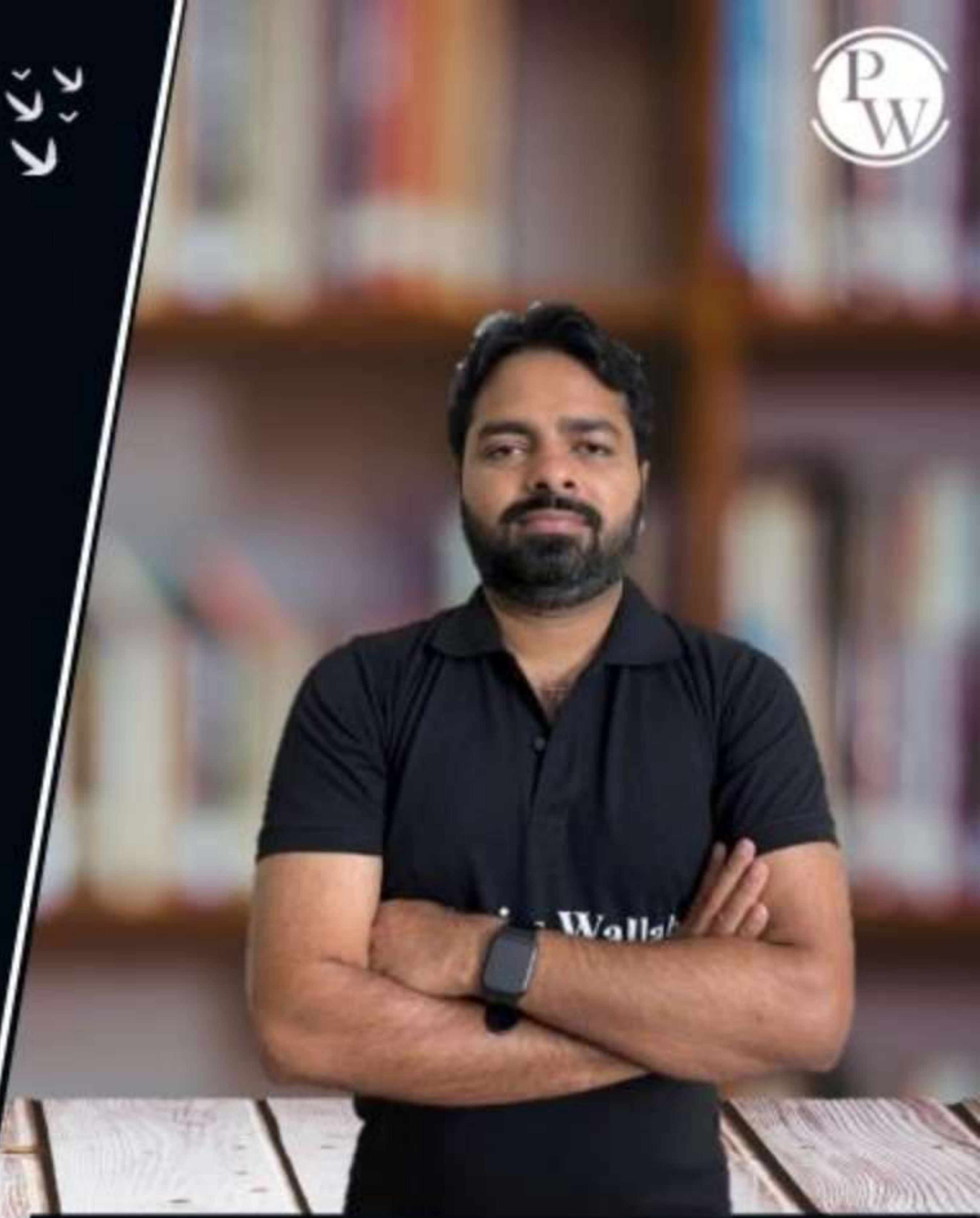
Computer Science

Theory of Computation

Regular Languages and Non-regular Languages



Lecture No.- 7

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Recap of Previous Lecture

Topic Closure Properties for Finite Languages : $\cup, \cap, \overline{L}, -$ ✓ ✓ ✗ ✓

Topic Closure Properties for Infinite Languages : $\cup, \cap, \overline{L}, -$ ✓ ✗ ✗ ✗

Topics to be Covered

Topic Closure Properties for Regular Languages

Closure Properties for regular languages: $L_i \to$ Regular lang

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $\overline{L}$
4. $L_1 - L_2$
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+
9. $\boxed{\text{Subset}(L)}$
10. prefix (L)
11. suffix (L)
12. Substring (L)
13. $f(L)$ = Substitution
14. $h(L)$ = Homomorphism
15. $h^{-1}(L)$
16. L_1 / L_2 Quotient
17. $L_1 \oplus L_2$ Symmetric Difference
18. Half $(L) = \frac{1}{2}(L)$
19. Second Half (L)
20. one third (L)
21. Middle $\frac{1}{3}(L)$
22. Last $\frac{1}{3}(L)$
23. Finite union
24. " $\cap$
25. " Difference
26. " Concatenation
27. " Subset
28. " Substitution
29. Inf $\cup$
30. Inf $\cap$
31. Inf $-$
32. Inf concatenation
33. Inf subset
34. Inf substitution

Closure Properties for Regular Languages

1) Union

↳ closed for regulars

$\underbrace{Reg_1 \cup Reg_2}$

⇓

Regular

proof 1 : Use Reg exps

proof 2 : Use NFA

proof 3 : compound FA

proof 4 : Use LLGs

proof 5 : Use RLGs

i) $L_1 = a^*$, $L_2 = b^*$ $\Rightarrow L_1 \cup L_2 = a^* + b^* = \varepsilon + a^+ + b^+$

ii) $L_1 = \phi$, $L_2 = \text{Any}$ $\Rightarrow L_1 \cup L_2 = L_2$

iii) $L_1 = \Sigma^*$, $L_2 = \text{Any}$ $\Rightarrow L_1 \cup L_2 = L_1 = \Sigma^*$

iv) $L_1 = \{a\}$, $L_2 = a^*$ $\Rightarrow L_1 \cup L_2 = L_2 = a^*$

*** Note :

I) Regular $\cup$ Regular $\Rightarrow$ Regular

II) Regular $\cup$ Non-regular $\Rightarrow$ either Regular or non-regular

$\Phi \cup$ Non-reg $\Rightarrow$ Non-reg

$\Sigma^* \cup$ Non-reg $\Rightarrow \Sigma^*$ (reg)

III) Non-regular $\cup$ Non-regular $\Rightarrow$ "

i) $a^n b^n \cup a^n b^n \Rightarrow a^n b^n$ (non-reg)

$\boxed{L \cup \bar{L} = \Sigma^*}$

ii) $a^n b^n \cup \overline{a^n b^n} \Rightarrow (a+b)^*$ regular

IV) If $\underbrace{L_1 \cup L_2 \text{ is Regular}}$ then L_1 is ______ either regular or non-reg

V) If $L_1 \cup L_2$ is Non-regular then L_1 is ______ either regular or not reg

$\phi \cup a^n b^n$ $\qquad a^n b^n$

$a^n b^n \cup a^n b^n$ $\qquad$ "

$\phi \rightarrow$ Reg

$\Sigma^* \rightarrow$ Reg

$a^n b^n \rightarrow$ Non-regular

$\overline{a^n b^n} \rightarrow$ Non-regular

Closure Properties for Regular Languages

2) Intersection

↳ closed for regulars

$$Reg_1 \cap Reg_2 \Rightarrow Regular$$

$$\underbrace{FA_1 \times FA_2}_{\text{Compound FA}}$$

i) $$\left.\begin{aligned} L_1 &= a^* \\ L_2 &= b^* \end{aligned}\right\} \Rightarrow L_1 \cap L_2 = \{\varepsilon\}$$

ii) $$\left.\begin{aligned} L_1 &= \phi \\ L_2 &= \text{Any} \end{aligned}\right\} \Rightarrow L_1 \cap L_2 = \phi$$

iii) $$\left.\begin{aligned} L_1 &= \Sigma^* \\ L_2 &= \text{Any} \end{aligned}\right\} \Rightarrow L_1 \cap L_2 = L_2$$

iv) $$L \cap \overline{L} = \phi$$

Closure Properties for Regular Languages

Note:

I) If L_1 and L_2 are regulars then $L_1 \cap L_2$ is Regular

II) If L_1 is Regular and L_2 is non-regular then $L_1 \cap L_2$ is either Reg or Not Reg

III) If L_1 and L_2 are non-regulars then $L_1 \cap L_2$ is "

IV) If $L_1 \cap L_2$ is Regular then L_1 is "

V) If $L_1 \cap L_2$ is Non-regular then L_1 is "

$\Sigma^* \cap$ Not reg

$a^n b^n \cap a^n b^n$

Reg $\cap$ Not Reg $\Longrightarrow$ either Regular or not regular

i) $\phi \cap$ Not Reg $\Rightarrow \phi$ Reg

ii) $\Sigma^* \cap$ Not Reg $\Rightarrow a^n b^n$ Not Reg

$a^n b^n$

Non-reg $\bigcap$ Non-reg $\Longrightarrow$?

i) $a^n b^n \bigcap a^n b^n \Longrightarrow a^n b^n$ not reg

ii) $a^n b^n \bigcap b^n a^n \Longrightarrow \{\varepsilon\}$ Reg

3) Complement

↳ closed for regular languages

$\overline{\text{Regular}} \Rightarrow$ Always Regular

Given

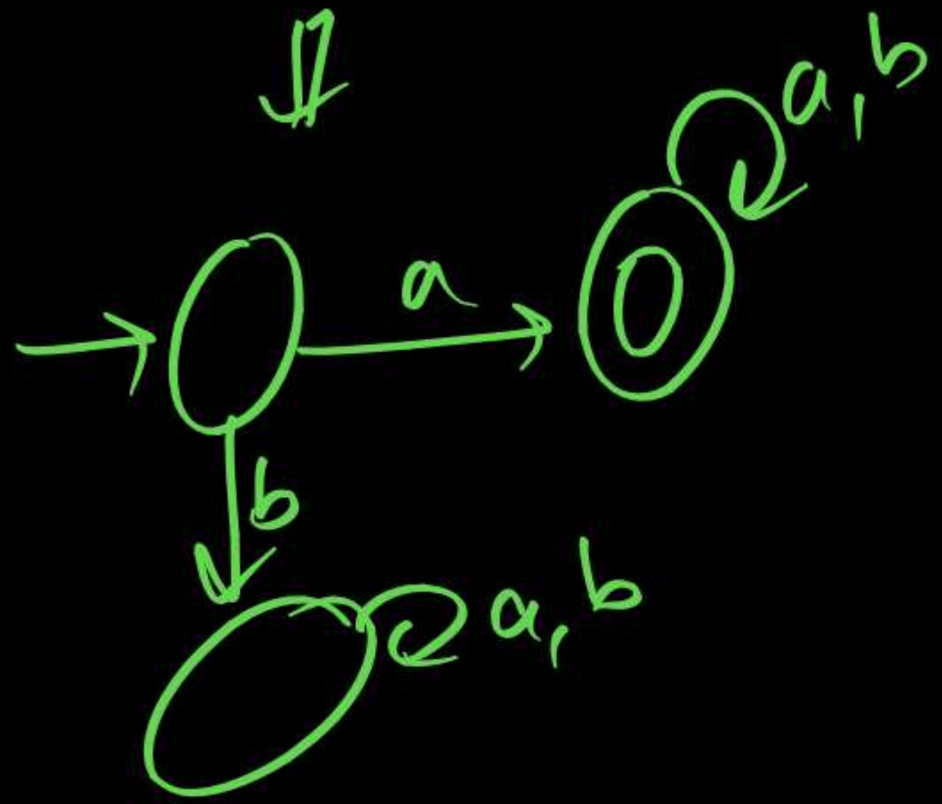
$\boxed{L \text{ is Reg}} \Rightarrow \boxed{L \text{ has DFA}} \xRightarrow{f \leftrightarrow nf} \boxed{\text{new DFA}} \Rightarrow \boxed{\text{It accepts } \overline{L}}$

$\Downarrow$

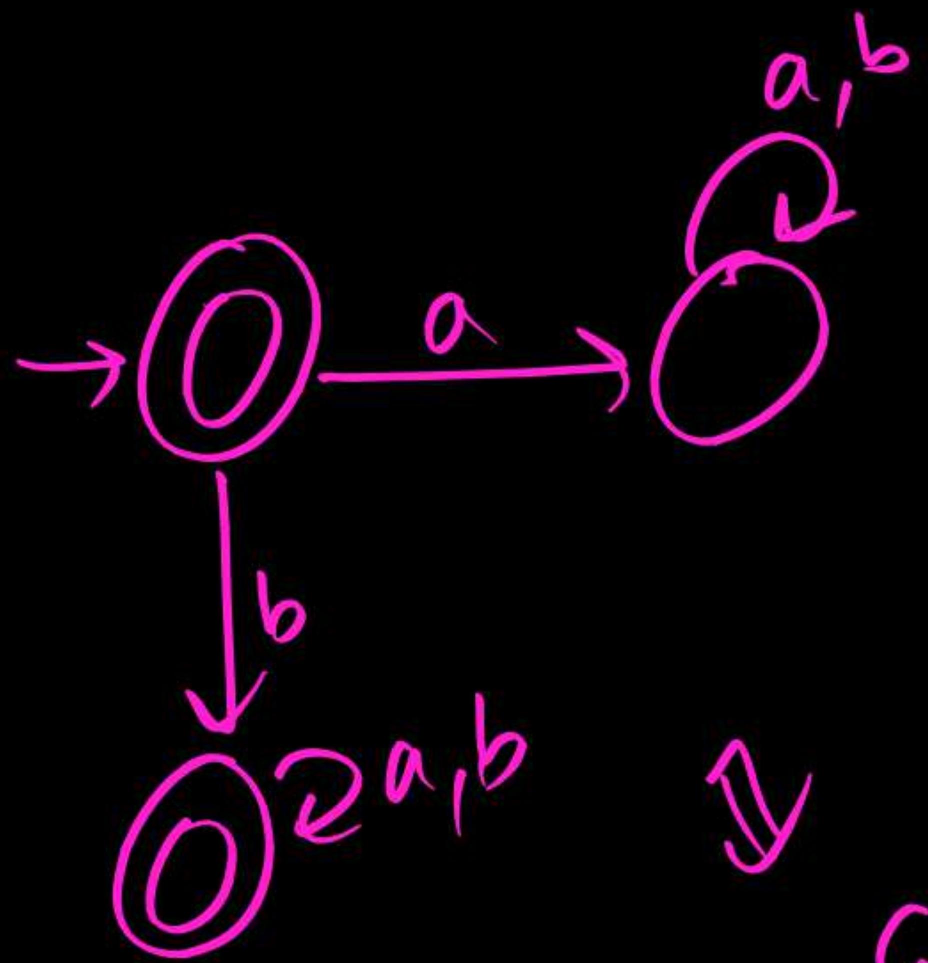
$\overline{L}$ is Regular

$L = a(a+b)^*$

$\Downarrow$



$\Longrightarrow$



$\Downarrow$

$L = \epsilon + b(a+b)^*$

i) $L = \phi \Rightarrow \bar{L} = \Sigma^*$

ii) $L = \Sigma^* \Rightarrow \bar{L} = \phi$

iii) $L = a(a+b)^* \Rightarrow \bar{L} = \varepsilon + b(a+b)^*$

iv) $L = b(a+b)^* \Rightarrow \bar{L} = \varepsilon + a(a+b)^*$

v) $L = (a+b)^* a \Rightarrow \bar{L} = \varepsilon + (a+b)^* b$

vi) $L = (a+b)^* b \Rightarrow \bar{L} = \varepsilon + (a+b)^* a$

vii) $L = (a+b)^* a (a+b)^* \Rightarrow \bar{L} = b^*$

Closure Properties for Regular Languages

Note:

I) If L is Reg then $\bar{L}$ is Regular

II) If $\bar{L}$ is Reg then L is Regular

V) L is Reg iff $\bar{L}$ is Reg

III) If L is not reg then $\bar{L}$ is not reg

IV) If $\bar{L}$ is not reg then L is not reg

VI) L is not reg iff $\bar{L}$ is not reg

Closure Properties for Regular Languages

4) Difference

↳ closed for regulars

$$Reg_1 - Reg_2 \implies \text{Regular}$$

$$Reg_1 - Reg_2 = Reg_1 \cap \boxed{\overline{Reg_2}}$$

$$\Downarrow$$

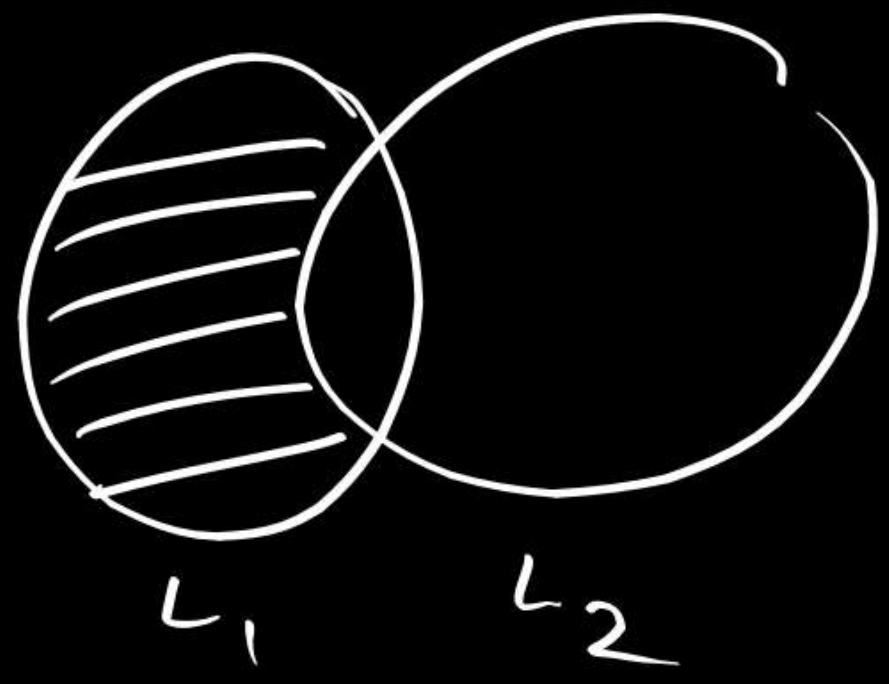
$$Reg_1 \cap Reg \implies Reg$$

i) $\left.\begin{array}{l} L_1 = a^* \\ L_2 = b^* \end{array}\right\} \Rightarrow \begin{array}{l} L_1 - L_2 = a^+ \\ L_2 - L_1 = b^+ \end{array}$

ii) $\left.\begin{array}{l} L_1 = \phi \\ L_2 = L \end{array}\right\} \Rightarrow \begin{array}{l} L_1 - L_2 = \phi - L = \phi \\ L_2 - L_1 = L - \phi = L \end{array}$

iii) $\left.\begin{array}{l} L_1 = \Sigma^* \\ L_2 = L \end{array}\right\} \Rightarrow \begin{array}{l} L_1 - L_2 = \Sigma^* - L = \bar{L} \\ L_2 - L_1 = L - \Sigma^* = \phi \end{array}$

$$L_1 - L_2 = L_1 \cap \overline{L_2}$$



2 mins Summary

Topic

$L_1 \cup L_2$

$L_1 \cap L_2$

$\bar{L}$

$L_1 - L_2$

THANK - YOU