

Computer Science

Theory of Computation

Regular Languages & Non Regular Languages

Lecture No.- 4

A man with a beard and mustache, wearing a black polo shirt, stands with his arms crossed in front of a bookshelf. The background is slightly blurred, showing various books on the shelves.

Malleham Devasane Sir

Recap of Previous Lecture



Topic

Regular Languages



Topic

Non-regular Languages



Topics to be Covered



Topic

Regular Grammar



Regular Language (Type-3 Language)

- It is a language represented by regular expression
- " " " " FA
- " " " " Regular Grammar (Type-3 Grammar)

Grammar $(G) = (V, T, P, S)$

→ start symbol

→ Set of production rules

→ Set of terminals

→ Set of variables
(non-terminals)

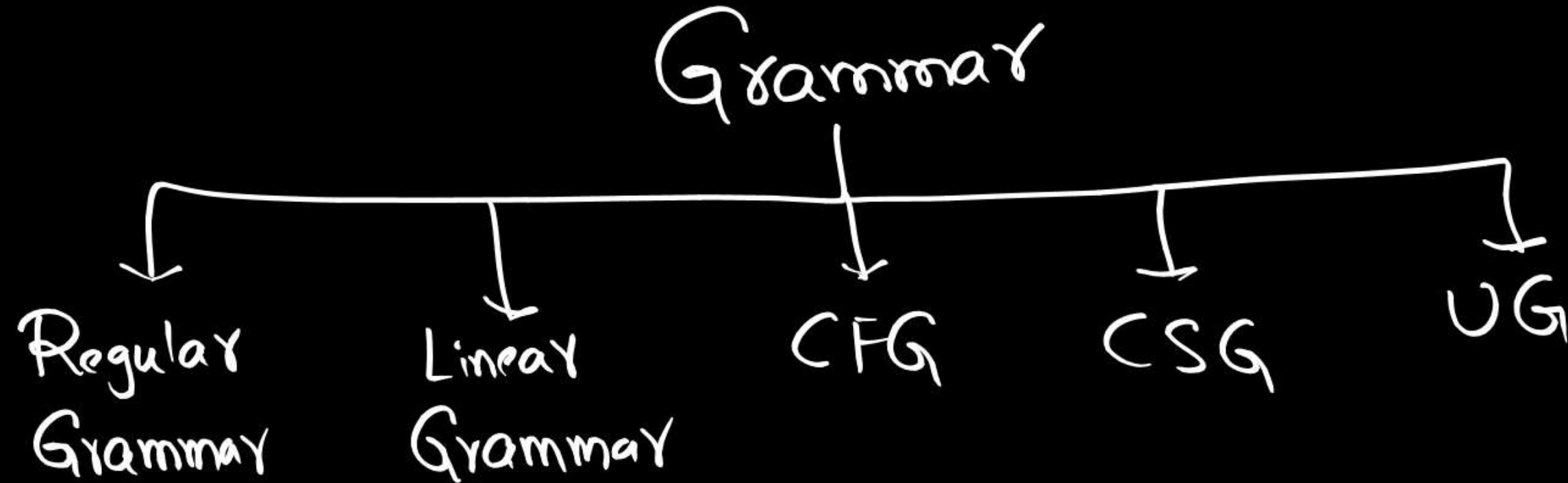
$S \rightarrow a \mid bS$

$V = \{S\}$ $P = \{S \rightarrow a, S \rightarrow bS\}$

$T = \{a, b\}$ $S = S$

Grammar

↳ It is set of rules which represents a language.



$$V = \{S, A\} = S + A$$

$$T = \{a, b, c\} = a + b + c$$

VV
 SS
 SA
 AS
 AA

TTT
 $a a a$
 $a a b$
 $a a c$
 $a b a$
 $a b b$
 $a b c$
 $:$
 $c c c$

VT
 Sa
 Sb
 Sc
 Aa
 Ab
 Ac

$V T^*$	$T^* V$
V	V
VT	TV
VTT	TTV
$VTTT$	$TTTV$

Regular Grammar



↳ It is RLG or LLG

Right Linear Grammar

Each rule appears in

$$V \rightarrow T^* V \mid T^*$$

$$S \rightarrow abA$$

$$A \rightarrow aS \mid \epsilon \mid ab$$

RLG

Left Linear Grammar

$$V \rightarrow VT^* \mid T^*$$

$$S \rightarrow Sab \mid \epsilon$$

Linear Grammar

$$V \rightarrow T^* V T^* \text{ or } T^*$$

$$S \rightarrow aaSb \mid c$$

$$\textcircled{1} \quad S \rightarrow \varepsilon \quad \overset{\checkmark}{\text{I}}, \overset{\checkmark}{\text{II}}, \overset{\checkmark}{\text{III}}$$

$$\textcircled{2} \quad S \rightarrow S | ab \quad \overset{\checkmark}{\text{I}}, \overset{\checkmark}{\text{II}}, \overset{\checkmark}{\text{III}}$$

$$\textcircled{3} \quad S \rightarrow \underbrace{aS} | \underbrace{abS} | \underbrace{cd} | \underbrace{\varepsilon} \quad \overset{\checkmark}{\text{I}}, \overset{\times}{\text{II}}, \overset{\checkmark}{\text{III}}$$

$$\textcircled{4} \quad S \rightarrow aS | bba | Saa \quad \overset{\times}{\text{I}}, \overset{\times}{\text{II}}, \overset{\times}{\text{III}}$$

$$\textcircled{1} \quad S \rightarrow a \quad L = L(S) = \{a\}$$

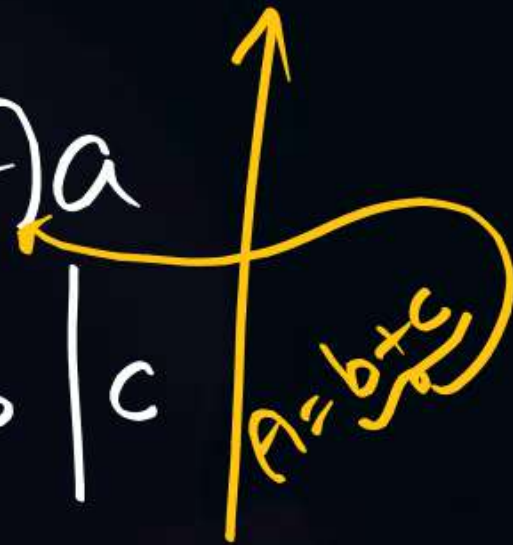
$$\textcircled{2} \quad S \rightarrow ab \mid \epsilon \quad L = L(S) = \{\epsilon, ab\}$$

$$\textcircled{3} \quad A \rightarrow a \mid b \mid \epsilon \quad L = L(A) = \{\epsilon, a, b\}$$

$$\textcircled{4} \quad S \rightarrow ab \mid aaa \mid a \quad L = \{a, ab, aaa\}$$

$$\textcircled{5} \quad \begin{array}{l} S \rightarrow Aa \\ A \rightarrow b \end{array}$$

$$L = L(S) = \{ba\}$$

$$\textcircled{6} \quad \begin{array}{l} S \rightarrow Aa \\ A \rightarrow b \mid c \end{array}$$


$$\begin{aligned} L &= (b+c)a \\ &= \{ba, ca\} \end{aligned}$$

⑦ $\boxed{P} \xrightarrow{\text{start}} aQ$
 $Q \rightarrow x \mid \epsilon$

$$L = L(P) = \{a, ax\}$$

$$Q = x + \epsilon$$

$$P = a.Q = a(x + \epsilon) \\ = ax + a$$

$$(8) \quad S \rightarrow Aa$$

A is useless

$$L = \phi = \{ \}$$

$$(9) \quad S \rightarrow Aa \mid \cancel{Bb}$$

$$A \rightarrow d$$

$$L = \{ da \}$$

$$(10) \quad S \rightarrow Aa \mid \cancel{Bb}$$

$$A \rightarrow d$$

$$\boxed{C \rightarrow e}$$

$$L = S = Aa = da$$

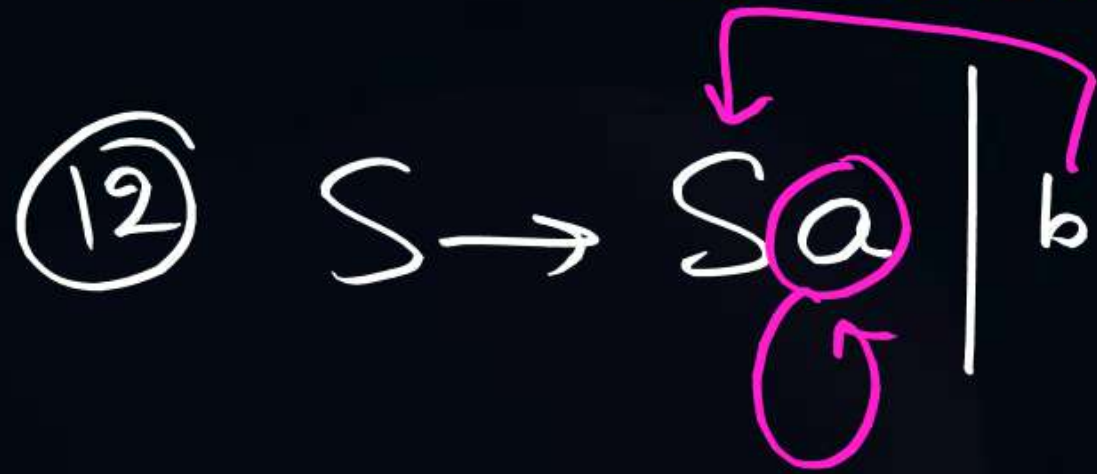
$$L = \{ da \}$$

$A \rightarrow Aab$
 Left Recursion

$A \rightarrow aabA$
 Right Recursion

$$(11) \quad S \rightarrow Sa$$

$$L = \emptyset = \{ \}$$

$$(12) \quad S \rightarrow Sa \mid b$$


$$L = ba^*$$

$$b \checkmark = ba^0$$

$$ba \checkmark = ba^1$$

$$baa \checkmark = ba^2$$

$$baaa \checkmark = ba^3$$

S
 Sa
 Saa
 $Saaa$

$$(13) \quad S \rightarrow S \overset{Q}{\underbrace{a}} | \epsilon$$

$$L = \epsilon a^* = a^*$$

$$(14) \quad S \rightarrow Sa | a$$

$$L = aa^* = a^+ = a^+$$

$$(15) \quad S \rightarrow Saa | \epsilon$$

$$L = (aa)^* = \{a^n \mid n = \text{even}\}$$

$$(16) \quad S \rightarrow Saa | a$$

$$L = \{a^n \mid n = \text{odd}\} = a(aa)^* = (aa)^*a$$

$$X \rightarrow X \boxed{\alpha} \mid \beta$$

The diagram shows a handwritten grammar rule $X \rightarrow X \boxed{\alpha} \mid \beta$. A green box highlights the α and the vertical bar. A green arrow points from the bottom of the box to the X on the left, indicating a recursive derivation.

$$L = \beta \alpha^*$$

$$(17) \quad S \rightarrow \textcircled{a} S \mid b$$

$$L = a^* b$$

$$(18) \quad S \rightarrow \textcircled{a} S \mid \epsilon$$

$$L = a^*$$

$$(19) \quad S \rightarrow \textcircled{a} S \mid a$$

$$L = a^* a = a^+$$

$$(20) \quad S \rightarrow \textcircled{ab} S \mid \epsilon$$

$$L = (ab)^*$$

$$(21) \quad S \rightarrow \textcircled{aa} S \mid a$$

$$L = (aa)^* a = a(aa)^*$$

$$= \{a^n \mid n = \text{odd}\}$$

(22) $S \rightarrow Sa \mid b \mid c$

Diagram illustrating the derivation of the language $L = (b+c)a^*$ from the grammar $S \rightarrow Sa \mid b \mid c$. The expression Sa is underlined, and b and c are grouped under a brace labeled $b+c$. An arrow points from Sa to Sa^* , and another arrow points from the $b+c$ group to Sa^* .

$$L = (b+c)a^*$$

$$= ba^* + ca^*$$

(23) $S \rightarrow Sa \mid b \mid c \mid d$

Diagram illustrating the derivation of the language $L = (b+c+d)a^*$ from the grammar $S \rightarrow Sa \mid b \mid c \mid d$. The expression Sa is underlined, and b , c , and d are grouped under a brace labeled $b+c+d$. An arrow points from Sa to Sa^* , and another arrow points from the $b+c+d$ group to Sa^* .

$$L = (b+c+d)a^*$$

(24)

$$S \rightarrow S\underbrace{a}_{\text{in}} \mid S\underbrace{b}_{\text{in}} \mid c$$

$$S(\underbrace{a+b}_{\text{in}})$$

$$= c(a+b)^*$$

c ✓

ca ✓

cb ✓

caa ✓

cab ✓

cba ✓

cbb ✓

⋮

$$(25) \quad S \rightarrow Sa \mid Sb \mid \varepsilon$$

$$L = (a+b)^*$$

$$(26) \quad S \rightarrow aS \mid bS \mid \varepsilon$$

$$L = (a+b)^*$$

$$(27) \quad S \rightarrow Sa \mid Sb \mid a$$

$$L = a(a+b)^*$$

$$(28) \quad S \rightarrow Sa \mid Sb \mid b$$

$$L = b(a+b)^*$$

(29) $S \rightarrow Sa | Sb | a | b$ $L = (a+b)(a+b)^* = (a+b)^+$

(Note: In the original image, a bracket under 'Sa' is labeled 'S(a+b)' and a bracket under 'a|b' is labeled 'a+b'.)

(30) $S \rightarrow aS | bS | a | b$ $L = (a+b)^+$

(31) $S \rightarrow aS | aaS | \epsilon$ $L = (a+aa)^* = a^*$

(32) $S \rightarrow aS | aaaS | \epsilon$ $L = a^*$

(33)

$$S \rightarrow Aa$$

$$A \rightarrow Aa | Ab | \epsilon$$

$$\left[A = (a+b)^* \right]$$

$$\left. \vphantom{\left[A = (a+b)^* \right]} \right\} L = S = Aa = (a+b)^* a$$

(34)

$$S \rightarrow aA$$

$$A \rightarrow aA | bA | \epsilon$$

$$L = a(a+b)^*$$

(35)

$$S \rightarrow aA \mid bA$$

$$A \rightarrow bB$$

$$B \rightarrow aB \mid bB \mid \epsilon$$

$$B = (a+b)^*$$

$$A = bB = b(a+b)^*$$

$$S = (a+b)A$$

$$= (a+b)b(a+b)^*$$

$$= (ab+bb)(a+b)^*$$

$$\textcircled{36} \quad S \rightarrow aA \mid bA \\ A \rightarrow a \mid b$$

$$L = \{w \mid w \in \{a, b\}^*, |w| = 2\} \\ = (a+b)^2$$

$$\textcircled{37} \quad S \rightarrow aA \mid bA \\ A \rightarrow aB \mid bB \\ B \rightarrow aB \mid bB \mid \epsilon$$

$$L = (a+b)^2 (a+b)^* \\ = \{w \mid w \in \{a, b\}^*, |w| \geq 2\}$$

$$\textcircled{38} \quad S \rightarrow aA \mid bA \mid \epsilon \\ A \rightarrow a \mid b \mid \epsilon$$

$$L = (a+b+\epsilon)^2$$

(39) $S \rightarrow bS \mid aA$
 $A \rightarrow bA \mid aB$
 $B \rightarrow bB \mid \epsilon$



$$B = b^*$$

$$A \rightarrow bA \mid ab^*$$

$$A = b^*ab^*$$

$$L = \{w \mid w \in \{a, b\}^*, n_a(w) = 2\}$$

$$S \rightarrow \underbrace{b}_{\uparrow} S \mid \underbrace{ab^*ab^*}_{\uparrow}$$

$$S = L = b^*ab^*ab^*$$

*** (40)

$$S \rightarrow aA$$

$$A \rightarrow bS \mid c$$

$$A = bS + c \rightarrow \textcircled{1}$$

$\hookrightarrow S$ will be solved later

$$S \rightarrow a \textcircled{A}$$

$$S \rightarrow a(bS + c)$$

$$S \rightarrow \textcircled{ab}S \mid \textcircled{ac}$$

$$L = (ab)^* ac$$

④

$$S \rightarrow Aa/f$$

$$A \rightarrow Sb/Bc$$

$$B \rightarrow Se$$

$$\boxed{B = Se} \rightarrow ①$$

$$A = Sb + \textcircled{B}c$$

$$\boxed{A = Sb + Sec} \rightarrow ②$$

$$S \rightarrow \textcircled{A}a/f$$

$$S \xrightarrow{②} (Sb + Sec)a/f$$

$$S \xrightarrow{③} S\textcircled{ba} | S\textcircled{eca} | f$$

$$L = f(bat\textcircled{e}ca)^*$$

H.W.

(42)

$$S \rightarrow aA$$

$$A \rightarrow bB$$

$$B \rightarrow aB \mid bB \mid \epsilon$$

$$L = ab(a+b)^*$$

(44)

$$S \rightarrow aS \mid A$$

$$A \rightarrow bA \mid \epsilon$$

$$L = a^*b^*$$

(43)

$$S \rightarrow aS \mid bS \mid aA$$

$$A \rightarrow bB$$

$$B \rightarrow \epsilon$$

$$L = (a+b)^*ab$$

(45)

$$S \rightarrow Sb \mid A$$

$$A \rightarrow Aa \mid \epsilon$$

$$L = a^*b^*$$

(46)

$$S \rightarrow bS / bA$$

$$A \rightarrow aA / a$$

$$L = b^+ a^*$$

(49)

$$S \rightarrow aS / bS / A$$

$$A \rightarrow abB$$

$$B \rightarrow aB / bB / \epsilon$$

$$L = (a+b)^* ab(a+b)^*$$

(47)

$$S \rightarrow bS / A$$

$$A \rightarrow aA / a$$

$$L = b^+ a^*$$

(50)

$$S \rightarrow Aa$$

$$A \rightarrow Ba$$

$$B \rightarrow Ba / Bb / a$$

$$L = a(a+b)^* aa$$

(48)

$$S \rightarrow Sa / Ab$$

$$A \rightarrow Ab / b$$

$$L = b^+ a^*$$



2 mins Summary



Topic

Regular Grammar



THANK - YOU