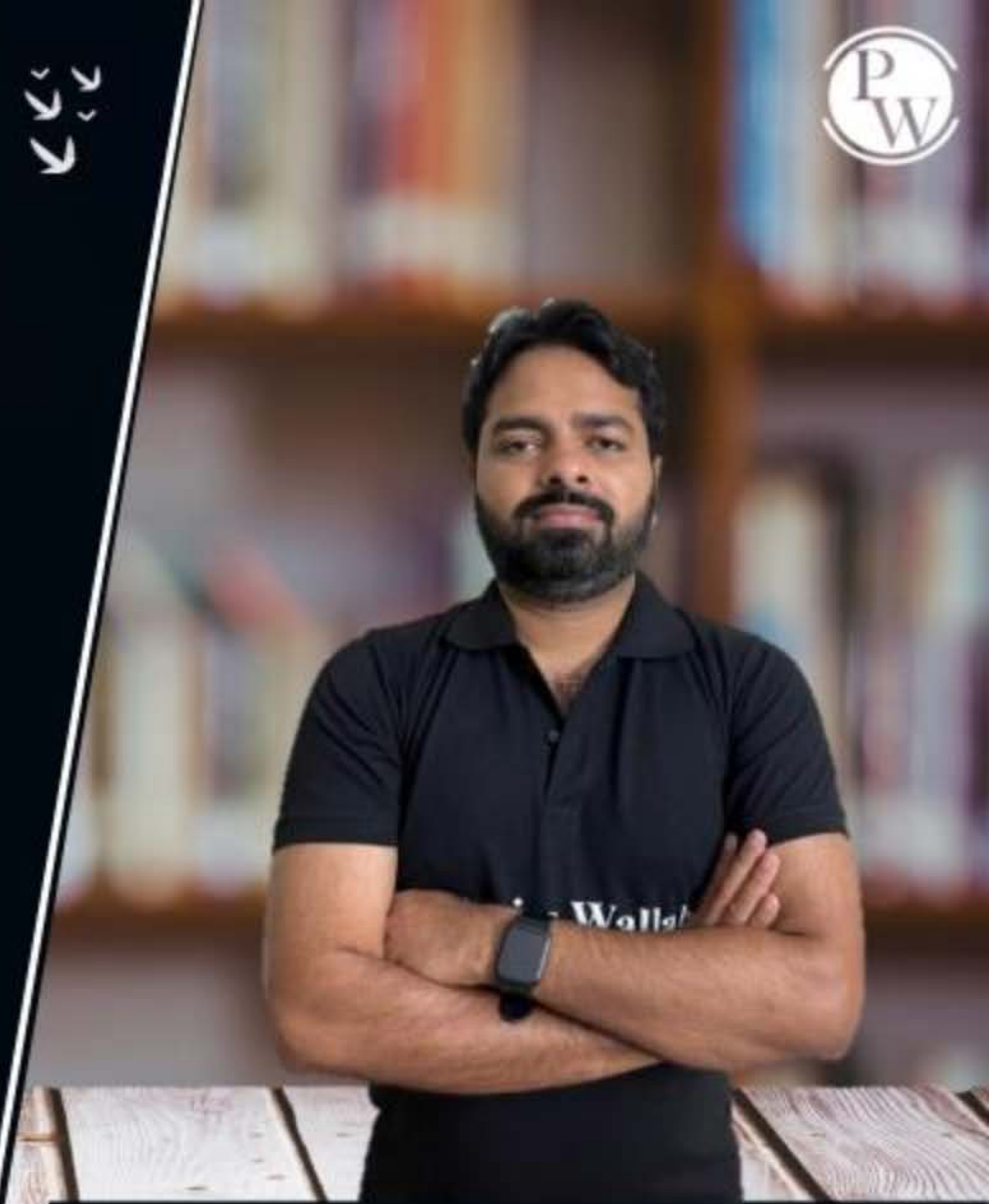
# Computer Science

## Theory of Computation

### Turing Machine

Lecture No.- 2



Mallesham Devasane Sir

# Recap of Previous Lecture

Topic

Turing Machine

# Topics to be Covered

Topic TM ✓

Topic LBA Vs HTM Vs TM ←

Topic TM Construction ✓

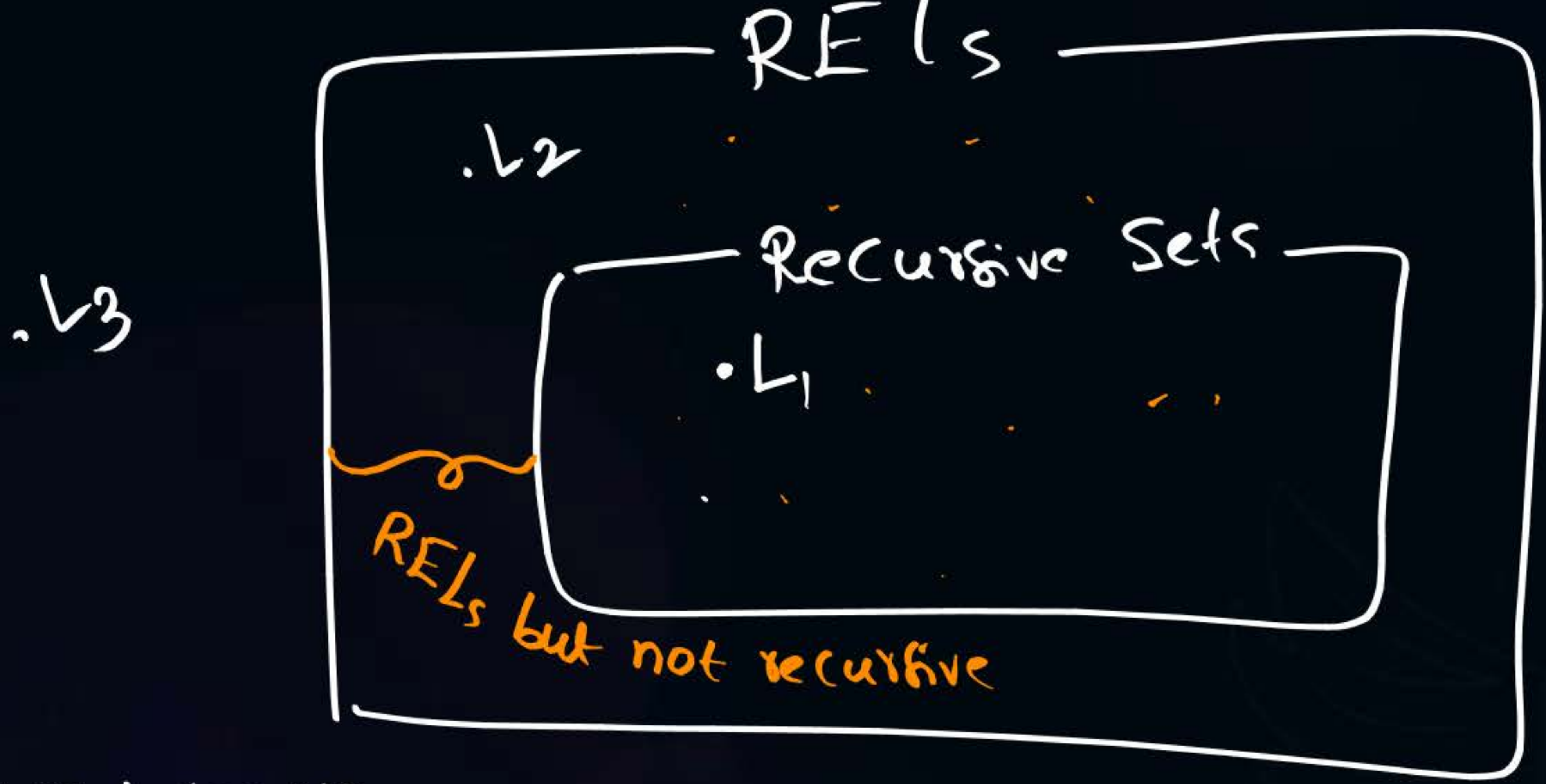
Topic Recursive Vs REL ←

Topic Closure properties ←

# Recursive Language Vs REL

| Recursive Language | RE Language |
|---|---|
| HTM exist<br>(TM that always halts) | TM exist |
| Valid ⇒ Halts at final<br>Invalid ⇒ Halts at nonfinal | Valid strings ⇒ logic exist<br>Invalid strings ⇒ logic may or may not exist |

# Recursive Language Vs REL



$L_1$ is Recursive

$L_1$ is REL

$L_2$ is REL

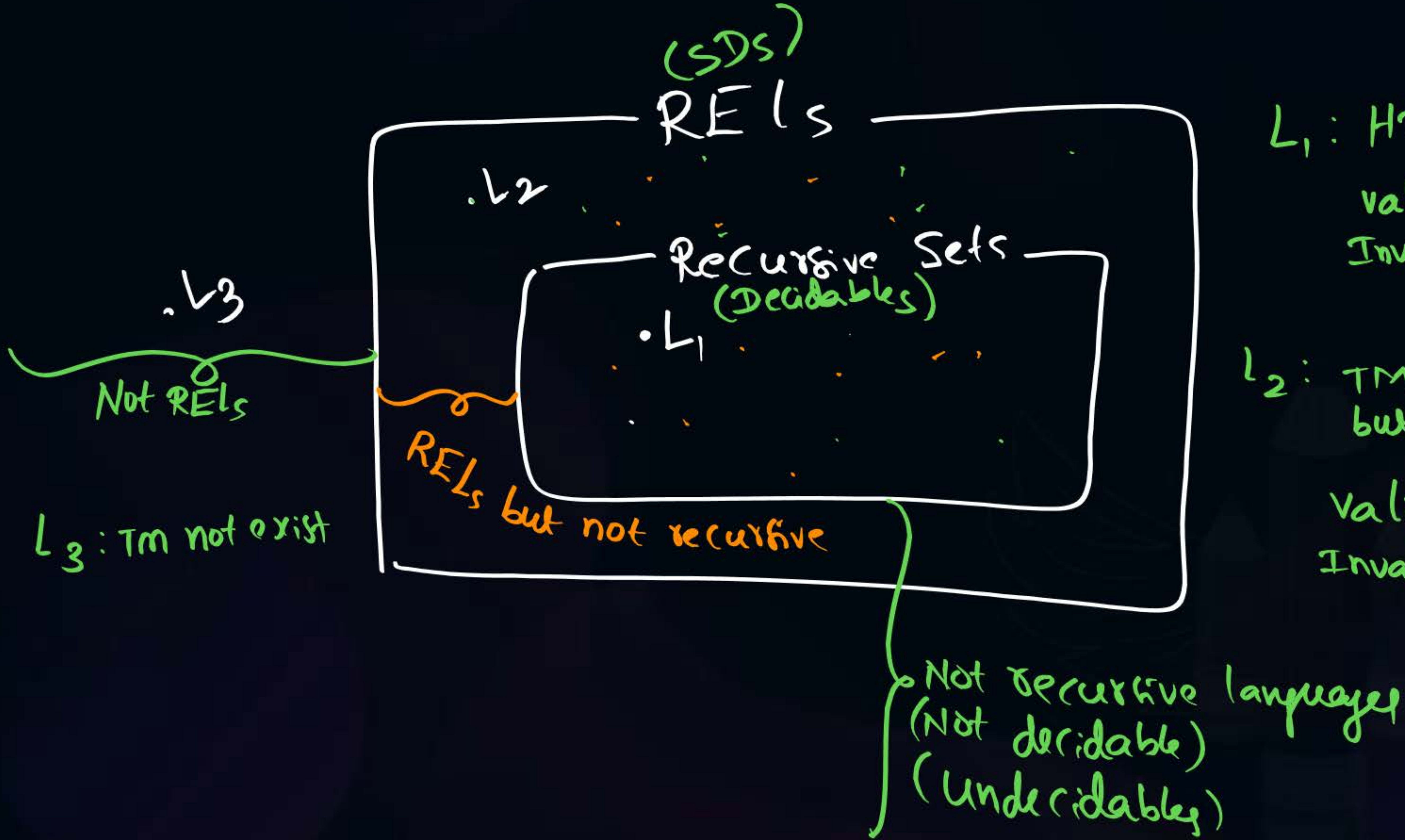
$L_2$ is REL but not rec

$L_2$ is not recursive

$L_3$ is not recursive

$L_3$ is not REL

# Recursive Language Vs REL



$L_3$ : TM not exist

$L_1$ : HTM exist

valid ⇒ logic ✓

Invalid ⇒ logic ✓

$L_2$ : TM exist
but HTM not exist

Valid ⇒ logic ✓

Invalid ⇒ logic not exist

# Recursive Language Vs REL

## CSL

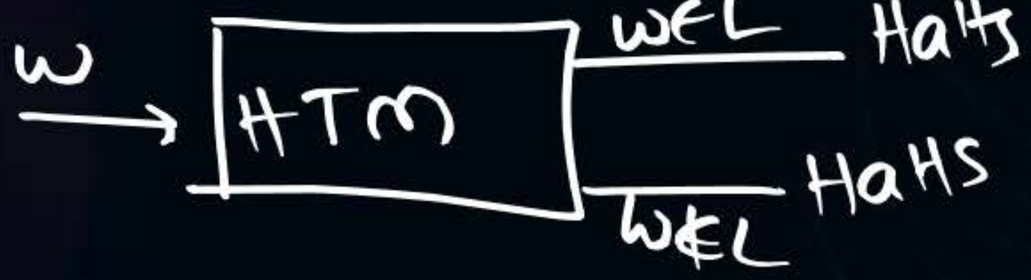
Linear Bound Automata exist

↳ HTM that uses linearly bounded tape

## Recursive Set

HTM exist

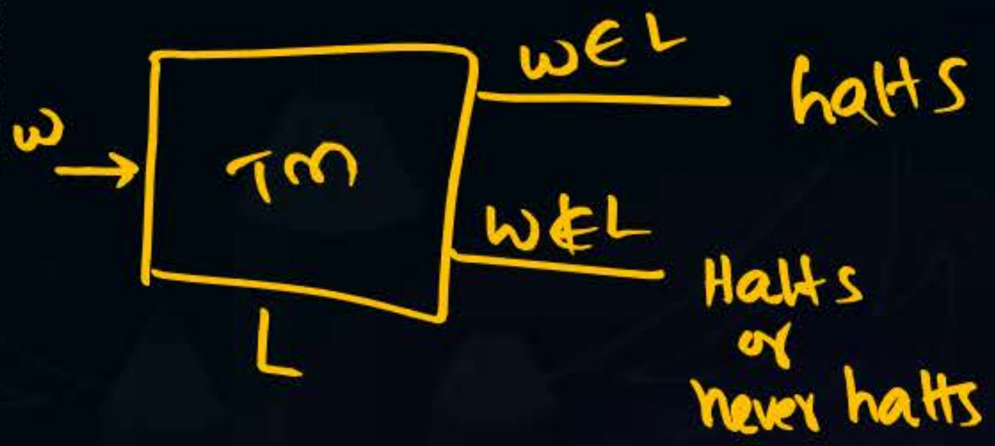
↳ TM that always halts



logic exist for both valid & invalid

## RE Set

TM exist



Logic exist for valid

Logic may or may not exist for invalid

# Closure properties for Recursive language

1. Union
2. Intersection
3. Complement

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4. Difference
5. Concatenation
6. Reversal
7. Kleene star
8. Kleene plus

~~9~~ Subset

10. prefix
11. suffix
12. substring

~~13~~ $f(L)$

~~14~~ $h(L)$

15. ε-free $h(L)$
16. $h^{-1}(L)$
17. Finite $\cup$
18. '' $\cap$
19. '' $-$
20. '' $\cdot$
21. '' $\subseteq$
22. '' $f$

~~23 to 28~~ : Infinite $(\cup, \cap, -, \cdot, \subseteq, f)$

For Recursive language

↳ Remember "Not closed"

↳ Subset

$f(L)$

$h(L)$

Infinite $(\cup, \cap, -, \cdot, \subseteq, f)$

SSHI$_{\text{All}}$

Subset

Substitution

Homomorphism

# Recursive Language Vs REL

## Closure properties for RELs

1 Union

2 Intersection

~~3~~ Complement

~~4~~ Difference

5 Concatenation

6 Reversal

7 Kleene star

8 Kleene plus

~~9~~ Subset

10 prefix

11 suffix

12 substring

13 $f(L)$

14 $h(L)$

15 $\varepsilon$-free $h(L)$

16 $h^{-1}(L)$

17 Finite $\cup$

18 " $\cap$

~~19~~ " $-$

20 " $\cdot$

21 " $\subseteq$

22 " $f$

~~23 to 28~~ : Infinite $(\cup, \cap, -, \cdot, \subseteq, f)$

For RELs :

↳ Remember "not closed"

- → Complement
- → Difference
- → Finite Difference
- → Subset
- → Infinite $(\cup, \cap, -, \cdot, \subseteq, f)$

# Recursive Language Vs REL

Note: (Regular closures)

I) Recursive Set $\cup$ / $\cap$ / $-$ Regular lang $\Longrightarrow$ Always Recursive (may or may not be regular)

II) REL $\cup$ / $\cap$ / $-$ Regular $\Longrightarrow$ Always REL (may or may not be reg)

# Recursive Language Vs REL

Rec → Recursive

RE → REL

1) $Rec_1 \cup Rec_2 \Rightarrow$ Recursive

2) $Rec_1 \cap Rec_2 \Rightarrow$ Recursive

3) $\overline{Rec} \Rightarrow$ Recursive

4) $RE_1 \cup REL_2 \Rightarrow$ REL

5) $RE_1 \cap RE_2 \Rightarrow$ REL

*** 6) $\overline{RE}$ ⟹ May or may not be RE

⟹ either recursive or not REL

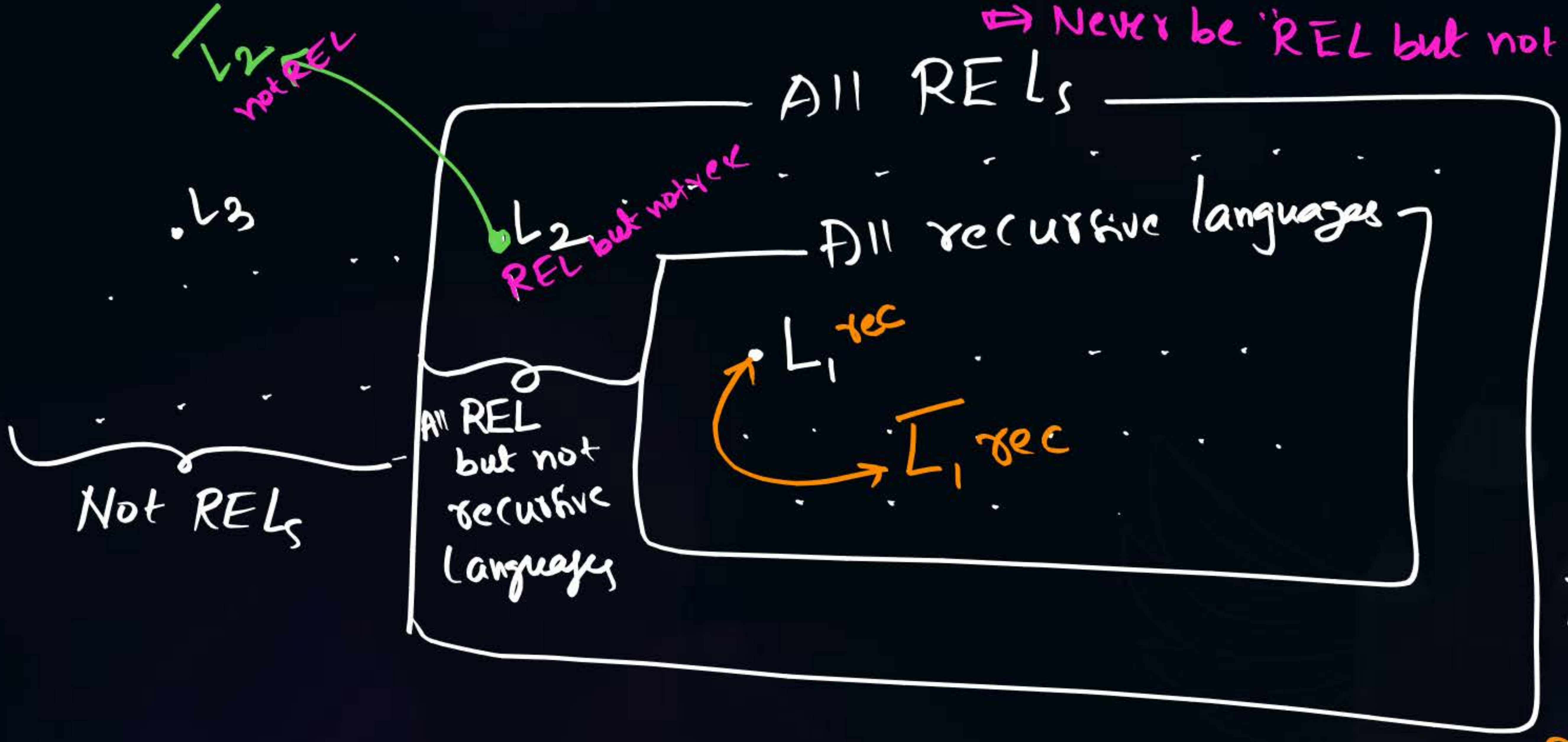
⟹ Never be "RE but not rec"

# Recursive Language Vs REL

$\overline{REL}$ ⇒ Need not be REL

⇒ either Recursive or not REL

⇒ Never be "REL but not recursive"



$\overline{REL}$ ⇒ Need not be REL

⇓

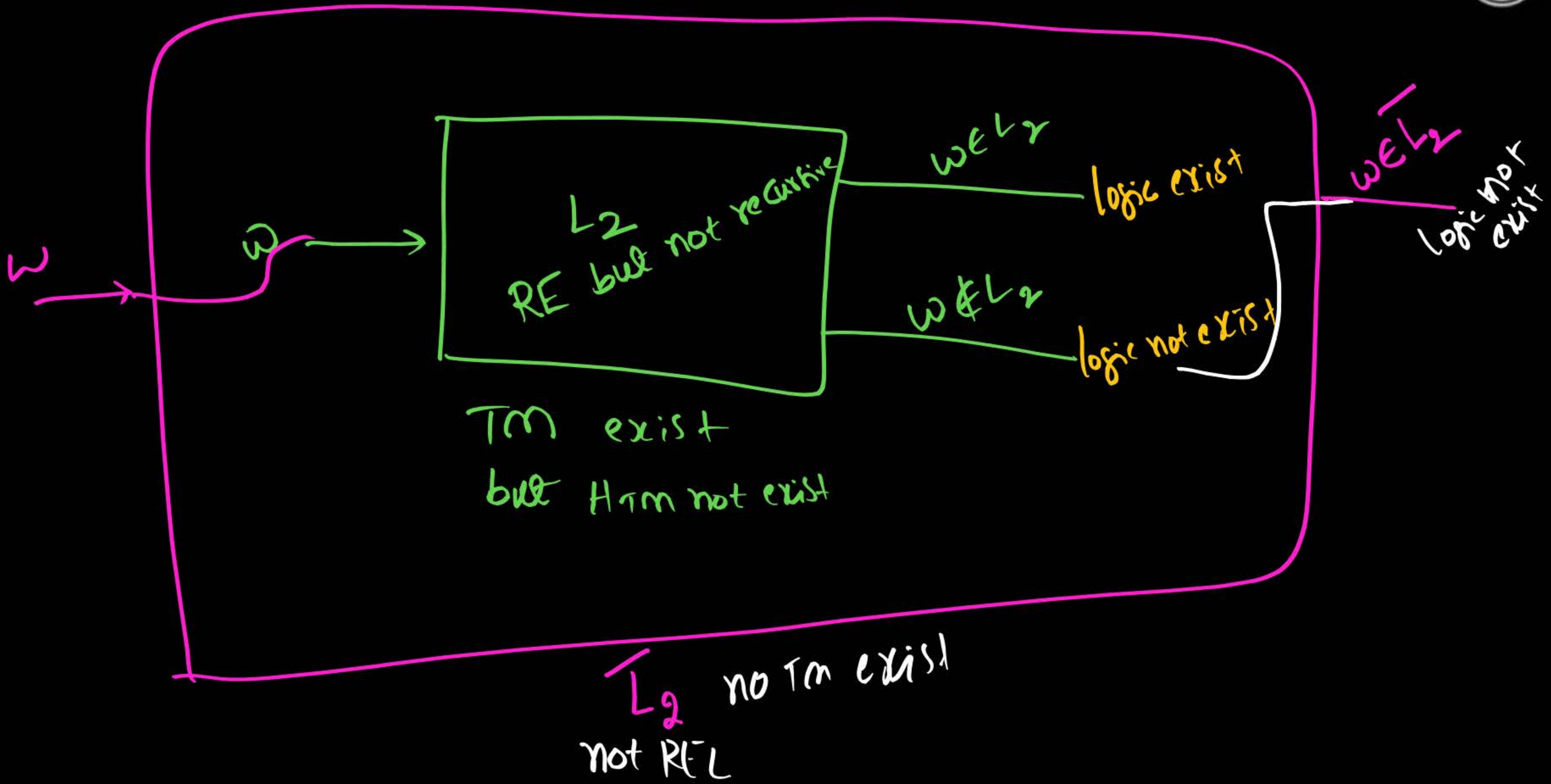
$\overline{L_1 \text{ or } L_2}$

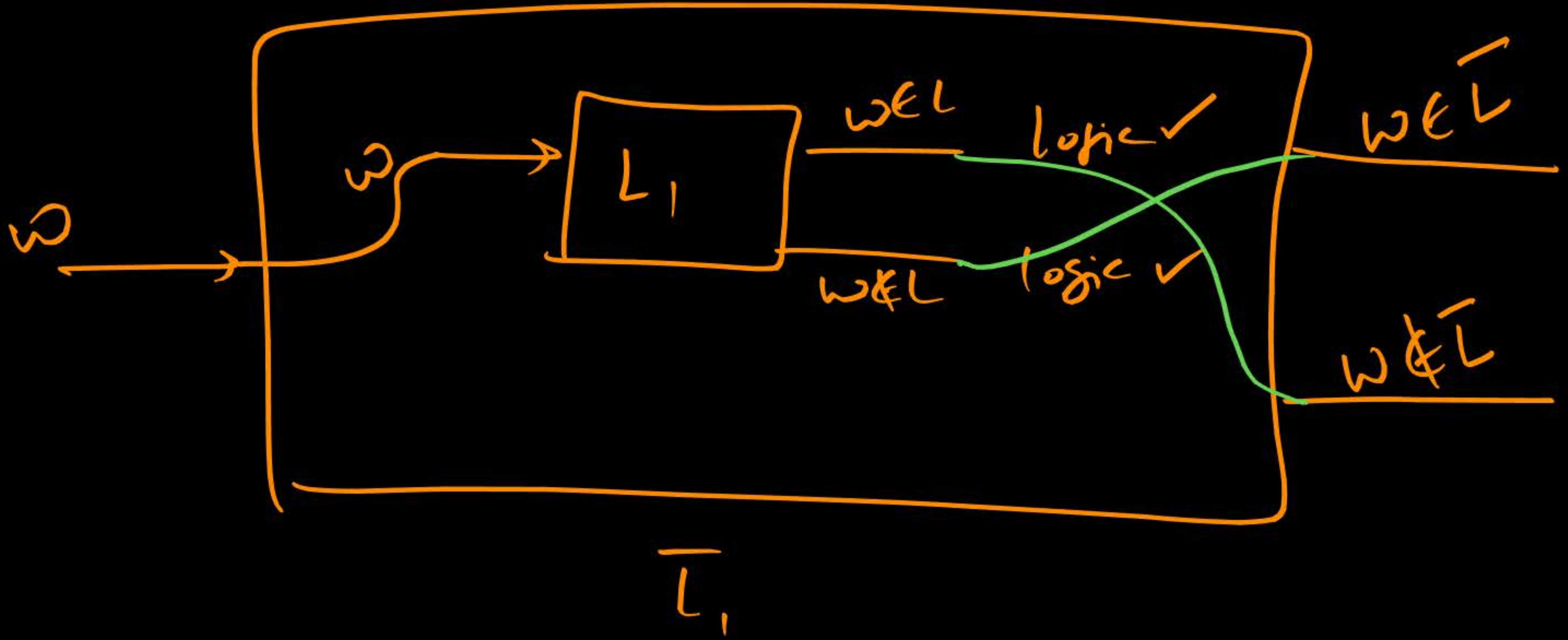
⇓

$\overline{L_1}$ , $\overline{L_2}$

$\overline{L_1}$ → Recursive

$\overline{L_2}$ → Not REL


w
w
$L_2$
RE but not recursive
TM exist
but HTM not exist
$w \in L_2$
logic exist
$w \notin L_2$
logic not exist
$w \in \overline{L_2}$
logic not exist
$\overline{L_2}$ no Tm exist
not REL



$\overline{\text{Rec}}$ ⟹ always Recursive

# Recursive Language Vs REL

Recursive language (L) (Decidable)

≅

HTM exist

≅

TM can enumerate L in lexicographic order (effective)

≅

Tm Decidable language

≅

Halting program exist

≅

Algorithm exist

RE language (L) (Semidecidable)

≅

Tm exist

≅

Tm can enumerate L

≅

Enumerable Set (Recognitable)

≅

program exist

# Recursive Language Vs REL

Turing Machine

$\cong$

Single Tape TM

$\cong$

Multi tape Tm

$\cong$

Multi head Tm

$\cong$

multi head & multi tape Tm

$\cong$

Multi Dimensional tape Tm

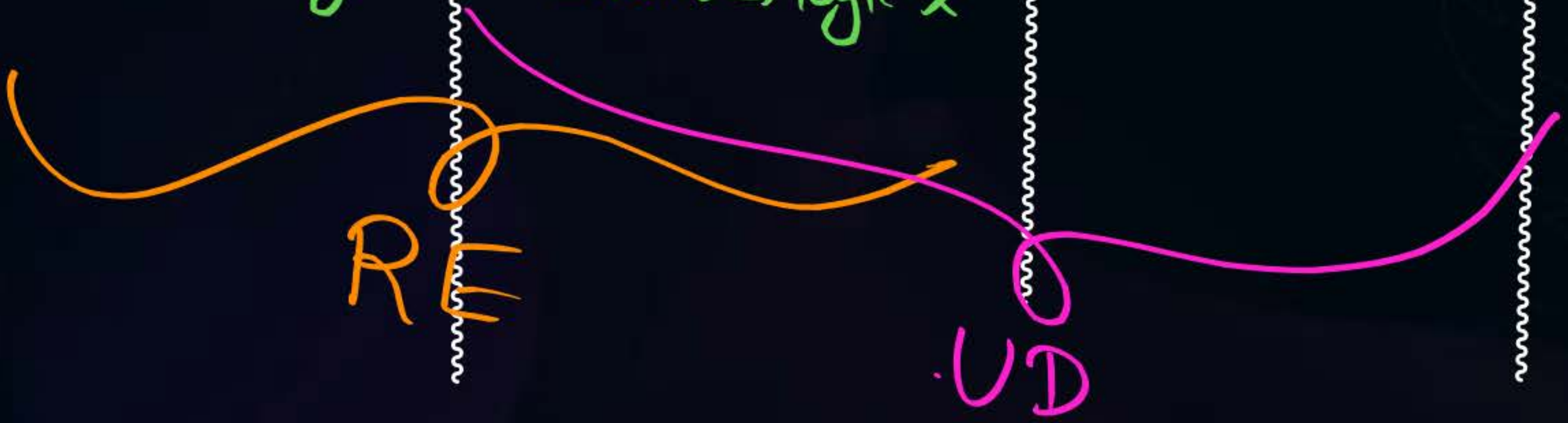
$\cong$ 2 stack PDA

$\cong$ n stack PDA $(n \geq 2)$

$\cong$ Universal Tm

# Recursive Language Vs REL

| (Decidable)<br>Recursive | (SDUD)<br>RE but not Recursive | Not RE | (SD)<br>RE | (not decidable)<br>(UD)<br>Undecidable |
|---|---|---|---|---|
| HTM ✓ | TM ✓<br>HTM X | TM X | TM ✓ | HTM X |
| valid ⇒ logic ✓<br>Invalid ⇒ logic ✓ | valid ⇒ logic ✓<br>Invalid ⇒ logic X | valid ⇒ logic X | valid ⇒ logic ✓ | eilter SDUD<br>or<br>Not RE |



UD ≅ not decidable ≅ not recursive

≅

$\boxed{\text{HTM not exist}}$

≅

|  |  |  |  |
|---|---|---|---|
| Valid | ✓ | × | × |
| Invalid | × | ✓ | × |

logic

|  | D | SDUD | Not RE |  |
|---|---|---|---|---|
| Valid | ✓ | ✓ | ✗ | ✗ |
| Invalid | ✓ | ✗ | ✓ | ✗ |

RE

UD

$\underbrace{\text{RE}}_{\text{SD}}$ but $\underbrace{\text{not recursive}}_{\text{Undecidable}}$

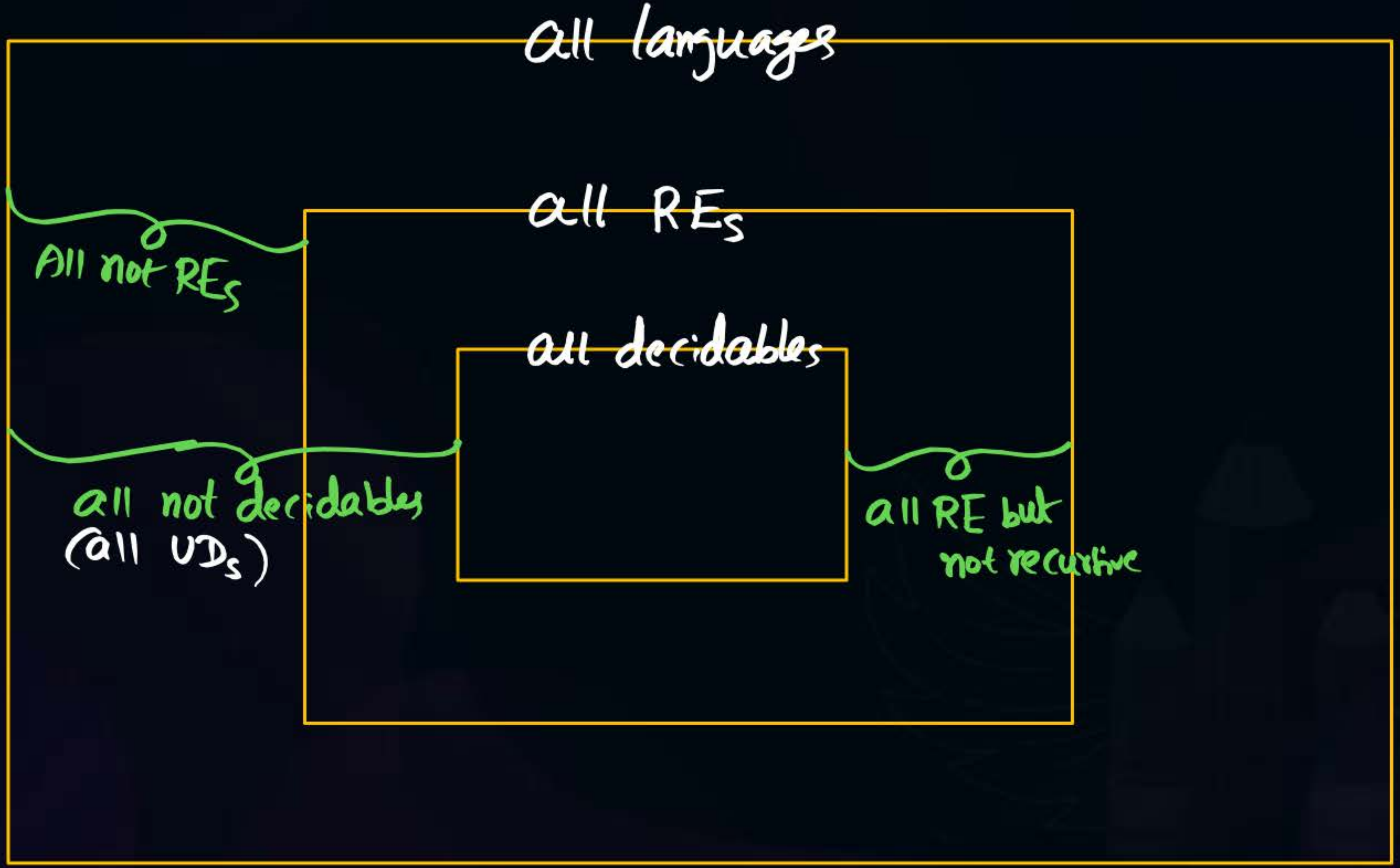
SD and UD

$\boxed{\text{SDUD}}$

RE ≅ SD

Not rec ≅ Not decidable ≅ undecidable

# Recursive Language Vs REL



# 2 mins Summary

Topic

- Closure properties ✓
- Import definitions ✓
- LBA, HTM, TM ✓

# THANK - YOU