Computer Science

Theory of Computation

Context Free Languages

Lecture No.- 4

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Recap of Previous Lecture

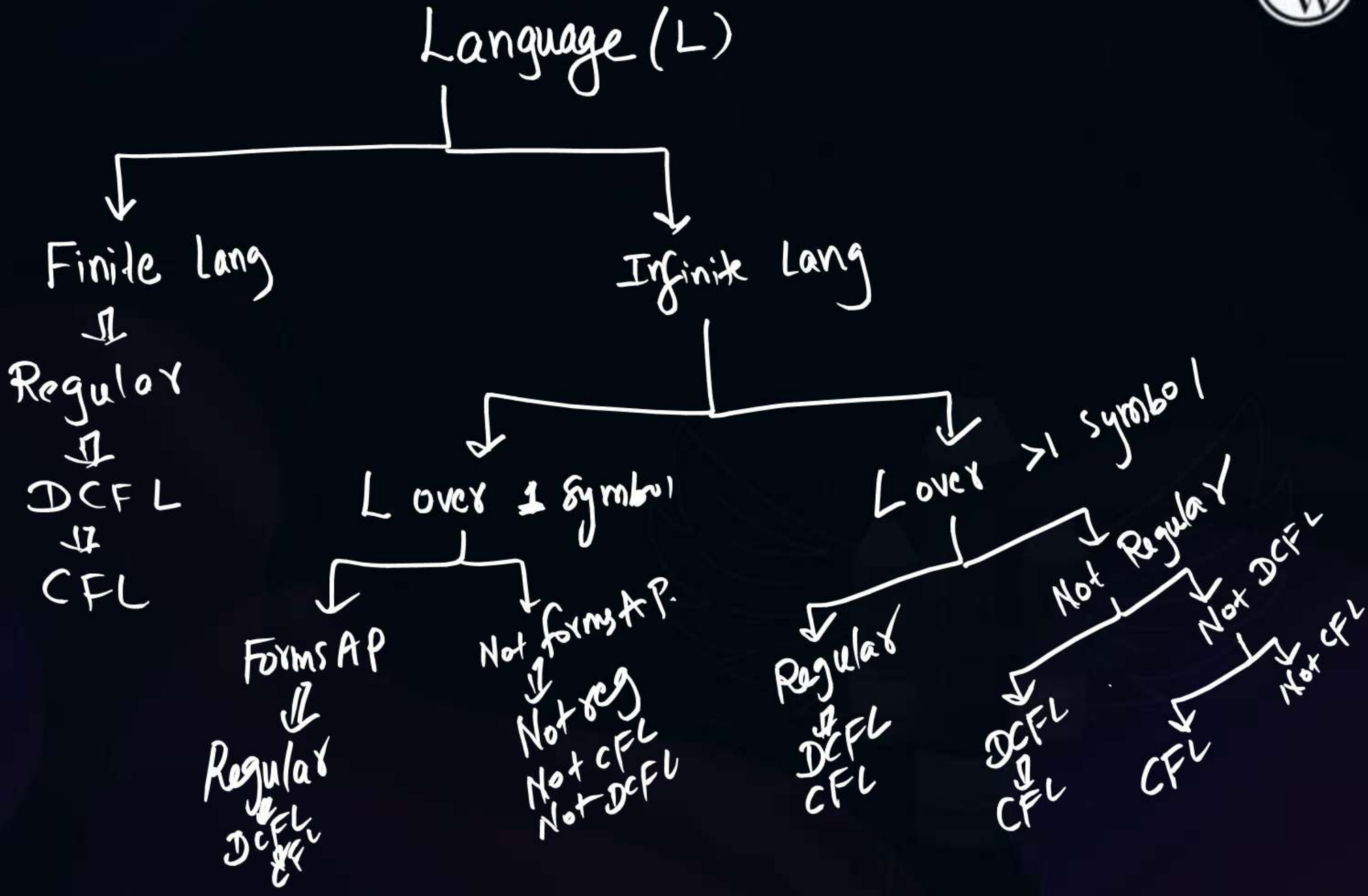
Topic PDA Construction

Topics to be Covered

Topic — Identifying CFLs and DCFLs

Topic — Closure Properties for CFLs and DCFLs

CFLs and DCFLs



CFLs and DCFLs

Regular, DCFL but not reg, CFL but not DCFL, Not CFL.

① $\{a^n b^n \mid n \geq 0\}$ ⇒ DCFL but not regular

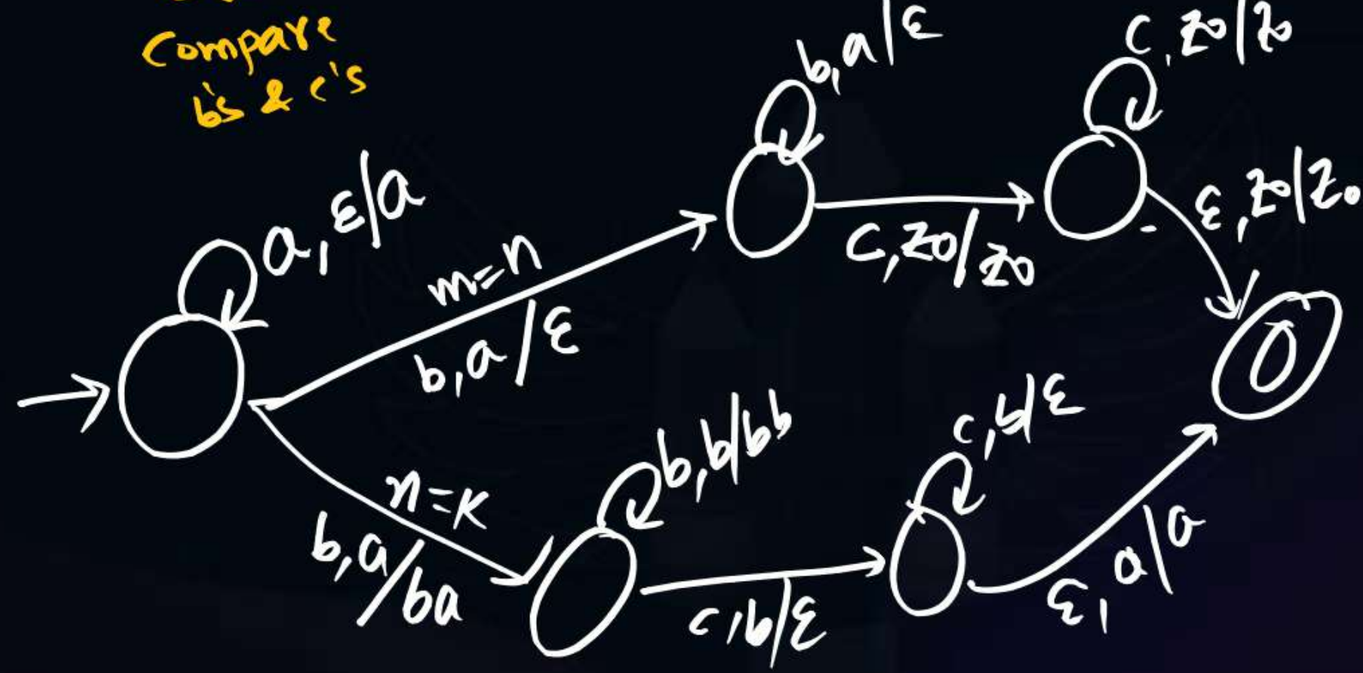
Push a's — Pop a's

② $\{a^m b^n c^k \mid \underbrace{m=n}_{\text{Compare a's \& b's}} \text{ OR } \underbrace{n=k}_{\text{Compare b's \& c's}}, m,n,k \geq 1\}$ is CFL but not DCFL

Push a's

Pop a's $m=n$

Push b's $n=k$



$$\{a^m b^n c^k \mid m=n \text{ OR } n=k, \quad m,n,k \geq 1\}$$

$$\Downarrow \qquad \Downarrow$$

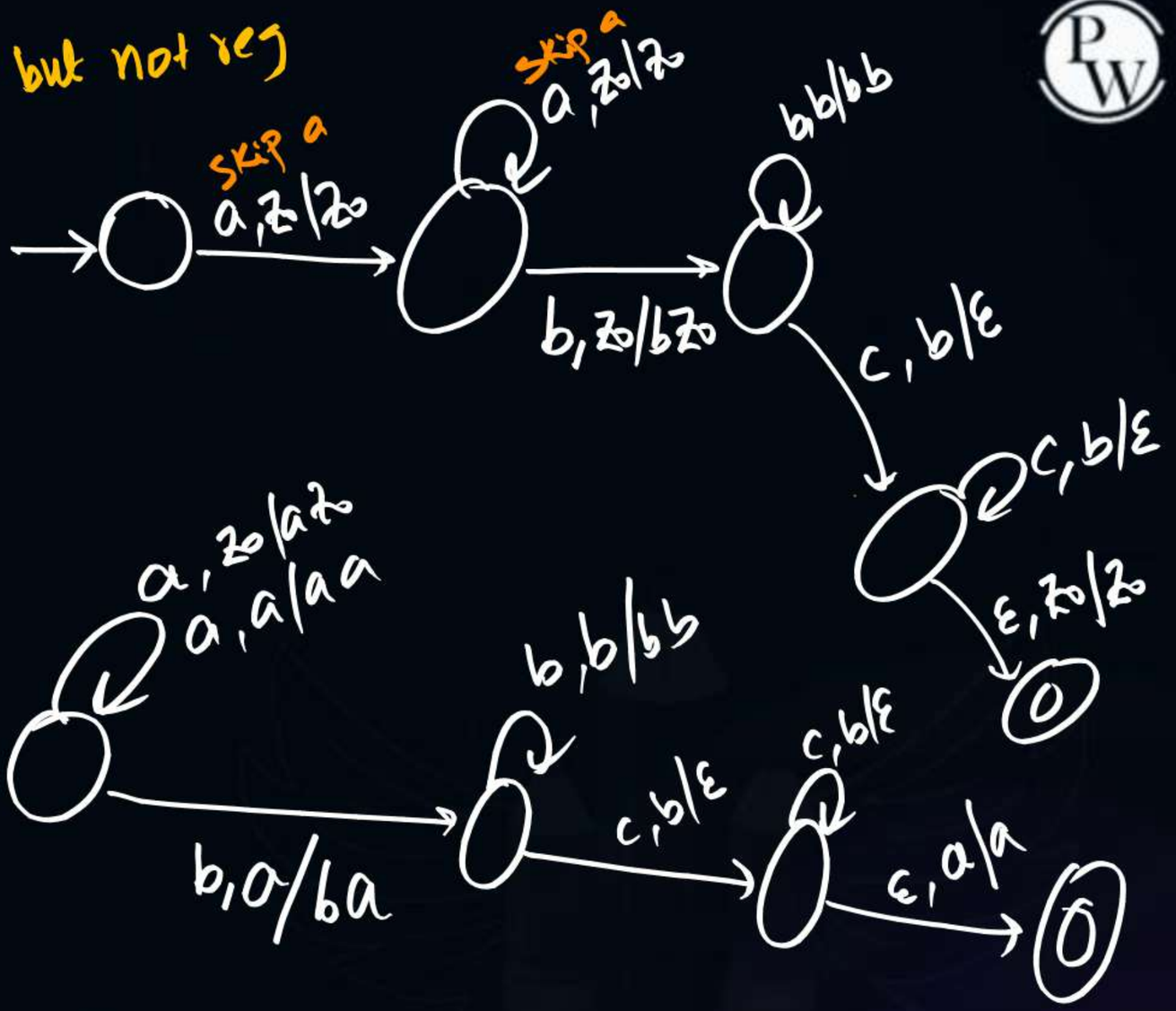
$$\{a^n b^n c^+ \mid n \geq 1\} \cup \{a^+ b^n c^n \mid n \geq 1\}$$

CFLs and DCFLs

(3) $\{a^+ b^n c^n / n \geq 1\}$ ⇒ DCFL but not reg

I II III

Skip | Push b's | Pop b's



A) Finite ✗

B) Infinite ✓

C) Regular ✗

D) Not Regular ✓

E) DCFL ✓

F) DCFL but not reg ✓

G) CFL ✓

CFLs and DCFLs

④ $\{a^m b^n c^{m+n}\}$ ⟹ DCFL but not reg

Push x's | Push x's | Pop x's

⑤ $\{a^m b^{m+n} c^n\} = \{a^m a^m b^n b^n c^n\}$ ⟹ DCFL, not reg

Push a's | Pop a's | Push b's | Pop b's

⑥ $\{\omega \# \omega^R \mid \omega \in \{a,b\}^*\}$ ⟹ DCFL but not reg

Push | Skip | Pop by match

CFLs and DCFLs

(7) $\{w\#w \mid w \in \{a,b\}^*\}$ ⇒ Not CFL

(8) $\{ww \mid w \in \{a,b\}^*\}$

(9) $\{a^n b^n c^n\}$

(10) $\{a^n b^{2n} c^{3n}\}$

(11) $\{a^n b^{100n}\}$ → DCFL, not reg

(12) $\{a^n \underline{b^{n^2}}\}$ → not CFL

not forms A.P.

(13) $\{a^m \underline{b^{n^2}}\}$ ⇒ not CFL

not forms A.P.

CFLs and DCFLs

(14) $\{a^{\text{prime}}\}$

(15) $\{a^{n!}\}$

(16) $\{a^{2^n}\}$

(17) $\{a^{n^2}\}$

(18) $\{a^{n^n}\}$

Not CFL
(All are CSLs)

Note: If Language is over one symbol then FA ≅ PDA.

CFLs and DCFLs

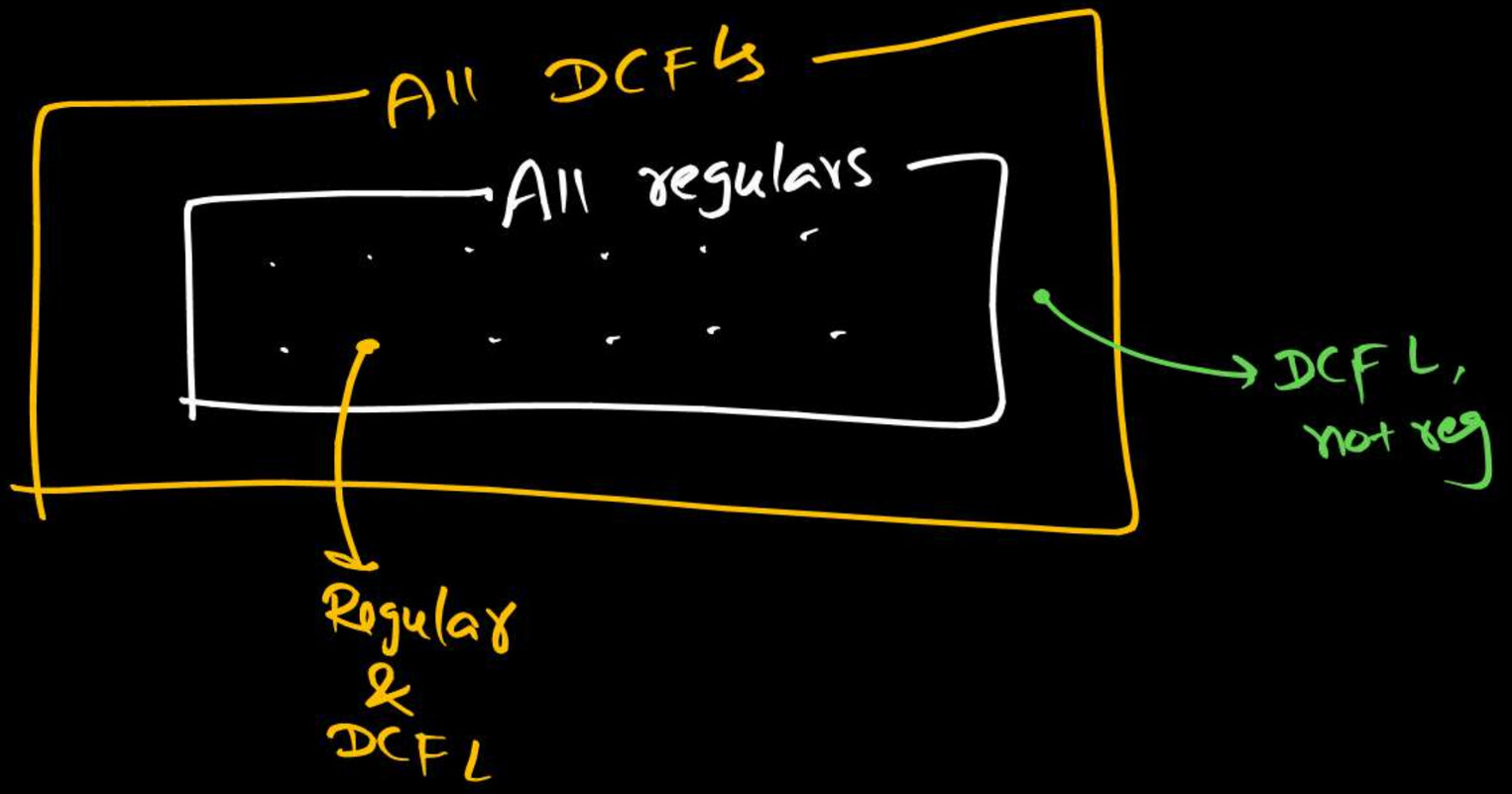
(19) $\{w_1 \# w_2 \mid w_1, w_2 \in \{a,b\}^*\} = (a+b)^* \# (a+b)^*$ → Regular, DCFL, CFL

(20) $\{\underline{w} \# \underline{w} \# \underline{w} \mid w \in \{a,b\}^*\}$ ⟶ not CFL

(21) $\{w_1 \# w_2 \# w_3 \mid w_1, w_2, w_3 \in \{a,b\}^*\}$ ⟶ Regular

(22) $\{a^n \# a^{2n}\}$ ⟶ DCFL, not reg

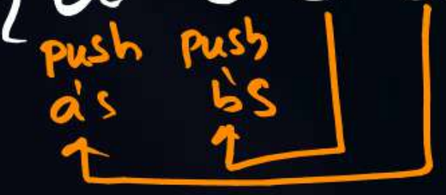
(23) $\{a^{n+1} b^{n+2} c^{n+3}\}$ ⟶ not CFL


All DCFLs
All regulars
DCFL, not reg
Regular & DCFL

CFLs and DCFLs

(24) $\{a^{n+i}\ b^{n+j}\ c^{n+k} \mid n,i,j,k \geq 0\} = a^*b^*c^*$ Regular

(25) $\{a^m\ b^n\ c^n\ d^m\}$ → DCFL, not reg



(26) $\{a^m\ b^n\ c^m\ d^n\}$ → not CFL

(27) $\{a^m\ b^m\ c^n\ d^n\}$ → DCFL, not reg

CFLs and DCFLs

Every Regular is DCFL

Every Regular is CFL

Every DCFL is CFL

(28) $\overline{\{a^n b^n\}}$ is DCFL but not reg

(29) $\{a^n b^n | n \geq 0\}^*$ is DCFL but not reg

(30) $\{a^n b^n\}^{Reversal}$ is DCFL but not reg

(31) $\overline{\{a^n b^n c^n\}}$ is CFL but not DCFL

(32) $\overline{\{ww / w \in \{a,b\}^*\}}$ is CFL but not DCFL

(33) $\overline{\{ww^R / w \in \{a,b\}^*\}}$ is CFL but not DCFL

Closure properties for Regulars

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $\overline{L}$
4. $L_1 - L_2$
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+

~~9~~ Subset(L)

10. Prefix(L)
11. Suffix(L)
12. Substring(L)
13. Quotient(L_1, L_2)
14. $f(L)$
15. $h(L)$
16. $h^{-1}(L)$
17. Finite Union
18. " Intersection
19. " Difference
20. " Concatenation
21. " Subset
22. " Substitution

~~23~~ to ~~28~~ : Infinite($\cup, \cap, -, \cdot, \subseteq, f$)

CFLs and DCFLs

Closure properties for DCFLs : CPIFs

✗ ① $L_1 \cup L_2$

✗ ② $L_1 \cap L_2$

closed ③ $\overline{L}$

✗ ④ $L_1 - L_2$

✗ ⑤ $L_1 \cdot L_2$

✗ ⑥ L^{Rev}

✗ ⑦ L^*

✗ ⑧ L^+

✗ ⑨ Subset(L)

closed ⑩ prefix(L)

✗ ⑪ Suffix(L)

✗ ⑫ Substring(L)

✗ ⑬ Quotient(L_1, L_2)

✗ ⑭ f(L)

✗ ⑮ h(L)

closed ⑯ $h^{-1}(L)$

✗ ⑰ Finite Union

✗ ⑱ " Intersection

✗ ⑲ " Difference

✗ ⑳ " Concatenation

closed ㉑ " Subset

✗ ㉒ " Substitution

✗ ㉓ to ㉘ : Infinite($\cup, \cap, -, \cdot, \subseteq, f$)

Closure properties for CFLs

1. $L_1 \cup L_2$
2. ~~$L_1 \cap L_2$~~
3. ~~$\overline{L}$~~
4. ~~$L_1 - L_2$~~
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+
9. ~~Subset(L)~~
10. Prefix(L)
11. Suffix(L)
12. Substring(L)
13. ~~Quotient(L_1, L_2)~~
14. $f(L)$
15. $h(L)$
16. $h^{-1}(L)$
17. Finite Union
18. ~~" Intersection~~
19. ~~" Difference~~
20. " Concatenation
21. " Subset
22. " Substitution

23. ~~to 26~~: Infinite$(\cup, \cap, -, \cdot, \subseteq, f)$

For Regulars: Remember Not closed

- Subset
- Infinite $(\cup, \cap, -, \cdot, \subseteq, f)$

For DCFLs: Remember closed

- Complement
- prefix
- h^{-1}
- Finite Subset

For CFLs: Remember Not closed:

$L_1 \cap L_2$ $\overline{L}$ $L_1 - L_2$	Subset Quotient	Finite $\cap$ Finite $-$ Infinite $(\cup, \cap, -, \cdot, \subseteq, f)$

CFLs and DCFLs

R_i is Regular

D_i is DCFL

C_i is CFL

① $R \cup D$ is always DCFL (may or may not Reg)

② $R \cup C$ is always CFL

③ $D \cup C$ is CFL (may or may not be DCFL)

④ $R_1 \cup R_2$ is Regular

⑤ $D_1 \cup D_2$ is always CFL (may or may not DCFL)

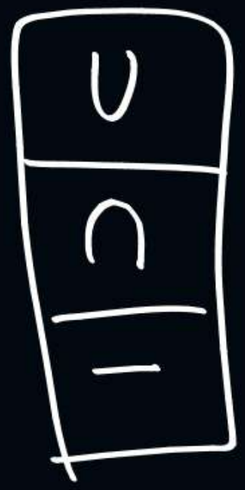
⑥ $C_1 \cup C_2$ is CFL

$L_1 = \{a^n b^{2n}\}$ is DCFL

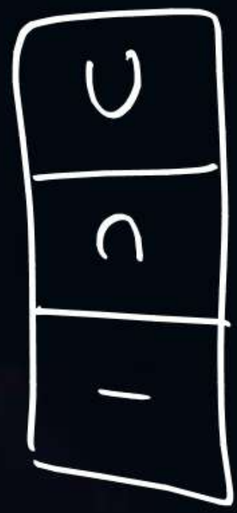
$L_2 = \{a^{2n} b^n\}$ is DCFL

$\{a^i b^j \mid i=2j \text{ or } j=2i\}$ CFL not DCFL

CFLs and DCFLs

Note: I) DCFL 

Regular $\Rightarrow$ Always DCFL
(either Reg or not reg)

II) CFL 

Regular $\not\Rightarrow$ Always CFL
(either reg or not reg)

III) Regular − DCFL ⇒ Always DCFL

IV) Regular − CFL ⇒ Need not be CFL

$L_1 - L_2$

$L_1 \cap \overline{L_2}$

CFLs and DCFLs

I) DCFL $\cap$ Regular $\Rightarrow$ Always DCFL

$a^n b^n \cap a^* b^* \Longrightarrow a^n b^n$ (not reg)

$a^n b^n \cap \phi \Rightarrow \phi$ (reg)

} Always DCFL

II) Set of all DCFLs $\cap$ Set of all regs $\Rightarrow$ Set of all regs

CFLs and DCFLs

① $\{a^n b^n c^*\} \cap \{a^* b^k c^k\} \Rightarrow \{a^n b^n c^n\}$

$\{a^n b^n c^*\}$: $a^n b^n$ DCFL

$\{a^* b^k c^k\}$: $b^k c^k$ DCFL

$\{a^n b^n c^n\}$: Not CFL

② $\{a^n b^n c\} \cap \{a b^k c^k\} \Rightarrow \{abc\}$

$\{a^n b^n c\}$: DCFL

$\{a b^k c^k\}$: DCFL

$\{abc\}$: Finite, Regular, DCFL, CFL

Note: DCFL$_1$ $\cap$ DCFL$_2$ $\Rightarrow$ Need not be DCFL (always CSL)

I) $DCFL_1 \cap DCFL_2 \Rightarrow$ Always CSL

(need not be CFL)

(need not be DCFL)

(need not be regular)

II) $CFL_1 \cap CFL_2 \Rightarrow$ Always CSL

(need not be CFL)

(need not be DCFL)

(need not be Reg)

2 mins Summary

Topic

- Identifying CFLs, DCFLs
- Closure properties for CFLs, DCFLs

Next: TM

THANK - YOU