CS & IT

Algorithms

DPP: 02

Analysis of algorithms

Q1 Which of the following natation is/are transitive but not reflexive
(A) Big oh (O)
(B) Big omega (Ω)
(C) Small oh (O)
(D) Small omega (ω)

Q2 If f(n) =$\sum_{i=1}^{n} x^3$
Then which of the following choices is/are true for f(n)?
(A) θ (n^4) (B) Ω (n^4)
(C) θ(n^5) (D) Ω (n^3)

Q3 Consider the following program:

```
main ( )
{
 P = n!
 for (i = 1; i <= n ; ++i)
     for (j = 1 ; j <= P ; 2*j )
 C = C + 1;
 }
```

What is the time complexity of above code?
(A) O(n^2) (B) O (n^2 logn)
(C) O (n logn) (D) O(n)

Q4 Consider the following code:

```
main ( )
{
i = 1; j = 1
while (j <= n )
{
  ++ i;
  j  = j + i;
}
```

What is the time complexity of above code?
(A) θ(n)
(B) $\theta\left(\sqrt{n}\right)$
(C) $\theta\left(\log\right)$
(D) θ(nlog (logn)

Q5 Consider the following code:

```
Algorithm T(n)
{
if (n = 1) return;
else
{
T(n/2);
}
}
```

What is the space complexity of above code?
(A) $\theta\left(\log n\right)$
(B) $\theta\left(n\right)$
(C) $\theta\left(n \log\log n\right)$
(D) $\theta\left(\sqrt{n}\right)$

Q6 $f\left(n\right) = 2^{n^2}$, $g\left(n\right) = n!$ $\mathrm{h}\left(n\right) = 2^{\log n^2}$
Which of the following is/are correct?
(A) f(n) = Ω (g(n))
(B) h(n) = Ω (g(n))
(C) h(n) = O (g(n))
(D) g(n) = Ω (f(n))

Q7 Consider the following rotations:
1. $\sqrt{\log n} = \mathrm{O}\left(\log\log n\right)$
2. $\log n = \Omega\left(\frac{1}{n}\right)$
3. $n^2 = \theta\left(2^{2\log n}\right)$
4. $(0.061)^n = \theta(1.02)^n$

How many rotations is/are correct?_____.

Q8 Consider the following functions:

$f_1 = 2^n$

$f_2 = n!$

$f_3 = n^n$

$f_4 = e^n$

What is the correct increasing order of above function?

(A) $f_1\ f_4\ f_2\ f_3$ (B) $f_2\ f_1\ f_4\ f_3$

(C) $f_2\ f_4\ f_1\ f_3$ (D) $f_2\ f_2\ f_4\ f_3$

Answer Key

Q1	(C, D)
Q2	(A, B, C, D)
Q3	(C)
Q4	(B)
Q5	(A)
Q6	(A, C)
Q7	2
Q8	(A)

Hints & Solutions

Q1 Text Solution:

	O	Ω	θ	o	ω
Reflective	✓	✓	✓	x	x
Transitive	✓	✓	✓	✓	✓

Small oh (o) and small omega(ω) comes under transitive but not reflective.

Q2 Text Solution:

f(n) = $\sum_{i=1}^{n} x^3$

i = 1

= $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + ...n^3$

= $\left[\frac{n(n+1)}{2}\right]^2$

= $O(n^4)$, $\theta(n^4)$, $\Omega(n^4)$

All options are correct.

Q3 Text Solution:

Time complexity = n * log p

↓ ↓

first loop second loop

= n* logn !

= n * nlogn

= $O(n^2 \log n)$

Q4 Text Solution:

i = 1, 2 3.....x

k =(1+2) (1+2) (1+2+3) (1+2+3+...x)

Time complexity = 1 + 2 + 3 + 4 +.....x $\leq$n

$\Rightarrow \frac{x(x+1)}{2} \leq n$

$\Rightarrow x^2 + x = 2n$

$\Rightarrow x^2 \cong 2n$

$\Rightarrow x = \sqrt{n}$

$\Rightarrow x = \theta(\sqrt{n})$

Q5 Text Solution:

Time complxity = $T\left(\frac{n}{2}\right) + 1$

$= \theta(\log n)$

Space complixity = we are pushing k activation record for n = 2^k

$n = 2^k$

$\boxed{k = (\log n_2)}$

Space = $\theta(\log n_2)$

Q6 Text Solution:

$f(n) = 2^{n^2}$

$g(n) = n!$

$h(n) = 2^{\log n^2}$

$= (n^2)^{\log_2}$

= n^2

- $2^{n^2} = n!$

$2^{n^2} = n^n$

$\log 2^{n^2} = \log n^n$

$n^2 = n \log n$

$n^2 = \Omega(n \log n)$

$f(n) = \Omega(g(n))$

- $h(n) = \Omega(f(n))$

$n^2 \leq n!$ (True)

- $g(n) = \Omega(f(n))$

$g(n) \geq 2^{n^2}$

$n! \geq 2^{n^2}$ (False)

Q7 Text Solution:

1. $\sqrt{\log n} = 0(\log\log n)$

$(\log n)^{\frac{1}{2}} \leq \log\log n$

$\frac{1}{2}\log\log n \leq \log(\log\log n)$ (False)

2. $\log n = \Omega\left(\frac{1}{n}\right)$

$\log n \geq \frac{1}{n}$ (True)

3. $n^2 = \theta\left(2^{2\log n}\right)$

$n^2 = 2^{2\log n}$

$n^2 = 2^{\log n^2}$

$n^2 = (n^2)^{\log_2^2}$

$n^2 = n^2$ (True)

4. $(0.0161)^n = \theta(1.02)^n$ (False)

Q8 Text Solution:

- $2^n < e^n$
- $n! < n^n$
- $(2^n, e^n) < n!$
- $2^n < e^n < n! < n^n$

$f_1 < f_4 < f_2 < f_3$