ASSIGNMENT 01

Q:1 (a) $\begin{vmatrix} 42 & 1 & 6 \\ 28 & 7 & 4 \\ 14 & 3 & 2 \end{vmatrix}$

* Minors :

$42\ (M_{11}) = \begin{vmatrix} 7 & 4 \\ 3 & 2 \end{vmatrix} = 14-12 = 2$

$1\ (M_{12}) = \begin{vmatrix} 28 & 4 \\ 14 & 2 \end{vmatrix} = 56-56 = 0$

$6\ (M_{13}) = \begin{vmatrix} 28 & 7 \\ 14 & 3 \end{vmatrix} = 84-98 = -14$

$28\ (M_{21}) = \begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} = 2-18 = -16$

$7\ (M_{22}) = \begin{vmatrix} 42 & 6 \\ 14 & 2 \end{vmatrix} = 84-84 = 0$

$4\ (M_{23}) = \begin{vmatrix} 42 & 1 \\ 14 & 3 \end{vmatrix} = 126-14 = 112$

$14\ (M_{31}) = \begin{vmatrix} 1 & 6 \\ 7 & 4 \end{vmatrix} = 4-42 = -38$

$3\ (M_{32}) = \begin{vmatrix} 42 & 6 \\ 28 & 4 \end{vmatrix} = 168-168 = 0$

$2\ (M_{33}) = \begin{vmatrix} 42 & 1 \\ 28 & 7 \end{vmatrix} = 294-28 = 266$

MINOR determinant

$$\begin{vmatrix} 2 & 0 & -14 \\ -16 & 0 & 112 \\ -38 & 0 & 266 \end{vmatrix}$$

* Cofactors : $(-1)^{i+j} * \text{Minor}$

$C_{11} = (-1)^{1+1} * 2 \rightarrow 1 * 2 = 2$

$C_{12} = (-1)^{1+2} * 0 \rightarrow -1 * 0 = 0$

$C_{13} = (-1)^{1+3} * -14 \rightarrow 1 * -14 = -14$

$C_{14} = (-1)^{2+1} * -16 \rightarrow -1 * -16 = 16$

$C_{22} = (-1)^{2+2} * 0 \rightarrow 1 * 0 = 0$

$C_{23} = (-1)^{2\times3} * 112 \rightarrow -1 * 112 = -112$

$C_{31} = (-1)^{3+1} * -38 \rightarrow 1 * -38 = -38$

$C_{32} = (-1)^{3+2} * 0 \rightarrow -1 * 0 = 0$

$C_{33} = (-1)^{3+3} * 266 \quad * \quad 1 * 266 = 266.$

(b) $\begin{vmatrix} 5 & 0 & 7 \\ 8 & -6 & -4 \\ 2 & 3 & 9 \end{vmatrix}$

* Minors :

$5\ (M_{11}) = \begin{vmatrix} -6 & -4 \\ 3 & 9 \end{vmatrix} = -54 + 12 = 42$

$0\ (M_{12}) = \begin{vmatrix} 8 & -4 \\ 2 & 9 \end{vmatrix} = 72 + 8 = 80$

$7\ (M_{13}) = \begin{vmatrix} 8 & -6 \\ 2 & 3 \end{vmatrix} = 24 + 12 = 36$

$8\ (M_{21}) = \begin{vmatrix} 0 & 7 \\ 3 & 9 \end{vmatrix} = 0 - 21 = -21$

$-6\ (M_{22}) = \begin{vmatrix} 5 & 7 \\ 2 & 9 \end{vmatrix} = 45 - 14 = 31$

$-4\ (M_{23}) = \begin{vmatrix} 5 & 0 \\ 2 & 3 \end{vmatrix} = 15 - 0 = 15$

$2\ (M_{31}) = \begin{vmatrix} 0 & 7 \\ -6 & -4 \end{vmatrix} = 0 + 42 = 42$

$3\ (M_{32}) = \begin{vmatrix} 5 & 7 \\ 8 & -4 \end{vmatrix} = -20 - 56 = -76$

$9\ (M_{33}) = \begin{vmatrix} 5 & 0 \\ 8 & -6 \end{vmatrix} = -30 - 0 = -30$

MINORS determinant

$\begin{vmatrix} 42 & 80 & 36 \\ -21 & 31 & 15 \\ 42 & -76 & -30 \end{vmatrix}$

* Cofactors :

$C_{11} = (-1)^{1+1} * 42 = 1 * 42 = 42$

$C_{12} = (-1)^{1+2} * 80 = -1 * 80 = -80$

$C_{13} = (-1)^{1+3} * 36 = 1 * 36 = 36$

$C_{21} = (-1)^{2+1} * -21 = -1 * -21 = 21$

$C_{22} = (-1)^{2+2} * 31 = 1 * 31 = 31$

$C_{23} = (-1)^{2+3} * 15 = -1 * 15 = -15$

$C_{31} = (-1)^{3+1} * 42 = 1 * 42 = 42$

$C_{32} = (-1)^{3+2} * -76 = -1 * -76 = 76$

$C_{33} = (-1)^{3+3} * -30 = 1 * -30 = -30$

Q:2 $x+y+z=1$

$3x+5y+6z=4$

$9x+2y-36z=17$

$$D=\begin{vmatrix}1&1&1\\3&5&6\\9&2&-36\end{vmatrix} \qquad D_1=\begin{vmatrix}1&1&1\\4&5&6\\17&2&-36\end{vmatrix}$$

$$D_2=\begin{vmatrix}1&1&1\\3&4&6\\9&17&-36\end{vmatrix} \qquad D_3=\begin{vmatrix}1&1&1\\3&5&4\\9&2&17\end{vmatrix}$$

7	= 0+42
-4	= 42
7	= -20-56
-4	= -76
0	= -30-0
-6	= -30

iant

$$\begin{vmatrix}\\\\\end{vmatrix}$$

* $D = 1\begin{vmatrix}5&6\\2&-36\end{vmatrix} - 1\begin{vmatrix}3&6\\9&-36\end{vmatrix} + 1\begin{vmatrix}3&5\\9&+02\end{vmatrix}$

» $1[-180-12]-1[-108-54]+1[6-45]$

» $1[-192]-1[-162]+1[-39]$

» $-192+162-39$ » $-231+162=-69.$

* $D_1 = 1\begin{vmatrix}5&6\\2&-36\end{vmatrix} - 1\begin{vmatrix}4&6\\17&-36\end{vmatrix} + 1\begin{vmatrix}4&5\\17&2\end{vmatrix}$

» $1[-180-12]-1[-144-102]+1[8-85]$

» $1[-192]-1[-246]+1[-77]$

» $-192+246-77$ » $-269+246=-23$

* $D_2 = 1\begin{vmatrix}4&6\\17&-36\end{vmatrix} - 1\begin{vmatrix}3&6\\9&-36\end{vmatrix} + 1\begin{vmatrix}3&4\\9&17\end{vmatrix}$

» $1[-144-102]-1[-108-54]+1[51-36]$

» $1[-246]-1[-162]+1[15]$

» $-246+162+15$ » $-246+177=-69$

* $D_3 = 1\begin{vmatrix}5&4\\2&17\end{vmatrix} - 1\begin{vmatrix}3&4\\9&17\end{vmatrix} + 1\begin{vmatrix}3&5\\9&2\end{vmatrix}$

» $1[85-8]-1[51-36]+1[6-45]$

$= 1[77] - 1[15] + 1[-39]$

$= 77 - 15 - 39 = \underline{\underline{23}}$

** $x = \frac{D_1}{D} = \frac{-23}{-69} = \frac{1}{3}$ ✓

** $y = \frac{D_2}{D} = \frac{-69}{-69} = 1$ ✓

** $z = \frac{D_3}{D} = \frac{23}{-69} = \frac{1}{-3}$ ✓

Q:3 $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 0 & 2 \\ 4 & -3 & 2 \end{bmatrix}$

verify that $(A+B)^2 = A^2 + AB + BA + B^2$

* $A^2 = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1\times1 + 2\times2 + -1\times0 & 1\times2 + 2\times0 + -1\times1 & 1\times-1 + 2\times3 + -1\times2 \\ 2\times1 + 0\times2 + 3\times0 & 2\times2 + 0\times0 + 3\times1 & 2\times-1 + 0\times3 + 3\times2 \\ 0\times1 + 1\times2 + 2\times0 & 0\times2 + 1\times0 + 2\times1 & 0\times-1 + 1\times3\times2\times2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1+4+(-1) & 2+0+(-1) & -1+6+(-2) \\ 2+0+0 & 4+0+3 & -2+0+6 \\ 0+2+0 & 0+0+2 & 0+3+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 4 & 1 & 3 \\ 2 & 7 & 4 \\ 2 & 2 & 7 \end{bmatrix}$

* $AB = \begin{bmatrix} \quad \end{bmatrix} * \begin{bmatrix} \quad \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

2

$$\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2\times 3 + -1\times 2 \\ +0\times 3 + 3\times 2 \\ +1\times 3\times 2\times 2 \end{bmatrix}$$

$$\begin{bmatrix} +6+(-2) \\ +0+6 \\ +3+4 \end{bmatrix}$$

$$\begin{bmatrix} \\ \\ \end{bmatrix}$$

Q:4 $A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix}$ Adjoint of A ?

* Minors :

$M_{11} = 2-4 = -2$
$M_{12} = 6-4 = 2$
$M_{13} = 3-1 = 2$
$M_{21} = 4-5 = -1$
$M_{22} = 2-5 = -3$
$M_{23} = 1-2 = -1$
$M_{31} = 8-5 = 3$
$M_{32} = 4-15 = -11$
$M_{33} = 1-6 = -5$

* Cofactors

$C_{11} = (-1)^{1+1} * -2 = -2$
$C_{12} = (-1)^{1+2} * 2 = -2$
$C_{13} = (-1)^{1+3} * 2 = 2$
$C_{21} = (-1)^{2+1} * -1 = 1$
$C_{22} = (-1)^{2+2} * -3 = -3$
$C_{23} = (-1)^{2+3} * -1 = 1$
$C_{31} = (-1)^{3+1} * 3 = 3$
$C_{32} = (-1)^{3+2} * -11 = 11$
$C_{33} = (-1)^{3+3} * -5 = -5$

- Adjoint of A = $\begin{bmatrix} -2 & -2 & 2 \\ 1 & -3 & 1 \\ 3 & 11 & -5 \end{bmatrix}$

Q:5 $A = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ Reverse of Matrix

* Minors :

$M_{11} = 1-4 = -3$
$M_{12} = 3-2 = 1$
$M_{13} = 6-1 = 5$
$M_{21} = 5-6 = -1$
$M_{22} = 2-3 = 1$
$M_{23} = 4-5 = -1$
$M_{31} = 10-3 = 7$
$M_{32} = 4-9 = -5$
$M_{33} = 2-15 = -13$

* Cofactors :

$C_{11} = (-1)^{1+1} * -3 = -3$
$C_{12} = (-1)^{1+2} * 1 = -1$
$C_{13} = (-1)^{1+3} * 5 = 5$
$C_{21} = (-1)^{2+1} * -1 = 1$
$C_{22} = (-1)^{2+2} * 1 = 1$
$C_{23} = (-1)^{2+3} * -1 = 1$
$C_{31} = (-1)^{3+1} * 7 = 7$
$C_{32} = (-1)^{3+2} * -5 = 5$
$C_{33} = (-1)^{3+3} * -13 = -13$

* Adjoint of A = $\begin{bmatrix} -3 & -1 & 5 \\ 1 & 1 & 1 \\ 7 & 5 & -13 \end{bmatrix}$

* Determinant of A :

$$2\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 5\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$$

$$2[1-4] - 5[3-2] + 3[6-1]$$

$$2[-3] - 5[1] + 3[5]$$

$$-6 - 5 + 15 = \underline{4}$$

determinant = $|A| = |4|$

* Inverse of Matrix = $\frac{\text{adj. } A}{|A|}$

$$A = \frac{\begin{bmatrix} -3 & -1 & 5 \\ 1 & 1 & 1 \\ 7 & 5 & -13 \end{bmatrix}}{4}$$

OR

$$A = \frac{1}{4}\begin{bmatrix} -3 & -1 & 5 \\ 1 & 1 & 1 \\ 7 & 5 & -13 \end{bmatrix}.$$