Computer Science & IT

Discrete Mathematics

Combinatorics

Lecture No. 05

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Recap of Previous Lecture

Topic Solution of recurrence relation using method of characteristic roots

Topic Method of undetermined coefficient for particular solution

Slide

Topics to be Covered

- **Topic** Generating Function
- **Topic** Extended binomial coefficient
- **Topic** Questions
- **Topic** Pigeonhole principle

Slide

Topic : Method of undetermined coefficient

Method of undetermined coefficient can be used to identify the solution of non-homogeneous part of linear recurrence relation when function f(n) is neither a constant nor an exponential function

Constant = C

$f(n) = C \cdot (1)^n$

Exponential function = $\underset{\text{Constant}}{C} \cdot (b)^n$

$f(n) = C \cdot (b)^n$

Topic : Method of undetermined coefficient

- Let, $\lambda_0 a_n + \lambda_1 \cdot a_{n-1} + \lambda_2 a_{n-2} + \cdots + \lambda_k \cdot a_{n-k} = f(n)$ be the linear recurrence relation.

- and after replacing n by 'n+k'. and after using shift operator eqⁿ becomes,

$$\boxed{\phi(E) \cdot a_n = F(n)}$$

function after substituting 'n' by 'n+k'

- We also know $\phi(t) = 0$ is the ch eqⁿ, and ch. root are roots of the eqⁿ $\phi(t) = 0$.

Topic : Method of undetermined coefficient

Rules for writing the particular Solnⁿ:

$f(n)$: function in original linear recurrence relⁿ	$\phi(E)$	Particular Solnⁿ
$\underbrace{(b)^n}_{\text{Exponential}} \cdot \underbrace{\left(C_0 n^s + C_1 n^{s-1} + \dots + C_s\right)}_{\text{Polynomial of deg}=s}$ $C_0, C_1, \dots$ etc are constants	if $\phi(b) = 0$, i.e. 'b' is a characteristic root (let with multiplicity $= m$)	$(b)^n \cdot (A_0 n^s + A_1 n^{s-1} + \dots + A_s)(n)^m$ $A_0, A_1, A_2, \dots$ etc are unknown coefficient
	if $\phi(b) \neq 0$ i.e., 'b' is not a characteristic root	$(b)^n \cdot (A_0 n^s + A_1 n^{s-1} + \dots A_s)\cancel{n^0}$ '1' $A_0, A_1, \dots A_s$ are unkown

Topic : Method of undetermined coefficient

Rules for writing the particular Solu^n :

$f(n)$: function in original linear recurrence rel^n	$\phi(E)$	Particular Solu^n.
$\left(\overbrace{C_0 n^S + C_1 n^{S-1} + \dots + C_S}^{\text{Polynomial of deg}=S}\right)$ $C_0, C_1, \dots$ etc are constants If nothing is there, then We can assume that $(1)^n$ is there	if $\phi(1) = 0$, i.e. '1' is a characteristic root (let with multiplicity $= m$)	$(A_0 n^S + A_1 n^{S-1} + \dots + A_S)(n)^m$ $A_0, A_1, A_2, \dots$ etc are unknown coefficient
	if $\phi(1) \neq 0$ i.e. '1' is not a characteristic root	$(A_0 n^S + A_1 n^{S-1} + \dots A_S)\cancel{n^0}$ '1' $A_0, A_1, \dots A_S$ are unkown

#Q. Find the solution of recurrence relation

$$a_n - 3a_{n-1} = n+3$$

$$a_{n+1} - 3a_n = (n+1)+3$$

$$E \cdot a_n - 3 \cdot a_n = n+4$$

$$\underbrace{(E-3)}_{\phi(E)} \cdot a_n = n+4$$

Ch. eq^n is $t-3=0$

Ch. root is $t=3$

$\therefore$ $\boxed{C.F. = C_1 \cdot (3)^n}$

Particular Solu^n:

$f(n) = (n+3)$

Polynomial of deg $= 1$

No. Exponential function $\therefore$ $(1)^n$ is there

Check $\phi(1)$

$\phi(1) = 1-3 = -2 \neq 0$

$\therefore$ Particular Solu^n $a_n^{(p)} = \overbrace{An+B}^{\text{Polynomial of deg}: 1}$

unknown

Particular Solu^n $a_n^{(P)}$ satisfies the given recurrence relation

i.e.; $a_n^{(P)} - 3 \cdot a_{n-1}^{(P)} = n+3$

$a_n^{(P)} = An+B$

$\therefore a_{n-1}^{(P)} = A(n-1)+B$

$\therefore (An+B) - 3(A(n-1)+B) = n+3$

Put $n=0$,

$B - 3(-A+B) = 3$

$3A - 2B = 3$ —— ①

Put $n=1$,

$(A+B) - 3(A \cdot 0 + B) = 4$

$A - 2B = 4$ —— ②

By eq^n ① & ②

$A = \frac{-1}{2}$ $B = \frac{-9}{4}$

$\therefore P.S = An+B$

$= \frac{-1}{2} \cdot n + \left(\frac{-9}{4}\right)$

$P.S = \left(\frac{-n}{2} - \frac{9}{4}\right)$

Complete Solu^n

$a_n = CF + PS$
$\quad\ \ a_n^{(H)} + a_n^{(P)}$

$$\boxed{a_n = C_1 \cdot (3)^n + \left(\frac{-n}{2} - \frac{9}{4}\right)}$$

C_1 Can be Calculated using initial Cond^n

#Q. Find the solution of recurrence relation

$$a_n - a_{n-1} = (n+2)(1)^n$$

Ch root $t=1$

'1' is root with multiplicity $=1$

$\therefore CF = C_1 \cdot (1)^n$

$$\boxed{a_n^{(H)} = C_1}$$

$\therefore P.S = (An+B)\cdot(n)^1$

$$\underbrace{a_n^{(P)} = (An+B)\cdot n}_{\text{it will satisfy given Recurrence relation}}$$

i.e. $a_n^{(P)} - a_{n-1}^{(P)} = n+2$

$$\underbrace{n(An+B)} - \underbrace{(n-1)\cdot(A(n-1)+B)} = n+2$$

H.W. Calculate A & B

#Q. Find the solution of recurrence relation

$$a_n - 2a_{n-1} + a_{n-2} = 3n+5$$

$(3n+5)\cdot(1)^n$

Ch. eq^n $t^2-2t+1=0$,

'1' is a root with multiplicity $= 2$

$\therefore$ Ch roots

$\underbrace{t_1 = 1,\ t_2 = 1}_{\text{two equal roots}}$

$\therefore$ $P.S = (An+B)\cdot n^2$

$a_n^{(P)} = (An+B)\cdot n^2$

$C.F = (C_1 + C_2 n)\cdot(1)^n$

$\boxed{a_n^{(H)} = C_1 + n\cdot C_2}$

$(An+B)\cdot n^2 - 2\cdot\{(A(n-1)+B)\cdot \cancel{n^2}\} + ((A(n-2)+B)\cdot\cancel{n^2}) = 3n+5$

$(n-1)^2$ $\quad$ $(n-2)^2$

H.W. Calculate A & B

#Q. Find the solution of recurrence relation

$$\underbrace{a_n - a_{n-1}}_{\text{C.h root } t=1} = 2^n.n$$

$= 2^n.(\underbrace{1.n+0}_{\text{Polynomial of deg}=1})$

2 is not a root

C.F = $C_1.(1)^n$

$\boxed{a_n^{(H)} = C_1}$

$\therefore$ P.S = $(2)^n.(An+B).\cancel{(n)^0} \rightarrow 1$

$a_n^{(P)} = 2^n.(An+B)$

#Q. Find the solution of recurrence relation

$$\underbrace{a_n - 3.a_{n-1}}_{} = 3^n(n+2)$$

Ch root $t=3$

$\therefore C.F = C_1.(3)^n$

$$\boxed{a_n^{(H)} = C_1.(3)^n}$$

3 is a root with multiplicity = 1

$\therefore P.S = (3)^n.(An+B).n^1$

$$a_n^{(P)} = n.3^n.(An+B)$$

Topic : Generating function

Generating function is a way of generating an infinite sequence of numbers (a_n) by treating them as the coefficients of a formal power series.

Consider the following sequence of numbers $\Rightarrow$ $a_0, \quad a_1, \quad a_2, \quad a_3, \ldots$

Generating function for above sequence of numbers will be $= \underbrace{a_0 x^0 + a_1 \cdot x^1 + a_2 \cdot x^2 + a_3 \cdot x^3 + \ldots}_{\text{Solution of this series is called generating function in closed form}}$

Consider the following sequence $\Rightarrow$

$$\begin{array}{cccccc} a_0 & a_1 & a_2 & a_3 & a_n & \\ 1, & 1, & 1, & 1, & 1, & \ldots \end{array}$$

Generating function for above sequence will be

$$= \sum_{i=0}^{\infty} 1.(x)^i$$

general term

$$= 1 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2 + 1 \cdot x^3 + \ldots$$

$$= \underbrace{1 + x + x^2 + x^3 + \ldots}$$

it is an infinite G.P.

1st term = 1

Common ratio = x

Summation of an infinit GP = $\dfrac{a}{1-r}$

1st term

Common ratio

$$= \frac{1}{1-x}$$

generating function in closed form for above sequence

Consider the following sequence of numbers $\Rightarrow$ $a_0, \ a_1, \ a_2, \ a_3, \ ----.$

Generating function for above sequence of numbers will be $= a_0 x^0 + a_1 \cdot x^1 + a_2 \cdot x^2 + a_3 \cdot x^3 + ------$

$$= \sum_{i=0}^{\infty} (a_i) \cdot (x)^i$$

'a_i' is called general term of the sequence

Topic : Generating function

Binomial Expansion :-

$$(a+b)^n = {}^nC_0 (a)^n \cdot (b)^0 + {}^nC_1 \cdot (a)^{n-1} \cdot (b)^1 + {}^nC_2 \cdot (a)^{n-2} \cdot (b)^2 + \dots + {}^nC_r (a)^{n-r} (b)^r + \dots$$

$$(1+x)^n = {}^nC_0 (1)^n \cdot x^0 + {}^nC_1 (1)^{n-1} \cdot x^1 + {}^nC_2 (1)^{n-2} \cdot x^2 + \dots + {}^nC_r (1)^{n-r} \cdot x^r + \dots$$

$$= {}^nC_0 \cdot x^0 + {}^nC_1 \cdot x^1 + {}^nC_2 \cdot x^2 + \dots + {}^nC_r \cdot x^r + \dots$$

general term

it is known as binomial Coefficient

$$= \sum_{i=0}^{\infty} ({}^nC_i) \cdot x^i$$

general term

If sequence of number is $= {}^nC_0, {}^nC_1, {}^nC_2, {}^nC_3, \ldots$

then generating function will be $= (1+x)^n$

Topic : Generating function

Q:- Write the generating function for following sequence of numbers

$$\underset{a_0}{1},\ \underset{a_1}{2},\ \underset{a_2}{3},\ \underset{a_3}{4},\ \underset{a_4}{5},\ \ldots$$

Generating function

$$Gf(x) = 1\cdot x^0 + 2\cdot x^1 + 3\cdot x^2 + 4\cdot x^3 + \ldots + \underbrace{(r+1)\cdot x^r}_{\text{general term}} + \ldots$$

$$= \sum_{i=0}^{\infty} (i+1)\cdot x^i$$

$$\boxed{\frac{d}{dx}\left(x^n\right) = n \cdot x^{n-1}}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^r + x^{r+1} + \cdots$$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}\left\{1 + x + x^2 + x^3 + \cdots + x^r + x^{r+1} + \cdots\right\}$$

$$= \underbrace{0 + 1 \cdot x^0 + 2 \cdot x^1 + 3 \cdot x^2 + \cdots + r \cdot x^{r-1} + (r+1) \cdot x^r + \cdots}$$

$$\boxed{\frac{d}{dx}\left(\frac{1}{1-x}\right) = Gf(x)}$$

$$Gf(x) = \frac{d}{dx}\left\{\frac{1}{1-x}\right\}$$

$$\frac{d}{dx}\left(\frac{p}{q}\right) = \frac{p'q - pq'}{(q)^2}$$

$$= \frac{0\cdot\left(\frac{1}{1-x}\right) - 1\cdot\frac{d}{dx}(1-x)}{(1-x)^2}$$

$$\boxed{Gf(x) = \frac{1}{(1-x)^2}}$$

$$\boxed{\frac{1}{(1-x)^2} = 1\cdot x^0 + 2\cdot x^1 + 3\cdot x^2 + \cdots + (r+1)\cdot x^r + \cdots}$$

$\frac{1}{(1-x)^2}$ is generating function for sequence $1, 2, 3, 4, \ldots$

Topic : Generating function

$$\boxed{\frac{1}{(1-x)^2} = 1\cdot x^0 + 2\cdot x^1 + 3\cdot x^2 + \cdots + r\cdot x^{r-1} + (r+1)\cdot x^r + \cdots}$$

Multiply both side by x

$$\frac{x}{(1-x)^2} = 1\cdot x^1 + 2\cdot x^2 + 3\cdot x^3 + \cdots + r\cdot x^r + (r+1)\cdot x^{r+1} + \cdots$$

$$\boxed{\frac{x}{(1-x)^2} = 0\cdot x^0 + 1\cdot x^1 + 2\cdot x^2 + 3\cdot x^3 + \cdots + r\cdot x^r + \cdots}$$

$\frac{x}{(1-x)^2}$ is generating function for sequence $0, 1, 2, 3, 4, \ldots$

Topic : Extended binomial coefficient

We know, $^nC_r = \dfrac{n!}{(n-r)!\, r!} = \dfrac{n\cdot(n-1)\cdot(n-2)\cdots(n-r+1)\,\cancel{(n-r)(n-r-1)\cdots 3\cdot2\cdot1}}{\cancel{\left((n-r)\cdot(n-r-1)(n-r-2)\cdots 3\cdot2\cdot1\right)}\left(r\cdot(r-1)\cdot(r-2)\cdots3\cdot2\cdot1\right)}$

$$\boxed{^nC_r = \frac{(n-0)(n-1)(n-2)\cdots\cdots(n-(r-1))}{r\cdot(r-1)\cdot(r-2)\cdots 3\cdot2\cdot1}}$$

$$\boxed{^6C_3 = \frac{(6-0)\cdot(6-1)\cdot(6-2)}{3!}}$$

$r=3$

$r!$

$(3-1)=(r-1)$ $\{0 \text{ to } (r-1)\}$

$$ {}^{-5}C_3 = \frac{(-5-0)(-5-1)(-5-2)}{3!} $$

$(3-1) = (r-1)$

$r = 3$

Topic : Extended binomial coefficient

0 to $(r-1)$

'r' term in numerator

$$^{-n}C_r = \frac{(-n-0)(-n-1)(-n-2)\ldots\ldots(-n-(r-1))}{r!}$$

'-1' common from 'r' terms

$$= \frac{(-1)^r\{n.(n+1).(n+2)\ldots\ldots(n+r-1)\}}{r!}$$

$$= \frac{(-1)^r.\{(n+r-1).(n+r-2)\ldots\ldots(n+2).(n+1).n\}}{r!} \times \frac{(n-1)(n-2)\ldots3.2.1}{(n-1)(n-2)\ldots3.2.1}$$

$$= \frac{(-1)^r.(n+r-1)!}{(n-1)!\,r!} = \frac{(-1)^r.(n+r-1)!}{((n+r-1)-r)!\cdot r!} = \boxed{(-1)^r.\,^{(n+r-1)}C_r}$$

Topic : Extended binomial coefficient

$$\boxed{{}^{-n}C_r = (-1)^r \cdot {}^{(n+r-1)}C_r}$$

Where 'n' is non-negative integer

① $(1+x)^n = {}^nC_0 \cdot x^0 + {}^nC_1 \cdot x^1 + {}^nC_2 \cdot x^2 + \cdots + {}^nC_r \cdot x^r + \cdots$

${}^nC_r$ is general term

$$Gf(x) = \sum_{i=0}^{\infty} \left({}^nC_i\right) \cdot (x)^i$$

general term

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② $(1-x)^{-n} = {}^{-n}C_0 \cdot (-x)^0 + {}^{-n}C_1 (-x)^1 + {}^{-n}C_2 \cdot (-x)^2 + \cdots + {}^{-n}C_r \cdot (-x)^r + \cdots$

$(1+(-x))^{-n}$

$$+ {}^{-n}C_r \cdot (-1)^r \cdot (x)^r +$$

$$+ \left\{(-1)^r \cdot {}^{n+r-1}C_r\right\}\left\{(-1)^r \cdot (x)^r\right\}$$

$$+ \cancel{(-1)^{2r}}^{\,1} \cdot \left\{{}^{n+r-1}C_r\right\} (x)^r +$$

$$+ {}^{(n+r-1)}C_r (x)^r +$$

general term

$$Gf(x) = \sum_{i=0}^{\infty} \left({}^{n+i-1}C_i\right) \cdot x^i$$

③ $(1-x)^n = {}^nC_0 \cdot (-x)^0 + {}^nC_1 \cdot (-x)^1 + {}^nC_2 \cdot (-x)^2 + \cdots + {}^nC_r \cdot (-x)^r + \cdots$

$(1+(-x))^n$

$$G.f(x) = \sum_{i=0}^{\infty} \left\{ (-1)^i \cdot {}^nC_i \right\} \cdot (x)^i$$

$$+ \underbrace{(-1)^r \cdot {}^nC_r \cdot (x)^r}_{\text{general term}} +$$

④ $(1+x)^{-n} = {}^{-n}C_0 (x)^0 + {}^{-n}C_1 \cdot (x)^1 + {}^{-n}C_2 \cdot (x)^2 + \cdots + {}^{-n}C_r \cdot (x)^r + \cdots$

$$+ \underbrace{\left\{ (-1)^r \cdot {}^{(n+r-1)}C_r \right\}}_{\text{general term}} (x)^r +$$

$$G.f(x) = \sum_{i=0}^{\infty} \left\{ (-1)^i \cdot {}^{(n+i-1)}C_i \right\} \cdot (x)^i$$

⑤ $(1+ax)^n = {}^nC_0 \cdot (ax)^0 + {}^nC_1 \cdot (ax)^1 + \cdots + {}^nC_r \cdot (ax)^r + \cdots$

$$= \sum_{i=0}^{\infty} \{\underbrace{(a)^i \cdot {}^nC_i}_{\text{general term}}\} \cdot (x)^i \qquad + (a)^r \cdot {}^nC_r \cdot (x)^r +$$

⑥ $(1-ax)^{-n} =$

$$= \sum_{i=0}^{\infty} \{\underbrace{(a)^i \cdot {}^{(n+i-1)}C_i}_{\text{general term}}\} \cdot (x)^i$$

7. $(1-ax)^n =$

$$= \sum_{i=0}^{\infty} \{\underbrace{(-1)^i . (a)^i . {}^nC_i}_{\text{general term}}\} . (x)^i$$

8. $(1+ax)^{-n} =$

$$= \sum_{i=0}^{\infty} \{\underbrace{(-1)^i . (a)^i . {}^{(n+i-1)}C_i}_{\text{general term}}\} . (x)^i$$

#Q. Let $G(x) = \frac{1}{(1-x)^2} = \sum_{i=0}^{\infty} \underbrace{g(i)}_{(i+1)} x^i$, where $|x| < 1$. What is $g(i)$?

$$Gf(x) = \frac{1}{(1-x)^2} = \underbrace{(1-x)}^{-2}$$

general term: $\left\{ {}^{n+i-1}C_i \right\} = \left({}^{2+i-1}C_i \right) = {}^{(i+1)}C_i = {}^{(i+1)}C_1 = (i+1)$

#Q. If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\frac{1+z}{(1-z)^3}$, then $a_3 - a_0$ is equal to?

$a_3 - a_0 = ?$

$$Gf(z) = \frac{1+z}{(1-z)^3} = a_0 \cdot z^0 + a_1 \cdot z^1 + a_2 \cdot z^2 + a_3 \cdot z^3 + \ldots\ldots$$

$$= (1+z)(1-z)^{-3}$$

$$= \{(1-z)^{-3}\} + \{z^1 \cdot (1-z)^{-3}\}$$

$$\left(\text{Coefficient of } z^3 + \text{Coefficient of } z^2\right) = a_3$$

$$\left({}^{3+3-1}C_3\right) + \left({}^{3+2-1}C_2\right)$$

$$(10 + 6) = 16$$

$$\left(\text{Coefficient of } z^0 + \text{Coefficient of } z^{0-1}\right) = a_0$$

$$\left({}^{3+0-1}C_0 + 0\right) = (1+0) = 1$$

$a_3 = 16$

$a_0 = 1$

$a_3 - a_0 = 16 - 1 = 15$

general term i.e. Coefficient of x^n

#Q. Which one the following is a closed from expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0,1,2, \ldots$?

A) $\frac{3}{(1-x)^2}$

B) $\frac{3x}{(1-x)^2}$

C) $\frac{2-x}{(1-x)^2}$

D) $\frac{3-x}{(1-x)^2}$

$$Gf(x) = \sum_{n=0}^{\infty} (a_n) \cdot (x)^n$$

$$= \sum_{n=0}^{\infty} (2n+3) \cdot (x)^n$$

$$= \sum_{n=0}^{\infty} 2n \cdot (x)^n + \sum_{n=0}^{\infty} 3 \cdot (x)^n$$

$$= \sum_{n=0}^{\infty} (2n)\cdot(x)^n + \sum_{n=0}^{\infty} 3\cdot(x)^n$$

$$= 2 * \underbrace{\sum_{n=0}^{\infty} (n)(x)^n} + 3\cdot \underbrace{\sum_{n=0}^{\infty} 1\cdot(x)^n}$$

general term

general term

$$= 2 * \left\{ \frac{x}{(1-x)^2} \right\} + 3 * \left\{ \frac{1}{1-x} \right\}$$

$$= \frac{2x + 3*(1-x)}{(1-x)^2} = \frac{3-x}{(1-x)^2}$$

#Q. Find the solution of the recurrence relation

$\boxed{a_k = 3a_{k-1}, \quad \text{where } \underbrace{k = 1, 2, 3, \ldots\ldots\ldots}} \text{ and } \boxed{a_0 = 2}$

$$Gf(x) = \sum_{k=0}^{\infty} a_k (x)^k$$

$$= a_0 (x)^0 + \sum_{k=1}^{\infty} a_k \cdot (x)^k$$

2 1

$$= 2 + \sum_{k=1}^{\infty} a_k \cdot (x)^k$$

$$= 2 + \sum_{k=1}^{\infty} 3 \cdot a_{k-1} \cdot (x)^k$$

$$Gf(x) = 2 + 3 \cdot \sum_{k=1}^{\infty} a_{k-1} \cdot (x)^k$$

$$= 2 + 3 \cdot x \sum_{k=1}^{\infty} a_{k-1} \cdot (x)^{k-1}$$

for $k=1$ to ∞
it is actually from $a_0(x)^0$ to $a_\infty(x)^\infty$

$$Gf(x) = 2 + 3 \cdot x \; Gf(x)$$

$$Gf(x)(1-3x) = 2$$

$$Gf(x) = \left(\frac{2}{1-3x}\right) = 2 * \left\{\frac{1}{1-3x}\right\} = 2 * (1-3x)^{-1} = 2 * \Big[\overbrace{3^k \cdot \cancel{{}^{1+k-1}C_k}}^{\text{general term}}\Big] = \underbrace{2 \cdot (3)^k}$$

general term is the solution of recurrence relation.

Q:- Find the Coefficient of x^{12} in the Expansion of

$$\left\{\underbrace{x^3+x^4+x^5+x^6+\cdots}\right\}^3$$

Soln

$$\left\{\frac{x^3}{(1-x)}\right\}^3$$

$$\left\{\text{Sum of GP} = \frac{a}{1-r}\right\}$$

$$= \left\{\frac{x^3}{(1-x)}\right\}^3 = x^9 \left\{(1-x)^{-3}\right\}$$

(Coefficient of x^9 * Coefficient of x^3) = Coefficient of x^{12}

$$1 * \left\{{}^{3+3-1}C_3\right\}$$

$$= 1 * {}^5C_3 = 10 \text{ Ans}$$

Aptitude

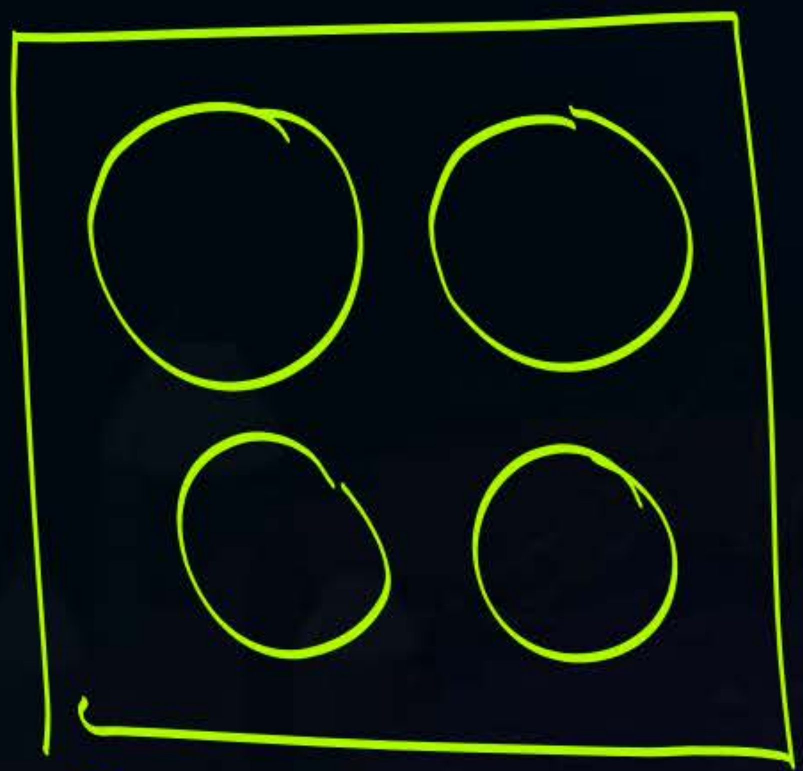
Topic : Pigeonhole principle

Let 'A' is the average number of pigeons per pigeonhole

i.e. $A = \dfrac{\text{No. of Pigeons}}{\text{No. of Pigeonholes}}$

$\therefore$ Some pigeonholes will contain atleast $\lceil A \rceil$ pigeons

& Some pigeonholes will contain at most $\lfloor A \rfloor$ pigeons



$\left.\begin{array}{l}\#\ \text{Pigeonhole} = 4 \\ \#\ \text{Pigeons} = 5\end{array}\right\} \therefore A = 1.25$

Topic : Pigeonhole principle

Let $(2n+1)$ pigeons are distributed in 'n' pigeonholes.

$$\therefore A = \frac{2n+1}{n} = 2 + \frac{1}{n}$$

$$\lceil A \rceil = 3 = 2 + 1$$

$$\lfloor A \rfloor = 2 = 2 + 0$$

Some pigeonhole will contain at least $(2+1)$ pigeons

Some pigeonhole will contain at most $(2+0)$ pigeons

Topic : Pigeonhole principle

Let $(kn+1)$ pigeons are distributed in 'n' pigeonholes.

then $$A = \frac{kn+1}{n} = k + \frac{1}{n}$$

$$\lceil A \rceil = k+1$$

$$\lfloor A \rfloor = k$$

Some pigeonhole will contain atleast $(k+1)$ pigeons

Some pigeonhole will contain at most 'k' pigeons

Topic : Pigeonhole principle

Q: Suppose we have 'n' pigeonholes, then what is the minimum number of pigeons required to ensure that some pigeonhole contains

(i) at least k pigeons = $((k-1) \cdot n+1)$

(ii) at least $(k+1)$ pigeons $\Rightarrow (k \cdot n+1)$

There are some months in which at least '6' students are born

Students = # Pigeons

#Q. Consider a group of 61 students, which of the following is/are true?

There are some months in which at most '5' students are born.

Months in a year = 12 = # Pigeon holes

A) At least 6 students are born is same month.

$\therefore A = \frac{\text{\# Pigeons}}{\text{\# Pigeonholes}}$

Avg no. of Pigeons per Pigeonhole

$= \frac{61}{12} = 5.08$

$\lceil A \rceil = 6$

$\lfloor A \rfloor = 5$

B) At most 6 students are born in same month.

"When D is true", B is automatically true

C) At least 5 students are born in same month.

{If statement A is true, then statement 'C' is automatically true}

D) At most 5 students are born in same month.

Pigeonholes ✓

#Q. Suppose there are 7 branches in a college and each branch has exactly 50 students, what must be the minimum number of students chosen such that there are at least 10 students from at least one branch?

$=(9+1)$

at least 10 students in some pigeonhole

To ensure that at least 'k+1' pigeon in some pigeon hole $(9+1)=10$
We need $(k \cdot n+1)$ pigeons

$(9 * 7+1)=64$

$B_1 \quad B_2 \quad B_3 \quad B_4 \quad B_5 \quad B_6 \quad B_7$

$$\underbrace{9+9+9+9+9+9+9}_{\text{Upto 63 students it may be the case that we don't have 10 students from any branch}}$$

But if select 64 students, then there will be at least 10 students from at least one branch

#Q. Suppose we have 8 red color balls, 6 blue color balls, 19 green color balls and 10 yellow color balls, then what must be the minimum number of balls chosen such that we have at least 5 balls of same color?

number of balls → No. of pigeons

at least 5 → (4+1)

same color → Pigeonhole → 4 Pigeon holes → Red, Blue, green, yellow

Red(8) Blue(6) Green(19) yellow(10)

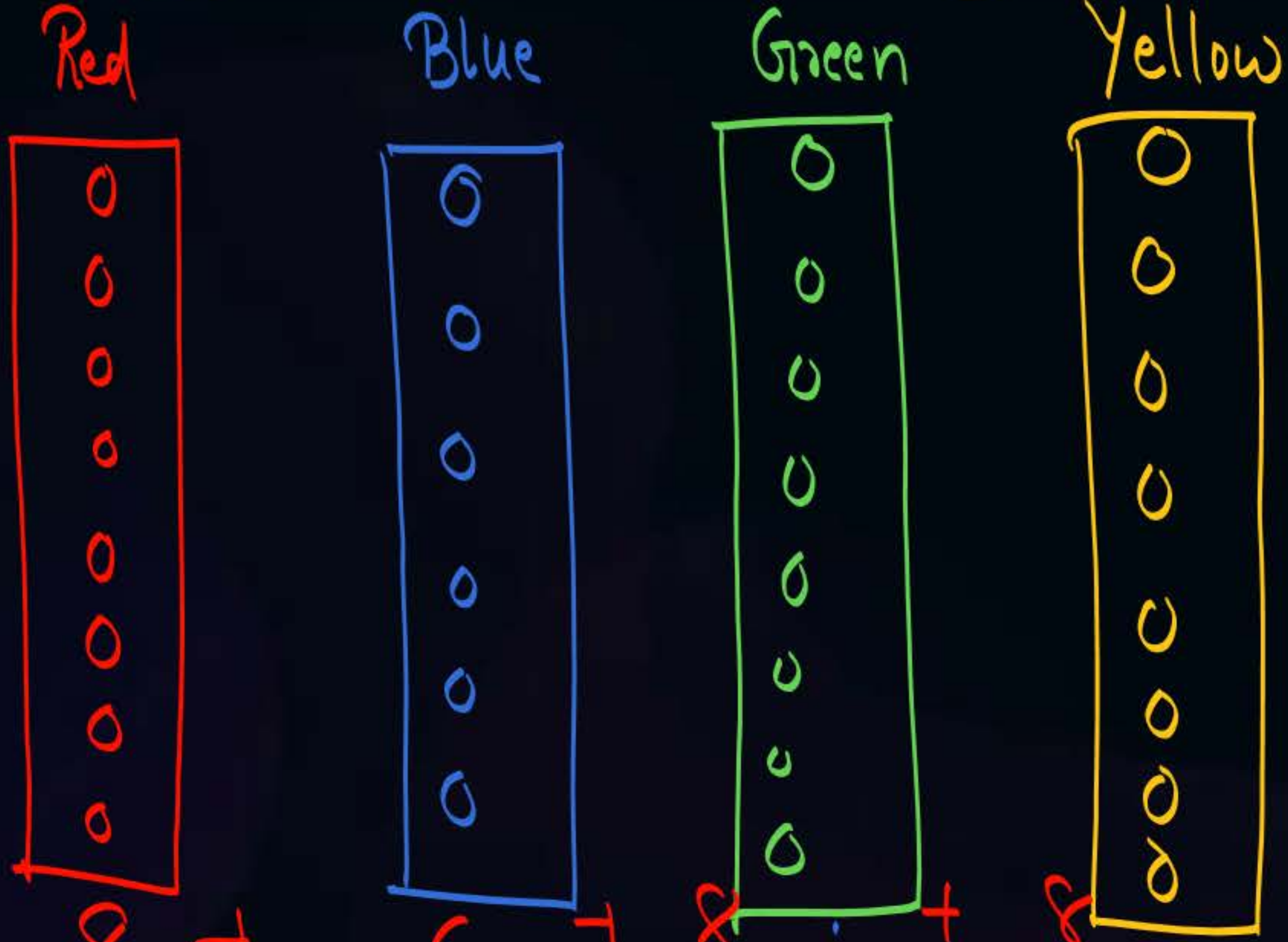
$\underbrace{4 + 4 + 4 + 4}$

{Upto 16 balls we may not have 5 balls of any Color. But If one more ball is Selected we have '5' balls of at least one Color.}

∴ Min no. of Balls selected = $(4 * 4 + 1)$

$= 17$

#Q. Suppose we have 8 red color balls, 6 blue color balls, 19 green color balls and 10 yellow color balls, then what must be the minimum number of balls chosen such that we have at least 9 balls of same color?



$8 + 6 + 8 + 8 = 30$

If no. of balls selected are '31', then it is guaranteed, that we will have at least '9' balls of same color

Upto "30" balls it may be the case that we don't have '9' balls of same color

2 mins Summary

Topic Generating Function

Topic Extended binomial coefficient

Topic Questions

Topic Pigeonhole principle

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THANK - YOU

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