

CS & IT ENGINEERING

Theory of Computation

Regular Languages

Lecture No.- 04

A man with a beard and mustache, wearing a black polo shirt, standing with his arms crossed in front of a bookshelf.

Malleham Devasane Sir

Recap of Previous Lecture



Topic

Basics of TOC



$\rightarrow \Sigma = \{a, b\}$
 $\rightarrow \text{String} = aaab$
 $\rightarrow \text{Language} = \{ab, aaa, aaab\}$

Topics to be Covered



Topic

Regular Expressions



problem



Language (Set)

How to check given number is even or not?

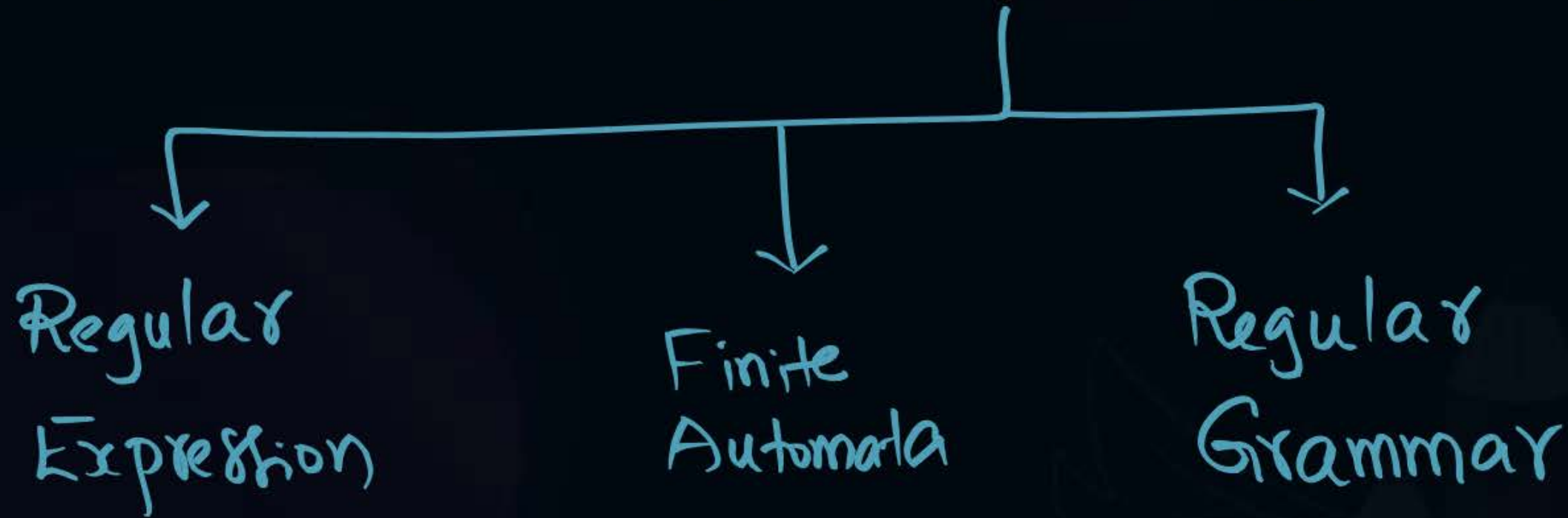
$L = \{0, 2, 4, 6, 8, \dots\}$ in maths

$L = \{\epsilon, aa, a^4, a^6, a^8, \dots\}$ in TPC over $\Sigma = \{a\}$



TOPIC: Regular Language :

Regular Language [Type-3 Language]





TOPIC:

Regular Expression :

→ It represents a regular language
(generates)
(describes)

→ It uses 4 operators





TOPIC: Operators :

Unary operator :

↳ one operand associated with unary operator

Binary

↳ 2 operands



TOPIC:

OR +	Concatenation •	Kleene star *	Kleene plus +
$a + b$ $aa + b$ $ab + aaa$ $R_1 + R_2$	$a.b$ $aa.bbb$ $ab.bba$ $R_1 R_2$	a^* $(ab)^*$ $(aaa)^*$ R^*	a^+ $(aaa)^+$ R^+

$(a+)b$ X

a_* X



TOPIC: OR +

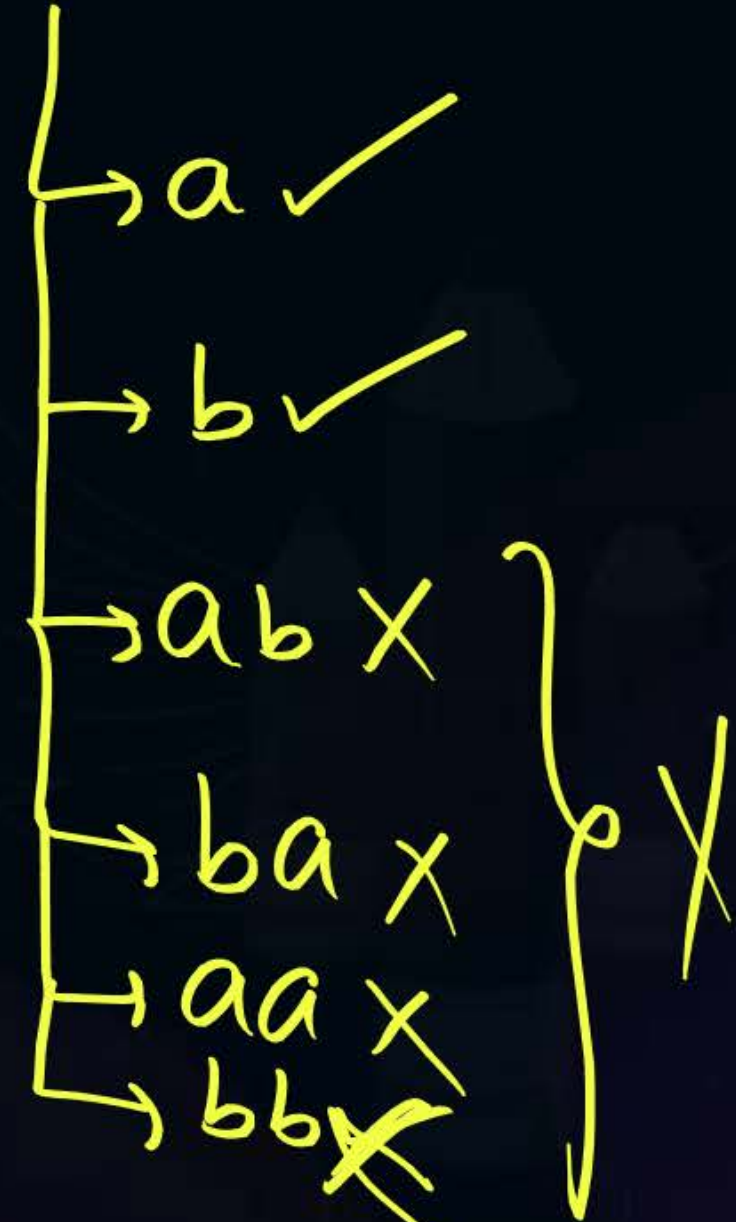


$R_1 + R_2$

either R_1 or R_2

$a+b$

$\{a, b\}$



$$L(R_1 + R_2) = L(R_1) \cup L(R_2)$$

$$L(a+b) = L(a) \cup L(b)$$

$$= \{a\} \cup \{b\}$$

$$= \{a, b\}$$

**TOPIC:**

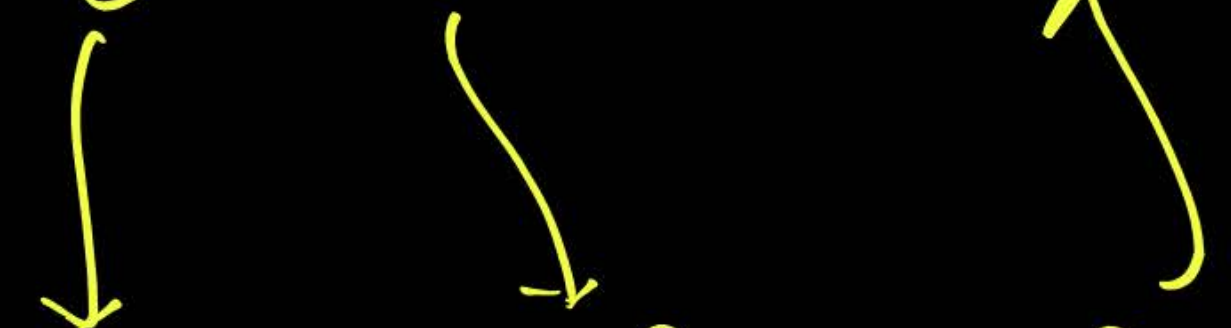
Expression

Vs

Language

ϵ	_____	$\{\epsilon\}$
a	_____	$\{a\}$
b	_____	$\{b\}$
ϕ	_____	$\{\} = \phi$
$a+b$	_____	$\{a, b\}$
$a+\epsilon$	_____	$\{a, \epsilon\}$
$a+a$	_____	$\{a\}$
$\epsilon+\phi$	_____	$\{\epsilon\}$

Reg Exp: $\epsilon + \phi = \Sigma$



Set:
(Language)

$$\{\epsilon\} \cup \{\} = \{\epsilon\}$$

ϕ
 → empty expression
 → empty language
 → no string generated

ϵ
 → non empty exp
 → empty string

$$L(\phi) = \{ \}$$

$$L(\epsilon) = \{ \epsilon \}$$



TOPIC: Simplify the expression.

$$a + a = a \checkmark$$

$$\phi + \phi = \phi \checkmark$$

$$\phi + \varepsilon = \varepsilon \checkmark$$

$$a + b = a + b \checkmark$$

$$a + \varepsilon = a + \varepsilon \checkmark$$

$$a + b$$

$$a \text{ or } b$$

$$a \mid b$$

$$a \cup b$$

$$L = \{a, b\}$$



TOPIC:



$$a + b \quad \checkmark$$

$$(a) + b \quad \checkmark$$

$$a + (b) \quad \checkmark$$

$$(a + b) \quad \checkmark$$

$$(a) + (b) \quad \checkmark$$

all

$$a(+)b \quad \times$$

$$(a +)b \quad \times$$

$$a(+ b) \quad \times$$

not all



TOPIC: Concatenation •



$R_1 \cdot R_2$

$R_1 R_2$

R_1 followed by R_2

ab
→ ax
→ bx
→ ab ✓
→ bax



TOPIC:



$$I) \quad a \cdot \epsilon =$$

$$a = a \epsilon$$

$$= \epsilon a$$

$$= \epsilon \epsilon \epsilon \epsilon a$$

$$= a \epsilon \epsilon \epsilon$$

$$= \epsilon a \epsilon$$

zero symbols

$$\checkmark A) a$$

$$B) aa$$

$$\checkmark C) \epsilon a$$

$$\checkmark D) a \epsilon \epsilon$$

$$|\epsilon| = 0$$

$$|\epsilon| = 0$$

(Note: In the original image, there are arrows pointing from the epsilon symbol to the words "string" and "length")

$$|\phi| = 0$$

(Note: In the original image, there are arrows pointing from the phi symbol to the words "size" and "set")

ϕ is set

ϵ is string

$|\text{Set}| = \text{Size of set} = \text{no. of objects in set}$

$$|\{a, bb, ab\}| = 3$$

$|\text{String}| = \text{Length of string}$

$$|aa| = 2$$



TOPIC:



$$\mathcal{E} + \mathcal{E} = \mathcal{E}$$

Set

↳ It is collection of ^{unordered} distinct objects

$$\boxed{\{a, b\} = \{b, a\}} \neq \underbrace{\{a, a, b\}}_{\text{multiset}}$$



TOPIC:



$$\text{II)} \quad a \cdot \phi = \phi$$

$$\text{III)} \quad \phi \cdot a = \phi$$

$$\text{IV)} \quad a \cdot a = aa = a^2$$

$$\text{V)} \quad \phi \cdot \varepsilon = \phi$$

$$a \cdot \phi = \phi$$

$$a \cdot \varepsilon = a$$

$$a + a = a$$

$$a \cdot a = aa$$



TOPIC:

Simplify expressions



$$\textcircled{1} \quad \varepsilon + \varepsilon = \varepsilon$$

$$\textcircled{2} \quad \phi + \phi = \phi$$

$$\textcircled{3} \quad a + a = a$$

$$\textcircled{4} \quad \varepsilon \cdot \varepsilon = \varepsilon$$

$$\textcircled{5} \quad \phi \cdot \phi = \phi$$

$$\textcircled{6} \quad a \cdot a = aa$$

$$\textcircled{7} \quad \varepsilon + a = \varepsilon + a$$

$$\textcircled{8} \quad \varepsilon + \phi = \varepsilon$$

$$\textcircled{9} \quad \phi + a = a$$

$$\textcircled{10} \quad \varepsilon \cdot a = a$$

$$\textcircled{11} \quad \varepsilon \cdot \phi = \phi$$

$$\textcircled{12} \quad \phi \cdot a = \phi$$

$$\mathcal{E} + \phi = \mathcal{E}$$

$$\mathcal{E} \cup \mathcal{E} = \mathcal{E}$$

$$\phi.R = R.\phi = \phi$$

$$\varepsilon.R = R.\varepsilon = R$$

#Q1. $a + \varepsilon + \phi =$

A a

~~**B**~~ $a + \varepsilon$

C ϕ

D aa

#Q2. $\phi + (\epsilon \cdot \phi) \cdot a$

☒ **A** ϕ

☐ **B** a

☐ **C** ϵ

☐ **D** aa

#Q3. $a \cdot \phi \cdot \varepsilon \cdot a$

A a

B ε

~~**C**~~ ϕ

D aa

#Q4. $(a \cdot \varepsilon) + \varepsilon \cdot \phi$

☒ **A** a

☐ **B** ε

☐ **C** ϕ

☐ **D** aa



2 mins Summary



Topic

Regular Language

Topic

Regular Expression

Topic

Operators



OR :

Concatenation

THANK - YOU