CS & IT

Algorithms

DPP: 1

Analysis of Algorithms

Q1 Sort the functions in ascending order of asymptotic(big-O) complexity.
$f_1(n) = n$, $f_2(n) = 80$, $f_3(n) = n^{\log n}$, $f_4(n) = \log\log^2 n$, $f_5(n) = (\log n)^{\log n}$
(A) $f_2(n), f_4(n), f_1(n), f_5(n), f_3(n)$
(B) $f_2(n), f_1(n), f_4(n), f_5(n), f_3(n)$
(C) $f_2(n), f_1(n), f_4(n), f_3(n), f_5(n)$
(D) $f_1(n), f_1(n), f_4(n), f_3(n), f_2(n)$

Q2 Consider two function $f(n) = 10n + 2\log n$ and $g(n) = 5n + 2(\log n)^2$, then which of the following is correct option?
(A) $f(n) = \theta(g(n))$
(B) $f(n) = O(g(n))$
(C) $f(n) = \omega(g(n^2))$
(D) None of the above

Q3 Consider two function $f(n) = \sqrt{n}$ and $g(n) = n \log n + n$ then $f(n)/g(n)$ is equivalent to how many of the following given below? _____.
(a) $o(n^{-1/2})$
(n) $O(n^{-1/2})$
(c) $\Omega(1/\log n)$
(d) $\theta(n^{-1/2})$

Q4 Consider the following C-code

```
void foo (int x)
  {
     int a = 1;
     if (n = = 1)
     return;
for (a=1; a ≤ n; a++)
     {
        printf("GATEWALLAH");
        break;
     }
  }
```

What is the worst time complexity of above program?
(A) O(1) (B) O(n)
(C) O (log n) (D) $O\sqrt{n}$

Q5 Consider the following asymptotic functions :
$f1 = 2^n$
$f2 = 1.001^n$
$f3 = e^n$
$f4 = n!$
Which of the following is correct increasing order of above functions?
(A) f3, f4, f1, f2 (B) f2, f4, f1, f3
(C) f3, f2, f1, f4 (D) f2, f1, f3, f4

Q6 Consider the following functions
$f_1(n) = 4^{2^n}$
$f_2(n) = n!$
$f_3(n) = 4^{e^n}$
$f_4(n) = n^{nn}$
Which of the following is/are correct?
(A) $f_1(n) = O(f_2(n))$
(B) $f_1(n) = O(f_4(n))$
(C) $f_1(n) = O(f_3(n))$
(D) $f_2(n) = O(f_3(n))$

Q7 Consider two function $f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$ then which of the following is true.
(A) $f_1(n) = O(f_2(n))$
(B) $f_1(n) = \theta(f_2(n))$
(C) $f_1(n) = \omega(f_2(n))$

(D) None of these

Q8 $f(n) = \Sigma_{i=1}^{n} i^3 = x$, choices for x

I. $\theta(n^4)$ II. $\theta(n^5)$

III. $O(n^5)$ IV. $\Omega(n^3)$

(A) I, II, III

(B) II, III, IV

(C) I, II,. III, IV

(D) I, III, IV

Answer Key

Q1	(A)
Q2	(A, B)
Q3	2
Q4	(A)
Q5	(D)
Q6	(B, C, D)
Q7	(C)
Q8	(D)

Hints & Solutions

Q1 Text Solution:

80 < n

log $\log^2 n$ < n

put n = 10^{100}

$\log(\log n)^2 = 10^{100}$

$\log(100)^2 < 10^{100}$

$4 < 10^{100}$

$n < n^{\log n}$

taking log on both side

logn < lognlogn

we know that $(\log n)^2 > \log n$

now, $(\log n)^{\log n} < n^{\log n}$

as we can see that logn in LHS and n on RHS.

n (logn)$^{\log n}$

taking log on both sides

logn < logn*log logn

From above we conclude that growth of log*logn is higher than 1.

∴ option (a) is correct.

Q2 Text Solution:

f(n) = 10n + 2 logn = O(n)

$g(n) = 5n + 2(\log n)^2 = O(n)$

f(n) = O(g(n))

g(n) = O(f(n))

f(n) = g(n)

As we know that if θ is possible then O, Ω is possible.

Hence option a, b are correct.

Q3 Text Solution:

$f(n) = \sqrt{n}$

g (n) = n log n + n

$\frac{f(n)}{g(n)} = \frac{\sqrt{n}}{n(1+\log n)} = \frac{1}{\sqrt{n}(1+\log n)} = \frac{1}{\sqrt{n}+\sqrt{n}\log n}$

Option (a)

f(n) < c.g (n) - o

f(n) ≤ c.g (n) – O

$0\left(n^{\frac{-1}{2}}\right) = \frac{1}{\sqrt{n}}$

f(n) g(n)

$\frac{1}{\sqrt{n}+\sqrt{n}\log n} = o\left(\frac{1}{\sqrt{n}}\right)$ True

Option (b)

- If o possible then O also possible, but vice versa is not possible.
- If w is possible then Ω also possible, but vice versa is not possible.

∴ Option i, ii are true.

Hence (2) is the correct answer.

Q4 Text Solution:

If we see carefully, loop will execute only one time because of break statement, therefore time complexity will be O(1)

Q5 Text Solution:

By observing options and given options, we can conclude that-

$f_4 > f_3 > f_1 > f_2$ increasing order is $f_2\ f_1\ f_3\ f_4$.

∴ Option (d) is correct.

Q6 Text Solution:

The increasing order of given functions is f2, f2, f3, f4.

Therefore (b,c,d) are correct.

Q7 Text Solution:

$f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$

$n^{2^n} = n^{n^2}$

Taking log on both side

$2^n \log n$ $n^2 \log n$

as we can see that

2^n has more growth rate than

n^2 ∴ we conclude

$f_2(n) < O(f_1(n))$ or $f_1(n) = \omega(f_2(n))$

∴ (c) is correct.

Q8 Text Solution:

$$\sum_{i=1}^{n} = \left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{n^2(n+1)^2}{4}$$

$$= \frac{n^2[n^2+2n+1]}{4}$$

$$f(n) = \frac{n^4+2n^3+n^2}{4}$$

$O(n^4)$ $\Omega(n^4)$