Supplement for "Analyzing the effect of equal-angle spatial discretization for sound event localization and detection"

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1 Mathematics of equal-angle spatial discretization

1.1 background

A 3D sphere can be discretized along its axes of inclination $\theta = [0, \pi] \in \mathbb{R}$ and azimuth $\phi = [0, 2\pi) \in \mathbb{R}$.

The discretization is done using a train of Dirac deltas on the sampling function:

$$s(\theta, \phi) = \sum_{e=0}^{N_e-1} \sum_{a=0}^{N_a-1} \alpha(\theta)\delta(\cos(\theta) - \cos(\theta_e))\delta(\phi - \phi_a). \tag{1}$$

Where $\alpha(\theta)$ is a coefficient that varies as a function of inclination θ, reaching a peak value at $\theta = \frac{\pi}{2}$, as shown in Boaz Rafaely, 2015.

N_E is the number of points sampled along the inclination axis and N_A is the number points along the azimuth axis. The points are:

$$\theta_e = e\frac{\pi}{N_E}, e = 0, 1, 2, ..., N_e - 1 \tag{2}$$

$$\phi_a = a\frac{2\pi}{N_A}, a = 0, 1, 2, ..., N_a - 1. \tag{3}$$

1.2 proof that equal-angle spatial discretization is irregularly spaced as a function of inclination

Neighboring equal-angle points on the azimuth axis are

$$\begin{aligned} s(\theta,\phi)_{a=x} &= \alpha(\theta)\delta(\phi-\phi_x) \\ s(\theta,\phi)_{a=x+1} &= \alpha(\theta)\delta(\phi-\phi_{x+1}) \end{aligned} \tag{4}$$

Proposition
$\phi \in (0, \frac{\pi}{2})$

$$||s(\theta,\phi)_{e=y,a=x} - s(\theta,\phi)_{e=y,a=x+1}|| < ||s(\theta,\phi)_{e=y+1,a=x} - s(\theta,\phi)_{e=y+1,a=x+1}|| \tag{5}$$

Proof
fixate $\bar{\theta} \in (0, \frac{\pi}{2})$

$$||s(\theta,\bar{\phi})_{a=x} - s(\theta,\bar{\phi})_{a=x+1}|| = \bar{c} \tag{6}$$

now let $\tilde{\theta} < \bar{\phi}$

$$||s(\theta,\tilde{\phi})_{a=x} - s(\theta,\tilde{\phi})_{a=x+1}|| = \tilde{c} < \bar{c} \quad \square \tag{7}$$

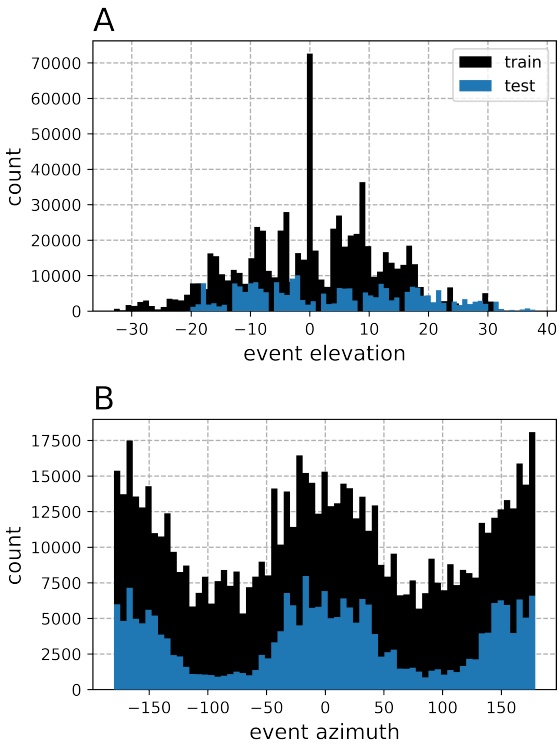

Figure 1: Density of sound events locations in the STARS2022 (+ supplemental synthetic) dataset along the elevation (A) and azimuth axes (B). Data is highly non-uniform along the elevation axis.

2 Data distribution for baseline data