Computer Science

Theory of Computation

Context Free Languages

Lecture No.- 1

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Recap of Previous Lecture

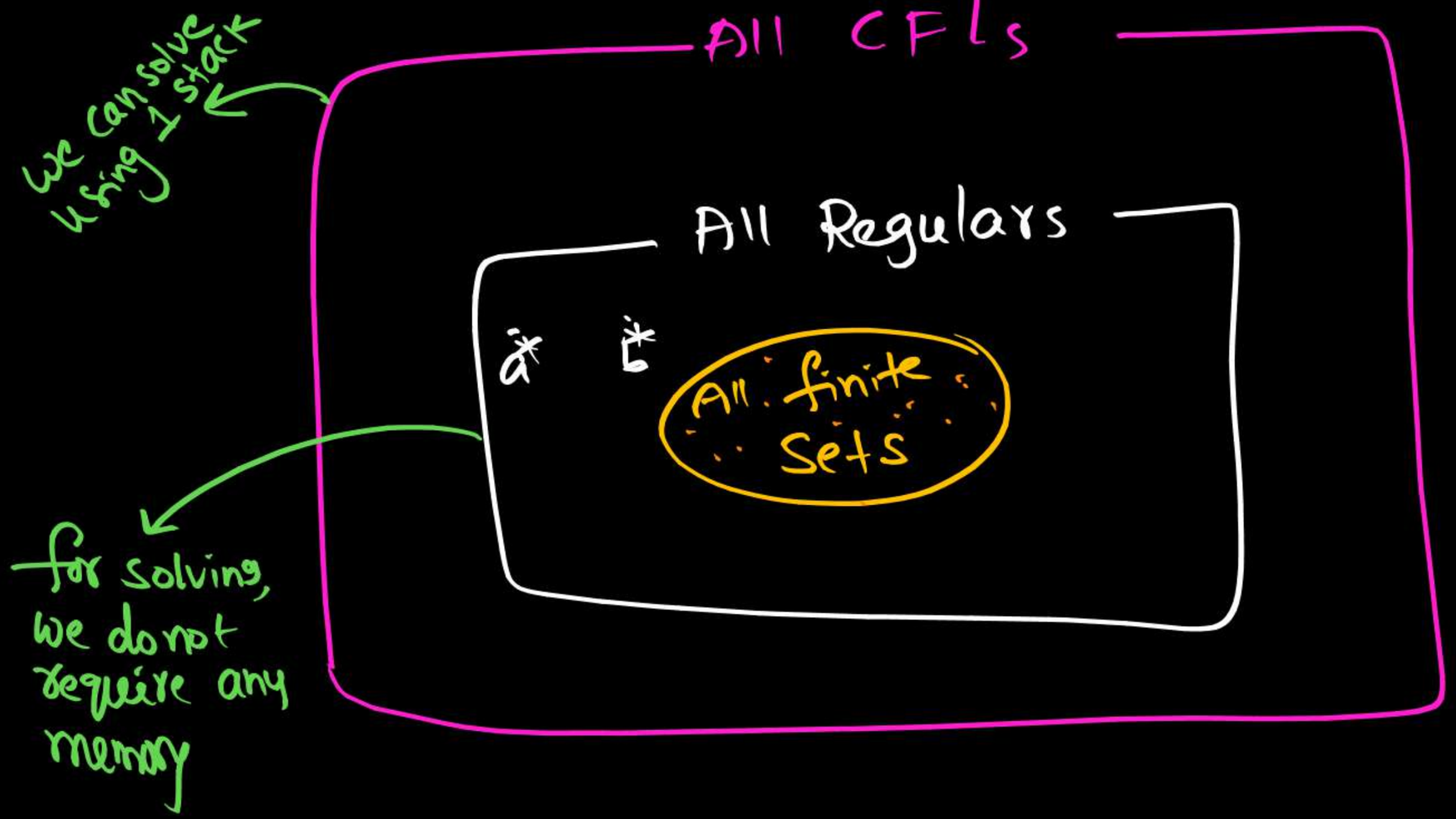
Topic Regular Languages and Non-regular Languages

Topics to be Covered

Topic	CFG
Topic	Types of CFGs
Topic	CFG Vs CFL
Topic	Types of Normal Forms
Topic	Simplification of CFGs


Context Free Language
CFG
PDA

Regular language need not be Subset of CFL.



Every finite is Reg

Every Reg set is CFL.

Every finite set is CFL.

Set of all Regulars is subset of Set of all CFLs

I) Regular language is CFL

II) Regular language need not be subset of CFL

*** III) "Set of all regulars" is subset of "Set of all CFLs"

Class of Regulars $\subset$ Class of CFLs

$X = \{\phi, a^*, \{\varepsilon\}, b^*, a^*b^*, \ldots\} \subset L = \{\phi, a^*, \{\varepsilon\}, b^*, a^*b^*, \ldots, a^n b^n, a^n b^{2n}, \ldots\}$

Applications of CFLs

- All applications of Regulars
- Any application that depends on 1 stack
  - Mobile phone call history
  - web Browsing History
  - Recursion
  - Syntax Verification of Programs

Understanding — Knowing

↳ Revision — Remembering

↳ Practice — Applying

Context Free Grammar

$$CFG = (V, T, P, S)$$

- V → Set of variables (non-terminals)
- T → Set of terminals
- P → Set of rules (productions)
- S → Start symbol, $S \in V$

Each rule in CFG:

$V \rightarrow$ Any

$$\boxed{V \rightarrow (V \cup T)^*}$$

Context Free Grammar

① $S \to AB$
$A \to aA \mid \varepsilon$
$B \to bB \mid \varepsilon$

It is CFG

$V = \{S, A, B\}$

$T = \{a, b\}$

$P = \{S \to AB,\ A \to aA,\ A \to \varepsilon,\ B \to bB,\ B \to \varepsilon\}$

$S = S$

② $S \to AaB$
$Aa \to aa$
$B \to \varepsilon$

It is not CFG

Context Free Grammar

RG

LLG	RLG	LG	CFG
$V \to VT^* \mid T^*$	$V \to T^*V \mid T^*$	$V \to T^*VT^* \mid T^*$	$V \to (V+T)^*$
Example $S \to Sab \mid \varepsilon$	$S \to abS \mid \varepsilon$	$S \to aSb \mid \varepsilon$	$S \to AaBS \mid \varepsilon$ $A \to a$ $B \to bb$

I) Every LLG is LG
II) Every RLG is LG
III) Every LLG is CFG
IV) Every RLG is CFG

→ V) Every RG is LG
VI) Every RG is CFG
VII) Every LG is CFG

Derivation of a String:

I) Left Most Derivation (LMD)

II) Right " " (RMD)

III) Parse Tree (Derivation Tree)

Context Free Grammar

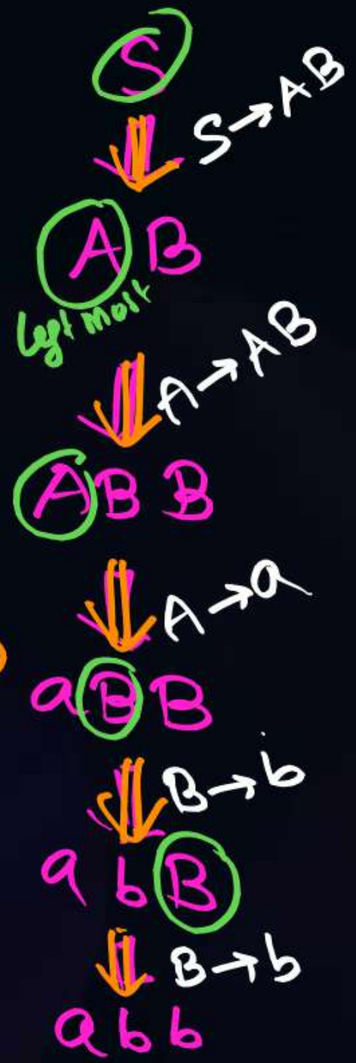
LMD

$S \to AB$

$A \to AB \mid a$

$B \to b$

$w = abb$



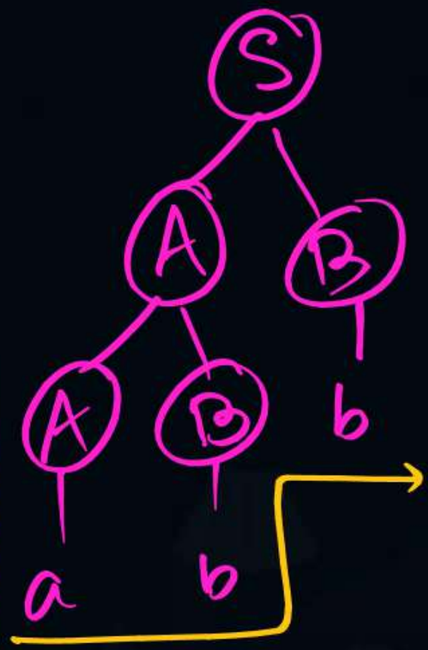
5 steps in LMD
(5 Substitutions)

LMD order:
S, A, A, B, B

RMD

5 steps
RMD order:
S, B, A, B, A

Parse Tree



$=$ 5 steps
$=$ 5 non leaf nodes

5 non leaf nodes
3 leaf nodes
Every leaf is either terminal or ε
Every non leaf is non-terminal

Context Free Grammar

LMD

Sequence derived from S either directly or indirectly using LMD

Left Sentential form: (Each step)

S
AB
ABB
aBB
abB
abb

S → AB
AB (Left most)
A → AB
ABB
A → a
aBB
B → b
abB
B → b
abb

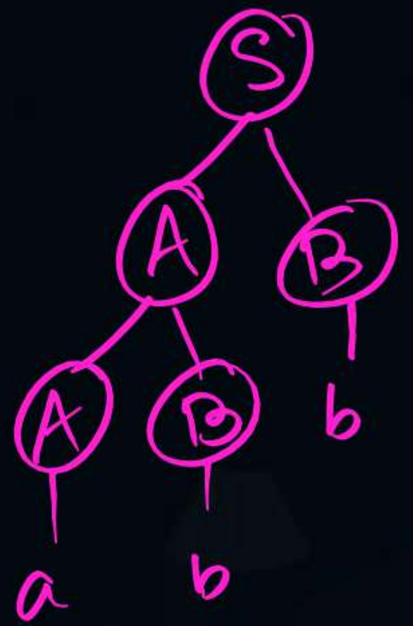
RMD

S
AB (Right most)
Ab
ABb
Abb
abb

Right Sentential form

S
AB
Ab
ABb
Abb
abb

Parse Tree



LMD: In each step, left most non-terminal is Substituted with corresponding production to derive given string.

RMD: In each step, right most non-terminal

Note:
In general, LMD and RMD need not be same

Note :

For given string :

No. of derivations = No. of LMDs = No. of RMDs = No. of Parse Trees.

Length of derivation = No. of steps in LMD = No. of steps in RMD = No. of non leaf nodes in PT

$S \to AB \mid ab$

$A \to a \mid b$

$B \to b \mid a$

$\omega = ab$

$\Downarrow$

2 derivations
(2 LMDs)
(2 RMDs)
(2 PTs)

$S \Rightarrow AB \Rightarrow aB \Rightarrow ab$

OR

$S \Rightarrow ab$

No. of steps = 3 or 1

$S \Rightarrow AB \Rightarrow Ab \Rightarrow ab$

OR

$S \Rightarrow ab$

No. of steps = 3 or 1

No. of steps = 3 or 1

Types of CFGs

I) Unambiguous CFG :

Every string derived from given CFG has exactly one derivation.

II) Ambiguous CFG :

Some string derived from CFG has more than one derivation.

Identify Ambiguous and Unambiguous CFGs.

① $S \to a \mid b$

Unambiguous CFG

a: S ⇓ a ⇓ 1 PT

b: S ⇓ b ⇓ 1 PT

② $S \to A \mid ab$

$A \to ab \mid c$

Amb. CFG

i) c ⇒ 1 PT ✓

ii) ab ⇒ >1 PT (=2 PTs) ⇒ Amb CFG

Context Free Grammar

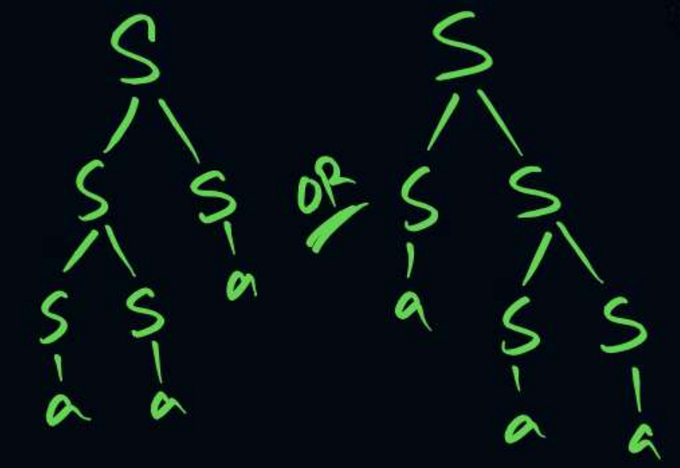
③ $S \rightarrow SS \mid a$

Ambiguous CFG

i) a ⇒ 1 PT

ii) aa ⇒ 1 PT

iii) aaa ⇒ >1 PT



④ $S \rightarrow aS \mid Sb \mid c$

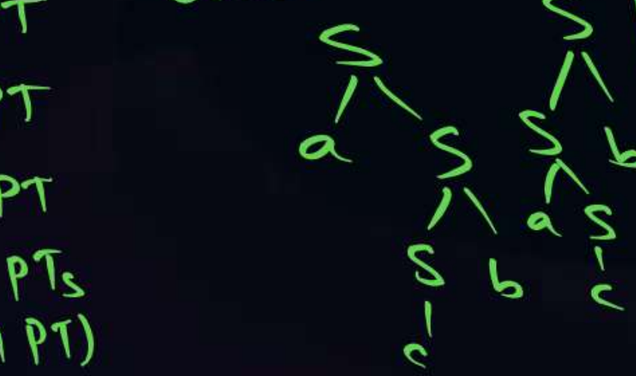
c ⇒ 1 PT

ac ⇒ 1 PT

cb ⇒ 1 PT

acb ⇒ 2 PTs (>1 PT)

acb:



⑤ $E \rightarrow E+E \mid a$ Ambiguous CFG

⑥ $S \rightarrow AB$

$A \rightarrow aA \mid Ab \mid \varepsilon$ AmL CFG

$B \rightarrow d$

CFG Vs CFL

① $S \to a \mid b \mid \varepsilon$ $\quad L = \{\varepsilon, a, b\}$

② $S \to Sa \mid b$ $\quad L = ba^*$

③ $S \to aS \mid b$ $\quad L = a^*b$

④ $S \to aS \mid \varepsilon$ $\quad L = a^*$

⑤ $S \to Sa \mid \varepsilon$ $\quad L = a^*$

⑥ $S \to aS \mid bS \mid \varepsilon$ $\quad L = (a+b)^*$

⑦ $S \to Sa \mid Sb \mid \varepsilon$ $\quad L = (a+b)^*$

⑧ $S \to abS \mid \varepsilon$ $\quad L = (ab)^*$

(9) $S \rightarrow aS \mid bS \mid c \mid d \mid e$

$(a+b)S$ $\quad c+d+e$

$L = (a+b)^* (c+d+e)$

10)

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b$

Not Regular Grammar

$L = \{ab\}$

Regular lang

Note:

Note: I) Every Regular Grammar always generates regular language.

II) If grammar is not regular then it may (or may not) generate regular language.

11) $S \rightarrow AB$

$A \rightarrow aA \mid \varepsilon$

$B \rightarrow Bb \mid \varepsilon$

$B = b^*$

$A = a^*$

$S = A \cdot B = a^* b^*$

$L = a^* b^*$

12) $S \rightarrow aS \mid Sb \mid \varepsilon$

$L = a^* b^*$

$a^* S b^*$

ε ✓

a ✓

b ✓

aa ✓

ab ✓

ba ✗

Context Free Grammar

13) $S \to aS \mid Sb \mid a$ $\quad L = a^* a\, b^* = a^+ b^* = \{a^m b^n \mid m \geq 1, n \geq 0\}$

14) $S \to aS \mid Sb \mid b$ $\quad L = a^* b\, b^* = a^* b^+$

15) $S \to aS \mid Sb \mid a \mid b$ $\quad L = a^*(a+b)b^* = a^* a b^* + a^* b b^* = a^+ b^* + a^* b^+$

16) $S \to aaS \mid Sb \mid \varepsilon$ $\quad L = (aa)^* b^*$

$= \{a^{2m} b^n \mid m, n \geq 0\}$

Context Free Grammar

17) $S \to aSb \mid \varepsilon$

equal

$L = a^n \varepsilon b^n = \{a^n b^n \mid n \geq 0\}$

$= \{\varepsilon, ab, a^2b^2, a^3b^3, \ldots\}$

18) $S \to aSb \mid a$

$L = a^n a b^n = a^{n+1} b^n = \{a^{n+1} b^n \mid n \geq 0\}$

19) $S \to aSb \mid b$

$L = a^n b^{n+1}$

20) $S \to aSa \mid \#$

$L = a^n \# a^n$

(21) $S \to aaSb \mid \varepsilon$ $\quad L = a^{2n} b^{n}$

(22) $S \to aSbb \mid \varepsilon$ $\quad L = a^{n} b^{2n}$

(23) $S \to aaSbb \mid \varepsilon$ $\quad L = a^{2n} b^{2n}$

(24) $S \to aaaSb \mid \varepsilon$ $\quad L = a^{3n} b^{n}$

(25) $S \to aSa \mid \varepsilon$

$L = a^{n} a^{n}$
$= a^{2n}$
$= (aa)^{*}$

(26) $S \to AB$
$A \to aAb \mid \varepsilon$
$B \to cBd \mid \varepsilon$

$B = \{c^n d^n\}$

$A = \{a^k b^k\}$

$L = a^k b^k c^n d^n$

$= \{a^i b^j c^k d^l \mid i=j, k=l\}$

(27) $S \to AB$
$A \to aA \mid \varepsilon$
$B \to aBb \mid \varepsilon$

$L = a^* a^n b^n$
$= \{a^i b^j \mid i \geq j\}$
$= \{a^{m+n} b^n\}$

(28) $S \to AB$
$A \to aAb \mid \varepsilon$
$B \to bB \mid \varepsilon$

$L = a^n b^n b^*$
$= \{a^i b^j \mid i \geq j\}$
$= \{a^n b^{m+n}\}$

(29) $S \to aSb \mid A$
$A \to aAd \mid \varepsilon \Rightarrow A = a^n d^n$

$L = a^k A b^k = a^k a^n d^n b^k$
$= \{a^i b^j c^k \mid i = j+k\} = a^{k+n} d^n b^k$

(30) $S \to aSb \mid A$
$A \to cAb \mid \varepsilon$

$A = c^n b^n$
$S = a^k A b^k = a^k c^n b^n b^k$
$= a^k c^n b^{n+k} = \{a^p c^q b^r \mid r = p+q\}$

2 mins Summary

Topic

CFG ✓

THANK - YOU