

CS & IT ENGINEERING

Theory of Computation

Regular Languages

Lecture No.- 22



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Recap of Previous Lecture



Topic

Model-I (Easy: Phi, Sigma*, only epsilon, Sigma⁺)

Topic

Model II (Length)

Topic

Model III (No. of symbols)

Topic

Model IV (Over 1 symbol)

Topic

Model V (Sequence based)

Topic

Model VI (Length & Remainder)

Topic

Model VII (Symbols & Remainder)

Topic

Model VIII (Multiple Conditions on symbols)

Topic

Model IX (Start, End , Contain)

Topics to be Covered



Topic

Model X (Position based)

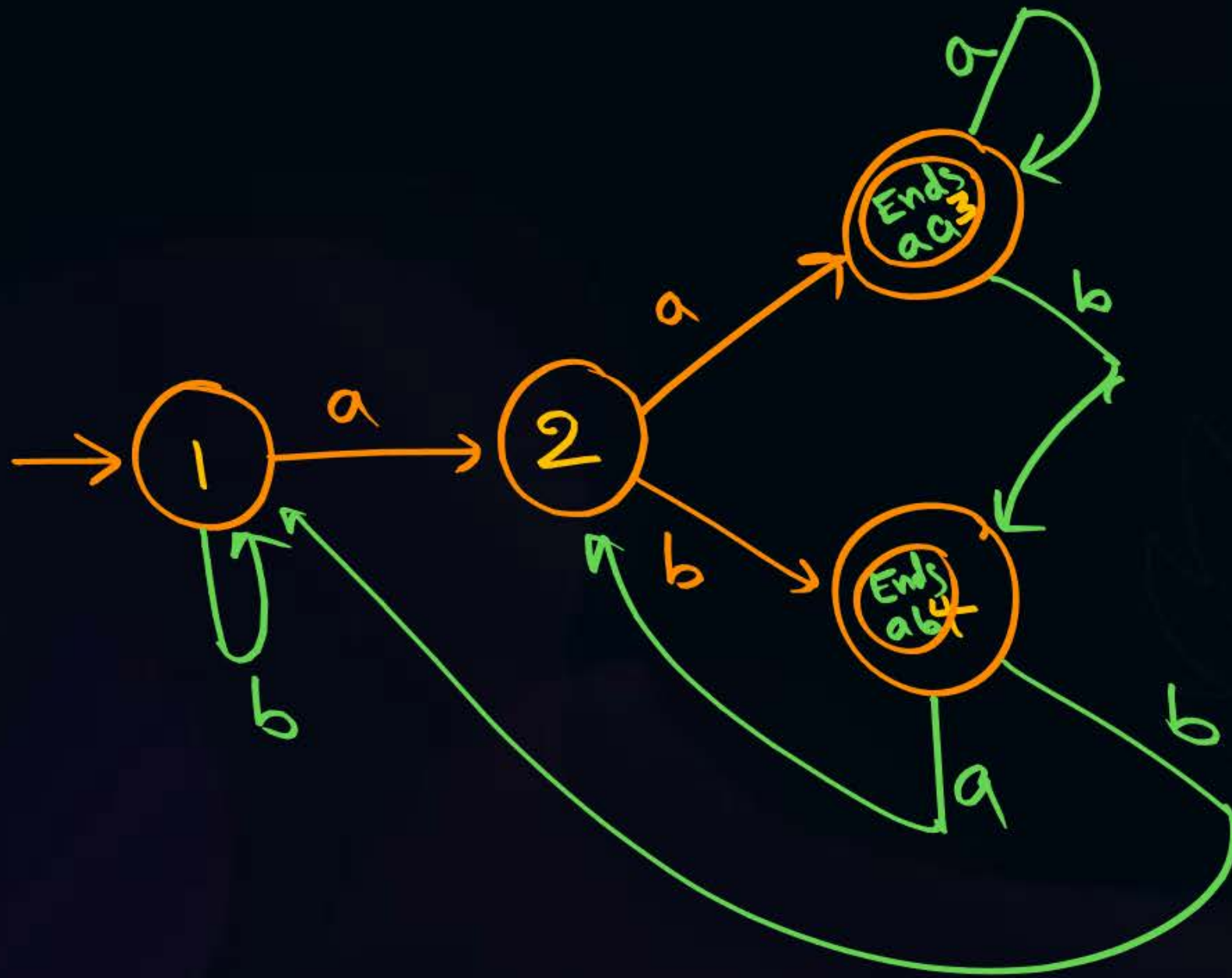
Topic

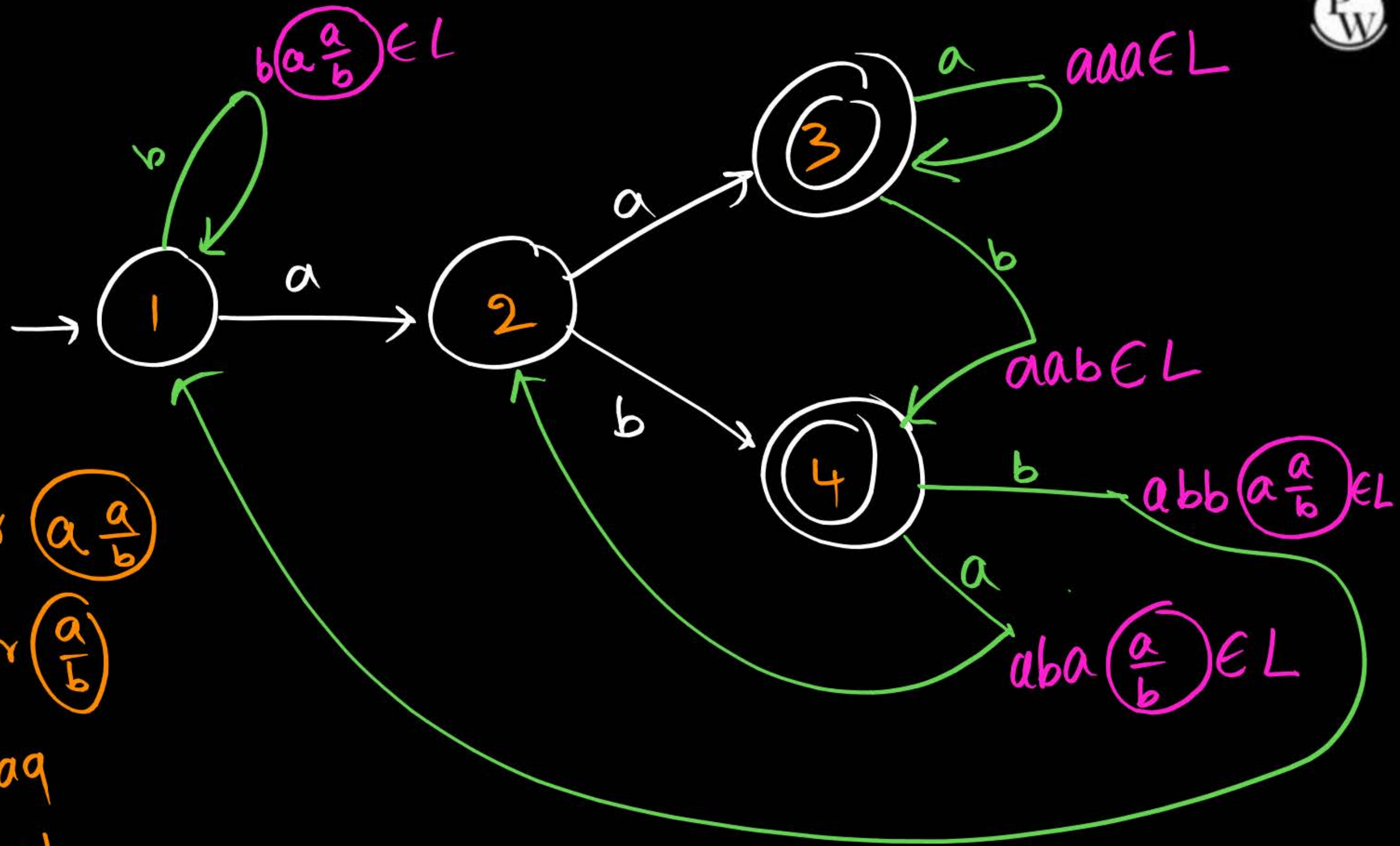
Model XI (Multiple Conditions-Remainder)

Model XII (" " -simple)

(75) $L = (a+b)^* \underline{a} (a+b)$

Min = aa or ab





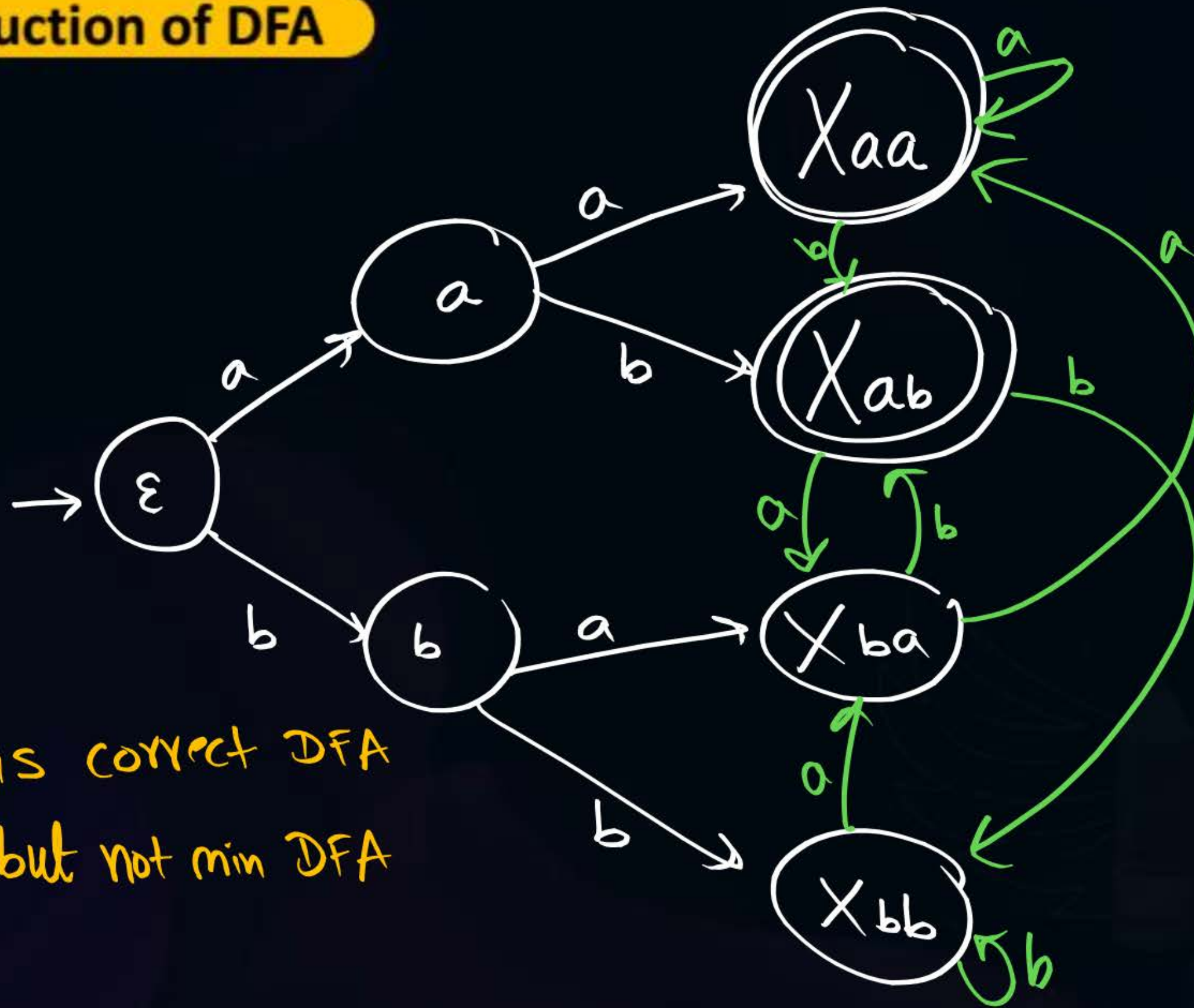
1: waits for $\left(\frac{a}{b} \right)$

2: waits for $\left(\frac{a}{b} \right)$

3: Ends with a

4: " " b

Construction of DFA



It is correct DFA
but not min DFA

aa/a
aa/b
ab/a
ab/b
ba/a
ba/b
bb/a
bb/b

(76)

$$L = (a+b)^* b (a+b)^9$$

10^{10} symbol from end is 'b'
(RHS)



$$= 2^{10} \text{ states}$$

$$= 1024 \text{ states in min DFA}$$

$$(77) \quad L = (a+b)^9 b (a+b)^*$$

10^k symbol from begin is 'b'
(LHS)



$(10+1)+1 = 12$ states in min DFA

Note:

$\Sigma = \{a, b\}$

- I) K^{th} symbol from begin is 'a' $\Rightarrow (K+2)$ states in min DFA
- II) K^{th} symbol from end is 'a' $\Rightarrow 2^K$ states

*** Model - II [Multiple conditions - remainder based]

(78) $\{w \mid w \in \{a,b\}^*, n_a(w) \text{ is div by 2, } n_b(w) \text{ is div by 2}\}$
 even even

(79) $\{w \mid \text{" , " OR " }\}$

(80) $\{w \mid \text{" , " But not " }\}$

(81) $\{w \mid \text{" , } n_b(w) \text{ is div by 2 But not } n_a(w) \text{ is div by 2}\}$

Construction of DFA



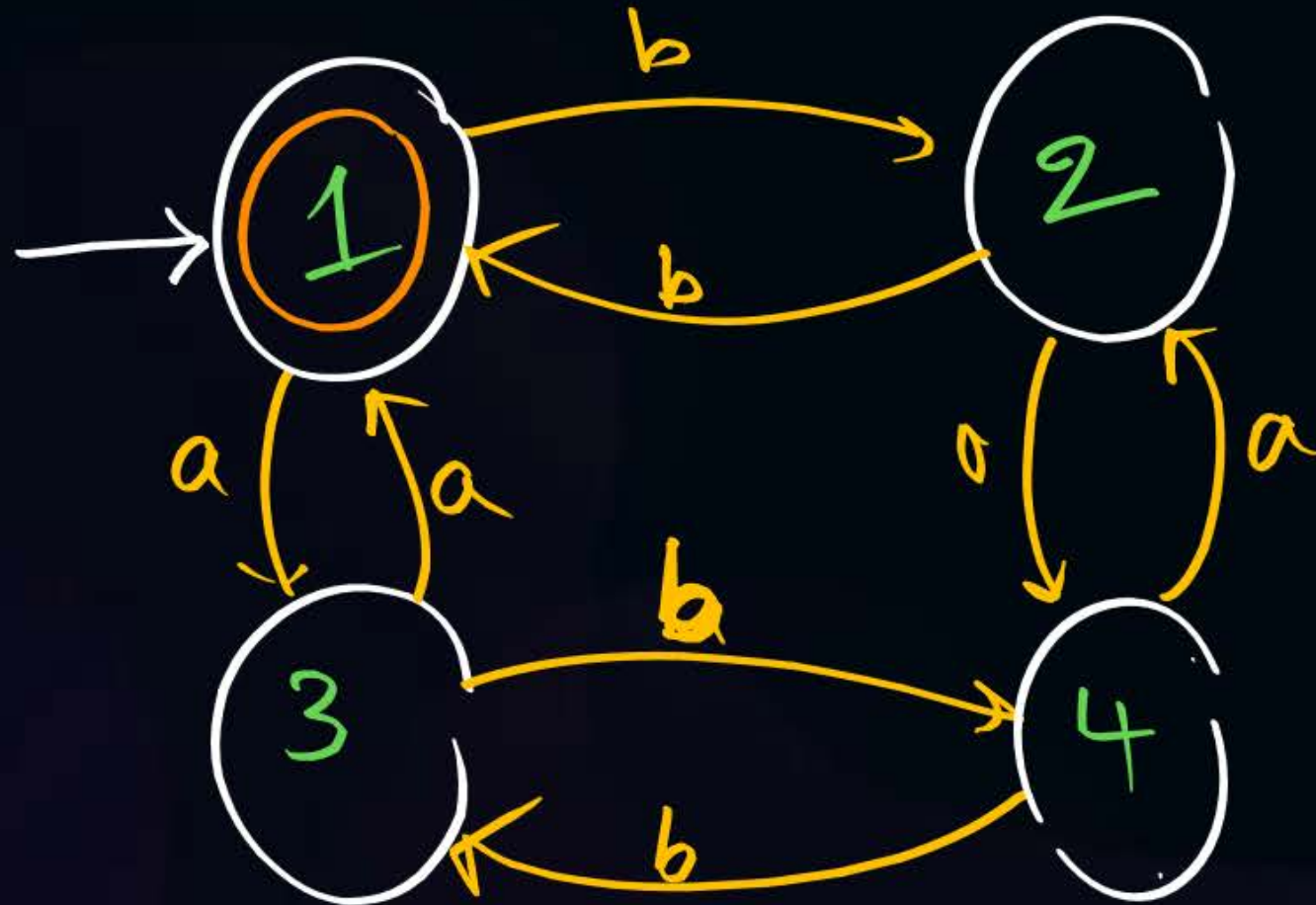
78

Remainders
0, 1

$\#a's \% 2$

AND

$\#b's \% 2$



a_0 : #a's is even
 a_1 : #a's is odd
 b_0 : #b's is even
 b_1 : #b's is odd

Construction of DFA



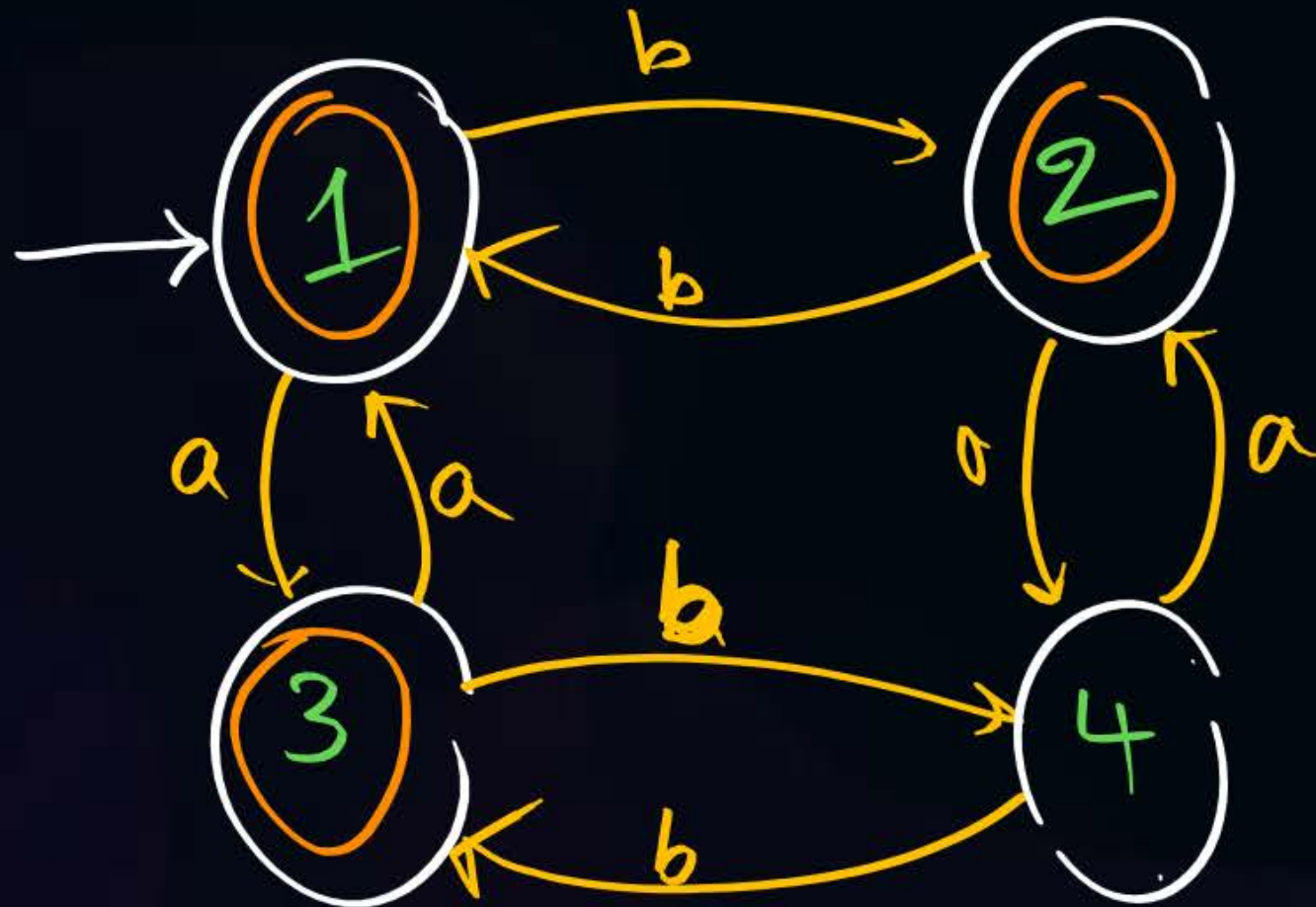
19

Remainders
0, 1

$\#a's \% 2$

OR

$\#b's \% 2$



a_0 : $\#a's$ is even
 a_1 : $\#a's$ is odd
 b_0 : $\#b's$ is even
 b_1 : $\#b's$ is odd

Construction of DFA



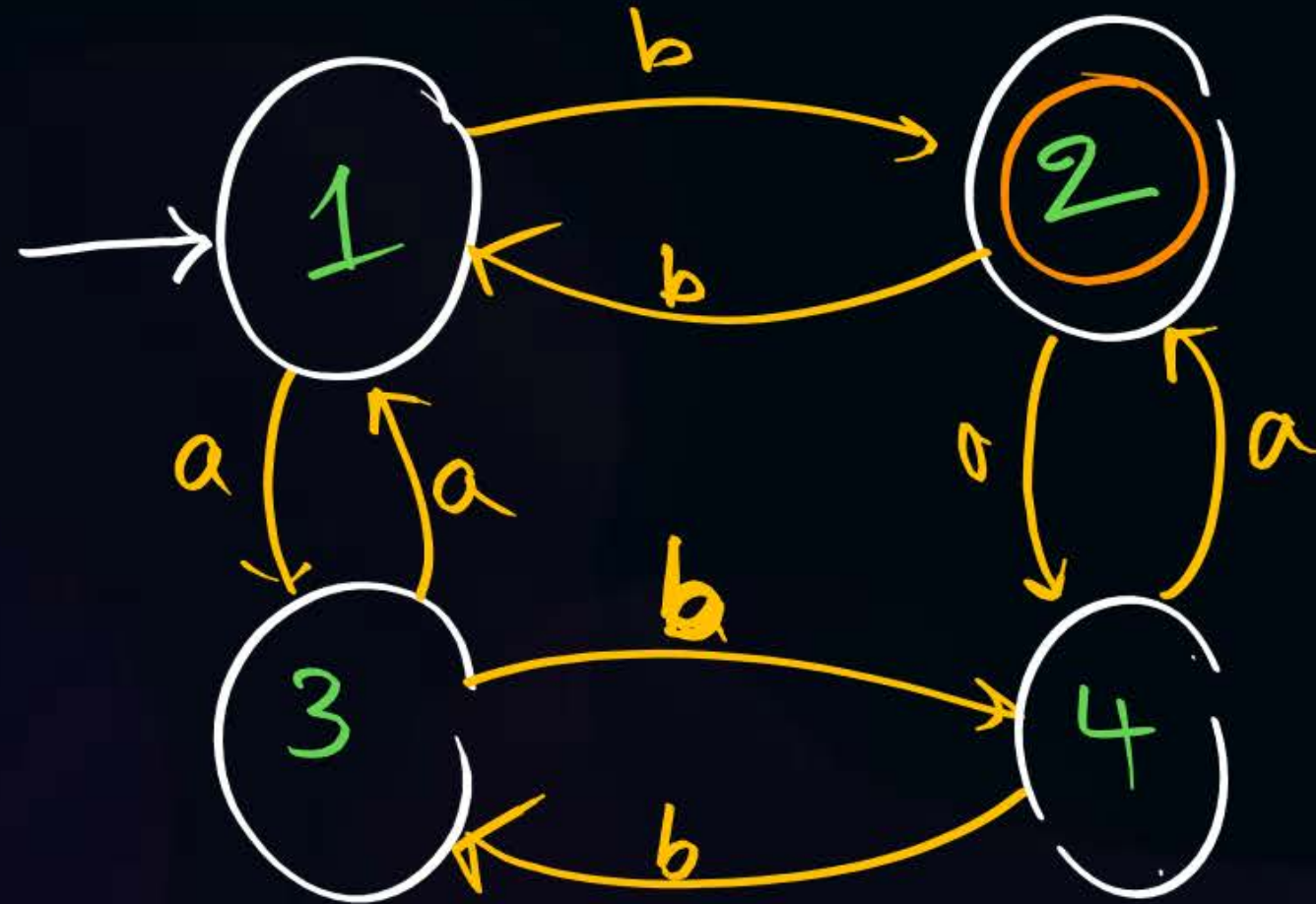
Q^0

Remainders
0, 1

#a's % 2
0, 1

But not

#b's % 2
0, 1



$a_0 b_1$

a_0 : #a's is even

a_1 : #a's is odd

b_0 : #b's is even

b_1 : #b's is odd

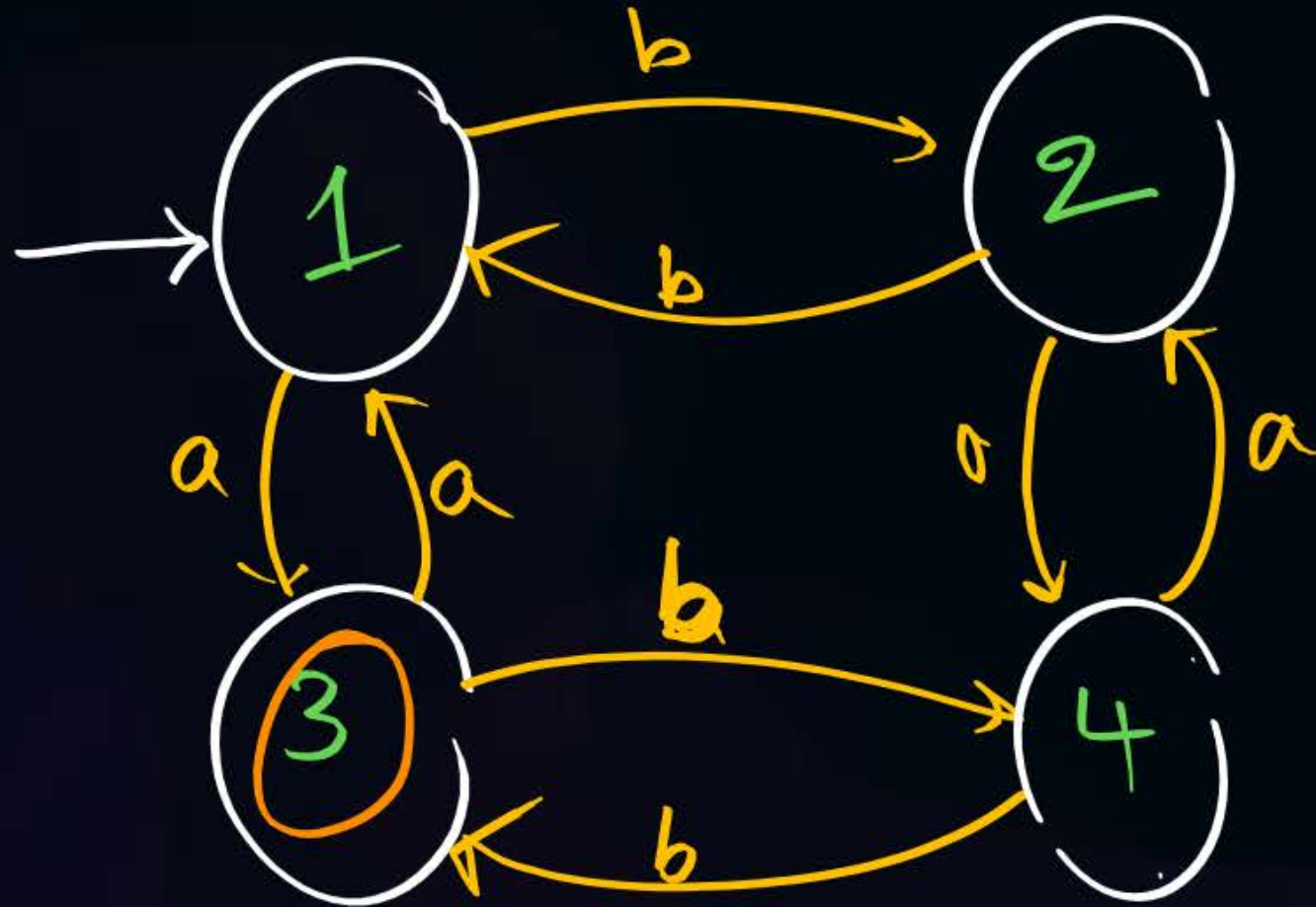
Construction of DFA



(81)

Remainders
0, 1

$\#b's \% 2 = 0$ AND $\#a's \% 2 \neq 0$
 $\#a's \% 2 = 1$



a_0 : $\#a's$ is even
 a_1 : $\#a's$ is odd
 b_0 : $\#b's$ is even
 b_1 : $\#b's$ is odd

(89) $\{w \mid w \in \{a,b\}^*, \underbrace{\#a's \text{ div by } 2}_{\substack{0,1 \\ \uparrow \\ 2 \text{ remainders}}} \text{ AND } \underbrace{\#b's \text{ div by } 3}_{\substack{0,1,2 \\ \uparrow \\ 3 \text{ remainders}}}\}$

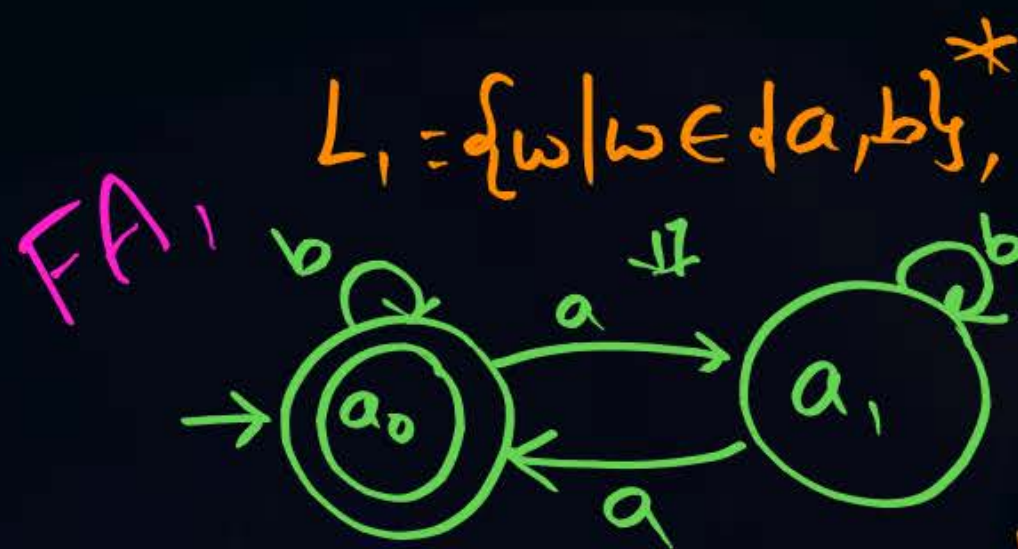
$\Rightarrow 2 \times 3 = 6 \text{ states}$

Construction of DFA

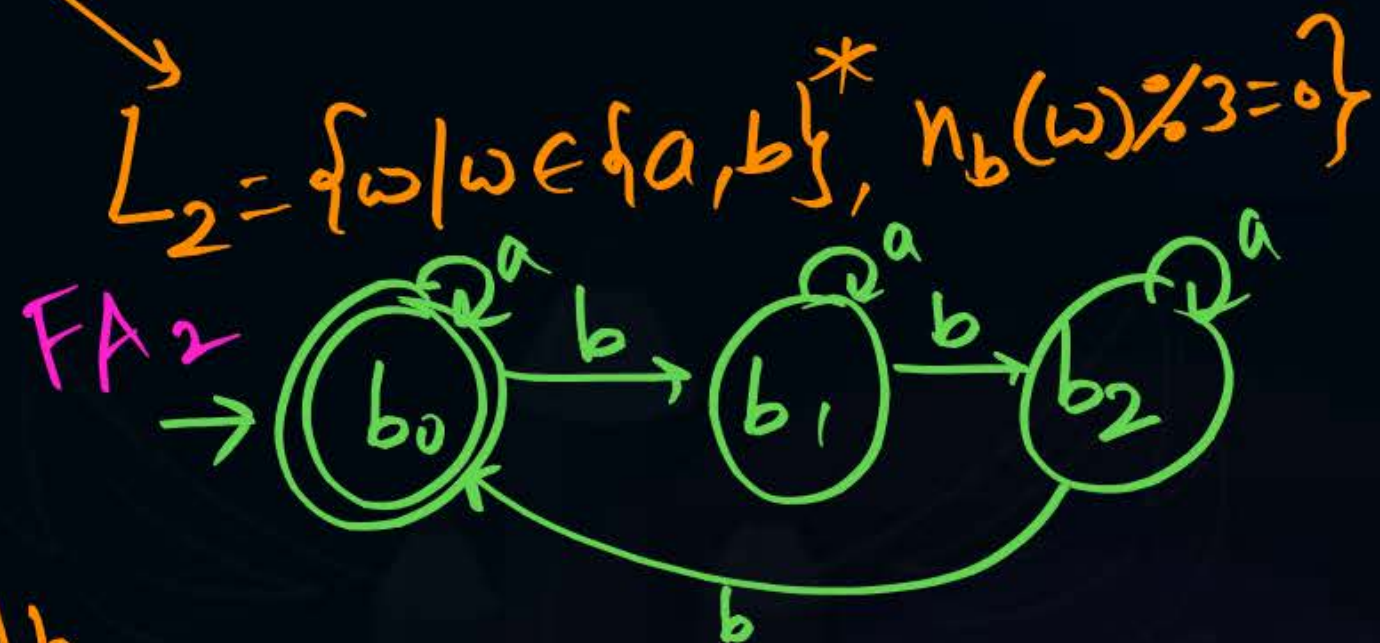
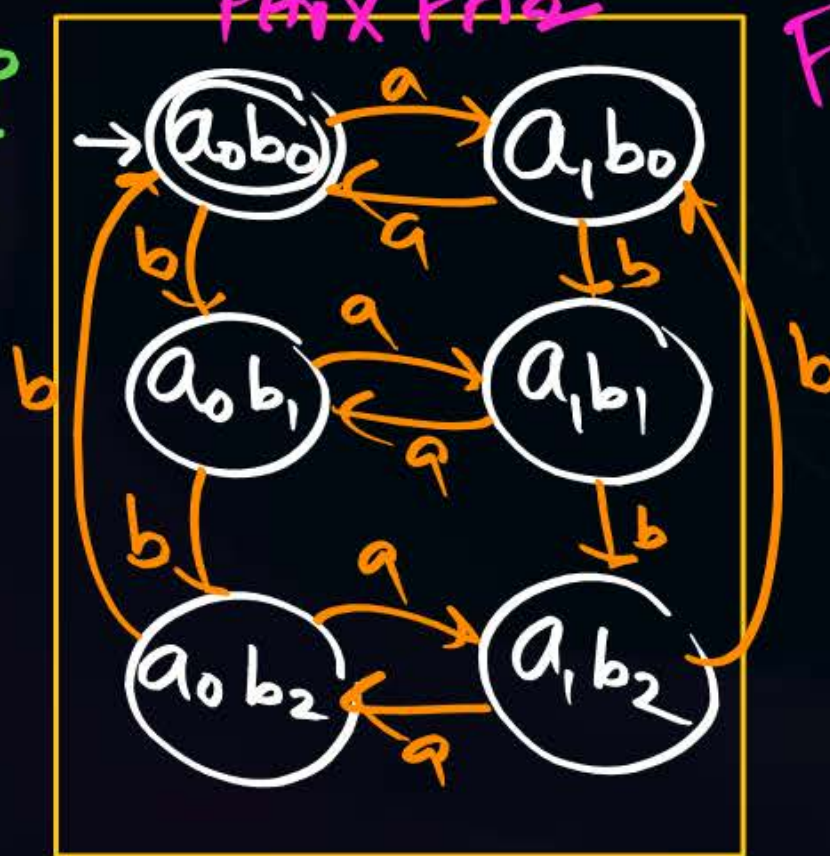
Composition Algorithm ($FA_1 \times FA_2$)



$$L = \{w \mid w \in \{a,b\}^*, n_a(w) \% 2 = 0 \text{ AND } n_b(w) \% 3 = 0\}$$



$a_0 b_0 \xrightarrow{a} a_1 b_0$



Note:

$$\{w \mid w \in \{a,b\}^*, \text{ \textcircled{\#a's} \% K_1 = 0 } \text{ \textbf{AND/OR} } \text{ \textcircled{\#b's} \% K_2 = 0 } \}$$

BUT NOT



$$\text{No. of states in min DFA} = K_1 \times K_2$$

Model - XII [Multiple Conditions — Remainder but Simple]

(83) $\{w \mid w \in \{a,b\}^*, \text{ \#a's is div by 2 AND \#a's is div by 4} \}$ #a's is div by 4

4 states

div by 2 ✓
#a's = 0, 2, 4, 6, 8, ...
div by 4 ✓
#a's = 0, 4, 8, ...
AND
= 0, 4, 8, ...

Model - XII [Multiple Conditions — Remainder but Simple]

(84) $\{w \mid w \in \{a, b\}^*, \text{ \#a's is div by 2 OR \#a's is div by 4} \}$ $\#a's \text{ is div by 2}$

$\text{div by 2} \checkmark$
 $\#a's = 0, 2, 4, 6, 8, \dots$
 div by 4
 $\#a's = 0, 4, 8, \dots$

OR
 $= 0, 2, 4, 6, 8, \dots$

 \Downarrow
2 states

Model-XIII [Multiple conditions — start/end/contain]

(85) $L = (aa + bb)(a+b)^*$

→ starts with aa or bb

(86) $L = (a+b)^*(aa+bb)$

→ ends with aa or bb

(87) $L = (a+b)^*(aa+bb)(a+b)^*$

→ contains aa or bb

(88) $L = (a+b)^*aa(a+b)^*bb(a+b)^* + (a+b)^*bb(a+b)^*aa(a+b)^*$

→ contains both aa and bb



2 mins Summary



Topic

Model X (Position based)

Topic

Model XI (Multiple Conditions-Remainder)

Model XII (

"

- simple)

Model XIII (

"

- start/end/contain)

THANK - YOU