CS & IT ENGINEERING

Theory of Computation

Regular Languages [redacted]

Lecture No.- 12

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Recap of Previous Lecture

Topic Regular Language Vs Regular Expression

Topics to be Covered

Topic Practice on Regular Expressions

Q

Let L = { w ∈ (0 + 1)* | w has even number of 1s}, i.e., L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L? [2010: 2 Marks]

(A) $(0^*10^*1)^*$ → 0 ×

(B) $0^*(10^*10^*)^*$

(C) $0^*(\underline{10^*1})^*\underline{0}^*$ → 11011 × valid

(D) $0^*1(10^*1)^*10^*$ → ε ×

$0^*(0^*10^*10^*)^*0^*$

Q

Let P be a regular language and Q be a context- free language such that $Q \subseteq P$. (For example, let P be the language represented by the regular expression $p^* q^*$ and Q be $\{p^n q^n \mid n \in N\}$. Then which of the following is ALWAYS regular?

[2011: 1 Mark]

(A) $P \cap Q$

(B) $\Sigma^* - P$

(C) $P - Q$

(D) $\Sigma^* - Q$

CFLs

Q

Given the language L = {ab, aa, baa}, which of the following strings are in L^*?

1. abaabaaabaa $\in L^5$
2. aaaabaaaa $\in L^4$
3. baaaaabaaaab $\notin L^*$
4. baaaaabaa $\in L^4$ [2012: 1 Mark]

(A) 1, 2 and 3

(B) 2, 3 and 4

(C) 1, 2 and 4

(D) 1, 3 and 4

$$L^* = (ab \overset{or}{+} aa \overset{or}{+} baa)^*$$

$$= (baa)^1 (aa)^1 (ab)^1 (aa)^1 (aa)^1 \cancel{b}$$

Q

Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$?

[2013: 1 Mark]

(A) $\{\varepsilon\}$

(B) ϕ

(C) a^*

(D) (ε, a)

$$\underbrace{\phi . a^*}_{} + \underbrace{\phi^*}_{}$$

$$\underbrace{\phi + \varepsilon}_{\varepsilon}$$

$$\phi^+ = \phi$$

$$\phi^* = \varepsilon$$

$$\phi^* = \phi^0 + \phi^1 + \phi^2 + \ldots$$
$$= \varepsilon + \phi + \phi + \ldots$$
$$= \varepsilon + \phi = \varepsilon$$

$$\boxed{\begin{aligned} \phi^0 &= \varepsilon \\ a^0 &= \varepsilon \\ \varepsilon^0 &= \varepsilon \end{aligned}}$$

Q

The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ________.

$L = a^*b^*(ba)^*a^*$ $= 3$

[2014-Set3: 1 Mark]

$\varepsilon = a^0 b^0 (ba)^0 a^0$
$a = a^1 b^0 (ba)^0 a^0$
$b = a^0 b^1 (ba)^0 a^0$
$aa = a^2$ $\underline{\varepsilon}$
$ab = a^1 b^1$ $\underline{\varepsilon}$
$ba = a^0 b^0 (ba)^1 a^0$
$bb = a^0 b^2 (\ )^0 a^0$

aaa ✓
aab ✓
aba ✓
abb ✓
baa ✓
$\boxed{bab}$ ✗

Len
0 → ε
1 → a, b
2 → aa, ab, ba, bb
3 → aaa, . . . , bbb
4
5
⋮

Q

Consider alphabet $\Sigma = \{0, 1\}$, the null/empty string λ and the sets of strings X_0, X_1 and X_2 generated by the corresponding non-terminals of a $\boxed{\text{regular grammar.}}$ X_0, X_1 and X_2 are related as follows:

[2015-Set2: 2 Marks]

$$X_0 = 1X_1$$
$$X_1 = 0X_1 + 1X_2$$
$$X_2 = 0X_1 + \{\lambda\}$$

Which one of the following choices precisely represents the strings in X_0?

Regular Grammar

(A) $10\,(0^* + (10)^*)1$

(B) $10\,(0^* + (10)^*)^*1$

(C) $1(0 + 10)^*1$

(D) $10\,(0 + 10)^*1 + 110\,(0 + 10)^*1$

Q

Which one of the following regular expressions represents the language: *the set of all binary strings having two consecutive 0s and two consecutive 1s*?

[2016-Set1: 1 Mark]

(A) $(0 + 1)^*0011(0 + 1)^* + (0 + 1)^*1100(0 + 1)^*$ → contains 0011 or 1100

(B) $(0 + 1)^*(00(0 + 1)^*11 + 11(0 + 1)^*00)(0 + 1)^*$

(C) $\underbrace{(0 + 1)^*00(0 + 1)^*}_{\text{contain 00}} + \underbrace{(0 + 1)^*11(0 + 1)^*}_{\text{contain 11}}$

contain 00 or contain 11

(D) $\underbrace{00(0 + 1)^*11}_{\text{starts with 00 and ends with 11}} + \underbrace{11(0 + 1)^*00}_{\text{starts with 11 and ends with 00}}$

starts with 00 and ends with 11 OR starts with 11 and ends with 00

$$\Sigma^* \underline{00\Sigma^*11}\Sigma^* + \Sigma^* \underline{11\Sigma^*00}\Sigma^*$$

$$\Sigma^*(00\Sigma^*11 + 11\Sigma^*00)\Sigma^*$$

Q

Which one of the following regular expression represents the set of all binary strings with an odd number of 1's?

[2020: 1 Mark]

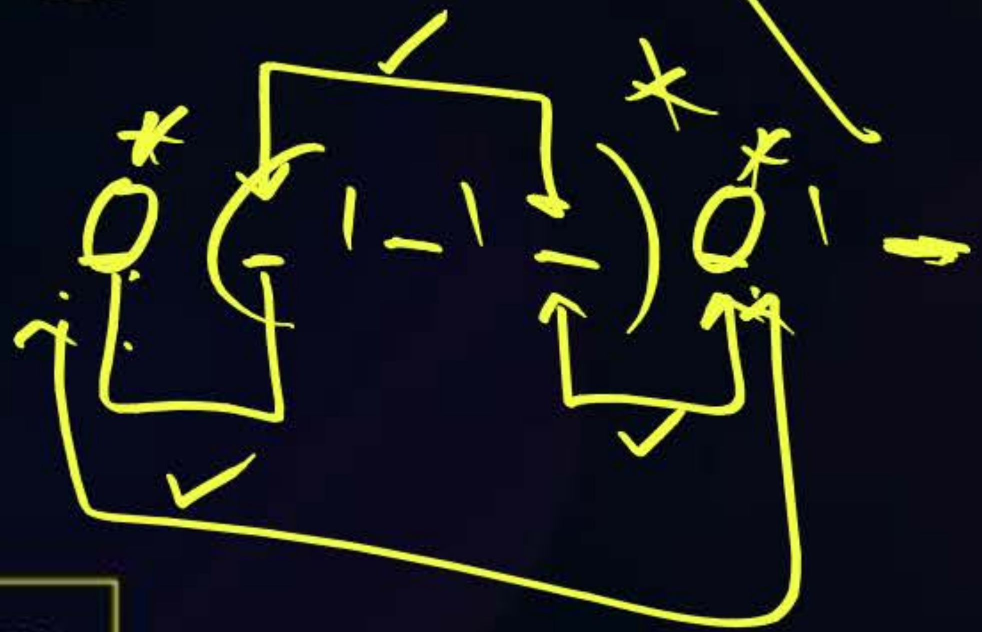
(A) $(0^*10^*10^*)^*0^*1 \rightarrow 10$ ✗

(B) $10^*(0^*10^*10^*)^* \rightarrow 01$ ✗

(C) $((0 + 1)^*1(0 + 1)^*1)^*10^* \rightarrow 01$ ✗

(D) $(0^*10^*10^*)^*10^* \rightarrow 01$ ✗

all options are wrong



$$= 0^*(0^*1\,0^*1\,0^*)^*0^*1\,0^*$$

$$= 0^*1\,0^*(0^*1\,0^*1\,0^*)^*0^*$$

TOPIC:

(61) If $L = \underbrace{a(a+b)^*}_{\text{starting with 'a'}}$ then $\overline{L} = ?$

$= \varepsilon + b\Sigma^*$

$= (ba^*)^+$

$= \underbrace{(b\Sigma^*)^*}_{\text{Not starting with 'a'}}$

$\Sigma^* = \{\varepsilon, \underline{a\Sigma^*}, b\Sigma^*\}$

Not starting with 'a'

$$\overline{L} = \Sigma^* - L$$

universal language

$\Sigma^* =$

$$\boxed{L = a\Sigma^*}$$

$$\overline{L} = \epsilon + b\Sigma^*$$

$$\boxed{L \cup \overline{L} = \Sigma^*}$$

TOPIC:

(62) If $L = \Sigma^*$ then $\bar{L} = \phi$

(63) If $L = \Sigma^+$ then $\bar{L} = \epsilon$

TOPIC:

(64) $L = b(a+b)^* \Longrightarrow \bar{L} = a\Sigma^* + \varepsilon$

(65) $L = (a+b)^* a \Longrightarrow \bar{L} = \Sigma^* b + \varepsilon$

(66) $L = (a+b)^* b \Longrightarrow \bar{L} = \Sigma^* a + \varepsilon$

(67) $L = \underbrace{(a+b)^* a (a+b)^*}_{\text{Contains `a'}} \Longrightarrow \bar{L} = \underbrace{b^*}_{\text{Not contains `a'}}$

TOPIC:

(67) $L = \underbrace{(a+b)^* a (a+b)^*}_{\text{Containing `a' as substring}}$

$\bar{L} = \underbrace{b^*}_{\text{not containing `a' as substring}}$

TOPIC:

I) $L \cup \bar{L} = \Sigma^*$

II) $L \cap \bar{L} = \phi$

L and $\bar{L}$ are disjoint

III) $\bar{L} = \Sigma^* - L$

TOPIC:

$\Sigma = \{a\}$

68. $L = a^* = \Sigma^* \Rightarrow \bar{L} = \phi$

69. $L = (aa)^* = \{\varepsilon, a^2, a^4, a^6, \ldots\} \Rightarrow \bar{L} = a(aa)^* = (aa)^* a$

70. $L = \underbrace{a(aa)^*}_{\text{odd no. of a's}} \Rightarrow \bar{L} = (aa)^* = \{\varepsilon, a^2, a^4, \ldots\}$

2 mins Summary

Topic Regular Languages

Topic Regular Expressions

THANK - YOU