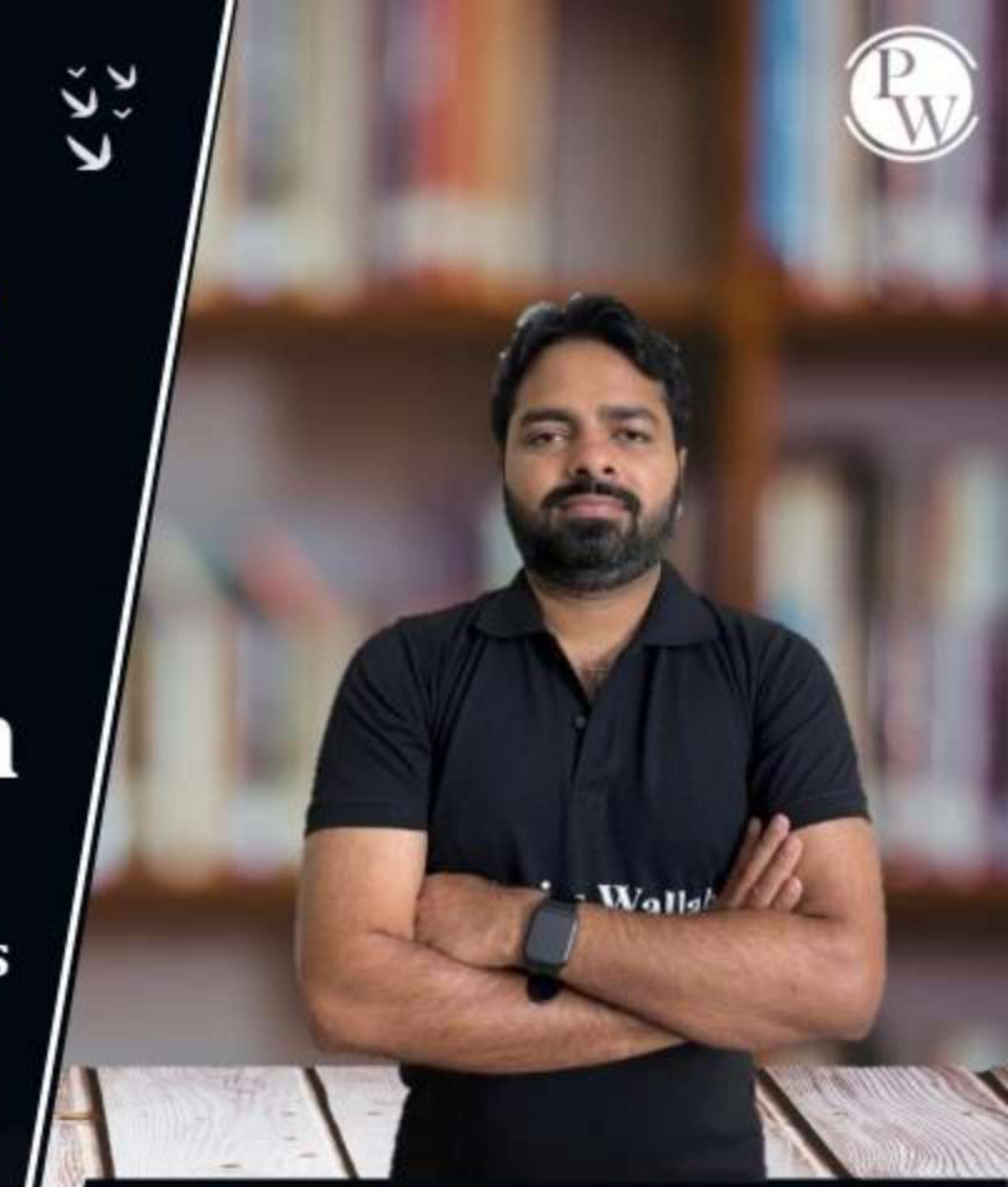
Computer Science

Theory of Computation

Regular Languages and Non-regular Languages

Lecture No.- 3



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Recap of Previous Lecture

- **Topic** — DFA Vs NFA
- **Topic** — NFA Construction
- **Topic** — Conversion from NFA to DFA
- **Topic** — NFA with epsilon Moves

Topics to be Covered

Topic	**Regular Languages**
Topic	**Non-regular Languages**

H.W.:

(45) $\{ww \mid w \in \{a,b\}^*\}$

(46) $\{ww^R \mid w \in \{a,b\}^*\}$

(47) $\{w\#w \mid w \in \{a,b\}^*\}$

(48) $\{w\#w^R \mid$ " $\}$

(49) $\{ww \mid w \in a^*\}$

(50) $\{w\#w \mid w \in a^*\}$

w^R : Reverse of w

If $w = abb$.
then $w^R = bba$

Identification of Regulars

(45) $\{ww \mid w \in \{a,b\}^*\} = \{\varepsilon, aa, bb, aaaa, abab, baba, bbbb, \ldots\}$

$$\underbrace{aabab}_{w}\,\underbrace{aabab}_{w}$$

46 $\{ww^R \mid w \in \{a,b\}^*\} = \{\varepsilon, aa, bb, aaaa, abba, \ldots\}$

$=$ Set of all even length palindromes

$\underbrace{aabba}_{w}\underbrace{abbaa}_{w^R}$

ww^R

$\varepsilon\varepsilon = \varepsilon$

$a\,a$

$b\,b$

$aa\,aa$

$ab\,ba$

$ba\,ab$

$bb\,bb$

$\vdots$

$w = ab$

$w^R = ba$

47 $\{w\#w \mid w \in \{a,b\}^*\}$

$\times$

$= \{\#, a\#a, b\#b, aa\#aa, ab\#ab, \ldots\}$

$\Sigma = \{\#, a, b\}$

$abaa \# abaa$

48 $\{w\#w^R \mid w \in \{a,b\}^*\}$

X

aab # baa

(49) $\{ww \mid w \in a^*\} = \{ww^R \mid w \in a^*\}$

$\boxed{w = w^R}$ over 1 symbol

ww^R

ww

$$\left.\begin{array}{l} \varepsilon\varepsilon = \varepsilon \\ a.a = a^2 \\ aaaa = a^4 \\ aaaaaa = a^6 \\ \vdots \end{array}\right\} = a^{2n} = a^{even} = (aa)^*$$

50 $\{w\#w \mid w \in a^*\} = \{\#, a\#a, a^2\#a^2, \ldots\}$

✗

$= \{a^n\#a^n\}$

(51) $\{w_1 w_2 \mid w_1, w_2 \in \{a,b\}^*\} = (a+b)^*$

$w_1 w_2$

$\varepsilon w_2 = w_2 = (a+b)^*$

$\vdots$

Identification of Regulars

(52) $\{ \underset{\varepsilon}{w} \underset{\text{any}}{\#} \underset{\varepsilon}{w} \mid w, \# \in \{a,b\}^* \} = (a+b)^*$

$$\underbrace{\qquad\text{any}\qquad}$$

$= (a+b)^*$

Identification of Regulars

(53) $\{w_1 w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = |w_2|\}$ ✓

$= ((a+b)^2)^*$

$w_1 \; w_2$

$$
\text{all even length}
\begin{cases}
\varepsilon \;\; \varepsilon = \varepsilon \longrightarrow 0 \text{ len} \\
\left.\begin{matrix} a \;\; a \\ a \;\; b \\ b \;\; a \\ b \;\; b \end{matrix}\right\} \text{all } 2 \text{ length} \\
2\text{len} \;\; 2\text{len} \longrightarrow \text{all } 4 \text{ len} \\
\vdots
\end{cases}
$$

Identification of Regulars

54. $\{ww x \mid w, x \in \{a,b\}^*\} = x$
55. $\{w x w \mid \text{''} \} = x$
56. $\{x w w \mid \text{''} \} = x$
57. $\{w w^R x \mid \text{''} \} = x$
58. $\{w x w^R \mid \text{''} \} = x$
59. $\{x w w^R \mid \text{''} \} = x$

$= (a+b)^*$

Identification of Regulars

× (60) $\{wwx \mid w, x \in \{a,b\}^+\}$ $\xrightarrow{w=a/ab}$ $axa + bxb$

× (61) $\{wxw \mid$ " $\}$ $\xrightarrow{w=aa/ab/ba/bb}$ $aaxaa + abxab + \dots$

× (62) $\{xww \mid$ " $\}$ → $xaa + xbb$ → $xabab + \dots$

× (63) $\{ww^Rx \mid$ " $\}$ → $aax + bbx$ → $aaaax + abbax + \dots$

*** (64) $\{wxw^R \mid$ " $\}$ $= a(a+b)^+a + b(a+b)^+b$

× (65) $\{xww^R \mid$ " $\}$ $\xrightarrow{|w|=1}$ $xaa + xbb$ $\xrightarrow{|w|=2}$ $xaaaa + xabba + \dots$

Identification of Regulars

60) $wwx \mid w,x \in \{a,b\}^+$

$w=a \Rightarrow$ new $aa(a+b)^+$

$w=b \Rightarrow$ new $bb(a+b)^+$

$w=aa \Rightarrow \boxed{aaaa(a+b)^+}$ already

$w=ab \Rightarrow abab(a+b)^+$ new

$w=ba \Rightarrow$ new $baba(a+b)^+$

$w=bb \Rightarrow \boxed{bbbb(a+b)^+}$ already covered

⋮

64) $wxw^R \mid w,x \in \{a,b\}^+$

$w=a \Rightarrow a(a+b)^+a$ new

$w=b \Rightarrow b(a+b)^+b$ new

$w=aa \Rightarrow aa(a+b)^+aa$ covered

$w=ab \Rightarrow abxba$ covered

$w=ba \Rightarrow baxab$

$w=bb \Rightarrow bbxbb$

Identification of Regulars

(61) $\{wxw \mid w, x \in \{a,b\}^+\}$

$w = a \Rightarrow a(a+b)^+a$

$w = b \Rightarrow b(a+b)^+b$

$w = aa \Rightarrow aaxaa$ ✓

$= ab \Rightarrow abxab$ ✗

$= ba$ ⋮

$= bb$ ⋮

(64) $wxw^R \mid w, x \in \{a,b\}^+$

$w = a \Rightarrow \underset{\text{new}}{a}(a+b)^+a$

$w = b \Rightarrow \underset{\text{new}}{b}(a+b)^+b$

$w = aa \Rightarrow aa(a+b)^+aa$ covered

$w = ab \Rightarrow abxba$ covered

$w = ba \Rightarrow baxab$

$w = bb \Rightarrow bbxbb$

Identification of Regulars

66. $\{ wwx \mid w, x \in \{a,b\}^+ ,\ |w| = 2 \}$
67. $\{ wxw \mid$ " , " $\}$
68. $\{ xww \mid$ " , " $\}$
69. $\{ ww^R x \mid$ " , " $\}$
70. $\{ wxw^R \mid$ " , " $\}$
71. $\{ xww^R \mid$ " , " $\}$

Identification of Regulars

✗ 72 $\{ wwx \mid w, x \in \{a,b\}^+ , |x| = 2 \}$

✗ 73 $\{ wxw \mid$ " , " $\}$

✗ 74 $\{ xww \mid$ " , " $\}$

✗ 75 $\{ ww^R x \mid$ " , " $\}$

✗ 76 $\{ wxw^R \mid$ " , " $\}$

✗ 77 $\{ xww^R \mid$ " , " $\}$

(66) $\{wwx \mid w, x \in \{a,b\}^+, |w| = 2\}$

$= \{aaaax, ababx, babax, bbbbx\}$

$= (aaaa + abab + baba + bbbb)\, x$

$= (aaaa + abab + baba + bbbb)(a+b)^+$

reg

(72) $\{wwx \mid w, x \in \{a,b\}^+, |x| = 2\}$

$= \{\underbrace{wwaa}_{I}, \underbrace{wwab}_{I}, \underbrace{wwba}_{I}, \underbrace{wwbb}_{I}\}$

not reg

Identification of Regulars

✓ 78 $\{wxwy \mid w, x, y \in \{a,b\}^+\} = axay + bxby$

✓ 79 $\{xwyw \mid$ " $\} = xaya + xbyb$

✓ 80 $\{xww^Ry \mid$ " $\} = xaay + xbby$

$\boxed{x, y \in \{a,b\}^+}$

W W

put min w

$|w|=1$

form

next min of w

$|w|=2$

form

$|w|=3$

. . .

Identification of Regulars

(81) $\{xwwy \mid x, w, y \in \{a,b\}^+\}$

$= \boxed{xaay + xbby + xababy + xbabay}$

$|w| = 1$

new $xaay$

new $xbby$

$|w| = 2$

$\cancel{xaaaay}$

$xababy$ new

$xbabay$ new

$\cancel{xbbbby}$

$|w| = 3$ (aaa, aab, aba, abb, baa, bab, bba, bbb)

$\cancel{xaaaaaay}$

$\cancel{xaabaaby}$

$\cancel{xabaabay}$

$\cancel{xabbabby}$

$\cancel{xbaabaay}$

$\cancel{xbabbaby}$

$\cancel{xbbabbay}$

$\cancel{xbbbbbby}$

× (82) $\{a^n b^{2n} c^{3n}\}$

✓ (83) $\{ww \mid w \in \{a,b\}^*, |w| < 100\}$

✓ (84) $\{w\#w \mid w \in \{a,b\}^*, |w| < 100\}$

✓ (85) $\{ww^R \mid$ " " $\}$

✓ (86) $\{w\#w^R \mid$ " " $\}$

Finite language

H.W.

✓ 87 $\{w \mid w \in \{0,1\}^*,\ \text{Decimal}(w) \text{ is divisible by } \overbrace{2^9 \ldots 2^0}\ \underbrace{1024}_{2^{\boxed{10}}}\} \Rightarrow 10+1 = 11 \text{ states}$

✓ 88 $\left\{w \mid w \in \{0,1\}^*,\ \text{every prefix } s \text{ of } w \text{ satisfies } |n_0(s) - n_1(s)| \le 2\right\}$

✗ 89 $\{w \mid w \in \{0,1\}^*,\ |n_0(w) - n_1(w)| \le 2\}$

✓ 90 $\{w \mid w \in \{0,1\}^*,\ n_{\underline{01}}(w) = n_{\underline{10}}(w)\}$

✗ 91 $\{w \mid w \in \{0,1\}^*,\ n_{00}(w) = n_{11}(w)\}$

✓ 92 $\{w \mid w \in \{0,1\}^*,\ n_{\underline{001}}(w) = n_{\underline{100}}(w)\}$

✗ 93 $\{w \mid w \in \{0,1\}^*,\ n_{000}(w) = n_{001}(w)\}$

(90) $\{w \mid w \in \{0,1\}^*,\ \underbrace{n_{01}(w) = n_{10}(w)}\}$

No. of 01's = no. of 10's

	#01's	#10's	=
ε	0	0	✓
0	0	0	✓
1	0	0	✓
00	0	0	✓
01	1	0	×
10	0	1	×
11	0	0	✓
000	0	0	✓
001	1	0	×
010	1	1	✓
011			
100			
101			
110			
111			

01010 → two 01's, two 10's

valid

2 mins Summary

Topic — Regular Languages ✓

Topic — Non-regular Languages ✓

THANK - YOU