CS & IT ENGINEERING

Theory of Computation

Regular Languages

Lecture No.- 15

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Recap of Previous Lecture

Topic What is FA?

Topic FA representations: Graph, Table, and Set

- Q : Set of states
- Σ : " " symbols
- δ : Transition Function
- q_0 : Initial state
- F : Set of final states

Topics to be Covered

Topic What is DFA, and NFA?

$w \longrightarrow$ FA

Accepts L

If $w \in L$, Halts at final state

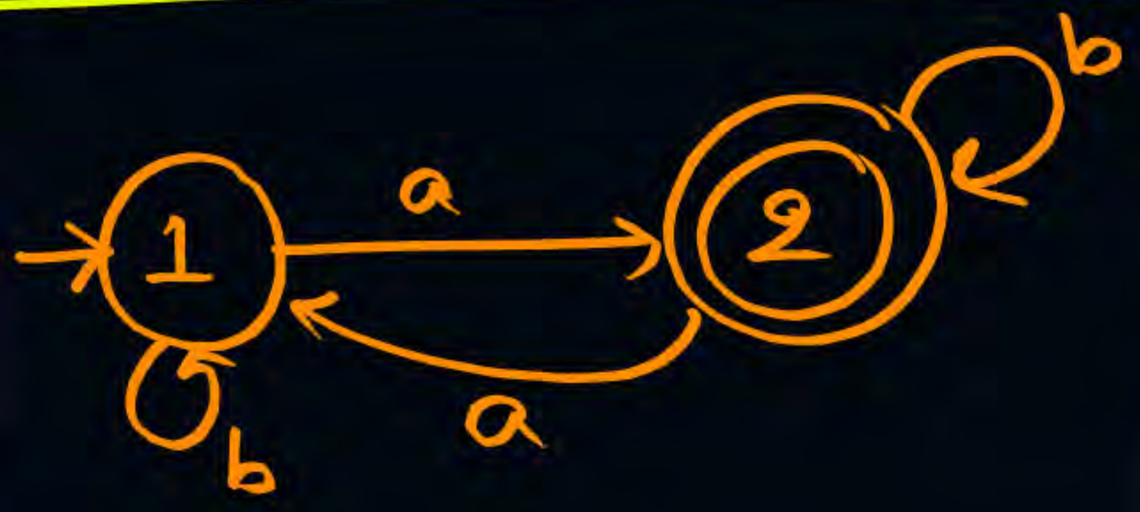
If $w \notin L$, Halts at nonfinal state

aba (Invalid)

$1 \xrightarrow{a} 2 \xrightarrow{b} 2 \xrightarrow{a} 1$

$1 \xrightarrow[w]{} \underbrace{1}_{\text{Halt}}$

ε, a, b, aa, ab



$w = \varepsilon \Rightarrow$ Halts at 1

Path : 1

$w = a \Rightarrow$ Halts at 2

Path: $1 \xrightarrow{a} 2$

$w = b \Rightarrow$ Halts at 1

Path: $1 \xrightarrow{b} 1$

aa: $1 \xrightarrow{a} 2 \xrightarrow{a} \underbrace{1}_{\text{Halt}}$

$1 \xRightarrow{aa} 1$

$w \in L$: Halts at final state

$w \notin L$: " " non-final state

$$\Sigma^* = (a+b)^* = \{\overset{\times}{\varepsilon}, \overset{\checkmark}{a}, \overset{\times}{b}, \overset{\times}{aa}, \overset{\checkmark}{ab}, \overset{\checkmark}{ba}, \overset{\times}{bb}, \ldots\}$$

$\varepsilon \Downarrow$ Halts at nonfinal

$a \Downarrow$ Halts at final

$$L = \{a, ab, ba, \ldots\}$$

Accepted : Halts at final after reading whole i/p string

Not Accepted : Halts at nonfinal

ε : Empty String : Zero symbols

$\xrightarrow{\varepsilon}$

$\xrightarrow{\text{no i/p symbol}}$

Path : Sequence of moves
(Zero or more no. of moves)

ε : $\underline{1}$

$\hat{\delta}(1, \varepsilon) = 1$ (start, string, end)

extended transition

Move = Transition

a : $1 \xrightarrow{a} 1$

$\hat{\delta}(1, a) = 1$

aab : $\underbrace{1 \xrightarrow{a}_{\delta} 1 \xrightarrow{a}_{\delta} 1 \xrightarrow{b}_{\delta} 2}_{\hat{\delta}}$

$\hat{\delta}(1, aab) = 2$ (start, string, end) aab :

$$\hat{\delta}(1, aab) = 2$$

$$1 \xrightarrow{a} 1 \xrightarrow{a} 1 \xrightarrow{b} 2$$

$$\underset{\Downarrow \atop \text{Path}}{\hat{\delta}}(1, aab) = \delta\left(\underbrace{\delta\left(\underbrace{\underset{\Downarrow}{\delta}(1, a)}_{\text{Transition} \atop 1}, a\right)}_{1}, b\right)_{2}$$

$$\underbrace{\delta(\text{state}, \text{symbol})}_{\text{transition}} \longrightarrow$$

$$\underbrace{\hat{\delta}(\text{state}, \underbrace{\text{String}})}_{\text{Path}} \Longrightarrow$$

$1 \xrightarrow{a} 1 \xrightarrow{a} 1 \xrightarrow{b} 2$ Path

$1 \xrightarrow{aab} 2$ path

Finite Automata

FA

DFA

(Deterministic FA)

$$\boxed{\delta : Q \times \Sigma \rightarrow Q}$$

NFA ($\underbrace{\text{Non-deterministic}}_{\text{one word}}$ FA)

NFA without ε moves

$$\boxed{\delta : Q \times \Sigma \rightarrow 2^Q}$$

NFA with ε moves

$$\boxed{\delta : Q \times \Sigma \cup \{\varepsilon\} \rightarrow 2^Q}$$

$Q = \{1, 2, 3\}$ = set of states

$\Sigma = \{a, b\}$

$\cancel{(a, 1)}$

$$Q \times \Sigma = \underbrace{\{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}}_{\text{From every state, for every i/p symbol}}$$

$\underbrace{P(Q)}_{\text{Power set of } Q} = 2^Q$ = Set of all subsets of Q

$= \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, Q\}$

$Q = \{1, 2, 3\}$

Subsets of Q :

$$\boxed{2^Q = P(Q) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}}$$

$$\left.\begin{array}{l} \{\} \\ \{1\} \\ \{2\} \\ \{3\} \\ \{1,2\} \\ \{1,3\} \\ \{2,3\} \\ \{1,2,3\} \end{array}\right\} \text{8 Subsets}$$

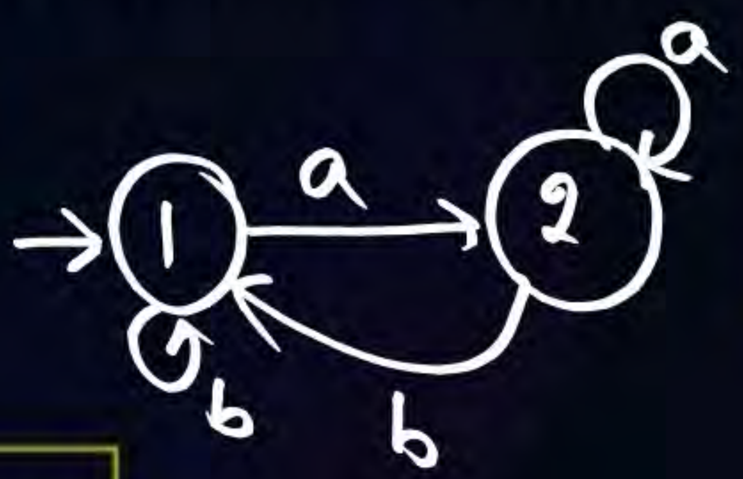
Finite Automata

DFA

I) $\delta : Q \times \Sigma \to Q$

II) From every state, for every i/p symbol, exactly one transition present to the next state

III)

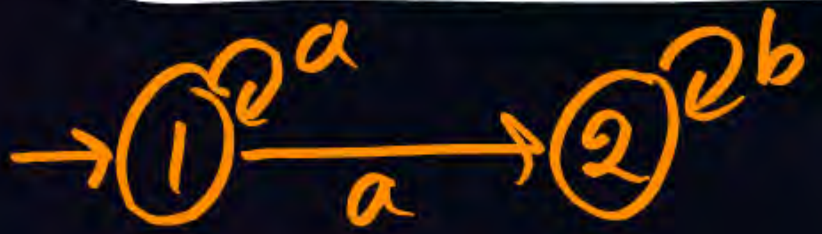


NFA without ε moves

I) $\delta : Q \times \Sigma \to 2^Q$

II) From every state, for every i/p symbol, any no. of transitions possible

III) Every DFA is NFA
But NFA need not be DFA



NFA with ε moves

I) $\delta : Q \times \Sigma_\varepsilon \to 2^Q$

II) From every state, for every i/p symbol or no i/p symbol, any no. of transitions possible

$\delta(1, \varepsilon) = 2$

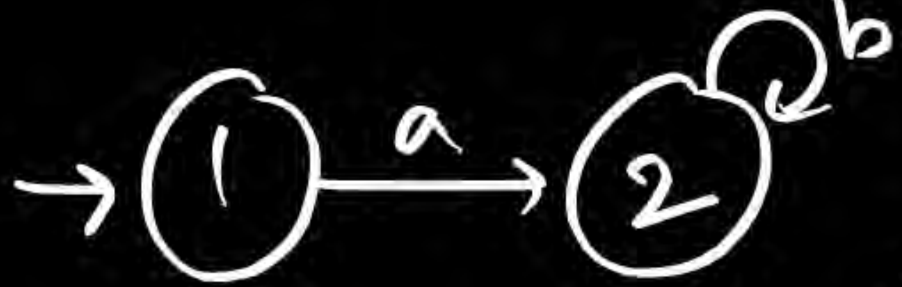
$Q \times \Sigma \cup \{\varepsilon\}$

$Q = \{1, 2\}$

$\Sigma = \{a, b\}$

$Q \times \underbrace{\Sigma \cup \{\varepsilon\}} = \{(1,a), (1,b), (1,\varepsilon), (2,a), (2,b), (2,\varepsilon)\}$

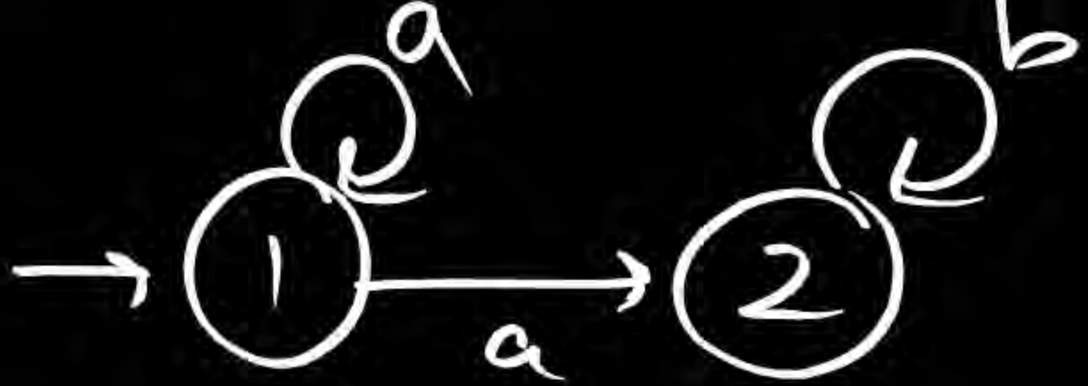
$\{1,2\} \times \{a, b, \varepsilon\}$



not DFA

$\delta(1,b) = X$

$\delta(2,a) = X$



$\delta(1,a) = 1 \text{ or } 2$

$= \{1,2\}$

two transitions

Every DFA is NFA

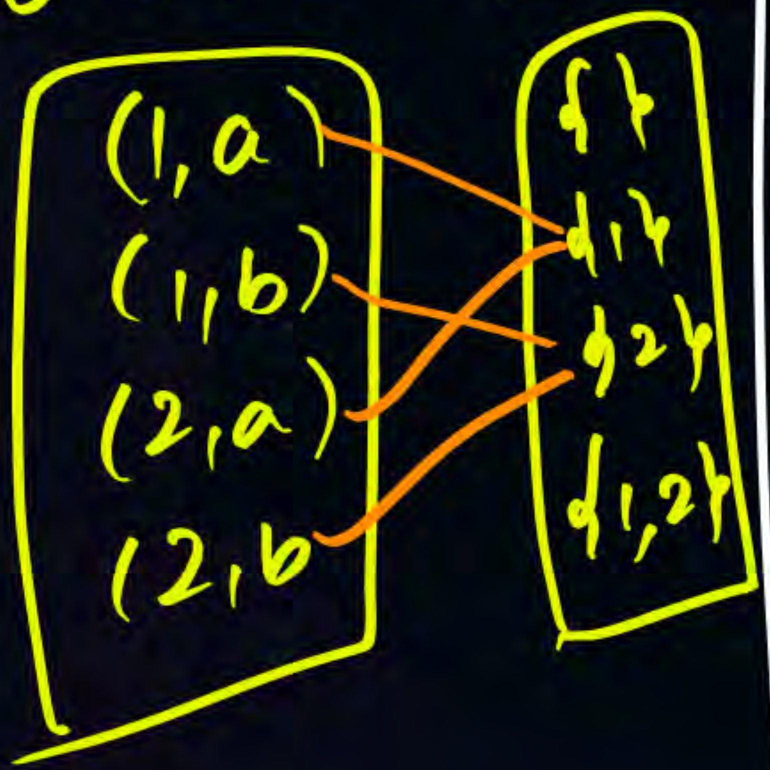
Every DFA is NFA without ε moves

Every DFA is NFA with ε-moves

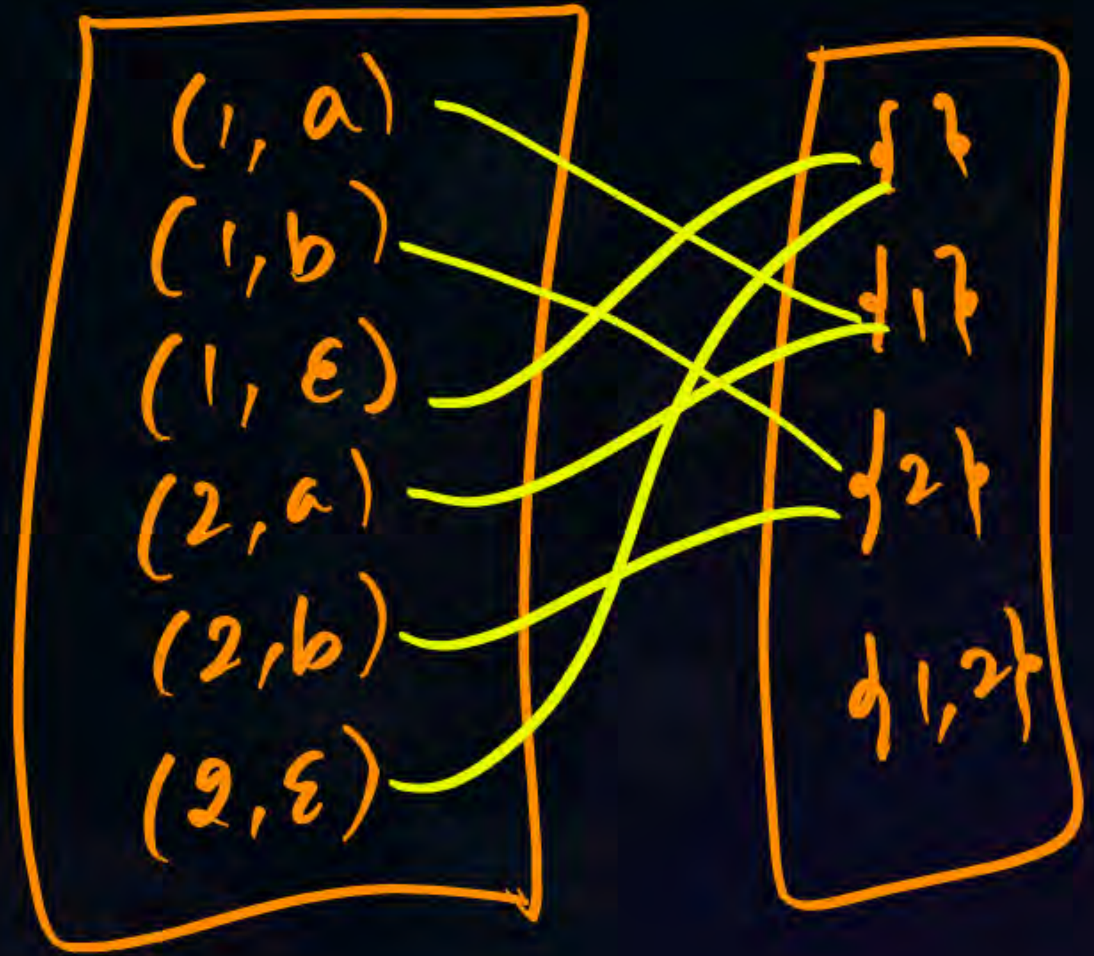
DFA

$Q \times \Sigma \rightarrow Q$

$\delta : Q \times \Sigma \rightarrow 2^Q$



$\delta : Q \times \Sigma \cup \{\varepsilon\} \rightarrow 2^Q$



I) Every DFA is NFA
(with/without ε moves)

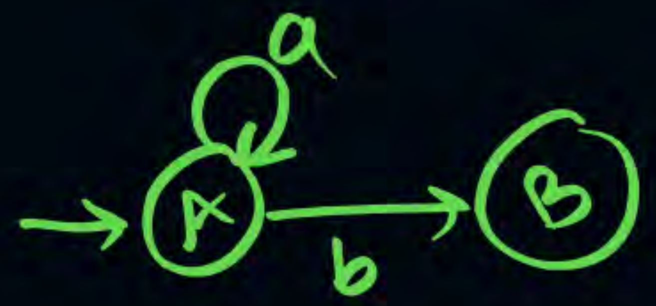
II) Every NFA without ε moves is also NFA
with ε-moves

III) Every NFA need not be DFA

DFA ①, NFA without ε moves ②, NFA with ε moves ③

$\Sigma = \{a, b\}$

(I)

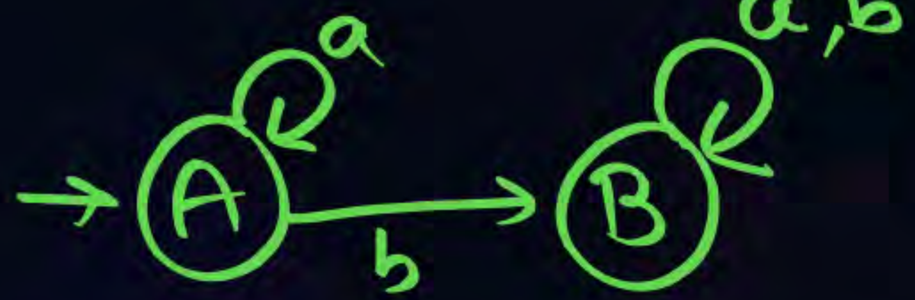


✗ ✓ ✓
1, 2, 3

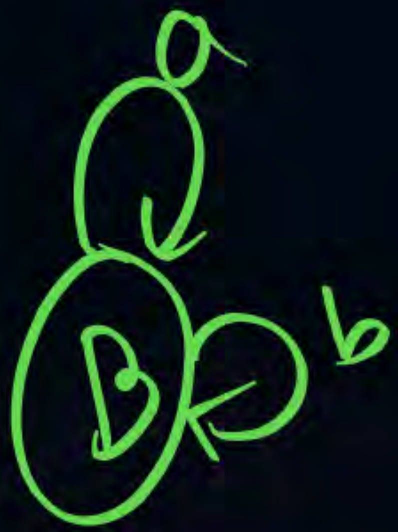
(II)

✗ ✗ ✓
1, 2, 3

(III)



✓ ✓ ✓
1, 2, 3



2 mins Summary

Topic DFA

Topic NFA without epsilon moves

Topic NFA with epsilon moves

✓

Next : DFA Construction

THANK - YOU