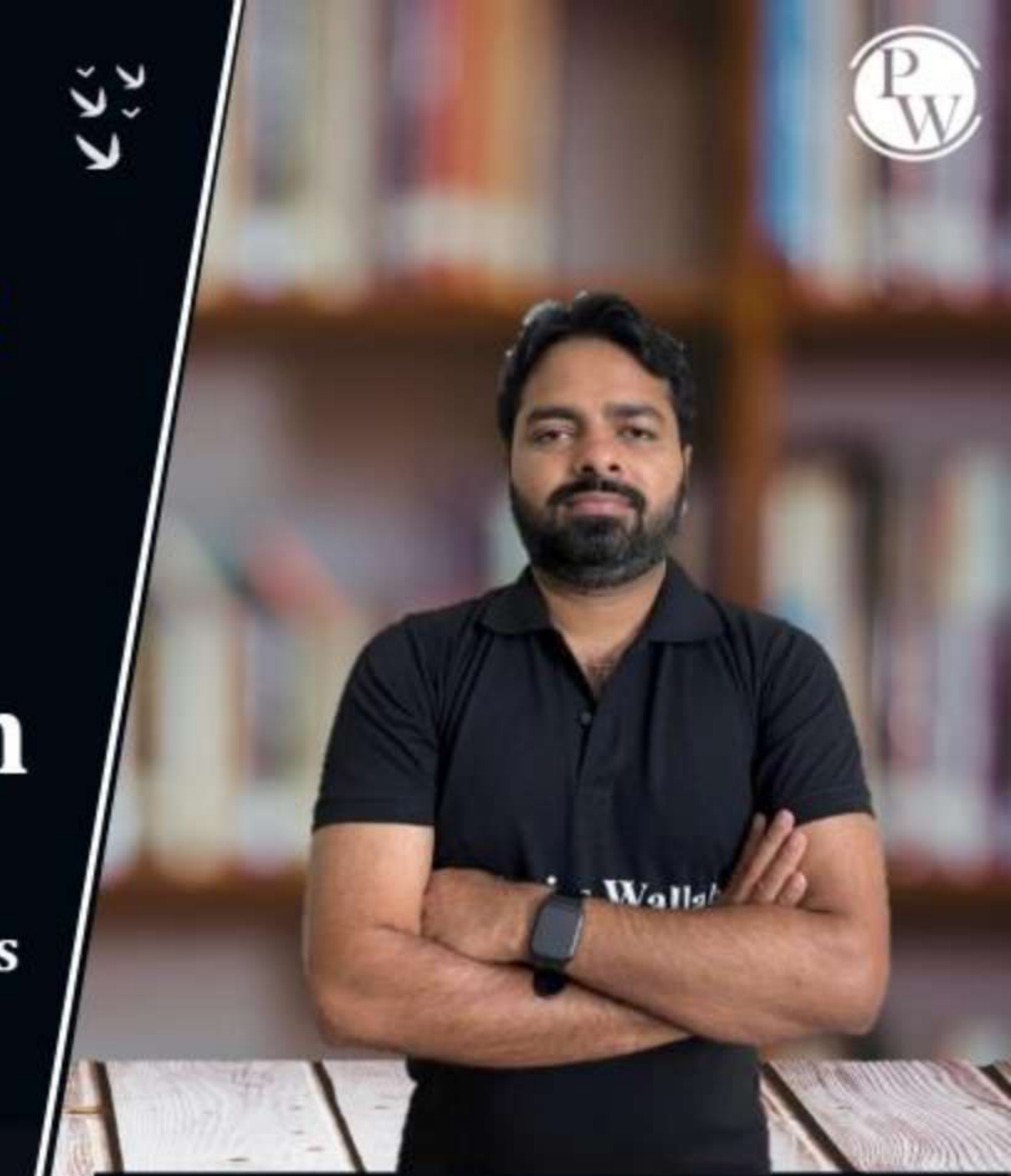
Computer Science

Theory of Computation

Regular Languages and Non-regular Languages

Lecture No.- 9



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Recap of Previous Lecture

Topic Closure Properties for Finite Languages

Topic Closure Properties for Infinite Languages

Topic Closure Properties for Regular Languages

Topics to be Covered

Topic Closure Properties for Regular Languages

Closure Properties for Regular Languages

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $\overline{L}$
4. $L_1 - L_2$
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+
9. $\boxed{\text{Subset}(L)}$
10. prefix (L)
11. suffix (L)
12. Substring (L)
13. f (L) = Substitution
14. h (L) = Homomorphism
15. $h^{-1}(L)$
16. L_1 / L_2 Quotient
17. $L_1 \oplus L_2$ Symmetric Difference
18. Half (L) = $\frac{1}{2}(L)$
19. Second Half (L)
20. one third (L)
21. Middle $\frac{1}{3}(L)$
22. Last $\frac{1}{3}(L)$
23. Finite union
24. " $\cap$
25. " Difference
26. " Concatenation
27. " Subset
28. " Substitution
29. Inf $\cup$
30. Inf $\cap$
31. Inf $-$
32. Inf concatenation
33. Inf Subset
34. Inf Substitution

(16) Quotient [$L_1/L_2 = \{u \mid uv \in L_1, v \in L_2\}$

$L_1 = \{\varepsilon, abb\}$

$L_2 = \{a, bb\}$

$L_1/L_2 = \{\varepsilon/\boxed{a}, \varepsilon/\boxed{bb}, abb/\boxed{a}, a\boxed{bb}/\boxed{bb}\}$

× × × a

$= \{a\}$

$a\cancel{\boxed{bb}}/\cancel{bb} = a$

$$\boxed{uv/v = u}$$

$abb/\boxed{a} = \phi$

$abb/\varepsilon = abb$

Closure Properties for Regular Languages

$L/\varepsilon = L$

$\varepsilon/\varepsilon = \varepsilon$

$a/\varepsilon = a$

$\varepsilon/a = \phi$

① $L_1 = a^*$

$L_2 = \{a\}$

i) $L_1/L_2 = a^*/a = \{\varepsilon, a, aa, aaa, \cdots\}/a$

$= \{\underset{\times}{\varepsilon/a}, \underset{=\varepsilon}{a/a}, \underset{=a}{aa/a}, \underset{=aa}{aaa/a}, \cdots\}$

$= \{\varepsilon, a, aa, \cdots\} = a^*$

ii) $L_2/L_1 = a/a^*$

$= \{\underset{=a}{a/\varepsilon}, \underset{=\varepsilon}{a/a}, \underset{\times}{a/aa}, \underset{\times}{a/aaa}, \cdots\}$

$= \{a, \varepsilon\}$

2) $L_1 = ab^*$

$L_2 = a^*b$

i) $L_1/L_2 = ab^*/a^*b = \{a, ab, abb, \ldots\}/\{b, ab, aab, \ldots\}$

$= \{\cancel{a/b}, \cancel{a/ab}, \cancel{a/aab}, \ldots,$

$\overset{=a}{ab/b}, \overset{=\varepsilon}{ab/ab}, \cancel{ab/aab}, \ldots,$

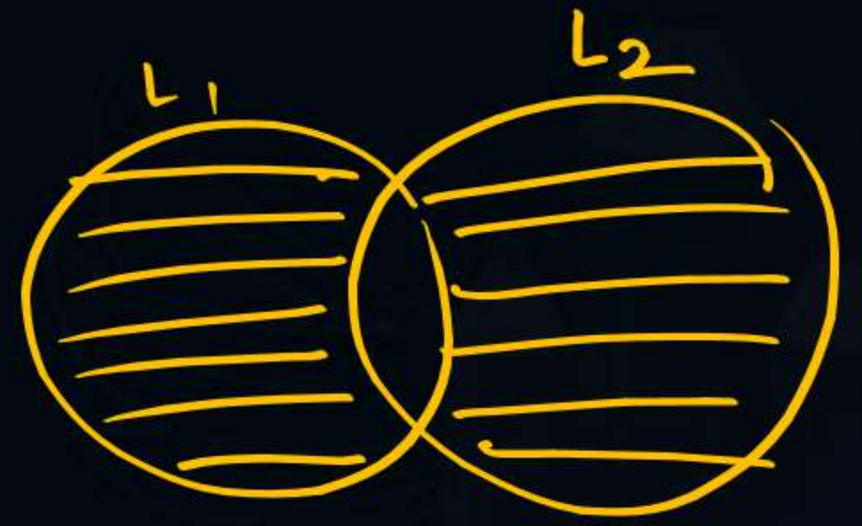
$\overset{=ab}{abb/b}, \cancel{abb/ab}, \cancel{abb/aab}, \ldots\} = \varepsilon + ab^*$

H.W. ii) $L_2/L_1 =$

(17) Symmetric Difference

$$L_1 \oplus L_2 = L_1 \Delta L_2 = (L_1 \cup L_2) - (L_1 \cap L_2)$$

$$= (L_1 - L_2) \cup (L_2 - L_1)$$



Closure Properties for Regular Languages

① $L_1 = a^*$

$L_2 = b^*$

$$L_1 \Delta L_2 = (L_1 \cup L_2) - (L_1 \cap L_2)$$
$$= (a^* \cup b^*) - (\{\varepsilon\})$$
$$= a^+ + b^+$$

② $L_1 = a^+$

$L_2 = b^+$

$$L_1 \Delta L_2 = (L_1 \cup L_2) - (L_1 \cap L_2)$$
$$= (a^+ \cup b^+) - \phi$$
$$= a^+ + b^+$$

Closure Properties for Regular Languages

(18) $Half(L) = \frac{1}{2}(L) = \text{First Half}(L)$

$$= \{u \mid uv \in L, |u| = |v|\}$$

$\text{First Half}(L) = \{\varepsilon, a, aa, abb\}$

(19) $\text{Second Half}(L) = \{v \mid uv \in L, |u| = |v|\}$

$\text{Second Half}(L) = \{\varepsilon, b, bb, aaa\}$

$$L = \{\varepsilon, a, ab, abb, aabb, aabab, abbaaa\}$$

ε : $\varepsilon \cdot \varepsilon$ — 0, 0

a : two equal lengths ×

ab : a, b — 1, 1

abb : ×

$aabb$: aa, bb — 2, 2

$aabab$: ×

$abbaaa$: abb, aaa — 3, 3

(20) one-third(L) $= \frac{1}{3}(L) = \{ x \mid xyz \in L, \underbrace{|x| = |y| = |z|}_{\text{3 equal parts}} \}$

(21) Middle $\frac{1}{3}(L) = \{ y \mid$ " $\}$

(22) Last $\frac{1}{3}(L) = \{ z \mid$ " $\}$

$L = \{ \varepsilon, a, ab, aaa, abaa, abaaa, babbaa \}$

$\varepsilon \to \boxed{\varepsilon}\boxed{\varepsilon}\boxed{\varepsilon}$, a ×, ab ×, $aaa \to \boxed{a}\boxed{a}\boxed{a}$, abaa ×, abaaa ×, $babbaa \to \boxed{ba}\boxed{bb}\boxed{aa}$

$\frac{1}{3}(L) = \{\varepsilon, a, ba\}$, Middle $\frac{1}{3}(L) = \{\varepsilon, a, bb\}$, Last $\frac{1}{3}(L) = \{\varepsilon, a, aa\}$

(23) Finite Union

$L_i \to$ Regular

$L_1 \cup L_2 \cup L_3 \cup \cdots \cup \underbrace{L_K}_{K \text{ is constant}} \Rightarrow$ Always Regular

Each L_i Regular
K Regulars

Union

$L_1 \cup L_2 = \{ w \mid w \in L_1 \text{ or } w \in L_2 \}$

(24) Finite Intersection : $L_1 \cap L_2 \cap L_3 \cap \cdots \cap L_K \Rightarrow$ Always Regular

(25) Finite Differenc : $L_1 - L_2 - L_3 - \cdots - L_K \Rightarrow$ Always Regular

(26) Finite concatenation : $L_1 \cdot L_2 \cdot L_3 \cdot \cdots \cdot L_K \Rightarrow$ Always Regular

*** (27) Finite Subset

↳ closed for Regulars

Finite subset of Regular is always finite lang (Regular)

$L = a^*$

finite subsets of a^*

ϕ — Fin

$\{\epsilon\}$ — Fin

$\{a^n \mid n < 10\}$ — Fin

$\{a^{2n} \mid n < 1000\}$ — Fin

Subset

↳ Not closed for Regulars

Subset of Regular is either Regular or not Regular

$L = a^*$

subsets of a^*

ϕ $\{\epsilon\}$ a^* $(aa)^*$ a^{prime} ...

(28) Finite substitution

↳ closed for regulars

L is Regular over $\Sigma = \{a, b\}$

$L = a^* b$

$f(a)$ = Finite lang $= \{\epsilon\}$

$f(b)$ = Finite lang $= \{0, 11\}$

$f(L)$ is Regular

$f(L) = \epsilon^*(0+11) \Rightarrow$ Regular

Closure Properties for Regular Languages

Not closed for Regulars

******* 29 Infinite Union : $\underbrace{L_1 \cup L_2 \cup L_3 \cup L_4 \cup \ldots}_{\text{Infinite no. of regulars}}$ ⇒ either Regular or not reg

Case 1: $\underset{reg}{\Sigma^*} \cup \underset{reg}{L_2} \cup \underset{reg}{L_3} \cup \ldots \Rightarrow \underset{reg}{\Sigma^*}$

Case 2: $\underset{Reg}{\{\varepsilon\}} \cup \underset{reg}{\{ab\}} \cup \underset{reg}{\{a^2b^2\}} \cup \underset{reg}{\{a^3b^3\}} \cup \ldots \Rightarrow \underset{\text{Not reg}}{a^n b^n}$

30 Infinite ∩

31 Infinite −

32 Infinite concatenation

33 Infinite subset

34 Infinite substitution

Shortcut:

For Regular Languages:

- → Subset
- → Infinite($\cup, \cap, -, \cdot, \subseteq, f$)

Not closed

1) $L_1 = (aa)^* = \{\varepsilon, a^2, a^4, a^6, \ldots\}$

$\Sigma = \{a\}$

$L_2 = a^* = \{\varepsilon, a, a^2, a^3, \ldots\}$

i) $L_1 \cup L_2 = (aa)^* + a^* = a^*$

ii) $L_1 \cap L_2 = (aa)^* \cap a^* = (aa)^*$

iii) $\overline{L_1} = \overline{(aa)^*} = \underbrace{\Sigma^*}_{\text{All}} - \underbrace{(aa)^*}_{\text{even}} = \underbrace{a(aa)^*}_{\text{odd}}$

iv) $\overline{L_2} = \overline{a^*} = \phi$

v) $L_1 - L_2 = \phi$

vi) $L_2 - L_1 = a(aa)^*$

vii) $L_1 / L_2 = a^*$

$L_1/L_2 = (aa)^*/a^*$

$= \{\varepsilon, aa, aaaa, aaaaaa, \ldots\}/a^*$

✓ ε/ε
× ε/a
× ε/aa
$\vdots$
$= \varepsilon$

$aa/\varepsilon = aa$
$aa/a = a$
$aa/aa = \varepsilon$
$aa/aaa = \times$
$\vdots$

2) Which of the following are $\boxed{\text{closed}}$ for Regular languages ?

i) Union

ii) Subset

iii) Inverse Homomorphism

iv) Finite Intersection

3) Which of the following is TRUE?

✓ A) $\text{Prefix}(L) = \{x \mid xy \in L,\ y \in \Sigma^*\}$

✓ B) $\text{Half}(L) = \{x \mid xy \in L,\ |x| = |y|\}$

✓ C) $L_1 \cap L_2 = \{x \mid x \in L_1,\ x \in L_2\}$

× D) $L_1 - L_2 = \{x \mid x \in L_2,\ x \notin L_1\}$

4) $L_1 \rightarrow$ finite language

$L_2 \rightarrow$ Regular language

$L_3 \rightarrow$ Infinite language

$L_4 \rightarrow$ Non-Regular language

TRUE ?

~~A)~~ $L_1 \cup L_2$ is Regular

~~B)~~ $L_1 \cup L_3$ is Infinite

~~C)~~ $L_1 \cap L_4$ is Regular

✗ D) $L_2 \cap L_3$ is Infinite

I) Non-reg $\cap$ Finite $\Rightarrow$ always Finite set

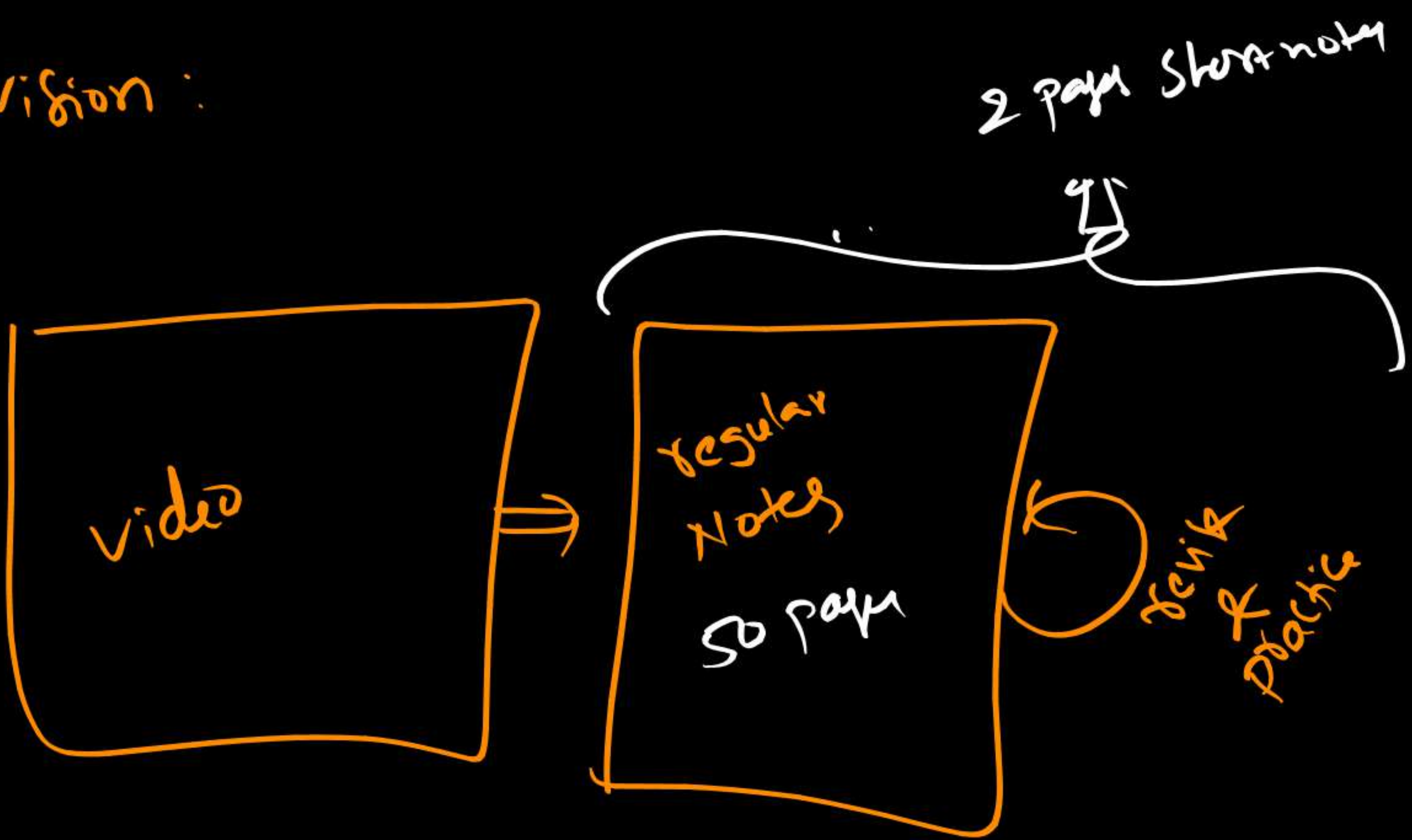
Note: Finite set $\cap$ Any $\Rightarrow$ Finite set

II) Non-regular $\cap$ Infinite $\Rightarrow$ either fin or Inf
either reg or nonreg

i) $\underbrace{a^n b^n}_{\text{nonreg}} \cap \underbrace{a^*}_{\text{Inf}} \Rightarrow \underset{\text{reg, fin}}{\{\varepsilon\}}$

ii) $\underbrace{a^n b^n}_{\text{nonreg}} \cap \underbrace{a^n b^n}_{\text{Inf}} \Rightarrow \underset{\text{nonreg, Inf}}{a^n b^n}$

Revision :


2 page short notes
video
regular Notes
50 page
revisit & practice

$S \to bA$

$A \to bS \mid c$

$S \to bA$
$S = b(bb)^*c$

$L = (bb)^* bc$

$= b(bb)^* c$

method 1:

$A = c + bS$

$S \to bA$

$S \to bc \mid bbS$

$\boxed{L = (bb)^* bc}$

method 2:

$A \to c \mid bS$

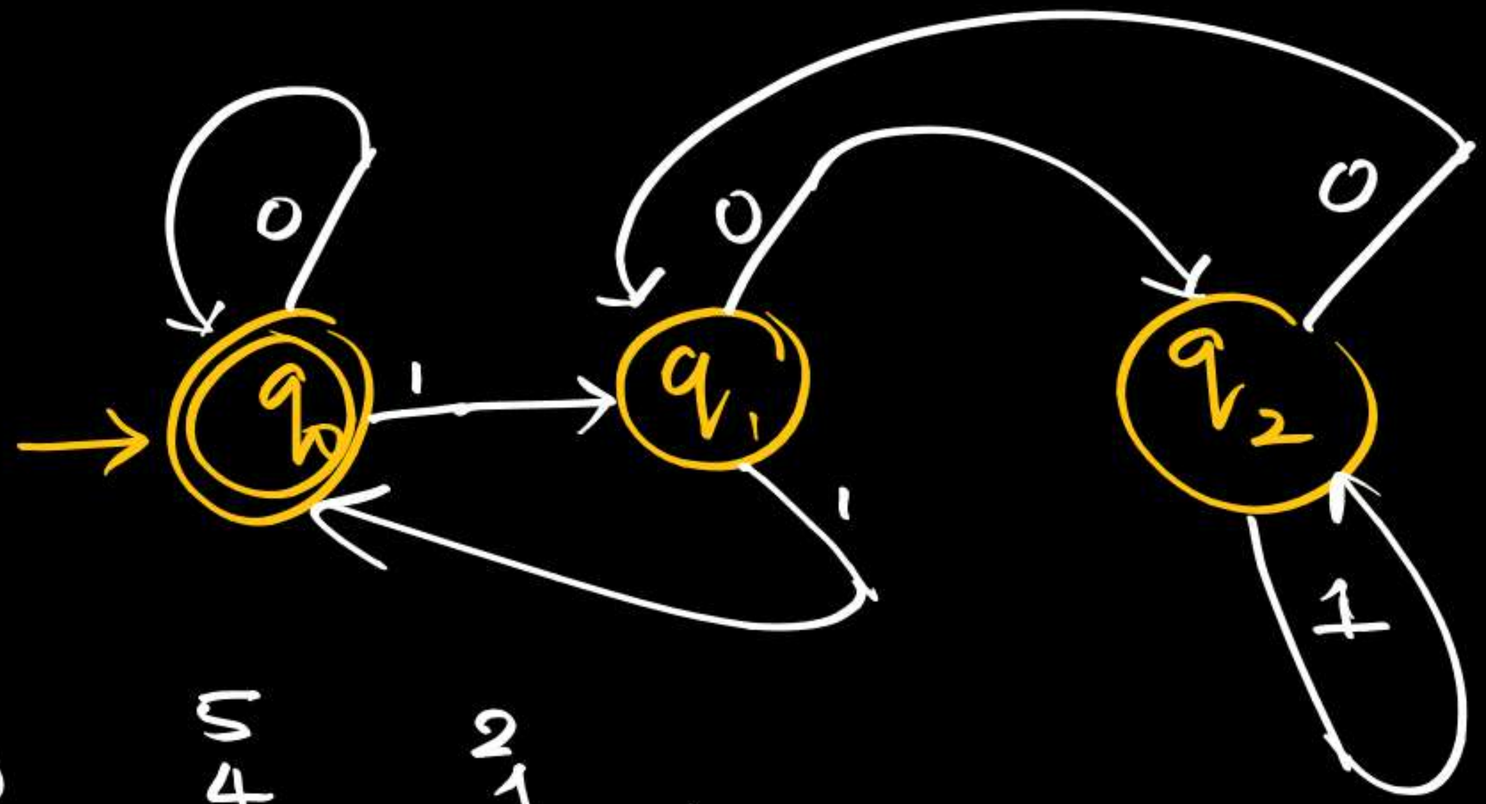
$A \to c \mid bbA$

$A = (bb)^* c \to (1)$

$L = \{w \mid w \in \{0,1\}^*, \text{Dec}(w) \text{ is div by } 3\}$

3 ~ prime or odd → 3 states

	0	1
$\rightarrow {}^*q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2



101	5	2
100	4	1
Bin	Dec	Remainder
0	0	$0 \rightarrow q_0$
1	1	$1 \rightarrow q_1$
10	2	$2 \rightarrow q_2$
11	3	0

2 mins Summary

Topic

Closure properties for regular languages

Next : Pumping lemma

&

FA with o/p

THANK - YOU