CS & IT ENGINEERING

Theory of Computation

Regular Languages

Lecture No.- 08

Mallesham Devasane Sir

Recap of Previous Lecture

Topic Regular Expressions: Basics

Topic Simplification of Regular Expressions

Topics to be Covered

Topic Regular Language Vs Regular Expression

TOPIC: Home Work :

15) $(a+b)^* + ab = (a+b)^*$

16) $(a+b)^+ + \varepsilon = (a+b)^*$

17) $(a+b+\varepsilon)^+ = (a+b)^*$

Important 18) $(a^* b^*)^* = (a+b)^*$

19) $(b^* a^*)^* = (a+b)^*$

20) $(a^* + b)^* =$ "

21) $(a + b^*)^* =$ "

22) $(a^* + b^*)^* =$ "

ε

a

b

aa

ab

ba

bb

aaa

aab

aba

abb

baa

bab

bba

bbb

$\Sigma = \{a, b\}$

. . . .

$a^* b^* \neq (a^* b^*)^*$

ab

$(\quad)^2 = a^* b^* a^* b^*$

$abab$

$(a^* b^*)^* = (b^* a^*)^* = (a+b)^*$

ε ✓

a ✓

b ✓

aa ✓

ab ✓

ba ✓

bb ✓

⋮

ε ✓

a ✓

b ✓

aa ✓

ab ✓

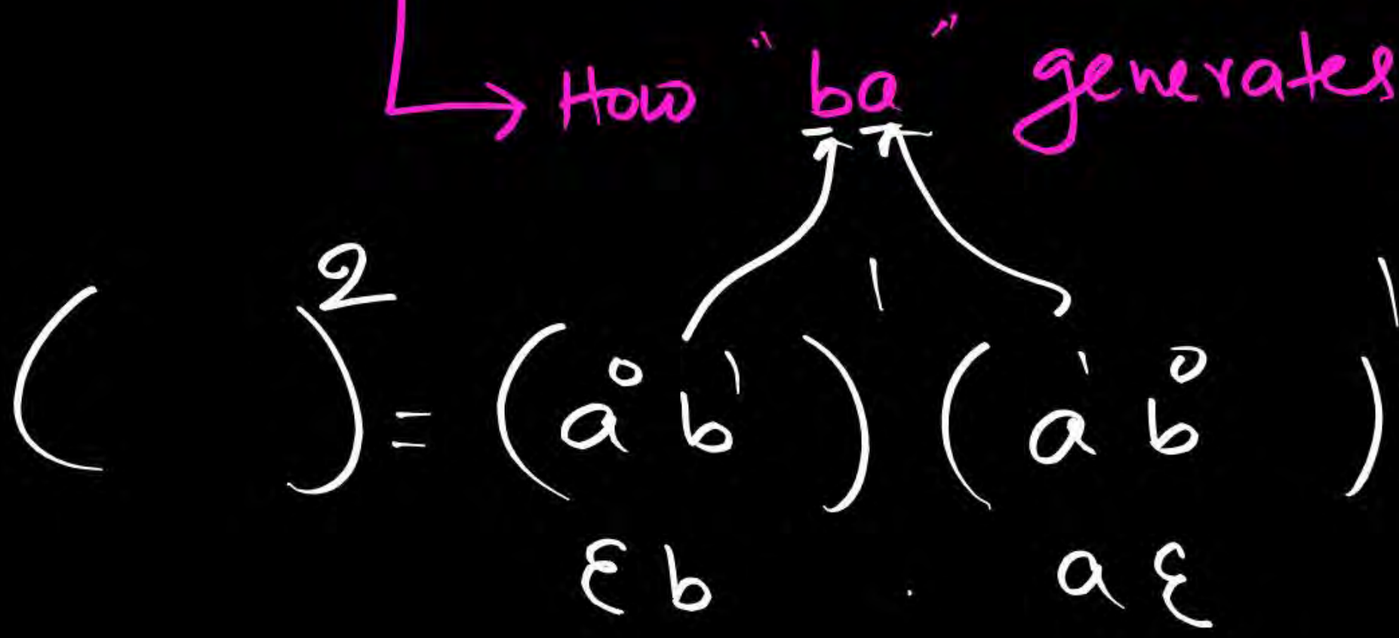
ba ✓

bb ✓

⋮

$(a^*b^*)^*$

↳ How "ba" generates ?



TOPIC:

$$(a^*b^*)^+ = (b^*a^*)^+ = (a+b)^*$$

$(a^\circ b^\circ)'$ → ε ✓, a ✓, b ✓, aa ✓, ⋮

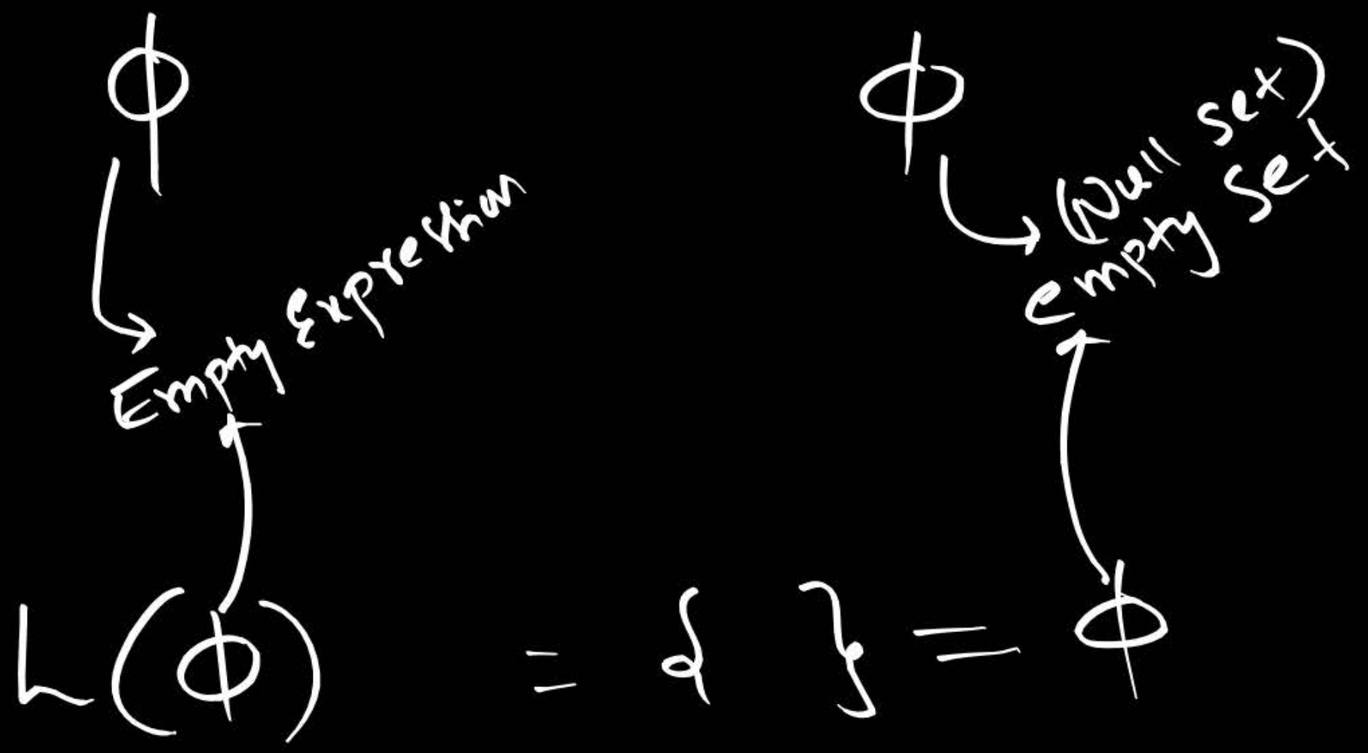
ε ✓ = $(b^\circ a^\circ)'$

a ✓, b ✓, aa ✓, ⋮

$$(a^+ b^+)^* \neq (b^+ a^+)^*$$

ε ✓
ax
bx
aax
ab ✓

ε ✓
ax
bx
aax
abx


ϕ
Empty Expression
L(ϕ) = { } = ϕ
ϕ
(Null set)
Empty Set

$(a+b)^2 = a^2+b^2+2ab$ In maths

$(a+b)^2 = aa+ab+ba+bb$ In TOC

$= (a+b)\cdot(a+b)$

$a \cdot a$

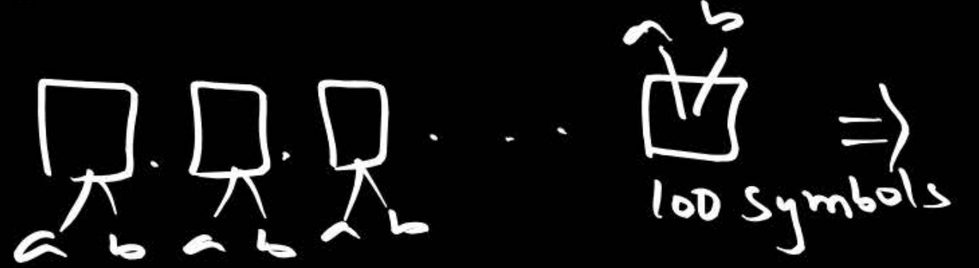
$a \cdot b$

$b \cdot a$

$b \cdot b$

$(a+b)^2 = (a+b)(a+b) = aa+ab+ba+bb$

$(a+b)^3 = (a+b).(a+b).(a+b) = aaa+aab+aba+ \cdots +bbb$

$(a+b)^{100} =$  $\Rightarrow$ Set of all 100 length strings

$2 \times 2 \times \cdots$ 100 times $= 2^{100}$ strings over $\Sigma = \{a, b\}$

Note: If $\Sigma = \{a, b, c, d, e\}$

then how many 100 length strings?

$= 5^{100}$

$$\{w \mid w \in \{a,b\}^*,\ |w| = 2\}$$

$w \in (a+b)^*$

$w \in \{\varepsilon, a, b, \underline{aa}, \underline{ab}, \underline{ba}, \underline{bb}, \ldots\}$

$= \{aa, ab, ba, bb\}$

$$L = \{w \mid w \in (ab)^*,\ |w| \le 2\}$$

$$= \{w \mid \underbrace{w \in (ab)^*}_{w \in \{\varepsilon, ab, abab, \ldots\}},\ |w| = 0, 1, \text{or } 2\}$$

$$= \{\varepsilon, ab\}$$

$w \in (ab)^*$

- $\varepsilon \rightarrow$ 0 len
- $ab \rightarrow$ 2 len

$abab \rightarrow$ 4 len

$(ab)^3$

TOPIC:

	Regular Exp.	Regular Language	Meaning
(8)	a^*	$\{\varepsilon, a, a^2, \ldots\} = \{a^n \mid n \geq 0\}$	Set of all strings over a's
(9)	$(aa)^*$	$\{\varepsilon, a^2, a^4, a^6, \ldots\} = \{a^n \mid n = \text{even}\}$	= Set of all even length strings over a's
(10)	$a(aa)^*$	$\{a, a^3, a^5, \ldots\}$	= Set of all odd length strings over a's
(11)	$(aa)^*a$	$\{a, a^3, a^5, \ldots\}$	
(12)	a^+	$\{a^n \mid n \geq 1\}$ =	Set of all strings over a's excluding zero length
(13)	$a.a^+$	$\{a^n \mid n \geq 2\}$ =	Set of all strings over a's with atleast 2 length.
(14)	$(aaa)^*$	$\{\varepsilon, a^3, a^6, a^9, \ldots\} = \{a^n \mid n\%3 = 0\} = \{a^{3K} \mid K \geq 0\}$	⇒ No. of a's is multiple of 3

$(ab)^* \Rightarrow \{\varepsilon, ab, abab, ababab, \cdots\}$

$w \in (ab)^*$

$|w| \leq 2$

TOPIC:

$\Sigma = \{a, b\}$

	Regular Exp.	Regular Language (Set)	Meaning
①	ϕ	$\{\ \} = \phi$	Set of zero no. of strings
②	ε	$\{\varepsilon\}$	Set of zero length strings $= \{w \mid \|w\| = 0\}$
③	a	$\{a\}$	$\{w \mid w = a\}$
④	$a+b$	$\{a, b\}$	Set of one length strings $= \{w \mid \|w\| = 1, w \in \{a,b\}^*\}$
⑤	$\varepsilon+a+aa$	$\{\varepsilon, a, a^2\}$	$\{w \mid w \in \{\varepsilon, a, aa\}\}$
⑥	$(a+b)^2$	$\{aa, ab, ba, bb\}$	$\{w \mid \|w\| = 2, w \in (a+b)^*\}$ = Set of all 2 len strings
⑦	$(a+b)^{100}$	$\{w \mid w \in (a+b)^*, \|w\| = 100\}$	= Set of all 100 length strings

TOPIC:

$\Sigma = \{a, b\}$

	Regular Exp.	Regular Language	Meaning
(15)	$(a+b)^*$	$\{a,b\}^*$	Set of all strings over $\Sigma = \{a,b\}$ ⇒ Universal Language
(16)	$(a+b)^+$	$\{a,b\}^+ = \{w \mid w \in (a+b)^*, \lvert w\rvert > 0\}$	= Set of all non zero length strings
(17)	$a(a+b)^*$		
(18)	$b(a+b)^*$		
(19)	$(a+b)^*a$		
(20)	$(a+b)^*b$		

TOPIC:

	Regular Exp.	Regular Language	Meaning
21	$(a^*b)^+$		
22	$(b^*a)^+$		
23	$(ab^*)^+$		
24	$(ba^*)^+$		
25	$(a^*b)^*a^*$		
26	$(b^*a)^*b^*$		
27	$a^*(ba^*)^*$		

TOPIC:

(28) $b^*(ab^*)^*$

$\Sigma = \{\varepsilon\}$ → symbol

$\Sigma^* = \{\lambda, \varepsilon, \varepsilon\varepsilon, \varepsilon\varepsilon\varepsilon, \ldots\}$

λ → empty string; ε → one length string; $\varepsilon\varepsilon$ → 2 len string

ε is not empty string

$\Sigma = \{a\}$

$\Sigma^* = \{\varepsilon, a, aa, aaa, \ldots\}$

[MSQ]

#Q1. $(a+b)^* a^* =$

A a^*

B $(a+b)^*$

C $a^* (a+b)^*$

D $a^* (a+b)^* a^*$

[MSQ]

#Q2. (ab)* (a+b)* =

A ab*

B (ab)*

C a* (a+b)*

D (a+b)* (ab)*

[MSQ]

#Q3. (a*b)*a* =

A a*(a*b)

B (b*a)*b*

C b*(ab*)*

D a*(ba*)*

[MSQ]

#Q4. $(a+b)^* a =$

A b^*a

B $(b^*a)^+$

C $(b^*a^+)^+$

D $(a+b)^* a^+$

[MSQ]

#Q5. Let L= a(a+b)*. Then Complement of L is ___

A b(a+b)*

B epsilon + b(a+b)*

C (a*b)*

D (a+b)*

$$\bar{L} = \Sigma^* - L$$
$$= (a+b)^* - L$$

2 mins Summary

Topic Operators

Topic Properties

Topic Simplification

Regular Language

THANK - YOU