No. of non zero length prefixes of w = ? $= |w|$



TOPIC:

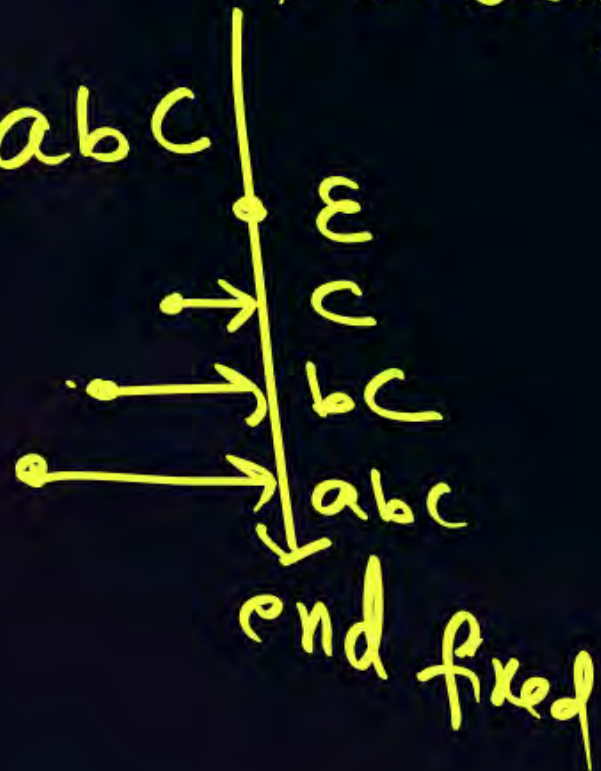


③ Suffix of a string:

$$\text{Suffix}(w) = \{ v \mid uv = w \}$$

⇒ ending sequences

$w = abc$





TOPIC:



Let w be a string.

Q1) No. of suffixes of w = ? $= |w| + 1$

Q2) No. of non empty suffixes of w = ? $= |w|$

Q3) No. of non zero length suffixes of w = ? $= |w|$



TOPIC:



$w = \overset{0}{a}bcd$

Prefixes: ϵ , a , ab , abc , $abcd$
Trivial, Non trivial, Trivial

Suffixes:

ϵ , d , cd , bcd , $abcd$
Trivial, Proper suffixes (other than given w), Trivial

w



is prefix of w } trivial



TOPIC:

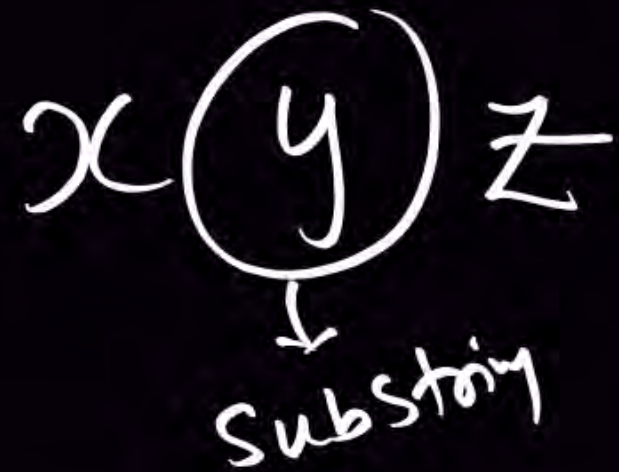


④ Substring of a String:

↳ part of a string

Substrings of $w = \{ y \mid xyz = w \}$
x and z anything

$w = abc$



Substrings of abc

↓
 ϵ
 a
 b
 c

ab
 bc
 abc

different lengths
 $0, 1, 2, 3$
 4 diff lengths

$w = x \cdot y \cdot z = abc$

| | | |
|------------|------------|-------------|
| ϵ | ϵ | $abc = abc$ |
| ϵ | a | $bc = "$ |
| a | b | $c = "$ |
| ab | c | ϵ |
| ϵ | ab | c |
| a | bc | ϵ |
| ϵ | abc | ϵ |



TOPIC:

$w = aaaaa$



No. of Substrings = ? = 5

$$= 4 + 1$$

$$= |w| + 1$$

(min)

ϵ
a
aa
aaa
aaaa

$w = abcd$



No. of Substrings = ? = 11

$$= \frac{4(4+1)}{2} + 1$$

ϵ

a

b

c

d

ab
bc
cd

abc
bcd

abcd

$$1 + 2 + 3 + \dots + |w| + 1$$

$\sum n$

If 4 length string given

then

No. of Substrings = ?

$w = aaaa \Rightarrow \text{Min} = 5$ (if all symbols are same)

$w = abcd \Rightarrow \text{Max} = 11$ (if all symbols are distinct)

Max no. of Substrings for n length string

$$= \sum n + 1$$

$$= \frac{n(n+1)}{2} + 1$$

**TOPIC:**

Assume n length string given

Q1) No. of substrings \Rightarrow Min = $n+1$, Max = $\sum n+1$

Q2) No. of non empty substrings \Rightarrow Min = n , Max = $\sum n$

* * * Q3) No. of ^{different} non zero length substrings = n

Q4) No. of different length substrings = $n+1$



TOPIC:



$w = abcd$

Substrings

$\rightarrow \epsilon, a, b, c, d, ab, bc, cd$
0 len 1 len 2 len

$abc, bcd, abcd$
3 len 4 len

Lengths: 0, 1, 2, 3, 4

5 different lengths

$w = aaaa$

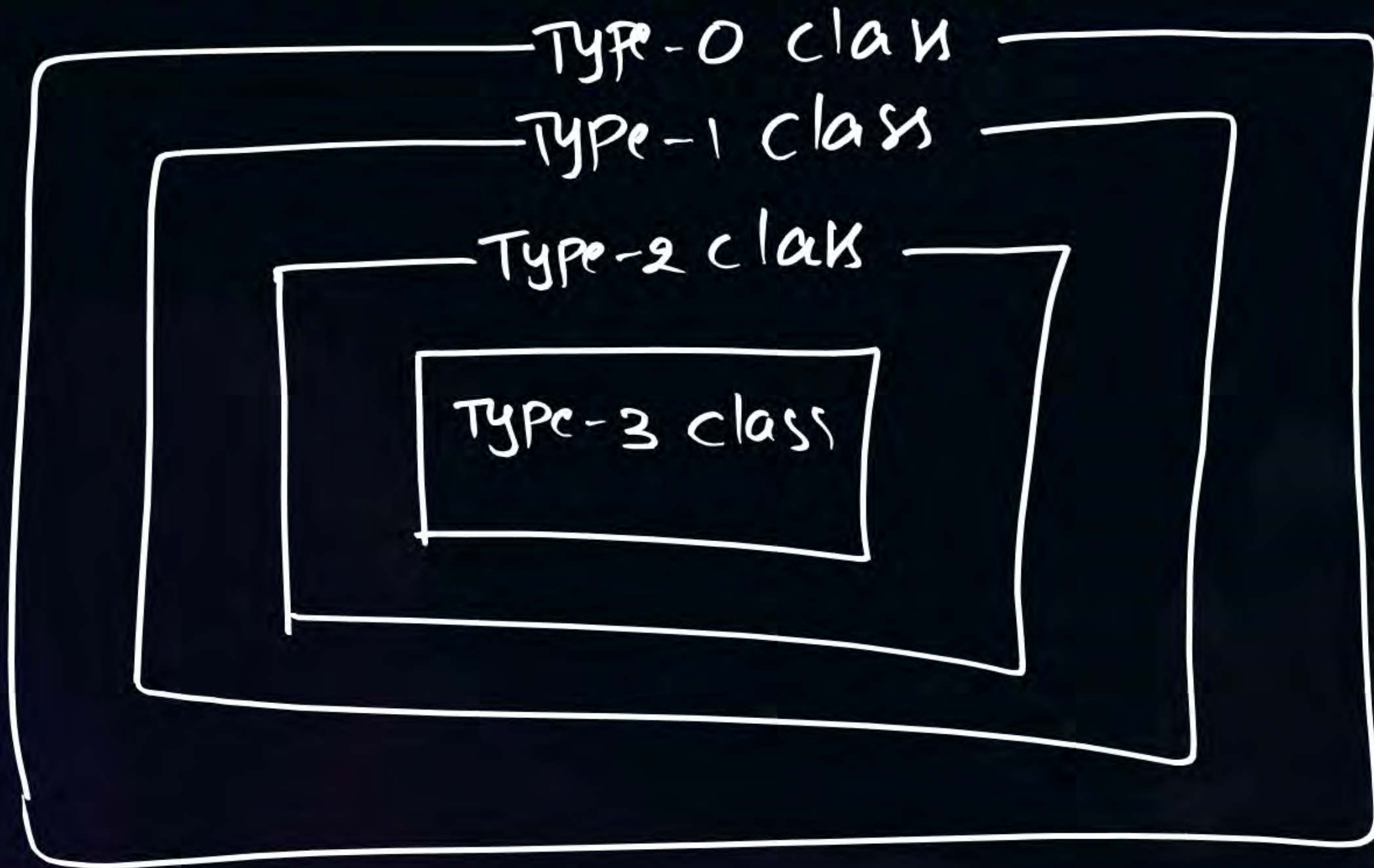
Substrings

$\rightarrow \epsilon, a, aa, aaa, aaaa$
0 1 2 3 4 len

Lengths: 0, 1, 2, 3, 4
non zero
5 diff lengths



TOPIC: CHOMSKY Hierarchy:





TOPIC:



Type-3 \subset Type-2 \subset Type-1 \subset Type-0 class

Smallest Largest

FA $<$ PDA $<$ LBA $<$ TM

Finite Automata (Push Down) (Linear Bound) (Turing M/c)



TOPIC:



Language s

- Finite Languages
- Infinite Languages
- Regular Languages
- Deterministic Context Free Languages
- Context Free Languages
- Context Sensitive "
- Recursive Languages
- Recursively Enumerable Languages



TOPIC:



Language

Finite language

Infinite Language

$\Sigma = \{a, b\}$

$$F_1 = \{ \}$$

$$F_2 = \{ \epsilon \}$$

$$F_3 = \{ a, b \}$$

$$F_4 = \{ aa, ba, aab \}$$

$$I_1 = \{ a^n \mid n \geq 0 \} = \{ a^0, a^1, a^2, \dots \}$$
$$= \{ \epsilon, a, aa, \dots \}$$

$$I_2 = a^n b^n$$

#Q1. Which of the following is not a prefix for “abc”?

Handwritten notes:
An arrow points from the word "not" to the word "abc".
The word "abc" is written vertically three times.

A a

B ab

~~**C**~~ bc

D abc

#Q2. Which of the following is proper prefix of "abb".

A ϵ ✓

C ab ✓

B abb ✗

D bb ✗
not a prefix

prefix { ϵ
a
ab } proper
{ abb } not proper

#Q3. If $w = a^{17}$ then no. of Substrings of w is —

A 17

B $17 \times (17+1)/2$

C ✓ 18

D $17 \times (17+1)/2 + 1$

$w = a^4 = aaaa \Rightarrow 5$ Substrings



2 mins Summary



Topic

Operations on Strings ✓

Topic

Types of Languages ✓

Topic

Chomsky Hierarchy ✓

Topic

Grammar

Topic

Automata



THANK - YOU