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UNIT 4

NUMBER REPRESENTATION :

Signed Numbers :
a) Signed Magnitude
b) One's Compliment
c) Two's Compliment

DRAWBACKS of Signed Magnitude :
a) there are two zeroes = +0 and -0 (double representation)
b) Addition cannot be performed correctly. Complictions in aritmetic operations.

ONE'S COMPLEMENT :

```
  0101    = +5 ]
+ 1100    = -3 ]+  = +2
 10001
+↓
  0001
+    1
  0010
```

```
  1011    = -4 ]
  0111    = +7 ]+ = +3
 10010
+↓
  0010
+    1
  0011
```

By using One's complement, the second problem of signed Magnitude is resolved or removed.

TWO'S COMPLEMENT : Negetively Biased

```
 0001
  ↓
 1110        or        0 0 0 1
+   1                  ↓ ↓ ↓ ↓
 1111                  1 1 1 1
```

Two Methods

ex: 0 0 0 0

↓

1 1 1 1

\+ 1

(1)0 0 0 0

↓

carry discarded

Carry or overloaded bit is being discarded in Two's complement.

Here the complement of 0 is 0, so by using 2's complement we can remove the drawback (1) of signed magnitude.

2's comp.		Binary 0-7	Binary 8-15		2's comp.	
+0	0	0000	1000	1		-1
+1	0	0001	1001	1		-2
+2	0	0010	1010	1		-3
+3	0	0011	1011	1		-4
+4	0	0100	1100	1		-5
+5	0	0101	1101	1		-6
+6	0	0110	1110	1		-7
+7	0	0111	1111	1		-8

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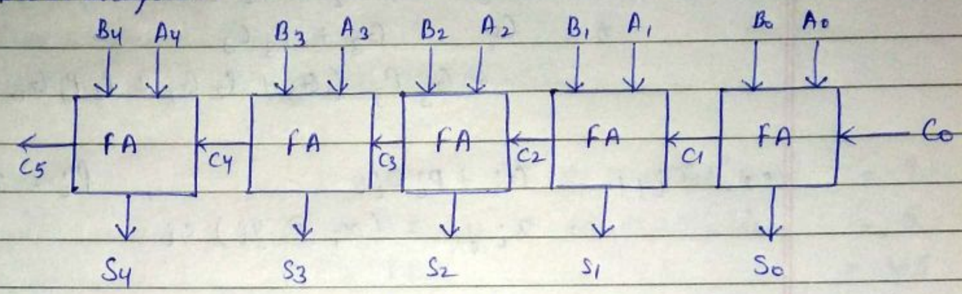
* FULL ADDER

Block Diagram =



(5 bits)

- Sum = $2n \Rightarrow 2_5 = \underline{10}$

Delay of 10 clock cycles in Full Adder of 5 bits

- Carry : $2(n-1)+1$
  $\Rightarrow 2n-1$

* ADDER :

Difference between half adder and Full Adder :

	HALF ADDER	FULL ADDER
i)	No. of Inputs = 2	i) No. of Inputs = 3
ii)	No. of Outputs = 2	ii) No. of Outputs = 2
iii)	Previous Carry is not used	iii) Previous carry is used.
iv)	$S = A \oplus B$ $C = A \cdot B$	iv) $S = A \oplus B \oplus C$ $C = AB + BC + CA$

* CARRY LOOK AHEAD ADDER :

i) Generate function = $x_i * y_i$ or $x_i \cdot y_i$

ii) Propogate function = $x_i + y_i$

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* $C_{i+1} = G_i + P_i C_i$

* $C_3 = G_2 + P_2 C_2$
* $C_4 = G_3 + P_3 C_3$
$= G_3 P_3 (G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0)$

• * $c_{i+1} = G_i + P_i C_i$, $P_i = x_i \oplus y_i$
$= x_i y_i + (x_i \oplus y_i) c_i$

• $C_{i+1} \Rightarrow x_i y_i + (x_i \overline{y_i} + \overline{x_i} y_i) c_i$
$\Rightarrow x_i y_i + (x_i \overline{y_i} c_i + \overline{x_i} y_i c_i)$
$\Rightarrow x_i (y_i + \overline{y_i} c_i) + \overline{x_i} y_i c_i$
$\Rightarrow x_i (y_i + c_i) + \overline{x_i} y_i c_i$
$\Rightarrow x_i y_i + x_i c_i + \overline{x_i} y_i c_i$
$\Rightarrow y_i (x_i + \overline{x_i} c_i) + x_i c_i$
$\Rightarrow y_i (x_i + c_i) + x_i c_i$
$\Rightarrow y_i x_i + y_i c_i + x_i c_i$ ✓

* Drawback :
i) we can use only 4-bit adder.

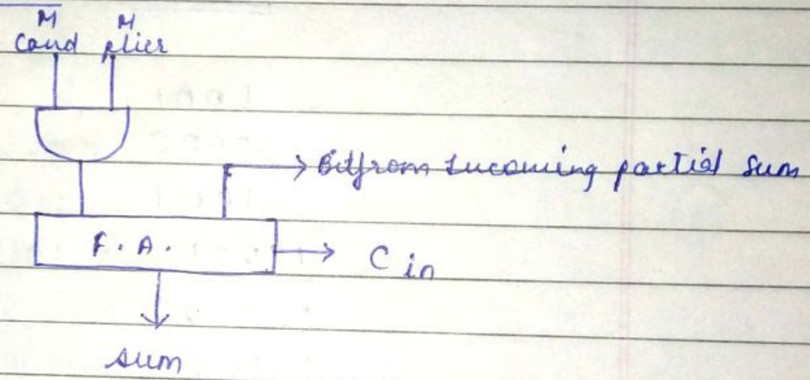
17.01.19

Multipliers

i) Array Multiplier
ii) Sequential Multiplier
iii) Booth's Algorithm for Multiplication

1 0 0 1	← Multiplicand	= 9
× 0 1 0 1	← Multiplier	= 5
0 1 0 1 1 0 1		= 45

A) ARRAY MULTIPLIER :



Examples :

- 11 = 3
  × 11 = 3
  1001 = 9

- 111 = 7
  × 111 = 7
  110001 = 49

- 1001 = 9
  × 0101 = 5
  1001
  0000X
  1001XX
  0000XXX
  0101101 = 45

```
   1001
   0101
   ----
   0000   ← Initial Partial Sum
 + 1001
   ----
   1001   → Partial Product

   1001
   0101
   ----
   1001
   0000
   ----
   1001

    1001
    0000
    ----
    1001
   11001
  ------
  101101
    0000
 -------
 0101101
```

Drawbacks:

i) Complexity Increases
ii) Cost Increases
iii) Cannot increase efficiency of a system.

B) SEQUENTIAL MULTIPLIER:

It is also called Add and Shift Multiplier

$\boxed{1011}$ → Multiplicand

Carry ← $\boxed{0}$ $\boxed{0000}$ $\boxed{1110}$ → multiplier

Ex:1

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Multiplicand = 1011 → B ⇒ 11
Multiplier = 1110 → Q ⇒ 14

Operation	Carry	A	Q	Count
Initial	$\boxed{0}$	0000	1110	4
⇒ 0 LSB × B ⇒		+ 0000		
		0000		
	0	0000	1110	
Right shift	$\boxed{0}$	0000	0111	3
		+ 1011		
		1011		
	0	1011	0111	
Right shift	$\boxed{0}$	0101	1011	2
		1011		
replaces 0 carry		10000		
	$\boxed{1}$	0000	1011	
Right shift	0	1000	0101	1
		1011		
		10011		
	$\boxed{1}$	0011	0101	
Right shift	0	1001	1010	0

⇓

010011010 Answer

```
  11
 ×14
 ---
  44
 11×
 ---
 154
```

Multiplicand = 1010 → B ⇒ 10
Multiplier = 0111 → Q ⇒ 7

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Ex: 2

Operation	Carry	A	Q	Count
Initial	$\boxed{0}$	0000 + 1010 = 1010	0111	4
	0	1010	0111	
Right shift	$\boxed{0}$	0101 + 1010 = 1111	0011	3
	0	1111	0011	
Right Shift	$\boxed{0}$	0111 + 1010 = 10001	1001	2
replaces 0 carry	$\boxed{1}$	0001	1001	
Right shift	0	1000 + 0000 = 1000	1100	1
Right Shift	0	0100	0110	0

Answer = 001000110

$$\begin{array}{r} 10 \\ \times\ 7 \\ \hline 70 \end{array}$$

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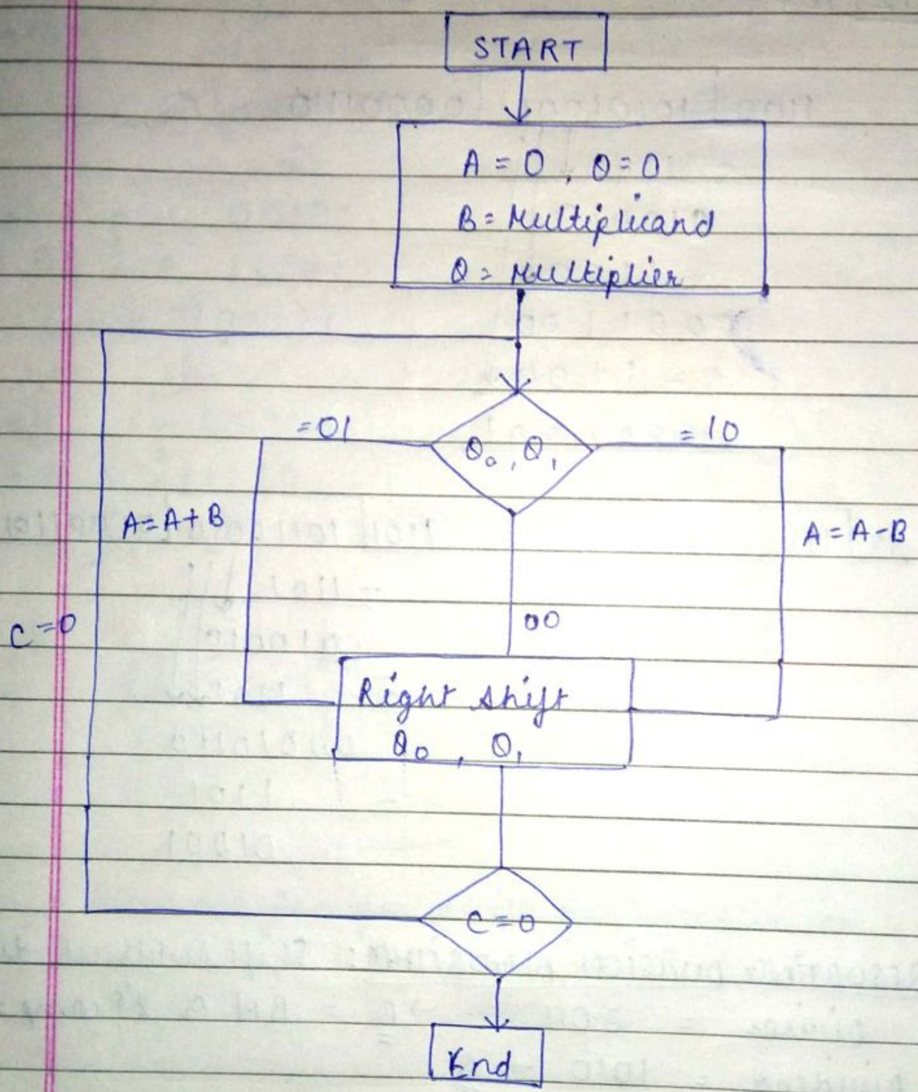
* Flowchart for sequential multiplier working:



- Operation - Right shift after addition of multiplicand.

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Division

Ex : ① 12 ⟌ 169 ?

$$
\begin{array}{r}
1100 \overline{)\,10101001\,}\; 00001110 \\
-\;1100 \\
\hline
010010 \\
-\;1100 \\
\hline
0001100 \\
-\;1100 \\
\hline
00000001
\end{array}
$$

② 13 ⟌ 178 ?

$$
\begin{array}{r}
1101 \overline{)\,10110010\,}\; 00001101 \\
-\;1101 \\
\hline
010010 \\
-\;1101 \\
\hline
00010110 \\
-\;1101 \\
\hline
01001
\end{array}
$$

A) RESORTING DIVISION ALGORITHM : Shift Subtract division

Divisor = 0011 + 0 → B = B+1 or 2's comp = 11101

Dividend = 1010 → Q

Operations	A	Q	Count
Initialization	00000	1010	4
Left shift	00001	010☐	
A = A − B	+11101		
	11110	010[0]	
A = A + B	+00011		
discard	(1)00001	0100	

left shift
A = A - B
$$\begin{array}{r} 00010 \\ +11101 \\ \hline 11111 \end{array} \qquad 100\boxed{} \to 100\boxed{0}$$

A = A + B
$$\begin{array}{r} 11111 \\ +00011 \\ \hline 1\,00010 \end{array} \qquad 1000$$

discard 1 | 00010 | 1000 | 2

Left shift
A = A - B
$$\begin{array}{r} 00101 \\ +11101 \\ \hline 1\,00010 \end{array} \qquad 000\boxed{} \to 000\boxed{1}$$

discard 1 | 1

left shift
A = A - B
$$\begin{array}{r} 00100 \\ +11101 \\ \hline 1\,00001 \end{array} \qquad 001\boxed{} \to 001\boxed{1}$$

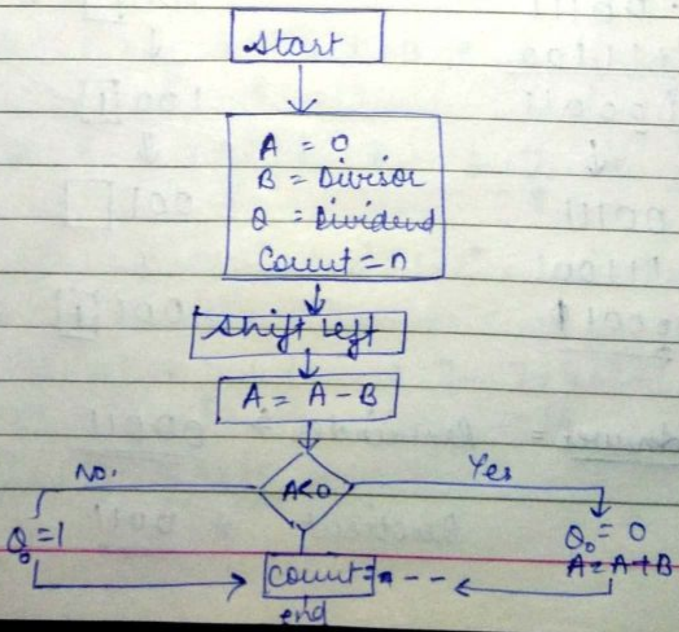
discard 1 | 0

$\boxed{00001}\boxed{0011}$

Remainder Quotient

$$\begin{array}{r|l} 3 & 10 \\ & -9 \\ \hline & 01 \end{array} \quad 3$$

* Flow diagram for Shift Subtract Division :



A = Dividend = 1111

B = Divisor = 0100 ⇒ 00100 → $\bar{B}+1$ ⇒ 11100

Operations	A	Q	Count
~~Initialize~~	~~00000~~	~~1111~~	
~~A = A-B~~	~~+11100~~		
	~~11100~~		
Initialize	00000	1111	4
Left Shift	00001	111 $\boxed{\ }$	
A = A-B	+ 11100		
	11101	111 $\boxed{0}$	
A = A+B	+ 00100		
discard	1 00001		
	00001	1110	3
Left shift	00011	110 $\boxed{\ }$	
A = A-B	+ 11100		
A = A+B	11111	110 $\boxed{0}$	
	+ 00100		
discard	1 00011	1100	
Left shift	00111	100 $\boxed{\ }$	2
A = A-B	+ 11100		
discard	1 00011	100 $\boxed{1}$	
Left shift	00111	001 $\boxed{\ }$	1
A = A-B	+ 11100		
discard	1 00011	001 $\boxed{1}$	0

Answer = Remainder ⇒ 00011

Quotient ⇒ 0011

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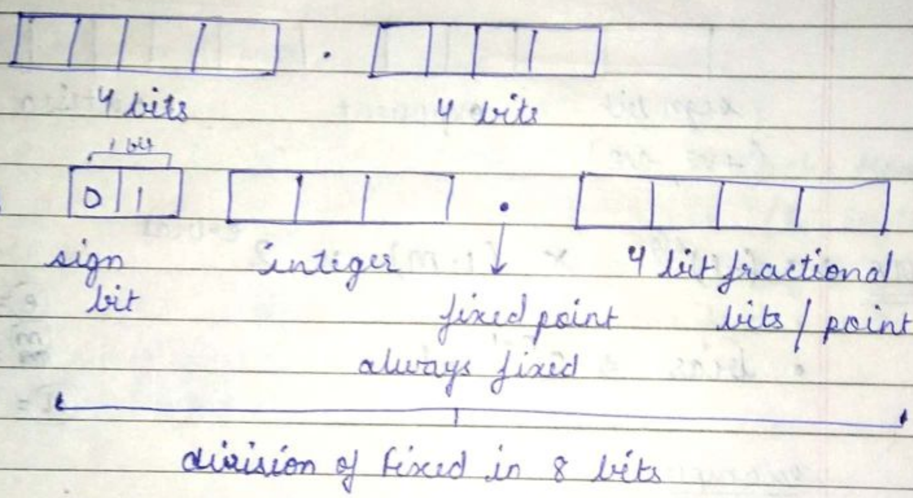
FIXED POINT and FLOATING POINT REPRESENTATION

• **FIXED** ⇒ MSB = denotes the number is positive or negitive

$\boxed{0}$ = +ve number
$\boxed{1}$ = -ve number
sign bit



Example 01 :

① $110 \cdot 1110 \Rightarrow 1\times2^2+1\times2^1+0\times2^0 \cdot 1\times2^{-1}+1\times2^{-2}+1\times2^{-3}+0\times2^{-4}$

↓

$6 \cdot \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\times0 \Rightarrow 6 \cdot \frac{1}{2}+\frac{1}{4}+\frac{1}{8}$

$\Rightarrow 6 \cdot \frac{4+2+1}{8} \Rightarrow 6 \cdot \frac{7}{8} \Rightarrow \underline{6.8}$

② $111 \cdot 1111 \Rightarrow 1\times2^2+1\times2^1+1\times2^0 \cdot 1\times2^{-1}+1\times2^{-2}+1\times2^{-3}+1\times2^{-4}$

↓

$7 \cdot \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16} \Rightarrow 7 \cdot \frac{8+4+2+1}{16} \Rightarrow 7 \cdot \frac{15}{16}$

$\Rightarrow \underline{7.9375}$

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* if 32 bits :

signed	integer	.	fractional
1 bit	15 bits		16 bits

- **FLOATING POINT :**

sign bit (+ve, -ve)	exponent	Mantissa

- FORMULA ⇒ • $(-1)^{sign} \times (1.M) \times 2^{e-bias}$
  - bias ⇒ $2^{n-1} - 1$

e = decimal value of exponent
m = mantissa
n = no. of bits in exponent

example :

sign bit	exponent	Mantissa
1	0 0 1	1 0 0 1

- bias = $2^{3-1} - 1$
  ⇒ $2^2 - 1$
  ⇒ $4 - 1 ⇒ 3$

⇒ $(-1)^1 \times (1.1001) \times 2^{1-3}$
⇒ $(-1)^1 \times (1.1001) \times 2^{-2}$
⇒ $(-1.1001) \times 2^{-2}$

- conversion in decimal ⇒
  $-1 + 1\times 2^{-1} + 0\times 2^{-2} + 0\times 2^{-3} + 0 \times 2^{-4}$
  ⇒ $\frac{-1}{1} + \frac{1}{2} + \frac{1}{16}$ ⇒ $\frac{-16+8+1}{16}$ ⇒ $\frac{-7}{16} = -0.4375$