


CS & IT ENGINEERING

Theory of Computation

Regular Languages

Lecture No.- 11

A man with a beard and mustache, wearing a black polo shirt, stands with his arms crossed in front of a blurred bookshelf. He is wearing a black watch on his left wrist.

Malleham Devasane Sir

Recap of Previous Lecture



Topic

Regular Language Vs Regular Expression



Topics to be Covered



Topic

Regular Language Vs Regular Expression





TOPIC:



$$(45) \quad \{w \mid w \in \{a, b\}^*, n_a(w) \% 3 = 2\}$$

$$= b^* (b^* a b^* a b^* a b^*)^* b^* a b^* a b^*$$

$$(46) \quad \{w \mid w \in \{a, b\}^*, n_a(w) \% 5 = 1\}$$

$$= b^* a b^* (b^* a b^* a b^* a b^* a b^*)^* b^*$$

$$L = \{w \mid w \in a^*, n_a(w) \% 3 = 2\}$$

$$= (aaa)^* aa$$

$$= aa(aaa)^*$$

$$= a(aaa)^* a$$

$$n_a(w) = 3k + 2$$

$k \geq 0$

$$= 2, 5, 8, 11, 14, \dots$$



TOPIC:



$$\textcircled{47} \quad \{w \mid w \in \{a,b\}^*, w \text{ starts with } aa \text{ or } bb\} \\ = aa\Sigma^* + bb\Sigma^* = (aa+bb)\Sigma^*$$

$$\textcircled{48} \quad \{w \mid w \in \{a,b\}^*, w \text{ ends with } aa \text{ or } bb\} \\ = \Sigma^*(aa+bb)$$

$$\textcircled{49} \quad \{w \mid w \in \{a,b\}^*, w \text{ contains } aa \text{ or } bb \text{ as substring}\} \\ = \Sigma^*(aa+bb)\Sigma^* = \Sigma^*aa\Sigma^* + \Sigma^*bb\Sigma^*$$



TOPIC:

(50) $\{w \mid w \in \{a,b\}^*, w \text{ starts with } aa \text{ and } w \text{ starts with } bb\}$
 $= \phi$

There is no string

(51) $\{w \mid w \in \{a,b\}^*, w \text{ ends with } aa, w \text{ ends with } bb\}$
 $= \phi$

*** (52) $\{w \mid w \in \{a,b\}^*, w \text{ contains } aa \text{ and } w \text{ contains } bb\}$

$$= \Sigma^* aa \Sigma^* bb \Sigma^* + \Sigma^* bb \Sigma^* aa \Sigma^*$$
$$= \Sigma^* (aa \Sigma^* bb + bb \Sigma^* aa) \Sigma^*$$

aabb ✓
bbaa ✓
... aa ... bb ... ✓
... bb ... aa ... ✓



TOPIC:



$$(53) \{w \mid w \in \{a,b\}^*, w \text{ contains 'aabb'}\}$$

$$= (a+b)^* aabb (a+b)^*$$

$$(54) \{w \mid w \in \{a,b\}^*, w \text{ contains 'aabb' or 'bbaa'}\}$$

$$= \Sigma^* (aabb + bbaa) \Sigma^*$$

$$= \Sigma^* aabb \Sigma^* + \Sigma^* bbaa \Sigma^*$$



TOPIC:



(55) $\{w \mid w \in \{a, b\}^*, \text{ 2}^{\text{nd}} \text{ symbol of } w \text{ is 'a'}\}$

$\overline{\text{1st}} \quad \text{2nd} \quad \text{_____}$

$$= (a+b) a (a+b)^*$$

$$= \Sigma a \Sigma^*$$

Min string:

$\begin{matrix} a & \underline{a} & \checkmark \\ b & \underline{a} & \checkmark \end{matrix}$

$\begin{matrix} \epsilon x \\ ax \\ bx \\ \cdot aa \checkmark \\ abx \\ \cdot ba \checkmark \\ bbx \end{matrix}$



TOPIC:



(56) $\{w \mid w \in \{a, b\}^*, \text{2}^{\text{nd}} \text{ symbol from end in } w \text{ is 'a'}\}$

$\overbrace{\hspace{1.5cm}}^{\text{2}^{\text{nd}}}$ a either a or b

$$= \Sigma^* a (a+b) = \Sigma^* aa + \Sigma^* ab$$

$$= \Sigma^* a \Sigma$$

$$= (a+b)^* a (a+b)$$

2nd symbol from end is 'a'

any sequence $\frac{a}{2^{\text{nd}} \text{ last}}$ $\frac{\text{last}}{\text{any symbol}}$



TOPIC:



$$(57) \quad \{a^m b^n \mid m, n \geq 0\} \Rightarrow a^* b^*$$

$$(58) \quad \{a^m b^n \mid m, n \geq 1\} \Rightarrow a^+ b^+$$

$$(59) \quad \{b^m a^n \mid m, n \geq 2\} \Rightarrow b b b^* a a a^* = b b^+ a a^+ = b^+ b a^+ a$$

$$(60) \quad \{a^m b^n c^k \mid \underbrace{m \geq 1}_{\substack{\text{min } 1a \\ n=0 \text{ or } 1}}, \underbrace{n \leq 1}_{\text{max } 1b}, \underbrace{k \geq 2}_{\text{min } 2c}\} = \underline{a^+} (\underline{\varepsilon + b}) \underline{c c^+}$$

$$= a^{\overbrace{m}^{\geq 1}} b^{\overbrace{n}^{\geq 0}} c^{\overbrace{k}^{\geq 2}}$$

$$= a^+ (b^0 + b^1) c c c^*$$

$$= a^+ (\epsilon + b) c c^+$$



$$\{a^m b^n \mid m, n \geq 0\}$$

$$= \{a^0 b^0, a^1 b^0, a^0 b^1, a^1 b^1, a^2 b^0, a^0 b^2, \dots\}$$

$$= \{\epsilon, a, b, ab, aa, bb, \dots\}$$

$$= a^* b^*$$



TOPIC:



Note: $\{ \overbrace{a^n} \overbrace{b^n} \mid n \geq 0 \} = \{ \epsilon, ab, aabb, aaabbb, \dots \}$

$\overbrace{a^n} \overbrace{b^n}$
equal no. of a's & b's

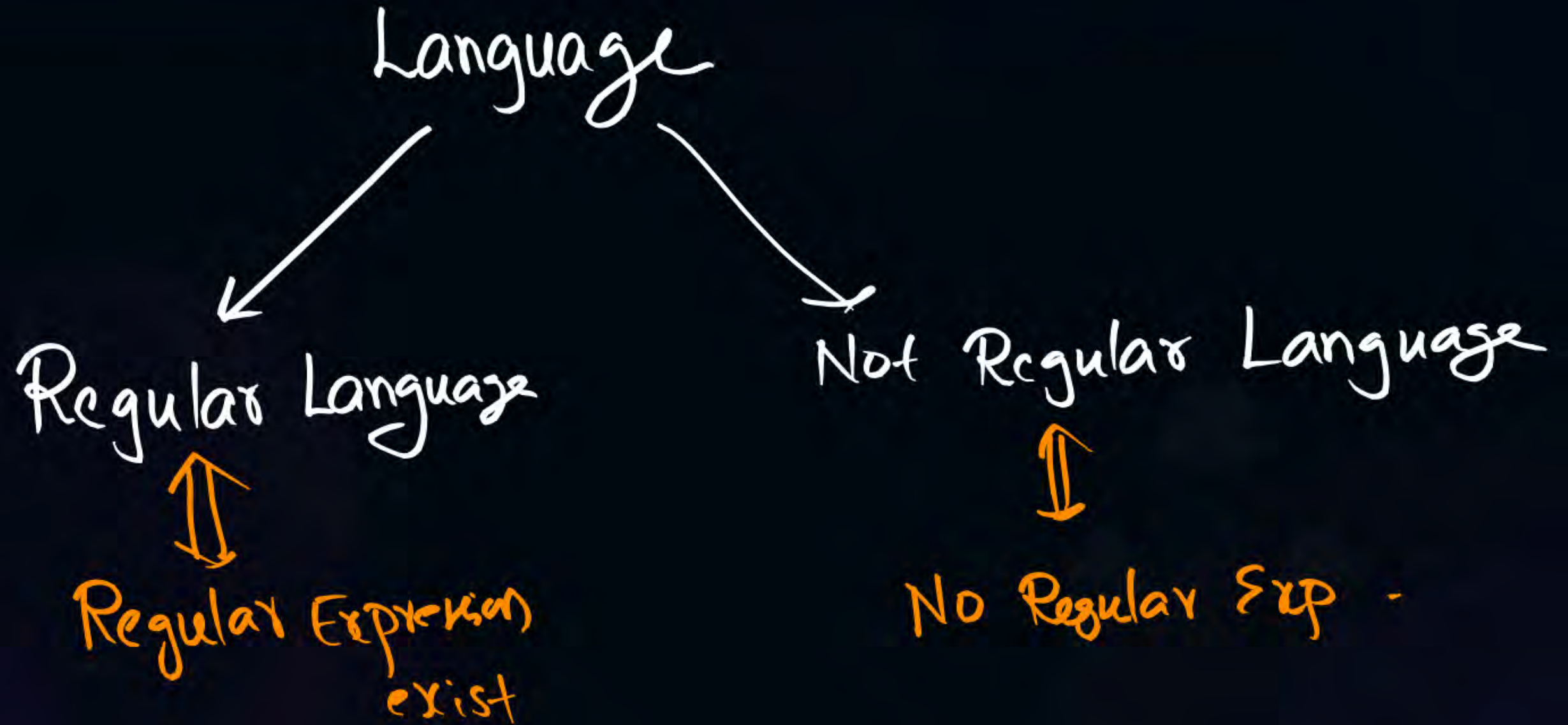
Can you write Regular Expression?

There is no regular expression

→ It is not regular language.



TOPIC:





TOPIC:



4^k symbol from end is 'a'

$$= \Sigma^* a \Sigma \Sigma \Sigma$$

$$= (a+b)^* \underbrace{a}_{\substack{4^k \\ \text{last}}} \underbrace{(a+b)^3}_{\text{last 3}}$$



2 mins Summary



Topic

Regular Languages

Topic

Regular Expressions

THANK - YOU