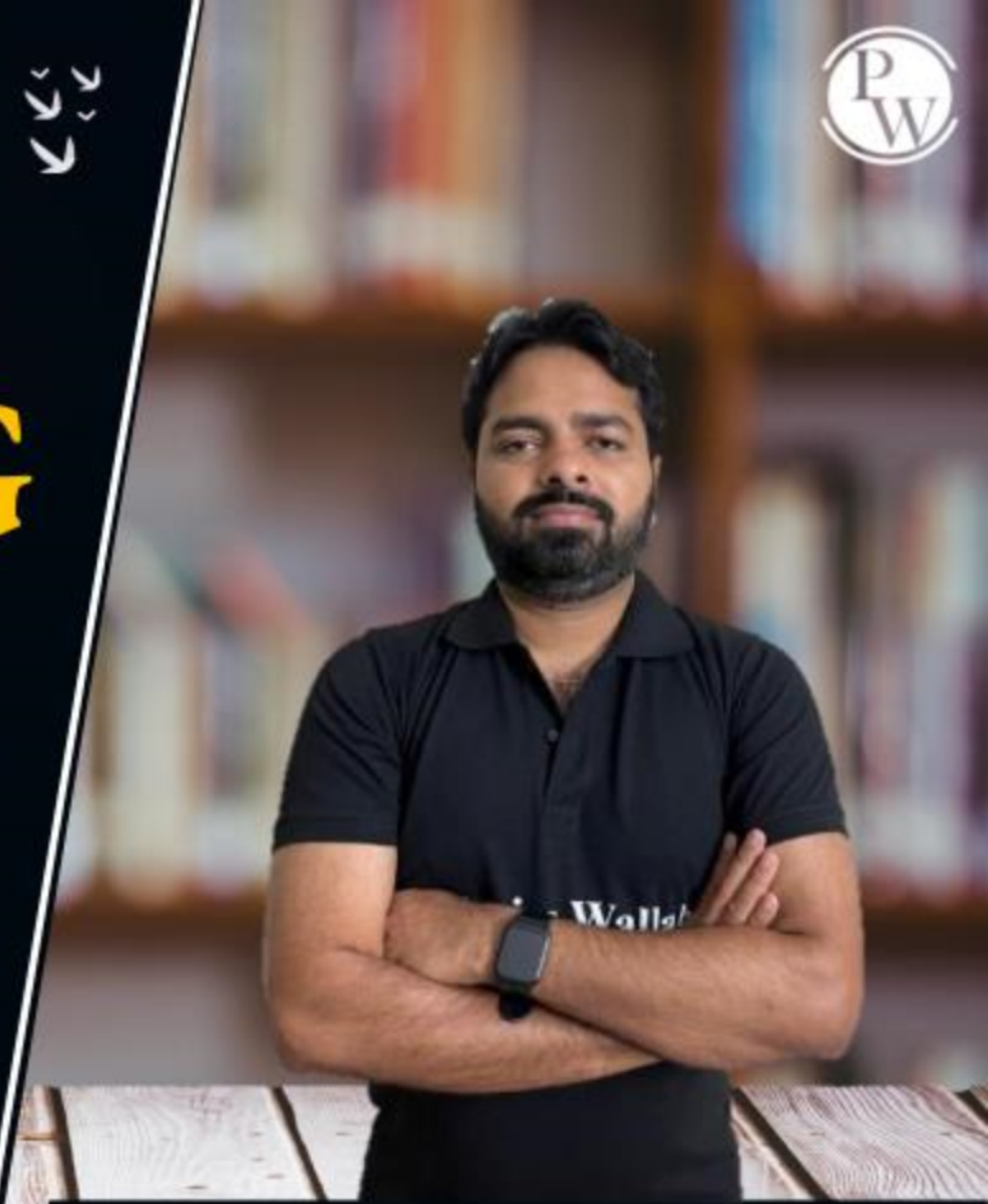
CS & IT ENGINEERING

Theory of Computation

Regular Languages [redacted]

Lecture No.- 09



Mallesham Devasane Sir

Recap of Previous Lecture

Topic Regular Language Vs Regular Expression

Topics to be Covered

Topic Regular Language Vs Regular Expression

Universal Set

TOPIC:

$\Sigma = \{a, b\}$

	Regular Exp.	Regular Language	Meaning
(15)	$(a+b)^*$	$\{a,b\}^*$	Set of all strings over $\Sigma = \{a,b\}$ $\Rightarrow$ Universal Language
(16)	$(a+b)^+$	$\{a,b\}^+ = \{w \mid w \in (a+b)^*, \lvert w \rvert > 0\}$	= Set of all non zero length strings
(17)	$a.(a+b)^*$	$= a\Sigma^* = \{aw \mid w \in \Sigma^*\}$	= Set of all strings where every string begins with a.
(18)	$b(a+b)^*$	Starting with b	
(19)	$(a+b)^* a$	ends with a	
(20)	$(a+b)^* b$	" " b	

① ab^* $\neq$ ② $a(a+b)^*$

$\underbrace{ab^*}$

a followed by any b's

Set of (all) strings starting with 'a' over $\Sigma = \{a, b\}$

$$ab^* \subset a\Sigma^*$$

(1) $ab^* = \{a, ab, abb, ab^3, \ldots\}$

(2) $a(a+b)^* = \{a, aa, ab, aaa, aab, \ldots\}$

$$(1) \subset (2)$$

TOPIC:

Regular Exp.	Regular Language	Meaning
(21) $(a^*b)^+$	$= (a+b)^*b$	= Set of all strings ending with b
(22) $(b^*a)^+$	$= (a+b)^*a = \Sigma^*a$	= Set of all strings ending with a
(23) $(ab^*)^+$	$= a\Sigma^*$	
(24) $(ba^*)^+$	$= b\Sigma^*$	
(25) $(a^*b)^*a^*$	$= (a+b)^* = \Sigma^*$	Universal Set over Σ
(26) $(b^*a)^*b^*$	$= (a+b)^* = \Sigma^*$	Universal Set over Σ
(27) $a^*(ba^*)^*$	$= (a+b)^* = \Sigma^*$	Universal Set over Σ

$$(a^*b)^+ = (a+b)^*b$$

b ✓	b ✓
ab ✓	ab ✓
bb ✓	bb ✓
⋮	⋮

$\Sigma = \{a, b\}$

$$a\Sigma^* = (ab^*)^+$$

$$b\Sigma^* = (ba^*)^+$$

$$\Sigma^* a = (b^* a)^+$$

$$\Sigma^* b = (a^* b)^+$$

$$(a^*b)^+ = (a+b)^*b$$

$(a^0b)^1 = b$ ✓ $\qquad$ b ✓

ab ✓ $\qquad$ ab ✓

$(\quad)^2 = \underbrace{(a^0b)^1}_{b}\underbrace{(a^0b)^1}_{b} = bb$ ✓ $\qquad$ bb ✓

⋮ $\qquad$ ⋮

TOPIC:

(28) $b^*(ab^*)^* = (a+b)^*$

$b^0(ab^*)^0 = \varepsilon$ ✓

$b^0(ab^0)^1 = a$ ✓

$b^1(\ \ )^0 = b$ ✓

ε ✓

a ✓

b ✓

TOPIC: Write Regular Exp

(29) $L = \{ \underbrace{aaw}_{\text{form}} \mid w \in a^* \}$

$a^* \to \varepsilon, a, a^2$

$= \{aa\varepsilon, aaa, aaaa, \ldots\}$

$= \{a^2, a^3, a^4, \ldots\}$

$= \{a^n \mid n \geq 2\}$

$= aaa^*$

$= a^*aa$

$= aa^*a$

$= aa^+$

$= a^+a$

(30) $L = \{ w_1 aa w_2 \mid w_1, w_2 \in (a+b)^* \}$

$$= (a+b)^* aa (a+b)^*$$

Set of all strings containing 'aa' as substring.

$$= \Sigma^* aa \Sigma^*$$

$$(a+b)^* aa (a+b)^*$$

$$(a^* b^*)^* aa (b^* a^*)^+$$

$$(a+b)^* = (a^* b^*)^*$$

$$= (a^* b^*)^+$$

$$= (b^* a^*)^*$$

$$= (b^* a^*)^+$$

$$= \quad \vdots$$

TOPIC:

(31) $\{w \mid w \in \{a,b\}^*, |w| = 2\}$

$= (a+b)^2 = (aa+ab+ba+bb) = \Sigma^2$

(32) $\{w \mid w \in \{a,b\}^*, |w| \geq 2\}$

$= (a+b)^2 \cdot (a+b)^* = \Sigma^2 \Sigma^* = \Sigma \Sigma^+ = \Sigma^+ \Sigma$

(33) $\{w \mid w \in \{a,b\}^*, |w| \leq 2\}$

$= \epsilon + a + b + aa + ab + ba + bb = \epsilon + a + b + (a+b)^2 = (\epsilon + a + b)^2$

$$(\varepsilon+a+b)^2$$

len

$$(\varepsilon+a+b)\cdot(\varepsilon+a+b)$$

$\varepsilon \rightarrow 0$

$a+b \rightarrow 1$

$\varepsilon+a+b \rightarrow \leq 1$

$(\varepsilon+a+b)^2 \rightarrow \leq 2$

$$\varepsilon+a+b+aa+ab+ba+bb$$

TOPIC:

(34) $\{w \mid w \in \{a,b\}^*, |w| \equiv \text{even}\}$ $\equiv$ divisible by 2

$= \{\varepsilon, aa, ab, ba, bb, aaaa, aaab, \ldots\}$ $= (\Sigma\Sigma)^*$

$= (aa+ab+ba+bb)^* = [(a+b)^2]^* = (\Sigma^2)^*$

(35) $\{w \mid w \in \{a,b\}^*, |w| = \text{odd}\}$

$(\Sigma^2)^*.\Sigma = \Sigma.(\Sigma^2)^*$

$= ((a+b)^2)^*(a+b) = (a+b).((a+b)^2)^*$

$$\left((a+b)^2\right)^* = (aa+ab+ba+bb)^*$$

$$\left((\varepsilon+a+b)^2\right)^* = (\varepsilon+\underline{a}+\underline{b}+aa+ab+ba+bb)^*$$

$$= (a+b)^*$$

$$n_a(w) = 2$$

no. of a's in w is 2

$$\#_a(w) = 2$$

TOPIC:

(36) $\{w \mid w \in \{a,b\}^*, n_a(w) = 2\}$

$= b^* a b^* a b^*$

(37) $\{\boxed{w} \mid w \in \{a,b\}^*, n_a(w) \geq 2\}$

$b^* a b^* a (a+b)^* = (a+b)^* a (a+b)^* a (a+b)^* = \Sigma^* a \Sigma^* a \Sigma^*$

(38) $\{w \mid w \in \{a,b\}^*, n_a(w) \leq 2\}$

$b^* (a+\varepsilon) b^* (a+\varepsilon) b^*$

$n_a(w) = 0 \Rightarrow b^*$

OR

$n_a(w) = 1 \Rightarrow b^* a b^*$

OR

$n_a(w) = 2 \Rightarrow b^* a b^* a b^*$

$$n_a(w) \le 2 = \underbrace{b^*} + \underbrace{b^* a b^*} + \underbrace{b^* a b^* a b^*}$$

$$= b^* (a+\varepsilon)\, b^* \,(a+\varepsilon)\, b^*$$

TOPIC: Home work

(39) $\{w \mid w \in \{a,b\}^*,\ n_a(w) = \text{even}\}$

(40) $\{w \mid w \in \{a,b\}^*,\ n_a(w) = \text{odd}\}$

2 mins Summary

Topic Operators

Topic Properties

Topic Simplification

THANK - YOU