

CS & IT ENGINEERING

Theory of Computation

Regular Languages

Lecture No.- 07

A man with a beard and mustache, wearing a black polo shirt, stands with his arms crossed in front of a blurred bookshelf. He is wearing a black watch on his left wrist.

Malleham Devasane Sir

Recap of Previous Lecture



Topic

Regular Expressions

→ 4 operators
→ properties



Topics to be Covered



Topic

Regular Expressions

↳ Simplification





TOPIC:


① MSQ $a + a^* = a^*$

\downarrow
 ϵ ✓
 a ✓
 aa
 \vdots

A large yellow bracket groups the terms ϵ , a , aa , and \vdots , with an arrow pointing to the a^* in the equation.

- A) $a^+ \rightarrow \epsilon$ not possible
- ~~B) a^*~~
- C) $\frac{a}{a} \cdot \frac{a^*}{\epsilon} = a^+$
- ~~D) $a + aa + a^*$~~
- Green arrows in D point from a and aa to the a^* .

$$a + a^2 + a^*$$

$$\{a\} \cup \{a^2\} \cup \{\epsilon, a, a^2, a^3, \dots\} = a^*$$




TOPIC:



② $\underline{a} \cdot \underline{a^*} =$

$a \cdot \{\epsilon, a, a^2, \dots\}$

$a \cdot \epsilon \rightarrow a$

$a \cdot a \rightarrow a^2$

$a \cdot a^2 \rightarrow a^3$

$a \cdot a^3 \rightarrow a^4$

$= a^+$

~~A) a^+~~

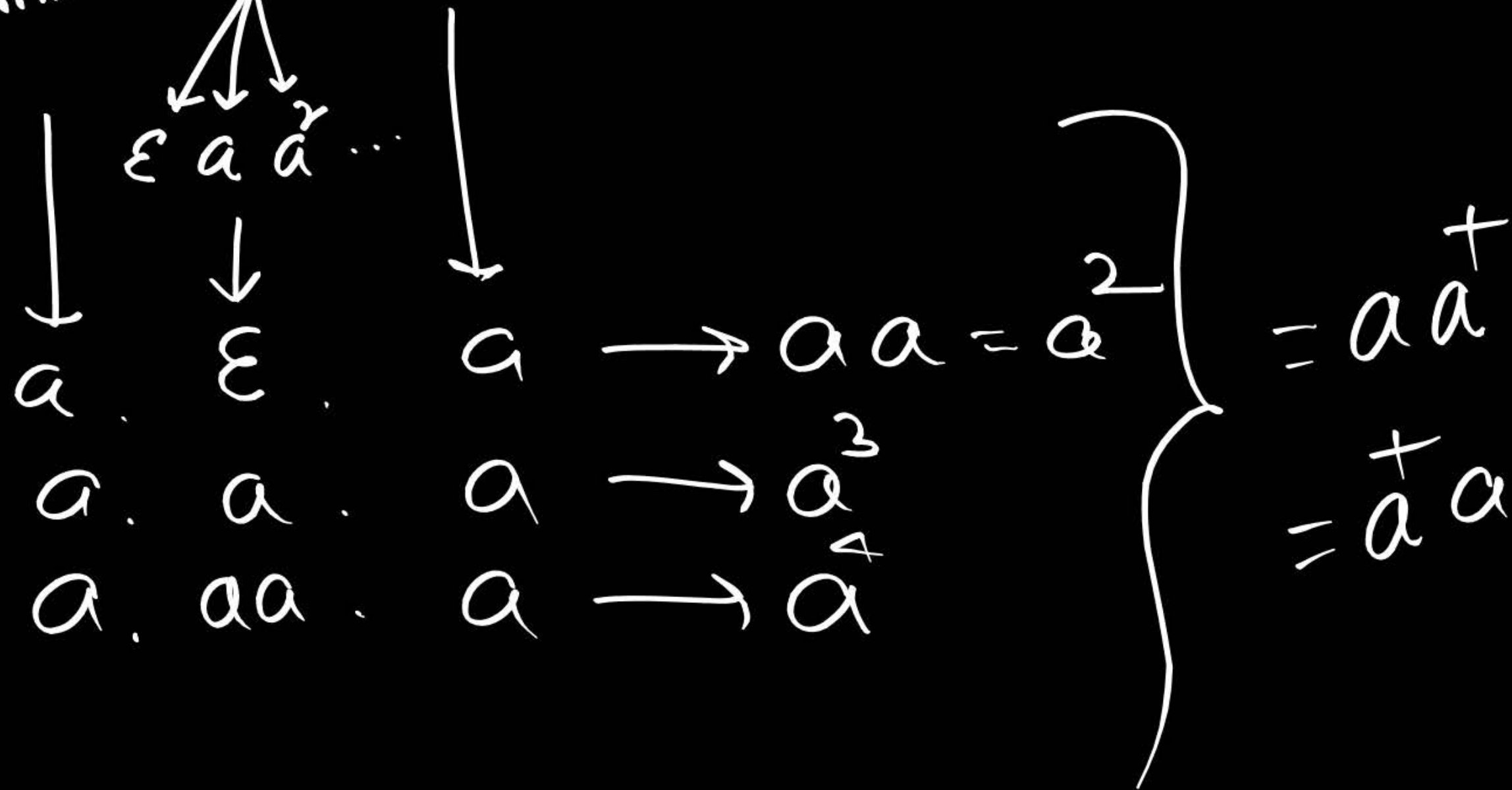
~~B) $a^* \cdot a$~~

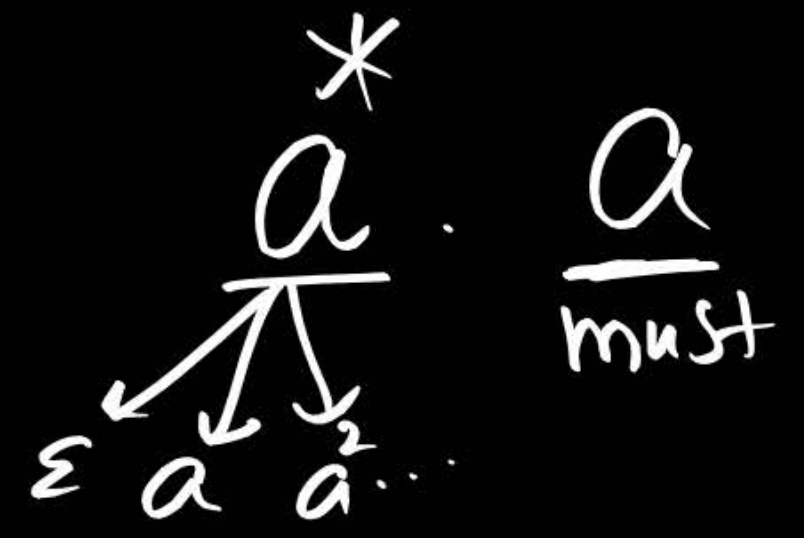
~~C) aa^*a~~

~~D) a^*a^*a~~

$$\begin{array}{l}
 \begin{array}{ccc}
 * & * & \\
 a & a & a \\
 \downarrow & \downarrow & \downarrow \\
 \varepsilon & \varepsilon & a \rightarrow a
 \end{array} \\
 \left. \begin{array}{ccc}
 a & \varepsilon & a \\
 & \text{or} & \\
 \varepsilon & a & a
 \end{array} \right\} \rightarrow a^2 \\
 \begin{array}{ccc}
 \varepsilon & a^2 & a \rightarrow a^3 \\
 a & a & a
 \end{array}
 \end{array}
 \left. \vphantom{\begin{array}{ccc} \varepsilon & a^2 & a \\ a & a & a \end{array}} \right\} = a^+$$

$$\underbrace{a}_{\text{must}} \cdot \underbrace{a^*}_{\text{must}} \cdot \underbrace{a}_{\text{must}} = \{a^2, a^3, a^4, a^5, \dots\}$$





$$\left. \begin{array}{l} \epsilon \cdot a \rightarrow a \\ a \cdot a \rightarrow a^2 \\ a^2 \cdot a \rightarrow a^3 \\ a^3 \cdot a \rightarrow a^4 \end{array} \right\} = a^+$$



TOPIC:



$$a^* = \{a^n \mid n \geq 0\} = \{\epsilon, a, aa, aaa, \dots\}$$

It can generate ϵ

a

aa

aaa



TOPIC:

$$3) (a^*)^{100} =$$

$$\begin{array}{l} a^* \cdot a^* \cdot a^* \dots \text{100 times} \\ \varepsilon \cdot \varepsilon \cdot \varepsilon \dots \varepsilon \Rightarrow \varepsilon \\ a \cdot \varepsilon \cdot \varepsilon \dots \varepsilon \Rightarrow a \\ a^2 \cdot \varepsilon \cdot \varepsilon \dots \varepsilon \Rightarrow a^2 \end{array} \left. \vphantom{\begin{array}{l} a^* \cdot a^* \cdot a^* \dots \text{100 times} \\ \varepsilon \cdot \varepsilon \cdot \varepsilon \dots \varepsilon \Rightarrow \varepsilon \\ a \cdot \varepsilon \cdot \varepsilon \dots \varepsilon \Rightarrow a \\ a^2 \cdot \varepsilon \cdot \varepsilon \dots \varepsilon \Rightarrow a^2 \end{array}} \right\} = a^*$$

~~A) a^*~~

~~B) $(a^*)^2 = a^* \cdot a^* = a^*$~~

~~C) $(a^2)^* = (aa)^*$~~

~~D) $(a^{100})^*$~~

$$\begin{aligned}
 (a^{100})^* &= (aaaa \dots 100 \text{ times})^* \\
 &\downarrow \varepsilon \\
 &a^{100} \\
 &a^{200} \\
 &\vdots
 \end{aligned}$$

$$\underbrace{(a^*)^2}_{= a^*} \neq \underbrace{(a^2)^*}_{= (aa)^*}$$

$\{e, a, a^2, a^3, \dots\}$

$\{e, a^2, a^4, a^6, \dots\}$



TOPIC:



4) $a.(aa)^*$ =

$$a.\epsilon \longrightarrow a$$

$$a.aa \longrightarrow a^3$$

$$a.(aa)^2 \longrightarrow aaaaaa \longrightarrow a^7$$

$$a.(aa)^3 \longrightarrow a^9$$

⋮

A) $(aa)^* = \{\epsilon, aa, aaaa, \dots\}$

~~B) $(aa)^*a = \{a, aaaa, \dots\}$~~

C) $a(aa)^*a = \{aaa, aaaaa, \dots\}$

D) $a^* = \{\epsilon, a, aa, \dots\}$



TOPIC:



$$5) a^* \cdot a^* \cdot a^* = a^*$$

$$6) (a^*)^* = a^*$$

$$7) a^* + a^* = a^*$$

$$8) a^* + a^+ = a^*$$

$$9) (a^*)^+ = a^*$$

$$10) (a^+)^* = a^*$$

$$11) (a^+)^+ = a^+$$

$$12) a^+ \cdot a^* \cdot a^* = a^+$$

$$13) a^* \cdot a \cdot a^* = a^+$$

$$14) a^+ + \epsilon = a^*$$

$$a^* \cdot \boxed{a} \cdot a^* \xrightarrow{\text{must}} \varepsilon X$$

$$\varepsilon \cdot a \cdot \varepsilon \xrightarrow{\quad} a \checkmark$$

$$\varepsilon \cdot a \cdot a \xrightarrow{\quad} a^2$$

$$\varepsilon \cdot a \cdot a^2 \xrightarrow{\quad} a^3$$

$$\left. \begin{array}{l} a \\ a^2 \\ a^3 \end{array} \right\} = a^+$$

$$a^+ + \varepsilon = a + a^2 + a^3 + \dots + \varepsilon = a^*$$

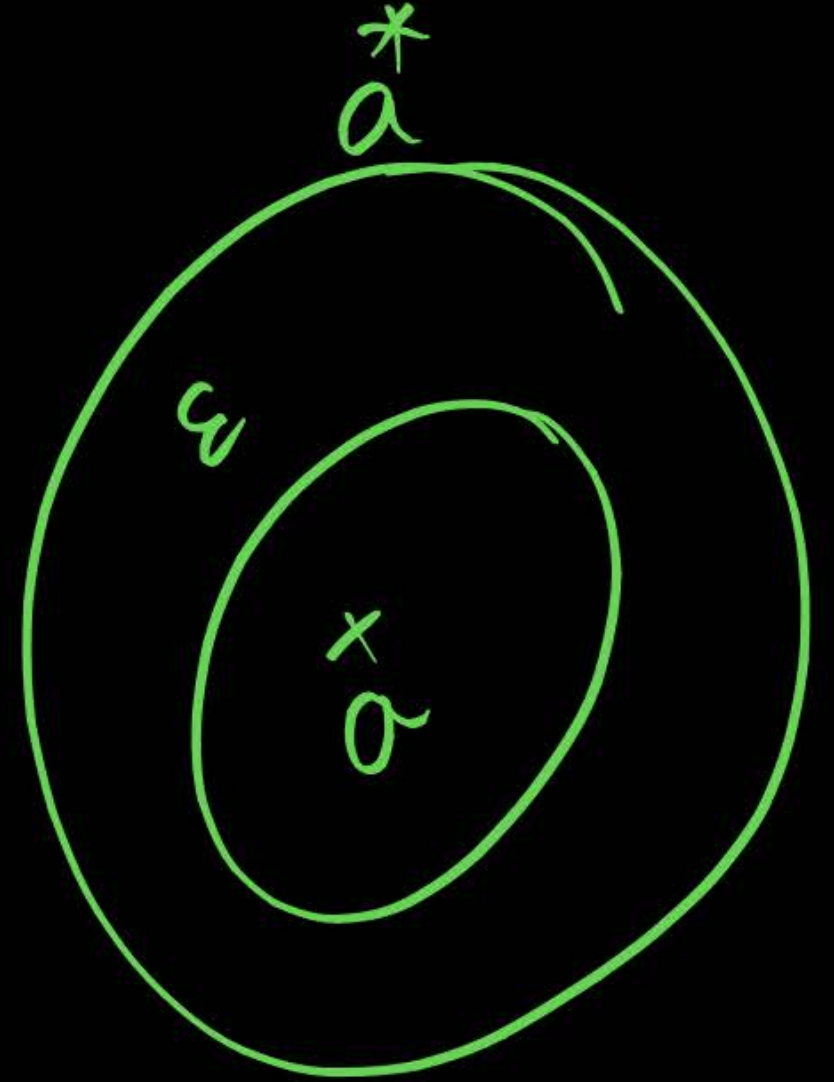
$$\underline{R^+} + \varepsilon = R^*$$

\downarrow
 R^1
 R^2
 R^3
 R^4
 \vdots

$$a^* + a^+$$

$$a^* \cup a^+$$

$$\{\varepsilon, a, a^2, \dots\} \cup \{a, a^2, \dots\} = a^*$$

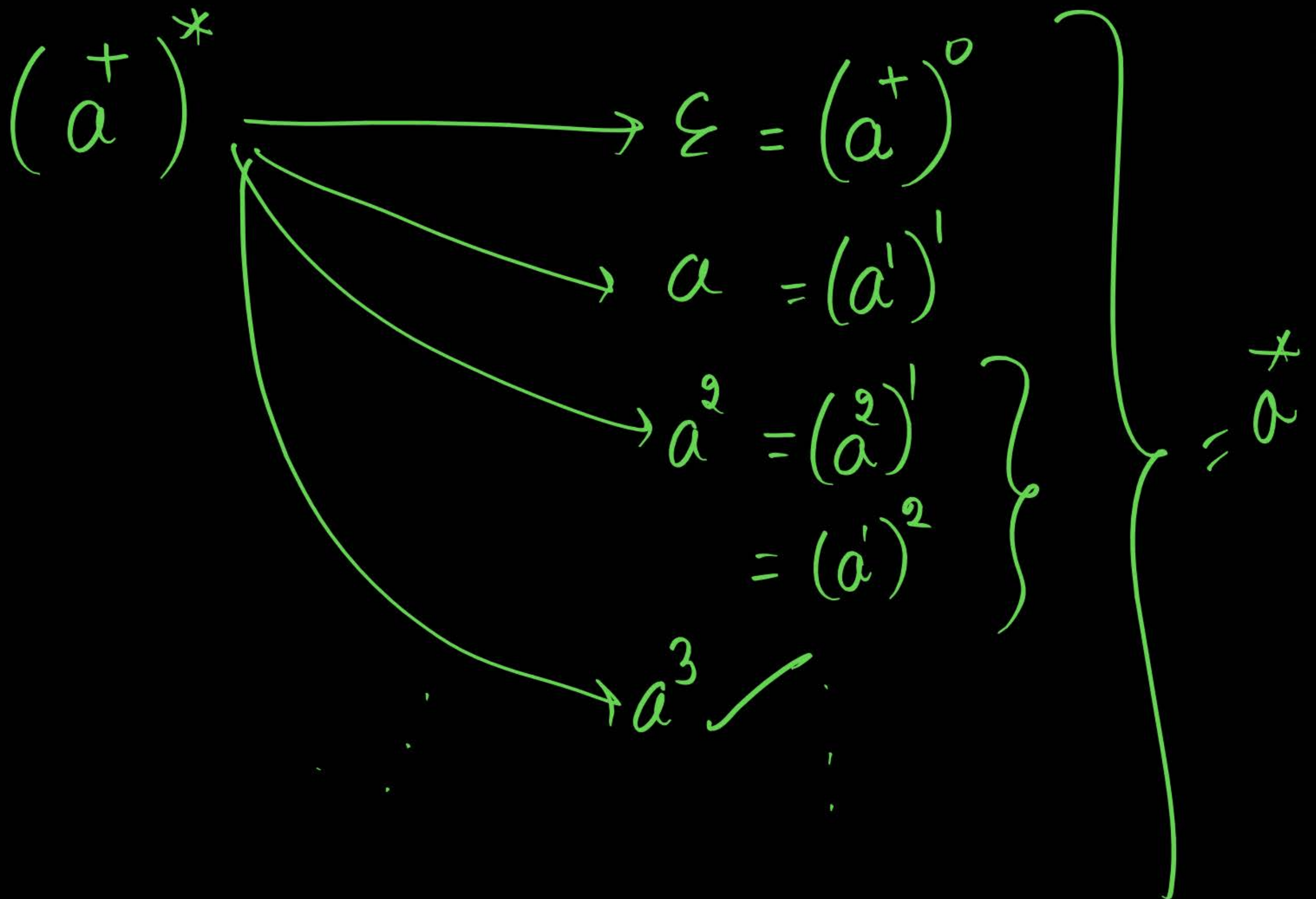


$$\Sigma^* \cup \text{Any} = \Sigma^*$$

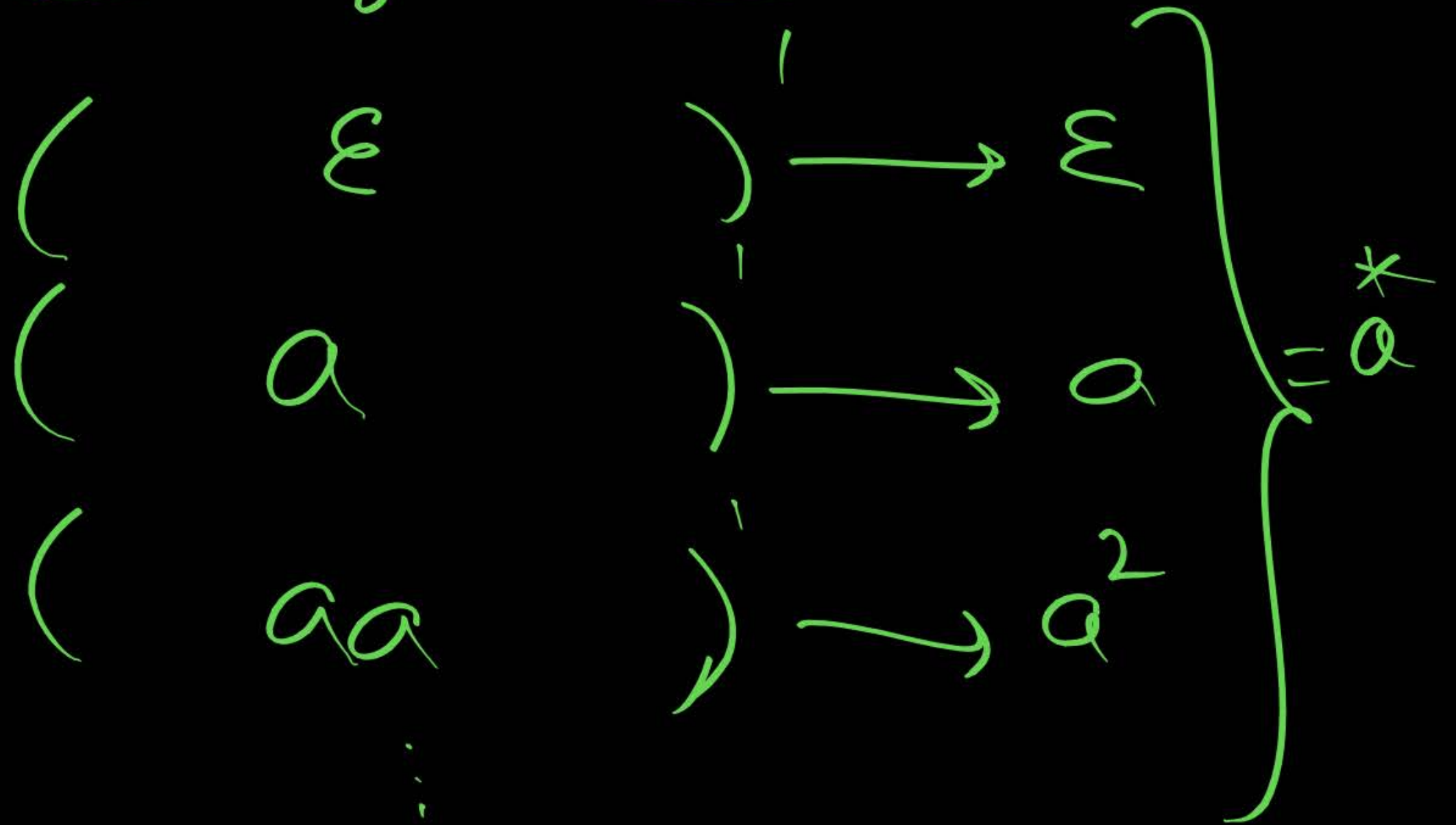
$$a^* + a = a^*$$

$$a^* + aa = a^*$$

$$a^* + a^+ = a^*$$



$$(a^*)^+ = \underbrace{(\varepsilon + a + a^2 + a^3 + \dots)^+}_{\text{NFA diagram}}$$





TOPIC:



$$\Sigma = \{a\} \Rightarrow \Sigma^* = a^*$$

$$\Sigma = \{a, b\} \Rightarrow \Sigma^* = (a+b)^*$$

$$\Sigma = \{a, b, c\} \Rightarrow \Sigma^* = (a+b+c)^*$$

$$\Sigma = \{0, +, \otimes\} \Rightarrow \Sigma^* = (0 + \otimes + \otimes)^*$$



TOPIC: Home work :



$$15) (a+b)^* + \boxed{ab} = (a+b)^*$$

$$16) (a+b)^+ + \epsilon = (a+b)^*$$

$$17) (a+b+\epsilon)^+ = (a+b)^*$$

Important

$$18) (a^* b^*)^* = (a+b)^*$$

$$19) (b^* a^*)^* = (a+b)^*$$

$$20) (a^* + b)^* = ''$$

$$21) (a + b^*)^* = ''$$

$$22) (a^* + b^*)^* = ''$$

$$(a^*b^*)^*$$

$$\rightarrow \varepsilon = (a^*b^*)^0$$

$$\rightarrow a = (a^1b^0)' = (a\varepsilon)' = a$$

$$\rightarrow b = (a^0b^1)'$$

$$\rightarrow aa = (a^2b^0)'$$

$$\rightarrow ab = (a^1b^1)'$$

$$\rightarrow ba = \left(\frac{a^0b^1}{b} \frac{a^1b^0}{a} \right)'$$

$$= (a+b)^*$$



TOPIC:



$$(a+b)^* = (a+b)^0 + (a+b)^1 + (a+b)^2 + (a+b)^3 + \dots$$
$$= \epsilon + a + b + aa + ab + ba + bb + \dots$$

$$R^* = R^0 + R^1 + R^2 + R^3 + \dots$$

$$(a+b)^0 = \epsilon$$

$$(a+b)^1 = a + b$$

$$(a+b)^2 = (a+b) \cdot (a+b)$$

$$= aa + ab + ba + bb$$

$$(a+b)^3 = aaa + aab + aba + \dots + bbb$$

$$(a+b)^* \neq \underbrace{a^* \cdot b^*}$$

\Downarrow
ba possible

\downarrow
ba not possible

$$a^*b^* \neq b^*a^*$$

$$(a^*b^*)^* = (b^*a^*)^* = (a+b)^*$$

$$a^* + b^* \neq a^* b^*$$

\downarrow
 ab not generated

$\{\epsilon, a, b, a^2, b^2, a^3, b^3, \dots\}$

$$a^* b^*$$

\downarrow
 ab generated

$\{\epsilon, a, b, a^2, ab, b^2, \dots\}$



2 mins Summary



Topic

Operators

Topic

Properties

Topic

Simplification



THANK - YOU