Computer Science

Theory of Computation

[redacted]

Regular Languages & Non Regular Languages

Lecture No.- 6

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Recap of Previous Lecture

Topic Conversions

Topics to be Covered

Topic Closure Properties

Operation :

Unary : $+2$

Binary : $5+7$

Ternary : 3 operands

⋮

K-ary : K operands

Closure property [operation]

I) Finite Languages
II) Infinite Languages
III) Regular Languages

Domains

Domain (D)

operation (o)

Is "o" is closed for "D" ?

Yes : D is closed under o

No : D is not closed under o

Closure Properties

(D, o) is closed

o is closed for D

D is closed under o

(D, o) is closed

iff

every

$\forall x_1, x_2 \in D$

such that

$x_1 \, o \, x_2 \in D$

In Maths:

$(N, +)$ is closed

(D, o) is not closed

iff

$\exists x_1, x_2 \in D$ such that $\underbrace{x_1 o x_2 \notin D}_{o}$

some

In Maths :

$(N, -)$ is not closed

proof by example :

$2 \in N$ $\quad$ $-2 \notin N$

$4 \in N$

ϕ $\{\varepsilon\}$ $\{a^n \mid n<10\}$

$\{a^n b^n \mid n<2\}$ $\{\varepsilon, a, b\}$

$\ldots$

$\ldots$

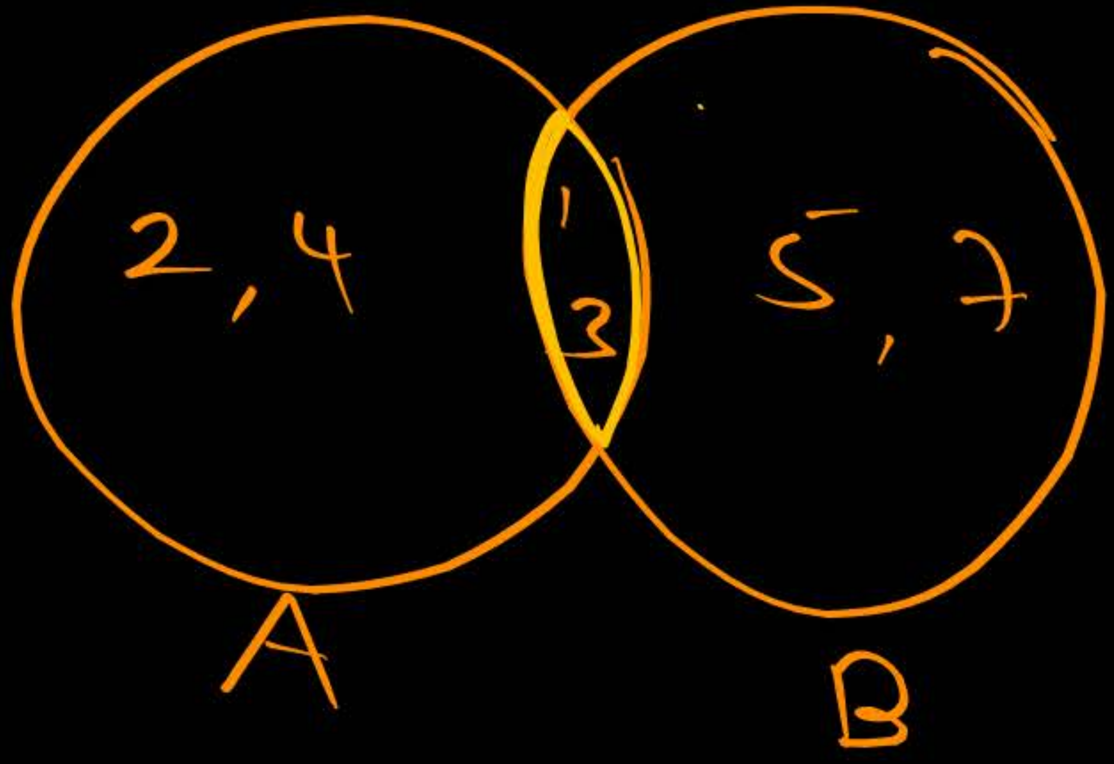
$\cup$

$\{\varepsilon\} \cup \{\varepsilon, a, b\} = \text{fin}$

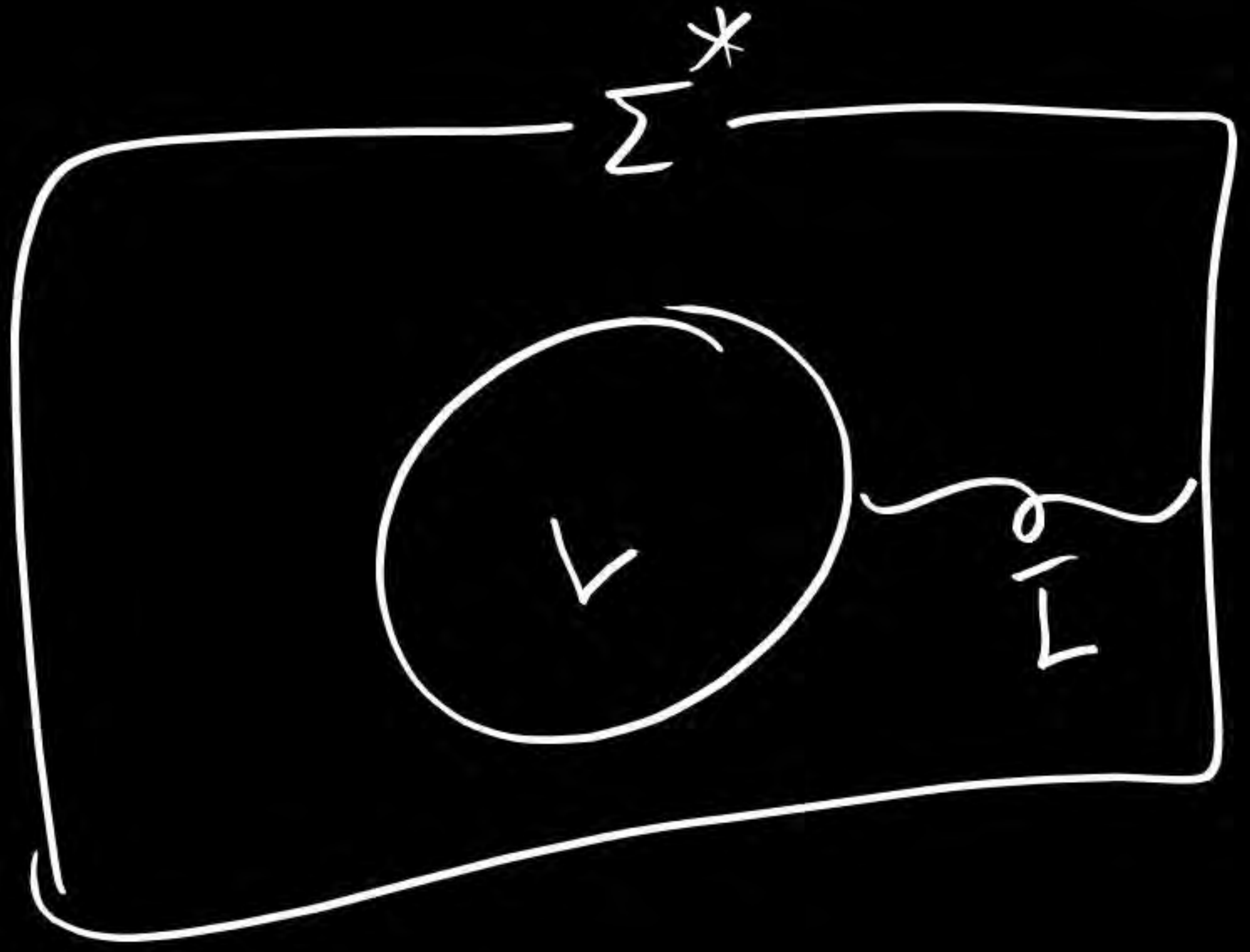
fin fin

$$\{1,2,3,4\} \cap \{1,3,5,7\} = \{1,3\}$$

$$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$$



$$\overline{L} = \Sigma^* - L$$



$L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$

$\overline{L} = (a+b)^+$

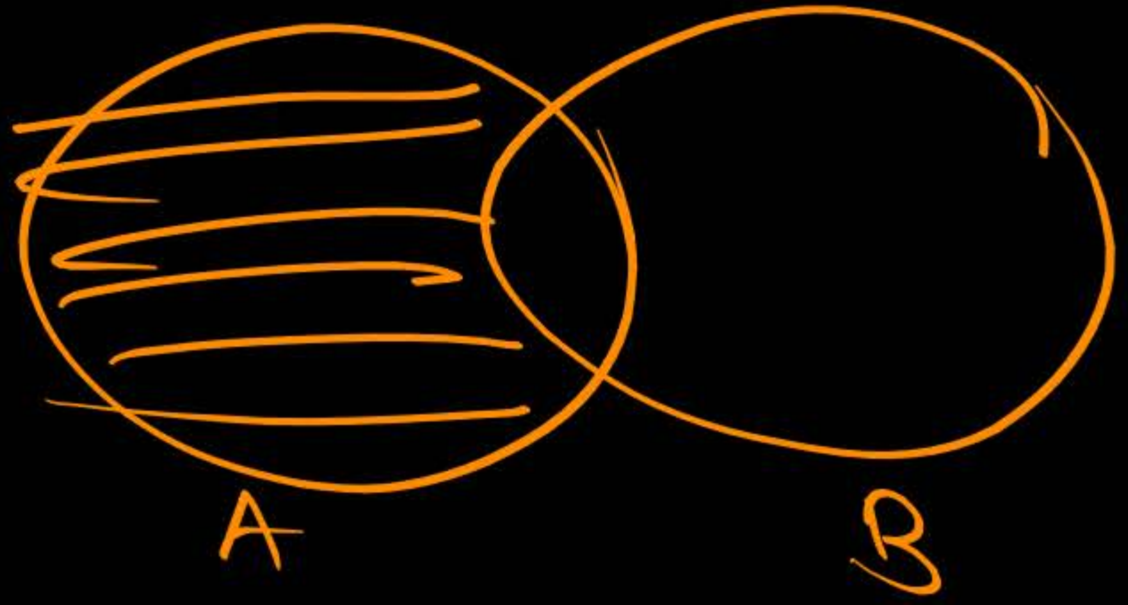
$$\overline{\text{Fin}} = \underbrace{\Sigma^* - \text{Fin}}_{\text{Always Infinite}}$$

$$\Sigma^* = \underbrace{(a+b)^*}_{\text{Infinite language}}$$

$$\text{Finite} \rightarrow \boxed{\overline{\text{Finite}}} \rightarrow \text{Infinite}$$

$$\{1, \cancel{2}, 5\} - \{2, 3\} = \{1, 5\}$$

$$\boxed{A} - B = \{x \mid x \in A \text{ AND } x \notin B\}$$



Closure Properties for Finite Languages:

① Union

$F_1, F_2 \rightarrow \boxed{\cup} \rightarrow \underbrace{F_1 \cup F_2}_{\text{always finite set}}$

Union is closed for finite languages

② Intersection

$F_1 \cap F_2 \Rightarrow$ always finite set

Intersection is closed for finite languages

③ Complement

$\overline{F} \Rightarrow$ always not finite

Complement is not closed for finite languages

④ Difference

$F_1 - F_2 \Rightarrow$ Always finite

Difference is closed for finite languages

$\Sigma = \{a\}$

$\overline{a^*} \Rightarrow \phi$

$\overline{a^+} \Rightarrow \{\varepsilon\}$

$\overline{aa^+} \Rightarrow \{\varepsilon, a\}$

$\overline{aaa^+} \Rightarrow \{\varepsilon, a, aa\}$

$\vdots$

Closure Properties for Infinite Languages:

① Union

$I_1 \cup I_2 \Rightarrow ?$

Always Infinite

$\Downarrow$

Closed

② Intersection

$I_1 \cap I_2 \Rightarrow ?$

Example:

$a^* \cap b^* \Rightarrow \{\varepsilon\}$

sometimes not infinite

$\Downarrow$

Not Closed

③ Complement

$\overline{I} \Rightarrow ?$

$\overline{\Sigma^*} \Rightarrow \phi$

Sometimes not infinite

$\Downarrow$

Not Closed

④ Difference

$I_1 - I_2 \Rightarrow ?$

$\underset{Inf}{a^*} - \underset{Inf}{a^*} \Rightarrow \phi$

Sometimes not Inf

$\Downarrow$

Not Closed

Closure Properties

✓ → closed

X → Not closed

	for Finite sets over Σ	for Infinite sets over Σ
∪	✓	✓
∩	✓	X
$\overline{L}$	X	X
Difference	✓	X

$\text{Inf} \cup \text{Any} \Rightarrow \overset{\text{Always}}{\text{Infinite set}}$

$\text{Finite} \cap \text{Any} \Rightarrow \overset{\text{Always}}{\text{Finite lang}}$

Closure Properties

F → Finite language

I → Infinite Language

PW

1. $F_1 \cup F_2 \Rightarrow$ Always Finite
2. $I_1 \cup I_2 \Rightarrow$ Always Infinite
3. $F \cup I \Rightarrow$ Always Infinite
4. $F_1 \cap F_2 \Rightarrow$ Always Finite lang
5. $I_1 \cap I_2 \Rightarrow$ either fin or Inf
6. $F \cap I \Rightarrow$ Always Finite
7. $F_1 - F_2 \Rightarrow$ Always Finite
8. $I_1 - I_2 \Rightarrow$ either finite or Infinite
9. $F - I \Rightarrow$ Always Finite
10. $I - F \Rightarrow$ Always Infinite

A) Always Finite

B) Always Infinite

C) either Finite or Infinite

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Closure Properties for regular languages:

$L_i \rightarrow$ Regular lang

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $\overline{L}$
4. $L_1 - L_2$
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+
9. $\boxed{\text{Subset}(L)}$
10. prefix (L)
11. suffix (L)
12. Substring(L)
13. $f(L)$ = Substitution
14. $h(L)$ = Homomorphism
15. $h^{-1}(L)$
16. L_1 / L_2 Quotient
17. $L_1 \oplus L_2$ Symmetric Difference
18. Half$(L) = \frac{1}{2}(L)$
19. Second Half(L)
20. one third(L)
21. Middle $\frac{1}{3}(L)$
22. Last $\frac{1}{3}(L)$
23. Finite Union
24. " $\cap$
25. " Difference
26. " Concatenation
27. " Subset
28. " Substitution
29. Inf $\cup$
30. Inf $\cap$
31. Inf $-$
32. Inf concatenation
33. Inf subset
34. Inf substitution

2 mins Summary

Topic Closed

Topic Not closed

Topic closure properties for finite language

Topic " " infinite "

Next: Topic closure properties for regular languages

Topic

THANK - YOU