

16. PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. Inlet : A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

Outlet : A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2. (i) If a pipe can fill a tank in x hours, then :

$$\text{part filled in 1 hour} = \frac{1}{x}$$

(ii) If a pipe can empty a full tank in y hours, then :

$$\text{part emptied in 1 hour} = \frac{1}{y}$$

(iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, the net part filled

$$\text{in 1 hour} = \left(\frac{1}{x} - \frac{1}{y} \right)$$

(iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$), then on opening both the pipes, the net part emptied

$$\text{in 1 hour} = \left(\frac{1}{y} - \frac{1}{x} \right)$$

SOLVED EXAMPLES

Ex. 1. Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Sol. Part filled by A in 1 hour = $\frac{1}{36}$. Part filled by B in 1 hour = $\frac{1}{45}$.

$$\text{Part filled by (A + B) in 1 hour} = \left(\frac{1}{36} + \frac{1}{45} \right) = \frac{9}{180} = \frac{1}{20}$$

Hence, both the pipes together will fill the tank in 20 hours.

Ex. 2. Two pipes can fill a tank in 10 hours and 12 hours respectively while a third pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?

$$\text{Sol. Net part filled in 1 hour} = \left(\frac{1}{10} + \frac{1}{12} - \frac{1}{20} \right) = \frac{8}{60} = \frac{2}{15}$$

$$\text{The tank will be full in } \frac{15}{2} \text{ hrs} = 7 \text{ hrs } 30 \text{ min.}$$

Ex. 3. If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?

Sol. Let the reservoir be filled by first pipe in x hours.
 Then, second pipe will fill it in $(x + 10)$ hours.

$$\begin{aligned} \therefore \frac{1}{x} + \frac{1}{(x+10)} &= \frac{1}{12} & \Leftrightarrow \frac{x+10+x}{x(x+10)} &= \frac{1}{12} \\ \Leftrightarrow x^2 - 14x - 120 &= 0 & \Leftrightarrow (x-20)(x+6) &= 0 \\ \Leftrightarrow x &= 20. & & \text{[neglecting the -ve value of } x] \end{aligned}$$

So, the second pipe will take $(20 + 10)$ hrs i.e., 30 hrs to fill the reservoir.

Ex. 4. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Sol. Work done by the waste pipe in 1 minute

$$= \frac{1}{20} - \left(\frac{1}{12} + \frac{1}{15} \right) = -\frac{1}{10} \quad \text{[-ve sign means emptying]}$$

∴ Waste pipe will empty the full cistern in 10 minutes.

Ex. 5. An electric pump can fill a tank in 3 hours. Because of a leak in the tank, it took $3\frac{1}{3}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

$$\text{Sol. Work done by the leak in 1 hour} = \left[\frac{1}{3} - \frac{1}{\left(\frac{10}{3}\right)} \right] = \left(\frac{1}{3} - \frac{3}{10} \right) = \frac{1}{21}$$

∴ The leak will empty the tank in 21 hours.

Ex. 6. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?

$$\text{Sol. Work done by the two pipes in 1 hour} = \left(\frac{1}{14} + \frac{1}{16} \right) = \frac{15}{112}$$

$$\therefore \text{Time taken by these pipes to fill the tank} = \frac{112}{15} \text{ hrs} = 7 \text{ hrs } 28 \text{ min.}$$

Due to leakage, time taken = 7 hrs 28 min + 32 min = 8 hrs

$$\therefore \text{Work done by (two pipes + leak) in 1 hour} = \frac{1}{8}$$

$$\text{Work done by the leak in 1 hour} = \left(\frac{15}{112} - \frac{1}{8} \right) = \frac{1}{112}$$

∴ Leak will empty the full cistern in 112 hours.

Ex. 7. Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. After 7 minutes, C is also opened. In how much time, the tank is full?

$$\text{Sol. Part filled in 7 min.} = 7 \left(\frac{1}{36} + \frac{1}{45} \right) = \frac{7}{20}$$

$$\text{Remaining part} = \left(1 - \frac{7}{20} \right) = \frac{13}{20}$$

Net part filled in 1 min. when A, B and C are opened = $\left(\frac{1}{36} + \frac{1}{45} - \frac{1}{30}\right) = \frac{1}{60}$.

Now, $\frac{1}{60}$ part is filled in 1 min.

$\frac{13}{20}$ part is filled in $\left(60 \times \frac{13}{20}\right) = 39$ min.

Total time taken to fill the tank = $(39 + 7)$ min. = 46 min.

Ex. 8. Two pipes A and B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 minutes?

Sol. Let B be closed after x minutes. Then,

part filled by $(A + B)$ in x min. + part filled by A in $(18 - x)$ min. = 1

$$x\left(\frac{1}{24} + \frac{1}{32}\right) + (18 - x) \times \frac{1}{24} = 1 \quad \Leftrightarrow \quad \frac{7x}{96} + \frac{18 - x}{24} = 1$$
$$\Leftrightarrow 7x + 4(18 - x) = 96 \quad \Leftrightarrow \quad x = 8.$$

Hence, B must be closed after 8 minutes.

EXERCISE 16A

(OBJECTIVE TYPE QUESTIONS)

Directions : Mark (✓) against the correct answer :

1. Two pipes A and B can fill a tank in 20 and 30 minutes respectively. If both the pipes are used together, then how long will it take to fill the tank? (M.A.T. 2003)
(a) 12 min (b) 15 min (c) 25 min (d) 50 min
2. A cistern can be filled by a tap in 4 hours while it can be emptied by another tap in 9 hours. If both the taps are opened simultaneously, then after how much time will the cistern get filled? (Hotel Management, 1997)
(a) 4.5 hrs (b) 5 hrs (c) 6.5 hrs (d) 7.2 hrs
3. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?
(a) 3 hrs 15 min (b) 3 hrs 45 min (c) 4 hrs (d) 4 hrs 15 min
(S.S.C. 2003)
4. A water tank is two-fifth full. Pipe A can fill a tank in 10 minutes and pipe B can empty it in 6 minutes. If both the pipes are open, how long will it take to empty or fill the tank completely? (Bank P.O. 1999)
(a) 6 min. to empty (b) 6 min. to fill (c) 9 min. to empty
(d) 9 min. to fill (e) None of these
5. Pipe A can fill a tank in 5 hours, pipe B in 10 hours and pipe C in 30 hours. If all the pipes are open, in how many hours will the tank be filled? (C.B.I. 1997)
(a) 2 (b) 2.5 (c) 3 (d) 3.5
6. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in:
(a) $1\frac{13}{17}$ hours (b) $2\frac{8}{11}$ hours (c) $3\frac{9}{17}$ hours (d) $4\frac{1}{2}$ hours
(Bank P.O. 2002)

7. Three pipes A, B and C can fill a tank from empty to full in 30 minutes, 20 minutes and 10 minutes respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P, Q and R respectively. What is the proportion of solution R in the liquid in the tank after 3 minutes? (D.M.R.C. 2003)
- (a) $\frac{5}{11}$ (b) $\frac{6}{11}$ (c) $\frac{7}{11}$ (d) $\frac{8}{11}$
8. Two pipes A and B can separately fill a cistern in 60 minutes and 75 minutes respectively. There is a third pipe in the bottom of the cistern to empty it. If all the three pipes are simultaneously opened, then the cistern is full in 50 minutes. In how much time, the third pipe alone can empty the cistern? (S.S.C. 2003)
- (a) 90 min (b) 100 min (c) 110 min (d) 120 min
9. A pump can fill a tank with water in 2 hours. Because of a leak, it took $2\frac{1}{3}$ hours to fill the tank. The leak can drain all the water of the tank in : (S.S.C. 2002)
- (a) $4\frac{1}{3}$ hrs (b) 7 hrs (c) 8 hrs (d) 14 hrs
10. Two taps A and B can fill a tank in 5 hours and 20 hours respectively. If both the taps are open then due to a leakage, it took 30 minutes more to fill the tank. If the tank is full, how long will it take for the leakage alone to empty the tank?
- (a) $4\frac{1}{2}$ hrs (b) 9 hrs (c) 18 hrs (d) 36 hrs
11. Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A to fill the cistern separately? (NABARD, 2001)
- (a) 1 hr (b) 2 hrs (c) 6 hrs (d) 8 hrs
12. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 minutes, then the slower pipe alone will be able to fill the tank in : (C.B.I. 2003)
- (a) 81 min (b) 108 min (c) 144 min (d) 192 min
13. A tank is filled in 5 hours by three pipes A, B and C. The pipe C is twice as fast as B and B is twice as fast as A. How much time will pipe A alone take to fill the tank?
- (a) 20 hrs (b) 25 hrs (c) 35 hrs
(d) Cannot be determined (e) None of these (Bank P.O. 2003)
14. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is:
- (a) 6 hrs (b) 10 hrs (c) 15 hrs (d) 30 hrs (M.B.A. 2002)
15. 12 buckets of water fill a tank when the capacity of each tank is 13.5 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 9 litres?
- (a) 8 (b) 15 (c) 16 (d) 18
16. Bucket P has thrice the capacity as bucket Q. It takes 30 turns for bucket P to fill the empty drum. How many turns it will take for both the buckets P and Q, having each turn together to fill the empty drum?
- (a) 30 (b) 40 (c) 45 (d) 90
17. Two pipes A and B can fill a tank in 12 minutes and 15 minutes respectively. If both the taps are opened simultaneously and the tap A is closed after 3 minutes, then how much more time will it take to fill the tank by tap B?
- (a) 7 min 15 sec (b) 7 min 45 sec (c) 8 min 5 sec (d) 8 min 15 sec