CS & IT ENGINEERING

Theory of Computation

Regular Languages

Lecture No.- 07

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Recap of Previous Lecture

Topic Regular Expressions

- 4 operators
- properties

Topics to be Covered

Topic Regular Expressions

↳ Simplification

TOPIC:

① $a + a^* = a^*$

MSQ

a^* → ε, a, aa, ...

A) $a^+ \rightarrow \varepsilon$ not possible

~~B)~~ a^*

C) $a \cdot a^* = a^+$ (a → a, a^* → ε)

~~D)~~ $a + aa + a^*$

$$a + a^2 + a^*$$

$$\{a\} \cup \{a^2\} \cup \{\varepsilon, a, a^2, a^3, \ldots\} = a^*$$

TOPIC:

(2) $a \cdot a^*$ = .

$a \cdot \{\varepsilon, a, a^2, \cdots\}$

$a \cdot \varepsilon \rightarrow a$

$a \cdot a \rightarrow a^2$

$a \cdot a^2 \rightarrow a^3$

$a \cdot a^3 \rightarrow a^4$

$\vdots$

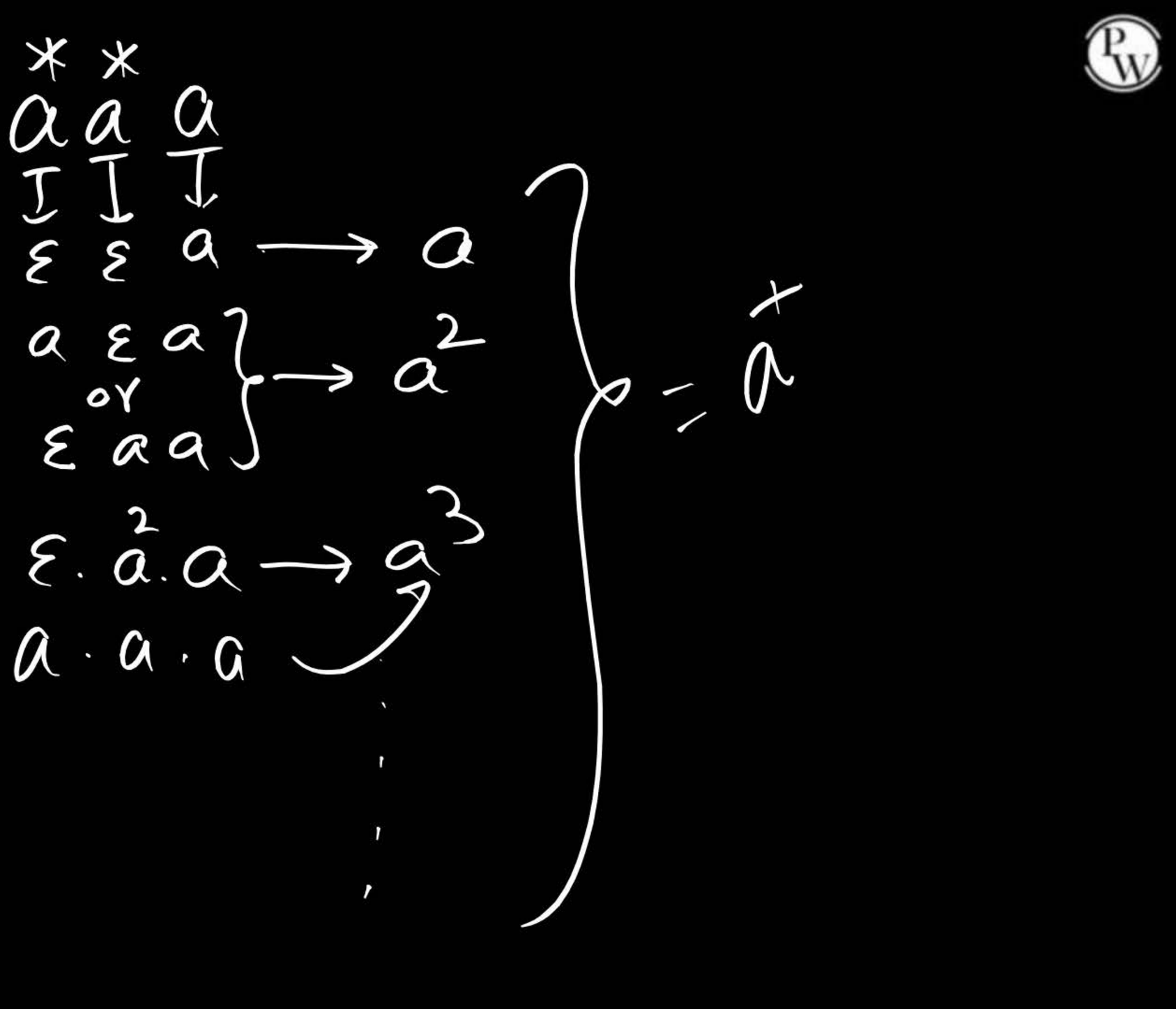
$= a^+$

A) a^+

B) $a^* \cdot a$

C) $a a^* a$

D) $a^* a^* a$


* *
a a a
ε ε a → a
a ε a
or
ε a a
→ a²
ε . a² . a → a³
a . a . a
= a⁺

$a \cdot a^* \cdot a = \{a^2, a^3, a^4, a^5, \cdots\}$

must must must

$\varepsilon \; a \; a^2 \cdots$

$a \cdot \varepsilon \cdot a \longrightarrow aa = a^2$

$a \cdot a \cdot a \longrightarrow a^3$

$a \cdot aa \cdot a \longrightarrow a^4$

$= aa^+$

$= a^+a$

a^* . $\underline{a}$

must

ε , a , a^2 ...

$$\left.\begin{aligned} \varepsilon \,.\, a &\to a \\ a \,.\, a &\to a^2 \\ a^2 \,.\, a &\to a^3 \\ aaa \,.\, a &\to a^4 \end{aligned}\right\} = a^+$$

TOPIC:

$$a^* = \{a^n \mid n \geq 0\} = \{\varepsilon, a, aa, aaa, \ldots\}$$

It can generate ε

a

aa

aaa

.

.

.

TOPIC:

3) $(a^*)^{100} =$

$a^* \cdot a^* \cdot a^* \ldots$ 100 times

$\varepsilon \cdot \varepsilon \cdot \varepsilon \ldots \varepsilon \Rightarrow \varepsilon$

$a \cdot \varepsilon \cdot \varepsilon \ldots \varepsilon \Rightarrow a$

$a^2 \cdot \varepsilon \cdot \varepsilon \ldots \varepsilon \Rightarrow a^2$

$= a^*$

A) ~~a^*~~

B) ~~$(a^*)^2 = a^* \cdot a^* = a^*$~~

C) ~~$(a^2)^* = (aa)^*$~~

D) ~~$(a^{100})^*$~~

$$(a^{100})^* = \underbrace{(aaaa\ldots \text{ 100 times})^*}$$

ε

a^{100}

a^{200}

$\vdots$

$$\underbrace{(a^*)^2}_{=a^*} \neq \underbrace{(a^2)^*}_{=(aa)^*}$$

$\{\varepsilon, a, a^2, a^3, \ldots\}$

$\{\varepsilon, a^2, a^4, a^6, \ldots\}$

TOPIC:

4) $\underline{a.(aa)^*} =$

$a \cdot \varepsilon \longrightarrow a$

$a \cdot aa \longrightarrow a^3$

$a \cdot (aa)^2 \longrightarrow aaaaa \rightarrow a^5$

$a \cdot (aa)^3 \longrightarrow a^7$

$\vdots$

A) $(aa)^* = \{\varepsilon, \ldots\}$

B) $(aa)^* a = \{a, a^3, a^5, \ldots\}$

C) $a(aa)^* a = \{a^2, \ldots\}$

D) $a^* = \{\varepsilon, \ldots\}$

TOPIC:

5) $a^* . a^* . a^* = a^*$

6) $(a^*)^* = a^*$

7) $a^* + a^* = a^*$

8) $a^* + a^+ = a^*$

9) $(a^*)^+ = a^*$

10) $(a^+)^* = a^*$

11) $(a^+)^+ = a^+$

12) $a^+ . a^* . a^* = a^+$

13) $a^* . a . a^* = a^+$

14) $a^+ + \varepsilon = a^*$

$a^* \boxed{a} a^*$ =

must

$\longrightarrow \varepsilon X$

$\varepsilon \; a \cdot \varepsilon \longrightarrow a$

$\varepsilon \cdot a \cdot a \longrightarrow a^2$

$\varepsilon \cdot a \cdot a^2 \longrightarrow a^3$

$= a^+$

$$a^{+} + \varepsilon = a + a^{2} + a^{3} + \cdots + \varepsilon = a^{*}$$

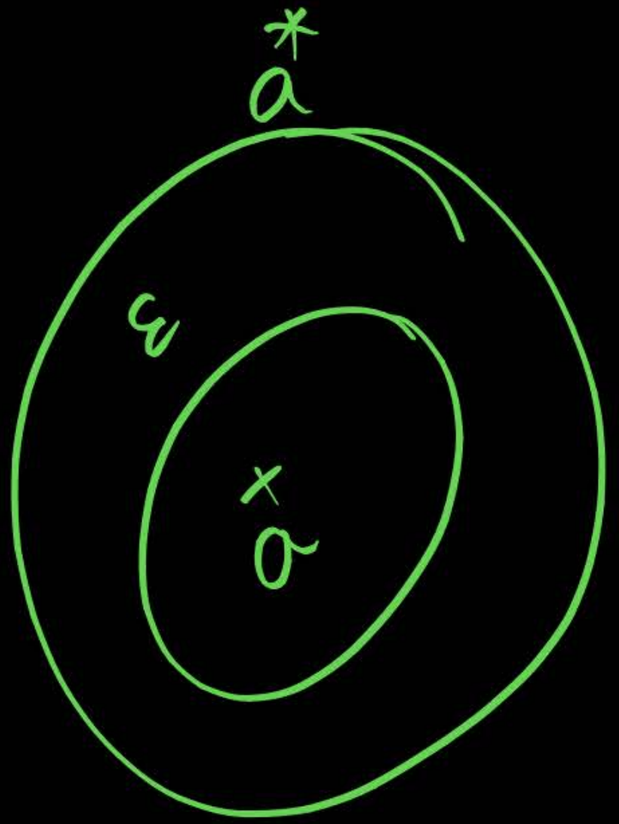
$$R^{+} + \varepsilon = R^{*}$$

R^{1}
R^{2}
R^{3}
R^{4}
.
.
.

$a^* + a^+$

$a^* \cup a^+$

$\{\varepsilon, a, a^2, \ldots\} \cup \{a, a^2 \ldots\} = a^*$

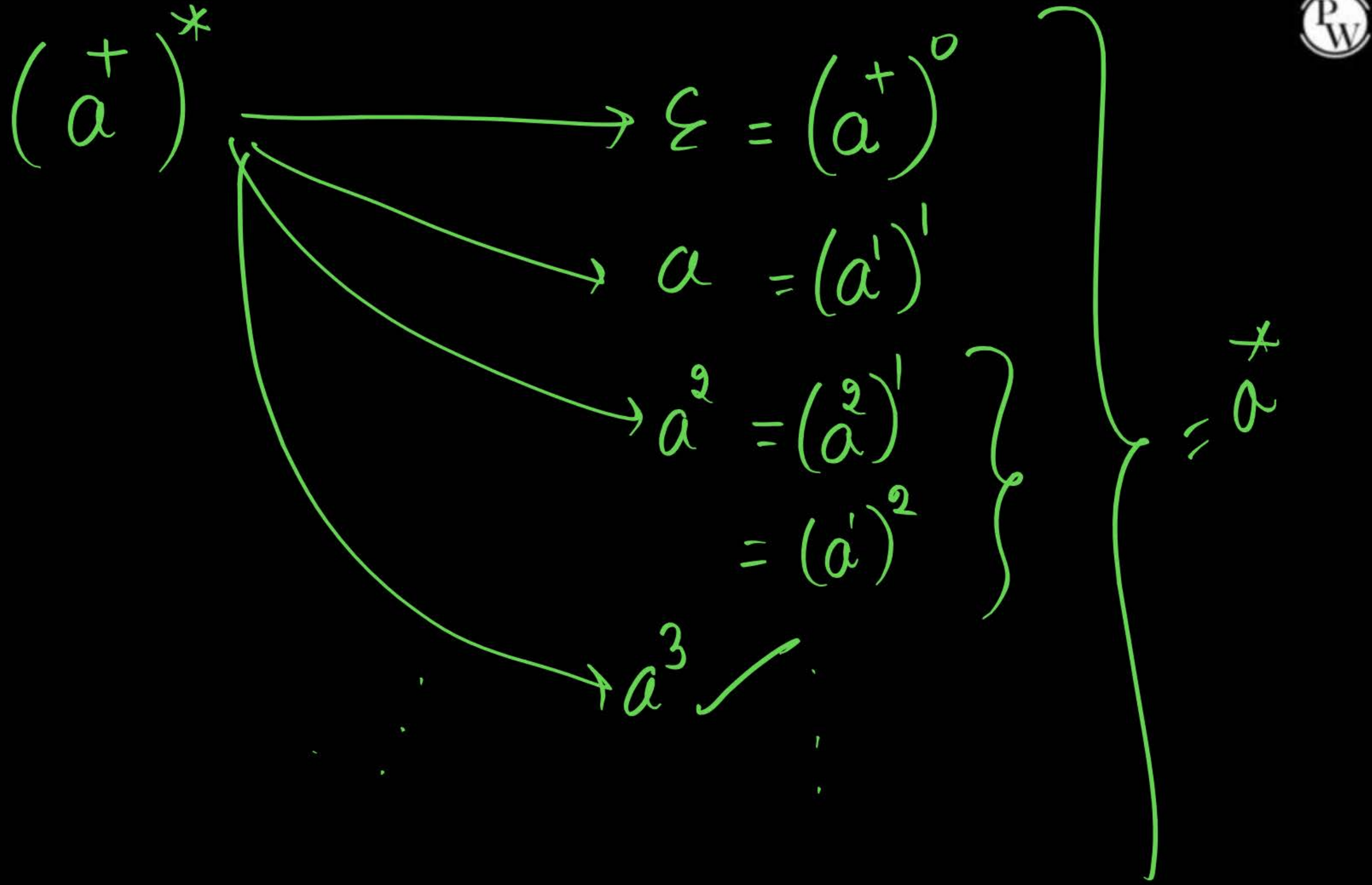


$$\Sigma^* \cup \text{Any} = \Sigma^*$$

$$a^* + a = a^*$$

$$a^* + aa = a^*$$

$$a^* + a^+ = a^*$$


$(a^+)^*$
$\varepsilon = (a^+)^0$
$a = (a')^1$
$a^2 = (a^2)'$
$= (a')^2$
a^3
$= a^*$

$$(a^*)^+ = (\varepsilon + a + a^2 + a^3 + \cdots)^+$$

$$(\varepsilon) \rightarrow \varepsilon$$

$$(a) \rightarrow a$$

$$(aa) \rightarrow a^2$$

$$\vdots$$

$$= a^*$$

TOPIC:

$\Sigma = \{a\} \Rightarrow \Sigma^* = a^*$

$\Sigma = \{a, b\} \Rightarrow \Sigma^* = (a+b)^*$

$\Sigma = \{a, b, c\} \Rightarrow \Sigma^* = (a+b+c)^*$

$\Sigma = \{\odot, \oplus, \otimes\} \Rightarrow \Sigma^* = (\odot + \oplus + \otimes)^*$

TOPIC: Home Work :

15) $(a+b)^* + \boxed{ab} = (a+b)^*$

16) $(a+b)^+ + \varepsilon = (a+b)^*$

17) $(a+b+\varepsilon)^+ = (a+b)^*$

Important 18) $(a^* b^*)^* = (a+b)^*$

19) $(b^* a^*)^* = (a+b)^*$

20) $(a^*+b)^* =$ "

21) $(a+b^*)^* =$ "

22) $(a^*+b^*)^* =$ "

$(a^*b^*)^*$

- $\varepsilon = (a^*b^*)^0$
- $a = (a^1b^0)^1 = (a\varepsilon)^1 = a$
- $b = (a^0b^1)^1$
- $aa = (a^2b^0)^1$
- $ab = (a^1b^1)^1$
- $ba = (\quad)^2 = \frac{(a^0b^1)^1}{b}\frac{(a^1b^0)^1}{a}$
- ⋮

$= (a+b)^*$

TOPIC:

$$(a+b)^* = (a+b)^0 + (a+b)^1 + (a+b)^2 + (a+b)^3 + \cdots$$

$$= \varepsilon + a+b + aa + ab + ba + bb + \cdots$$

$$R^* = R^0 + R^1 + R^2 + R^3 + \cdots$$

$(a+b)^0 = \varepsilon$

$(a+b)^1 = a+b$

$(a+b)^2 = (a+b).(a+b)$

$= aa + ab + ba + bb$

$(a+b)^3 = aaa + aab + aba + \cdots + bbb$

$$(a+b)^* \neq a^* . b^*$$

$(a+b)^*$ = ba possible

$a^* . b^*$ = ba not possible

$$a^* b^* \neq b^* a^*$$

$$(a^* b^*)^* = (b^* a^*)^* = (a+b)^*$$

$a^* + b^* \neq a^* b^*$

ab not generated

$\{\varepsilon, a, b, a^2, b^2, a^3, b^3, \ldots\}$

ab generated

$\{\varepsilon, a, b, a^2, ab, b^2, \ldots\}$

2 mins Summary

Topic Operators

Topic Properties

Topic Simplification

THANK - YOU