

PERMUTATIONS and COMBINATIONS

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the different arrangement of a given number of things but by taking same or all and at a time are called Permutations.

example:

all permutations make with the letters a, b, c by taking 2 at a time.

(ab, bc, ac, ca, cb, ba)

- No. of all permutations of n things taken r at a time is given by ${}^n P_r$.

$${}^n P_r = \frac{{}^n L_n}{{}^n L_{n-r}}$$

- NOTE: No. of all Permutations of n things taken all at a time is ${}^n L_n$

$${}^5 L_4 = \frac{{}^5 L_5}{{}^{5-4} L_1} \Rightarrow \frac{{}^5 L_5}{{}^1 L_1} \Rightarrow \frac{5 \times 4 \times 3 \times 2 \times 1}{1}$$

$$\Rightarrow 120$$

Q: Evaluate: $\frac{{}^3 L_{30}}{{}^2 L_{28}}$

$$\Rightarrow \frac{30 \times 29 \times {}^2 L_{28}}{{}^2 L_{28}} \Rightarrow 30 \times 29 \Rightarrow 870$$

Q: Find the value of ${}^{60} P_{28}$:

$$\Rightarrow {}^{60} P_{28} = \frac{{}^6 L_{60}}{{}^{60-28} L_{32}} \Rightarrow \frac{{}^6 L_{60}}{{}^{32} L_{32}} \Rightarrow \frac{60 \times \dots \times {}^6 L_{32}}{{}^{32} L_{32}}$$

* Factorial :

Let n be a positive number (integers) then $n!$ denoted by $n!$, $n!$ that is defined as :

$$n! = (n-1)(n-2) \dots$$

$$\Rightarrow n! = \frac{n!}{n! - n}$$

example : $3(3-1)(3-2)$

$$3! = 1$$

$$\Rightarrow 3 \times 2 \times 1 = \underline{\underline{6}}$$

$$\bullet \quad 2! = 2$$

$$\bullet \quad 4! = 24, \quad \bullet \quad 5! = 120$$

$$\bullet \quad 6! = 720$$

* Combinations :

Each of the different groups of selection which can be formed by taking some or all of a number of objects is called combinations.

example : suppose we want to select two out of three A, B, C, then possible selection (AB, BC, CA). Note that AB and BA represent the same selection.

• The number of all combinations of n things taken r at a time is nC_r .

$$\Rightarrow {}^nC_r = \frac{n!}{r! (n-r)!}$$

NOTE :

$$\bullet \quad {}^nC_n = 1$$

$$\bullet \quad {}^nC_0 = 1$$

Q: Find the value of ${}^{10}C_3$:

$$\Rightarrow {}^{10}C_3 = \frac{10!}{3! (10-3)!} \Rightarrow \frac{10!}{3! 7!}$$

$$\Rightarrow \frac{10!}{3! \times 7!} = \frac{10 \times 9 \times 8 \times \cancel{7!}}{3! \times \cancel{7!}} = \frac{10 \times 9 \times 8}{3!}$$

$$\Rightarrow \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = \frac{720}{6} = \underline{\underline{120}}$$

* In how many ways can a cricket team be selected out of 15 players :

$$\Rightarrow {}^{15}C_{11} = \frac{15!}{11! (15-11)!} = \frac{15!}{11! 4!}$$

$$\Rightarrow \frac{15 \times 14 \times 13 \times 12 \times \cancel{11!}}{\cancel{11!} 4!} = \frac{15 \times 14 \times 13 \times 12}{4!}$$

$$\Rightarrow \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = \frac{15 \times 7 \times 13 \times 1}{24} = 1365$$

$$\Rightarrow 15 \times 7 \times 13 = \underline{\underline{1365}}$$

Q: In how many ways a committee of 5 members can be selected from 6 men and 5 women, consisting of 3 men and 2 women.

$$\begin{matrix} 6 & \times & 5 \\ C & \times & C \\ 3 & \times & 2 \end{matrix}$$

and \bullet \bullet
or \bullet \bullet

$$\frac{L6}{L3L3} \times \frac{L5}{L3L2}$$

$$\frac{6 \times 5 \times 4 \times L3}{L3L3}$$

$$\frac{5 \times 4 \times 3 \times L2}{L3L2}$$

$$\frac{6 \times 5 \times 4}{L3} \times \frac{5 \times 4 \times 3}{L3} = \frac{6 \times 5 \times 4}{6} \times \frac{5 \times 4 \times 3}{6}$$

$$20 \times \frac{20}{2} = 20 \times 10 = \underline{\underline{200}}$$

Q: How many words can be formed by using all the letters of the word BIHAR?

$$L7 \cdot L5$$

$$L5 = 5 \times 4 \times 3 \times 2 \times 1$$

$$40 \times 3 = \underline{\underline{120}}$$

PERCENTAGE

12.10
2019

* to express $\frac{a}{b}$ as a percent :

$$\left(\frac{a}{b} \times 100 \right) \%$$

Ex: $\left(\frac{2}{3} \times 100 \right) \% = 33.33 = 33\frac{1}{3} \%$, $\left(\frac{1}{3} \times 100 \right) \% = 33\frac{1}{3} \%$

* To express $x\%$ as a expression :

$$(x\% = x \times \frac{1}{100})$$

Ex: $(2\% = 2 \times \frac{1}{100})$, $(5\% = 5 \times \frac{1}{100})$

Q: EVALUATE 28% of 450 (+) 45% of 280

$$450 \times \frac{28}{100} (+) 280 \times \frac{45}{100}$$

$$126 + 126 = \underline{\underline{252}}$$

Q: 2 is what percent of 50 :

$$\frac{2}{50} \times \frac{100}{1} = 2 \times 2 = \underline{\underline{4\%}}$$

Q: difference of 2 numbers is 1660, if 7.5% of one number is 12.5% of the other number. Find the 2 numbers :

Ans: $x - y = 1660$

$$\frac{7.5}{100} \times x = \frac{12.5}{100} \times y \Rightarrow 75x = 125y \Rightarrow 3x = 5y$$

$$x = \frac{5}{3}y$$

$$\frac{5}{3}y - y = 1660 \Rightarrow 5y - 3y = 4980$$

$$y = \frac{4980}{2} = \underline{\underline{2490}} \quad x = 2490 \times \frac{5}{3} = 830 \times 5 = \underline{\underline{4150}}$$

Q: if the GST be reduced from $\frac{7}{2}\%$ to $\frac{10}{3}\%$. then what difference does it make to a person to purchase an article with price ₹ 8400.

Ans: $8400 \times \frac{7}{2} \times \frac{1}{100} = ?$
 $8400 \times \frac{10}{3} \times \frac{1}{100} = ?$

$$8400 \left[\frac{7}{2} - \frac{10}{3} \right] = 84 \left[\frac{21-20}{6} \right]$$

$$84 \times \frac{1}{6} = \frac{84}{6} = 14$$

$$\rightarrow 8400 \left[\frac{7}{2} - \frac{10}{3} \right] = 8400 \left[\frac{1}{6} \right] = \frac{8400}{6}$$

$$= \underline{1400}$$

SIMPLE INTEREST

$$\left(\frac{P \times R \times T}{100} \right)$$

Principle Amount
Rate of Interest
Time

Q: Find the simple Interest on ₹ 68000 at the rate $\frac{50}{3}\%$ per annum for 9 Months.

Ans: $68000 \times \frac{50}{3} \times \frac{9}{12}$
 $\frac{68000 \times 50 \times 9}{3 \times 12} = \frac{68000 \times 50 \times 3}{4} = 170 \times 50$
 $= \underline{8500} = SI$

Q: Find the SI on ₹ 3000 @ $\frac{25}{4}\%$ per annum for the period from 4th february 2005 to 18th April 2005

Ans: $- 24 + 31 + 18 = 73 \text{ days}$

$$3000 \times \frac{25}{4} \times \frac{1}{5}$$

$$= \frac{3000 \times 25 \times 1}{4 \times 5} = \frac{3000 \times 5}{4} = 750 \times \frac{5}{2} = 1875$$

$$= \underline{37.5} = SI$$

COMPOUND INTEREST

• Compound Interest = $A - P$

• $A = P \left(1 + \frac{r}{100} \right)^n$

A = amount including the interest

P = Principle amount

R = Rate of Interest

T = Time

n = time Period

Q: Find C.I on ₹ 7500 at 4% per annum for 2 years, compounded annually?

Condition: annually

$$P = 7500, N = 2$$

$$R = 4$$

+ CASE-1

$$A = 7500 \left(1 + \frac{4}{100}\right)^2 = 7500 \left(1 + \frac{1}{25}\right)^2$$

$$= 7500 \left(\frac{26}{25}\right)^2 = 7500 \times \frac{26}{25} \times \frac{26}{25}$$

$$= 12 \times 26 \times 26 = 12 \times 676 = 8112$$

$$C.I = A - P = 8112 - 7500 = 612$$

+ CASE-2

• when the interest compounded half yearly:

$$A = P \left(1 + \frac{R/2}{100}\right)^{2n}$$

Q: Find C.I on ₹ 10000 in 2 years @ 4% per annum. the interest compounded half yearly.

$$P = 10000, R = 4/2$$

$$N = 2 \times 2 = 4, \text{ condition} = \text{half yearly.}$$

$$A = \frac{10000 + 2500}{10000} \left(1 + \frac{4}{200}\right)^4 = 10000 \left(1 + \frac{4}{200}\right)^4$$

$$= 10000 \left(\frac{51}{50}\right)^4 = 10000 \left(\frac{51}{50}\right)^4$$

$$= 10000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} = \frac{51 \times 51 \times 51 \times 51}{5 \times 5 \times 5 \times 5}$$

$$= \frac{6765201}{625} = 10824.32$$

$$C.I = A - P = 10824.32 - 10000 = 824.32$$

+ CASE-3

• when the interest compounded quarterly:

$$A = P \left(1 + \frac{R}{4}\right)^{4n}$$

Q: Find the C.I on ₹ 16000 at 20% per annum. Find the interest compounded quarterly: 9 months

$$P = 16000, N = 4 \times 9 = 36, N = 3 \text{ years}$$

$$R = 20\%, \text{ condition} = \text{quarterly.}$$

$$A = 16000 \left(1 + \frac{20}{4}\right)^3 = 16000 \left(1 + \frac{20}{400}\right)^3$$

$$= 16000 \left(\frac{21}{20}\right)^3 = 16000 \left(\frac{20+1}{20}\right)^3$$

$$= 16000 \left(\frac{21}{20}\right)^3 = 16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$= 2 \times 9261 = 18522$$

$$C.I = A - P$$

$$= 18522 - 16000$$

$$= 2522$$

Q: If the S.I. on the sum of Money @ 5% p.a. for 3 years is ₹ 1200. Find the C.I. on the same sum for the same time and the same interest. (condition - yearly).

$$\therefore \text{S.I.} = \frac{P \times R \times T}{100} \Rightarrow 1200 = \frac{P \times 5 \times 3}{100}$$

$$\Rightarrow 120000 = 15P \Rightarrow P = \frac{120000}{15} \Rightarrow P = 8000$$

$$\therefore \text{C.I. } A = P \left(1 + \frac{r}{100} \right)^n \Rightarrow 8000 \left(1 + \frac{5}{100} \right)^3$$

$$\Rightarrow 8000 \left(1 + \frac{1}{20} \right)^3 \Rightarrow 8000 \left(\frac{21}{20} \right)^3$$

$$\Rightarrow \cancel{8000}^4 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \Rightarrow 9261$$

$$\Rightarrow 9261 - 8000 \Rightarrow 1261$$

Q: In what time will ₹ 1000 become ₹ 1331 @ 10% p.a. compounded annually.

$$\therefore 1331 = 1000 \left(1 + \frac{10}{100} \right)^n \Rightarrow 1331 = 1000 \left(\frac{11}{10} \right)^n$$

$$\Rightarrow \frac{1331}{1000} = \left(\frac{11}{10} \right)^n \Rightarrow \left(\frac{11}{10} \right)^3 = \left(\frac{11}{10} \right)^n$$

$$\therefore n = 3$$

Q: A certain sum amounts to ₹ 7380 in 2 years and ₹ 8575 in 3 years. Find the sum and interest.

Ans: Let the sum be ₹

$$\therefore \text{S.I.} = \frac{7380 \times R \times 1}{100} \Rightarrow \frac{7380 R}{100} = 1195$$

$$\therefore R = \frac{16.2}{3} \Rightarrow \frac{16.2}{3} \Rightarrow \frac{50}{3} = 16.67$$

$$A = P \left(1 + \frac{r}{100} \right)^n \Rightarrow 7380 = x \left(1 + \frac{50}{3} \right)^2$$

$$\Rightarrow 7380 = x \left(1 + \frac{1}{6} \right)^2 \Rightarrow 7380 = x \left(\frac{7}{6} \right)^2$$

$$7380 = x \times \frac{7}{6} \times \frac{7}{6} \Rightarrow x = \frac{7380 \times 6 \times 6}{7 \times 7} \Rightarrow \frac{265680}{49}$$

$$x = \frac{265680}{49} \Rightarrow 5422$$

Q: A sum of money to ₹ 6690 after 3 years and to ₹ 10035 after 6 years on compound interest. Find the sum.

Ans: $10035 - 6690 = ₹ 3345$ First 3 years, 0-3.

$$\text{S.I.} = \frac{6690 \times R \times 3}{100} \Rightarrow 3345 = \frac{20070 \times R}{100}$$

$$\Rightarrow 3345 = 20.07 R \Rightarrow R = \frac{3345}{20.07}$$

$$\Rightarrow \frac{166.6}{100} = \frac{32}{100} = \frac{16}{50}$$

$$10035 = x \left(1 + \frac{16}{100} \right)^6$$