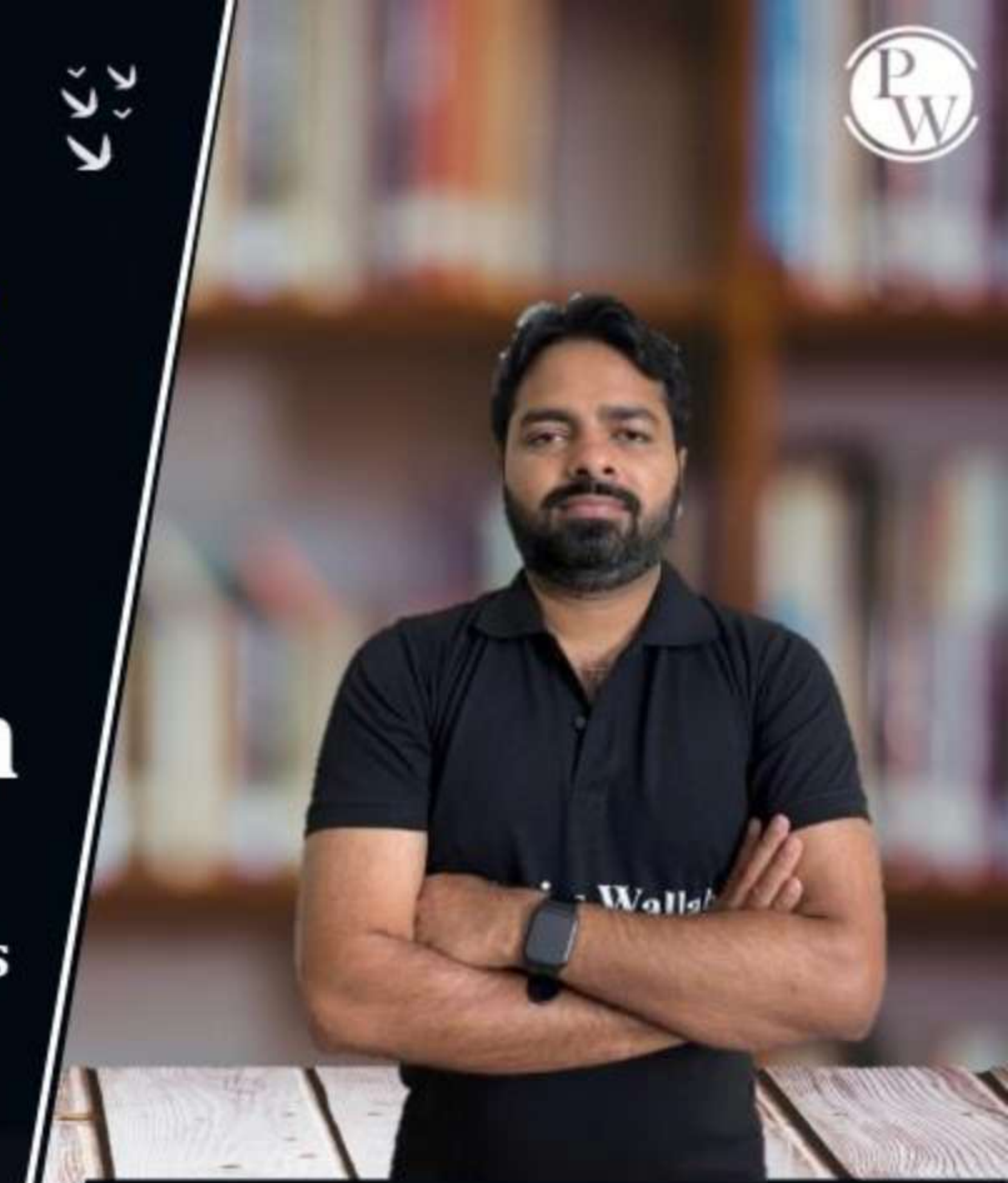
Computer Science

Theory of Computation

Regular Languages and Non-regular Languages

Lecture No.- 1



Mallesham Devasane Sir

Recap of Previous Lecture

- **Topic** — Model-I (Easy: Phi, Sigma*, only epsilon, Sigma^{+})
- **Topic** — Model II (Length)
- **Topic** — Model III (No. of symbols), Model IV (Over 1 symbol)
- **Topic** — Model V (Sequence based), Model VI (Length & Remainder)
- **Topic** — Model VII (Symbols & Remainder)
- **Topic** — Model VIII (Multiple Conditions on symbols)
- **Topic** — Model IX (Start, End , Contain), Model X (Position based)
- **Topic** — Model XI (Multiple Conditions-Remainder)
- **Topic** — Model XII (Multiple Conditions-Simple)
- **Topic** — Minimization of DFA

Topics to be Covered

Topic	DFA Vs NFA
Topic	NFA Construction
Topic	Conversion from NFA to DFA
Topic	NFA with epsilon Moves

DFA Vs NFA

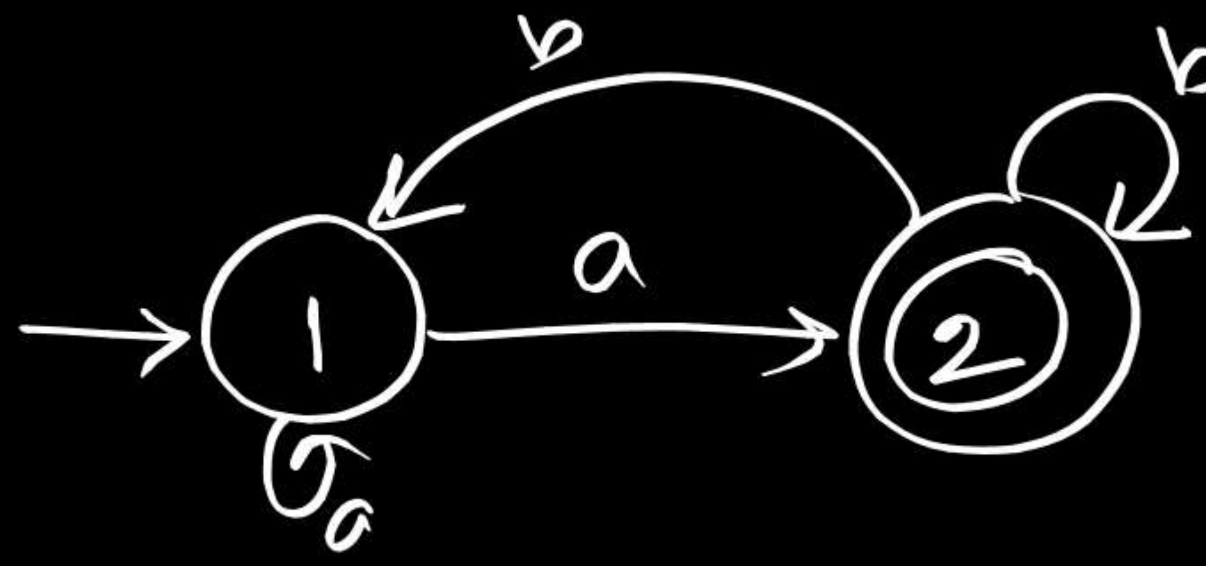
DFA	NFA
$\delta : Q \times \Sigma \rightarrow Q$	$\delta : Q \times \Sigma_{\varepsilon} \rightarrow 2^{Q}$

I) Every DFA is NFA.

II) NFA may or may not be DFA.

III) $DFA \cong NFA$ $\left[\begin{array}{l} ① \; DFA \Rightarrow NFA \\ ② \; NFA \Rightarrow DFA \end{array}\right]$

ε ✗
a ✓
b ✗
aa ✓
ab ✓
ba ✗
bb ✗



ε: 1

a: $1 \xrightarrow{a} 1$
OR
$\boxed{1 \xrightarrow{a} 2}$

b: $1 \xrightarrow{b}$ no transition

I) If the string is valid then atleast 1 path exist that halts at final.

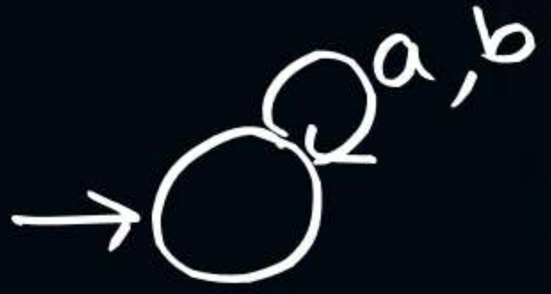
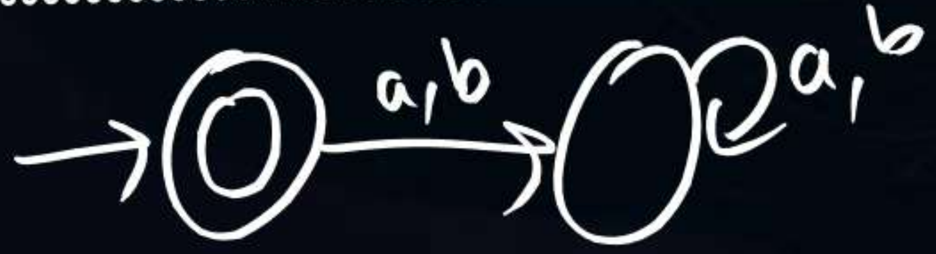
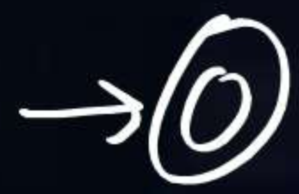
II) If the string is invalid then i) all paths halts at non final
OR
ii) no path

Different words

non-deterministic FA

not deterministic FA [not DFA]

DFA Vs NFA

		Min DFA	Min NFA
1	$L = \phi$ over $\Sigma = \{a, b\}$ $= \{\ \}$		
2	$L = (a+b)^*$		
3	$L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$		 ε ✓ a ✗ b ✗ aa ✗ ab ✗

Note :

I) For every regular language, No. of min DFAs $= 1$

II) " " " , No. of min NFAs ≥ 1

III) For a regular language

$$n(\text{min DFA}) \geq n(\text{min NFA})$$

DFA Vs NFA

Min DFA | Min NFA

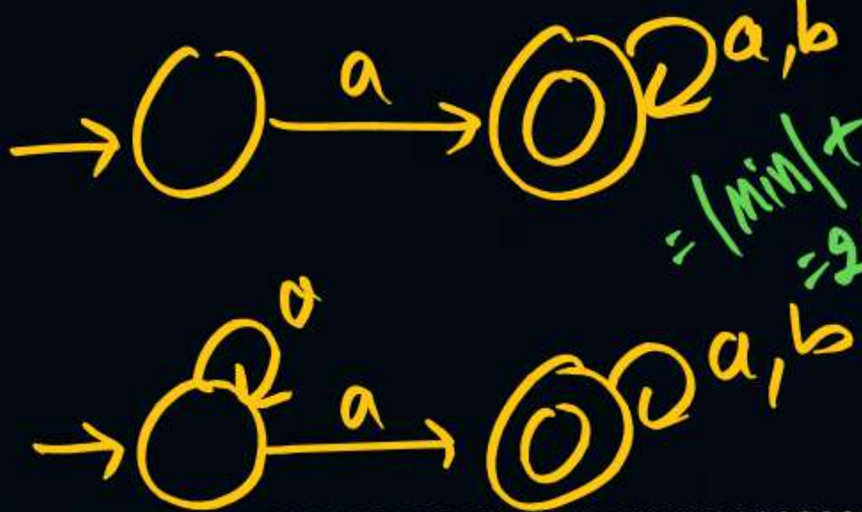
(4) $L = a(a+b)^*$

min = a

Min DFA:

|min string| + 2
= 3 states

Min NFA:

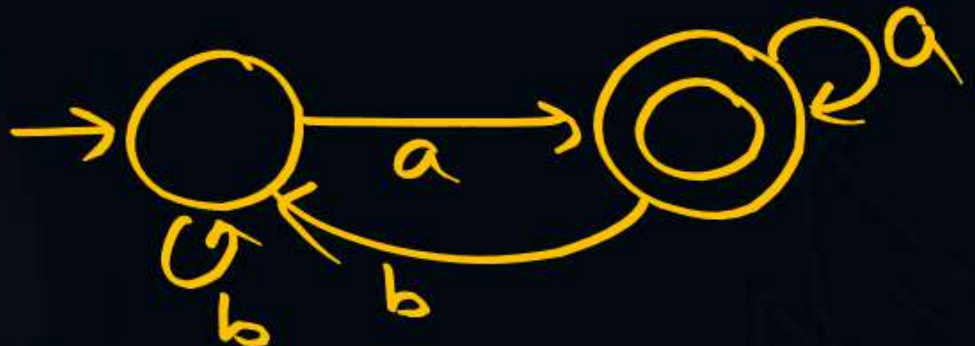


= |min| + 1
= 2 states

(5) $L = (a+b)^* a$

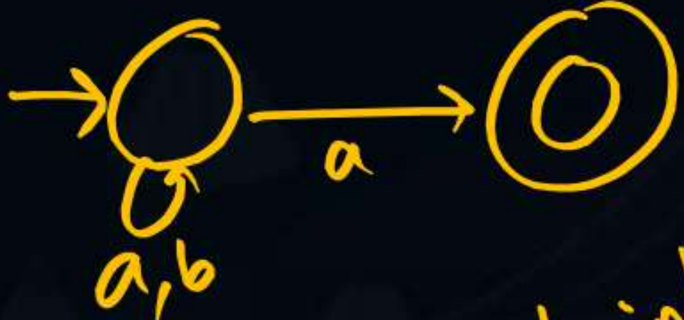
min = a

Min DFA:



|min| + 1 = 2 states

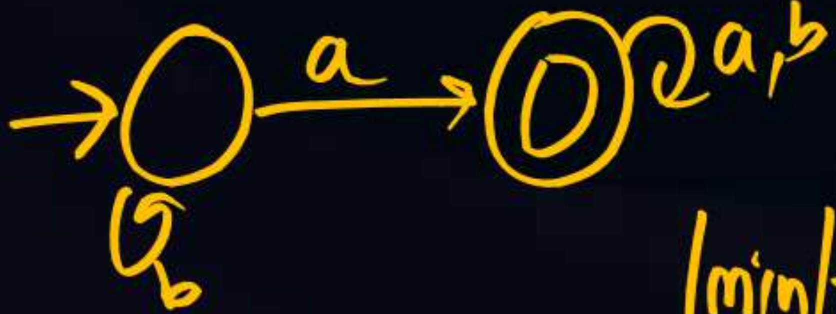
Min NFA:



|min| + 1 = 2 states

(6) $L = (a+b)^* a (a+b)^*$

Min DFA:

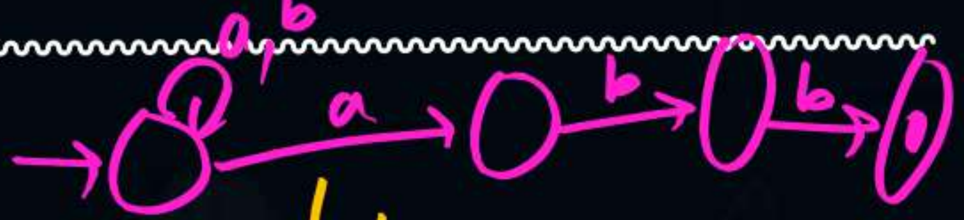


|min| + 1 = 2

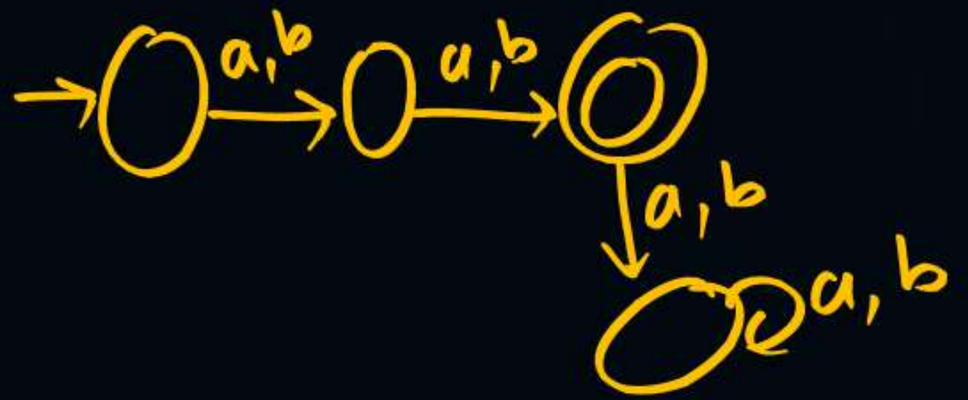
Min NFA:

|min| + 1 = 2 states

DFA Vs NFA

		Min DFA	Min NFA
(7)	$L = abb(a+b)^*$	5 states	 4 states
(8)	$L = (a+b)^* abb$	4 states	 4
(9)	$L = (a+b)^* abb (a+b)^*$	4 states	 4

DFA Vs NFA

		Min DFA	Min NFA
(10)	$L = (a+b)^2$ Exactly 2 length	 $=4$	 $=3$
(11)	$L = (a+b+\varepsilon)^2$ Atmost 2 length	$=4$	$=3$
(12)	$L = (a+b)^2(a+b)^*$ Atleast 2 length	$=3$	$=3$

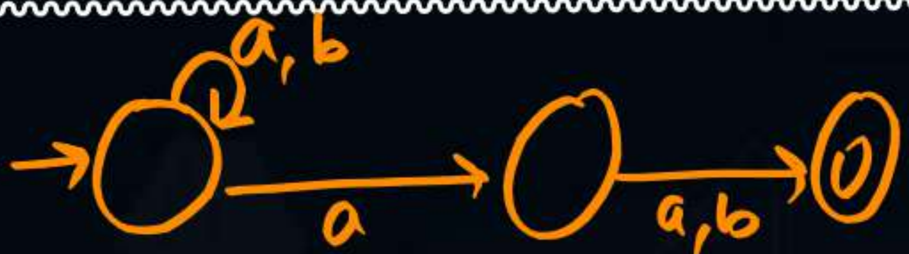
DFA Vs NFA

	Min DFA	Min NFA
(13) $L=(a+b)^K$ K is constant	$=K+2$ states	$=K+1$
(14) $L=(a+b+\varepsilon)^K$ K is constant	$=K+2$ states	$=K+1$
(15) $L=(a+b)^K(a+b)^*$ K is constant	$=K+1$ states	$=K+1$

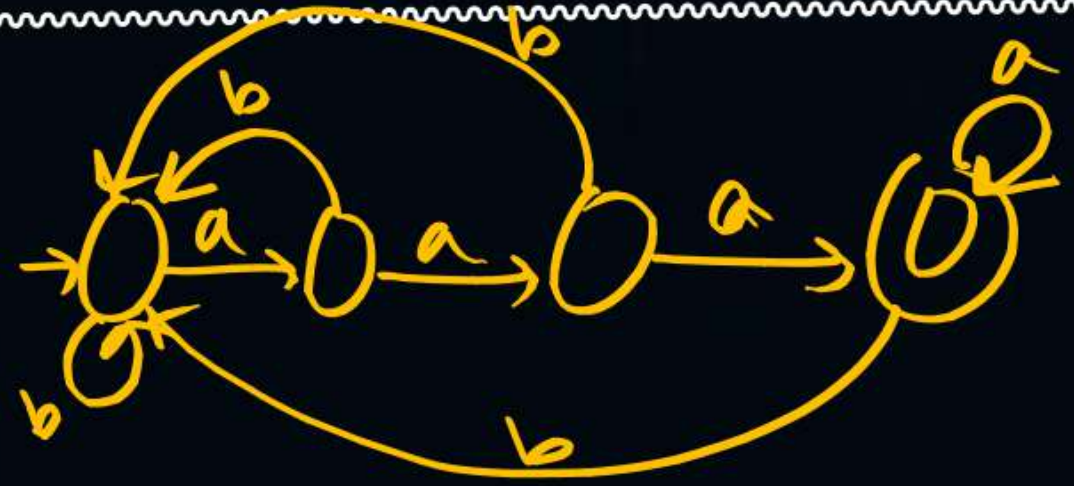
DFA Vs NFA

		Min DFA	Min NFA
(16)	$L = b^* a b^* a b^*$ $\boxed{\#a\text{'s} = 2}$	= 4 states	= 3
(17)	$L = b^*(a+\varepsilon)b^*(a+\varepsilon)b^*$ $\#a\text{'s} \le 2$	= 4	= 3
(18)	$L = (a+b)^* a (a+b)^* a (a+b)^*$ $\#a\text{'s} \ge 2$	= 3	= 3

DFA Vs NFA

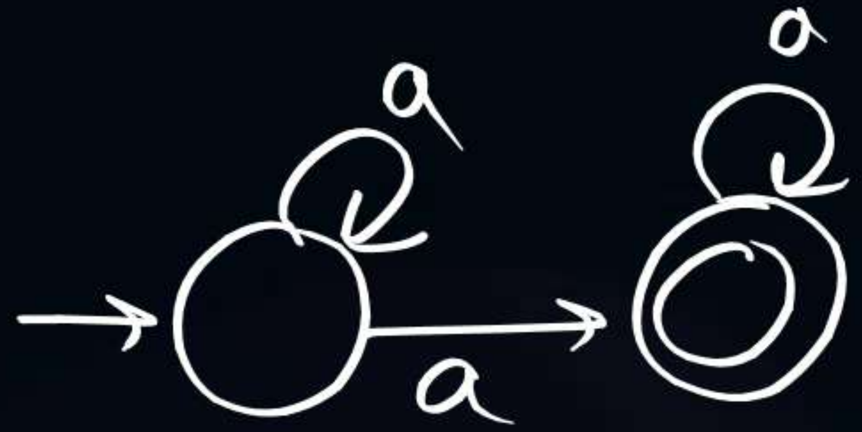
	Min DFA	Min NFA
(19) $L=(a+b)\,a\,(a+b)^*$ 2nd symbol from begin is 'a'	$=(2+1)+1=4$ states	 $=2+1=3$
(20) $L=(a+b)^*\,a\,(a+b)$	$=2^2=4$ states	 $=2+1=3$ states
(21) $L=(a+b)^*\,a\,(a+b)^9$	$=2^{10}=1024$ states	$=10+1=11$ states

DFA Vs NFA

	Min DFA	Min NFA
(22) $L = (a+b)^* aaa$		

Identify Language accepted by NFAs.

(1)



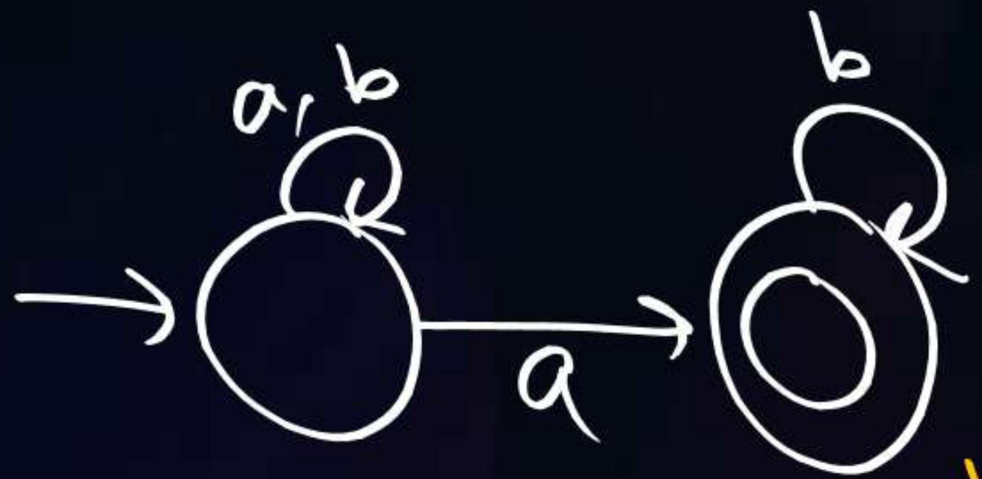
$L = a^+$

$= a^* a a^*$

(3)

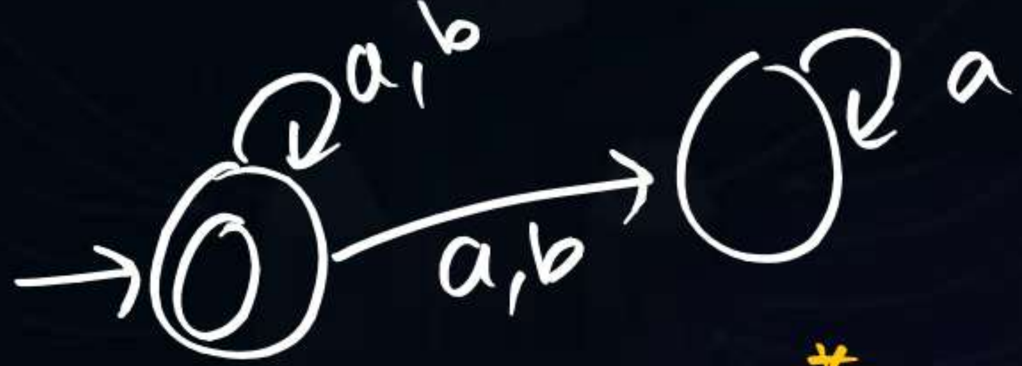
$L = a^*$

(2)



$L = (a+b)^* a b^*$

(4)



$L = (a+b)^*$


find path that halts at final

Given NFA

$Q = \{1, 2\}$ = set of states in NFA

state in NFA

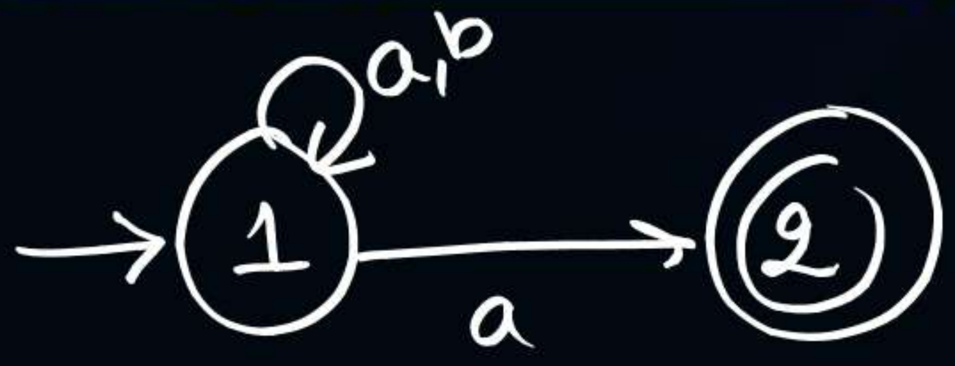
subset construction

DFA

$2^Q = \{\phi, \{1\}, \{2\}, \{1,2\}\}$ = set of states in DFA

State in DFA

Conversion From NFA to DFA :

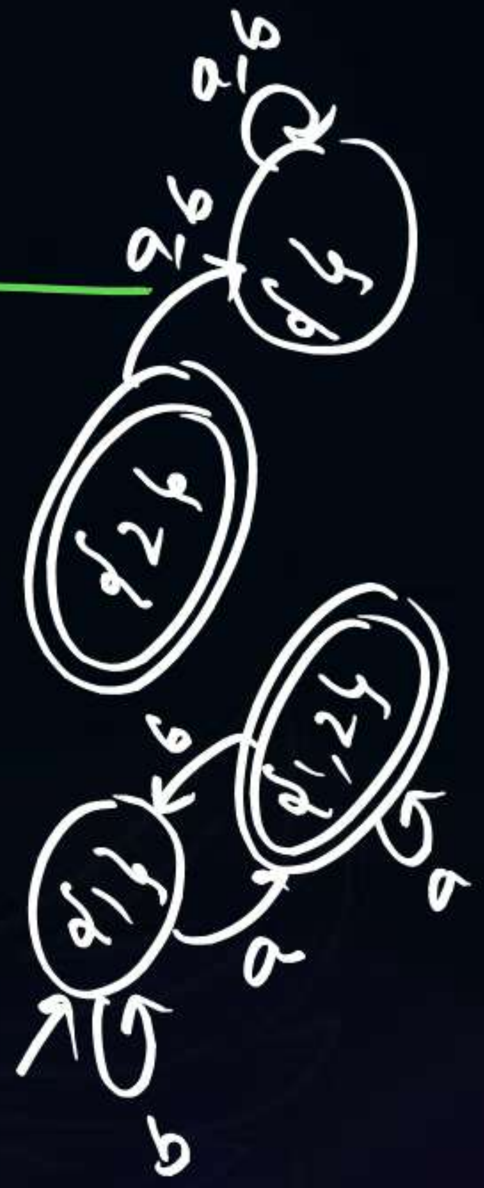


NFA

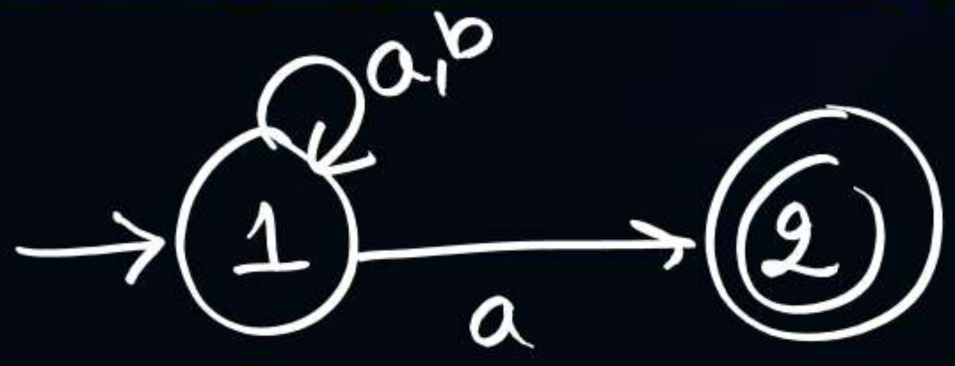
δ	a	b
$\rightarrow 1$	$\{1,2\}$	$\{1\}$
^{*2}	ϕ	ϕ

Subset construction $\Longrightarrow$

δ_{DFA}	a	b
$\{\ \}$	$\{\ \}$	$\{\ \}$
$\rightarrow \{1\}$	$\{1,2\}$	$\{1\}$
$*\{2\}$	$\{\ \}$	$\{\ \}$
$*\{1,2\}$	$\{1,2\}$	$\{1\}$



Conversion From NFA to DFA :

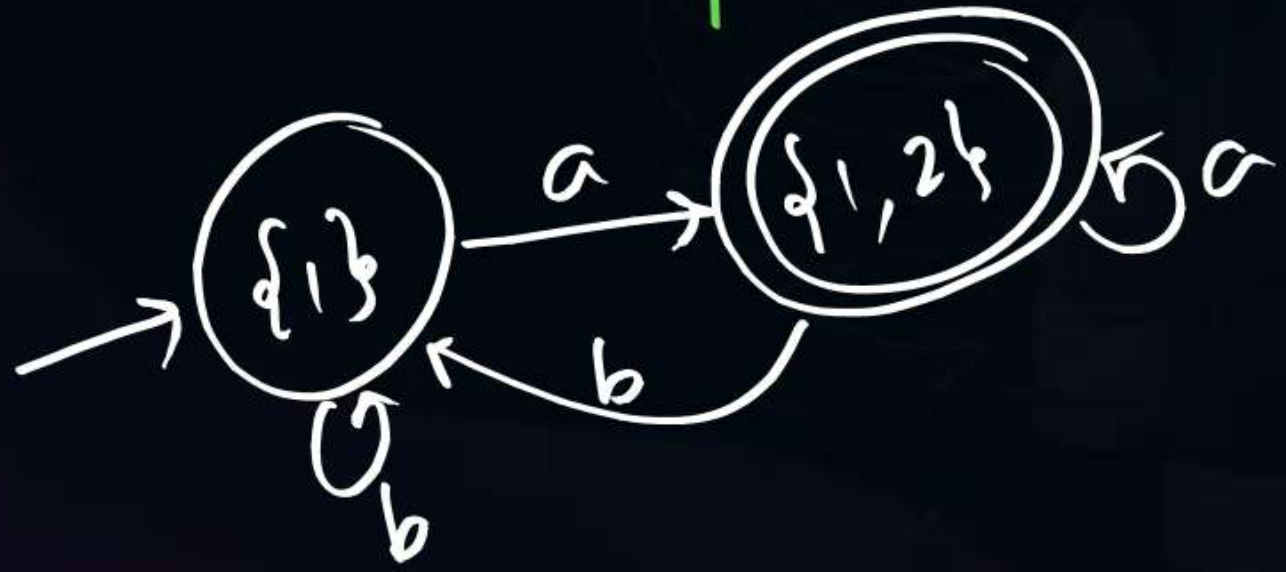


NFA

δ_N	a	b
$\rightarrow 1$	{1,2}	{1}
$*2$	ϕ	ϕ

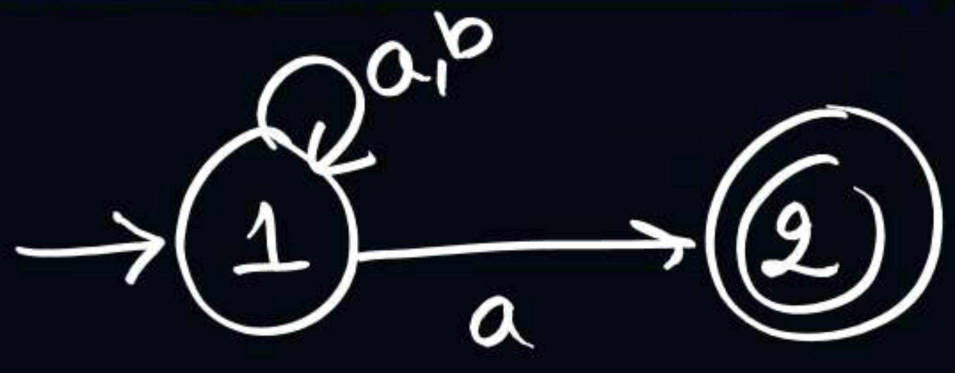
Subset construction $\Longrightarrow$

δ_{DFA}	a	b
$\rightarrow$ {1}	{1,2}	{1}
$*$ {1,2}	{1,2}	{1}



$$\delta_D(\{1,2\},a) = \delta_N(1,a) \cup \delta_N(2,a) = \{1,2\} \cup \phi = \{1,2\}$$

Conversion From NFA to DFA :



NFA

δ	a	b
$\rightarrow 1$	{1,2}	{1}
$*2$	ϕ	ϕ

Subset construction $\Longrightarrow$

δ_{DFA}	a	b
$\rightarrow$ {1}	{1,2}	{1}
$*$ {1,2}	{1,2}	{1}

I) Initial state of DFA is same as initial of NFA

II) $\delta_{DFA}(\{p,q\},x) = \delta_{NFA}(p,x) \cup \delta_{NFA}(q,x)$

III) If any subset contain final of NFA then make it as final in DFA

$$\text{NFA} \xrightarrow{\text{subset construction}} \text{DFA}$$

n states 2^n states

$$\text{NFA} \Longrightarrow \text{Min DFA}$$

n states   ($\le 2^n$ states)

Maximum 2^n states

Note: If NFA has 100 (n) states then how many states in min DFA?

⇓

atmost 2^{100} states

(2^n)

NFA with ε moves:

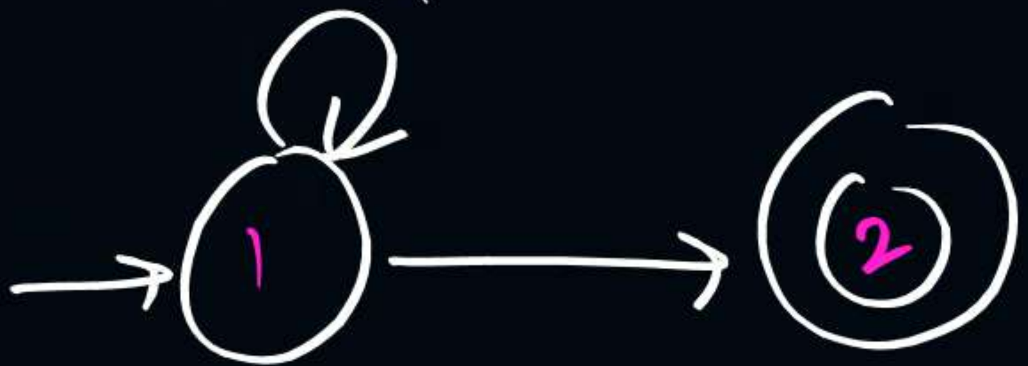
1)

$\varepsilon: 1$

$1 \xrightarrow{\varepsilon} 2$

$L = \{\varepsilon\}$

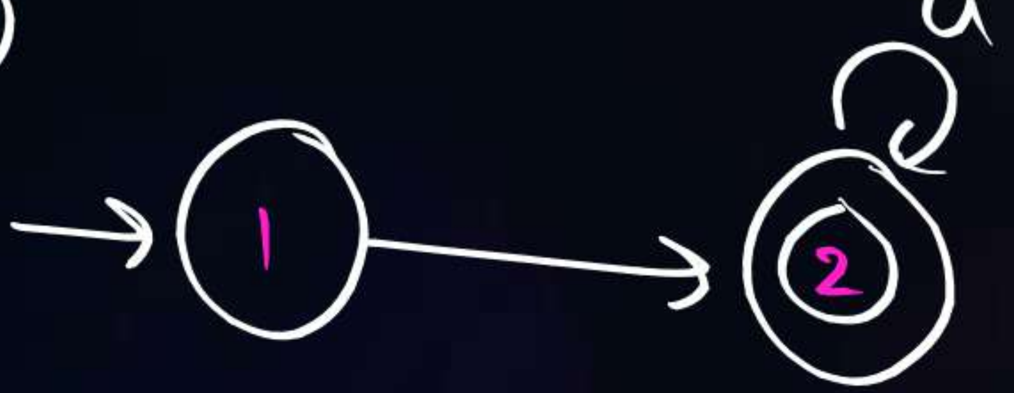
3)



$L = a^*$

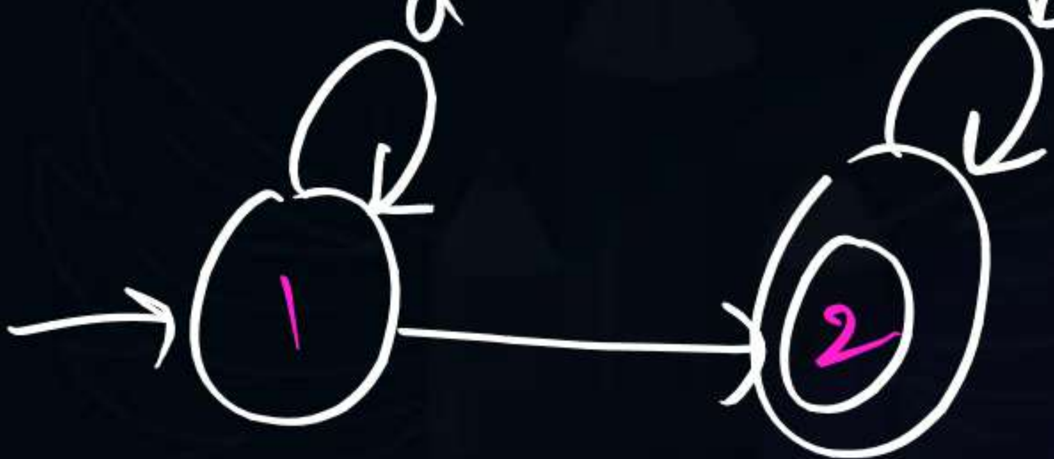
$W = aa \quad 1 \xrightarrow{\varepsilon} 2 \xrightarrow{a} 2 \xrightarrow{a} 2$

2)



$L = a^*$

4)



$L = a^* b^*$

$1 \longrightarrow 2$

$\underbrace{\delta}_{\text{Transition}}(1, \varepsilon) = 2$

$\underbrace{\hat{\delta}}_{\text{path}}(1, \varepsilon) = \{1, 2\}$

we can reach 2 from 1 $\underbrace{\text{without reading any symbol}}_{\text{(by reading } \varepsilon)}$

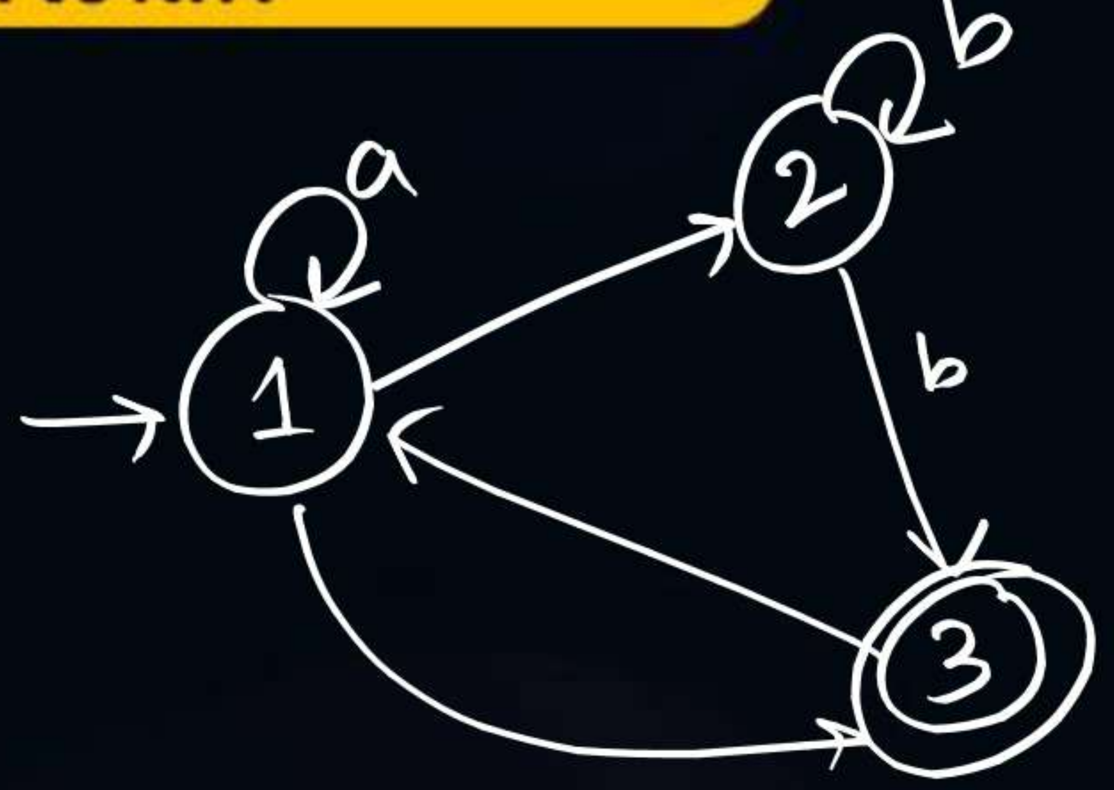
ε has 2 paths:

I) $1 \xrightarrow{\varepsilon} 2$

II) 1

$\delta(1, \varepsilon) = 2$

$\hat{\delta}(1, \varepsilon) = 1$ or 2



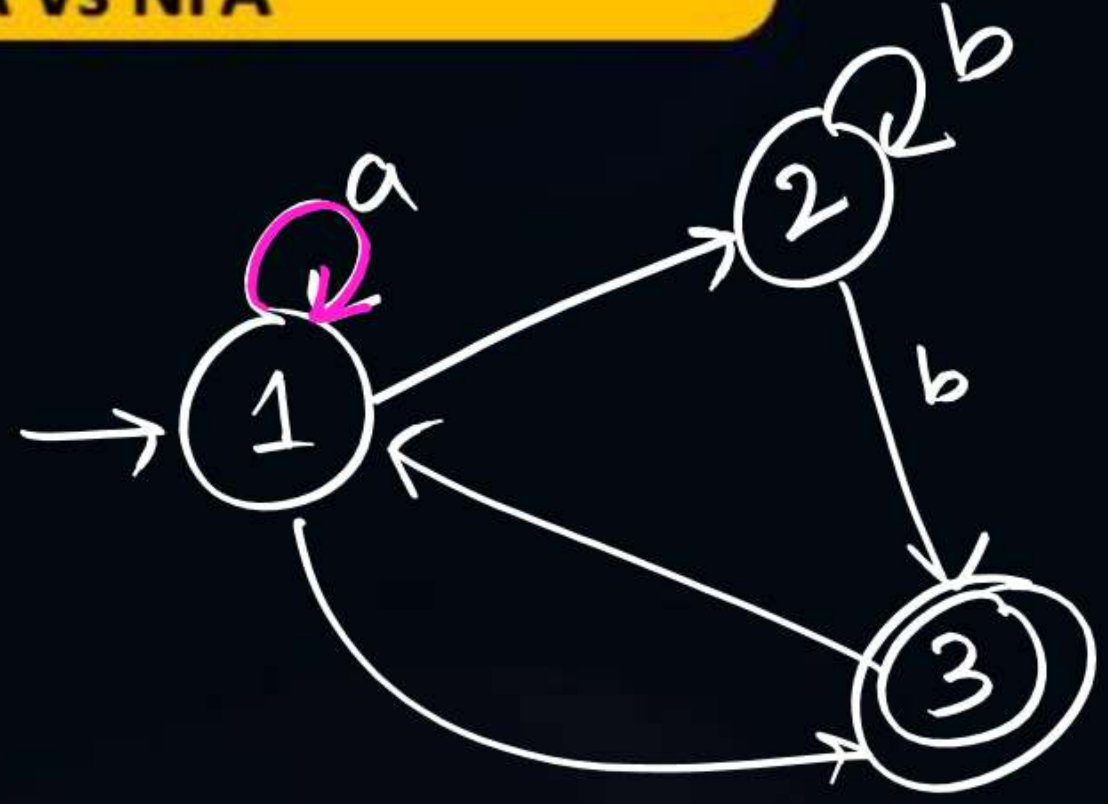
ε-NFA

δ	a	b	ε
→ 1	{1}	{ }	{2, 3}
2	{ }	{2, 3}	{ }
*3	{ }	{ }	{1}

Transition Table

ε-NFA :

$$\delta : \underbrace{Q}_{\text{rows}} \times \underbrace{\Sigma \cup \{\varepsilon\}}_{\text{columns}} \rightarrow \underbrace{2^{Q}}_{\text{entries}}$$



ε-NFA

δ	a	b	ε
→1	{1}	{ }	{2,3}
2	{ }	{2,3}	{ }
*3	{ }	{ }	{1}

Transition Table

Extended Transition

I) $\hat{\delta}(1, a) = \{1,2,3\}$

$1 \xrightarrow{a} 1$

$1 \xrightarrow{a} 1 \xrightarrow{\varepsilon} 2$

$1 \xrightarrow{a} 1 \xrightarrow{\varepsilon} 3$

$1 \xrightarrow{a} 1 \xrightarrow{\varepsilon} 3 \xrightarrow{\varepsilon} 1$

$1 \xrightarrow{a} 1 \xrightarrow{\varepsilon} 3 \xrightarrow{\varepsilon} 1 \xrightarrow{\varepsilon} 2$

II) $\delta(1, a) = \{1\}$

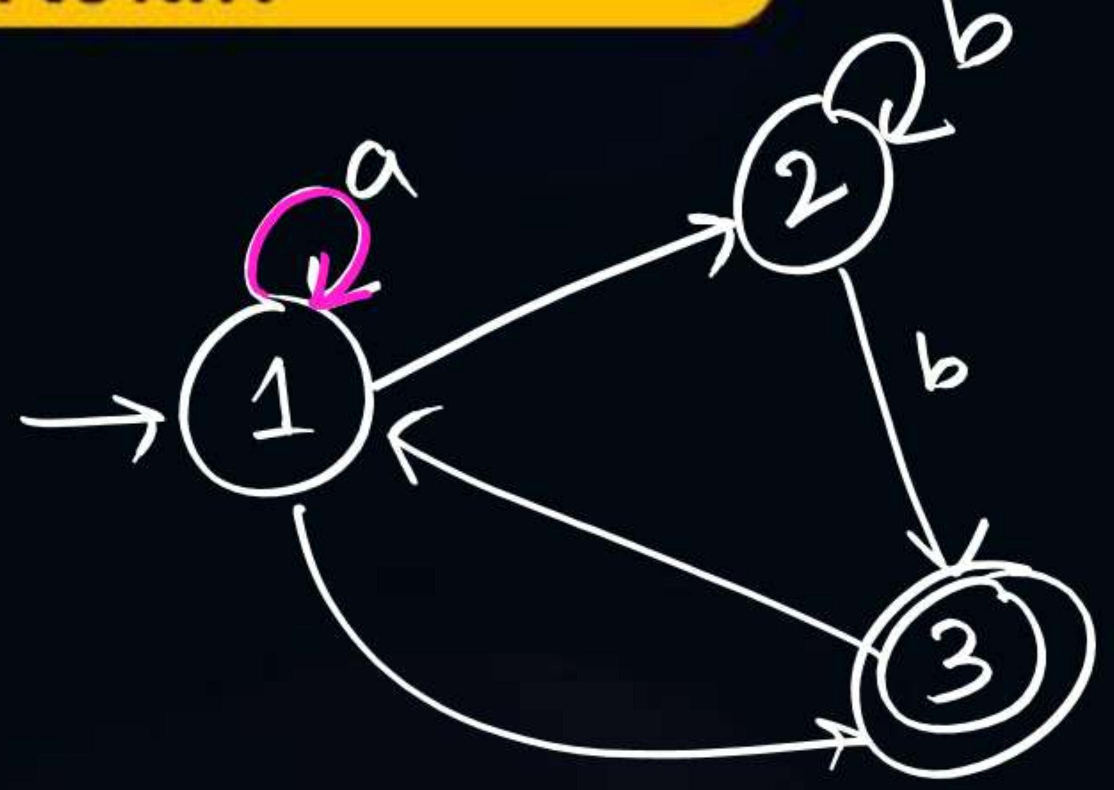
Transition

ε-closure(q) : Epsilon closure of state q

$= \{ x \mid x$ is a state reachable from q by reading $\varepsilon \}$

= Set of all states reachable by reading ε from q

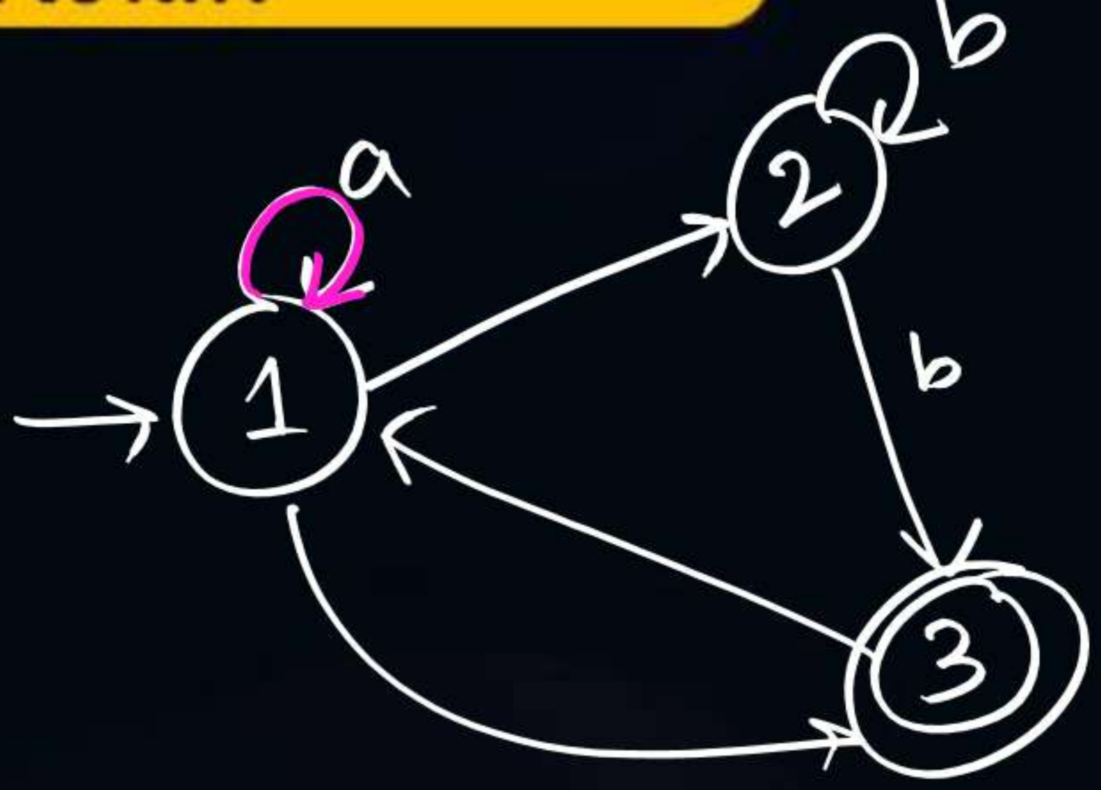
DFA Vs NFA



$\varepsilon\text{-closure}(1) = \{1, 2, 3\}$

$\varepsilon\text{-closure}(2) = \{2\}$

$\varepsilon\text{-closure}(3) = \{3, 1, 2\}$



$$\hat{\delta}(2, bb) = \{1, 2, 3\}$$

$2 \xrightarrow{b} 2 \xrightarrow{b} 2$ ✓

$2 \xrightarrow{b} 2 \xrightarrow{b} 3$ ✓

$2 \xrightarrow{b} 2 \xrightarrow{b} 3 \xrightarrow{\varepsilon} 1$ ✓

$2 \xRightarrow{bb} 2$ ✓

$2 \xRightarrow{bb} 3$ ✓

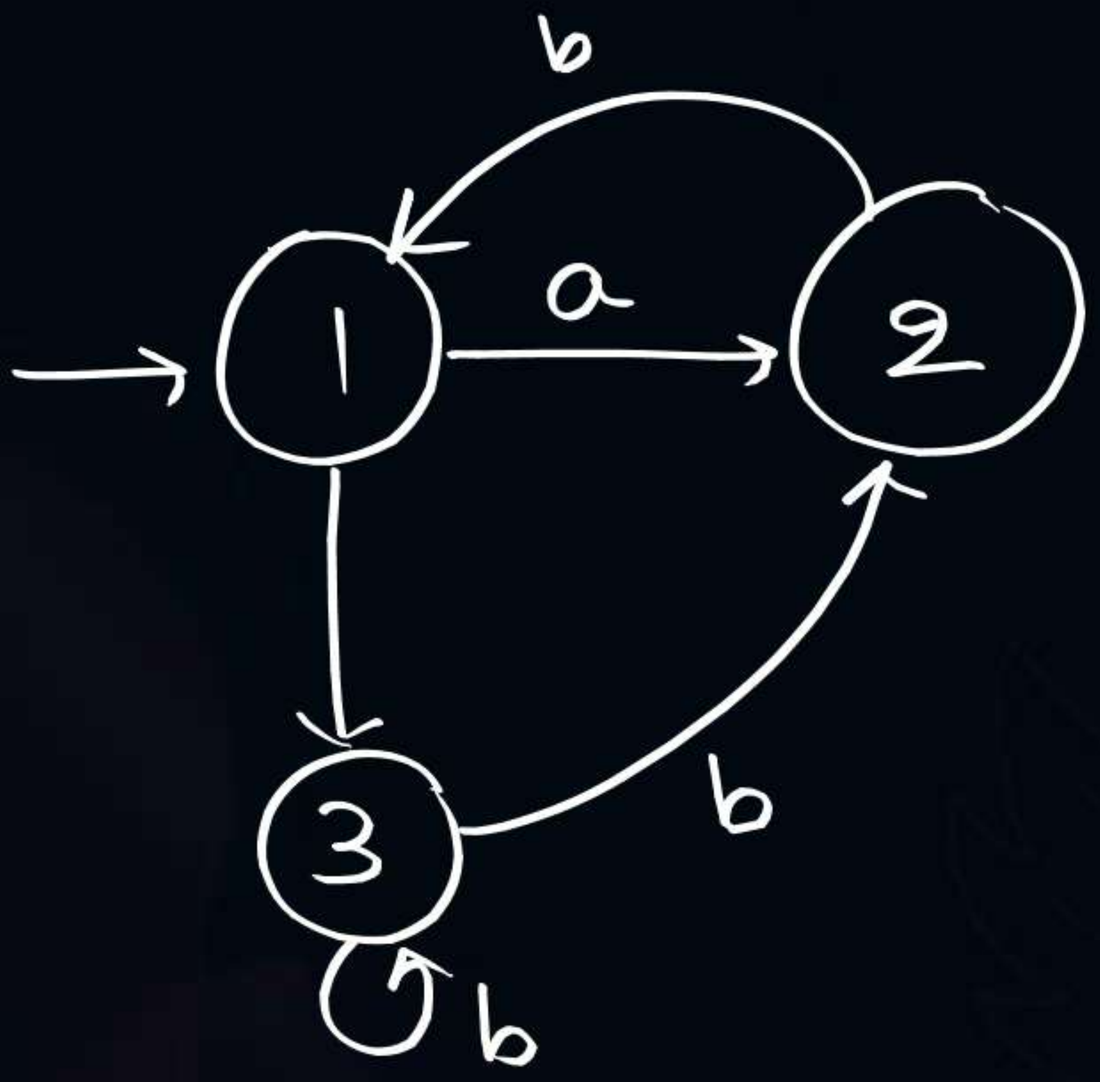
$2 \xRightarrow{bb} 1$ ✓

$$\hat{\delta}(1, abb) = \{2, 3, 1\}$$

$$\hat{\delta}(3, ab) = \{2, 3, 1\}$$

$$\hat{\delta}(2, ab) = \phi$$

H.W.:



i) $\varepsilon\text{-clo}(1) = ?$

ii) $\varepsilon\text{-clo}(2) = ?$

iii) $\varepsilon\text{-clo}(3) = ?$

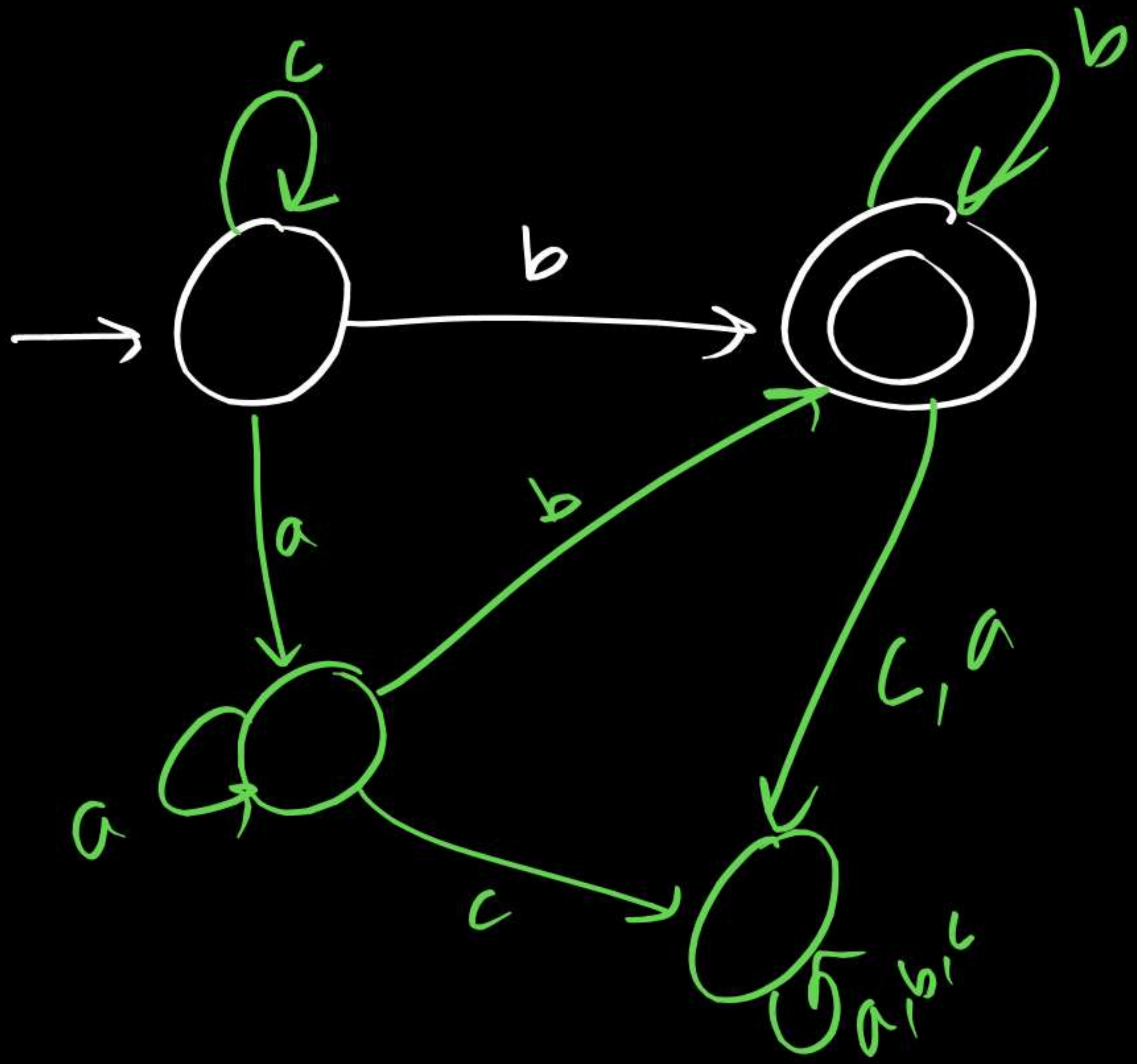
iv) $\hat{\delta}(1, ab) = ?$

v) $\hat{\delta}(2, bb) = ?$

vi) $\hat{\delta}(3, bbb) = ?$

$L = c^* a^* b^+$

$\boxed{\text{Min} = b}$



2 mins Summary

Topic	DFA Vs NFA ✓
Topic	NFA Construction ✓
Topic	Conversion from NFA to DFA ✓
Topic	NFA with epsilon Moves ✓

THANK - YOU