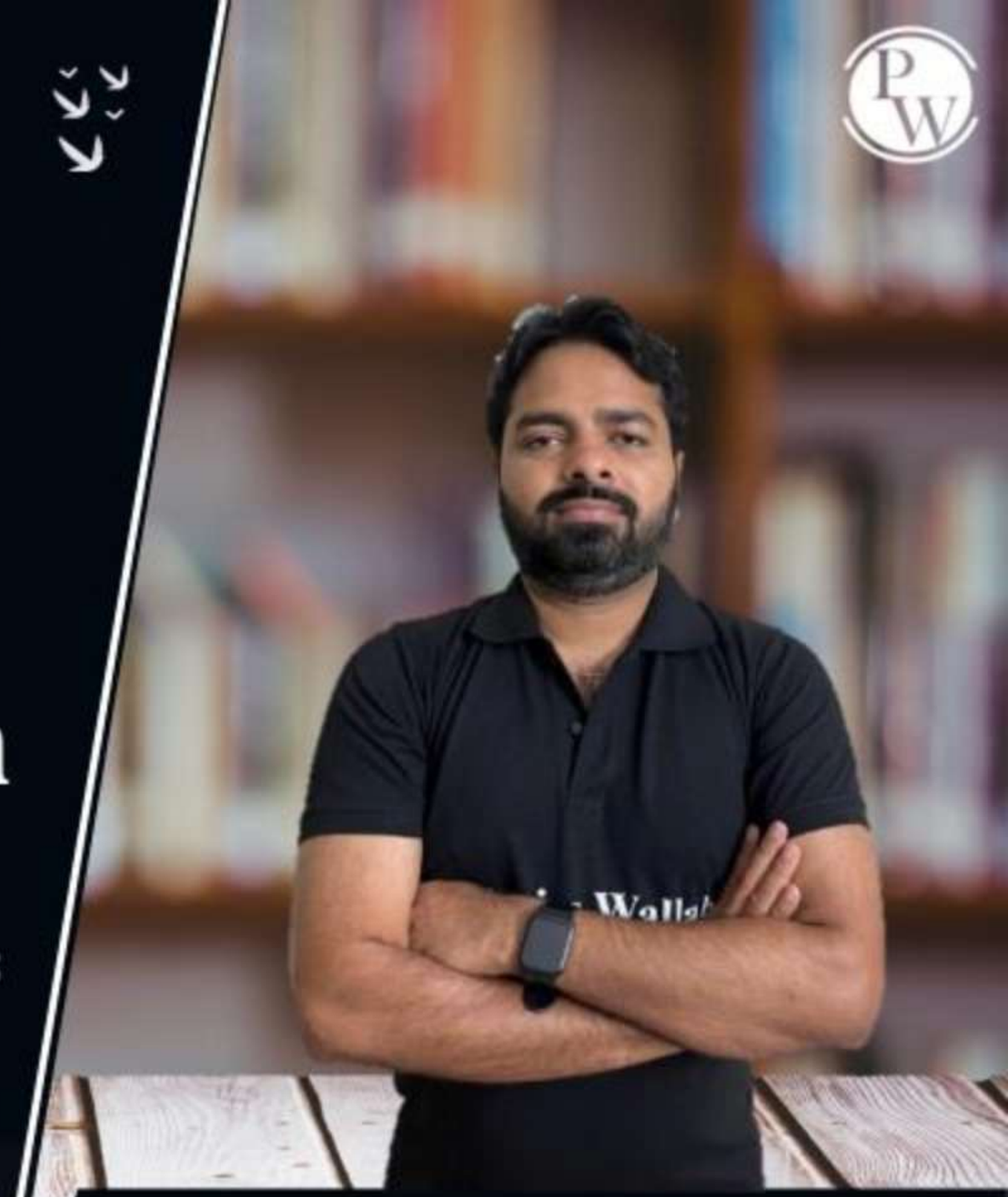
Computer Science

Theory of Computation

Regular Languages and Non-regular Languages

Lecture No.- 2



Mallesham Devasane Sir

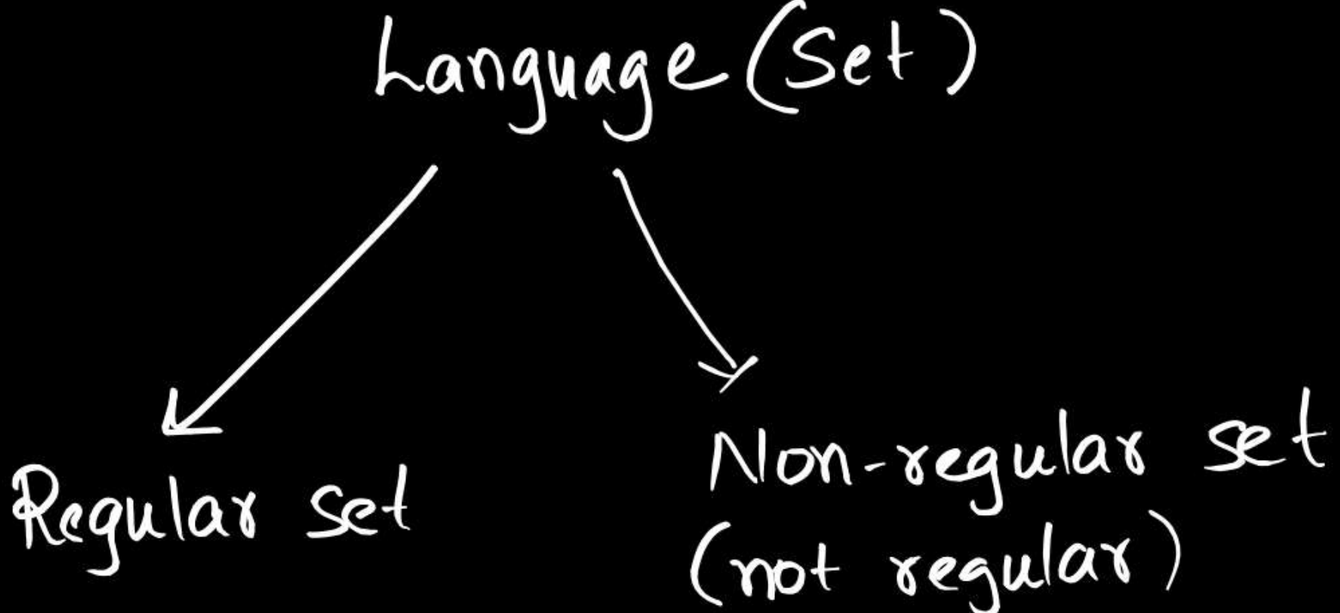
Recap of Previous Lecture

Topic	DFA Vs NFA
Topic	NFA Construction
Topic	Conversion from NFA to DFA
Topic	NFA with epsilon Moves

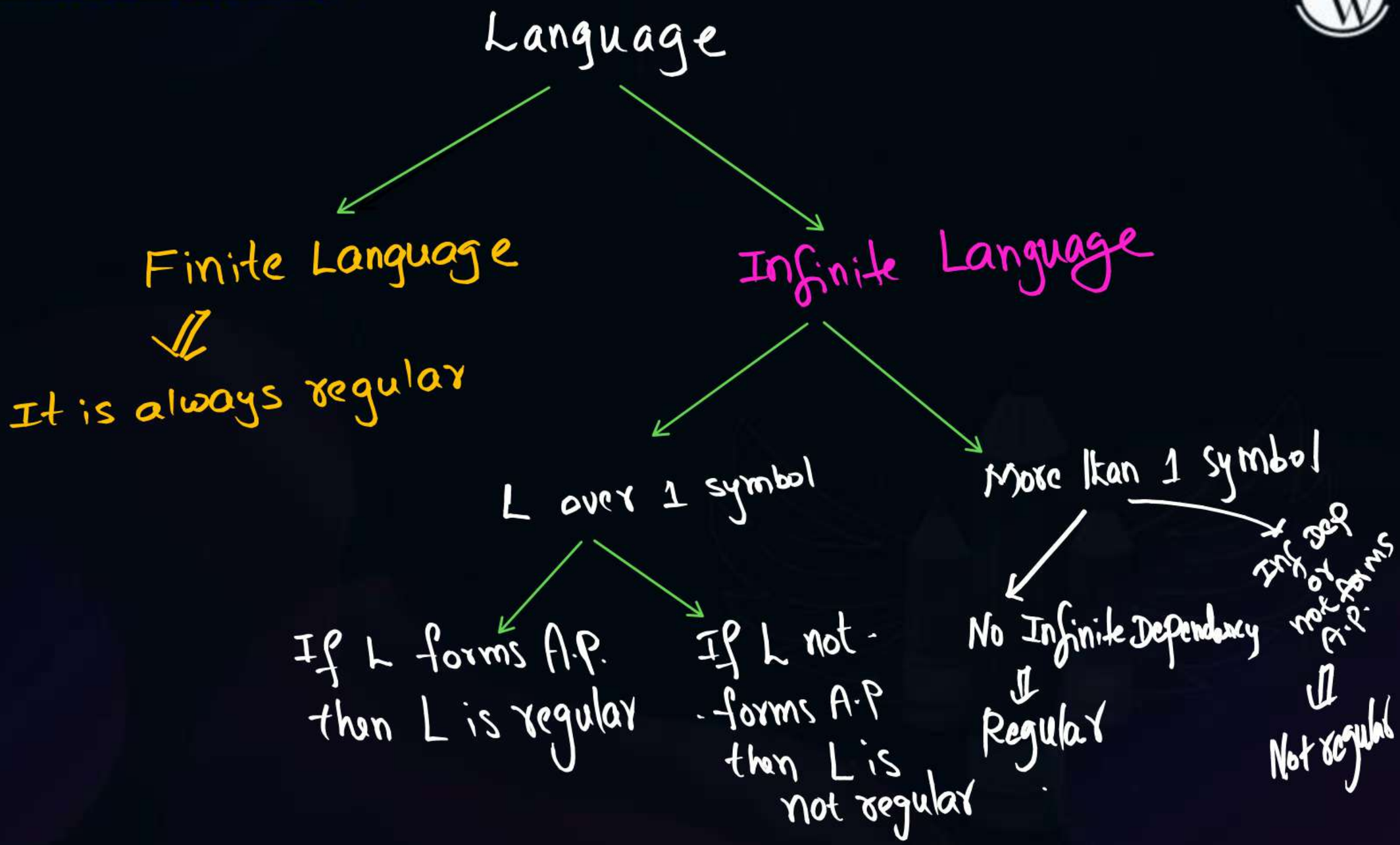
Topics to be Covered

Topic Regular Languages

Topic Non-regular Languages


Language (Set)
Regular set
Non-regular set
(not regular)

Identification of Regulars



$\{a^* b^*\} = \{a^m b^n\}$

No Infinite dependency

⇓

Regular

$\{a^n b^n\}$

$= \{\varepsilon, ab, a^2b^2, a^3b^3, \ldots\}$

Infinite Dependency

⇓

Not regular

$\{a^{2n} \mid n \geq 0\}$ is Regular

$\Downarrow$

$\varepsilon, a^2, a^4, a^6, \ldots$

$0, 2, 4, 6, \ldots$

A.P. series

$\{a^{2^n} \mid n \geq 0\}$ is not regular

$\Downarrow$

$a, a^2, a^4, a^8, \ldots$

$1, 2, 4, 8, \ldots$

not A.P. series

$a^{2^0} = a^1$

$a^{2^1} = a^2$

$a^{2^2} = a^4$

$a^{2^3} = a^8$

Note:

I) Every finite language is regular.

II) Regular language is either finite or infinite

III) Infinite language is either regular or not regular

Identification of Regulars

✓ ① $\{a^m b^n \mid m,n \geq 0\} = a^* b^*$

$m \geq 0, n \geq 0$

× ② $\{a^m b^n \mid m = n\} = \{a^n b^n\}$

✓ ③ $\{a^m b^n \mid m = n,\ m < 3\} = \{a^0 b^0, a^1 b^1, a^2 b^2\}$

Finite Dependency $= \varepsilon + ab + aabb$

× ④ $\{a^m b^n \mid m < n,\ m, n \geq 0\} = \{a^0 b^1, a^0 b^2, a^1 b^2, a^0 b^3, a^1 b^3, a^1 b^4, \ldots\}$

$0<1$, $0<2$, $1<2$, $0<3$

✓ ⑤ $\{a^m b^n \mid m < n < 10\}$

Finite Dep

✓ Regular

× Not regular

Identification of Regulars

✓ (6) $\{a^m b^n \mid m=n=10\} = \{a^{10} b^{10}\}$

✗ (7) $\{a^m b^n \mid m>n>5\}$

✓ (8) $\{a^m b^n \mid m>n,\ m<5\}$ Finite set

$n<m<5$

✓ (9) $\{a^m b^n \mid m=1,\ n=\text{even}\} = a^1 b^{\text{even}} = a(bb)^*$

✓ (10) $\{a^m b^n \mid m=\text{even},\ n=\text{odd}\} = a^{\text{even}} b^{\text{odd}} = (aa)^* b(bb)^*$

Identification of Regulars

✓ (11) $\{a^m b^n \mid m=\text{odd}, n=\text{odd}\}$

✓ (12) $\{a^m b^n \mid m+n=\text{even}\} = \underbrace{a^{\text{even}}\, b^{\text{even}}}_{\text{even}} + \underbrace{a^{\text{odd}}\, b^{\text{odd}}}_{\text{even}}$

✓ (13) $\{a^m b^n \mid m+n=\text{odd}\}$

✗ (14) $\{a^n b^{2n}\} = \{a^n b^{2n} \mid n \geq 0\}$

✗ (15) $\{a^m b^n \mid \underbrace{m=n}_{\text{Inf Dep}}=\text{even},\ m,n \geq 0\}$

$m+n=\text{even}$

I) $m=\text{even}, n=\text{even}$

II) $m=\text{odd}, n=\text{odd}$

$\underbrace{m+n=\text{odd}}_{\text{I)}}$

$a^m b^n$ |

$m = n =$ even

$a^0 b^0$ ✓

$a^2 b^2$ ✓

$a^4 b^4$

$a^6 b^6$

m = even, n = even

$a^{\text{even}} b^{\text{even}}$

$$\{a^m b^n \mid m+n = \text{even}\}$$

(+ : addition)

even + even
odd + odd

$$= a^{\text{even}}\, b^{\text{even}} + a^{\text{odd}}\, b^{\text{odd}}$$

$$= (aa)^*(bb)^* + a(aa)^* b(bb)^*$$

$m+n$

even + even = even ✓

even + odd = odd

odd + even = odd

odd + odd = even ✓

Identification of Regulars

✓ (16) $\{a^m b^n \mid LCM(m,n) = 1\} = \{a^1 b^1\}$

$LCM(m, n) = 1$

(m, n) ↓ ↓ (1, 1)

✗ (17) $\{a^m b^n \mid GCD(m,n) = 1\}$

dependency

$GCD(m,n) = 1$

1, 2
1, 3
2, 3
2, 2

Identification of Regulars

18) $\{ a^m b^n \mid \text{if } (m = \text{even}) \text{ then } (n = \text{odd}) \}$

I) $m = \text{even} \Rightarrow n = \text{odd}$
II) $m \neq \text{even} \Rightarrow n$ is anything

A) $a^{\text{even}}\, b^{\text{odd}}$

B) $a^{\text{even}}\, b^{\text{even}}$

C) $a^{\text{even}}\, b^{\text{odd}} + a^{\text{odd}}\, b^{\text{any}}$

D) $a^{\text{even}}\, b^{\text{odd}} + a^{\text{odd}}\, b^{\text{odd}}$

$(aa)^* b(bb)^* + a(aa)^* b^*$

Identification of Regulars

✓ (19) $\{a^m b^n c^k\} = \{a^m b^n c^k \mid m \geq 0, n \geq 0, k \geq 0\} = a^* b^* c^*$

$m, n, k \geq 0$

X (20) $\{a^n b^n c^n\}$

Identification of Regulars

✓ (21) $\{a^n\}$

✓ (22) $\{a^{2n}\}$

✓ (23) $\{a^{3n+2}\}$

✓ (24) $\{a^{1000n+354}\}$

✓ (25) $\{a^{100n}\}$

Forms A.P. series

$n \Rightarrow 0, 1, 2, \cdots$

$2n \Rightarrow 0, 2, 4, 6, \cdots$

$3n+2 \Rightarrow 2, 5, 8, 11, \cdots$

Identification of Regulars

(26) $\{a^{\text{prime}}\} = \{a^2, a^3, a^5, a^7, a^{11}, \ldots\}$ ✗

(27) $\{a^{n^2}\} = \{a^0, a^1, a^4, a^9, a^{16}, \ldots\}$ ✗

(28) $\{a^{n^5}\} = \{a^0, a^1, a^{32}, \ldots\}$ ✗

(29) $\{a^{2^n}\} = \{a^1, a^2, a^4, a^8, \ldots\}$ ✗ ***

(30) $\{a^{100^n}\}$ ✗

(31) $\{a^{n!}\} = \{a^1, a^2, a^6, a^{24}, \ldots\}$ ✗

(32) $\{a^{n^n}\} = \{a^1, a^4, a^{27}, \ldots\}$ ✗

(33) $\{a^{m^n}\} = \{a^0, a^1, a^2, a^3, a^4, \ldots\} = a^*$ ✓

Identification of Regulars

(34) $\{a^{\text{prime}}\}^* = \{a^2, a^3, a^5, a^7, a^{11}, \ldots\}^*$

$= \{\varepsilon, a^2, a^3, a^4, a^5, a^6, \ldots\}$

$\{\}^0 \quad \{a^2\}^1 \quad \{a^3\}^1$

$\{\}^2 = \{a^2\}^1 \{a^2\}^1$

$= \{a^n \mid n \neq 1\} = \varepsilon + aaa^*$

Identification of Regulars

✓ (35) $\{a^n \mid n=2 \text{ or } 3\}^* = (aa+aaa)^* = \{\varepsilon, a^2, a^3, a^4, a^5, \ldots\} \Rightarrow$ (34)

✓ (36) $\{a^{2n} \mid n \geq 0\}^* = \{\varepsilon, a^2, a^4, a^6, \ldots\}^* = (aa)^* = \{a^{2n}\}$

✓ (37) $\{a^{n^2} \mid n \geq 0\}^* = \{a^1\}^* \cup \underbrace{\ldots\ldots}_{\text{remaining}} = a^*$ ($n=1$)

✓ (38) $\{a^{2^n} \mid n \geq 0\}^* = \{a^{2^0}\}^* \cup \ldots\ldots = a^*$

✓ (39) $\{a^{n!}\}^* = a^*$

$(aa+aaa)^*$

$\Downarrow$

How a^5 generates?

$(\quad)^2 = (aa)'(aaa)'$

$(x+y)^*$

$\Downarrow$

xy

$(\quad)^2 = (x)'(y)'$

Identification of Regulars

(40) $\{a^n b^{n+1}\} = \{a^n b^n b\}$

(41) $\{a^{n+2} b^{n+3}\} = \{a^2 a^n b^n b^3\}$

(42) $\{a^{n+i} b^{n+j} \mid n, i, j \geq 0\} = \{a^* b^*\} \cup \underset{n>0}{\cdots} = a^* b^*$

$\{a^* b^*\}$ at $n=0$

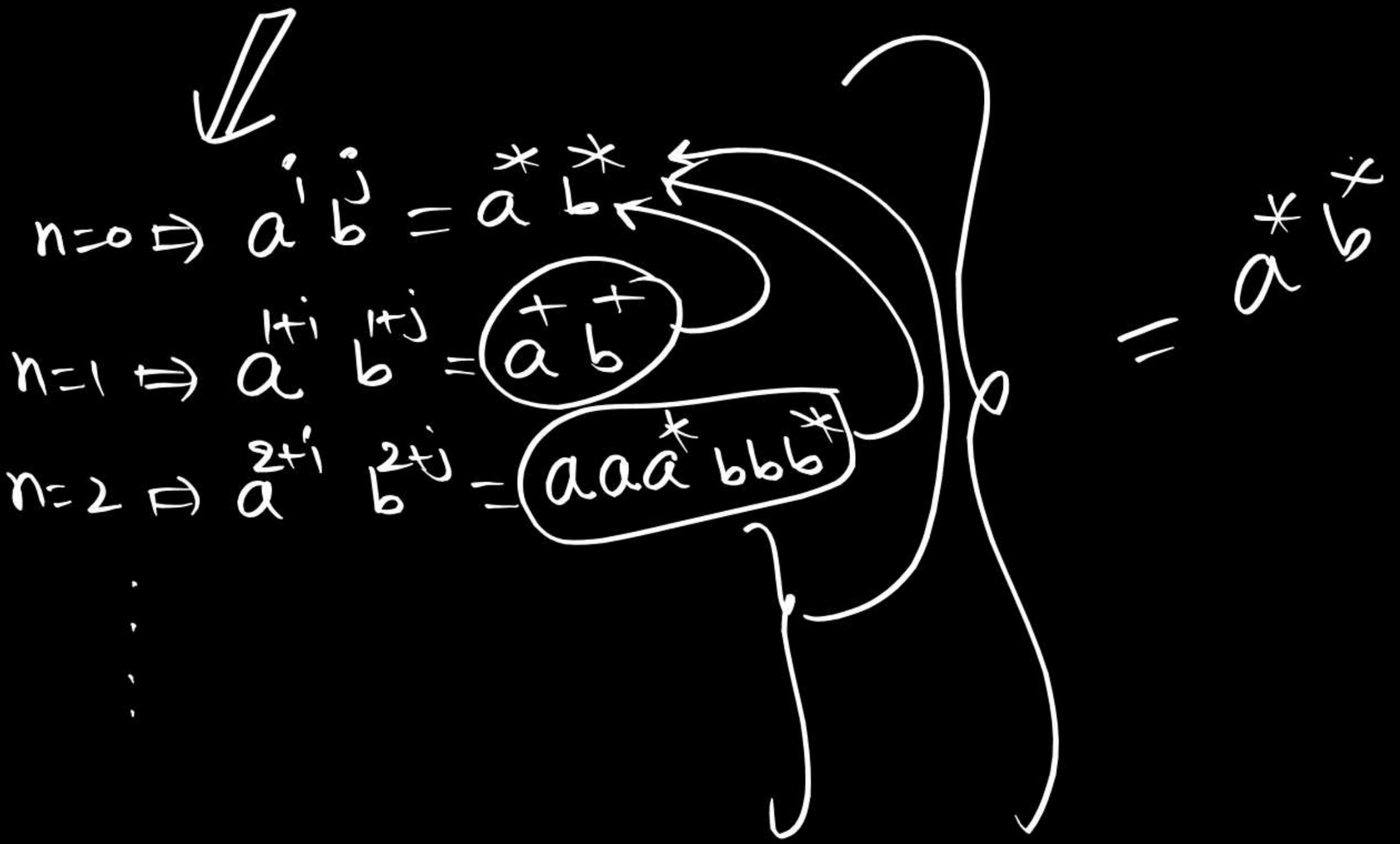
(43) $\{a^n b^{n^2}\}$

$b^{n^2} \rightarrow$ not R.P.

$a^k b \rightarrow$ Dependent

(44) $\{a^{2n} b^{k^2} \mid n, k \geq 0\}$

$$\{a^{n+i}\ b^{n+j}\}$$


$n=0 \Rightarrow a^i b^j = a^* b^*$
$n=1 \Rightarrow a^{1+i} b^{1+j} = a^+ b^+$
$n=2 \Rightarrow a^{2+i} b^{2+j} = aaa^* bbb^*$
$= a^* b^*$

H.W.:

(45) $\{ww \mid w \in \{a,b\}^*\}$

(46) $\{ww^R \mid w \in \{a,b\}^*\}$

(47) $\{w\#w \mid w \in \{a,b\}^*\}$

(48) $\{w\#w^R \mid$ " $\}$

(49) $\{ww \mid w \in a^*\}$

(50) $\{w\#w \mid w \in a^*\}$

w^R : Reverse of w

If $w = abb$.
then $w^R = bba$

2 mins Summary

Topic Regular Languages

Topic Non-regular Languages

THANK - YOU