PERMUTATIONS and COMBINATIONS

the different arrangement of a given number of things but by taking some or all and at a time are called Permutations.

example:

All permutations make with the letters a, b, c by taking 2 at a time.

(ab, bc, ac, ca, cb, ba)

- No. of all permutations of n things taken r at a time is given by ${}^nP_r$.

$${}^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

ABC

(n) total numbers

(2) numbers in pair

- NOTE: No. of all Permutations of n things taken all at a time is $\lfloor n$

$${}^5L_4 = \frac{\lfloor 5}{\lfloor 5-4} \Rightarrow \frac{\lfloor 5}{\lfloor 1} \Rightarrow \frac{5\times4\times3\times2\times1}{1}$$

$$\Rightarrow 120$$

Q: Evaluate: $\frac{\lfloor 30}{\lfloor 28}$:

$$\Rightarrow \frac{30\times29\times\lfloor 28}{\lfloor 28} \Rightarrow 30\times29 \Rightarrow 870$$

Q: Find the value of ${}^{60}P_{28}$:

$$\Rightarrow {}^{60}P_{28} = \frac{\lfloor 60}{\lfloor 60-28} \Rightarrow \frac{\lfloor 60}{\lfloor 32} \Rightarrow \frac{60\times\cdots\lfloor 32}{\lfloor 32}$$

★ Factorial :

Let n be a positive number (integers) than $\lfloor n$ denoted by $\lfloor n$, n! that is defined as :

$$\lfloor n = (n-1)(n-2) \ldots \ldots$$

$$\Rightarrow \lfloor n = \frac{\lfloor n}{\lfloor n-r}$$

example : $3(3-1)(3-2)$ $\qquad \lfloor 0 = 1$

$\Rightarrow 3 \times 2 \times 1 \Rightarrow \underline{6}$

- $\lfloor 2 = 2$ $\Rightarrow 2$
- $\lfloor 4 = 24$, • $\lfloor 5 = 120$
- $\lfloor 6 = 720$

★ Combinations :

Each of the different groups of selection which can be formed by taking some or all of a number of objects is called combinations.

example : suppose we want to select two out of three A, B, C. Than possible selection (AB, BC, CA). Note that AB and BA represent the same selection.

- The number of all combinations of n things taken r at a time is ^nC_r

$$\Rightarrow {}^nC_r = \frac{\lfloor n}{\lfloor r \, \lfloor n-r}$$

NOTE :

$\Rightarrow {}^nC_n = 1$

$\Rightarrow {}^nC_0 = 1$

Q: Find the value of $^{10}C_3$:

$$\Rightarrow {}^{10}C_3 = \frac{\lfloor 10}{\lfloor 3 \, \lfloor 10-3} \Rightarrow \frac{\lfloor 10}{\lfloor 3 \, \lfloor 7}$$

$$\Rightarrow \frac{\lfloor 10}{\lfloor 3 \times \lfloor 7} = \frac{10 \times 9 \times 8 \times \cancel{\lfloor 7}}{\lfloor 3 \, \cancel{\lfloor 7}} = \frac{10 \times 9 \times 8}{\lfloor 3}$$

$$\Rightarrow \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \Rightarrow \frac{720}{6} = \underline{120}$$

★ In how many ways can a cricket team be selected out of 15 players :

$$\Rightarrow {}^{15}C_{11} = \frac{\lfloor 15}{\lfloor 11 \, \lfloor 15-11} \Rightarrow \frac{\lfloor 15}{\lfloor 11 \, \lfloor 04}$$

$$\Rightarrow \frac{15 \times 14 \times 13 \times 12 \times \cancel{\lfloor 11}}{\cancel{\lfloor 11} \, \lfloor 4} = \frac{15 \times 14 \times 13 \times 12}{\lfloor 4}$$

$$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = \frac{15 \times \cancel{14}^{7} \times 13 \times \cancel{12}^{1}}{\cancel{24}_{2}}$$

$$\Rightarrow 15 \times 7 \times 13 \Rightarrow \underline{1365}$$

Q: In how many ways a committee of 6 members can be selected from 6 men and 5 womens. Consisting of 3 men and 2 womens.

$$^6C_3 \times {}^5C_2$$

and → ×
or → +

$\Rightarrow \frac{L6}{L3\,L3} \times \frac{L5}{L3\,L2} \Rightarrow \frac{6 \times 5 \times 4 \times L3}{L3\;L3} \times$

$\Rightarrow \frac{5 \times 4 \times 3 \times L2}{L3\;L2}$

$\Rightarrow \frac{6 \times 5 \times 4}{L3} \times \frac{5 \times 4 \times 3}{L3} \Rightarrow \frac{6 \times 5 \times 4}{6} \times \frac{5 \times 4 \times 3}{6_2}$

$\Rightarrow 20 \times \frac{20}{2} \Rightarrow 20 \times 10 \Rightarrow \underline{200}$

Q: How many words can be formed by using all the letters of the word BIHAR ?

$\Rightarrow L5 \cdot L5$

$\Rightarrow L5 \Rightarrow 5 \times 4 \times 3 \times 2 \times 1$

$\Rightarrow 40 \times 3 \Rightarrow \underline{120}$

* MUMBAI

$\frac{L6}{L2} \Rightarrow \underline{360}$

12.10.2019

PERCENTAGE

* to express $\frac{a}{b}$ as a percent :

$$\left(\frac{a}{b} \times 100\right)\%$$

- Ex: $\left(\frac{2}{3} \times 100\right)\% = 33+33 = 66\%$, $\left(\frac{1}{3} \times 100\right)\% = 33\%$

* to express $x\%$ as a expression :

$$\left(x\% = x \times \frac{1}{100}\right)$$

- Ex: $\left(2\% = 2 \times \frac{1}{100}\right)$, $\left(5\% = 5 \times \frac{1}{100}\right)$

Q: EVALUATE 28% of 450 (+) 45% of 280

$\Rightarrow 45\cancel{0} \times \frac{28}{10\cancel{0}} \;(+)\; 28\cancel{0} \times \frac{45}{10\cancel{0}}$

$\Rightarrow 126 + 126 = \underline{252}$

Q: 2 is what percent of 50 :

$\frac{2}{50} \times 100^{2} \Rightarrow 2 \times 2 = \underline{4\%}$

Q: difference of 2 numbers is 1660, if 7.5% of one number is 12.5% of the other number . Find the 2 numbers :

Ans: $x - y = 1660$

$\frac{7.5}{100} \times x = \frac{12.5}{100} \times y \Rightarrow 75x = 125x \Rightarrow 3x = 5y$

LCM

$\therefore x = \frac{5}{3}y \Rightarrow \frac{5}{3}y - y = 1660 \Rightarrow 5y - 3y = 4980$

$\Rightarrow y = \frac{4980}{2} \Rightarrow \underline{2490} \Rightarrow x = 2490 \times \frac{5}{3} = 830 \times 5 = \underline{4150}$

Q: if the GST be reduced from $\frac{7}{2}$ % to $\frac{10}{3}$ % then what difference does it make to a person to purchase an article with price ₹ 8400.

Ans: $8400 \times \frac{7}{2} \times \frac{1}{100} = ?$

$8400 \times \frac{10}{3} \times \frac{1}{100} = ?$] +

$\Rightarrow \frac{8400}{100}\left[\frac{7}{2} - \frac{10}{3}\right] = 84\left[\frac{21-20}{6}\right]$

$\Rightarrow 84 \times \frac{1}{6} \Rightarrow \frac{84}{6} \Rightarrow 14$

(or)

$\rightarrow 8400\left[\frac{7}{2} - \frac{10}{3}\right] = 8400\left[\frac{1}{6}\right] \Rightarrow \frac{8400}{6}$

$\Rightarrow 1400$.

12-10-2019

SIMPLE INTEREST

$+\left(\frac{P \times R \times t}{100}\right)$

P = Principle Amount
R = Rate of Interest
T = Time

Q: Find the simple Interest on ₹ 68000 at the rate $\frac{50}{3}$ % per annum for 9 months.

Ans: $\frac{68000 \times \frac{50}{3} \times \frac{9}{12}}{100} \Rightarrow \frac{680 \times \frac{50}{3} \times \frac{3}{4}}{1} \Rightarrow 170 \times 50$

$\Rightarrow 8500 = SI$

Q: Find the SI on ₹ 3000 @ $\frac{25}{4}$ % per annum for the period from 4th february 2005 to 18th April 2005

Ans - 24+31+18 = 73 days $\Rightarrow \frac{73}{365} \Rightarrow \frac{1}{5} \Rightarrow \frac{1}{5}$ yrs

$\Rightarrow \frac{3000 \times \frac{25}{4} \times \frac{1}{5}}{100} \Rightarrow 15 \times \frac{5}{2} \Rightarrow \frac{75}{2}$

$\Rightarrow 37.5 = SI$

COMPOUND INTEREST

- Compound Interest = A - P
- (A) $= P\left(1 + \frac{r}{100}\right)^n$

A = amount including the interest
P = Principle amount
R = Rate of Interest
T = Time

(n) = time Period

Q: Find C.I on ₹ 7500 at 4% per annum for 2 years, compounded anually?

Condition : annually

+ CASE-1

P = 7500 , N = 2

R = 4

$\therefore A = 7500\left(1+\frac{4}{100}\right)^2 \therefore 7500\left(1+\frac{1}{25}\right)^2$

$\therefore 7500\left(\frac{26}{25}\right)^2 \therefore \overset{300}{\cancel{7500}} \times \frac{26}{\cancel{25}_1} \times \frac{26}{\cancel{25}_1}$

$\therefore 12 \times 26 \times 26 \therefore 12 \times 676 \therefore 8112$

$\therefore C.I = A - P \therefore 8112 - 7500 \therefore 612$

+ CASE-2

- when the interest compounded half yearly :
- (A) $= P\left(1+\frac{r/2}{100}\right)^{2n}$

Q: Find C.I on ₹ 10000 in 2 years @ 4% per annum. the interest compounded half yearly.

P = 10000 , R = 4/2

N = 2×2 = 4 , condition = half yearly.

$\therefore A \therefore \frac{7500+2500}{10000}\left(1+\frac{\frac{4}{2}}{100}\right)^4 \therefore 10000\left(1+\frac{4}{200}\right)^4$

$\therefore 10000\left(\frac{1+50}{50}\right)^4 \therefore 10000\left(\frac{51}{50}\right)^4$

$\therefore 10000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{50}{50} \times \frac{51}{50} \therefore \frac{51}{5} \times \frac{51}{5} \times \frac{51}{5} \times \frac{51}{5}$

$\therefore \frac{6765201}{625} \therefore 10824.32$

$\therefore C.I \therefore A - P \therefore 10824.32 - 10000 \therefore 824.32$

+ CASE-3

- when the interest compounded quarterly :
- (A) $= P\left(1+\frac{\frac{r}{4}}{100}\right)^{4n}$

Q: Find the C.I on ₹ 16000 at 20% per annum. Find the interest compounded quarterly : 9 months

P = 16000 , N = 4×9 $\therefore$ N = 3 years

R = 20% , condition = quarterly.

- $\therefore A = 16000\left(1+\frac{\frac{20}{4}}{100}\right)^3 \therefore 16000\left(1+\frac{20}{400}\right)^3$

$\therefore 16000\left(\frac{1}{1}+\frac{1}{20}\right)^3 \therefore 16000\left(\frac{20+1}{20}\right)^3$

$\therefore 16000\left(\frac{21}{20}\right)^3 \therefore \overset{8}{\cancel{16000}} \times \frac{21}{\cancel{20}_1} \times \frac{21}{\cancel{20}_1} \times \frac{21}{\cancel{20}_1}$

$\therefore 2 \times 9261 \therefore 18522$

- $\therefore C.I \therefore A - P$

$\therefore 18522 - 16000$

$\therefore 2522$

Q: If the S.I. on the sum of Money @ 5% p.a. for 3 years is ₹ 1200. Find the C.I on the same sum for the same time and the same interest. (condition - yearly).

$\Rightarrow$ S.I. $= \frac{P \times R \times T}{100} \Rightarrow 1200 = \frac{P \times 5 \times 3}{100}$

$\Rightarrow 120000 = 15P \Rightarrow P = \frac{120000}{15} \Rightarrow P = 8000$

$\Rightarrow$ C.I $A = P\left(1 + \frac{r}{100}\right)^n \Rightarrow 8000\left(1 + \frac{5}{100}\right)^3$

$\Rightarrow 8000\left(1 + \frac{1}{20}\right)^3 \Rightarrow 8000\left(\frac{21}{20}\right)^3$

$\Rightarrow 8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \Rightarrow 9261$

$\Rightarrow 9261 - 8000 \Rightarrow 1261$

Q: In what time will ₹ 1000 become ₹ 1331 @ 10% p.a. compounded annually.

$\Rightarrow 1331 = 1000\left(1 + \frac{10}{100}\right)^n \Rightarrow 1331 = 1000\left(\frac{11}{10}\right)^n$

$\Rightarrow \frac{1331}{1000} = \left(\frac{11}{10}\right)^n \Rightarrow \left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n$

$\Rightarrow n = 3$

Q: A certain sum amounts to ₹ 7380 in 2 years and ₹ 8575 in 3 years. Find the sum and interest.

Ans: Let the sum be x ₹ $\quad 8575 - 7380 = 1195$

$\Rightarrow$ S.I $\Rightarrow \frac{7380 \times R \times 1}{100} \Rightarrow \frac{7380R}{100} = 1195$

$\Rightarrow R = 16.2 \Rightarrow 16\frac{2}{3} \Rightarrow \frac{50}{3} = 16.19$

$A = P\left(1 + \frac{r}{100}\right)^n \Rightarrow 7380 = x\left(1 + \frac{50}{\frac{3}{100}}\right)^2$

$\Rightarrow 7380 = x\left(1 + \frac{1}{6}\right)^2 \Rightarrow 7380 = x\left(\frac{7}{6}\right)^2$

$7380 \Rightarrow x \times \frac{7}{6} \times \frac{7}{6} \Rightarrow x \Rightarrow \frac{7380 \times 6 \times 6}{7 \times 7} \Rightarrow \frac{265680}{49}$

$x \Rightarrow \frac{265680}{49} \Rightarrow 5422.$

Q: A sum of money to ₹ 6690 after 3 years and to ₹ 10035 after 6 years on compound interest. Find the sum.

Ans: $10035 - 6690 \Rightarrow$ ₹ 3345 · First 3 years · 0-3.

SI $\Rightarrow \frac{6690 \times R \times 3}{100} \Rightarrow 3345 = \frac{20070 \times R}{100}$

$\Rightarrow 3345 = 20.07R \Rightarrow R = \frac{3345}{20.07}$

$\Rightarrow 16.66 \quad \frac{1666}{100} \quad \frac{32}{66} \quad 16\frac{33}{50}$

$10035 = x\left(1 + \frac{r}{100}\right)^6 \Rightarrow$

PROBABILITY

11/11/2019

$$P(A) = \frac{m}{n}$$

where m is the no. of cases favourable to the event and n is the total no. of cases.

Q: Two unbiased dices are thrown. Find the probability:

i) both the dices show the same number.

ii) the first dices throws 6.

iii) the total of numbers on the dices > 8.

iv) the total of the numbers on the dices is 13.

Ans:

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

i) $\frac{6}{36} = \frac{1}{6}$ $\therefore \frac{m}{n} = P(A)$

ii) $\frac{6}{36} = \frac{1}{6}$ $\therefore \frac{m}{n} = P(A)$

iii) $\frac{10}{36} = \frac{5}{18}$ $\therefore \frac{m}{n} = P(A)$

iv) The probability of impossible events is always 0.

Q: What is the probability of getting atmost 2 heads in tossing of 3 coins.

Total Case = 8.

(H,H,H) H,H,T H,T,H T,H,H H,T,T T,T,T T,T,H T,H,T

$\therefore \frac{m}{n} = \frac{7}{8} = P(A)$

Q: What is the chance that a leap year selected at random will contain 53 sunday.

∴ In a leap year there are 366 days, 52 complete weeks and 2 more days.

S, M	1
M, T	2
T, W	3
W, T	4
T, F	5
F, S	6
S, S	7

$\therefore \frac{m}{n} = \frac{2}{7} = P(A)$

Q: A, B and C are three events associated with a random experiment. Find P(A) given that P(B) is $\frac{3}{2}P(A)$ and P(C) is $\frac{1}{2}(P(B))$

Let $P(A) = P$

$P(B) = \frac{3}{2}P$

$P(C) = \frac{1}{2} \times \frac{3}{2}P$ ⇒ $P(C) = \frac{3}{4}P$.

NOTE: The probability of an event A is some number between and including 0 and 1.

Q: Probability of a certain event is always 1 therefore

$P(A) + P(\bar{A}) = 1$

$\therefore P(A) + P(B) + P(C) = 1$ ⇒ $P + \frac{3}{2}P + \frac{3}{4}P = 1$

$$\Rightarrow \frac{4P + 6P + 3P}{4} = 1 \quad \Rightarrow P = \frac{4}{13}$$

* **THEOREM** : If A and B are any two events then

$$P(A \cup B) = P(A) + (P(B)) - P(A \cap B)$$

OR

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Q: An integer is taken at random from the first 200 (+ve) integers. What is the probability that the integer is divisible by 6 and 8.

2, 4, 6, 8, 10, 12, 18 ... 200

6, 12, 18, (24), 30, 36, 42, (48), 54, 60, 66, (72), 78, 84, 90, (96), 102, 108, 114, (120), 126, 132, 138, (144), 150, 156, 162, (168), 174, 180, 186, (192), 198.

- $\Rightarrow P(A) = \frac{33}{200}$, $P(B) = \frac{25}{200}$
- $\Rightarrow P(A \cap B) = \frac{8}{200}$
- $\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow = \left(\frac{33}{200} + \frac{25}{200}\right) - \frac{8}{200} \Rightarrow \frac{50}{200} = \frac{1}{4}$