GATE

Computer Science & Information Technology

Discrete Mathematics

DPP: 1

Combinatorics

Q1 Let B_n represent the number of bit strings of length n that contains a substring "00".
Write the recurrence relation for B_n.
(A) $B_n = B_{n-1} + B_{n-2} + 2^{n-2}$
(B) $B_n = B_{n-1} + B_{n-2} - 2^{n-2}$
(C) $B_n = B_{n-1} - B_{n-2} + 2^{n-2}$
(D) $B_n = B_{n-1} - B_{n-2} - 2^{n-2}$

Q2 A population of bacteria has initial count 200. After one hour, the population reached 220. The population grows in such a way that the number of additional bacteria per hour, doubles each hour.
Write a recurrence relation 'b_n' to describe the number of bacteria after n hours.
We know $b_0 = 200$ and $b_1 = 220$
(A) $b_n = 2b_{n-1} + 3b_{n-2}$
(B) $b_n = 2b_{n-1} - 3b_{n-2}$
(C) $b_n = 3b_{n-1} + 2b_{n-2}$
(D) $b_n = 3b_{n-1} - 2b_{n-2}$

Q3 Find the solution of recurrence relation
$a_n = 2a_{n-1} + 3 \quad n \geq 1$
with initial terms $a_0 = 2$
(A) $5.2^n + 3$ (B) $5.2^n - 3$
(C) $3.2^n - 5$ (D) $3.2^n + 5$

Q4 Solve the recurrence relation $a_n = a_{n-1} + 8\, a_{n-2} - 12\, a_{n-3}$, subjected to the initial conditions $a_0 = 0$, $a_1 = 1$, $a_2 = 3$.
(A) $a_n = \left(\frac{1}{25} + \frac{2n}{5}\right)2^n - \left(\frac{2}{25}\right)(-3)^n$
(B) $a_n = \left(\frac{1}{25} + \frac{2n}{5}\right)2^n - \left(\frac{1}{25}\right)(3)^n$
(C) $a_n = \left(\frac{1}{25} - \frac{2n}{5}\right)2^n - \left(\frac{1}{25}\right)(-3)^n$
(D) $a_n = \left(\frac{1}{25} + \frac{2n}{5}\right)2^n - \left(\frac{1}{25}\right)(-3)^n$

Q5 The correct option is/are with respect to the recurrence relation $a_n = 11a_{n-1} - 30\, a_{n-2}$, subjected to the initial conditions $a_0 = 1$, $a_1 = 5$.
(A) The characteristic roots are 0 and 1.
(B) $a_n = 5^n$
(C) The characteristic roots are 5 and 6.
(D) $a_n = 1\text{-}5^n$

Q6 Solve the recurrence relation $a_n + 7a_{n-1} - 18a_{n-2} = 2^n + 1$, subjected to the initial conditions $a_0 = 1$, $a_1 = 2$.
(A) $a_n = \frac{4n}{11}2^{n-1} - \frac{1}{10}$
(B) $a_n = \frac{4n}{11}2^{n-1} + \frac{1}{10}$
(C) $a_n = \frac{4n}{11}2^{n-2} - \frac{1}{10}$
(D) $a_n = \frac{3n}{11}2^{n-1} - \frac{1}{10}$

Q7 Solve the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2} + 3^n$, subjected to the initial conditions $a_0 = 1$, $a_1 = 2$.
(A) $a_n = 3^{n-2}\left[9 + 3n + {}^nC_2\right]$
(B) $a_n = 3^{n-2}\left[9 - 3n - {}^nC_2\right]$
(C) $a_n = 3^{n-2}\left[9 - 3n + {}^nC_2\right]$
(D) $a_n = 3^{n-2}\left[9 + 3n - {}^nC_2\right]$

Q8 Solve the recurrence relation $a_n - 2a_{n-1} + a_{n-2} = n$, subjected to the initial conditions $a_0 = 1$, $a_1 = 2$.
(A) $a_n = 1 + \frac{n}{3} + \left(\frac{n}{6} + \frac{1}{2}\right)n^2$
(B) $a_n = 1 - \frac{n}{3} + \left(\frac{n}{6} + \frac{1}{2}\right)n^2$
(C) $a_n = 1 + \frac{n}{3} - \left(\frac{n}{6} + \frac{1}{2}\right)n^2$
(D) $a_n = 1 + \frac{n}{3} + \left(\frac{n}{6} - \frac{1}{2}\right)n^2$

Q9 Solve the recurrence relation $a_n - 3a_{n-1} - 10a_{n-2} = (2n+1).5^n$, subjected to the initial conditions $a_0 = 1$, $a_1 = 2$.
(A) $a_n = \left(\frac{156}{133}\right)5^n + \frac{23}{133}(-2)^n + 5^n\left(\frac{n}{38} + \frac{5}{38}\right)n$
(B) $a_n = \left(\frac{156}{133}\right)5^n - \frac{23}{133}(-2)^n - 5^n\left(\frac{n}{38} + \frac{5}{38}\right)n$
(C) $a_n = \left(\frac{156}{133}\right)5^n - \frac{23}{133}(-2)^n + 5^n\left(\frac{n}{38} + \frac{5}{38}\right)n$

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(D) $a_n = \left(\frac{156}{133}\right)5^n - \frac{23}{133}(2)^n$
$+ 5^n\left(\frac{n}{38} + \frac{5}{38}\right)n$

Q10 Solve the recurrence relation $a_n = 11a_{n-1} - 24a_{n-2} + 3^n$, subjected to the initial conditions $a_0 = 1$, $a_1 = 2$.

(A) $a_n = \left(\frac{77}{73}\right)3^n - \frac{4}{73}(8)^n - \frac{n}{5}(3)^{n-1}$
(B) $a_n = \left(\frac{77}{73}\right)3^n + \frac{4}{73}(8)^n - \frac{n}{5}(3)^{n-1}$
(C) $a_n = \left(\frac{77}{73}\right)3^n - \frac{4}{73}(8)^n + \frac{n}{5}(3)^{n-1}$
(D) $a_n = \left(\frac{77}{73}\right)3^n - \frac{4}{73}(8)^n - \frac{2n}{5}(3)^{n-1}$

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Answer Key

Q1	(A)
Q2	(D)
Q3	(B)
Q4	(D)
Q5	(B, C)
Q6	(A)
Q7	(C)
Q8	(A)
Q9	(C)
Q10	(A)

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Hints & Solutions

Q1 Text Solution:

$B_n = B_{n-1} + B_{n-2} + 2^{n-2}$, $n \geq 2$

$B_0 = 0$

$B_1 = 0$

$B_2 = 1$

Q2 Text Solution:

$b_1 - b_0 = 20$

$b_2 - b_1 = 40$

$b_3 - b_2 = 80$

$\vdots$

$b_n - b_{n-1} = 2(b_{n-1} - b_{n-2})$

$b_n = 3b_{n-1} - 2b_{n-2}$, $n \geq 2$

Q3 Text Solution:

$a_n = 2a_{n-1} + 3$

$= 2(2a_{n-2} + 3) + 3$

$= 2(2(2a_{n-2} + 3) + 3) + 3$

$\vdots$

$= 2^k a_{n-k} + 2^{k-1}.3 + 2^{k-2}.3 + ... + 3$

Let $n - k = 0$

$\Rightarrow n = k$

$2^n \times 2 + 3(2^n - 1)$

$= 2^n(2 + 3) - 3 = 5.2^n - 3$

Q4 Text Solution:

$a_n = a_{n-1} + 8a_{n-2} - 12a_{n-3}$

Characteristic equation is,

$t^3 - t^2 - 8t + 12 = 0$

$(t-2)^2(t+3) = 0$

Complementary function,

$a_n = (c_1 + c_2 n)2^n + c_3(-3)^n$

$a_0 = c_1 + c_3$

$0 = c_1 + c_3 \Rightarrow c_3 = -c_1$

$a_1 = (c_1 + c_2).2 + c_3.(-3)$

$\Rightarrow 1 = 2c_1 + 2c_2 - 3c_3$

$\Rightarrow 1 = 5c_1 + 2c_2$ ...(i)

$a_2 = (c_1 + 2c_2).4 + c_3.9$

$\Rightarrow 3 = 4c_1 + 8c_2 + 9c_3$

$\Rightarrow 3 = -5c_1 + 8c_2$ ...(ii)

using (i) and (ii)

$c_1 = \frac{1}{25}, c_2 = \frac{2}{5}, c_3 = \frac{-1}{25}$

$a_n = \left(\frac{1}{25} + \frac{2n}{5}\right)2^n - \left(\frac{1}{25}\right)(-3)^n$

Q5 Text Solution:

$a_n = 11a_{n-1} - 30\, a_{n-2}$

$t^2 - 11t + 30 = 0$

$(t-6)(t-5) = 0$

$t = 6, 5$

CF is

$a_n = C_1(6)^n + C_2(5)^n$

$a_0 = C_1 + C_2$

$1 = C_1 + C_2$ ...(i)

$a_1 = 6C_1 + 5C_2$

$5 = 6C_1 + 5C_2$ ...(ii)

$5 = 6C_1 + 5(1 - C_1)$

$5 = C_1 + 5$

$C_1 = 0$

Then, $C_2 = 1$

$a_n = 5^n$

Q6 Text Solution:

Put n = n+2

$a_{n+1} + 7a_{n+1} - 18a_n = 2^{n+2} + 1$

$\phi(E) = E^2 + 7E - 18$

$\Rightarrow \phi(t) = t^2 + 7t - 18$

$\Rightarrow t = 2, -9$

$a_n^{(H)} = C_1(2)^n + C_2(-9)^n$

Now, $f(n) = 2^n + 1$

$F(n) = 2^{n+2} + 1 = 4(2^n) + 1$

$PS = \text{Sol}^n \text{ of } \left(\frac{2^{n+2}+1}{(t-2)(t+9)}\right)$

$= 4\left(\frac{2^n}{(t-2)(t+9)}\right) + \frac{1^n}{(t-2)(t+9)}$

$= 4 \times \frac{1}{2+9} \times \left({}^nC_1 2^{n-1}\right) + \frac{1^n}{(-1).(10)}$

$a_n = \frac{4n}{11}2^{n-1} - \frac{1}{10}$

Q7 Text Solution:

$a_n - 6a_{n-1} + 9a_{n-2} = 3^n$

$\phi(E) = E^2 - 6E + 9$

$\phi(t) = t^2 - 6t + 9$

$\Rightarrow t = 3, 3$

$a_n^{(H)} = (C_1 + C_2 n)3^n$

$F(n) = 3^{n+2} = 9.3^n$

$a_n^{(P)} = 9.\ \text{Sol of}\left\{\frac{3^n}{\phi(E)}\right\}[b = 3]$

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$\phi(b) = \phi(3) = 0$

$\phi(E) = (E - b)^m . \psi(E)$

$\Rightarrow (E - 3)^2 = (E - 3)^2 . \psi(E)$

$\Rightarrow \psi(E) = 1$

$PS = {}^nC_2 (3)^{n-2}$

$a_n = a_n^{(H)} + a_n^{(P)}$

$a_n = (C_1 + C_2 n)3^n + {}^nC_2 3^{n-2}$

$C_1 = 1, C_2 = \frac{-1}{3}$

$a_n = \left(1 - \frac{n}{3}\right)3^n + {}^nC_2 3^{n-2}$

$a_n = (3 - n)3^{n-1} + {}^nC_2 3^{n-2}$

$a_n = 3^{n-2}\left[9 - 3n + {}^nC_2\right]$

Q8 Text Solution:

$\phi(E) = E^2 - 2E + 1$

$\Rightarrow \phi(t) = t^2 - 2t + 1$

$\Rightarrow t = 1, 1$

$a_n^{(H)} = C_1 + C_2 n$

$f(n) = n = 1^n (n)$

$\phi(1) = 0$

$a_n^{(P)} = 1^n (An + B)n^2 = (An + B)n^2$

$(An + B)n^2$

$- 2\left\{(A(n - 1) + B)(n - 1)^2\right\}$

$+ \left[\{A(n - 2) + B\}(n - 2)^2\right] = n$

Put n = 0, –3A + B = 0

Put n = 1, 2B = 1

$A = \frac{1}{6}, B = \frac{1}{2}$

$a_n^{(P)} = \left(\frac{n}{6} + \frac{1}{2}\right)n^2$

$a_n = C_1 + C_2 n + \left(\frac{n}{2} + \frac{1}{6}\right)n^2$

For a_0, $C_1 = 1$

For a_1, $C_2 = 1/3$

$a_n = 1 + \frac{n}{3} + \left(\frac{n}{6} + \frac{1}{2}\right)n^2$

Q9 Text Solution:

$\phi(E) = E^2 - 3E - 10$

$\phi(t) = t^2 - 3t - 10$

$\Rightarrow t = 5, -2$

$a_n^{(H)} = C_1(5)^n + C_2(-2)^n$

$f(n) = 5^n (2n + 1)$

$\phi(b) = 0$

$a_n^{(P)} = 5^n (An + b)n$

$\{5^n (An + b)n\} \qquad - \left[10.5^{n-}\right.$

$+ \left[3.5^{n-1}\{A(n - 1) + B\}.(n - 1)\right]$

Put n = 0, –5A + B = 0

Put n = 1, 3A + 7B = 1

$A = \frac{1}{38}, B = \frac{5}{38}$

$a_n = C_1(5)^n + C_2(-2)^n + 5^n (An + B)n$

$a_n = \left(\frac{156}{133}\right)5^n - \frac{23}{133}(-2)^n + 5^n\left(\frac{n}{38} + \frac{5}{38}\right)n$

Q10 Text Solution:

$\phi(E) = E^2 - 11E + 24$

$\phi(t) = t^2 - 11t + 24$

$\Rightarrow t = 3, 8$

$a_n^{(H)} = C_1(3)^n + C_2(8)^n$

$F(n) = 3^{n+2} = 9.3^n$

$a_n^{(P)} = 9 \text{ . Sol of} \left\{\frac{3^n}{\phi(E)}\right\}$

$\phi(3) = 0$

$\phi(E) = (E - 3)\psi(E)$

$\Rightarrow (E - 3)(E - 8) = (E - 3)\psi(E)$

$\Rightarrow \psi(E) = E - 8$

$\psi(3) = -5$

$a_n^{(P)} = \frac{-1}{5}\left\{{}^nC_1 3^{n-1}\right\}$

$a_n = C_1(3)^n + C_2(8)^n - \frac{n}{5}3^{n-1}$

For a_0, $1 = C_1 + C_2$

For a_1, $2 = 3C_1 + 8C_2 - \frac{1}{5}$

$C_1 = \frac{77}{73}, C_2 = \frac{-4}{73}$

$a_n = \left(\frac{77}{73}\right)3^n - \frac{4}{73}(8)^n - \frac{n}{5}(3)^{n-1}$

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