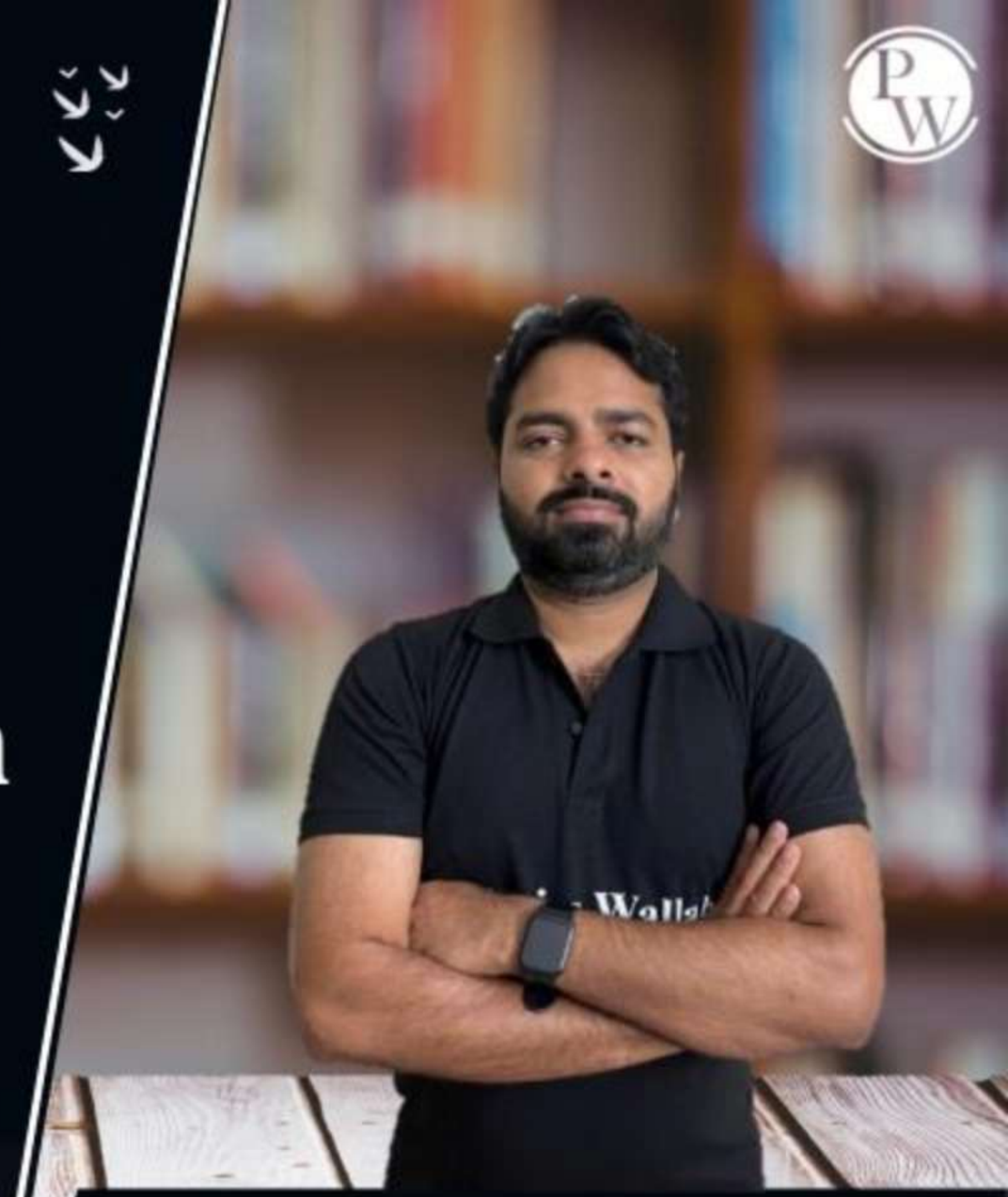
Computer Science

Theory of Computation

Context Free Languages

Lecture No.- 3



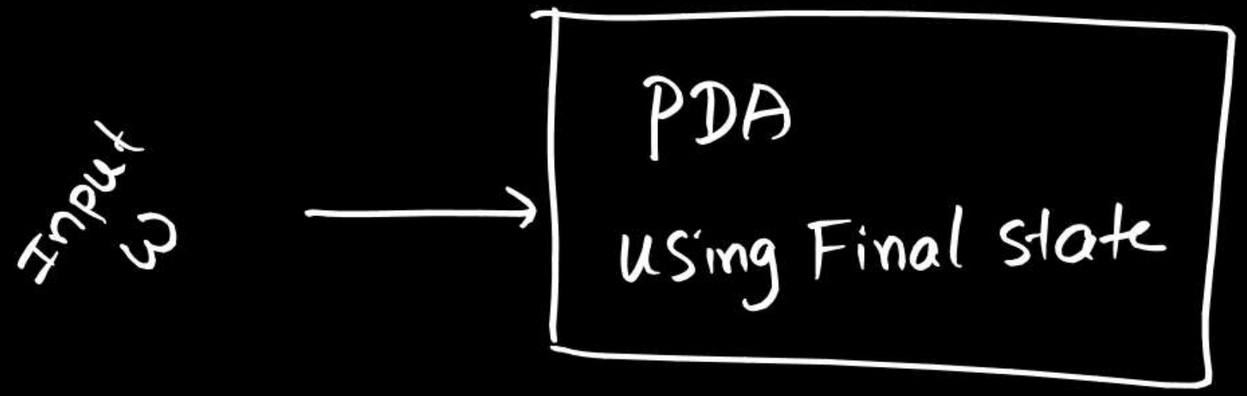
Mallesham Devasane Sir

Recap of Previous Lecture

Topic	Context Free Grammar
Topic	Push Down Automata Basics

Topics to be Covered

Topic	Push Down Automata Construction



If w is valid, atleast one path must halt at final state.

What is i/p ? Σ Σ_ε

What is tos ? Γ (DPDA) Γ^* (PDA)

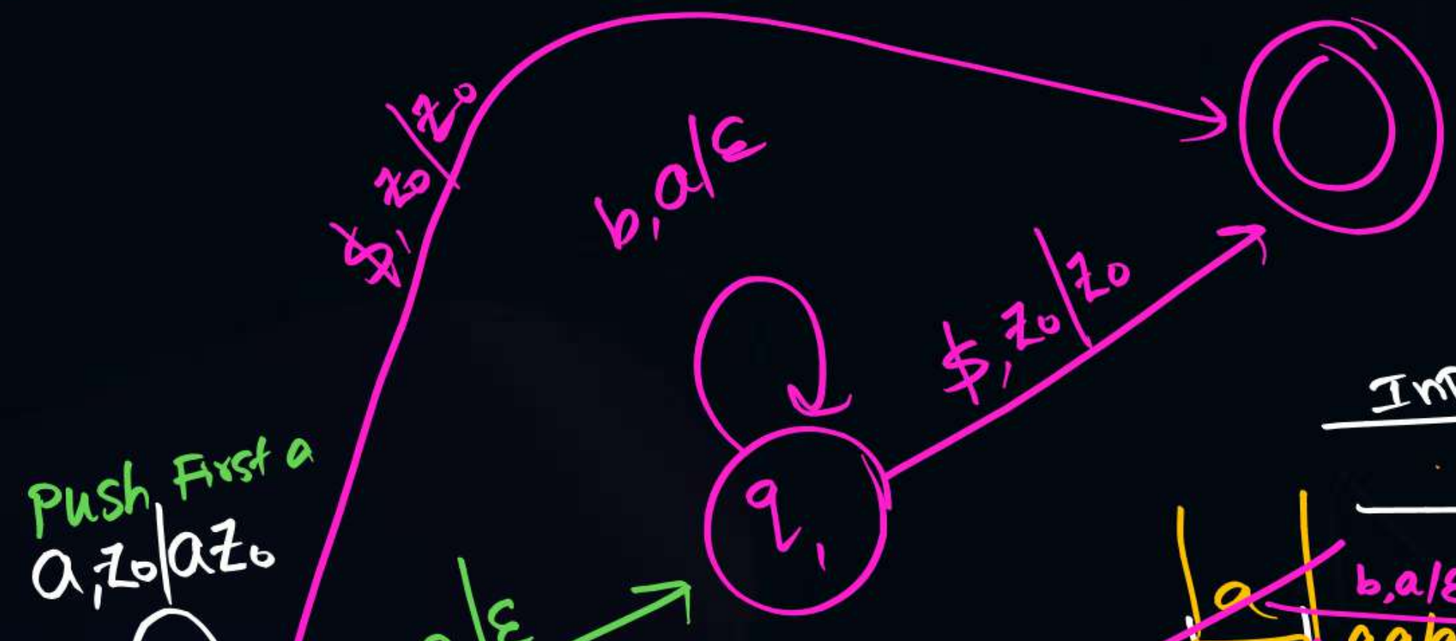
What is operation ? Γ/Γ^* Γ^*/Γ^*

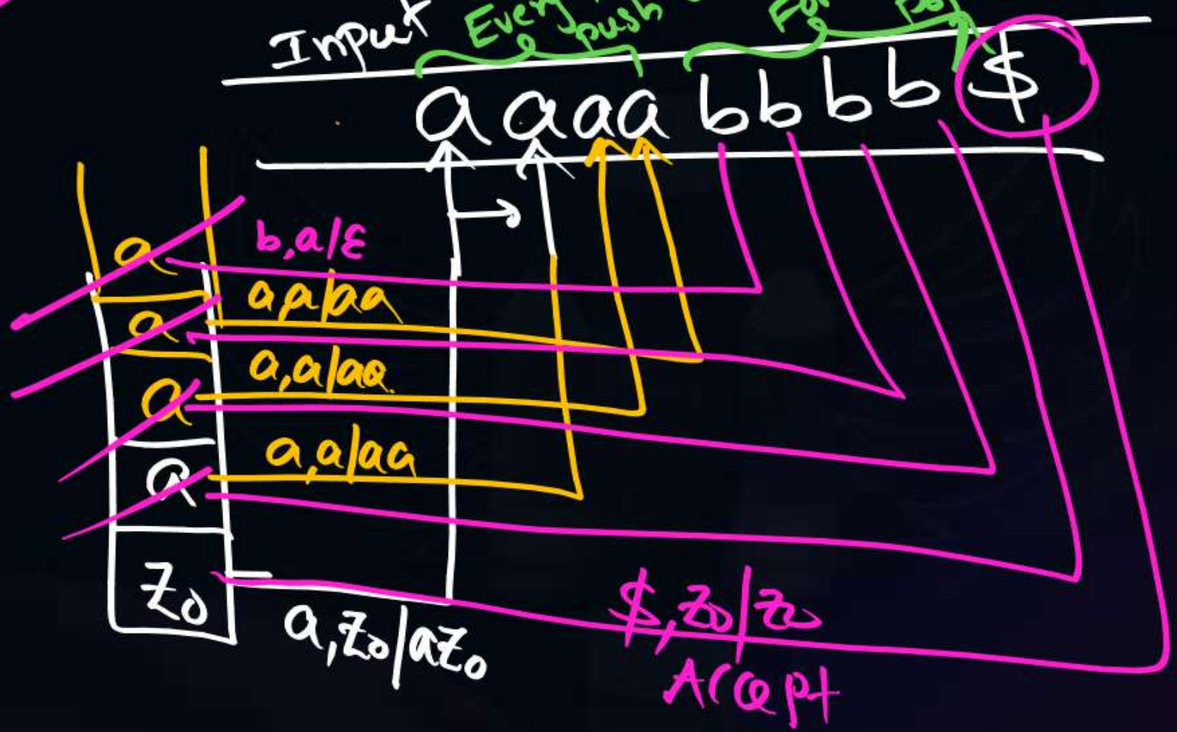
DPDA: $Q \times \Sigma \times \Gamma \longrightarrow Q \times \Gamma^*$

PDA: $Q \times \Sigma_\varepsilon \times \Gamma^* \longrightarrow 2^{Q \times \Gamma^*}$

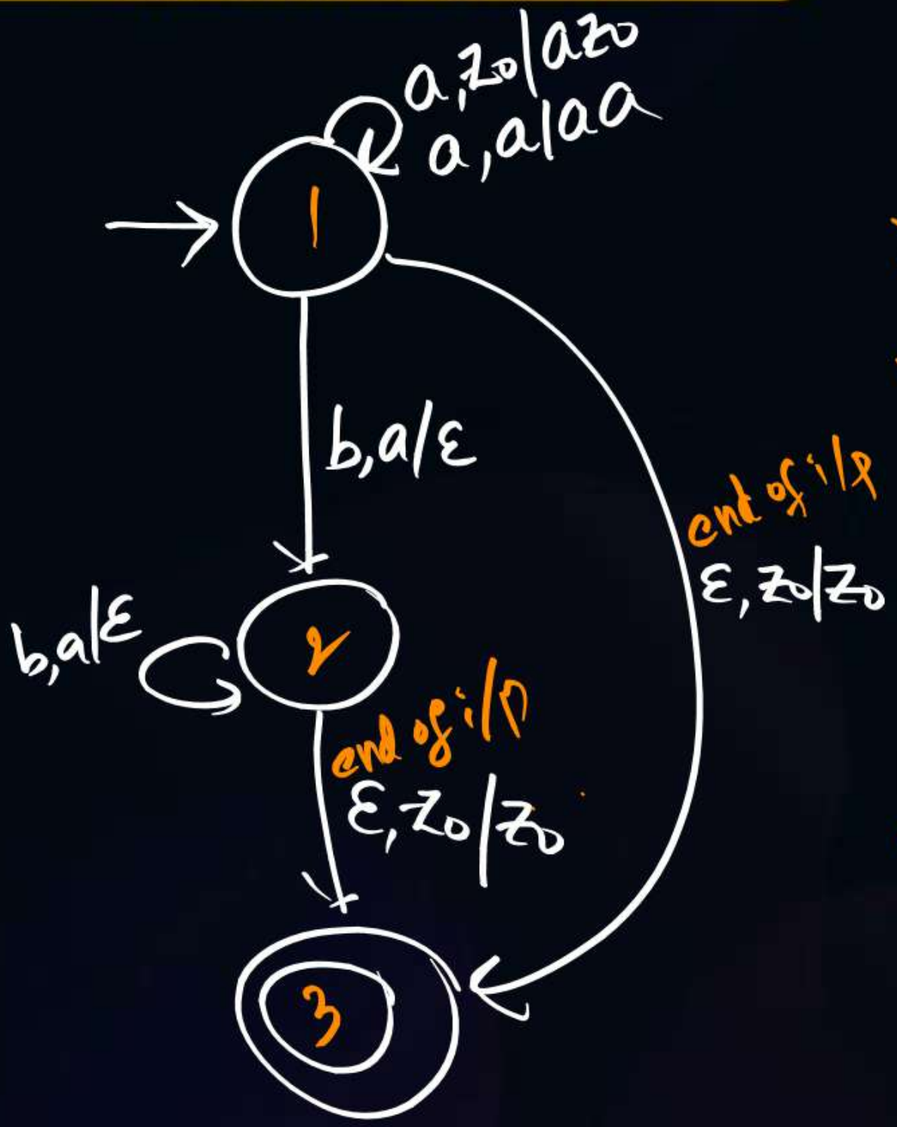
Push Down Automata

① $L = \{a^n b^n \mid n \geq 0\} = \{\varepsilon, ab, aabb, aaabbb, a^4 b^4, \ldots\}$





Push Down Automata



✓ ε : 1 $\xrightarrow{\varepsilon, Z_0}$ 3

✗ a : 1 $\xrightarrow[\text{Push } a]{a, Z_0}$ 1 $\xrightarrow{\varepsilon, a}$ no transition

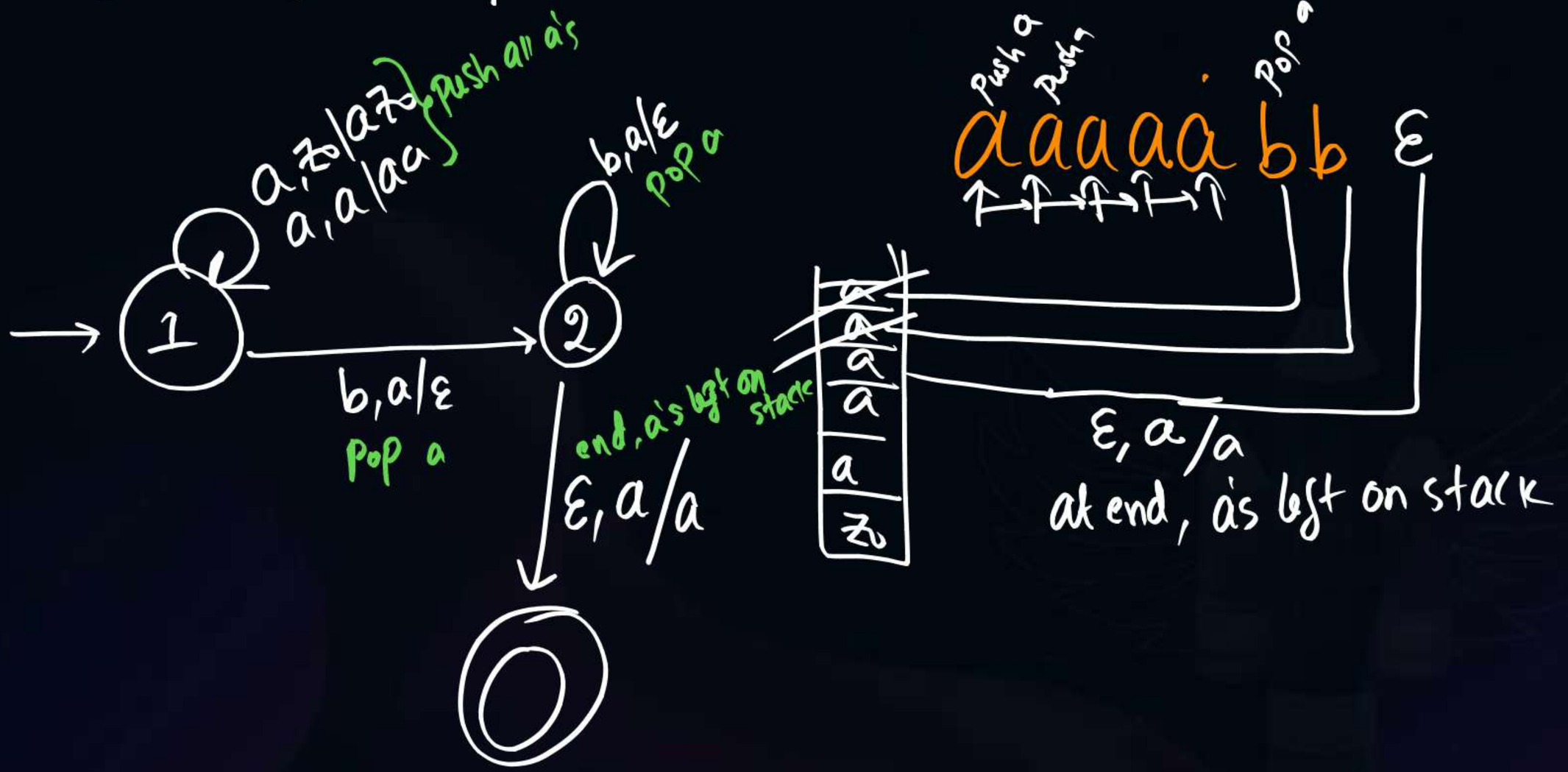
✗ b : 1 $\xrightarrow{b, Z_0}$ no transition

✗ aa : 1 $\xrightarrow[\text{push}]{a, Z_0}$ 1 $\xrightarrow[\text{push}]{a, a}$ 1 $\xrightarrow{\varepsilon, a}$ no tran

✓ ab : 1 $\xrightarrow[\text{Push}]{a, Z_0}$ 1 $\xrightarrow[\text{Pop}]{b, a}$ 2 $\xrightarrow{\varepsilon, Z_0}$ 3

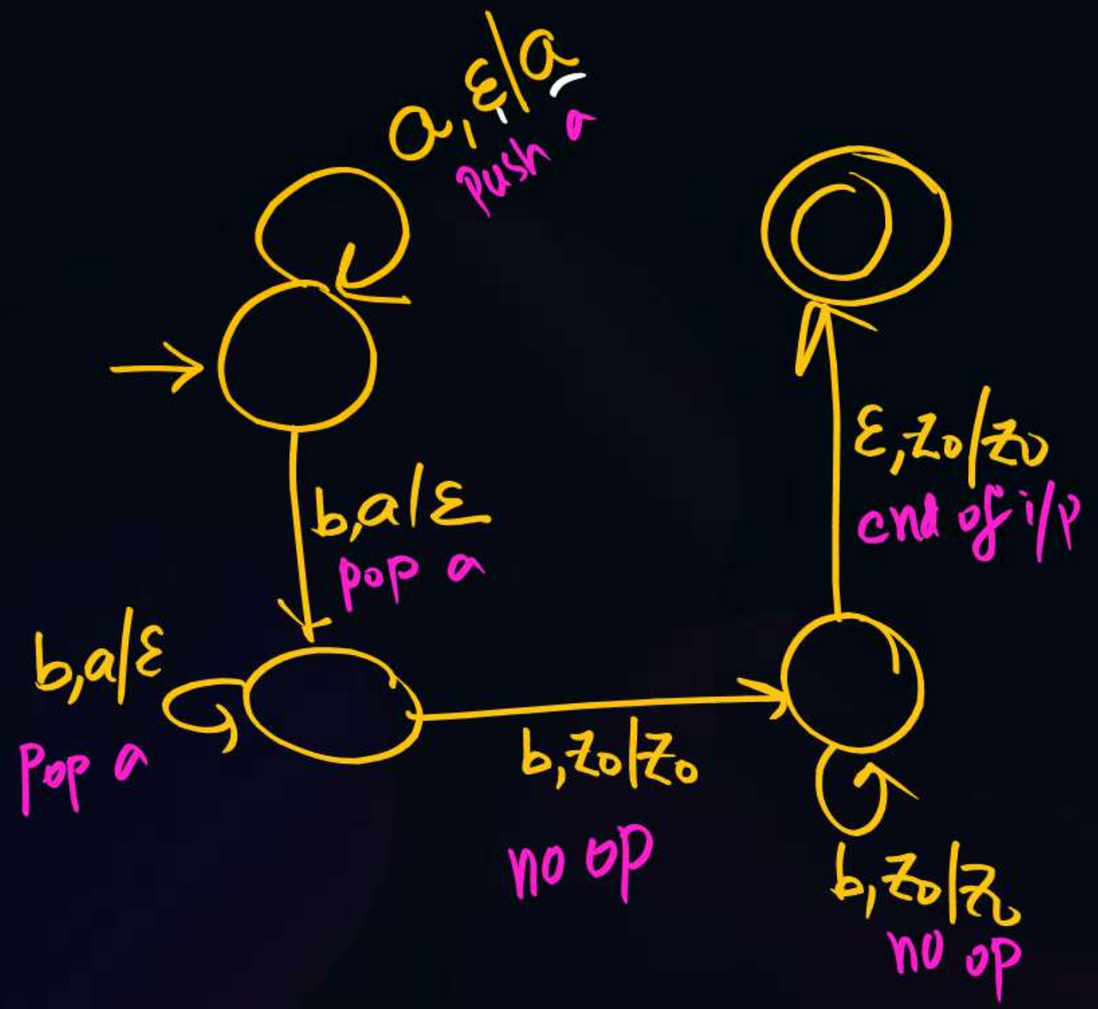
Push Down Automata

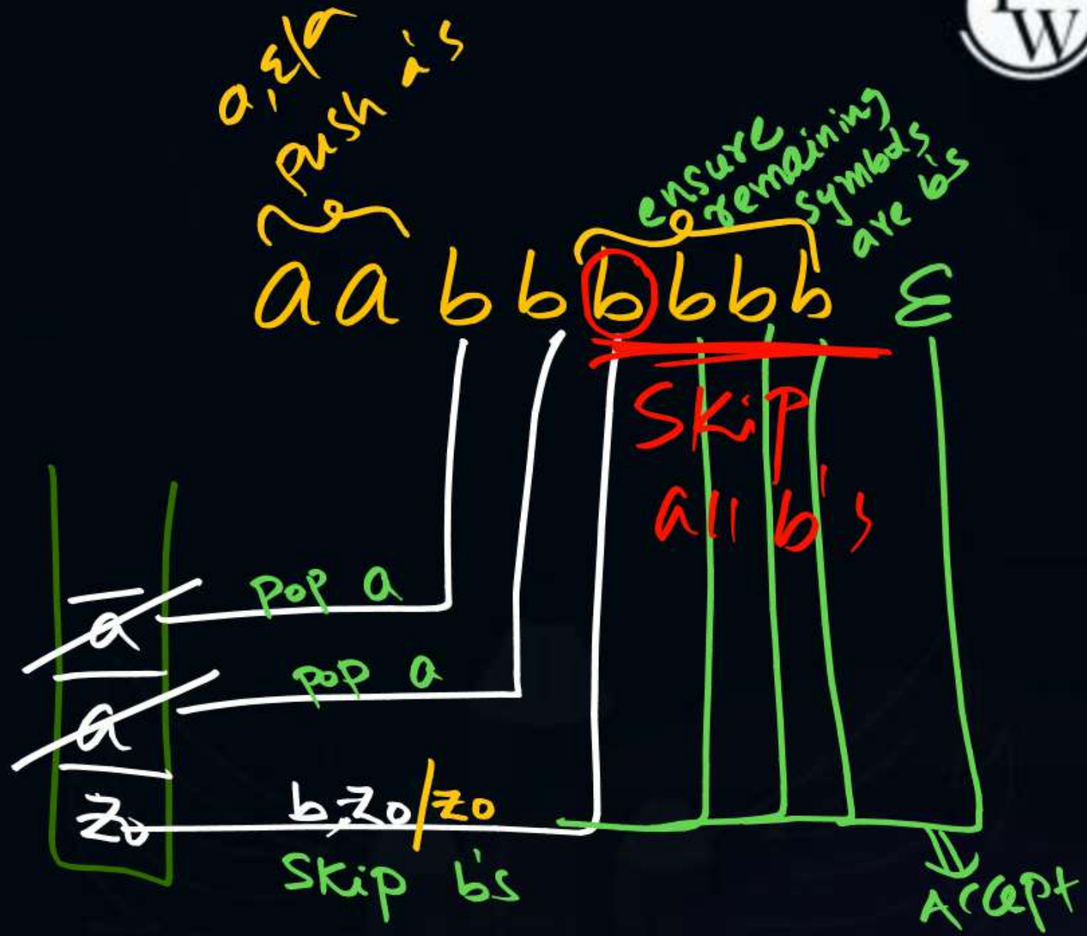
② $\{a^m b^n \mid m, n \geq 1,\ m > n\}$



Push Down Automata

(3) $\{a^m b^n \mid m,n \geq 1,\ m<n\}$





b, Z0/Z0

$a, \varepsilon / a$

Push 'a'

Read 'a'

Don't look at stack

Push a

$\varepsilon, \varepsilon / \varepsilon$

Don't read i/p
Don't look at stack
No operation

$\varepsilon, a / a$

Don't read input

tos is 'a'

No operation

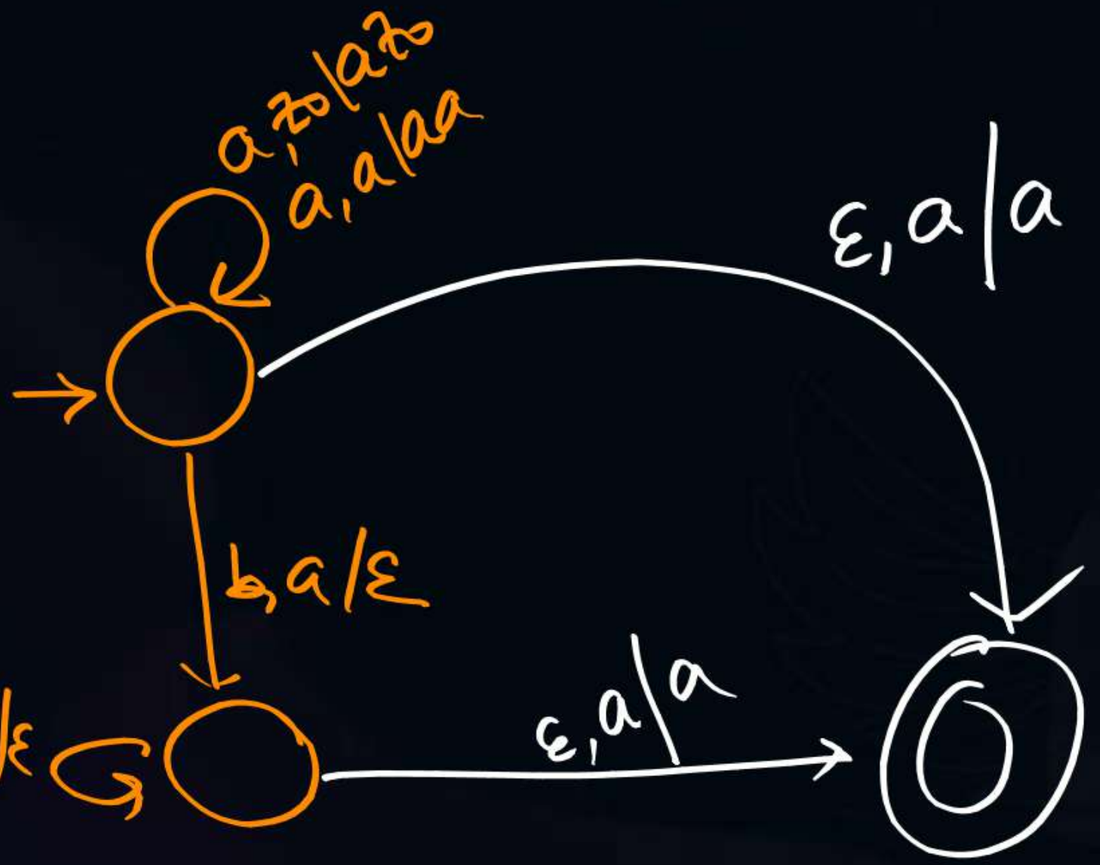
$a, b / \varepsilon$

Read a

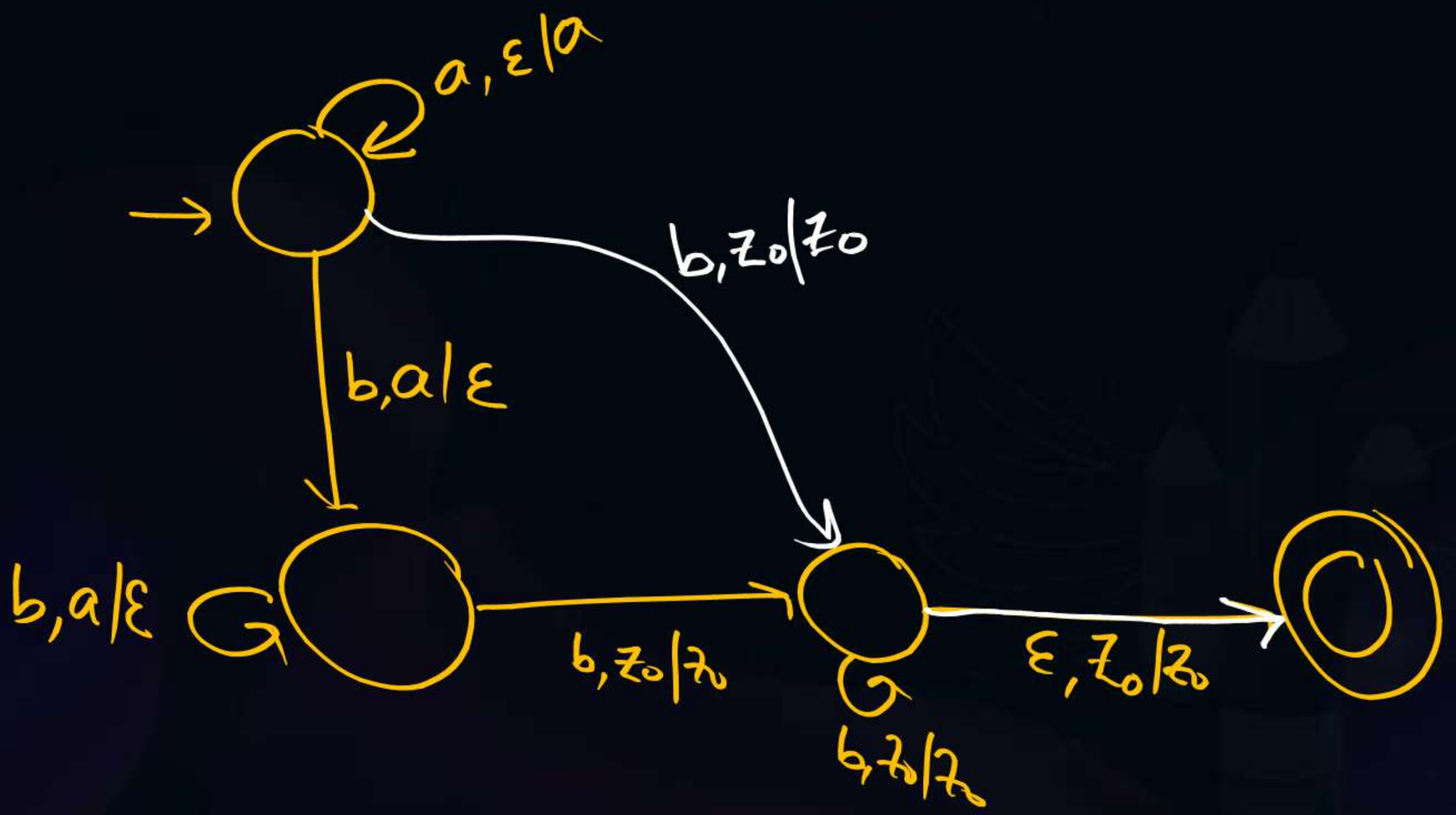
tos 'b'

Pop 'b'

④ $\{a^m b^n \mid m > n, \quad m, n \geq 0\} = ② \cup a^+$

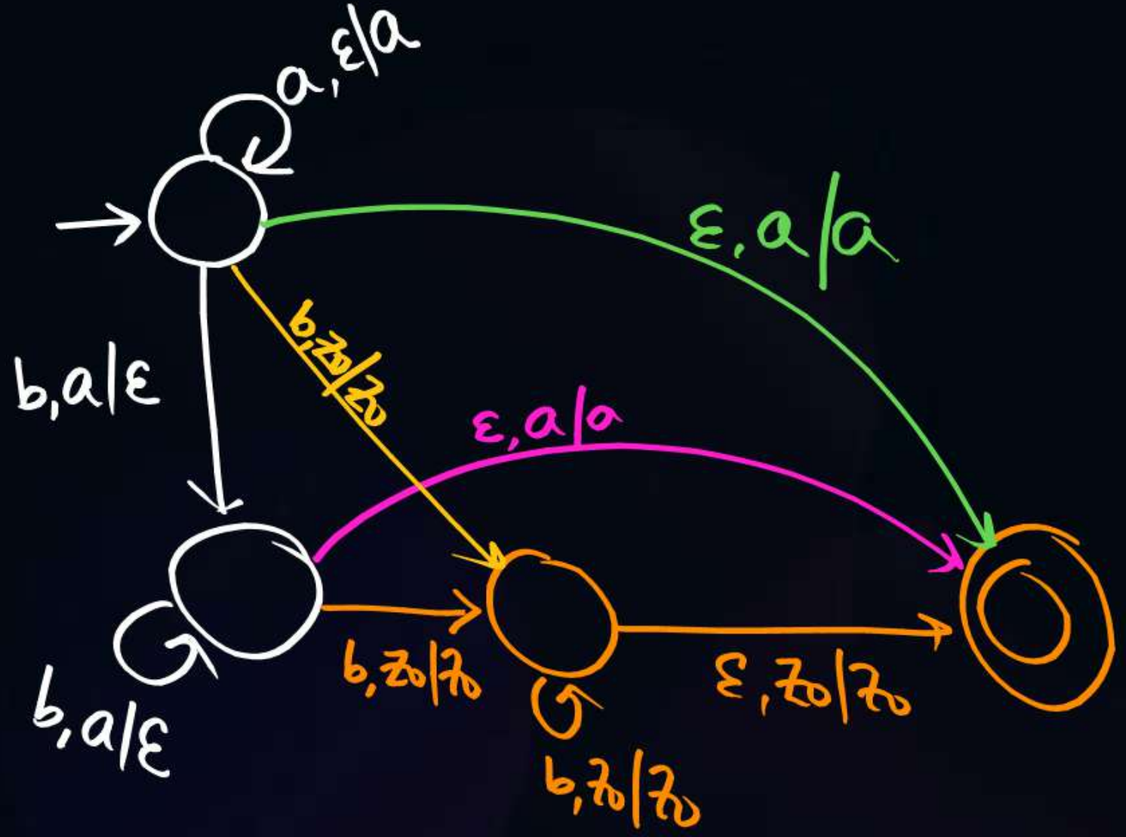


⑤ $\{a^m b^n \mid m<n\} = ③ \cup b^+$



⑥ $\{a^m b^n \mid m \neq n, m,n \geq 0\}$

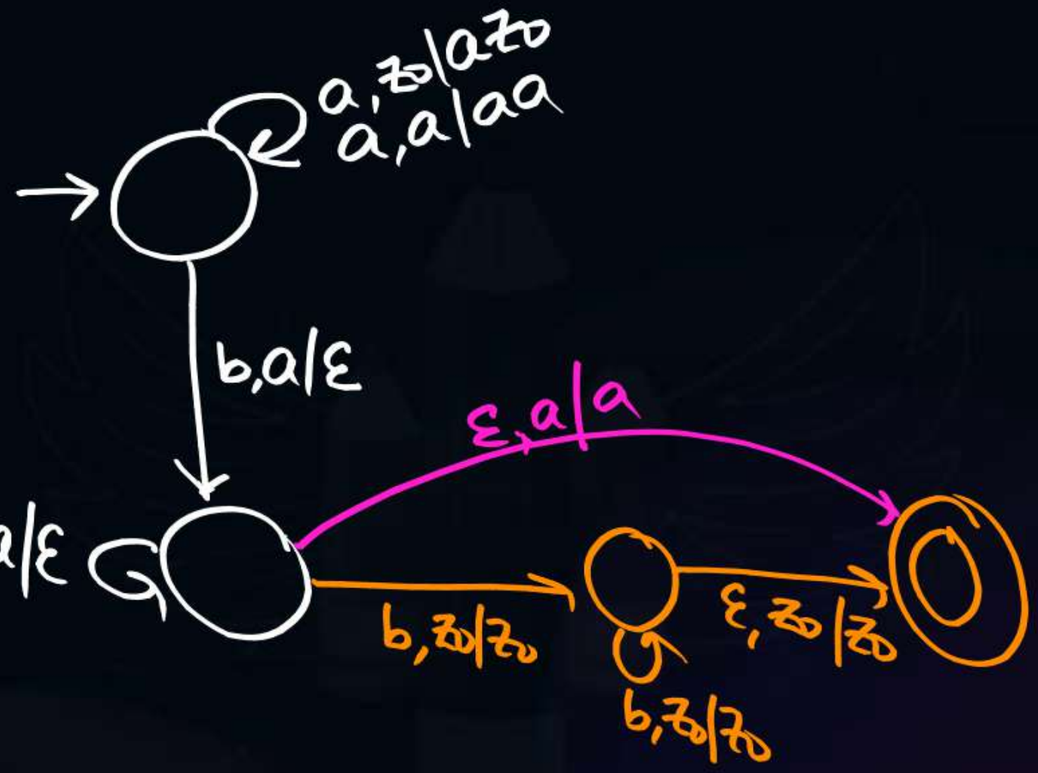
$= ⑦ \cup a^+ \cup b^+$



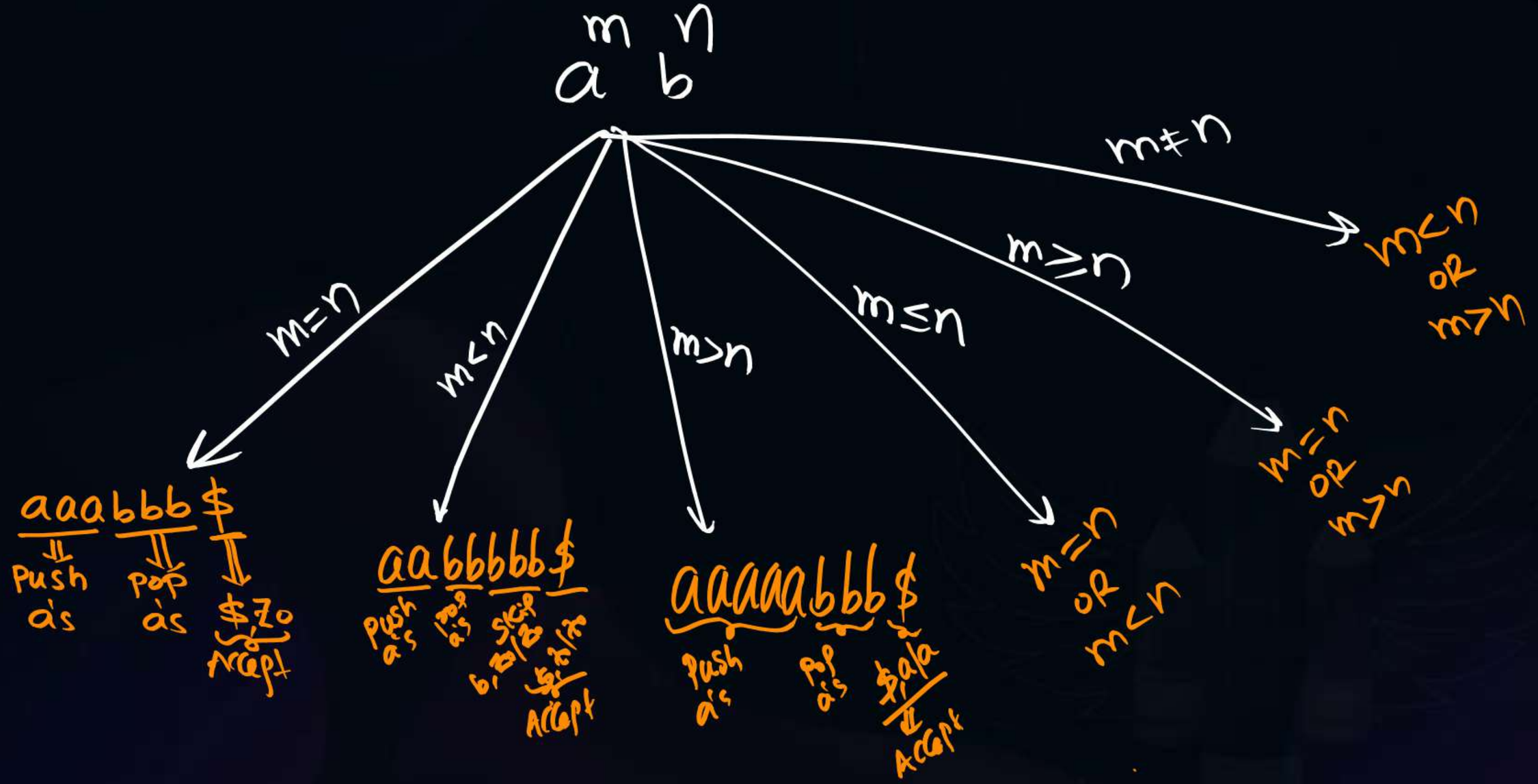
⑦ $\{a^m b^n \mid m \neq n, m,n \geq 1\}$

$m<n$ or $m>n$

$= ② \cup ③$



Push Down Automata



H.W.

(8) $\{b^n a^n \mid n \geq 1\}$

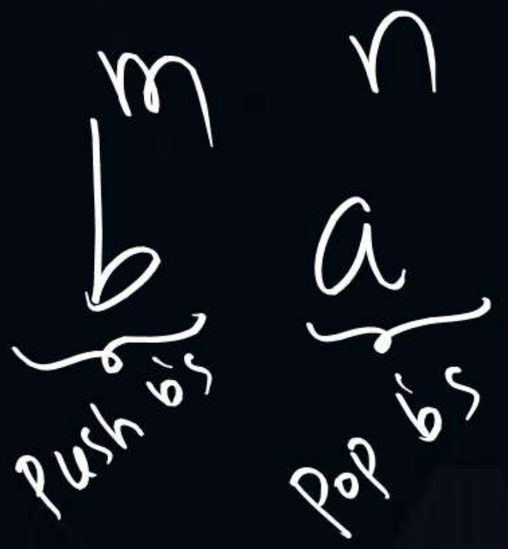
(9) $\{b^m a^n \mid m, n \geq 1,\ m > n\}$

(10) $\{b^m a^n \mid m, n \geq 1,\ m < n\}$

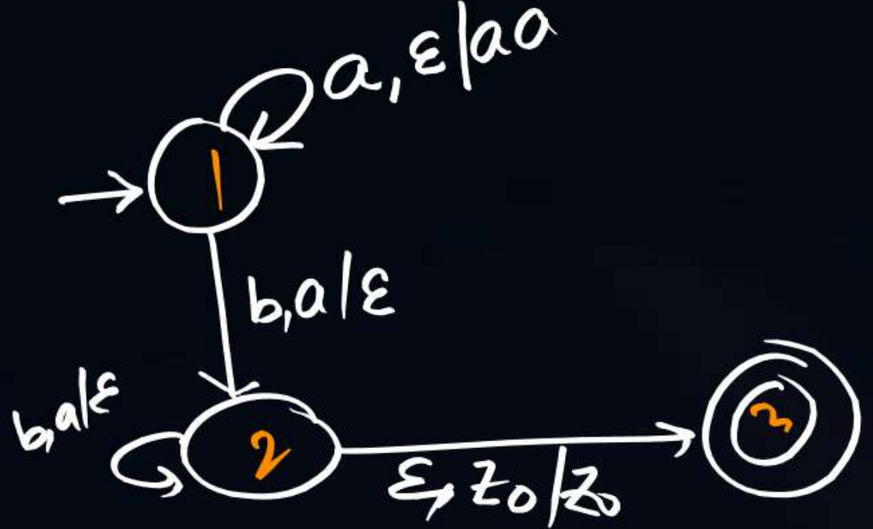
(11) $\{b^m a^n \mid m, n \geq 1,\ m \neq n\}$

(12) $\{b^m a^n \mid m, n \geq 1,\ m \geq n\}$

(13) $\{b^m a^n \mid m, n \geq 1,\ m \leq n\}$

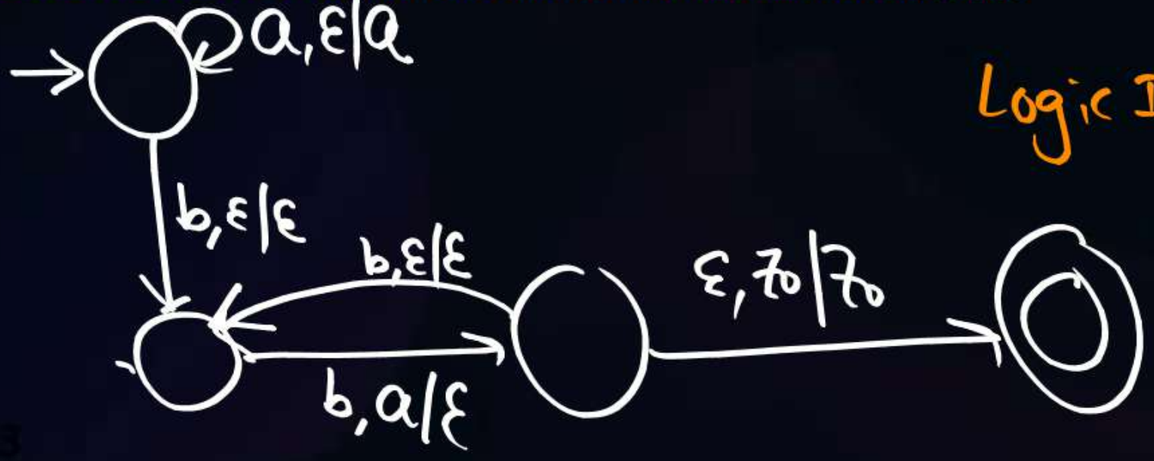


(14) $\{a^n b^{2n} \mid n \geq 1\}$



aaabbbbbb $

Logic I: Every a ⇒ push 2a's
Every b ⇒ pop 1 a
end ⇒ z0 left on stack



Logic II: Every a ⇒ push 1a
Every 2bs ⇒ pop 1a
end ⇒ z0 left on stack

15) $\{a^{2n} b^{n} \mid n \geq 1\}$

H.W.

Logic I: For every input a ⇒ Push 1 a
For every input b ⇒ pop 2 a's
at end of i/p ⇒ Stack only has Z_0

Logic II: For every 2 a's ⇒ Push 1 a
For every b ⇒ Pop 1 a
at end ⇒ Z_0 appears on stack

Push Down Automata

(16) $\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$

(I)

Push input

$a, Z_0 / aZ_0$

$b, Z_0 / bZ_0$

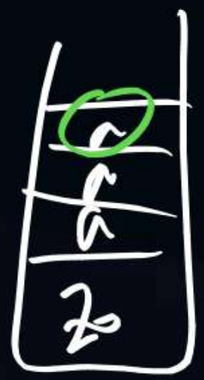
(II)

If tos == i/p
PUSH input

a,a/aa
b,b/bb

Push 'b'

(III)



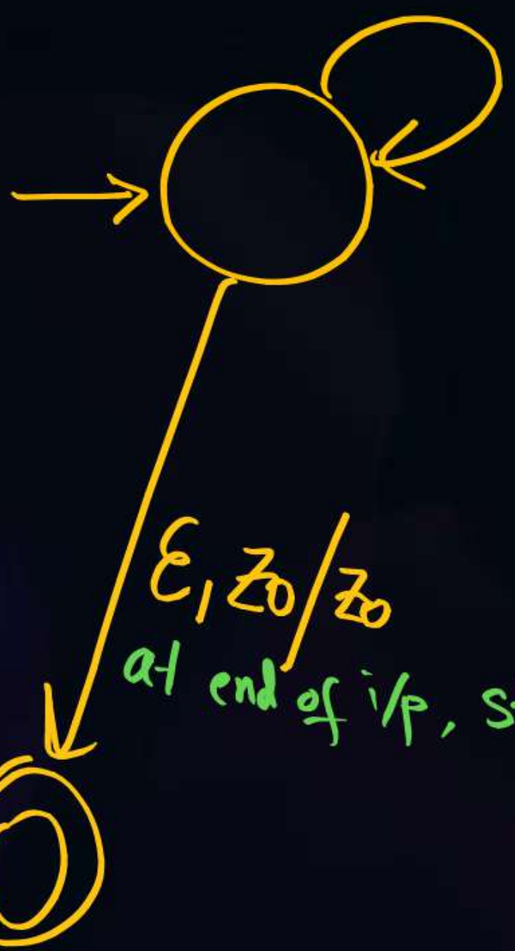
pop tos

pop tos

$a, b/\varepsilon$

$b, a/\varepsilon$

(16) $\{w \mid w \in \{a,b\}^*,\ n_a(w) = n_b(w)\}$



ε ✓

a ε X

b X

a a X

a b ε ✓

b a ✓

a a b b ✗

b b a a ✓

a b a b ✓

a b b a ✓

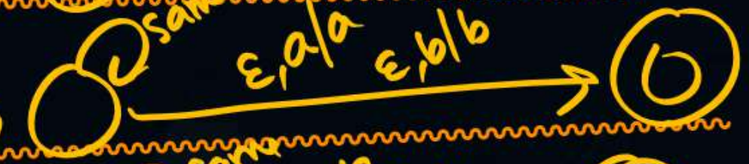
Push Down Automata

16) $\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$

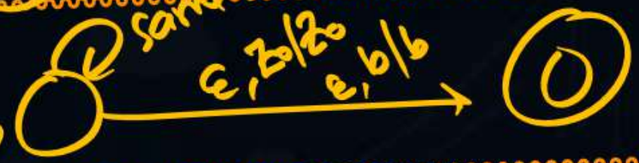
17) $\{w \mid \text{''}, n_a(w) > n_b(w)\}$

18) $\{w \mid \text{''}, n_a(w) < n_b(w)\}$

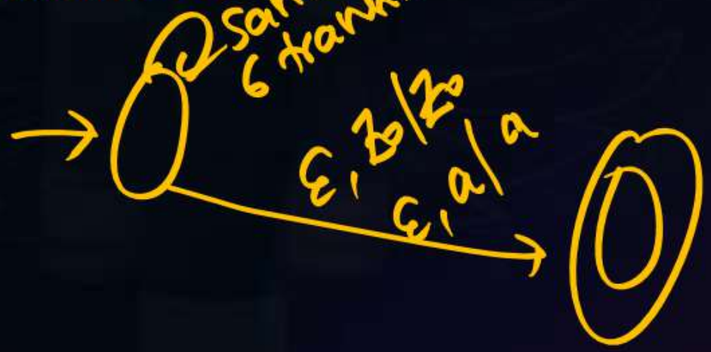
19) $\{w \mid \text{''}, n_a(w) \neq n_b(w)\}$



20) $\{w \mid \text{''}, n_a(w) \leq n_b(w)\}$



21) $\{w \mid \text{''}, n_a(w) \geq n_b(w)\}$



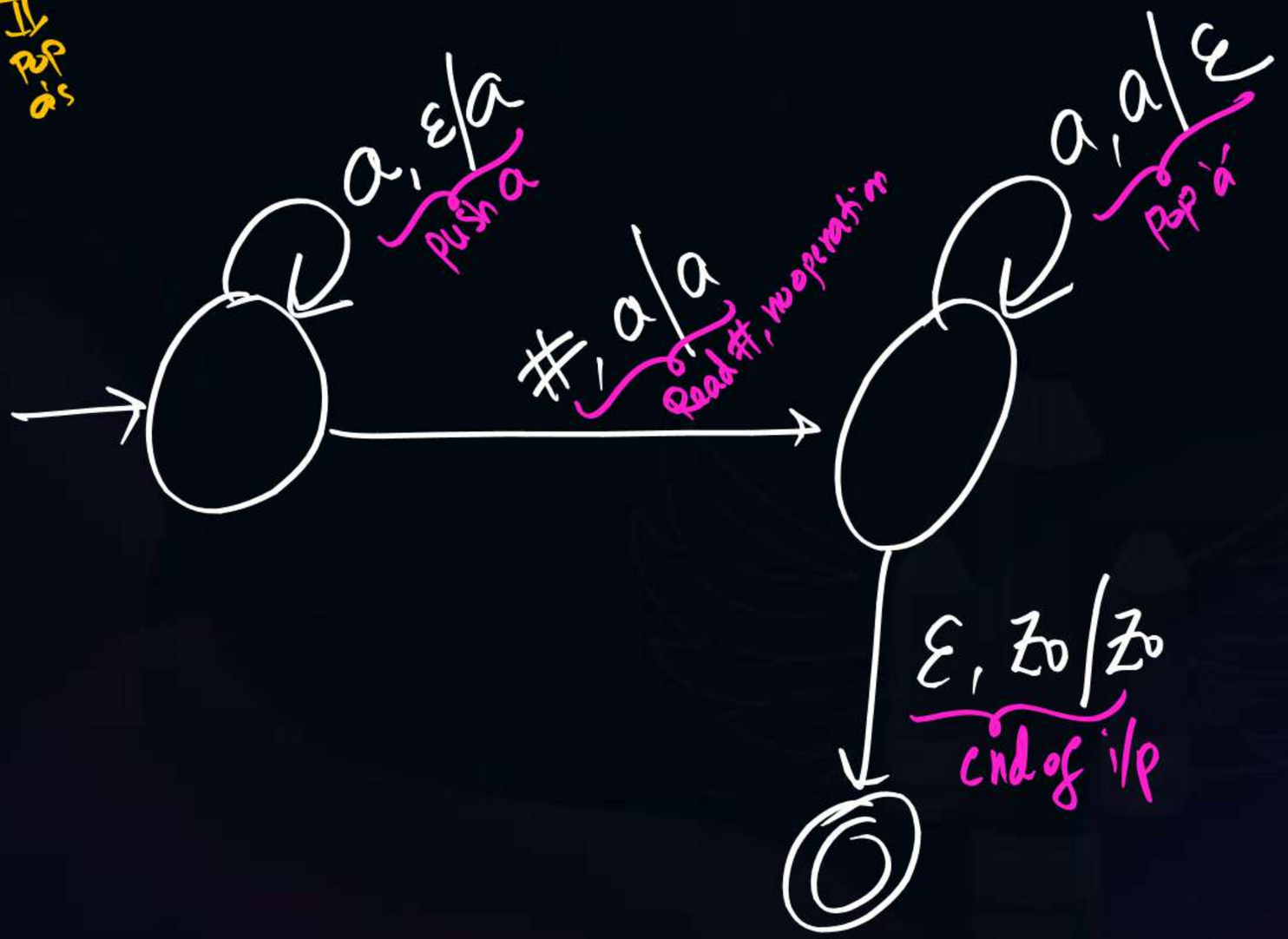
a b a a b b a b a b a b ε

Push Pop Push Push Pop Pop Push Pop Push Pop Push Pop

ε → end of input

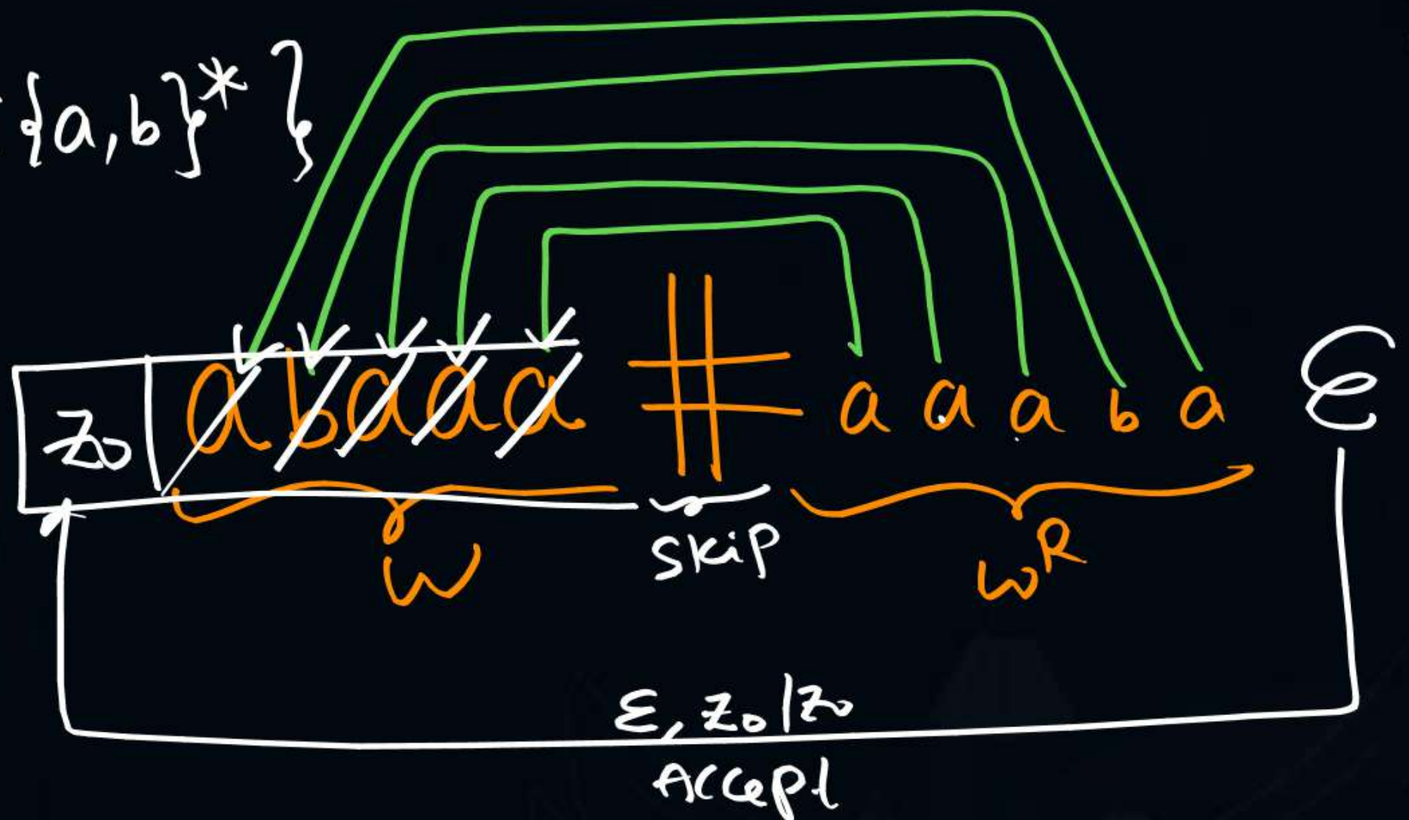
(22) $\{a^n \# a^n \mid n \geq 1\}$

Push a's || skip || pop a's



(23) $\{w \# w^R \mid w \in \{a,b\}^*\}$

#✓
a#a✓
b#b✓
aa#aa✓
ab#ba✓
ba#ab✓
bb#bb✓



Push all

#

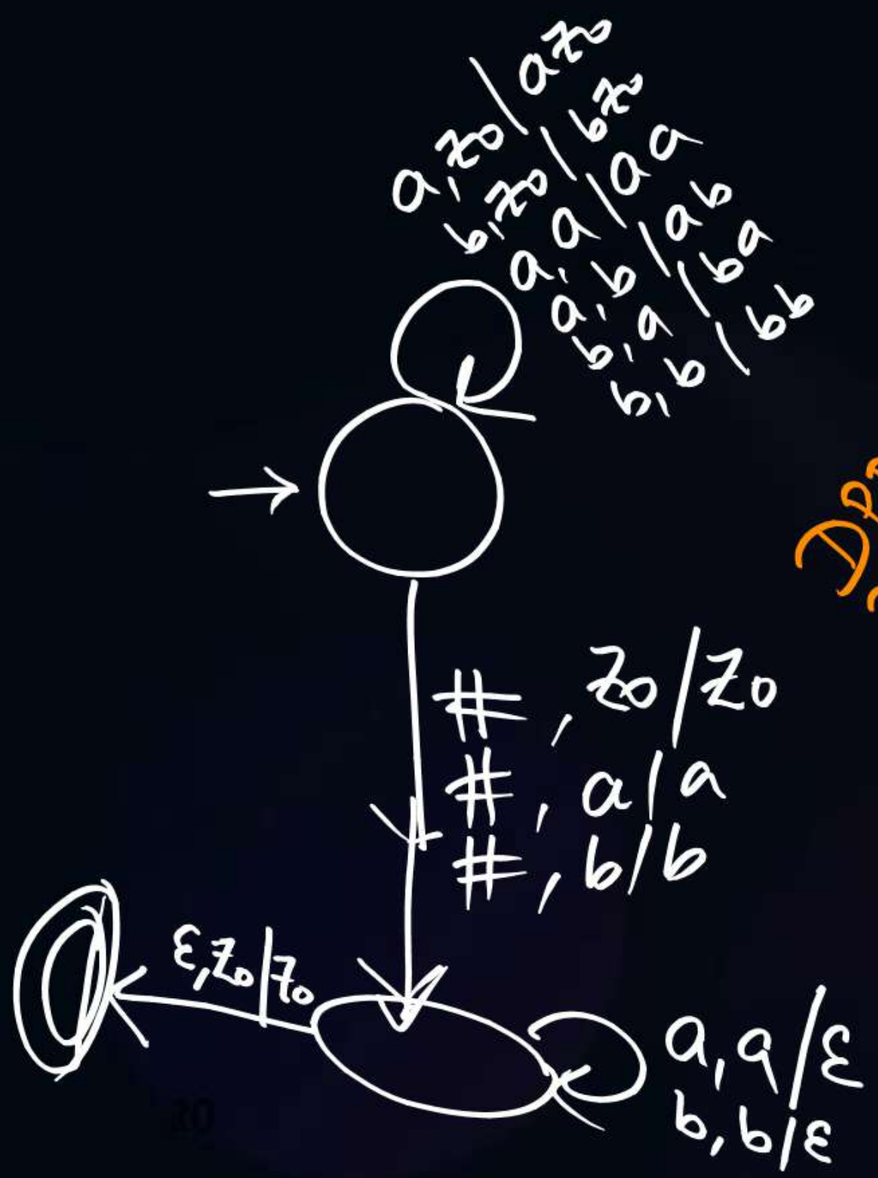
Pop only by match

a, a/ε
b, b/ε

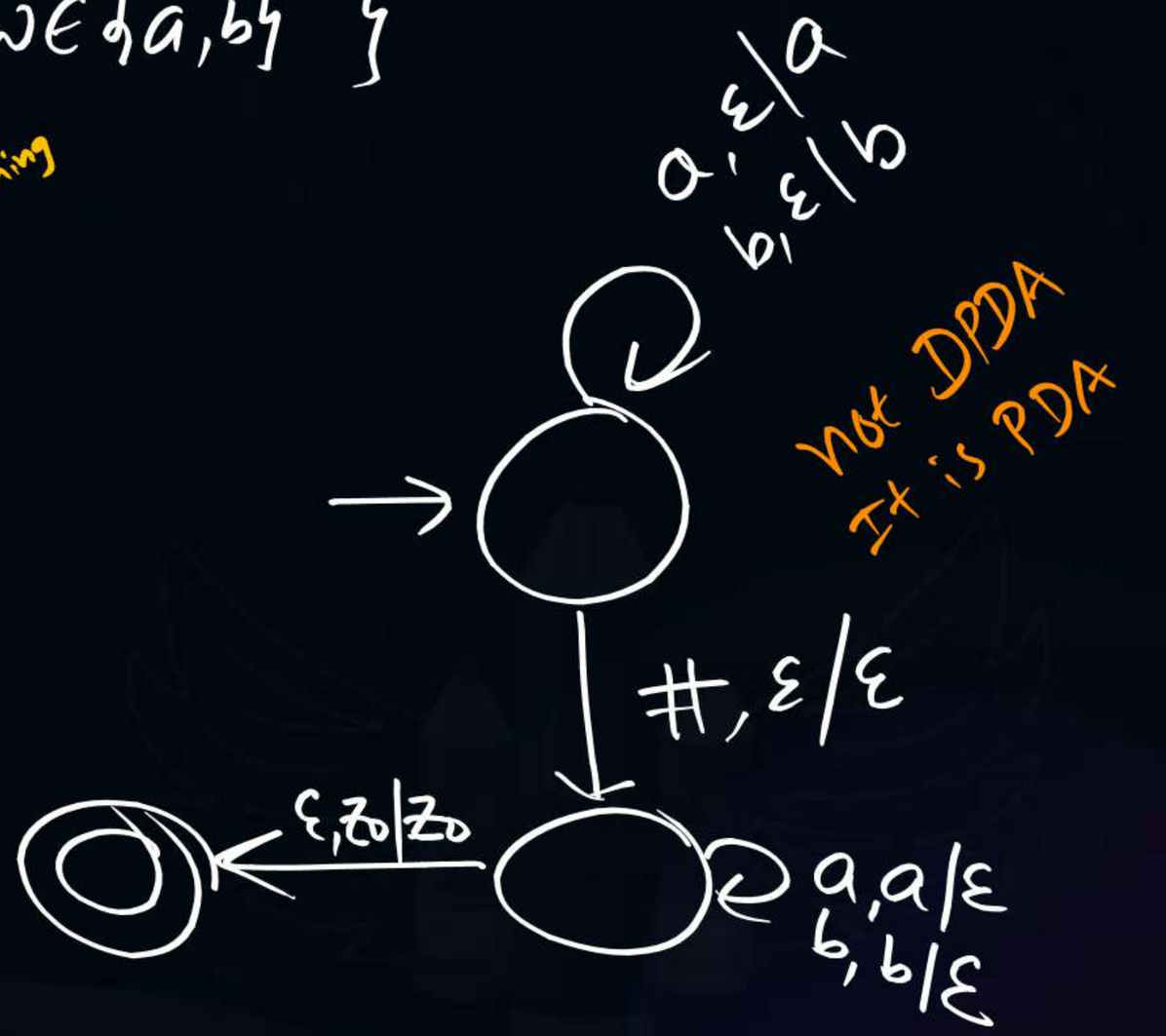
Push Down Automata

$$\{ w \# w^R \mid w \in \{a,b\}^* \}$$

Push | Skip | Pop by matching

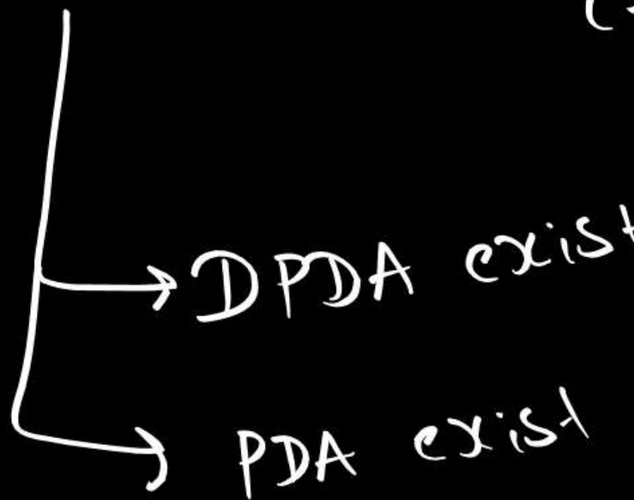


DPDA
PDA



Not DPDA
It is PDA

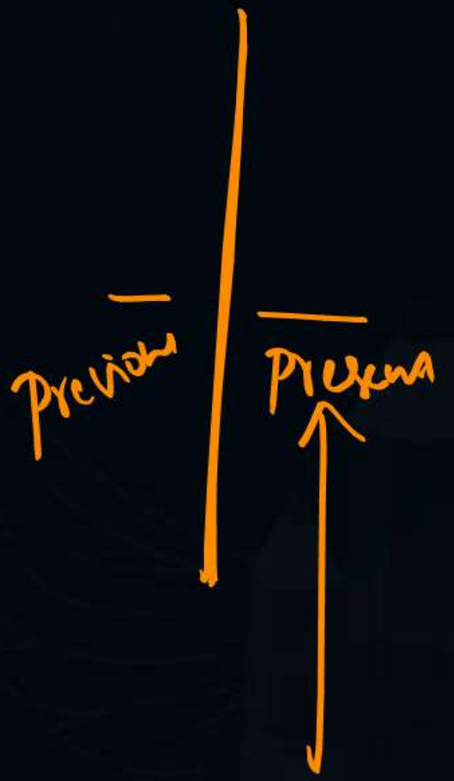
All previous 23 languages are DCFLs (so, CFLs)



*** (24) $\{ww^R \mid w \in \{a,b\}^*\}$

- DPDA not exist
- It has PDA
- It is CFL but not DCFL

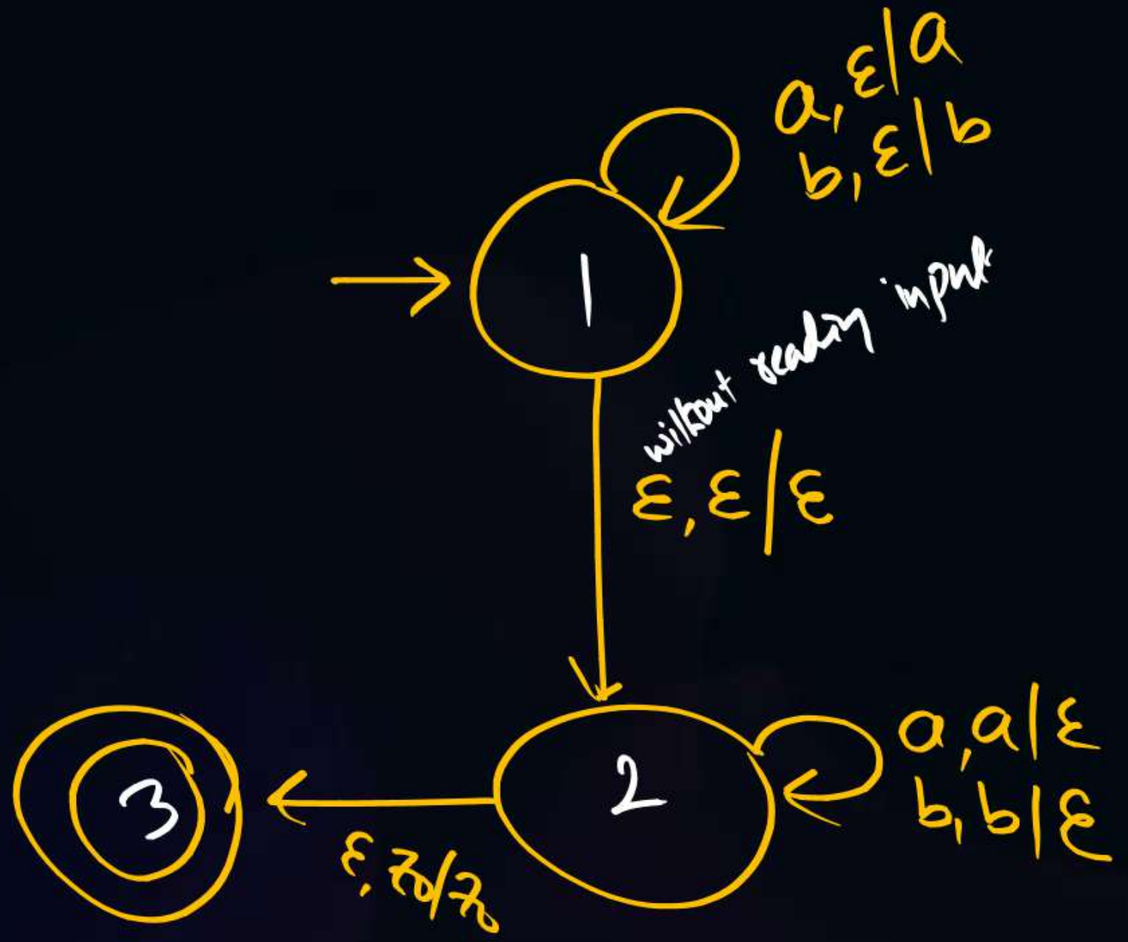
$\{ww^R / w \in \{a,b\}^*\} = \{\varepsilon, aa, bb, abba, aaaa, bbbb, baab, \ldots\}$



If present = previous (top)

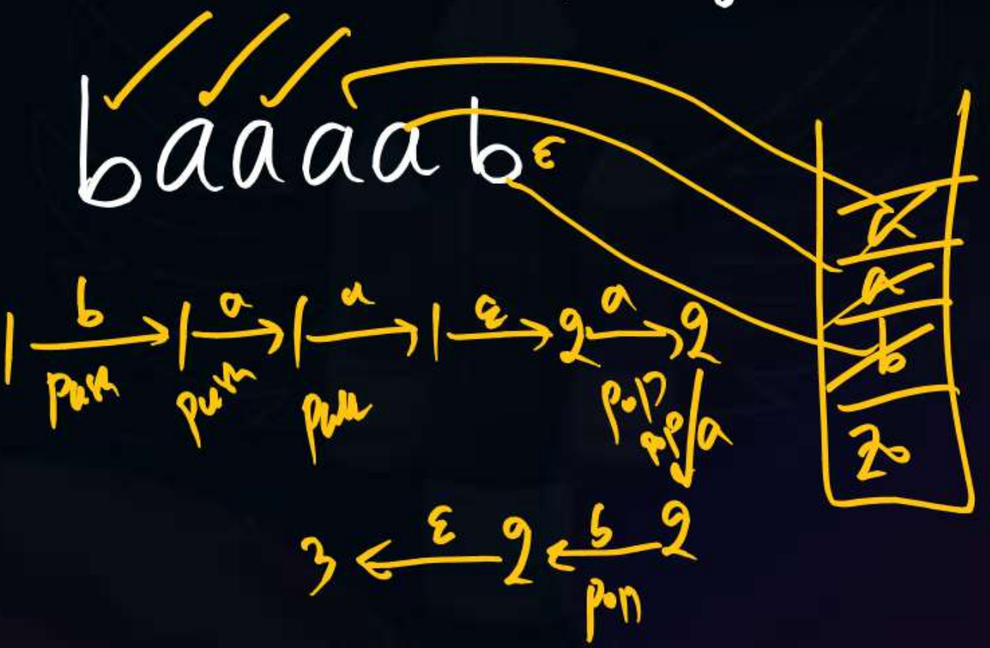
w^R may begin OR w^R may not begin

(24) $\{ww^R \mid w \in \{a,b\}^*\} = \{\varepsilon, aa, \ldots$



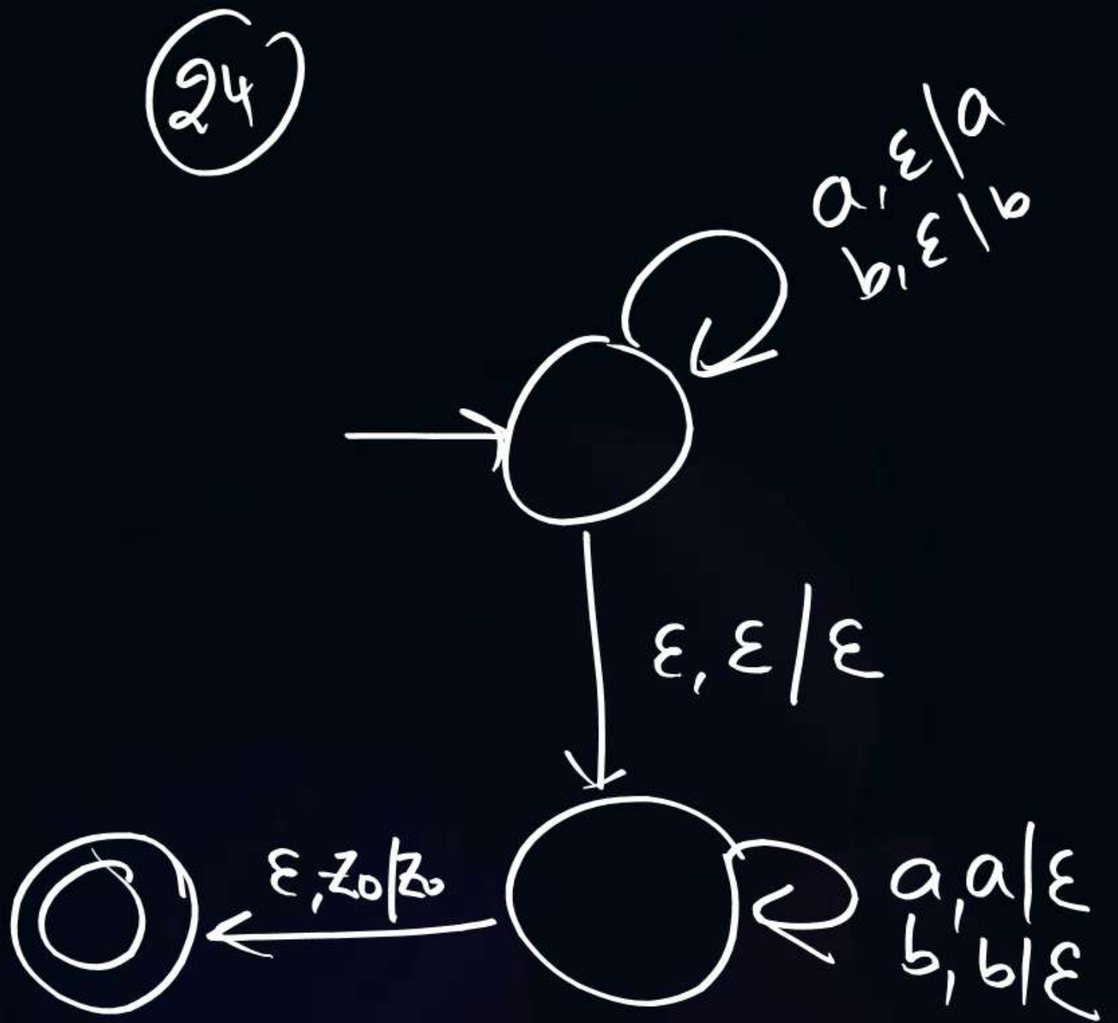
If string is valid,

Can we find a path that halts at final ?



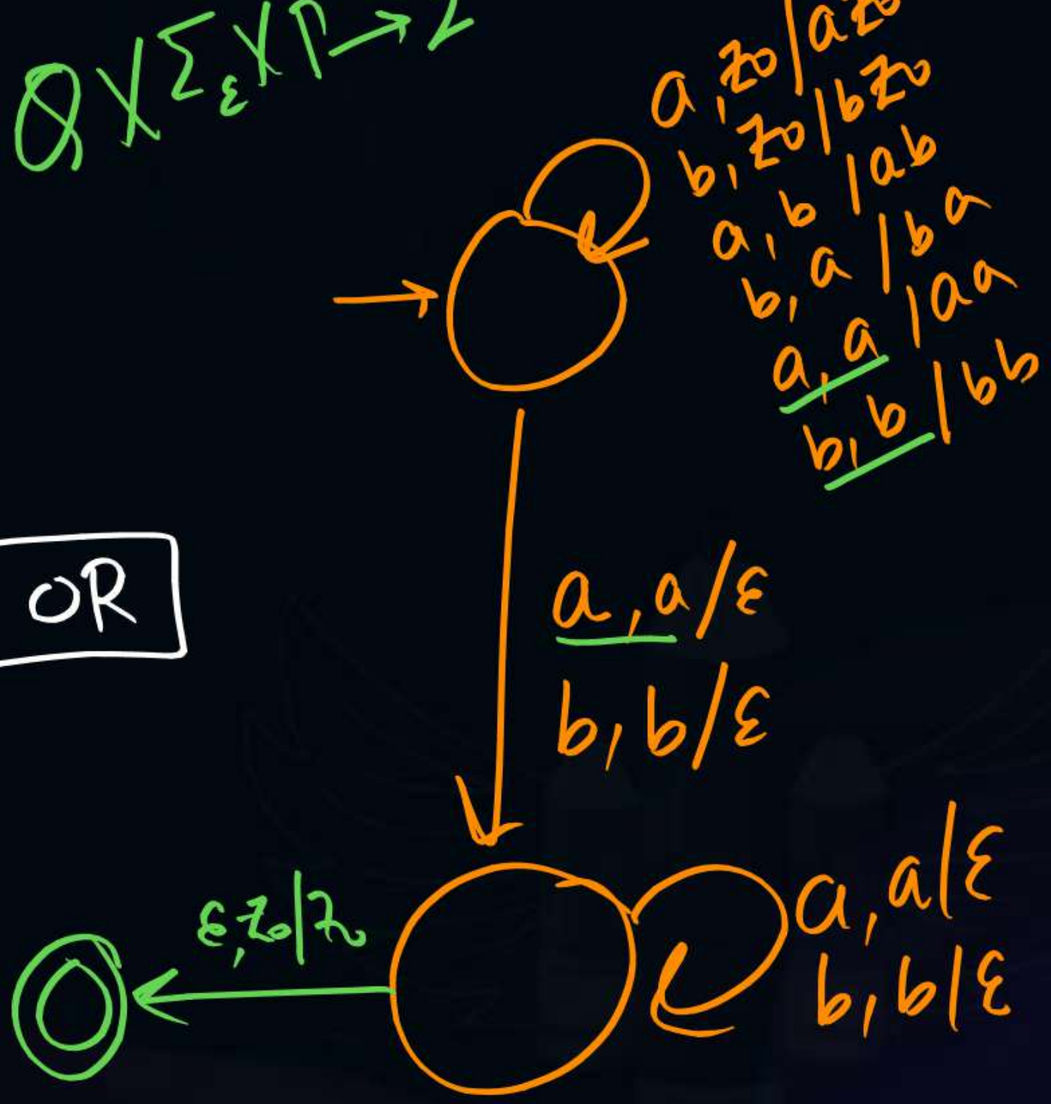
Push Down Automata

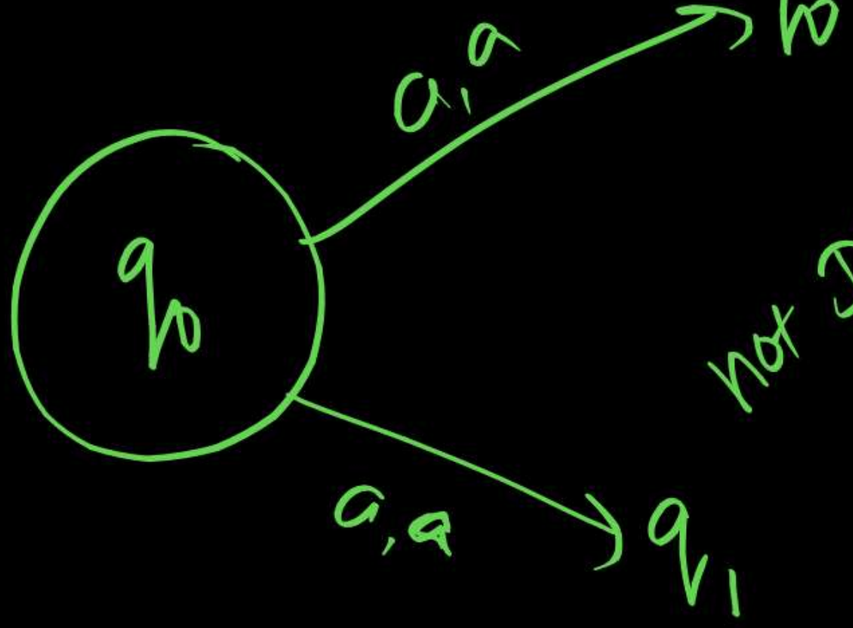
(24)



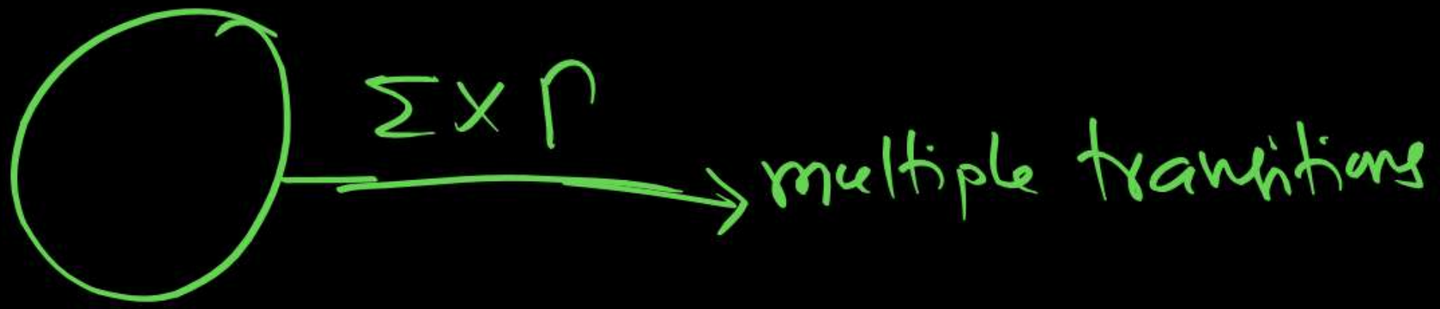
OR

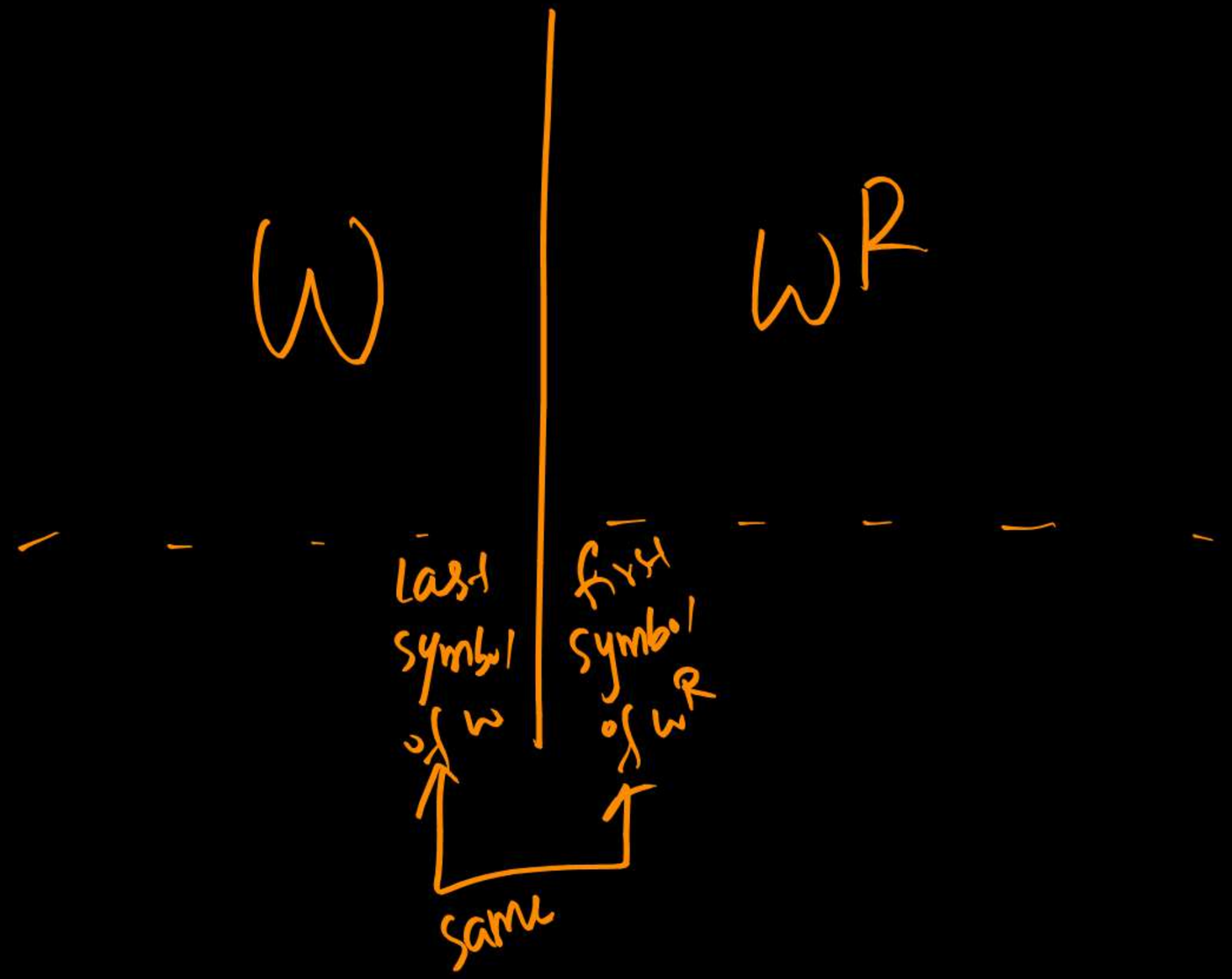
$Q \times \Sigma_\varepsilon \times \Gamma^* \to 2^{Q \times \Gamma^*}$





not DPDA




W
WR
Last symbol of W
First symbol of WR
same

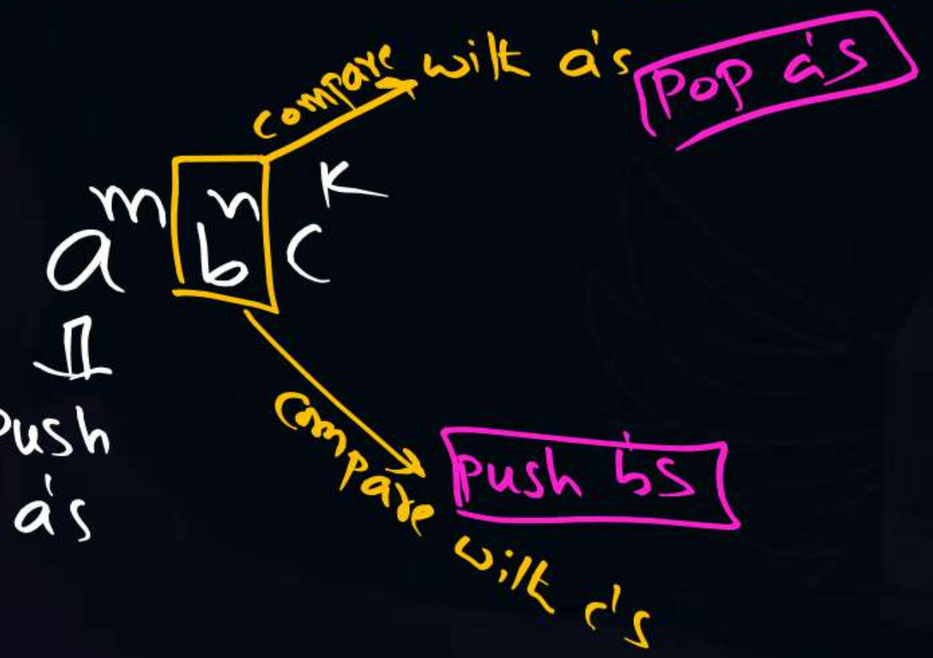
H.W ② $L = \{ a^m b^n c^K \mid m=n \text{ or } n=K, \; m,n,K \geq 1 \}$

$= \{ a^n b^n c^+ \mid n \geq 1 \} \cup \{ a^+ b^n c^n \mid n \geq 1 \}$

→ CFL but not DCFL

→ DPDA not exist

→ But PDA exist



DPDA exist

(26) $\{a^n b^n c^+ \mid n \geq 1\}$

(27) $\{a^+ b^n c^n \mid n \geq 1\}$ H.W.

(28) $\{a^n b^+ c^n \mid n \geq 1\}$

2 mins Summary

Topic

PDA ✓

DPDA ✓

Next: Identifying CFLs & DCFLs

Closure properties.

THANK - YOU