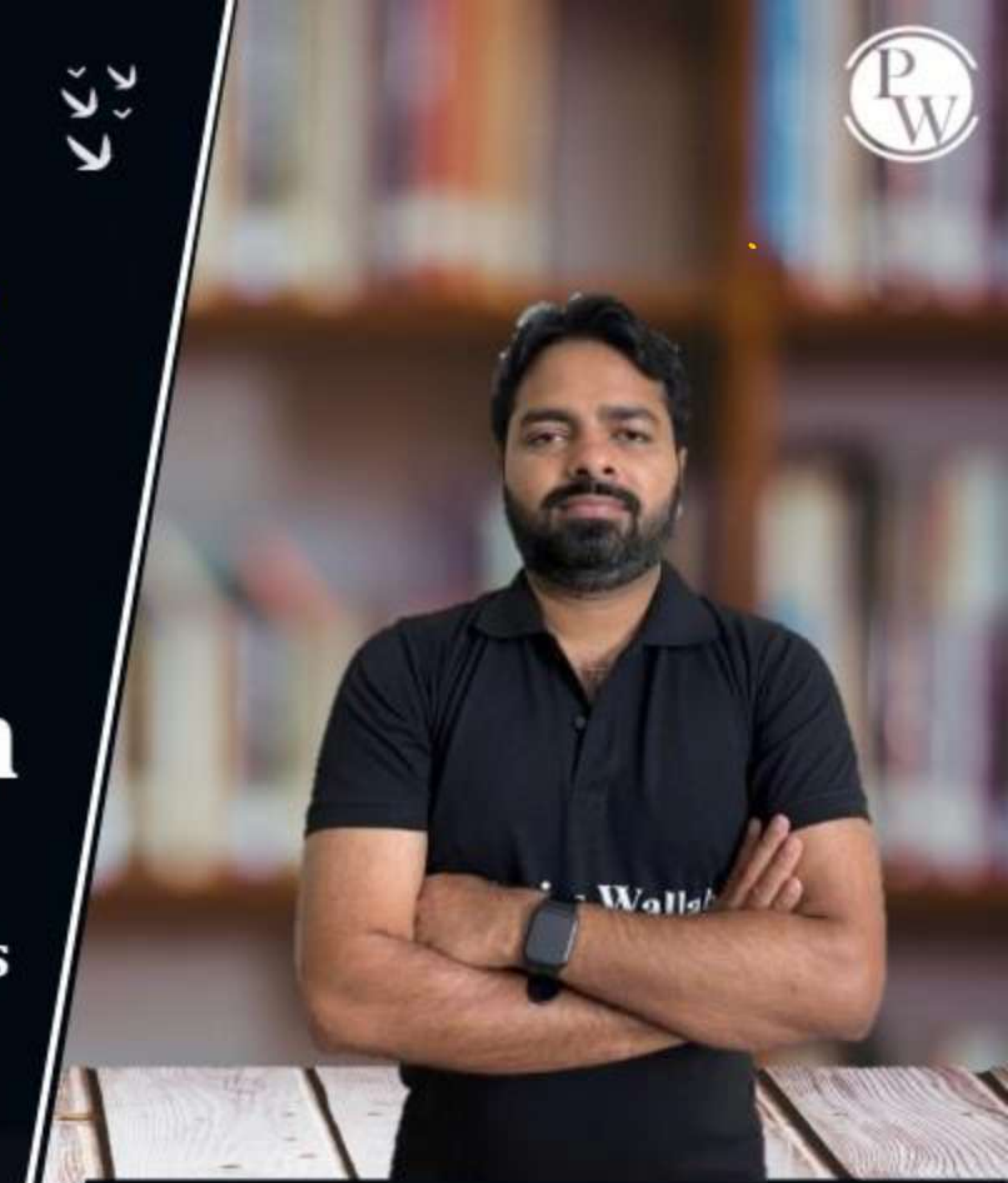
Computer Science

Theory of Computation

Regular Languages and Non-regular Languages

Lecture No.- 8



Mallesham Devasane Sir

Recap of Previous Lecture

Topic Closure Properties for Finite Languages

Topic Closure Properties for Infinite Languages

Topic Closure Properties for Regular Languages

$\cup$ ✓

$\cap$ ✓

$\overline{L}$ ✓

Diff ✓

Topics to be Covered

Topic Closure Properties for Regular Languages

Closure Properties for regular languages: $L_i \rightarrow$ Regular lang

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $\overline{L}$
4. $L_1 - L_2$
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+
9. $\boxed{\text{Subset}(L)}$
10. prefix(L)
11. suffix(L)
12. Substring(L)
13. $f(L)$ = Substitution
14. $h(L)$ = Homomorphism
15. $h^{-1}(L)$
16. L_1 / L_2
    Quotient
17. $L_1 \oplus L_2$
    Symmetric Difference
18. Half$(L) = \frac{1}{2}(L)$
19. Second Half(L)
20. one third(L)
21. Middle $\frac{1}{3}(L)$
22. Last $\frac{1}{3}(L)$
23. Finite union
24. " $\cap$
25. " Difference
26. " Concatenation
27. " Subset
28. " Substitution
29. Inf $\cup$
30. Inf $\cap$
31. Inf $-$
32. Inf concatenation
33. Inf subset
34. Inf substitution

5) Concatenation

$L_1 . L_2 = \{ w_1 . w_2 \mid w_1 \in L_1 , w_2 \in L_2 \}$

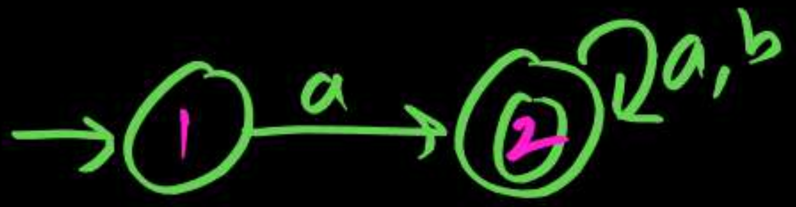
$\rightarrow$ closed for Regular Languages

$R_1 . R_2$

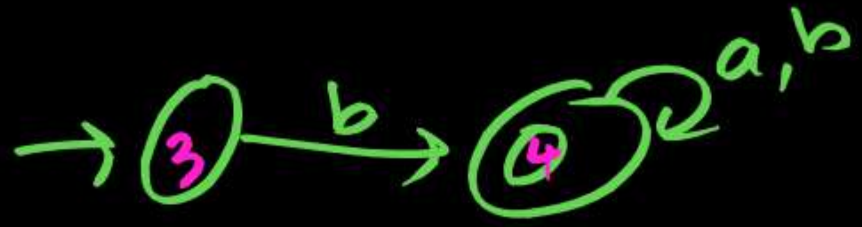
$\Downarrow \quad \Downarrow$

$FA_1 \quad FA_2$

$L_1 = a(a+b)^*$



$L_2 = b(a+b)^*$



$L_1 \cdot L_2 = a(a+b)^* \cdot b(a+b)^*$

Closure Properties for Regular Languages

① $\left.\begin{array}{l} L_1 = \phi \\ L_2 = L \end{array}\right\} \Rightarrow \begin{array}{l} L_1 . L_2 = \phi \\ L_2 . L_1 = \phi \end{array}$

② $\left.\begin{array}{l} L_1 = a^* \\ L_2 = b^* \end{array}\right\} \Rightarrow \begin{array}{l} L_1 L_2 = a^* b^* \\ L_2 L_1 = b^* a^* \end{array}$

③ $\left.\begin{array}{l} L_1 = a^* \\ L_2 = (aa)^* \end{array}\right\} \Rightarrow \begin{array}{l} L_1 . L_2 = a^* \\ L_2 . L_1 = a^* \end{array}$

$\underbrace{a^* . (aa)^*}_{} = a^*$

ε ✓
a ✓
aa ✓
aaa ✓

Closure Properties for Regular Languages

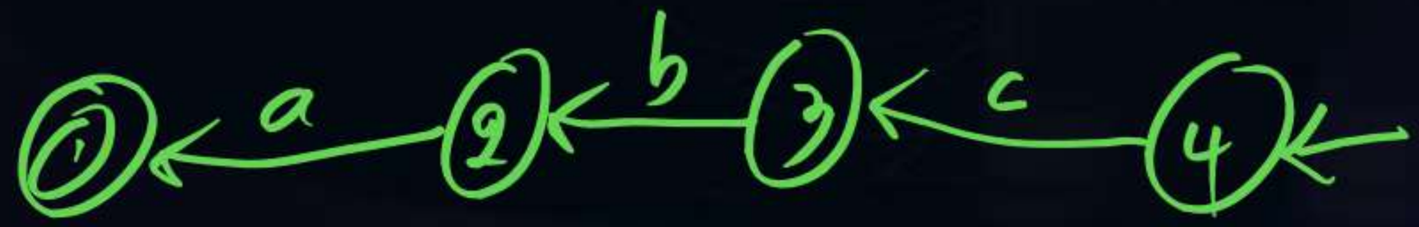
6) Reversal

↳ closed for Regulars

$$L^{Rev} = \{ w \mid w^{Rev} \in L \}$$

$L = \{abc\}$

$L^{Rev} = \{cba\}$



Closure Properties for Regular Languages

1) $L = \phi \Rightarrow L^{Rev} = \phi$

2) $L = \Sigma^* \Rightarrow L^{Rev} = \Sigma^*$

3) $L = a(a+b)^* \Rightarrow L^{Rev} = (a+b)^* a$

4) $L = (a+b)^* a \Rightarrow L^{Rev} = a(a+b)^*$

5) $L = (a+b)^* ab (a+b)^* \Rightarrow L^{Rev} = (b+a)^* ba (b+a)^*$

6) $L = a^* b^* \Rightarrow L^{Rev} = b^* a^*$

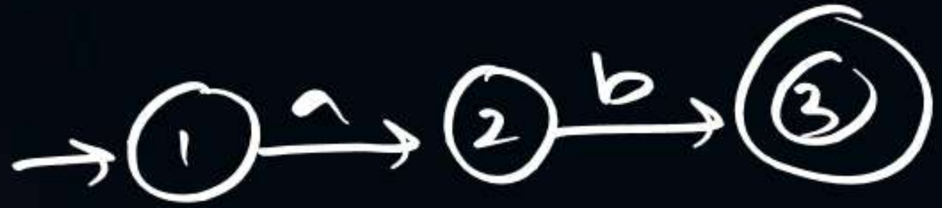
$\overline{\overline{L}} = (L^c)^c = L$

$(L^{Rev})^{Rev} = L$

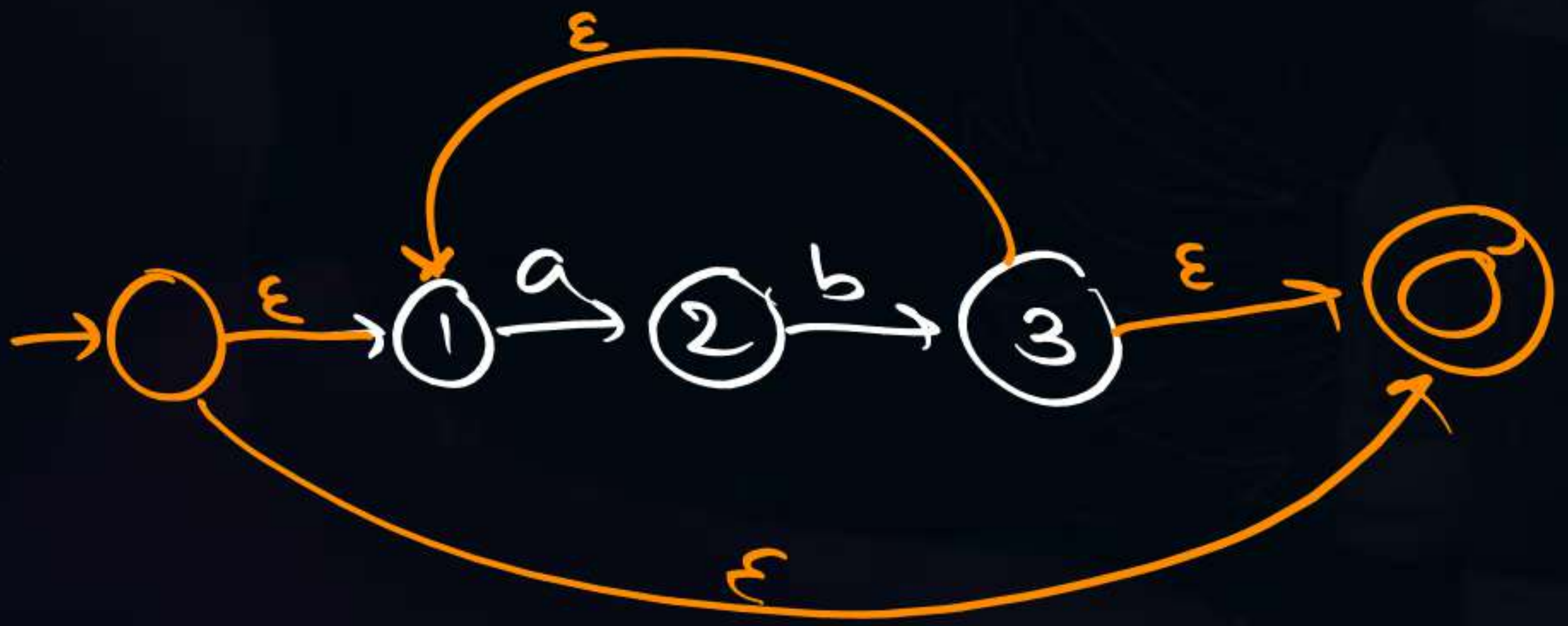
Closure Properties for Regular Languages

(7) L^*
(8) L^+ } $\Rightarrow$ closed for Regulars

$L = ab$



$L^* = (ab)^*$



*** (9) Subset

→ Not closed for regular languages

→ But closed for finite languages

Subset of a regular language is ____ May or may not be Regular

$L = a^*$

ϕ — reg

$(aa)^*$ — reg

a^* — reg

....

a^{prime} — nonreg

a^{n^2} — nonreg

Note: Subset of a finite language is always finite
(regular)

$$L = \{a, aa\}$$

ϕ $\{a\}$ $\{aa\}$ L

fin   fin   fin   fin

Closure Properties for Regular Languages

10) $prefix(L) = \{u \mid uv \in L\}$

11) $Suffix(L) = \{v \mid uv \in L\}$

$abc.\varepsilon = abc$

$ab.c = abc$

$a.bc = abc$

$\varepsilon abc = abc$

$\underbrace{abc}$

↳ prefixes:

ε

a

ab

abc

$L = \{ab, bbb\}$

$Prefix(L) = \{pref(ab), pref(bbb)\}$

$= \{\varepsilon, a, ab, b, bb, bbb\}$

string

$prefix(w) = \{u \mid uv = w\}$

$Suffix(w) = \{v \mid uv = w\}$

$w = abc$

suffixes:

ε

c

bc

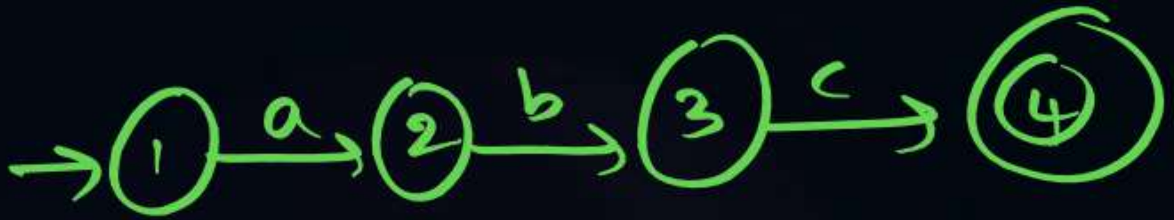
abc

10) $prefix(L) = \{u \mid uv \in L\}$

Init(L)

↳ closed for regulars ← (applies to both Prefix and Suffix)

$L = \{abc\}$

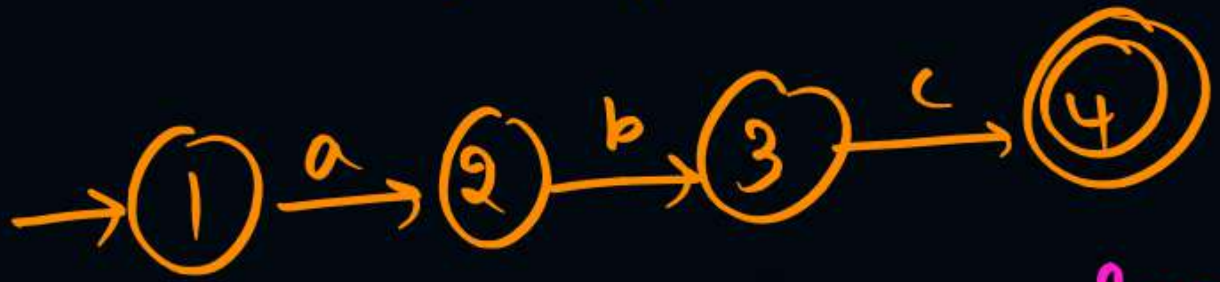


⇓ Make every state as final if it appears in initial to final path

$Prefix(L) = \{\varepsilon, a, ab, abc\}$

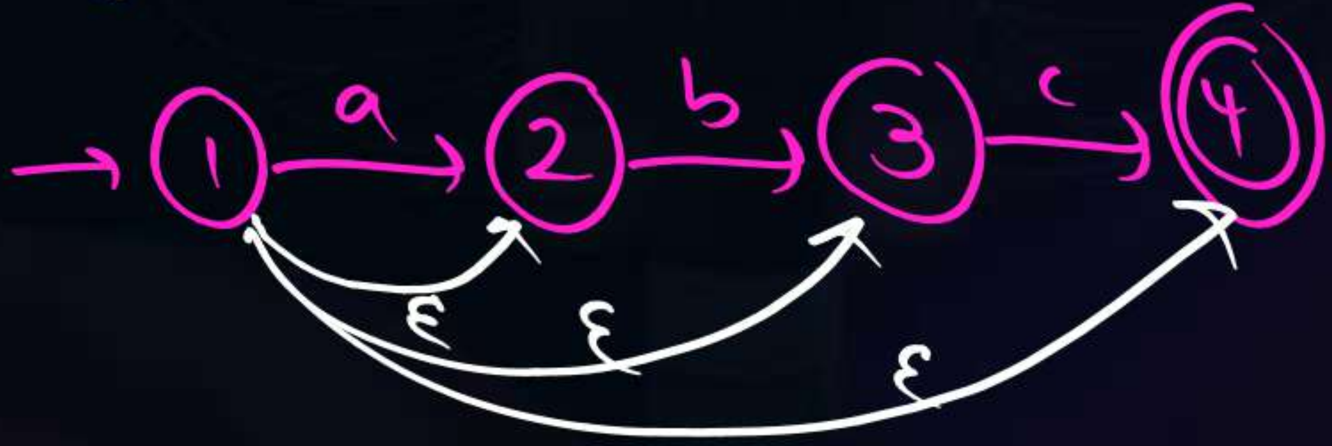
11) $Suffix(L) = \{v \mid uv \in L\}$

$L = \{abc\}$



⇓ Add ε move from initial to every state if it appears in initial to final path.

$suffix(L) = \{\varepsilon, c, bc, abc\}$



Note :

I) For n length string, no. of prefixes = $n+1$

II) " , " suffixes = $n+1$

III) " , no. of non-empty prefixes = n

IV) " , no. of non-empty suffixes = n

12) $Substring(L) = \{ y \mid xyz \in L \}$
(Subword)
→ closed for regulars

Substring:
Part of a string

$w = \underbrace{aaaa}_{4}$

Substrings of w:

$\left.\begin{array}{l} \varepsilon \\ a \\ aa \\ aaa \\ aaaa \end{array}\right\}$ 5 (min)

0, 1, 2, 3, 4 Diff lengths

$w = \underbrace{abcd}_{4}$

Substrings of w:

ε

$\left.\begin{array}{l} a \\ b \\ c \\ d \end{array}\right\} 4$ $\left.\begin{array}{l} ab \\ bc \\ cd \end{array}\right\} 3$ $\left.\begin{array}{l} abc \\ bcd \end{array}\right\} 2$ $\left. abcd \right\} 1$

0, 1, 2, 3, 4 Diff lengths

$\frac{4(4+1)}{2} + 1$ (max) " substring

Substring(w)
$= \{ y \mid xyz = w \}$

Note: I) For n length string, no. of substrings = $\left(\underset{\text{min}}{n+1}, \underset{\text{max}}{\sum n + 1} \right)$

min ↓ if all characters are same

max ↓ if all are distinct

II) For n length string, "no. of different length substrings" = $n+1$

III) For n length string, no. of nonzero different length substrings = n.

Closure Properties for Regular Languages

1) $L = a(a+b)^*$

i) $\text{prefix}(L) = \varepsilon + L$

ii) $\text{Suffix}(L) = (a+b)^*$

iii) $\text{substring}(L) = (a+b)^*$

2) $L = aab(a+b)^*$

i) $\text{prefix}(L) = \varepsilon + a + aa + L$

ii) $\text{Suffix}(L) = (a+b)^*$

iii) $\text{Substring}(L) = (a+b)^*$

3) $L = (a+b)^* aab$

i) prefix(L) = $(a+b)^*$

ii) suffix(L) = $\varepsilon + b + ab + L$

iii) substring(L) = $(a+b)^*$

$L = \{abcd\}$

From Initial to final path:

i) make all states final

ii) Add ε move from initial to every state

Substring(L) = $\{\varepsilon, a, b, c, d, ab, bc, cd, abc, bcd, abcd\}$

13) $f(L)$

Substitution(L)

In given regular over Σ, every symbol is substituted with some regular over Δ

$L = a^* ba \longrightarrow \boxed{f : \Sigma \to P(\Delta^*) \quad f(a) = \{01\} \quad f(b) = 1^*} \longrightarrow f(L)$

Regular

$\Sigma = \{a, b\}$ $\quad a \longrightarrow$ Regular lang

$b \longrightarrow$ Reg lang $\quad \Delta = \{0, 1\}$

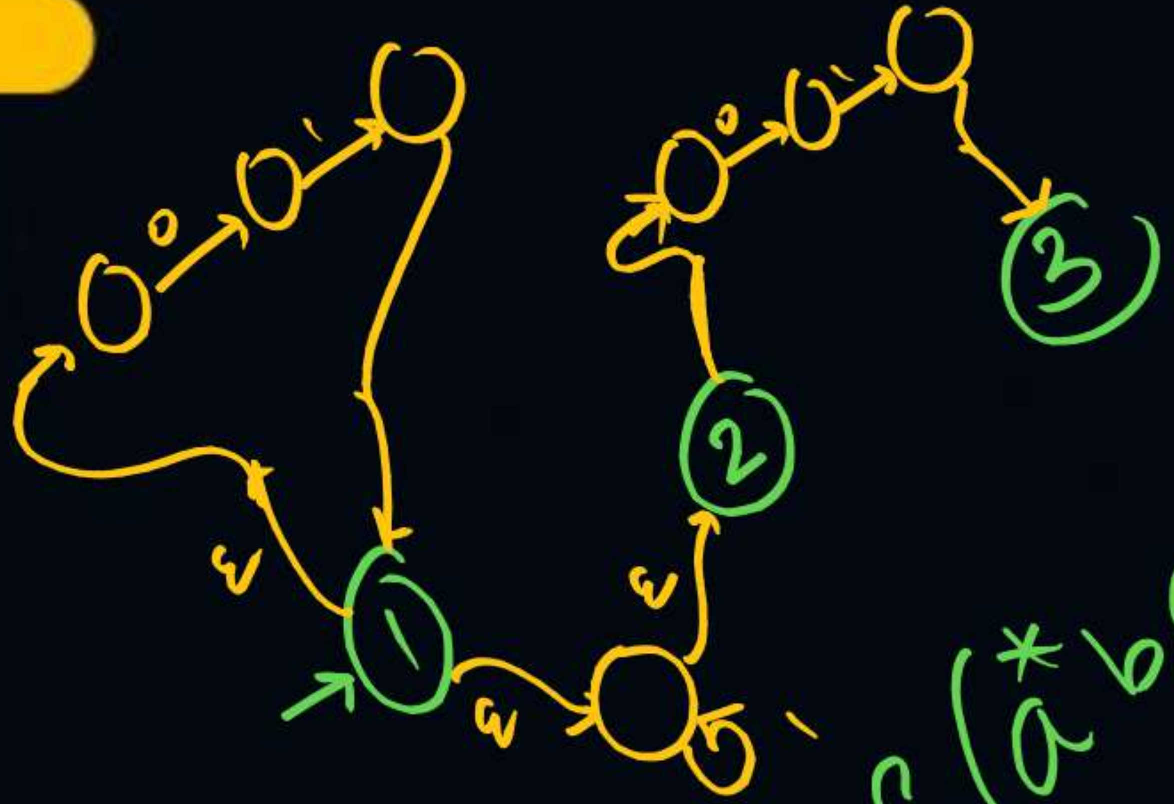
$f(L) = f(a^* b a)$

$= (f(a))^* \cdot f(b) \cdot f(a)$

$= (01)^* 1^* . 01$

Regular

13) $f(L)$



$L = a^* b a$ → $\boxed{f : \Sigma \to P(\Delta^*) \quad f(a) = \{01\} \quad f(b) = 1^*}$ → $f(L)$

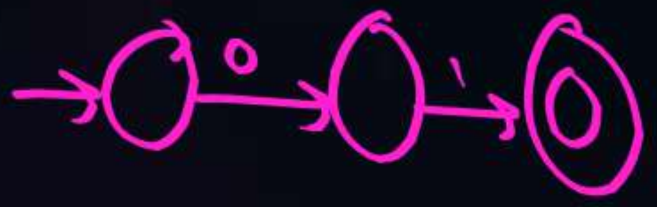
Regular

$f(L) = f(a^* b a)$

$= (f(a))^* \cdot f(b) \cdot f(a)$

$= (01)^* \cdot 1^* \cdot 01$

Regular



$f(a)$

$f(b)$

$\Sigma = \{a, b\}$

$\Delta = \{0, 1\}$

$\Sigma^* = (a+b)^*$

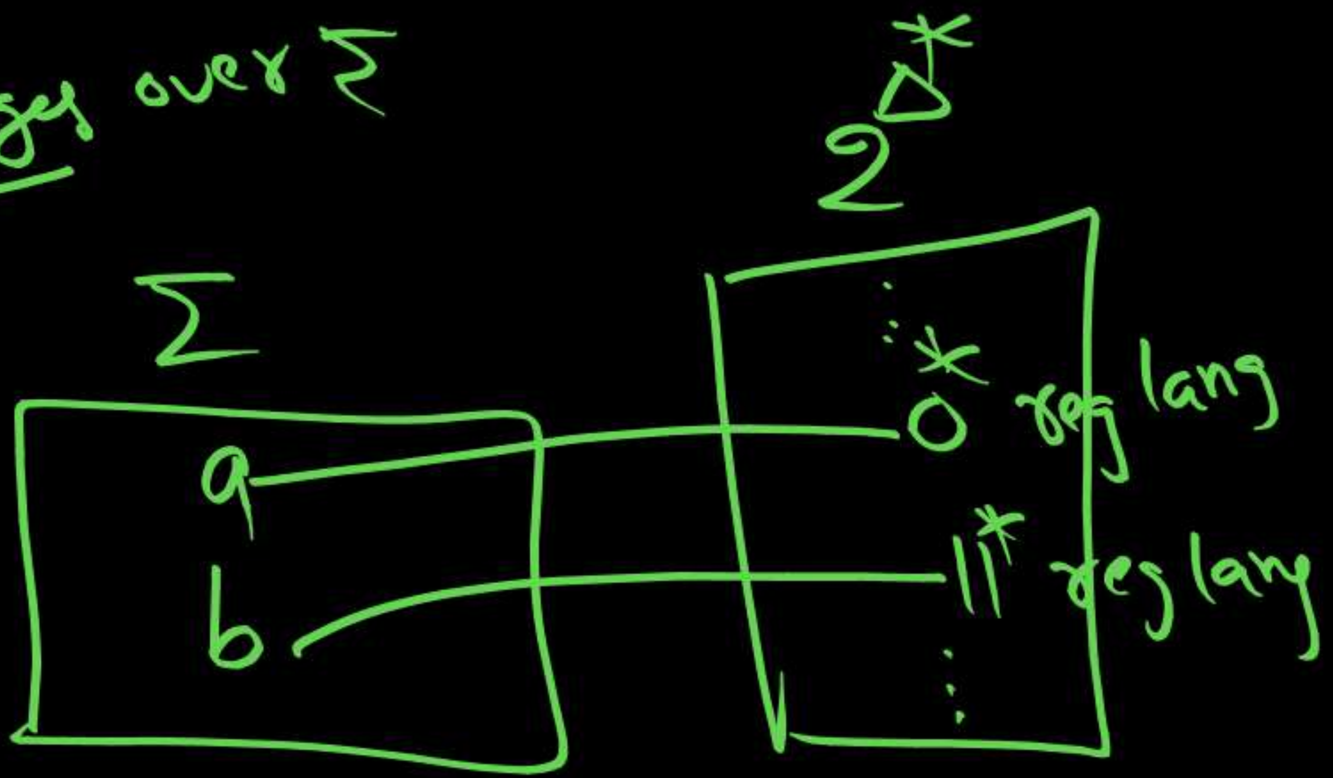
$P(\Delta^*) = 2^{\Delta^*}$

= set of all langs over Δ

$P(\Sigma^*) = 2^{\Sigma^*}$ = set of all subsets of Σ^*

= set of all languages over Σ

2^{Σ^*} = power set of Σ^*



$$A = \{1, 2\}$$

$$P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$$

$$= 2^{A}$$

Closure Properties for Regular Languages

$$L = (ab)^{+}aab$$

$$f(a) = 0^{*}$$

$$f(b) = 11^{*}$$

$$f(L) = ? \quad = \left(f(a).\, f(b)\right)^{+} f(a).\, f(a).\, f(b)$$

$$= \left(0^{*}\, 11^{*}\right)^{+} 0^{*}\, 0^{*}\, 11^{*}$$

Substitution:

$\text{Symbol} \xrightarrow{\text{mapped}} \text{Reg lang}$

f

Homomorphism

$\text{Symbol} \xrightarrow{\text{mapped}} \text{string}$

h

14) Homomorphism(L)

$$L = a^* b a \longrightarrow \boxed{h : \Sigma \to \Delta^*} \longrightarrow h(L) = h(a^* b a)$$

$$= [h(a)]^* \, h(b) . h(a)$$

$$= (01)^* \, \varepsilon . 01$$

$$= (01)^+$$

$h(a) = 01$

$h(b) = \varepsilon$

15) Inverse Homomorphism

Compute h^{-1} using h

$h^{-1}(0) = a$

$h^{-1}(1) = b$

$h^{-1}(11) = c$

Given

$L = \{01, 111\} \longrightarrow \boxed{h^{-1}} \longrightarrow h^{-1}(L)$

$h(a) = 0$

$h(b) = 1$

$h(c) = 11$

Given

$h^{-1}(L) = \{h^{-1}(01), h^{-1}(111)\}$

$h^{-1}(01) \rightarrow ab$

$h^{-1}(111) \rightarrow cb, bc, bbb$

$h^{-1}(L) = \{ab, bbb, bc, cb\}$

2 mins Summary

Topic

- $\cup, \cap, \bar{L}$, Diff
- $L_1 . L_2$

L^{Rev}

L^*, L^+

$\subseteq$

Pref, Suff, Substring

f, h, h^{-1}

THANK - YOU