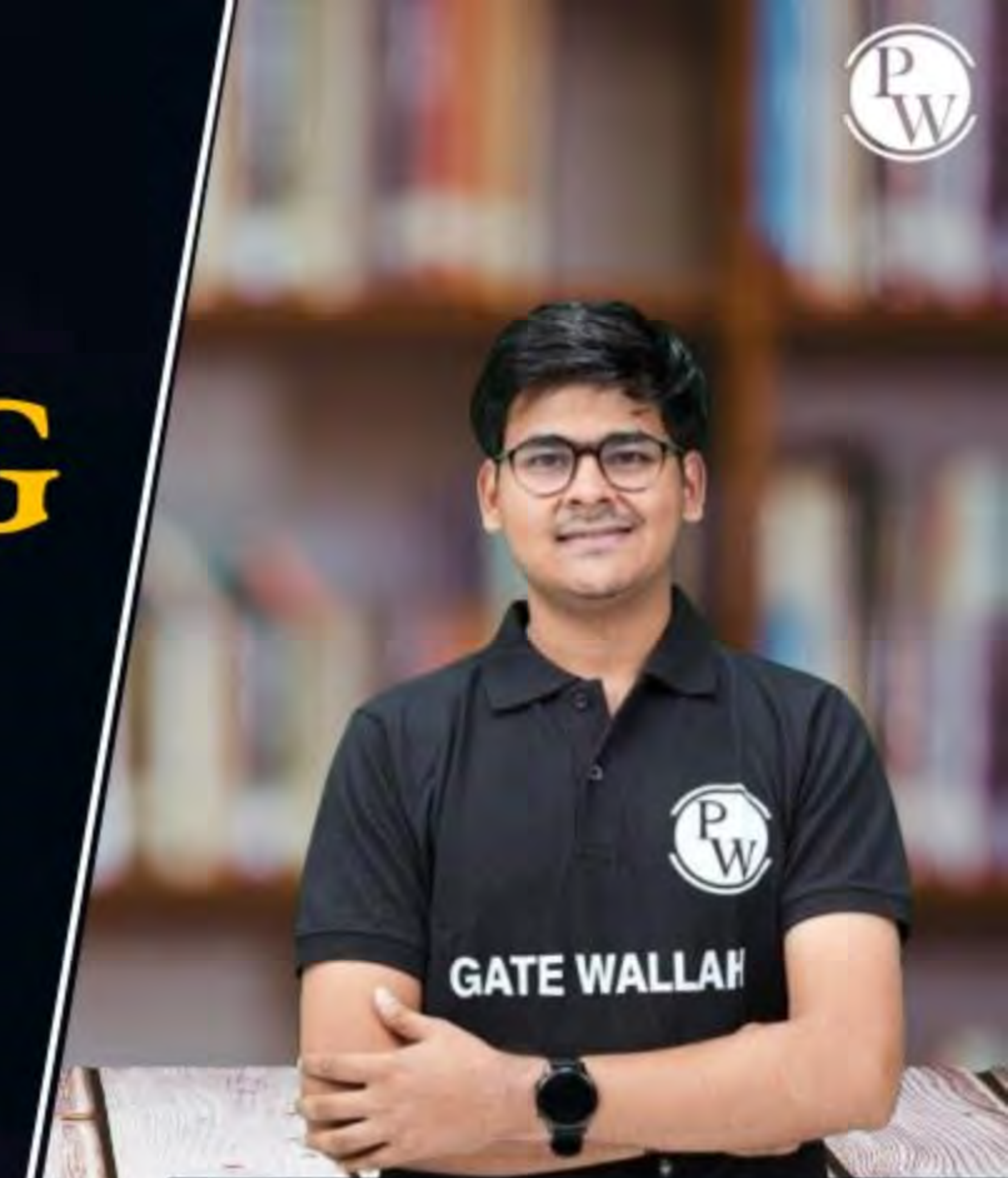
CS & IT ENGINEERING

Algorithms

Analysis of Algorithms



DPP- 01

Discussion Notes

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[MCQ]

increasing

#Q. Sort the functions in ascending order of asymptotic(big-O) complexity.

$f_1(n) = n$, $f_2(n) = 80$, $f_3(n) = n^{\log n}$, $f_4(n) = \log\log^2 n$, $f_5(n) = (\log n)^{\log n}$

A. $f_2(n), f_4(n), f_1(n), f_5(n), f_3(n)$

B. $f_2(n), f_1(n), f_4(n), f_5(n), f_3(n)$

C. $f_2(n), f_1(n), f_4(n), f_3(n), f_5(n)$

D. $f_2(n), f_1(n), f_4(n), f_3(n), f_2(n)$

A

Rate of growth:

decr < Const < Poly < Expo

Soln:

$f_1(n) = n \longrightarrow$ Poly

$f_2(n) = 80 \longrightarrow$ Const

$f_3(n) = n^{(\log n)} \longrightarrow$ Expo

$f_4(n) = \log((\log n)^2)$ — Poly

$f_5(n) = \log n^{(\log n)} \longrightarrow$ Expo

$F_2 < F_4 < F_1 < F_5 < F_3$

$n \qquad \log((\log n)^2)$

$n > 2\log(\log n)$

$$n^{\log n} > \log n^{\log n}$$

$$\cancel{\log n} * \log n \qquad \cancel{\log n} * \log(\log n)$$

Let $\log n \to x$

$$\log n \qquad \log(\log n)$$

$$x > \log(x)$$

[MSQ] → multiple soln can be correct

#Q. Consider two function $f(n) = 10n + 2\log n$ and $g(n) = 5n + 2(\log n)^2$, then which of the following is correct option?

$f \leq g$

A. $f(n) = \theta(g(n))$

B. $f(n) = O(g(n))$ → Big oh

C. $f(n) = \omega(g(n^2))$ → small omega

$f(n) > g(n^2) * c$

D. None of the above

A, B

Soln: $f(n) = 10 * n + 2 * \log(n)$, $g(n) = 5n + 2(\log n)^2$

$\boxed{f(n) = \Theta(n)}$ $\quad$ $\boxed{g(n) = \Theta(n)}$

$O \quad \Omega$ $\qquad$ $O \quad \Omega$

$f = \Theta(n)$

$g = \Theta(n)$

$f = \Theta(g(n))$

$g = \Theta(f(n))$

$$g(n) = 5n + 2(\log n)^2$$

$$g(n^2) = 5n^2 + 2(\log(n^2))^2$$

$$= 5n^2 + 2(2\log(n))^2$$

$$= \underline{5n^2} + 8\underline{(\log n)}^2 \longrightarrow \underline{\theta(n^2)}$$

$$f(n) = \theta(n), \quad g(n^2) = \theta(n^2)$$

$$f(n) < \underline{\underline{g(n^2)}}$$

~~[MCQ]~~ NAT $\rightarrow$ Numerical Answer Type $\rightarrow$ Int

#Q. Consider two function $f(n) = \sqrt{n}$ and $g(n) = n \log n + n$ then $f(n)/g(n)$ is equivalent to how many of the following given below? ______.

$\rightarrow$ Small oh

A $O(n^{-1/2})$

$\rightarrow$ Big oh

B $O(n^{-1/2})$

$$\frac{f(n)}{g(n)} < \frac{1}{\sqrt{n}}$$

C $\Omega(1/\log n)$

D $\theta(n^{-1/2})$

Ans: 2

Soln:-

$f(n) = \sqrt{n}$, $g(n) = n\log n + n$

$$\boxed{h(n)} = \frac{f(n)}{g(n)} = \frac{\sqrt{n}}{n\log n + n} = \frac{\cancel{\sqrt{n}}}{\cancel{n}(\log n + 1)} = \boxed{\frac{1}{\sqrt{n}(\log n + 1)}}$$

Check option A :- $h(n) = \dfrac{1}{\sqrt{n}\,(\log n+1)}$ is $O(n^{-1/2})$?

$\dfrac{1}{\log n+1} < 1$

$\dfrac{1}{\sqrt{n}(\log n+1)}$

$n^{-1/2} \rightarrow \dfrac{1}{\sqrt{n}}$

h(n) is not $\Omega(n^{-1/2})$

$\downarrow$

Hence not $O(n^{-1/2})$

$\dfrac{1}{\cancel{\sqrt{n}}(\log n+1)}$ $\quad \dfrac{1}{\cancel{\sqrt{n}}}$

Hence A & B both are True

$\dfrac{1}{(\log n+1)} < 1$

Check ($\dfrac{1}{\sqrt{n}(\log n+1)}$ is $\Omega\left(\dfrac{1}{\log n}\right)$?

$$\frac{1}{\sqrt{n}\log n + \sqrt{n}} < \frac{1}{\log n}$$

$$h(n) = \Omega\left(\frac{1}{\log n}\right)$$

X

$$\sqrt{n}\log n + \sqrt{n} > \log n$$

[MCQ]

#Q. Consider the following C-code

void foo (int n)
{
int a = 1;
if (n = = 1)
return; → exit
~~for (; a ≤ n; a++)~~ for(a=1, a≤n ;a++)
{
printf("GATEWALLAH");
break;
}
}

What is the worst time complexity of above program?

Ans :- A

A. O(1)

B. O(n)

C. O (log n)

D. $O\sqrt{n}$

[MCQ]

#Q.Consider the following asymptotic functions :

$f1 = 2^n$

$f2 = 1.001^n$

$f3 = e^n$

$f4 = n!$

Ans → D

Which of the following is correct increasing order of above functions?

A f3, f4, f1, f2

B f2, f4, f1, f3

C f3, f2, f1, f4

D f2, f1, f3, f4

Soln :

$e \longrightarrow \approx 2.71$

$f_1 = 2^n \longrightarrow$ Expo

$1.001 < 2 < 2.71(e)$

$f_2 = (1.001)^n \longrightarrow$ Expo

$(1.001)^n < 2^n < e^n < n^n$

$f_3 = e^n \longrightarrow$ Expo

$f_4 = n! \longrightarrow$ Expo (n^n)

$$\boxed{f_2 < f_1 < f_3 < f_4}$$

$\hookrightarrow n(n-1) \times (n-2) \ldots 1 \longrightarrow \approx n^n$

~~[MCQ]~~ MSQ → Multiple can be Correct options

#Q. Consider ~~two function~~ the following functions:

$f_1(n) = 4^{2^n}$

$f_2(n) = n!$

$f_3(n) = 4^{e^n}$

$f_4(n) = n^{n^n}$

Which of the following is/are correct?

Ans: B, C, D

$f_2 < f_1 < f_3 < f_4$

$f_1 = O(f_2)$

$f_1 \le f_2$ ✗

A. $f_1(n) = O(f_2(n))$ ✗

B. $f_1(n) = O(f_4(n))$ ✓

$f_1 = O(f_4)$

$\downarrow$

$f_1 \le f_4$

C. $f_1(n) = O(f_3(n))$ ✓

$f_1 \le f_3$

D. $f_2(n) = O(f_3(n))$ ✓

$f_2 \le f_3$

Soln :-

✓ $f_1(n) = (4)^{2^n} \longrightarrow$ expo

$f_2(n) = n! \rightarrow (n)^n \longrightarrow$ expo

✓ $f_3(n) = (4)^{e^n} \longrightarrow$ expo

$f_4(n) = (n)^{n^n} \longrightarrow$ expo

$$\boxed{f_2 < f_1 < f_3 < f_4}$$

$$(4)^{2^n} < (4)^{e^n}$$

$$2^n < e^n \qquad e \approx 2.71$$

$$f_1 < f_3$$

$$(n)^n < (n)^{n^n}$$

$$f_2 < f_4$$

$$(n)^n \quad < \quad (4)^{e^n}$$

$$\log(n^n) \qquad \log((4)^{e^n})$$

$$n * \log n \qquad e^n * \log(4)$$

$$n\log n \quad < \quad e^n$$

Poly

expo

$$(n)^n \quad < \quad (4)^{2^n}$$

$$\log(n^n) \qquad \log\left(\left((4)^{(2)^n}\right)\right)$$

$$n * \log n \qquad 2^n * \log(4)$$

$$n \log n \quad < \quad 2^n$$

Poly

Expo

[MCQ]

Ans :- C

#Q. Consider two function $f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$ then which of the following is true.

$f_1 \leq f_2$

$f_1 > f_2$

A $f_1(n) = (Of_2(n))$

B $f_1(n) = \theta(f_2(n))$

$f_1 \approx f_2$ (rate of growth)

small omega

C $f_1(n) = \omega(f_2(n))$

D None of these

$f_1 > f_2$

$2^n \neq \Theta(n^2)$

Soln:-

$$f_1(n) = n^{2^n} > f_2(n) = n^{n^2}$$

log on both sides

$$\log\left(n^{2^n}\right) \qquad \log\left(n^{n^2}\right)$$

$$2^n * \cancel{\log n} \qquad n^2 * \cancel{\log n}$$

$$2^n > n^2$$

expo Poly

[MCQ]

$\theta \to O$, $\theta \to \Omega$

#Q. $f(n) = \Sigma_{i=1}^{n} i^3 = x$, choices for x

I. $\theta(n^4)$ II. $\theta(n^5)$

III. $O(n^5)$ IV. $\Omega(n^3)$

A. I, II, III

B. II, III, IV

C. I, II. III, IV

D. I, III, IV

D

$x = \frac{1}{4}(n^4 + 2n^3 + n^2)$

$x \to O(n^4)$, $x \to \Omega(n^4)$ } $\theta(n^4)$

$x \le n^4 \to$ Tight

$x \le n^5$

$\le n^6$ loose

$x \to O(n^5)$

$x \ge n^5$

$x = \Omega(n^3)$

$x \ge C * n^3$

Soln:

$$x = \sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 \cdots n^3$$

$$= \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{n^2+n}{2}\right)^2$$

$$(n^2)^2 = n^{2\times 2} = \underline{\underline{n^4}}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \frac{1}{4}(n^2+n)^2 = \frac{1}{4}\left[(n^2)^2 + 2 * n^2 * n + n^2\right]$$

$$= \frac{1}{4}\underbrace{\left[n^4 + 2n^3 + n^2\right]}_{\Theta(n^4)}$$

Dominating

THANK - YOU

Practice all the Concepts taught in Class.