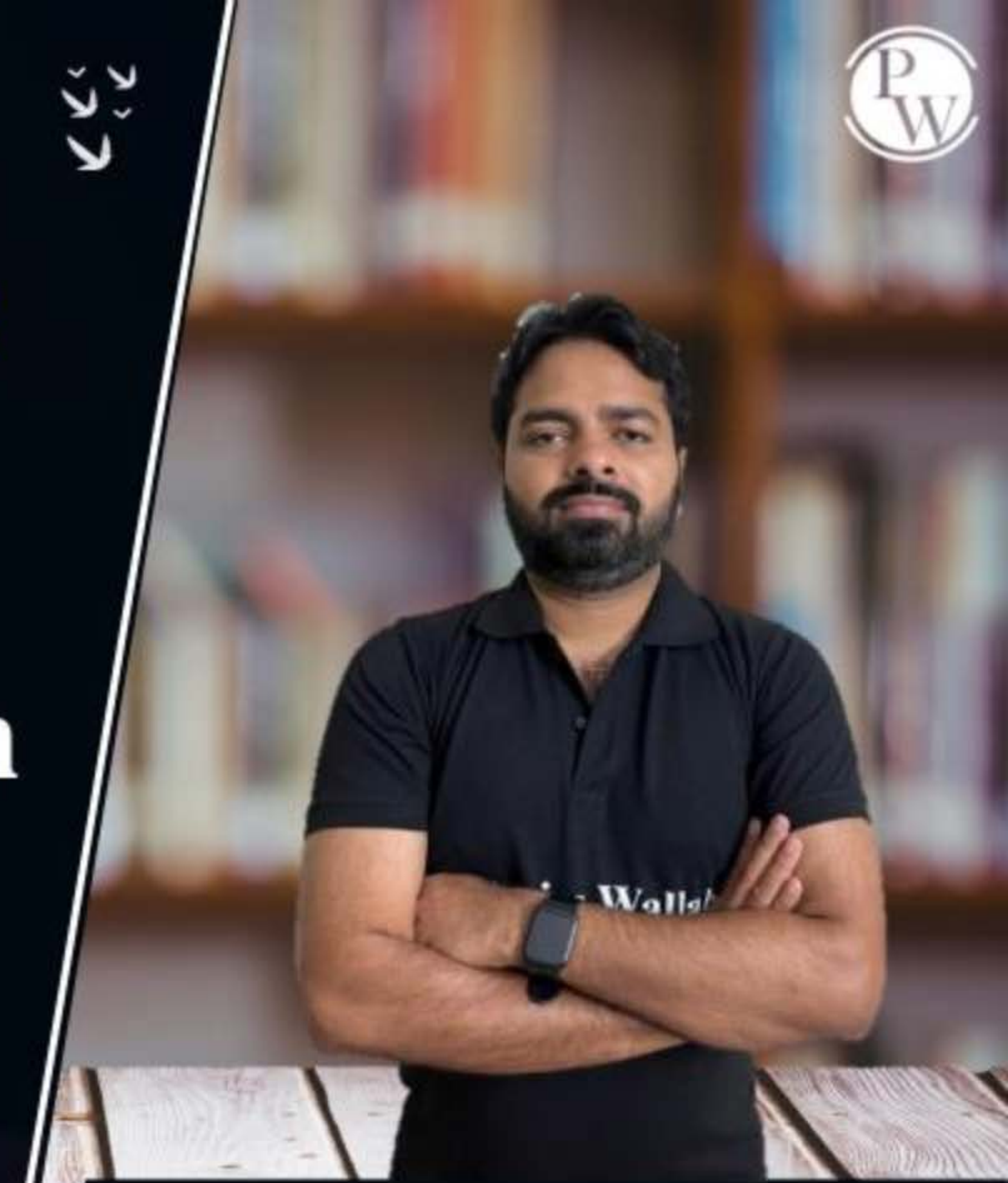
Computer Science

Theory of Computation

Turing Machine

Lecture No.- 1



Mallesham Devasane Sir

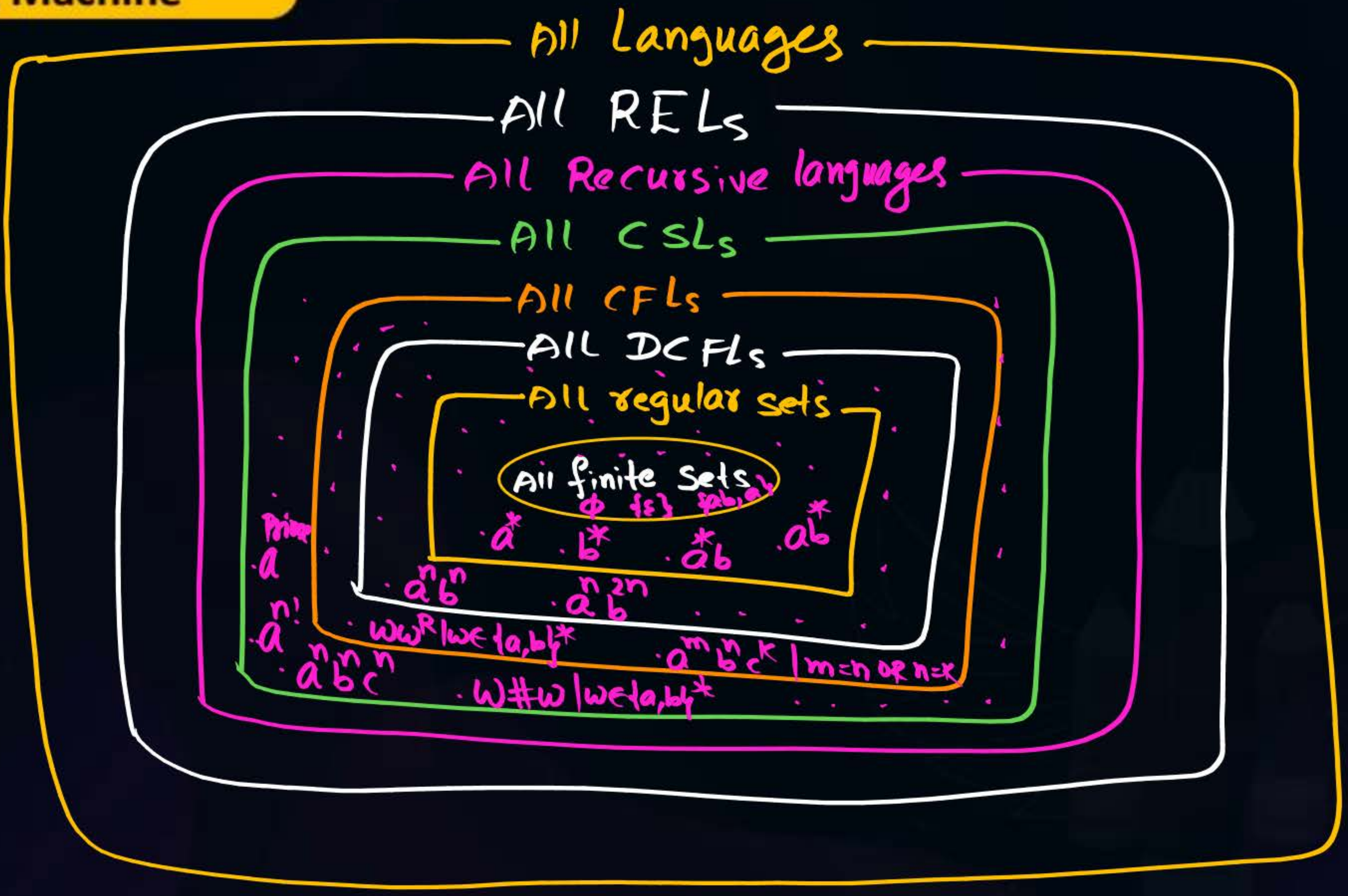
Recap of Previous Lecture

Topic Context Free Languages

Topics to be Covered

- **Topic**: TM
- **Topic**: LBA Vs HTM Vs TM
- **Topic**: TM Construction
- **Topic**: Recursive Vs REL
- **Topic**: Closure properties

Turing Machine

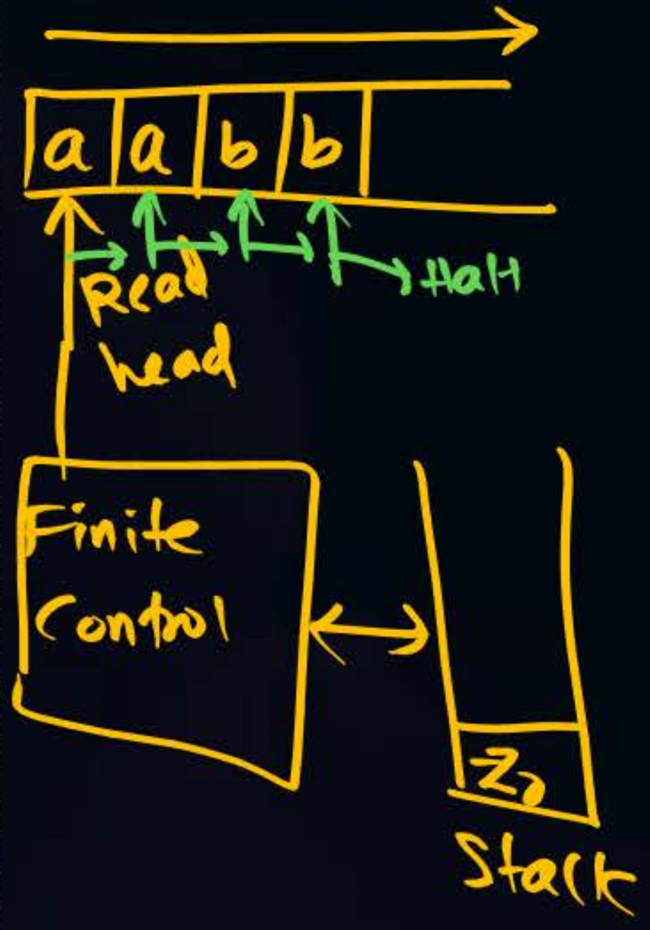


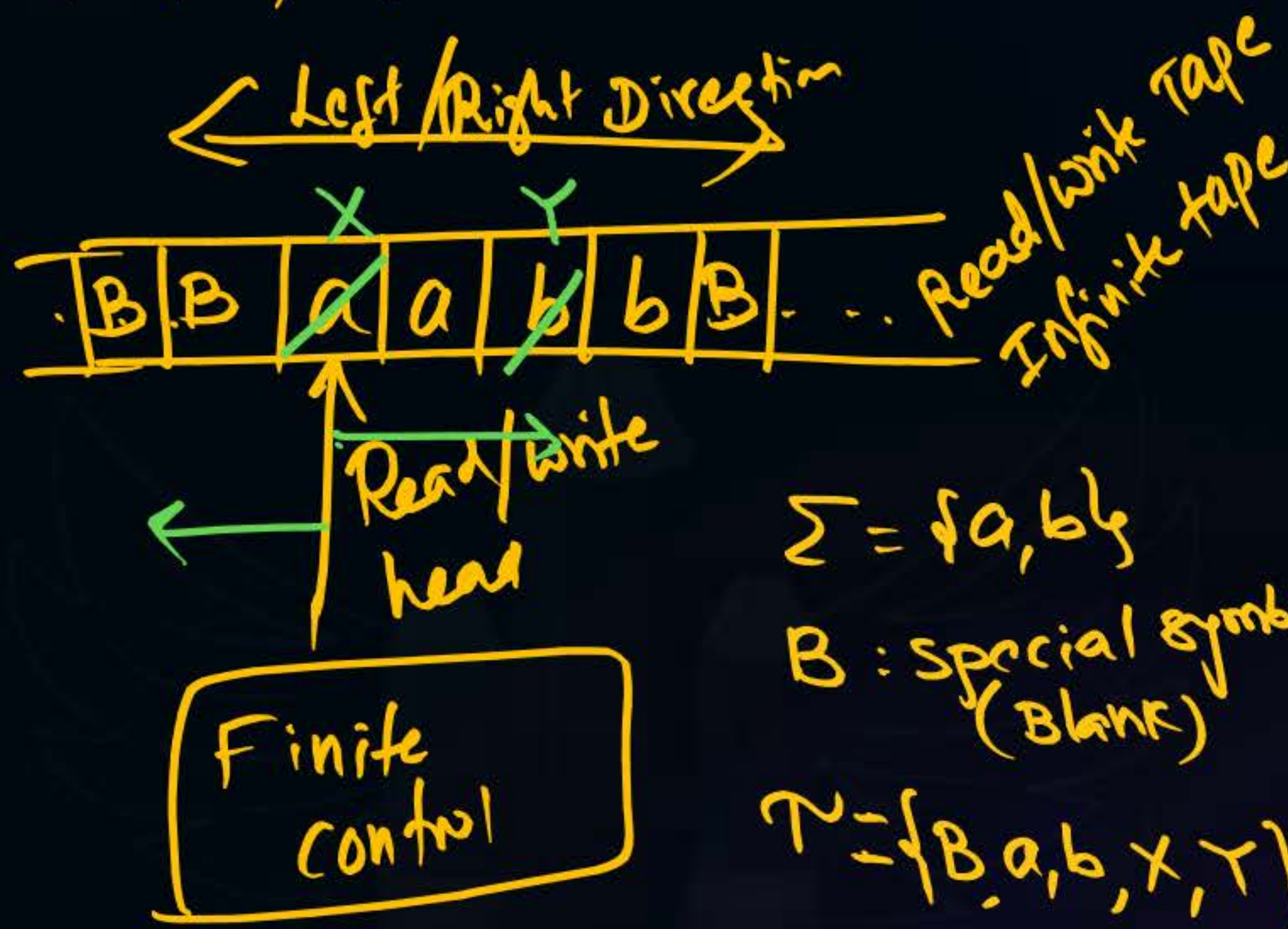
Turing Machine

FA	PDA	TM
$(Q, \Sigma, \delta, q_0, F)$	$(Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$	$(Q, \Sigma, \delta, q_0, F, B, \Gamma)$

Γ → stack alphabet (PDA)

Γ → Tape Alphabet (TM)





$\Sigma = \{a, b\}$

B : special symbol (Blank)

$\Gamma = \{B, a, b, X, Y\}$

Turing Machine

FA

$$(Q, \Sigma, \delta, q_0, F)$$

$$DFA \cong NFA$$

$$DFA: Q \times \Sigma \to Q$$

$$NFA: Q \times \Sigma_\varepsilon \to 2^Q$$

PDA

$$(Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$$

$$PDA: Q \times \Sigma_\varepsilon \times \Gamma^* \to 2^{Q \times \Gamma^*}$$

TM

$$(Q, \Sigma, \delta, q_0, F, B, \tau)$$

$$DTM \cong NTM$$

$$DTM: Q \times \overset{\text{Read}}{\tau} \longrightarrow Q \times \overset{\text{write}}{\tau} \times \{L, R\}$$

$$NTM: Q \times \tau \longrightarrow 2^{Q \times \tau \times \{L, R\}}$$

Turing Machine

FA

$$1 \xrightarrow{a} 2$$

$$\delta(1, a) = 2$$

$$Q \times \Sigma \rightarrow Q$$

$1 \in Q$ $a \in \Sigma$ $2 \in Q$

PDA

$$1 \xrightarrow{a, x / xx} 2$$

$$\delta(1, a, x) = (2, xx)$$

NPDA: $Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*$

$1 \in Q$, $a \in \Sigma$, $x \in \Gamma$

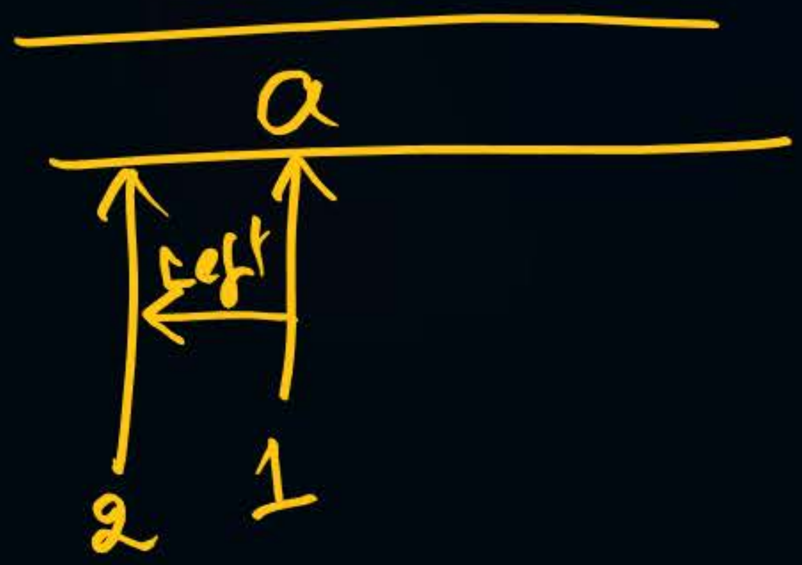
$2 \in Q$, $xx \in \Gamma^*$

TM

$$1 \xrightarrow{a, y, R} 2$$

$$\delta(\underset{\text{State}}{1}, \underset{\text{Read}}{a}) = (\underset{\text{State}}{2}, \underset{\text{Write}}{y}, \underset{\text{Direction}}{R})$$

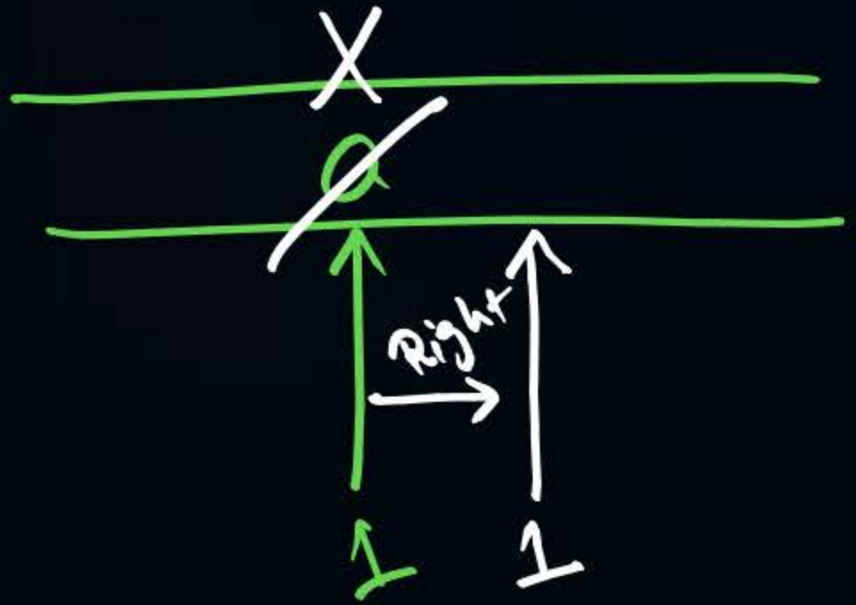
$$\delta(1, a) = (2, a, L)$$



from state 1,
by reading a,
goes to state 2
by writing with a
and moves to left direction.

Turing Machine

$$\delta(1, a) = (1, x, R)$$

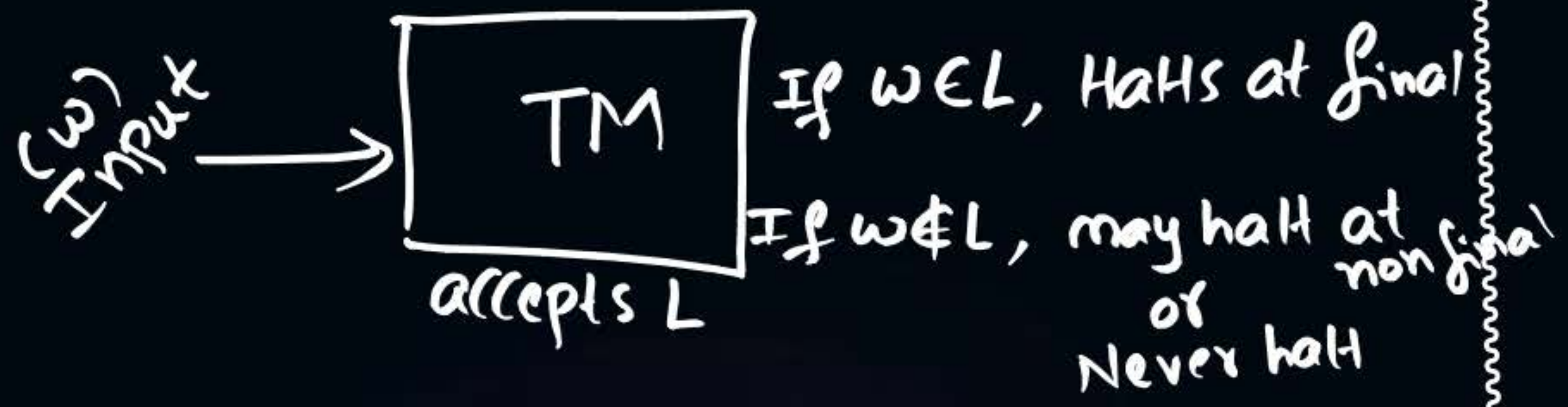


Turing Machine

- It is powerful machine
- It is equivalent to computer/program
- It represents "Recursively Enumerable Language" (REL)
  (Semi-decidable language)
  (Enumerable language)
  (Recognizable language)

Turing Machine

Acceptor

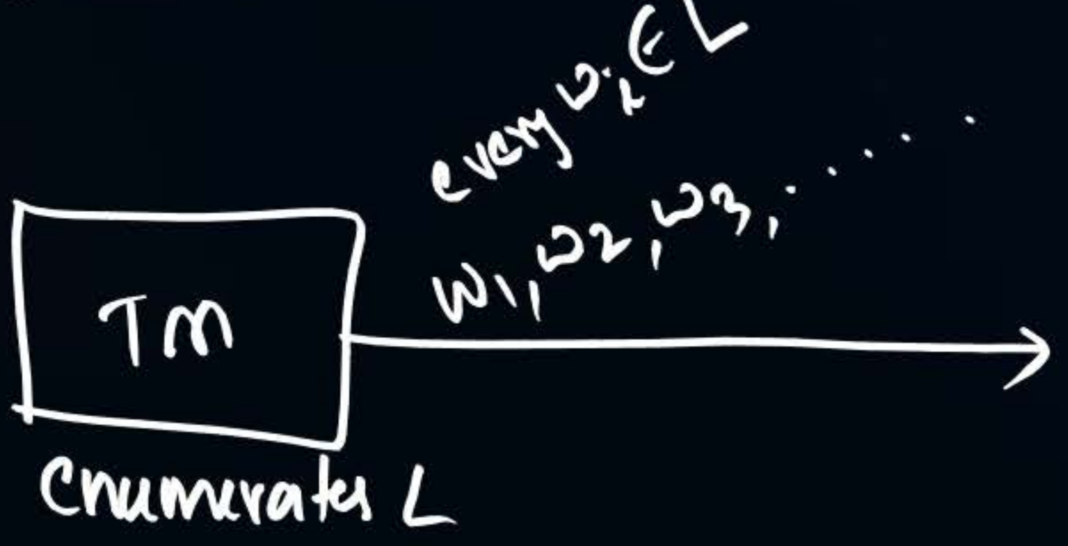


TM accepts REL.

L(TM) is REL

TM ≅ REL

Enumerator



TM enumerates REL

L(TM) is REL

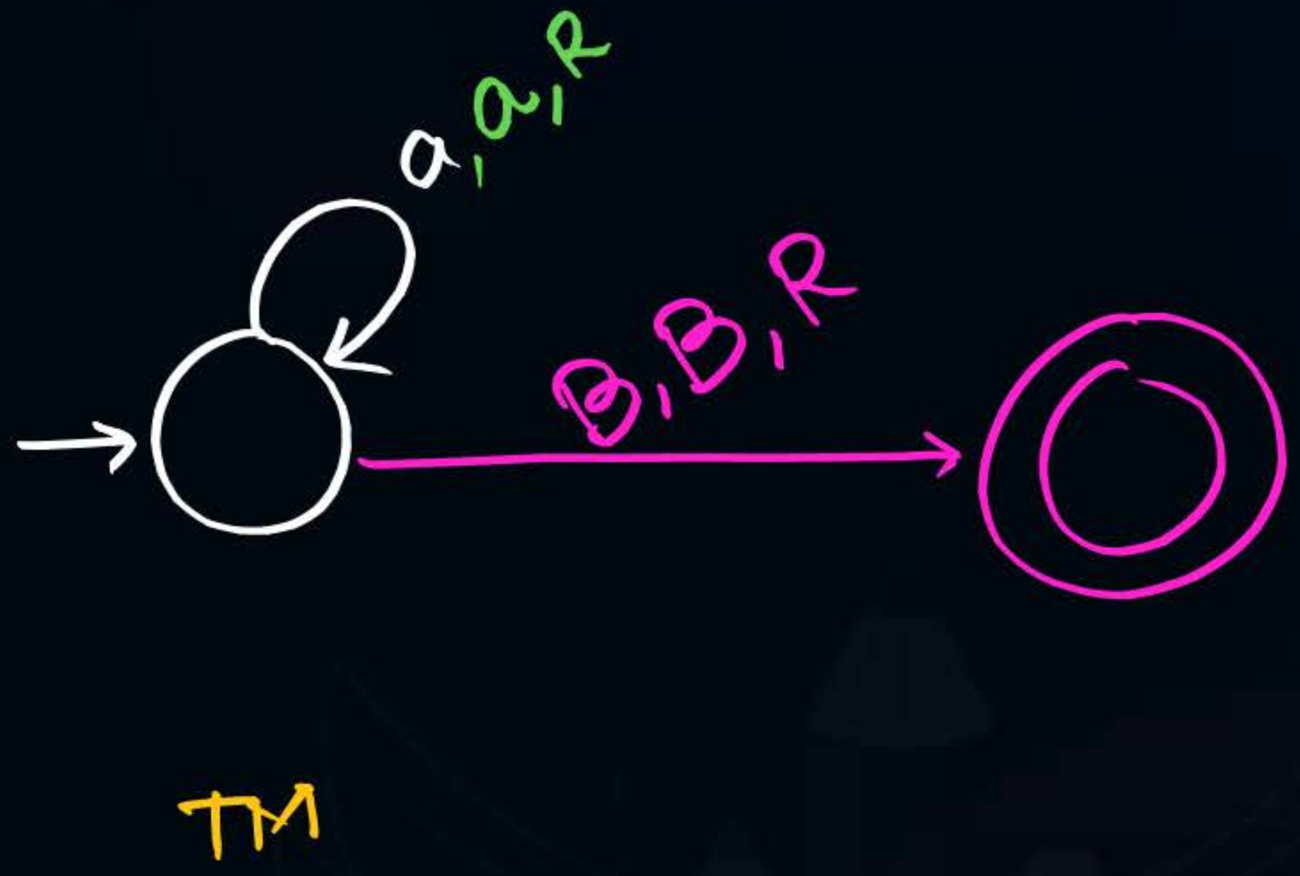
***

If $\underbrace{\overset{\text{Input}}{\text{String}} \text{ is valid}}_{w \in L}$, $\underbrace{\text{Always halts at final.}}_{\text{Logic exist}}$

If $\underbrace{\overset{\text{Input}}{\text{String}} \text{ is invalid}}_{w \notin L}$, $\underbrace{\text{either halts at nonfinal or never halts}}_{\text{Logic may or may not exist}}$

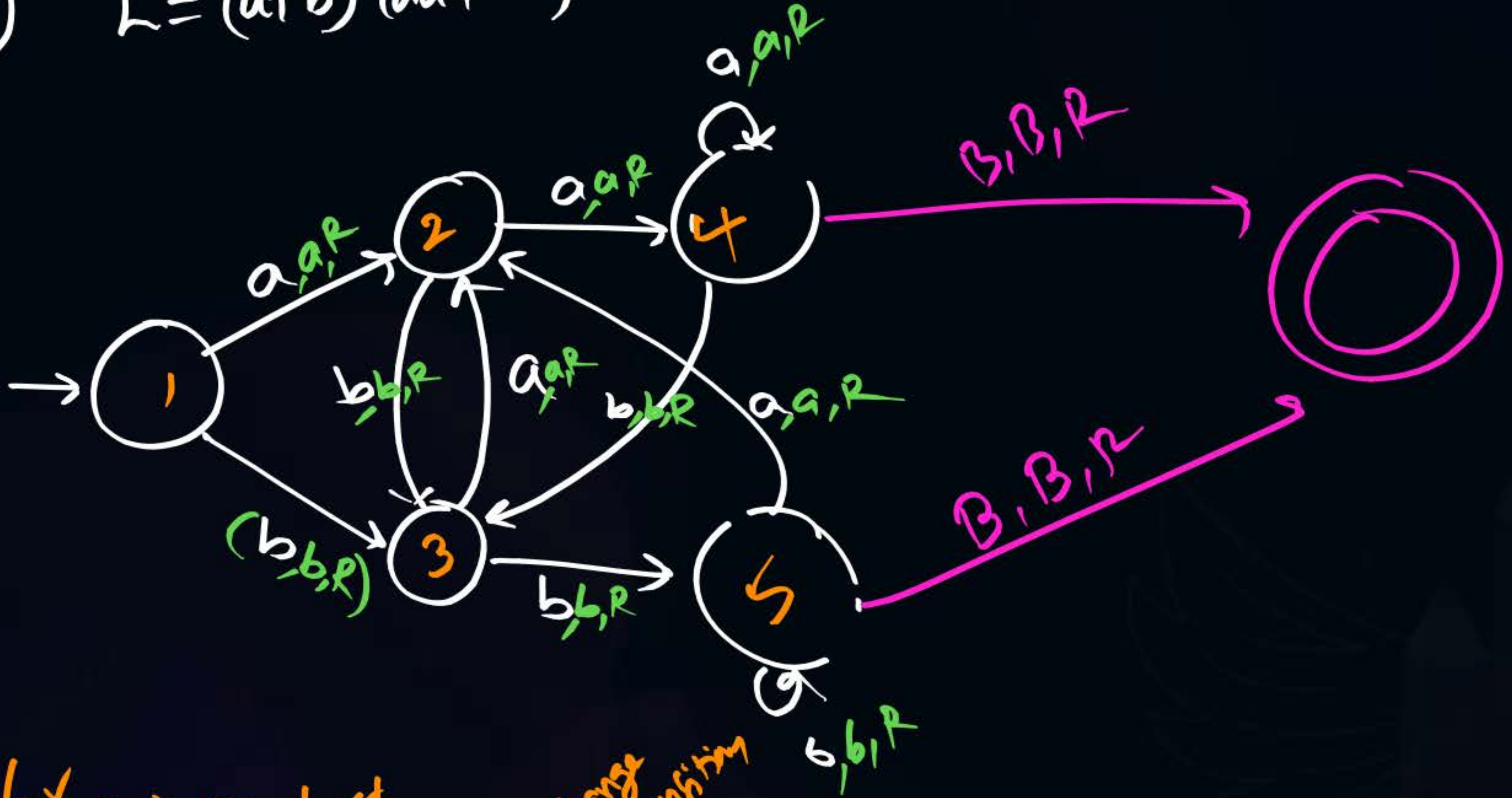
Construction of TM :

① $L = a^*$



For every regular, we can design TM

(2) $L = (a+b)^*(aa+bb)$



L is Regular $\Rightarrow$ construct DFA or NFA $\xrightarrow{\text{change transition}}$ TM

Note: We can convert any FA to TM

$$1 \xrightarrow{a} 2$$

$$\Downarrow$$

$$1 \xrightarrow{a,\,a,\,R} 2$$

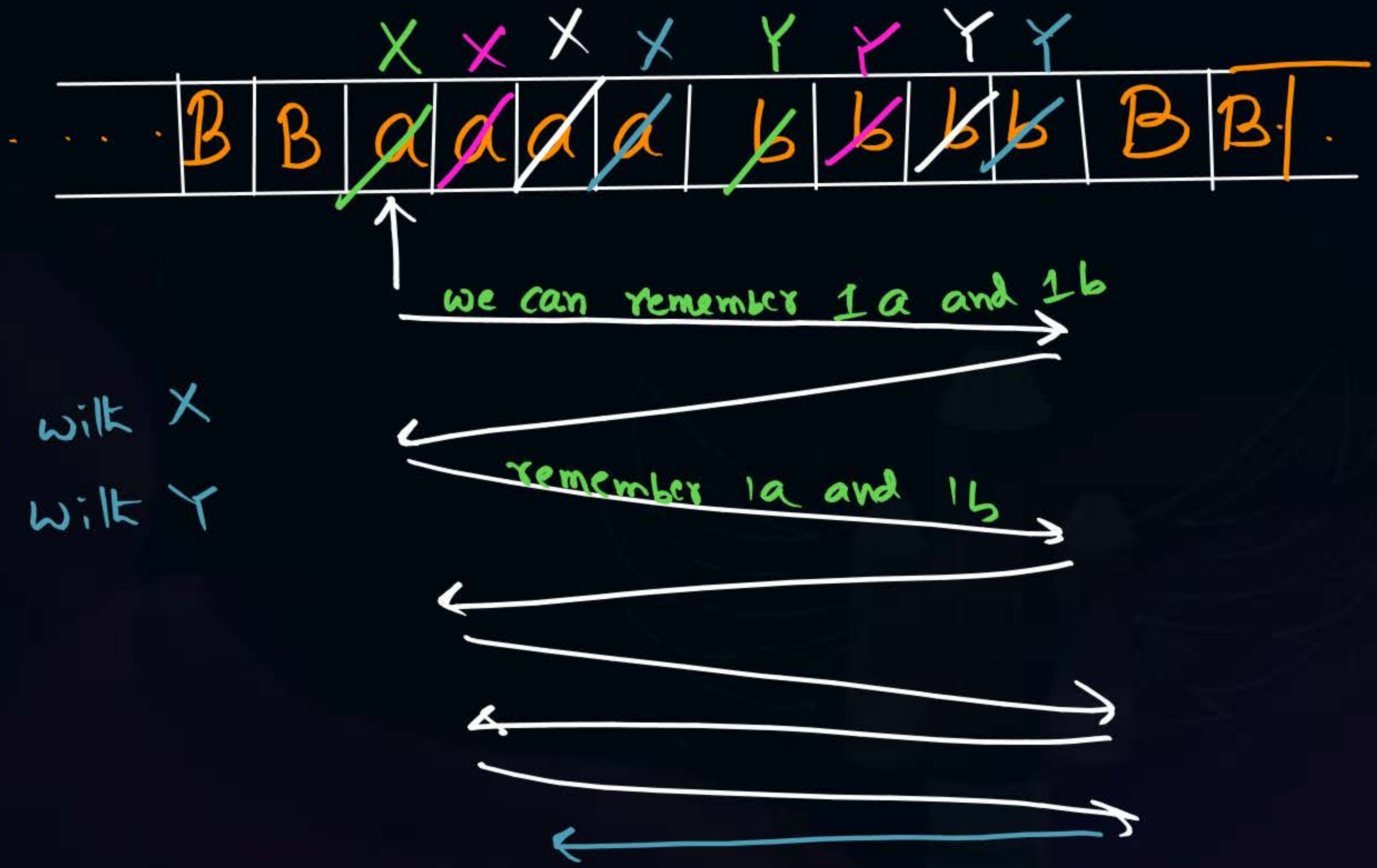
If q is final in FA

$$q$$

$$\Downarrow$$

$$q \xrightarrow{B,\,B,\,R} \text{new}$$

(3) $L = \{a^n b^n \mid n \geq 1\}$



In each scan,

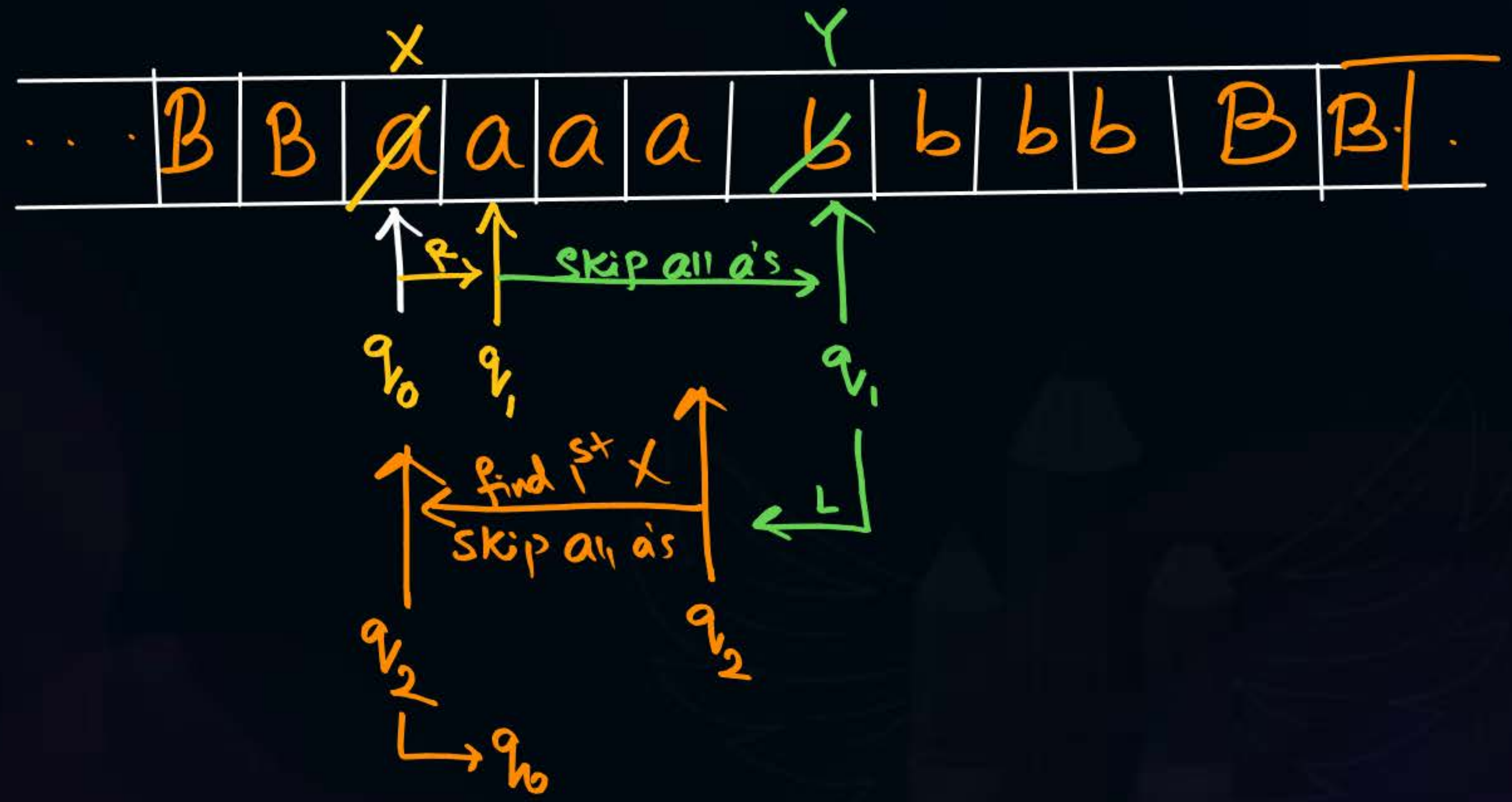
we replace a with X

b with Y

Turing Machine

(3) $L = \{a^n b^n \mid n \geq 1\}$

1^{st} scan:

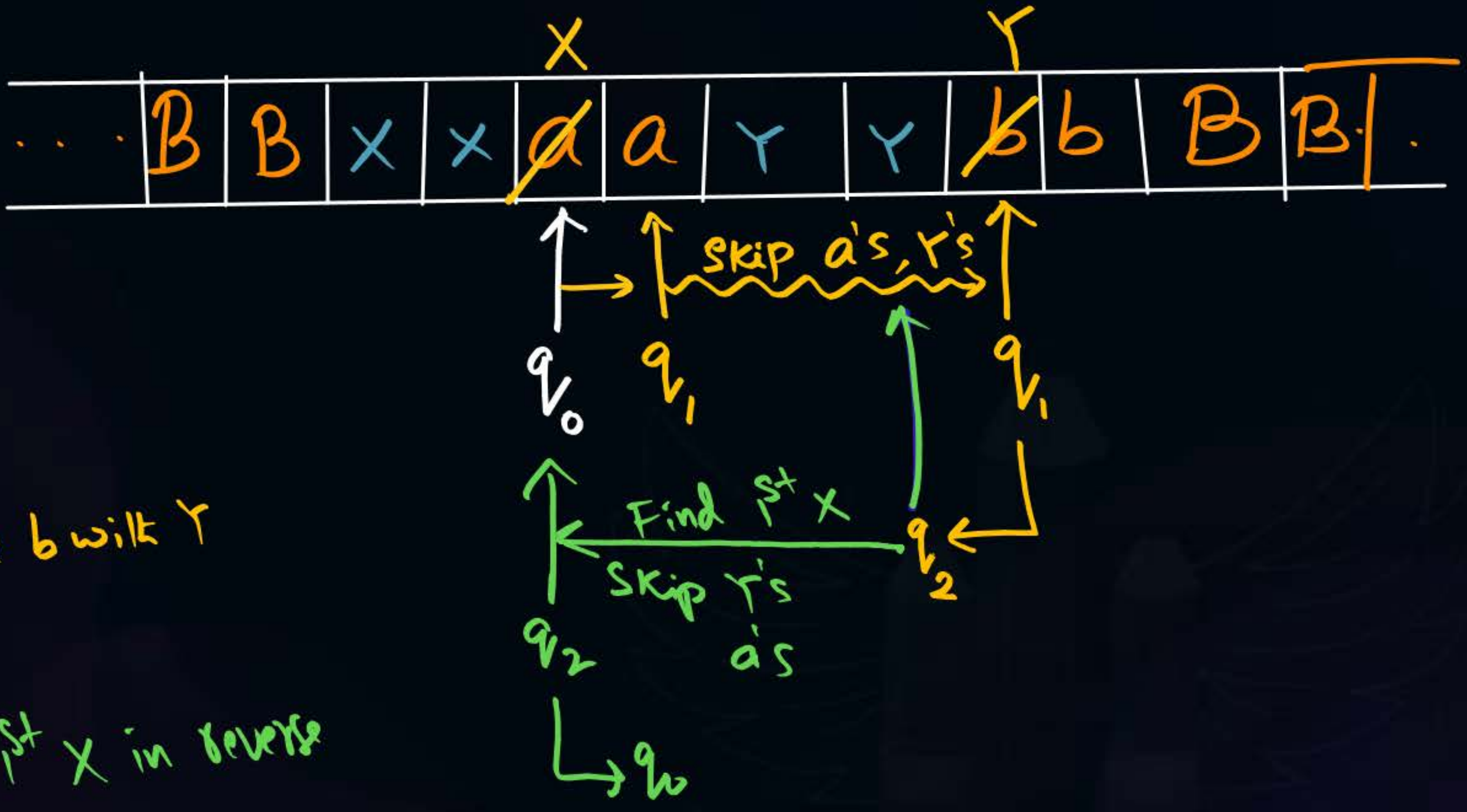


q_0: To replace a with X

q_1: Skip all a's
Replace b with Y

(3) $L = \{a^n b^n \mid n \geq 1\}$

3rd scan



q_0 : Replace a with X

q_1 : skip a's, Y's } Replace b with Y

q_2 : Skip a's, Y's } Find 1st X in reverse

(3) $L = \{a^n b^n \mid n \geq 1\}$

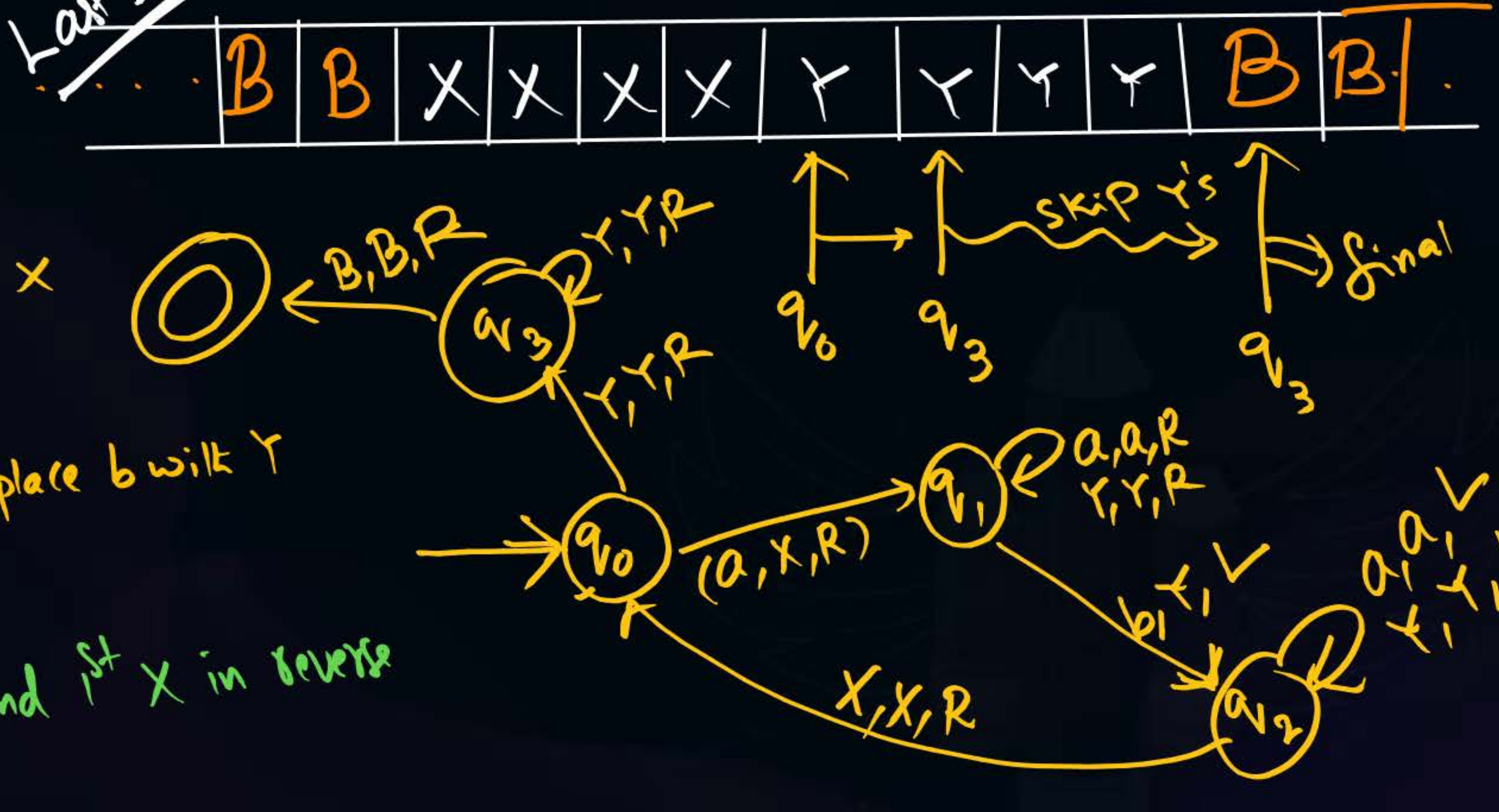
Last scan:

q_3: Skip all Y's to reach B.

q_0: Replace a with X

q_1: Skip a's, Y's } Replace b with Y

q_2: Skip a's, Y's } Find 1st X in reverse



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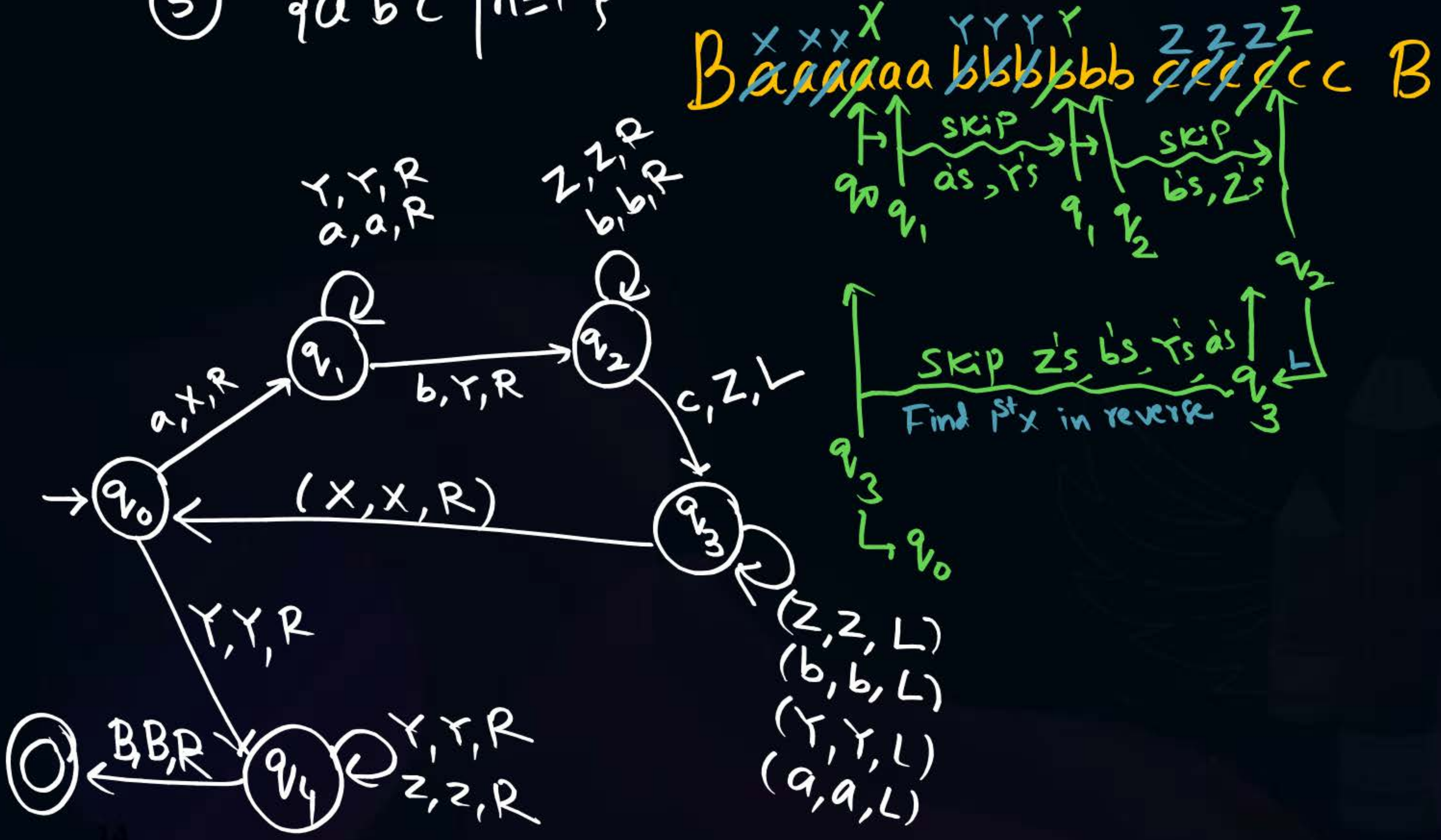
Turing Machine

(4) $L = \{a^n b^n \mid n \geq 0\}$

(5) $\{a^n b^n c^n \mid n \geq 1\}$



q_0 : $\cancel{a}$ X

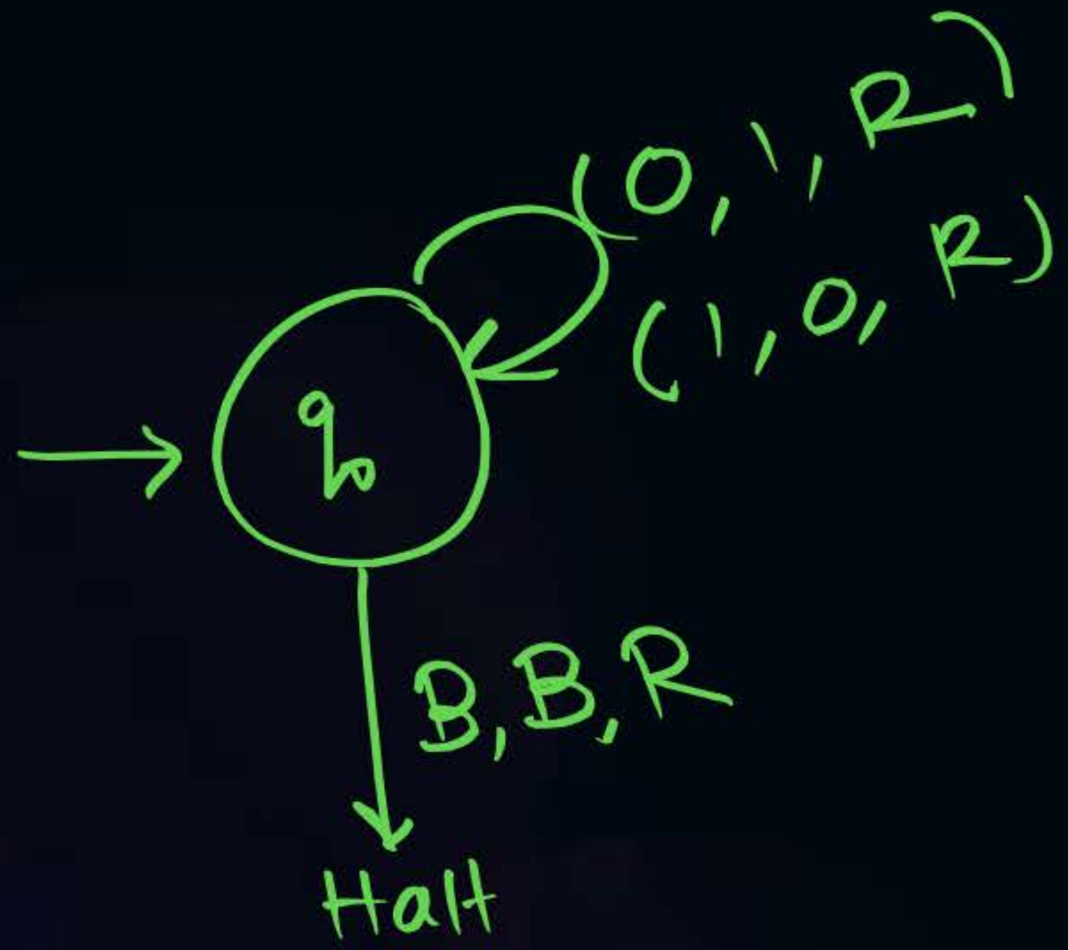
q_1 : $\cancel{b}$ Y

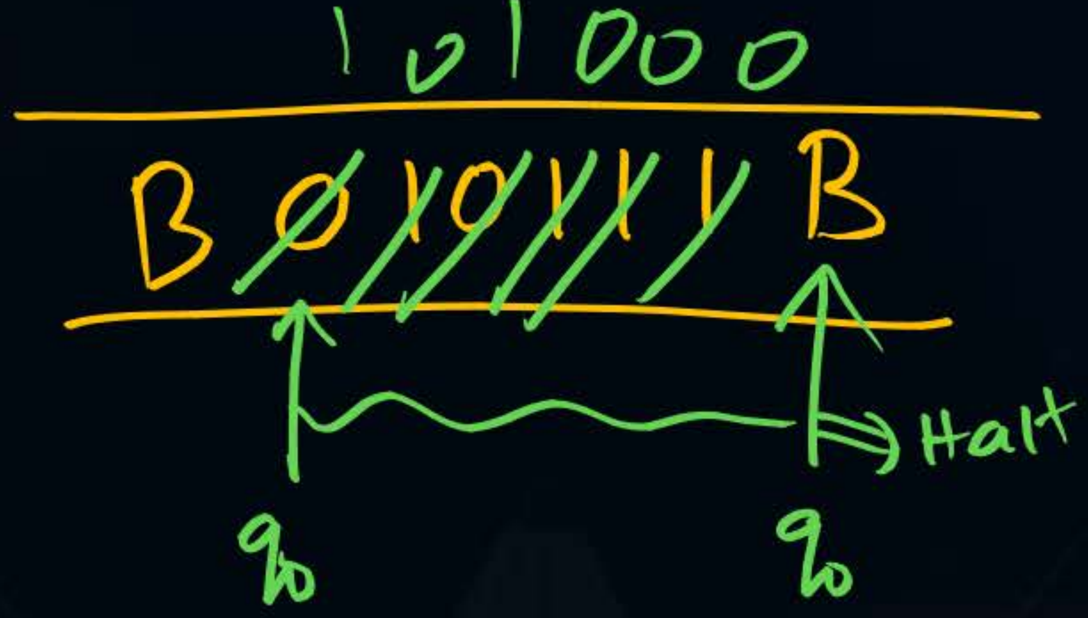
q_2 : $\cancel{c}$ Z

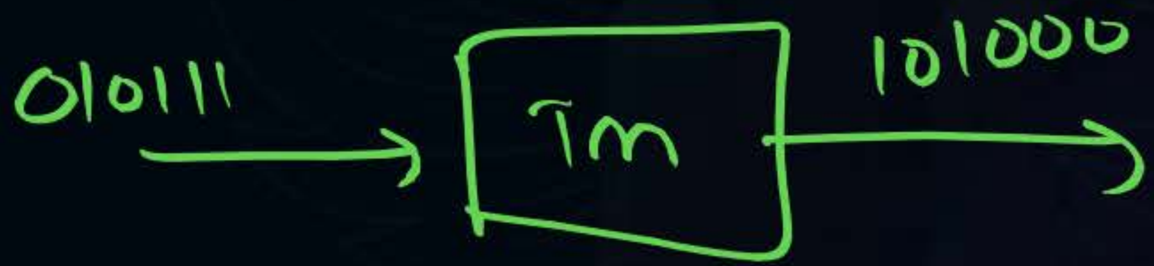
q_3 : To find 1st X in reverse

q_4 : To find B

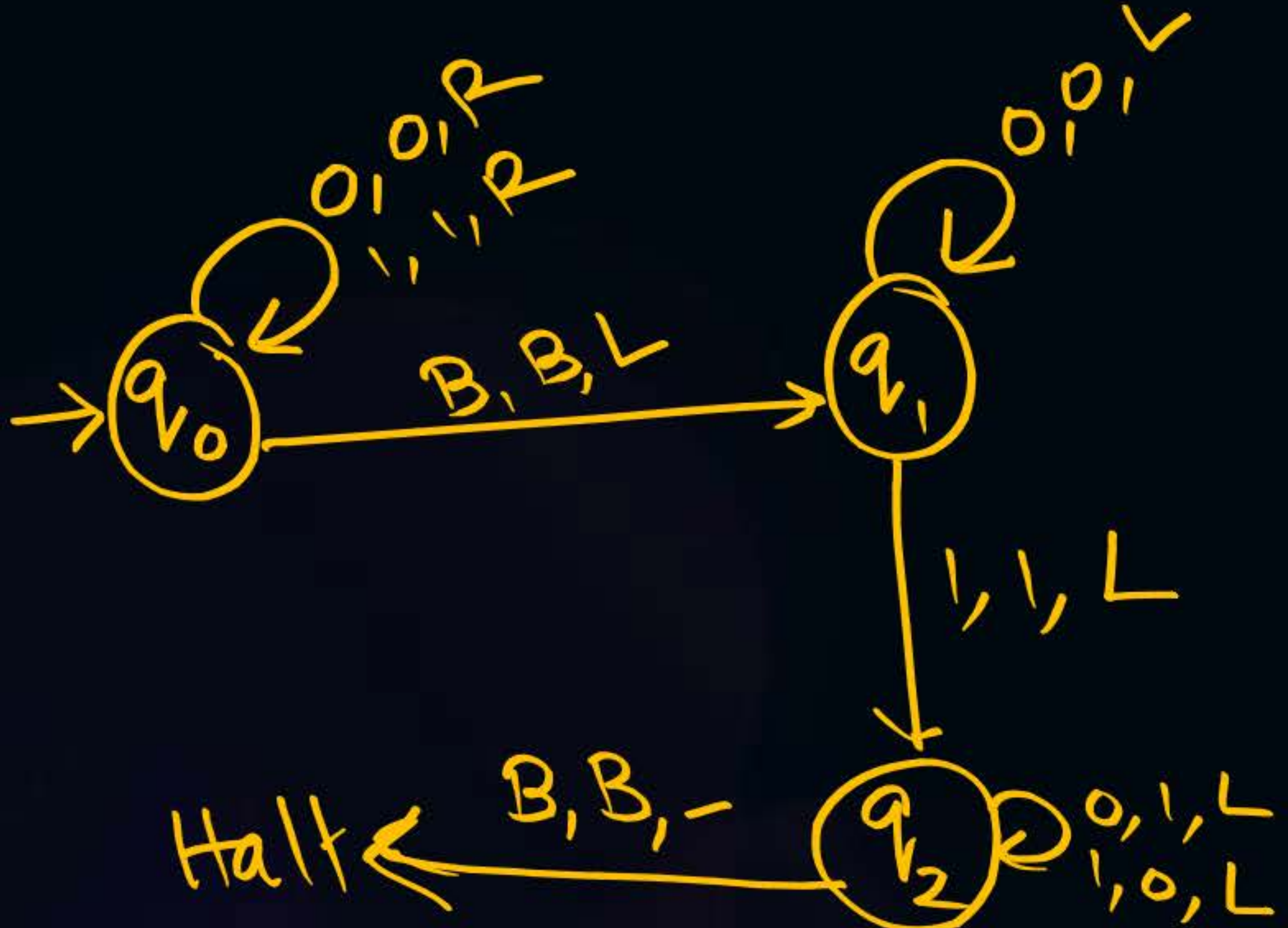
6 1's complement of binary

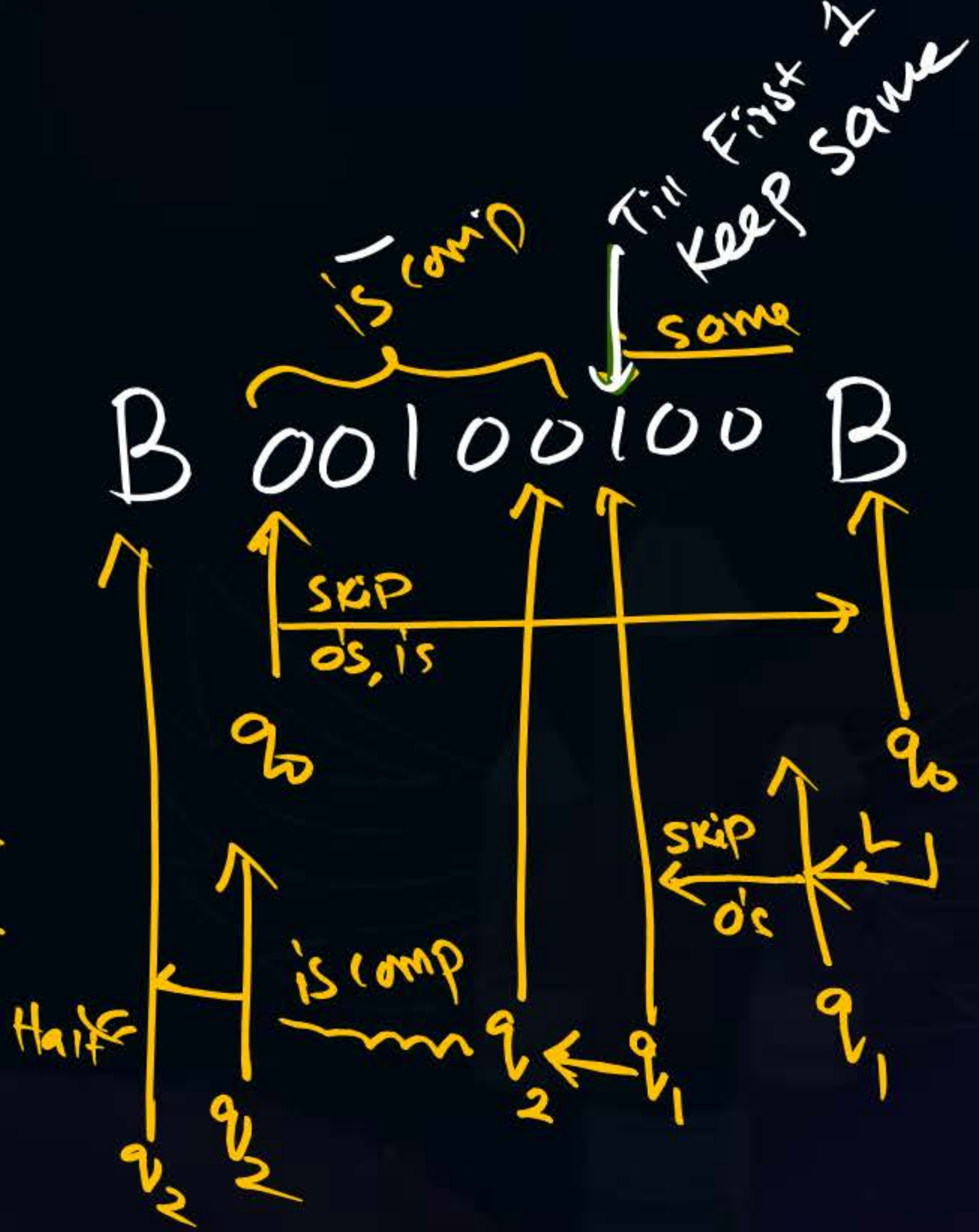




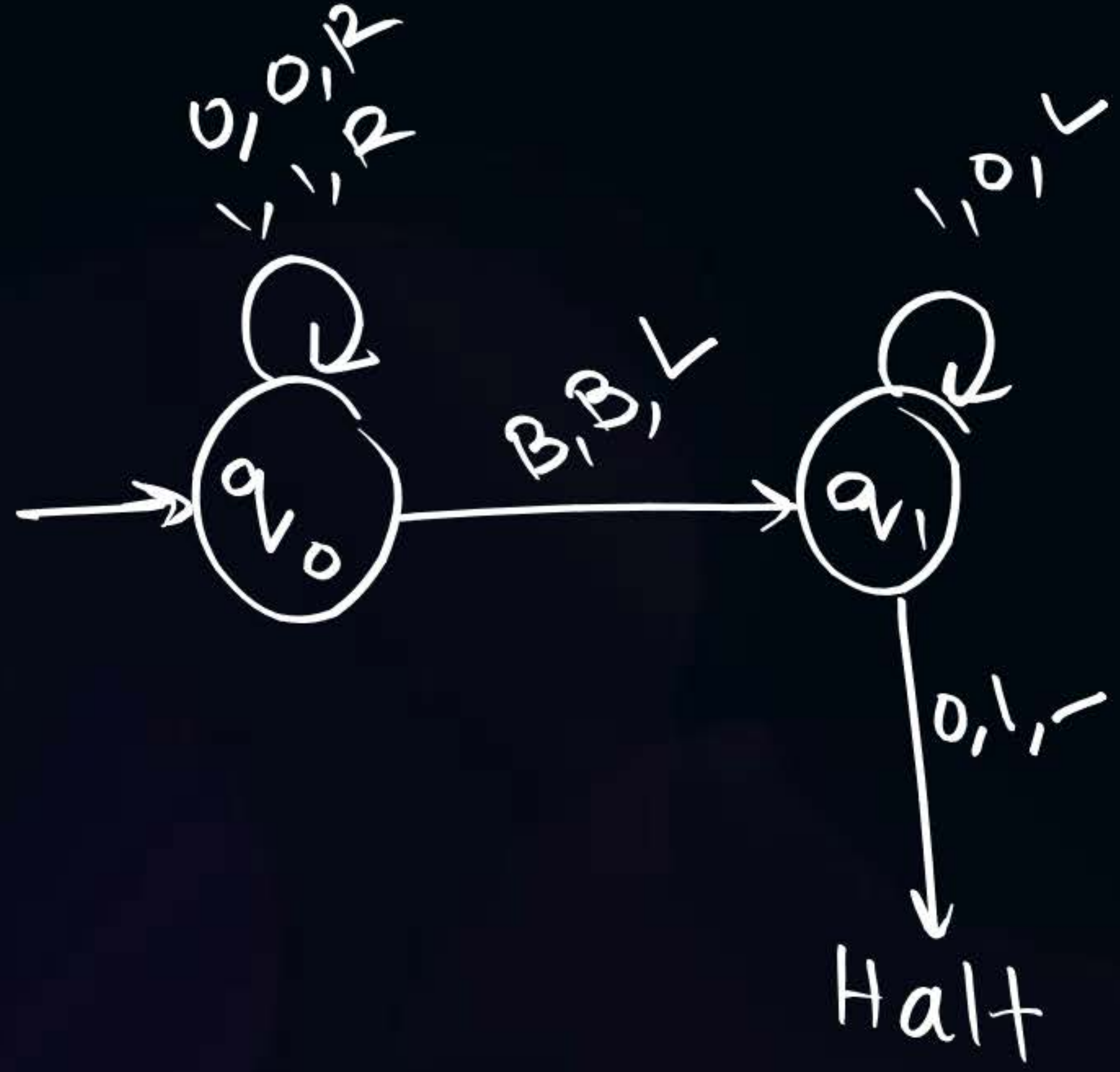


(7) 2's complement of binary





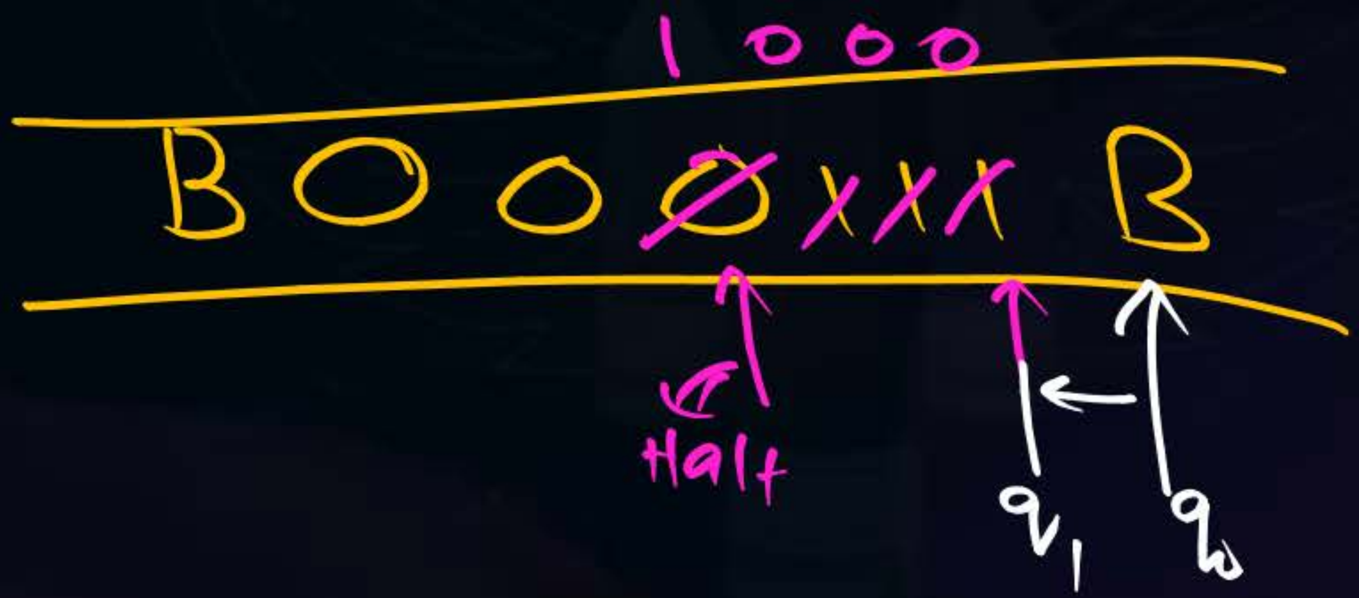
8. Increment of Binary $\quad f(x) = x + 1$



Input: $\underbrace{0\,0}_{\text{same}}\ \boxed{0}\ 1\ 1\ 1$

$\qquad\qquad\qquad\qquad 1$

$\quad 0\ 0\ 1\ 0\ 0\ 0$



Turing Machine

(9) $\{ww^R \mid w \in \{a,b\}^*\}$

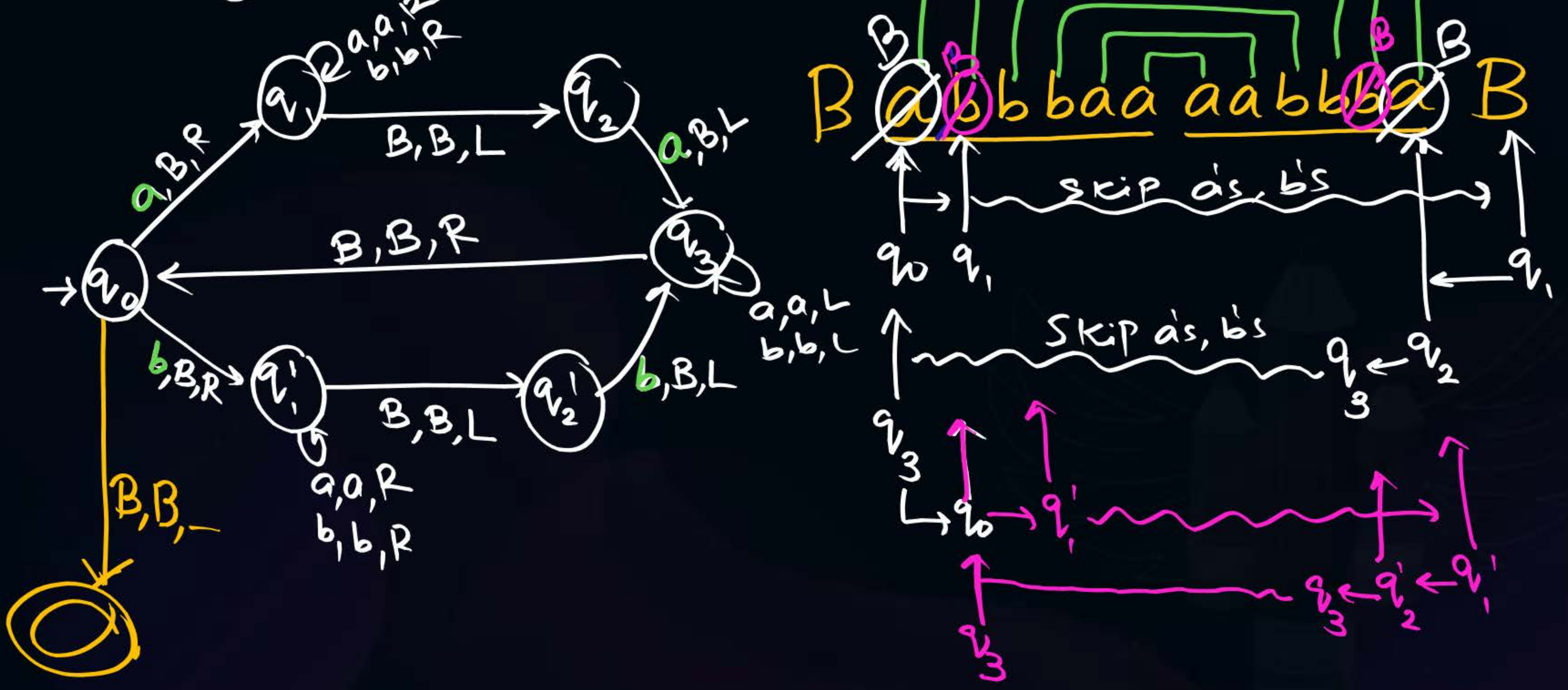
B abbbaa aabbba B

B B bbbaaaaabbb B B

B bbaaaaabb B

B baaaab B B

(9) $\{ww^R \mid w \in \{a,b\}^*\}$



H.W. =

(10) $\{w\#w^R \mid w \in \{a,b\}^*\}$

(11) $\{w\#w \mid w \in \{a,b\}^*\}$

2 mins Summary

Topic

↳ TM Construction

Next : closure properties

Recursive Vs REL

THANK - YOU