19/08/19

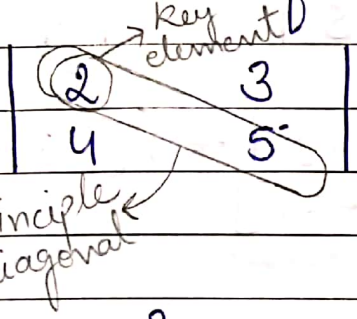
Determinant & Matrices

★ Symbols :

(i) | | → Determinant
(ii) [ ] → Matrices

★ m → no. of rows
n → no. of columns
} m x n

Ex



$2 = a_{11}$
$3 = a_{12}$
$4 = a_{21}$
$5 = a_{22}$

Multiple the Principle Diagonal

$a_{11} \times a_{22} - a_{12} \times a_{21}$
$2 \times 5 - 4 \times 3$
$10 - 12$
-2 Ans

★ Trignometry Property :

$\sin^2 x + \cos^2 x = 1$ ←

$\sin x$	$-\cos x$
$\cos x$	$\sin x$

Cross multiplied

It applied only in 3 X 3.

Ex $\begin{vmatrix} 2 & 6 & 1 \\ 3 & 7 & 4 \\ 4 & -1 & 5 \end{vmatrix}$ $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$= 2 \begin{vmatrix} 7 & 4 \\ -1 & 5 \end{vmatrix} - 6 \begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix} + 1 \begin{vmatrix} 3 & 7 \\ 4 & -1 \end{vmatrix}$

$= 2[35+4] - 6[15-16] + 1[-3-28]$

$= 2[39] - 6[-1] + 1[-31]$

$= 78 + 6 - 31$

$= 53$

★ Properties of Determinants:

1. In the determinant, if the row or a column interchange then the value of determinant will not be changed.
2. If two rows or two columns of a determinant are interchanged, the sign of the value of the determinant changes.
3. If two rows or two columns of a determinant are identical, then the value of the determinant is zero.

4. If all the elements of row or column of a determinant are zero, then the value of determinant is zero.

5. If the elements of any row or a column of a determinant be each multiplied by the same number. The determinant is multiply by that number.

★ Minor

- The minor of element is defined as a determinant obtained by deleting the row and column containing the element.

Ex

$$\begin{vmatrix} 2 & 3 & 4 \\ 6 & 2 & 8 \\ 7 & -1 & 10 \end{vmatrix}$$

$= M_{11} = \begin{vmatrix} 2 & 8 \\ -1 & 10 \end{vmatrix}$ $20-(-8)=28$

$M_{21} = \begin{vmatrix} 3 & 4 \\ -1 & 10 \end{vmatrix}$ $30-(-4)=34$

$= M_{12} = \begin{vmatrix} 6 & 8 \\ 7 & 10 \end{vmatrix}$ $60-56=4$

$M_{22} = \begin{vmatrix} 2 & 4 \\ 7 & 10 \end{vmatrix}$ $20-28=-8$

$= M_{13} = \begin{vmatrix} 6 & 2 \\ 7 & -1 \end{vmatrix}$ $-6-14=-20$

$M_{23} = \begin{vmatrix} 2 & 3 \\ 7 & -1 \end{vmatrix}$ $-2-21=-23$

$M_{31} = \begin{vmatrix} 3 & 4 \\ 2 & 8 \end{vmatrix}$

$24 - 8 = 16$

$M_{32} = \begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix}$

$16 - 24 = -8$

$M_{33} = \begin{vmatrix} 2 & 3 \\ 6 & 2 \end{vmatrix}$

$4 - 18 = -14$

★ Co-factor

Formula → Cofactor $= (-1)^{i+j} \times$ Minor

where,

i = no. of rows of element

j = no. of column of element

Ex

$C_{11} = (-1)^{1+1} \times 28 = 28$

$C_{12} = (-1)^{1+2} \times 4 = -4$

$C_{13} = (-1)^{1+3} \times -20 = -20$

$C_{21} = (-1)^{2+1} \times 34 = -34$

$C_{22} = (-1)^{2+2} \times -8 = -8$

$C_{23} = (-1)^{2+3} \times -23 = 23$

$C_{31} = (-1)^{3+1} \times 16 = 16$

$C_{32} = (-1)^{3+2} \times -8 = 8$

$C_{33} = (-1)^{3+3} \times -14 = -14$

→ Solution of simultaneously linear equation by using cramers rule.

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

Let us consider simultaneous linear equation when we solve such type of equation, first we find D using the coficient of unknown.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}$$

Ques 1) Solve the following equations by using cramer's rule:

$$2x - 3y + 4z = -9$$
$$-3x + 4y + 2z = -12$$
$$4x - 2y - 3z = -3$$

Ans $D = \begin{vmatrix} 2 & -3 & 4 \\ -3 & 4 & 2 \\ 4 & -2 & -3 \end{vmatrix}$

$D_1 = \begin{vmatrix} -9 & -3 & 4 \\ -12 & 4 & 2 \\ -3 & -2 & -3 \end{vmatrix}$

$D_2 = \begin{vmatrix} 2 & -9 & 4 \\ -3 & -12 & 2 \\ 4 & -3 & -3 \end{vmatrix}$

$D_3 = \begin{vmatrix} 2 & -3 & -9 \\ -3 & 4 & -12 \\ 4 & -2 & -3 \end{vmatrix}$

$D = 2[-12 + 4] + 3[9 - 8] + 4[6 - 16]$
$= -16 + 3 - 40$
$= -53$

$D_1 = -9[-12 + 4] + 3[36 + 6] + 4[24 + 12]$

$= 72 + 126 + 144$

$= 342$

$D_2 = 2[36+6] + 9[9-8] + 4[9+48]$

$= 84 + 9 + 228$

$= 93 + 228$

$= 321$

$D_3 = 2[-12-24] + 3[9+48] - 9[6-16]$

$= -72 + 171 + 90$

$= -72 + 261$

$= 189$

Ques 2) Solve the following equations using cramer's rule:

$5x - 7y + z = 11$

$6x - 8y - z = 15$

$3x + 2y - 6z = 7$

Ans $D = \begin{vmatrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{vmatrix}$

$= 5[48+2] + 7[-36+3] + 1[12+24]$

$= 250 - 231 + 36$

$= 286 - 231$

$= 55$

$$D_1 = \begin{vmatrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{vmatrix}$$

$$= 11[48+2] + 7[-90+7] + 1[30+56]$$
$$= 11[50] + 7[-83] + 1[86]$$
$$= 550 - 581 + 86$$
$$= 636 - 581$$
$$= 55$$

$$D_2 = \begin{vmatrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{vmatrix}$$

$$= 5[-90+7] - 11[-36+3] + 1[42-45]$$
$$= 5(-83) - 11(-33) - 3$$
$$= -415 + 363 - 3$$
$$= -418 + 363$$
$$= -55$$

$$D_3 = \begin{vmatrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{vmatrix}$$

$$= 5[-56-30] + 7[42-45] + 11[12+24]$$
$$= 5[-86] + 7[-3] + 11[36]$$
$$= -430 - 21 + 396 = -451 + 396$$
$$= -55$$

$$D = 55^{-}, \quad D_1 = 55, \quad D_2 = -55, \quad D_3 = -5^{-}5^{-}$$

$$x = \frac{D_1}{D} = \frac{55}{5^{-}5} = 1$$

$$y = \frac{D_2}{D} = \frac{-55}{55} = -1$$

$$z = \frac{D_3}{D} = \frac{-55^{-}}{55^{-}} = -1$$

Hence, $x = 1, \; y = -1, \; y = -1$

★ Matrices

- Let us consider the set of simultaneous equations:

$$3x + y + 4z = 0$$
$$2x + 3y + 8z = 0$$
$$5x + 4y + 7z = 0$$

Now we write down the co-efficient of x, y, z of the above equations & enclose them with a bracket & then we get:

$$\begin{bmatrix} 3 & 1 & 4 \\ 2 & 3 & 8 \\ 5^{-} & 4 & 7 \end{bmatrix}$$

The above system of numbers arranged in a rectangular array in rows and columns and bounded by the brackets is called a matrix.

* Types of Matrices

1. Row Matrix → If a matrix has only one row and any no. of columns is called row matrix.

Eg:

$$\begin{bmatrix} 2 & 4 & 6 & 7 \end{bmatrix}_{1\times 4}$$

2. Column Matrix → If a matrix has only one column and any no. of rows is called column matrix.

Eg:

$$\begin{bmatrix} 2 \\ 4 \\ 6 \\ 7 \end{bmatrix}_{4\times 1}$$

3. NULL Matrix / Zero Matrix → Any matrix in which all the elements are zero is called null or zero matrix.

Eg:

$$A \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$$

4. Square Matrix → A matrix in which the no. of columns is equal to the no. of rows is called square matrix.

Eg:

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

5. Diagonal Matrix → A square matrix is called diagonal matrix if all non-diagonal elements are zero.

Eg:

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

6. Unity / Identity Matrix → A square matrix is called unit / identity matrix if all the diagonal elements are unity (1) and non-diagonal elements are zero and this is denoted by I.

Eg:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$$

* Algebra Matrices

1.) Addition

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -7 & 10 \\ 8 & 6 \end{bmatrix}$

$$A+B = \begin{bmatrix} 2-7 & -1+10 \\ 3+8 & 4+6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 9 \\ 11 & 10 \end{bmatrix}$$

2.) Subtraction

$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -7 & 10 \\ 8 & 6 \end{bmatrix}$

$$A-B = \begin{bmatrix} 2+7 & -1-10 \\ 3-8 & 4-6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 11 \\ -5 & -2 \end{bmatrix}$$

3. Multiply → The product of two matrices is only possible if the no. of columns is equal to the no. of rows

If $A = \begin{bmatrix} -1 & 3 \\ 2 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

find A and B & show that $AB \neq BA$

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -1\times6+3\times7 & -1\times8+3\times9 \\ 2\times6+6\times7 & 2\times8+6\times9 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 19 \\ 54 & 70 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} \quad \begin{bmatrix} -1 & 3 \\ 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6\times-1+8\times2 & 6\times3+8\times6 \\ 7\times-1+9\times2 & 7\times3+9\times6 \end{bmatrix}$$

$$= \begin{bmatrix} -6+16 & 18+48 \\ -7+18 & 21+54 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 66 \\ 11 & 75 \end{bmatrix}$$

Ques:) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$

Obtain the products A & B and explain why BA is not defined

Ans

$$\begin{bmatrix} 0\times1+1\times-1+2\times2 & 0\times-2+1\times0+2\times-1 \\ 1\times1+2\times-1+3\times2 & 1\times-2+2\times0+3\times-1 \\ 2\times1+3\times-1+4\times2 & 2\times-2+3\times0+4\times-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+-1+4 & +0+-2 \\ 1+-2+6 & -2+0+-3 \\ 2+-3+8 & -4+0+-4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ 5 & -5 \\ 7 & -8 \end{bmatrix}$$

The no. of columns in matrix B is 2 & no. of matrix in A is 3 therefore, BA is not defined.

* Scalar Multiplications

Ques:) If $A = \begin{bmatrix} 5 & 10 & 12 \\ -2 & 6 & 13 \\ 3 & 8 & 4 \end{bmatrix}$

Find 5A ?

Ans $5A = 5\begin{bmatrix} 5 & 10 & 12 \\ -2 & 6 & 13 \\ 3 & 8 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 25 & 50 & 60 \\ -10 & 30 & 65 \\ 15 & 40 & 20 \end{bmatrix}$$

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Inverse of a Matrix

(Refer: Topic Minor & Co-factor)

Formula:

$$(A^{-1}) = \frac{adj.A}{|A|}, \quad |A| \neq 0$$

Ques Find the inverse of a matrix:

(a) $\begin{bmatrix} 2 & 3 & 4 \\ 6 & 2 & 8 \\ 7 & -1 & 10 \end{bmatrix}$

Sol $|A| = 2[20+8] - 3[60-56] + 4[-6-14]$

$= 56 - 12 - 80$

$= -36 \neq 0$

Co-factors:

$$\begin{bmatrix} 28 & -4 & -20 \\ -34 & -8 & 23 \\ 16 & 8 & -14 \end{bmatrix}$$

Matrix obtained by the co-factors of A.

$$\text{Adj of } A = \begin{bmatrix} 28 & -34 & 16 \\ -4 & -8 & 8 \\ -20 & 23 & -14 \end{bmatrix}$$

Note: Above matrix is formed by transpose of row into column.

$$A^{-1} = \frac{\text{adj. } A}{|A|}$$

$$A^{-1} = \frac{\begin{bmatrix} 28 & -34 & 16 \\ -4 & -8 & 8 \\ -20 & 23 & -14 \end{bmatrix}}{-36}$$

$$A^{-1} = \frac{-1}{36}\begin{bmatrix} 28 & -34 & 16 \\ -4 & -8 & 8 \\ -20 & 23 & -14 \end{bmatrix}$$

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Arithmetic Operation

★ Basic Formula :

(i) $(a+b)^2 = a^2 + b^2 + 2ab$
(ii) $(a-b)^2 = a^2 + b^2 - 2ab$
(iii) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
(iv) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
(v) $a^2 - b^2 = (a+b)(a-b)$
(vi) $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$
(vii) $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

★ Polynomials :

- $ax^2 + bx + c = 0$ (Quadratic Equation)

Formula :- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex $3x^2 + 9x - 4 = 0$

here,

$a = 3$
$b = 9$
$c = -4$

$= x = \frac{-9 \pm \sqrt{81 - 4(3)(-4)}}{6}$

$= \frac{-9 \pm \sqrt{81 + 48}}{6}$

$= \frac{-9 \pm \sqrt{129}}{6}$

$$x = \frac{-9 + \sqrt{129}}{6}, \quad x = \frac{-9 - \sqrt{129}}{6}$$

Ex: $x^2 - 5x + 6 = 0$

here,

$a = 1$

$b = -5$

$c = 6$

$$x = \frac{5 \pm \sqrt{25 - 4 \times 6}}{2}$$

$$= \frac{5 \pm \sqrt{1}}{2}$$

$$= \frac{5 \pm 1}{2}$$

$$x = \frac{5+1}{2}, \quad x = \frac{5-1}{2}$$

$x = 3, \quad x = 2$

- By Splitting the mid-term -

Ex: $(ax^2 + bx + c = 0)$

$x^2 - 5x + 6 = 0$

$= x^2 - 2x - 3x + 6 = 0$

$= x(x-2) - 3(x-2) = 0$

$= (x-2)(x-3) = 0$

$= x = 2, \; x = 3$

Note:
division = -1
Multiply = +1

Ex: $x^2 - 2x - 15 = 0$

$= x^2 - 5x + 3x - 15$

$= x(x-5) + 3(x-5)$

$= (x-5)(x+3)$

$= x = 5, \quad x = -3$

Ex: $x^2 + 10x + 25 = 0$

$= (x)^2 + 2(x)5 + (5)^2 = 0$

$= (x+5)^2 = 0$

$= x = -5, \quad x = -5$

Ex $x^2 - 48x + 576 = 0$

$= (x-24)^2 = 0$

$= x = 24, \quad x = 24$

Ex $x^2 - 49 = 0$

$= (x+7)(x-7) = 0$

$= x^2 = 49, \quad x = \pm 7$

Ex $(676 - 576)$

$= (26+24)(26-24)$

* Cubic Polynomials:

• Remainder Theorm -

Solution of cubic polynomial by using remainder Theorm.

Note: $(x-2=0)$ is the factor of given equation of Ex1:

Ex1: $x^3 - 12x^2 + 36x - 32 = 0$

$= (2)^3 - 12(2)^2 + 36(2) - 32$

$= 8 - 48 + 72 - 32$

$= 80 - 80 = 0$

Method 1:

$$
\begin{array}{r|l}
x^3 - 12x^2 + 36x - 32 & x^2 - 10x + 16 \\
\cline{2-2}
x^3 - 2x^2 & \\
+ \quad - & \\
\hline
-10x^2 + 36x - 32 & \\
-10x^2 + 20x & \\
+ \quad - & \\
\hline
16x - 32 & \\
16x - 32 & \\
\hline
0 &
\end{array}
$$

$\rightarrow (x-2)(x^2 - 10x + 16) = 0$

$= (x-2)(x^2 - 8x - 2x + 16) = 0$

$= (x-2)[x(x-8) - 2(x-8)] = 0$

$= (x-2)(x-8)(x-2) = 0$

$= x = 2, 2, 8$

Method 2: $x^2(x-2) - 10x(x-2) + 16(x-2) = 0$

$= (x-2)(x^2 - 8x - 2x + 16) = 0$

$= (x-2)[x(x-8) - 2(x-8)] = 0$

$= (x-2)(x-8)(x-2) = 0$

$x = 2, 2, 8$

Method 3: $x^3 - 2x^2 - 10x^2 + 20x + 16x - 32 = 0$

$= x^2(x-2) - 10x(x-2) + 16(x-2) = 0$

$= (x-2)(x^2 - 10x + 16) = 0$

$= (x-2)[x(x-8) - 2(x-8)] = 0$

$= (x-2)(x-8)(x-2) = 0$

$= x = 2, 2, 8$

Ex2: $x^3 + x^2 - 21x - 45 = 0$

$= (-3)^3 + (-3)^2 - 21(-3) - 45$

$= -27 + 9 + 63 - 45$

$= -72 + 72 = 0$

Note: $(x+3=0)$ is the factor of given equation.

Method 3: $x^3 + 3x^2 - 2x^2 - 6x - 15x - 45 = 0$

$= x^2(x+3) - 2x(x+3) - 15(x+3) = 0$

$= (x+3)(x^2 - 2x - 15) = 0$

$= (x+3)(x^2 - 5x + 3x - 15) = 0$

$= (x+3)(x-5)(x+3) = 0$

$= x = -3, -3, 5$

Ex3) $x^3 - 6x^2 + 9x - 4 = 0$

$= (1)^3 - 6(1)^2 + 9(1) - 4$

$= 1 - 6 + 9 - 4$

$= 10 - 10 = 0$

Note: $(x-1=0)$ is the factor of given equation

$$x-1 \overline{)\ x^3 - 6x^2 + 9x - 4\ (\ x^2 - 5x + 4}$$

$x^3 - 1x^2$

(−) (+)

$-5x^2 + 9x$

$-5x^2 + 5x$

(+) (−)

$4x - 4$

$4x - 4$

(−) (+)

0

$= (x-1)(x^2 - 5x + 4) = 0$

$= (x-1)(x^2 - 4x - 1x + 4) = 0$

$= (x-1)[x(x-4) - 1(x-4)] = 0$

$= (x-1)(x-1)(x-4) = 0$

$= x = 1, 1, 4$

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Number System

- Counting numbers 1, 2, 3, 4 - - - and so on. are called Natural Numbers.
- All counting numbers together with the 0 form the set of whole numbers.
- All natural numbers, 0 and negative of counting numbers together form the set of integers.

Ex :

1, 2, 3, - - - -

0, 1, 2, 3, - - - - -

- - -4, -3, -2, -1, 0, 1, 2, 3, 4, - -

- A number divisible by 2 is called an even number.

Ex :

2, 4, 6, 8, 10, - - - -

- A number not divisible by 2 is called an odd number.

Ex

1, 3, 5, 7, 9 - - - -

- A number greater than one is called a prime number if it has exactly 2 factors namely 1 and the number

itself

Ex: 2, 3, 7, 13, 17, 19 - - -

- Numbers greater than 1 which are not prime are known as composite numbers.

Ex: 8, 4, 6, 9, 12 - - -

Note (i) 1 is neither prime nor composite.

(ii) 2 is the only even number which is prime.

(iii) There are 25 prime numbers between 1 & 100.

* Types of Divisibility:

1.) Divisibility by 2:-

A Number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.

Ques Find the LCM:-

(i) 64
(ii) 128
(iii) 512
(iv) 624

Sol: P.T.O

2	64, 128, 512, 624
2	32, 64, 256, 312
2	16, 32, 128, 156
2	8, 16, 64, 78
2	4, 8, 32, 39
2	2, 4, 16, 39
2	1, 2, 8, 39
2	1, 1, 4, 39
2	1, 1, 2, 39
3	1, 1, 1, 39
13	1, 1, 1, 13
	1, 1, 1, 1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 13$

$= 19,968$

Ques Simplify:

Hint: $a^2 - b^2 = (a+b)(a-b)$

1.) $896 \times 896 - 204 \times 204$

$= (896 + 204)(896 - 204)$

$= 1100 \times 692$

$= 7,61,200$

Hint: $a^2 + b^2 + 2ab = (a+b)^2$

2.) $387 \times 387 + 114 \times 114 + 2 \times 387 \times 114$

$= (387 + 114)^2$

$= (501)^2$

$= 2,51,001$

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Arithmetic Progression

Ex: 2, 4, 6, 8, — — —

$= 4 - 2 = 2$

$= 6 - 4 = 2$

$= 8 - 6 = 2$

Formula:

(i) $T_n = a + (n-1)d$

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

Q. How many no. between 11 and 90 are divisible by 9?

Sol 18, 27, 36 — — — — 81

$T_n = a + (n-1)d$

$81 = 18 + (n-1)9$

$81 = 18 + 9n - 9$

$9n = 81 - 9 = 72$

$n = 8$

Q. Find the sum of all odd no. upto 100.

Sol 1, 3, 5, 7, 9, 11 — — — — — — 99

$T_n = a + (n-1)d$

$99 = 1 + (n-1)2$

$99 = 1 + 2n - 2$

$99 = 2n - 1$

$2n = 100$

$n = 50$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{50}{2}[2 + (50-1)2]$

$= 25[2 + 98]$

$= 2500$

Q. Find the sum of all 2 digit numbers which are divisible by 3.

Sol

$12, 15, 18, 21, 24 \ldots \ldots 99$

$T_n = a + (n-1)d$

$99 = 12 + (n-1)3$

$99 = 12 + 3n - 3$

$99 = 9 + 3n$

$3n = 99 - 9$

$n = \frac{90}{3}$

$n = 30$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{30}{2}[2 \times 12 + (30-1)3]$

$= 15[24 + 29 \times 3]$

$= 15[24 + 87]$

$= 15[111]$

$= 1665$

Q. How many numbers between 200 and 600 are divisible by 4, 5, And 6.?

Sol LCM of 4, 5, 6

2	4, 5, 6
2	2, 5, 3
3	1, 5, 3
5	1, 5, 1
.	1, 1, 1

$2 \times 2 \times 3 \times 5 = 60$

240, 300 — — — — 540

$T_n = a + (n-1)d$

$540 = 240 + (n-1)60$

$540 = 240 + 60n - 60$

$540 = 180 + 60n$

$60n = 540 - 180$

$60n = 360$

$n = \frac{360}{60}$

$n = 6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{6}{2}[2 \times 240 + (6-1)60]$

$= 3[480 + 5 \times 60]$

$= 3[480 + 300] = 3 \times 780 = 2340$

Geometric Progression

Ex: $a, ar, ar^2, ar^3, \ldots ar^n$ (G.P.)

$$\frac{a}{ar} = \frac{1}{r}$$

$$\frac{ar}{ar^2} = \frac{1}{r}$$

Formula:

(i) $T_n = ar^{n-1}$

(ii) $S_n = \frac{a(1-r^n)}{1-r}$

Q. How many terms are there in $2, 4, 8, 16 \ldots 1024$?

Sol $\frac{4}{2} = 2, \frac{8}{4} = 2, \frac{16}{8} = 2$

therefore, this a G.P.

$r = 2, a = 2, T_n = 1024$

$$T_n = ar^{n-1}$$

$$1024 = 2 \times (2)^{n-1}$$

$$1024 = 2 \times \frac{2^n}{2}$$

$$1024 = 2^n$$

$$2n = 1024 = 2^{10}$$

Q. Find the sum of the series:

$2 + 2^2 + 2^3 + \dots + 2^8$

$a = 2, \quad r = 2, \quad T_n = 256 \ (2^8)$

$T_n = ar^{n-1}$

$256 = 2 \times 2^{n-1}$

$256 = 2 \times \frac{2^n}{2}$

$256 = 2^n \Rightarrow 2^8 = 2^n$

$n = 8$

Note: $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \dots$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

Permutations and Combinations

- **Factorial Notations :**

Let n be a positive number (integer) then, factorial n denoted by $\underline{n}$ and is defined as,

$$\underline{n} = n(n-1)(n-2) - - - -$$

Ex: $\underline{3} = 3(3-1)(3-2)$
$= 3 \times 2 \times 1$
$= 6$

$\underline{2} = 2$
$\underline{0} = 1$
$\underline{4} = 24$
$\underline{5} = 120$
$\underline{6} = 720$

- **Permutation :**

* The different arrangements of a given numbers of things by taking some or all at a time are called permutation.

* Number of all permutations of n things taken are at a time is given by,

$$^{n}P_{n} = \frac{\underline{n}}{\underline{n-r}}$$

* Ex: All permutations made with the letters

a, b, c by taking two at a time are -

⇒ ab, ba, ac, ca, bc, cb Ans

Note: Number of all permutations of n things taken all at a time is factorial n ($\lfloor n$).

Ques Evaluate $\frac{\lfloor 30}{\lfloor 28}$

⇒ $\lfloor 30 = 30(30-1)(30-2)$

$= \frac{30 \times 29 \times \lfloor 28}{\lfloor 28}$

$= 30 \times 29$

$= 870$

Ques Find the value of ${}^{60}P_{28}$

⇒ $\frac{\lfloor 60}{\lfloor 60-28} = \frac{\lfloor 60}{\lfloor 32}$

$= \frac{60 \times ---- \times \lfloor 32}{\lfloor 32}$

- Combination :

★ Each of the different groups or selections which can be form by taking some or all of

a number of objects is called a combination.

* The number of all combinations of n things taken are at a time is ${}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$

* Ex: Suppose we want to select two out of 3 boys A, B, C then possible selections are,
AB, BC, CA
Note that AB and BA represent the same selection.

Note: ${}^NC_N = 1$, ${}^NC_0 = 1$

Ques Find the value of ${}^{10}C_3$.

⇒ here, $n = 10$
$r = 3$
therefore, ${}^{10}C_3 = \frac{\lfloor 10}{\lfloor 3 \lfloor 7}$

$= \frac{10 \times 9 \times 8 \times \lfloor 7}{\lfloor 3 \times \lfloor 7}$

$= \frac{720}{6}$

$= 120$

Qus In how many ways can a cricket team be selected out of 15 players.

$$\Rightarrow {}^{15}C_{11} = \frac{\lfloor 15}{\lfloor 11 \lfloor 4}$$

$$= \frac{15 \times 14 \times 13 \times 12 \times \lfloor 11}{4 \times 3 \times 2 \times 1 \times \lfloor 11}$$

$$= \frac{15 \times 14 \times 13 \times 12}{24}$$

$$= 15 \times 7 \times 13$$

$$= 1365$$

Qus In how many ways a committee of 5 members can be selected from 6 man and 5 women consisting of 3 man & 2 women.

⇒ Note : And = × and OR = +

$$= {}^{6}C_3 \times {}^{5}C_2$$

$$= \frac{\lfloor 6}{\lfloor 3 \times \lfloor 3} \times \frac{\lfloor 5}{\lfloor 3 \times \lfloor 2}$$

$$= \frac{6 \times 5 \times 4 \times \lfloor 3}{\lfloor 3 \times \lfloor 3} \times \frac{5 \times 4 \times 3 \times \lfloor 2}{\lfloor 3 \times \lfloor 2}$$

$$= 20 \times 10$$

$$= 200$$

Qus How many words can be formed by using

all the letters of the words BIHAR.

$\Rightarrow {}^{5}C_{5} = \frac{\lfloor 5}{} = 5 \times 4 \times 3 \times 2 \times 1$

$= 120$.

12/10/19

Percentage

- To express $\frac{a}{b}$ as a percent:

$$\left(\frac{a}{b} \times 100\right)\%$$

Ex:

$$\left(\frac{2}{3} \times 100\right)\% = 33 + 33$$

$$\left(\frac{1}{3} \times 100\right)\% = 33\% \quad = 66\%$$

- To express $x\%$ as a expression:

$$\left(x\% = x \times \frac{1}{100}\right)$$

Ex:

i) $2\% = 2 \times \frac{1}{100}$

ii) $5\% = 5 \times \frac{1}{100}$

Ques: Evaluate 28% of 450 (+) 45% of 280

$$\rightarrow \quad 450 \times \frac{28}{100} \quad (+) \quad 280 \times \frac{45}{100}$$

$$= \quad 126 \quad + \quad 126$$

$$= \quad 252$$

Ques: 2 is what percent of 50 :

→ $\frac{2}{50} \times \frac{100}{1}$

$= 2 \times 2 = 4\%$

Ques: Difference of 2 numbers is 1660, if 7.5% of one number is 12.5% of the other number. Find the 2 numbers:

→ $x - y = 1660$

$= \frac{7.5}{100} \times x = \frac{12.5}{100} \times y$

$= 75x = 125y$

$= 3x = 5y$

$= x = \frac{5}{3}y$

$= \frac{5}{3}y - y = 1660$ } LCM

$= 5y - 3y = 4980$

$= y = \frac{4980}{2} = 2490$ Ans

$= x = 2490 \times \frac{5}{3} = 830 \times 5 = 4150$ Ans

Ques: If the GST be reduced from $\frac{7}{2}\%$ to $\frac{10}{3}\%$

then, what difference does it make to a person to purchase an article with price ₹8400

→ $8400 \times \frac{7}{2} \times \frac{1}{100} = ?$

$8400 \times \frac{10}{3} \times \frac{1}{100} = ?$ } +

$= \frac{8400}{100}\left[\frac{7}{2} - \frac{10}{3}\right]$

$= 84\left[\frac{21-20}{6}\right]$

$= 84 \times \frac{1}{6}$

$= \frac{84}{6}$

$= 14$ Ans

1/10/19

Simple Interest

Formula: $\frac{P \times R \times T}{100}$

here,

P = principle amount

R = rate of interest

T = time

Ques: Find the S.I. on ₹68000 at the rate $\frac{50}{3}$% per annum for 9 months.

→ $\frac{68000 \times \frac{50}{3} \times \frac{9}{12}}{100}$

$= \frac{\overset{170}{\cancel{680}} \times \frac{50}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}}}{}$

$= 1.70 \times 50$

$= 85.00$ Ans

Ques: Find the S.I. on ₹3000 @ $\frac{25}{4}$% per annum for the period from 4th feb, 2005 to 18th April 2005.

→ 24 + 31 + 18 = 73 days

$= \frac{\cancel{73}}{\cancel{365}} \frac{1}{5}$

$= \frac{1}{5}$ years

$$= \frac{\cancel{3000}^{15} \times \frac{\cancel{25}^{5}}{\cancel{4}_{2}} \times \frac{1}{\cancel{5}_{1}}}{\cancel{100}}$$

$$= \frac{15 \times 5}{2}$$

$$= \frac{75}{2}$$

$$= 37.5 \quad \text{Ans}$$

10/19

Compound Interest

Formula: $A = P\left(1 + \frac{r}{100}\right)^n$

Here, A → amount including interest

$C.I. = A - P$

Note: i) When the interest compounded annually

Ques Find C.I. on rupees 7500 @ 4% per annum for 2 years compounded annually.

→ $P = 7500$

$R = 4$

$n = 2$

$$A = 7500\left(1 + \frac{4}{100}\right)^2$$

$$= 7500\left(\frac{26}{25}\right)^2 \rightarrow \left\{1 + \frac{1}{25} \rightarrow \frac{25+1}{25}\right\}$$

$$= 7500 \times \frac{26}{25} \times \frac{26}{25}$$

$$= 300 \times 26 \times \frac{26}{25}$$

$$= 12 \times 26 \times 26$$

$$= 12 \times 676$$

$$= 8112 \quad \text{Ans}$$

$CI = 8112 - 7500$

$= 612$ Ans

Note: ii) When the interest compounded half yearly.

$$A = P\left(1 + \frac{r/2}{100}\right)^{2n}$$

Ques Find the C.I. on rupees 10,000 in 2 years @ 4% p.a. The interest being compounded half yearly.

→ $P = 10000$

$R = 4 \rightarrow 4/2$

$T = 2 \rightarrow 2 \times 2$

$$A = P\left(1 + \frac{r}{2 \times 100}\right)^4$$

$$= 10000\left(1 + \frac{4}{2 \times 100}\right)^4$$

$$= 10000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50}$$

$$= 10824.32 \quad \text{Ans}$$

$$C.I. = 10824.32 - 10000$$

$$= \text{Rs. } 824.32 \quad \text{Ans}$$

Note iii) when the interest compounded quarterly.

$$A = P\left(1 + \frac{r}{4 \times 100}\right)^{4n}$$

Ques Find the C.I. on rupees 16000 @ 20% p.a. on 9 months compounded quarterly.

→ $P = 16000$

$R = 20$

$t = 9m. \rightarrow 9 \times 4 = 36m. \rightarrow 3 \text{ yrs.}$

$$A = 16000\left(1 + \frac{20}{4\times100}\right)^3$$

$$= 16000\left(\frac{21}{20}\right)^3$$

$$= 16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$= \text{Rs. } 18522 \quad \text{Ans}$$

$$\text{C.I.} = 18522 - 16000$$

$$= \text{Rs. } 2522 \quad \text{Ans}$$

Ques If the S.I. on a sum of money @ 5% p.a. for 3 years is rs. 1200. Find the C.I. on the same sum, for the same time at the same rate of interest. (Hint: yearly)

→ $$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$P = \frac{100 \times \text{S.I.}}{R \times T} = P = \frac{120000}{15}$$

$$P = 8000$$

$$A = P\left(1 + \frac{R}{100}\right)^n = 8000\left(1 + \frac{5}{20\cdot100}\right)^3$$

$$= 8000\left(1 + \frac{1}{20}\right)^3 = 8000\left(\frac{21}{20}\right)^3$$

$$= \cancel{8}\times\cancel{8000} \times \frac{21}{\cancel{20}} \times \frac{21}{\cancel{20}} \times \frac{21}{\cancel{20}} = 9261$$

$$= 9261 - 8000$$

$$= 1261 \quad \text{Ans}$$

Que) In what time will rs. 1000 become Rs 1331 at 10% p.a. compounded annually.

→ $A = P\left(1 + \frac{r}{100}\right)^n$

$1331 = 1000\left(1 + \frac{10}{100}\right)^n$

$\frac{1331}{1000} = \left(\frac{11}{10}\right)^n$

$\left(\frac{11}{10}\right)^3 = \left(\frac{11}{10}\right)^n$

$n = 3$ Ans

Que) A certain sum of amount to Rs 7380 in 2 yrs. and Rs 8575 in 3 yrs. Find the sum and rate of interest.

→ Let the sum be x

$SI = \frac{P \times R \times T}{100}$

$= \frac{7380 \times R \times 1}{100} = 1195$ ↓ (8575 − 7380) S.I. for 1 year

$7380 \times R = 119500$

$R = \frac{119500}{7380}$

$R = 16\frac{2}{3} = \frac{50}{3} = 16.19$

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$7380 = x\left(1 + \frac{50}{3 \times 100}\right)^2$$

$$7380 = x \cdot \frac{7}{6} \times \frac{7}{6}$$

$$x = \frac{7380 \times 6 \times 6}{7 \times 7}$$

$$= 5422.04$$

Ques A sum of money to Rs 6690 after 3 yrs. and to Rs 10035/- after 6 yrs. on compound interest. Find the sum.

→ $$A = P\left(1 + \frac{r}{100}\right)^n$$

$$10035 = x\left(1 + \frac{r}{100}\right)^6$$

$$6690 = x\left(1 + \frac{r}{100}\right)^3$$

$$\frac{10035}{6690} = \left(1 + \frac{r}{100}\right)^3$$

$$6690 = x \cdot \frac{3}{2}$$

$$x = \frac{6690 \times 2}{3}$$

$= 2230 \times 2$

$= 4460$

1/11/19

Probability

formula:- Probability : $P(A) = \frac{m}{n}$

where,

m = no. of cases favourable to the event and,

n = total no. of cases

Ques Two unbiased dice are thrown find the probability :

(i) both the dice show the same number

(ii) the first dice shows 6

(iii) the total of numbers on the dice is greater than 8.

(iv) the total of the numbers on the dice is 13.

Sol:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

→ (i) $= \frac{6}{36} = \frac{1}{6}$ (ii) $= \frac{6}{36} = \frac{1}{6}$

(iii) $= \frac{10}{36} = \frac{5}{18}$

(iv) The probability of impossible event is always 0.

Ques What is the probability of getting at most 2 heads in tossing of 3 coins.

Sol:

H	H	H	×
H	H	T	✓
H	T	H	✓
T	H	H	✓
T	T	T	✓
T	T	H	✓
T	H	T	✓
H	T	T	✓

$= \frac{7}{8}$

Ques What is the chance that a leap year selected at random will contain 53 sundays.

Sol In a leap year consist of 366 days, 52 complete week and 2 more days.

Sun	Mon
Mon	Tue
Tue	Wed
Wed	Thu
Thu	Fri
Fri	Sat
Sat	Sun

$= \frac{2}{7}$

Ques A, B and C are three events associated with a random experiment. find P(A). Given that $P(B) = \frac{3}{2} P(A)$ and

$$P(C) = \frac{1}{2} P(B)$$

Ans Let $P(A) = P$

Given,

$$P(B) = \frac{3}{2} P(A) = \frac{3}{2} P$$

$$P(C) = \frac{1}{2} P(B) = \frac{1}{2} \times \frac{3}{2} P = \frac{3}{4} P$$

Note: i) The probability of an event A is some number between and including 0 & 1.

ii) Probability of a certain event is always 1 therefore, $P(A) + P(\bar{A}) = 1$

$$\Rightarrow P(A) + P(B) + P(C) = 1$$

$$\Rightarrow P + \frac{3}{2} P + \frac{3}{4} P = 1$$

$$\Rightarrow \frac{4P + 6P + 3P}{4} = 1$$

$$\Rightarrow P = \frac{4}{13}$$

Theorm: If A and B are any two events then,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Ques An integer is taken at random from the first 200 (positive integer) numbers, what is the probability that the integer is taken is divisible by 6 and 8.

Sol Let A be the event of integer is divisible by 6,

$A = 6, 12, 18, 24, 30, 36, 42, 48, 198$
$= 33$

$B = 8, 16, 24 - - - - - - \cdot 200 = 25$

$\Rightarrow 24, 48, 72, 96, 120, 144, 168, 192 = 8$

Therefore,

$$P(A) = \frac{33}{200}$$

$$P(B) = \frac{25}{200}$$

$$P(A \cap B) = \frac{8}{200}$$

we know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{25}{200} + \frac{33}{200} - \frac{8}{200}$$
$$= \frac{50}{200} = \frac{1}{4}$$

16/11/19

Profit And Loss

Formula 1: Gain = SP − CP

Loss = CP − SP

here,

SP = Selling Price

CP = Cost Price

Formula 2: $\text{Gain \%} = \frac{\text{Gain} \times 100}{CP}$

$\text{Loss \%} = \frac{\text{Loss} \times 100}{CP}$

Formula 3: $SP = \frac{100 + \text{gain \%} \times CP}{100}$

$SP = \frac{100 - \text{Loss \%} \times CP}{100}$

- If an article is sold at a gain of say 35 %. then SP = 135 % of CP.
- If an article is sold at a loss of say 35 % then SP = 65 % of CP.

Ques A Man buy an article for Rs. 27.50 and sells it for Rs. 28.00. Find his gain %.

→ $\text{Gain \%} = \frac{\text{Gain} \times 100}{CP}$

Gain = SP - CP

$= 28 - 27.50$

$= 0.5$

$= \frac{0.5 \times 100}{27.5}$

$= \frac{100}{55} = \frac{20}{11}$

$= 1.81\ \%$

Ques If a pen is purchased for Rs. 490 and sold for Rs. 465.50. Find the loss %.

→ $\text{Loss \%} = \frac{\text{Loss} \times 100}{\text{CP}}$

Loss = CP - SP

$= 490 - 465.50$

$= 24.5$

$= \frac{24.5 \times 100}{490}$

$= 5\ \%$

Ques Find CP when SP = 56.25, Gain = 16%.

→ $SP = \frac{100 + \text{gain\%}}{100} \times CP$

$56.25 = \frac{100 + 16\%}{100} \times CP$

$100 \times 56.25 = 116 \times CP$

$\frac{5625}{116} = CP$

$CP = 48.49$

Ques A Book was sold for Rs 27.50 with a profit of 10%. If it were sold for Rs. 25.75. Find what is percentage of Profit or Loss.

→ $27.50 = \frac{100 + 10}{100} \times x$

$27.50 = \frac{110}{100} \times x$

$\frac{27.50 \times 100}{100} = 110 \times x$

$$2750 = 110 \times x$$

$$\frac{2750}{110} = x \quad = \quad x = 25 \text{ (CP)}$$

$$\text{Gain} = \frac{0.75 \times 10^{2}}{25} = 3\%$$

Ques The CP of 21 articles is equal to SP of 18 articles. Find the gain or loss.

→ Let The CP of 1 article is ₹1

Therfore,

SP of 18 articles = ₹21

CP of 18 articles = ₹18

21 − 18 = 3

Profit = 3

$$\text{Gain\%} = \frac{\text{Gain} \times 100}{\text{CP}}$$

$$= \frac{3 \times 100}{18} = \frac{300}{18} = 16.66$$

Ques Monica purchased a pressure cooker at $\frac{9}{10}$ of its selling price, and sold it at 8% more than its SP. Find Gain %.

→ Let SP = 100

$$CP = \frac{9}{10} \times 100 = 90$$

$$SP = \frac{108}{100} \times 100 = 108$$

Gain = 108 - 90 = 18

$$\text{Gain \%} = \frac{18 \times 100}{90}$$

$$= \frac{1800}{90}$$

$$= 20\ \%$$

Ques A Dealer sold $\frac{3}{4}$ of his articles at a gain of 20%. and the remaining at the cost prize. Find the gain by the dealer in the four transactions

→ Let CP = 100

$$SP = 100 \times \frac{3}{4} = 75$$

$$SP = 75 \times \frac{20}{100} = 15$$

SP = 100 + 15 - 115

Gain % = 15 %

18/11/19

Time and Distance

- If a man do a piece of work in 10 days then 1 day of work A = $\frac{1}{n}$

Ques Worker A and B works 8 hours & 10 hours to do the same work. How long should it take both A and B working together but independtly to do the same job.

→ $A = \frac{1}{8}$, $B = \frac{1}{10}$

$= \frac{1}{8} + \frac{1}{10}$

$= \frac{5+4}{40}$

$= \frac{9}{40}$ = ∴ 1hr = 9/40
Total work = 40/9

Therefore, A & B will finish the work at $\frac{40}{9}$ hrs.

Ques A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days. In how many days B can alone do that work

→ $A + B = \frac{1}{4} - \frac{1}{12}$

$B = \left(\frac{1}{A+B}\right) - \frac{1}{A}$

$= \frac{3-1}{12} = \frac{2}{12}$

$= \frac{1}{6} = 6$ days

Ques A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How will long they take to do it, working together $8\frac{2}{5}$ hours per day.

→ A can complete the work in,
$7 \times 9 = 63$ hours
B can complete the work in,
$6 \times 7 = 42$ h
Therefore, 1 hr work of $A = \frac{1}{63}$
Therefore, 1 hr work of $B = \frac{1}{42}$
$A + B = \frac{1}{63} + \frac{1}{42} = \frac{5}{126}$

$\therefore$ A + B can complete the work in $= \frac{126}{5}$

$= \frac{126}{5} \times \frac{5}{42} \quad \left(8\frac{2}{5} \Rightarrow \frac{42}{5}\right)$

$= 3$ days.

Ques A & B can do a piece of work in 18 days, B and C can do it in 24 days, A & C can do it in 36 days. How many days A, B, C finish it working together and separately?

→ 1 day work of A & B $= \frac{1}{18}$

1 day work of B & C $= \frac{1}{24}$

1 day work of A & C $= \frac{1}{36}$

$2 \times (A + B + C) = \frac{1}{16}$

They will finish the work together in 16 days.

(i) $A + B + C = \frac{1}{16}$

$\frac{1}{24} = B + C$

...

$$A = \frac{1}{16} - \frac{1}{24}$$

$$= \frac{9-6}{144} = \frac{3}{144} = \frac{1}{48}$$

$A = 48$ days.

(ii) $$B = \frac{1}{36} - \frac{1}{18}$$

$$= \frac{4-8}{144} = \frac{4}{144} = \frac{1}{36}$$

$B = 36$ days

(iii) $$C = \frac{1}{16} - \frac{1}{18}$$

$$= \frac{9-8}{144} = \frac{1}{144}$$

$C = 144$ days

Ans 45 men can complete the work in 16 or 18 days. 6 days after day started working 13 more men joined them. How many days will they now take to complete the remaining work.

→ $45 \times 18 = 810$ men can complete work in one day.

One day work of one man $= \frac{1}{810}$

$= \frac{6 \times 45}{810} = \frac{2 \times 45}{270} = \frac{2 \times 5}{30} = \frac{1}{3}$

Remaining work $= 1 - \frac{1}{3} = \frac{2}{3}$

one day work of 75 men $= \frac{75}{810} = \frac{5}{54}$

So, $\frac{2}{3} \times \frac{54}{5} = \frac{2 \times 18}{5} = \frac{36}{5} = 72$ days

Time and Distance

Formula: 1.) $V = \frac{D}{T}$

2.) $D = V \times T$

here,

V = Velocity / Speed
D = Distance
T = Time

3.) $km/h = \frac{5}{18}\ m/sec.$

Ques How many minutes does aditya take to cover a distance of 400 m, if he run at a speed of 20 km/h.

→ D = 400 m
V = 20 km/h

$= 20 \times \frac{5}{18} = \frac{100}{18} = \frac{50}{9}$

$= \frac{400}{50/9} = \frac{400 \times 9}{50} = 72\ sec$

$= \frac{72}{60} = \frac{6}{5} = 1.2\ minutes$

Ques While covering a distance of 24 km. A man notice that after walking for 1 hr 40 min., the distance covered by him was 5/7 of the remaining distance. what was his speed in m/sec.

→ Remaining distance = 24 - distance covered
So, Let the remaining distance be = x

$$= x \times \frac{5}{7} = 24 - x$$

$$= \frac{5x}{7} = 24 - x$$

$$= 5x = 168 - 7x$$

$$= 12x = 168$$

$$= x = \frac{168}{12} = 14 \text{ km}$$

⇒ Covered distance = 24 - 14
= 10 km

⇒ Distance in meters = 10 × 1000 = 10,000 m

⇒ Time in sec = 100 min × 60 = 6,000 sec

$$\text{Speed} = \frac{D}{T}$$

$$= \frac{10000}{6000}$$

$$= 1.67 \text{ m/sec.}$$

Ques Peter can cover a certain distance in 1 hr 24 min by covering 2/3 of the distance at 4 km/h and the rest at 5 km/h. Find the total distance.

→ Let the total distance be x km

$$= \frac{2/3}{4}x + \frac{1/3x}{5} = \frac{7}{5}\left(1+\frac{24}{60} \rightarrow 1+\frac{2}{5} = \frac{7}{5}\right)$$

$$= \frac{x}{6} + \frac{x}{15} = \frac{7}{5}$$

$$= \frac{(10+4)x}{60} = \frac{7}{5}$$

$$= x = \frac{7 \times 60^{30}}{5 \times 14_{2}} = 6$$