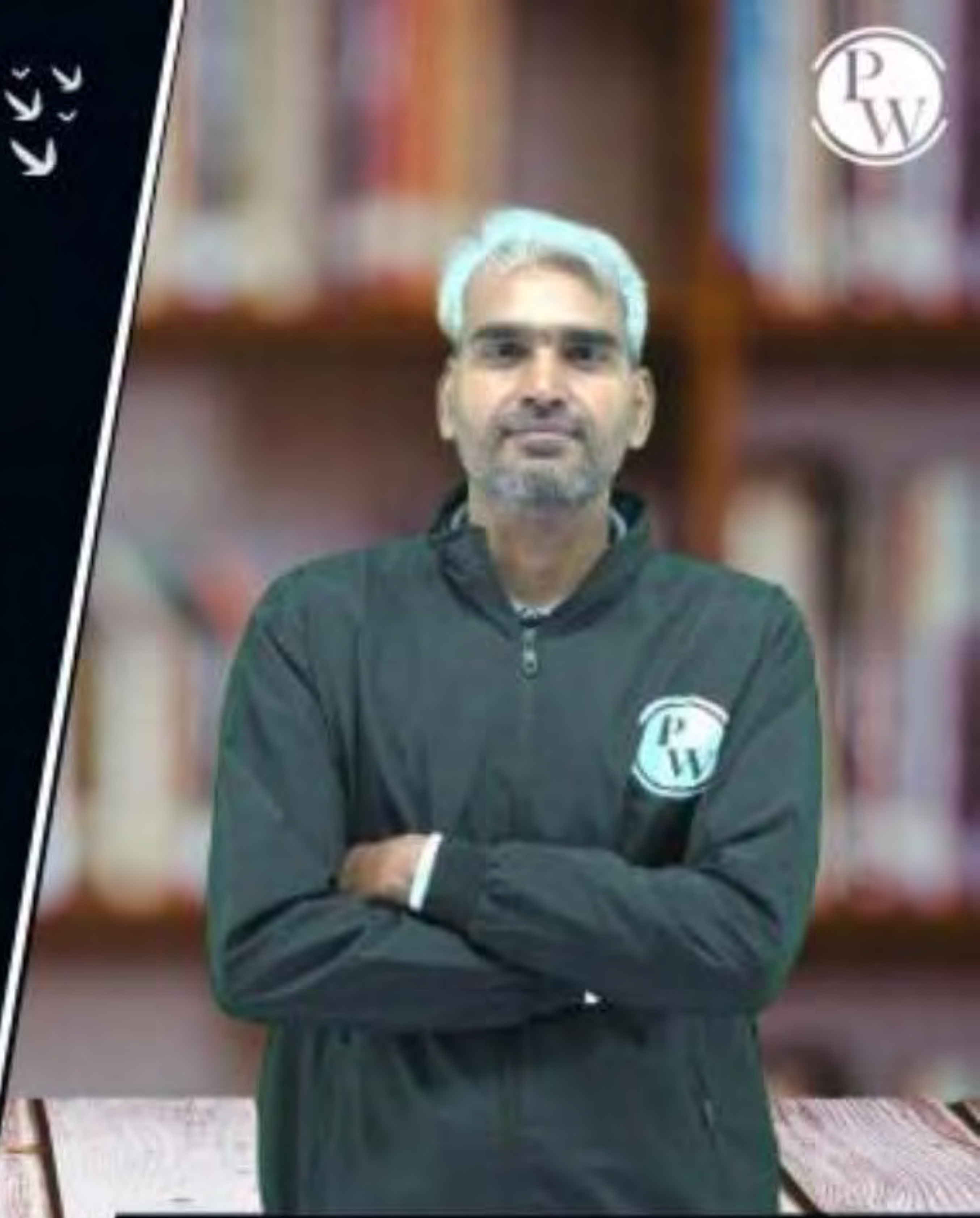
Computer Science & IT

Discrete Mathematics

Combinatorics

Lecture No. 03



By- Vishal Sir

Recap of Previous Lecture

Topic Formation of recurrence relation

Topic Solution of recurrence relation using substitution method

Slide

Topics to be Covered

- **Topic** Solution of recurrence relation using method of characteristic roots
- **Topic** Method of undetermined coefficient for particular solution

Slide

Topic : Substitution method

In this method we use recurrence relation repetitively for $n=0,1,2,\ldots\ldots..$, then we solve the expression to obtain the solution of recurrence relation.

#Q. Find the solution of the recurrence relation

$a_n = n\, a_{n-1}$, where $a_0 = 1$

$a_0 = 1$

$a_1 = 1 \cdot a_0 = 1 \cdot 1 = 1$

$a_2 = 2 \cdot a_1 = 2 \cdot 1 \cdot 1 = 2$

$a_3 = 3 \cdot a_2 = 3 \cdot 2 \cdot 1 \cdot 1 = 6$

$a_4 = 4 \cdot a_3 = 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 12$

$\vdots$

$a_n = n \cdot (n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \cdot 1 = n! \implies \boxed{a_n = n!}$

#Q. Find the solution of the recurrence relation

$$a_n = a_{n-1} + 3^{n-1}, \quad \text{where } a_1 = 2$$

$a_1 = 2$

$a_2 = a_1 + 3^{2-1} = 2 + 3^1$

$a_3 = a_2 + 3^{3-1} = 2 + 3^1 + 3^2$

$a_4 = a_3 + 3^{4-1} = 2 + 3^1 + 3^2 + 3^3$

$\vdots$

$a_n = 2 + 3^1 + 3^2 + 3^3 + \cdots + 3^{n-2} + 3^{n-1}$

$= 1 + 3^0 + 3^1 + 3^2 + \cdots + 3^{n-1}$

Summation of GP $= \dfrac{a(r^n - 1)}{r - 1}$

first term, Common ratio, No. of terms

$a_n = 1 + [3^0 + 3^1 + 3^2 + \cdots + 3^{n-1}]$

$= 1 + \left[\dfrac{1 \cdot (3^n - 1)}{3 - 1}\right]$

$= 1 + \dfrac{3^n - 1}{2}$

$= \left(\dfrac{3^n + 1}{2}\right)$

#Q. Find the solution of the recurrence relation

$$a_n = a_{n-1} + (2n+1), \text{ where } a_0 = 1$$

$a_0 = 1$

$a_1 = a_0 + (2\cdot1+1) = 1 + (2\cdot1+1)$

$a_2 = a_1 + (2\cdot2+1) = 1 + (2\cdot1+1) + (2\cdot2+1)$

$a_3 = a_2 + (2\cdot3+1) = 1 + (2\cdot1+1) + (2\cdot2+1) + (2\cdot3+1)$

$\vdots$

$a_n = 1 + (2\cdot1+1) + (2\cdot2+1) + (2\cdot3+1) + \cdots + (2\cdot n+1)$

$$a_n = 1 + (2\cdot1+1) + (2\cdot2+1) + (2\cdot3+1) + \cdots + (2\cdot n+1)$$

$$= \underset{(2\cdot 0+1)}{1} + 3 + 5 + 7 + 9 + \cdots (2n+1)$$

{ it is summation of first $(n+1)$ odd natural numbers

$$= (n+1)^2$$

Note: Summation of first 'n' odd natural number $= n^2$

$$a_n = 1 + (2\cdot1+1) + (2\cdot2+1) + (2\cdot3+1) + \cdots + (2\cdot n+1)$$

$(n+1)$ term $= 1 + 3 + 5 + 7 + \cdots + (2n+1)$

{ It is an Arithmatic progression with Common difference = 2 }

$$= \frac{(n+1)}{2}\left[1 + (2n+1)\right]$$

$$= \frac{(n+1)}{\cancel{2}} \cdot \cancel{2}\cdot(n+1) = (n+1)^2$$

Summation of AP $= \frac{n}{2}\left[2\cdot a + (n-1)d\right]$

$= \frac{n}{2}\cdot\left[\text{first term} + \text{last term}\right]$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{(n+1)}$$

#Q. Find the solution of the recurrence relation

$$a_n = a_{n-1} + \frac{1}{n(n+1)}, \quad \text{where } a_0 = 1$$

$$a_n = 2 - \frac{1}{(n+1)}$$

$$\boxed{a_n = \frac{(2n+1)}{(n+1)}}$$

$a_0 = 1$

$a_1 = a_0 + \left(\frac{1}{1} - \frac{1}{2}\right) = 1 + \left(\frac{1}{1} - \frac{1}{2}\right)$

$a_2 = a_1 + \left(\frac{1}{2} - \frac{1}{3}\right) = 1 + \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right)$

$a_3 = a_2 + \left(\frac{1}{3} - \frac{1}{4}\right) = 1 + \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right)$

$\vdots$

$a_n = 1 + \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \cancel{/} - \cancel{/} - \cancel{/} - \cancel{/} - \cancel{/} + \left(\cancel{\frac{1}{(n-1)}} - \cancel{\frac{1}{n}}\right) + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$

Topic : Solution of recurrence relation

1) Substitution method

Today's Topic 2) Method of characteristic roots

3) Using concept of generating function

Topic : Method of characteristic roots

Shift operator:

$$a_{n+k} = E^k \cdot a_n$$

$$a_{n+(k-1)} = E^{k-1} \cdot a_n$$

$$\vdots$$

$$a_{n+1} = E^1 \cdot a_n$$

$$a_n = E^0 \cdot a_n = a_n$$

Topic : Method of characteristic roots

Consider the following linear recurrence relation.

$$\lambda_0 \cdot a_n + \lambda_1 \cdot a_{n-1} + \lambda_2 \cdot a_{n-2} + \cdots + \lambda_k \cdot a_{n-k} = f(n) \quad \text{—— eq}^n \ (1)$$

Put $n-k = n$,

i.e., $n = n+k$,

$\therefore$ Equation becomes,

It is the new function obtain after replacing n by $(n+k)$ in $f(n)$.

$$\lambda_0 \cdot a_{n+k} + \lambda_1 \cdot a_{n+k-1} + \lambda_2 \cdot a_{n+k-2} + \cdots + \lambda_k a_n = F(n) \quad \text{—— eq}^n \ (2)$$

Topic : Method of characteristic roots

Use shift operator in eqⁿ (2) i.e. $a_{n+i} = E^i \cdot a_n$

$$\lambda_0 \cdot (E^k \cdot a_n) + \lambda_1 \cdot (E^{k-1} \cdot a_n) + \lambda_2 (E^{k-2} \cdot a_n) + \cdots + \lambda_k \cdot (\cancelto{1}{E^0} \cdot a_n) = F(n)$$

$$E^k \cdot \lambda_0 \cdot a_n + E^{k-1} \cdot \lambda_1 \cdot a_n + E^{k-2} \cdot \lambda_2 \cdot a_n + \cdots + \lambda_k \cdot a_n = F(n)$$

$$(\underbrace{E^k \cdot \lambda_0 + E^{k-1} \cdot \lambda_1 + E^{k-2} \cdot \lambda_2 + \cdots + E^0 \cdot \lambda_k}) \, a_n = F(n)$$

it is a polynomial of degree 'k', lets call it $\phi(E)$

i.e., $\boxed{\phi(E) \cdot a_n = F(n)}$ — eqⁿ (3)

Topic : Method of characteristic roots

* Characteristic Equation :-

$\phi(E)$ is a Polynomial of degree $= k$

$\boxed{\phi(t) = 0}$ is called Characteristic Equation

Roots of Characteristic Equation are called Characteristic roots.

Let the characteristic roots are.

$$\underbrace{t_1, \ t_2, \ t_3, \ \ldots \ldots, t_k}_{}$$

'k' characteristic roots, w.r.t. polynomial of degree $= k$

Topic : Method of characteristic roots

Complementary Function (C.F.)

Complementary function is the solution w.r.t. homogeneous part of linear recurrence relation. {i.e. w.r.t. $f(n) = 0$}

Particular Solution :-

Particular solution is the solution w.r.t non-homogeneous part of linear recurrence relation {i.e. w.r.t. $f(n) \neq 0$}

$\therefore$ $\boxed{\text{Complete Solution of linear recurrence Relation} = \text{Complementary Function} + \text{Particular Solution}}$

Note:- If given recurrence relation is a homogeneous linear recurrence relation {i.e. $f(n)=0$}, then Complementary function alone gives the complete solution of the homogeneous linear recurrence relation

$$a_n = a_n^H + a_n^P$$

$$3a_n + 5a_{n-1} + a_{n-2} = 2^n$$

$$= 0$$

$$0 \to a_n^H$$

$$2^n \to a_n^P$$

$$3\cdot(a_n^H + a_n^P) + 5(a_{n-1}^H + a_{n-1}^P) + (a_{n-2}^H + a_{n-2}^P)$$

$$\underbrace{\left(3\cdot a_n^H + 5a_{n-1}^H + a_{n-2}^H\right)}_{0} + \underbrace{\left(3\cdot a_n^P + 5a_{n-1}^P + a_{n-2}^P\right)}_{2^n} = 2^n$$

Topic : Method of characteristic roots

Rules to write Complementary function :-

Characteristic roots	Complementary function
(1) When all roots are real & distinct i.e., $t_1, t_2, t_3, \ldots, t_k$	$C_1 (t_1)^n + C_2 (t_2)^n + C_3 (t_3)^n + \ldots + C_k (t_k)^n$ $C_1, C_2, \ldots C_k$ are unknown constants
(2) When all roots are real and two roots are equal, let, $\underbrace{t_1, t_1}, t_3, t_4, \ldots, t_k$	$(C_1 + C_2 n)(t_1)^n + C_3 (t_3)^n + \ldots + C_k (t_k)^n$ $C_1, C_2, \ldots C_k$ are constants
(3) When roots are real & three roots are equal, let $\underbrace{t_1, t_1, t_1}, t_4, \ldots t_k$	$(C_1 + C_2 n + C_3 n^2)(t_1)^n + C_4 (t_4)^n + \ldots + C_k (t_k)^n$ $C_1, C_2, \ldots C_k$ are constants.

and so on

Note: We can write the Complementary function in the presence of Complex root as well.

{Not required for GATE}

Note:- In eqn (1) if $f(n) = 0$, then it becomes homogeneous linear recurrence relation, and Complementary function alone will give us the Complete solution of that linear recurrence relation.

Note: In eq^n (I) if $f(n) \neq 0$, then complete solution is defined as,

$$a_n = \text{Complementary function} + \text{Particular solution}$$

Solution w.r.t homogeneous part

Solution w.r.t. non-homogeneous part

i.e., if $f(n) \neq 0$, then we need to obtain the particular solution as well.

Particular Solution :-

From Eq^n ③ $\quad \phi(E) \cdot a_n = F(n)$

i.e., $a_n = \dfrac{F(n)}{\phi(E)}$

Particular solution (a_n^p) is the solution of $\left(\dfrac{F(n)}{\phi(E)}\right)$

P.S. = Solution of $\dfrac{F(n)}{\phi(E)}$

if $f(n) = 0$

then $F(n) = 0$

$\therefore \dfrac{F(n)}{\phi(E)} = \dfrac{0}{\phi(E)} = 0$

Hence when $f(n) = 0$, then Particular Solution $= 0$

Topic : Method of characteristic roots

Rules for writing Particular Solution When $F(n) = \underbrace{(b)^n}_{}$

Exponential Function with some base = b

$$\text{Particular Solution} = \text{Solution of} \left(\frac{F(n)}{\phi(E)}\right) = \text{Solution of} \left(\frac{b^n}{\phi(E)}\right)$$

Case ① When 'b' is not a characteristic root.

i.e, when $\phi(b) \neq 0$

then $\text{Solution of} \left(\frac{b^n}{\phi(E)}\right) = \frac{b^n}{\phi(b)}$

$$\boxed{\therefore \text{Particular Solution } (a_n^p) = \frac{b^n}{\phi(b)}}$$

Topic : Method of characteristic roots

Rules for writing Particular Solution When $F(n) = \underbrace{(b)^n}$

Exponential function with some base = b

$$\text{Particular Solution} = \text{Solution of} \left(\frac{F(n)}{\phi(E)}\right) = \text{Solution of} \left(\frac{b^n}{\phi(E)}\right)$$

Case ② :- When 'b' is the characteristic root with multiplicity 'm'.

i.e., $\phi(b) = 0$, and $\phi(E) = (E-b)^m \cdot \psi(E)$,

where, $\psi(b) \neq 0$

then Particular Solution $(a_n^p) = \text{Solution of} \left(\frac{b^n}{\phi(E)}\right) = \text{Solution of} \left(\frac{b^n}{(E-b)^m \cdot \psi(E)}\right) = \frac{1}{\psi(b)} \cdot \left\{ {}^nC_m \, (b)^{n-m} \right\}$

Q: Find the solution of recurrence relation,

$$a_n = 3 \cdot a_{n-1} \quad \text{where} \quad \boxed{a_0 = 1}$$

Solu^n.

$$a_n - 3a_{n-1} = 0 \leftarrow \text{homogeneous}$$

Put $n-1 = n$ {step-1 put $n-k=n$}

$$a_{n+1} - 3a_n = 0$$

Use shift operator i.e. $a_{n+i} = E^i a_n$ {step2: Use shift operator}

$$E^1 \cdot a_n - 3 \cdot E^0 \cdot a_n = 0$$

$$E \cdot a_n - 3 \cdot a_n = 0$$

$$\underbrace{(E-3)}_{\phi(E)} \cdot a_n = 0$$

$$\phi(E) = E - 3,$$

$\therefore$ Characteristic equation {$\phi(t) = 0$ is ch. eq^n} is $t - 3 = 0$

$$\boxed{t = 3}$$

Chacacteristic root is $t = 3$

$\therefore$ Complementary function $(C.F.) = \underbrace{C_1}_{\text{Constant}} \cdot (3)^n$

Given recurrense relation is homogeneous linear recurrence relation $\therefore$ Particular Solu^n = 0

Hence, Complete Solution

$$a_n = C.F. + \cancel{P.S.}^{\,0}$$

$$\boxed{a_n = C_1 \cdot (3)^n}$$

we know $a_0 = 1$

$$\therefore a_0 = C_1 \cdot (3)^0$$

$$1 = C_1 \cdot (3)^0$$

$$\boxed{C_1 = 1}$$

Hence,

$$a_n = 1(3)^n$$

$$\boxed{a_n = 3^n}$$

Q: Find solution of recurrence relation.

$$a_n = 2\cdot a_{n-1} - 1, \quad \text{where } \boxed{a_1 = 2}$$

$a_n - 2a_{n-1} = -1$

$\boxed{\text{Put } n-1 = n}$

$a_{n+1} - 2a_n = -1$

$\boxed{\text{Shift operator}}$

$E\cdot a_n - 2a_n = -1$

$(E-2)\cdot a_n = -1$

$\phi(E) = E-2$

Ch. eqn is
$t-2=0$
$\therefore$ Ch root is
$t=2$.

Hence,
Complementary Function $(C.F) = C_1(2)^n$

$\boxed{C.F = C_1(2)^n}$ → a_n^H

From
$(E-2)\cdot a_n = -1$

$(E-2)\cdot a_n = -1\cdot\{1^n\}$ ← base $b=1$

$a_n = \dfrac{-1\cdot(1)^n}{(E-2)}$

$\because$ $b=1$ is not a Characteristic root
i.e. $\phi(b) \neq 0$.

$\therefore P.S.(a_n^P) = \text{Sol}^n \text{ of } \left(\dfrac{-1\cdot(1)^n}{(E-2)}\right)$

$= -1\cdot \text{Sol}^n \text{ of } \left\{\dfrac{(1)^n}{(E-2)}\right\}$ → $\phi(E)$

$= -1\left\{\dfrac{(1)^n}{\phi(1)}\right\} = -1\left\{\dfrac{(1)^n}{(1-2)}\right\} = 1$

Complete Solution,

$a_n = a_n^H + a_n^P$
$= C.F + P.S$

$$\boxed{a_n = C_1\cdot(2)^n + 1}$$

$a_1 = 2$

$\therefore a_1 = 2 = C_1\cdot(2)^1 + 1$

$\boxed{C_1 = \dfrac{1}{2}}$

$\therefore a_n = \dfrac{1}{2}\cdot(2)^n + 1$

$$\boxed{a_n = 2^{n-1} + 1}$$

Q. Find the Solu^n of recurrence relation,

$$a_n = 2 \cdot a_{n-1} + 2^n \leftarrow f(n) \qquad \boxed{a_0 = 1}$$

$$a_n - 2a_{n-1} = 2^n$$

$\boxed{\text{Put } n-1 = n,\ (n+1) = n}$

$$a_{n+1} - 2a_n = 2^{n+1} \leftarrow F(n)$$

using shift operator

$$E \cdot a_n - 2 \cdot a_n = 2^{n+1}$$

$$(E-2) \cdot a_n = 2^{n+1}$$

$$\phi(E) = (E-2)$$

$$\therefore \boxed{\phi(E) \cdot a_n = 2(2)^n} \text{ — eq}^n \text{ ①}$$

Ch. eq^n is $t - 2 = 0$

$\therefore$ Ch. root is $t = 2$

Hence,

Complementary Function (C.F) $= C_1 \cdot (2)^n$

$$\boxed{C.F = a_n^H = C_1 \cdot (2)^n}$$

Particular Solution

from eq^n ①

$$\phi(E) \cdot a_n = 2(2)^n$$

Particular Solu^n $=$ Solu^n of $\left(\dfrac{2 \cdot (2)^n}{\phi(E)}\right)$

$$= 2 \cdot \left\{ \text{Solu}^n \text{ of } \frac{(2)^n}{\phi(E) = (E-2)} \right\}$$

$b = 2$, & $\phi(b) = \phi(2) = 2 - 2 = 0$

We know '2' is Ch. root with multiplicity $= 1$

$$\phi(E) = E - 2 = (E-2)^1 \cdot 1$$

$$\phi(E) = (E-2)^1 \cdot \psi(E) \quad \text{when } \psi(E) = 1$$

$$= 2 \times \left\{ \text{Solu}^n \text{ of } \left(\frac{(2)^n}{(E-2)^1 \cdot \cancel{\psi(E)}_{1}} \right) \right\}$$

$$= 2 \times \left[\frac{1}{\cancel{\psi(2)}_{=1}} \cdot \left({}^{n}C_{1} \cdot (2)^{n-1} \right) \right] \quad (b = 2,\ m = 1)$$

$$\therefore P.S.(a_n^P) = 2 \cdot \left\{ \frac{1}{1} \cdot \left(n \cdot (2)^{n-1} \right) \right\}$$

$$P.S.(a_n^P) = n \cdot 2^n$$

Complete Solu^n

$$a_n = CF + P.S$$

$$= C_1 \cdot 2^n + n \cdot 2^n$$

$$a_0 = 1$$

$$\therefore a_0 = 1 = C_1 \cdot 2^0 + 0 \cdot 2^0$$

$$\boxed{C_1 = 1}$$

Hence

$$a_n = 1 \cdot 2^n + n \cdot 2^n$$

$$= (n+1) \cdot 2^n$$

Q: Find the solution of recurrence relation,

$$a_n - 7 \cdot a_{n-1} + 12 \cdot a_{n-2} = 0$$

homogeneous

$\boxed{\begin{matrix} a_0 = 1 \\ a_1 = 2 \end{matrix}}$

$$a_{n+2} - 7a_{n+1} + 12a_n = 0$$

$$E^2 \cdot a_n - 7 \cdot E \cdot a_n + 12a_n = 0$$

$$\underbrace{(E^2 - 7E + 12)}_{\phi(E)} \cdot a_n = 0$$

Ch. eqn is $t^2 - 7t + 12 = 0$

& Ch. root are, $t_1 = 3$, $t_2 = 4$

$\therefore$ Complementary function

$$C \cdot F = c_1 \cdot (3)^n + c_2 \cdot (4)^n$$

$\therefore$ P.S. = 0

Complete Soln

$$a_n = CF + P.S.$$

$$a_n = \left(c_1 \cdot (3)^n + c_2 \cdot (4)^n\right) + 0$$

from, $a_0 = 1$ & $a_1 = 2$,

we get,

$c_1 = 2$ & $c_2 = -1$

$$\therefore a_n = 2 \cdot (3)^n - 1 \cdot (4)^n$$

$$\boxed{a_n = 2(3)^n - 4^n}$$

Q: Find the solution of recurrence relation.

$$T(3^k) = 2 \cdot T(3^{k-1}) + 1 \qquad \boxed{T(1) = 1}$$

Put $T(3^k) = a_k$

$\therefore T(3^{k-1}) = a_{k-1}$

i.e., $a_k = 2 \cdot a_{k-1} + 1$

$(E-2) \cdot a_k = 1$

Ch. root is $t = 2$

$C.F. = C_1 \cdot (2)^k$

recurrence relⁿ is a function of 'k'

From $(E-2) \cdot a_k = 1$

$$a_k = \frac{(1)^k}{(E-2)}$$

$\therefore P.S = $ Solⁿ of $\left(\frac{(1)^k}{(E-2)}\right)$

'1' is not a Ch. root.

$$= \frac{(1)^k}{\phi(1)} = \frac{(1)^k}{(1-2)} = -1$$

$\boxed{P.S = -1}$

Complete Solⁿ

$a_k = CF + P.S.$

$= C_1 \cdot 2^k + (-1)$

$\boxed{a_k = C_1 \cdot 2^k - 1}$

$a_k = T(3^k)$

$\therefore \boxed{T(3^k) = C_1 \cdot 2^k - 1}$ & $\boxed{T(1) = 1}$

$3^k = 1$ at $k = 0$.

$T(1) = T(3^0) = C_1 \cdot 2^0 - 1$

$1 = C_1 - 1 \Rightarrow \boxed{C_1 = 2}$

$\therefore T(3^k) = 2 \cdot 2^k - 1 \Rightarrow \boxed{T(3^k) = 2^{k+1} - 1}$

Q. Find the solu^n of recurrence relation

$$a_n - 2a_{n-1} + a_{n-2} = 0 \quad \text{homogeneous} \qquad \boxed{\begin{aligned} a_0 &= 1 \\ a_1 &= 2 \end{aligned}}$$

$$E^2 \cdot a_n - 2 \cdot E \cdot a_n + a_n = 0$$

$$\phi(E) = (E^2 - 2E + 1)$$

Ch. eq^n is $t^2 - 2t + 1 = 0$

$\therefore$ Ch. roots $t = \underbrace{1, 1}$

two equal roots

$$\therefore \; C \cdot F = (c_1 + c_2 n) \cdot (1)^n$$

$$\boxed{C \cdot F = c_1 + c_2 n}$$

$$\boxed{P \cdot S = 0}$$

Complete solution

$$a_n = CF + PS$$
$$= (c_1 + c_2 n) + 0$$

$$\boxed{a_n = c_1 + c_2 n}$$

$$a_0 = 1 = c_1 \Rightarrow \boxed{c_1 = 1}$$

$$a_1 = 2 = c_1 + c_2 \cdot 1$$

$$2 = 1 + c_2 \Rightarrow \boxed{c_2 = 1}$$

$$a_n = 1 + 1 \cdot n$$

$$\boxed{a_n = n + 1}$$

Q: Find the solution of the recurrence relation.

$$a_n - 3a_{n-1} + 2a_{n-2} = 2^n$$

$$\boxed{\begin{aligned} a_0 &= 1 \\ a_1 &= 2 \end{aligned}}$$

$$E^2 a_n - 3E\cdot a_n + 2a_n = 2^{n+2}$$

because $n = n+2$

$$\boxed{(E^2 - 3E + 2)\cdot a_n = 4(2)^n}$$

Ch eq^n is $t^2 - 3t + 2 = 0$

$\therefore$ Ch root are, $t_1 = 2$, $t_2 = 1$

Complementary function

$$\boxed{C.F = C_1\cdot(2)^n + C_2\cdot(1)^n}$$

$$P.S = \text{Sol}^n \text{ of } \left(\frac{4\cdot 2^n}{(E^2-3E+2)}\right)$$

$$= 4\cdot\left\{\text{Sol}^n \text{ of } \frac{(2)^n}{(E^2-3E+2)}\right\}$$

'2' is a root with multiplicity = 1

$$\therefore \phi(E) = (E-2)^1\cdot\underbrace{(E-1)}_{\psi(E)}$$

$$P.S : 4 * \left\{\text{Sol}^n \text{ of } \frac{(2)^n}{(E-2)^1\cdot\underbrace{(E-1)}_{\psi(E)}}\right\}$$

$$= 4 * \left[\frac{1}{\psi(2)}\left\{{}^nC_1\cdot(2)^{n-1}\right\}\right]$$

$$= 4 * \left[\frac{1}{(2-1)} * \left\{n\cdot(2)^{n-1}\right\}\right]$$

$$P.S = 2\cdot n\cdot 2^n$$

Complete Solu^n

$$a_n = CF + PS$$

$$= C_1(2)^n + C_2 + 2\cdot n\cdot 2^n$$

We know $a_0 = 1$ & $a_2 = 2$

$\therefore C_1 = -3$ & $C_2 = 4$

Hence, $\boxed{a_n = -3\cdot 2^n + n\cdot 2^{n+1} + 4}$

H.W. Find the solution of recurrence relation,

$$\underbrace{a_n^2 - 2a_{n-1}^2 = 1}$$ $\boxed{a_1 = 2}$

it is not a linear recurrence relⁿ,
but it can be converted into one
by putting $\boxed{a_n^2 = b_n}$

i.e, eqⁿ becomes

$$\boxed{b_n - 2b_{n-1} = 1}$$

in the end
$a_n = \pm\sqrt{b_n}$

H.W Find the solution of recurrence relation

$$\sqrt{a_n} - \sqrt{a_{n-1}} - 2\sqrt{a_{n-2}} = 0 \qquad \left\{ \begin{array}{l} a_0 = 1 \\ a_1 = 1 \end{array} \right\}$$

Put $\sqrt{a_n} = b_n$ $\left\{ \begin{array}{l} \text{in the eq}^n \\ a_n = (b_n)^2 \end{array} \right.$

$$b_n - b_{n-1} - 2\, b_{n-2} = 0$$

2 mins Summary

Topic Solution of recurrence relation using method of characteristic roots

Topic Method of undetermined coefficient for particular solution

Slide

THANK - YOU

Slide