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BASIC MATHEMATICS

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Integration symbol $\int$

* $\frac{d}{dt}(y)$ * $\frac{dy}{dx} = \frac{d}{dx}(y)$

* $\frac{d}{dx}(e^x) = e^x$

Exponential function

* $\frac{d}{dx}(\sin x) = \cos x$ (TRIGNOMETRY FUNCTION)

differentiation of sin x is cos x.

* $\frac{d}{dx}(\cos x) = -\sin x$

* $\frac{d}{dx}(\log x) = \frac{1}{x}$ (LOGARITHMATIC FUNCTION)

* $\frac{d}{dx}(\tan x) = \sec^2 x$

* $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

* $\frac{d}{dx}(\sec x) = \sec x \tan x$

* $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

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Q: find $\frac{d}{dx}[\log x + \tan x - e^x]$ (LINEARTY PROPERTY)

Ans: $\frac{d}{dx}(\log x) + \frac{d}{dx}(\tan x) + - \frac{d}{dx}(e^x)$

$\Rightarrow \frac{1}{x} + \sec^2 x - e^x$

DETERMINANT and MATRIX

$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$

determinant

$\begin{array}{c} \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} n \\ \langle \; M \; \rangle \end{array}$

Matrix

* $\underline{M} \times \underline{N}$
  No. of Rows, No. of Columns

$\Rightarrow 2 \times 2$ (MATRIX ORDER)

* $\begin{bmatrix} 2 & 9 & 10 \\ 3 & 7 & 8 \end{bmatrix}$ $\Rightarrow 2 \times 3$ (MATRIX ORDER)

PRINCIPLE DIAGONALS (DIAGONAL ELEMENTS)

$\begin{bmatrix} \overset{a_{11}}{2} & \overset{a_{12}}{3} \\ \underset{a_{21}}{4} & \underset{a_{22}}{5} \end{bmatrix}$

Key Element

Place Value of 2 $\Rightarrow a_{11}$

$\Rightarrow a_{11} \times a_{22} - a_{12} \times a_{21}$

$\Rightarrow 2 \times 5 - 3 \times 4$

$\Rightarrow 10 - 12 \Rightarrow -2$

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* $\begin{bmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{bmatrix}$ (2 × 2)

$\Rightarrow \sin^2 x - (-\cos^2 x) \Rightarrow \sin^2 x + \cos^2 x = 1$

* $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ (3 × 3)

- a_{11} rows and columns are excluded first.

$\Rightarrow a_{11} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

- EXAMPLE : $\begin{vmatrix} 2 & 6 & 1 \\ 3 & 7 & 4 \\ 4 & -1 & 5 \end{vmatrix}$

$\Rightarrow 2 \begin{vmatrix} 7 & 4 \\ -1 & 5 \end{vmatrix} - 6 \begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix} + 1 \begin{vmatrix} 3 & 7 \\ 4 & -1 \end{vmatrix}$

$\Rightarrow 2(35 + 4) - 6(15 - 16) + 1(-3 - 28)$

$\Rightarrow 2(39) - 6(-1) + 1(-31)$

$\Rightarrow 78 + 6 - 31 \Rightarrow 84 - 31 \Rightarrow 53$ Answer

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PROPERTIES OF DETERMINANTS

$$\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix}$$

i) In a determinant, if the row or a column interchange then the value of the determinant will not change.

$$\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} \neq \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} \Rightarrow \text{ROW INTERCHANGING}$$

$12-15 = -3$ $\qquad$ $12-15 = -3$

$$\begin{vmatrix} 2 & 6 & 1 \\ 3 & 7 & 4 \\ 4 & -1 & 5 \end{vmatrix} \neq \begin{vmatrix} 2 & 3 & 4 \\ 6 & 7 & -1 \\ 1 & 4 & 5 \end{vmatrix} \Rightarrow \text{ROW CONVERTED IN COLUMNS}$$

$= 53$

$\Rightarrow 2(35+4) - 3(30+1) + 4(24-7)$

$\Rightarrow 78 - 93 + 68$

$\Rightarrow 146 - 93 \Rightarrow 53$

ii) If 2 rows or 2 columns of a determinant are interchanged, the sign of the value of the determinant changes.

$$\begin{vmatrix} 2 & 7 \\ 6 & 3 \end{vmatrix} \neq \begin{vmatrix} 6 & 3 \\ 2 & 7 \end{vmatrix} \Rightarrow \text{INTERCHANGING ROWS}$$

$\Rightarrow 6 - 42$ $\qquad$ $\Rightarrow 42 - 6$

$= -36$ $\qquad$ $\Rightarrow 36$ $\qquad$ (SIGN CHANGES)

$\neq$ INTERCHANGING

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iii) If 2 rows or 2 columns of a determinant are identical, then the value of the determinant is 0.

$$\begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix} \Rightarrow 2\times2 - 2\times2$$

$$\Rightarrow 4-4 \Rightarrow 0$$

iv) If all the elements of row and column of a determinant are 0 then the value of the determinant is 0.

$$\begin{vmatrix} 2 & 0 & 6 \\ 3 & 0 & 8 \\ 4 & 0 & 9 \end{vmatrix} \quad \text{or} \quad \begin{vmatrix} 0 & 0 & 0 \\ 3 & 0 & 8 \\ 4 & 5 & 9 \end{vmatrix}$$

v) If the element of any row or column of the determinant be each multiplied by the same number, the determinant is multiplied by that number.

$$\begin{vmatrix} 2 & -6 \\ 3 & 8 \end{vmatrix} \neq \begin{vmatrix} 2\times2 & -6 \\ 3\times2 & 8 \end{vmatrix}$$

$\Rightarrow 16 + 18$ $\qquad\qquad$ $\Rightarrow 32 + 36$

$\Rightarrow 34$ $\qquad\qquad$ $\Rightarrow 68$ or 2×34

$$\begin{vmatrix} 2 & -4 \\ 3 & 5 \end{vmatrix} \neq \begin{vmatrix} 2 & -4 \\ 3\times3 & 3\times5 \end{vmatrix}$$

$\Rightarrow 10 + 12$ $\qquad\qquad$ $\Rightarrow 30 + 36$

$= 22$ $\qquad\qquad$ $\Rightarrow 66$ or 3×22

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MINOR

The Minor of element is defined as a determinant obtained by deleting the row and column containing the element.

$$\begin{vmatrix} 2 & 6 \\ 4 & 9 \end{vmatrix}$$

deleting row 2, 6 and column 2, 4
Minor for 2 is 9.

9 is the MINOR of **2**

$$\begin{vmatrix} 2 & 6 \\ 4 & 9 \end{vmatrix}$$

4 is the MINOR of **6**

$$\begin{vmatrix} 2 & 6 \\ 4 & 9 \end{vmatrix}$$

2 is the MINOR for 9

$(-1)^{+ve}$ always +ve
↓
EVEN

$(-1)^{-ve}$ always -ve
↓
ODD

$$\begin{vmatrix} 2 & 3 & 4 \\ 6 & 2 & 8 \\ 7 & -1 & 10 \end{vmatrix}$$

MINOR for **2** is $\begin{vmatrix} 2 & 8 \\ -1 & 10 \end{vmatrix}$

M_{11} $\begin{vmatrix} 2 & 8 \\ -1 & 10 \end{vmatrix}$ 28

M_{12} $\begin{vmatrix} 6 & 8 \\ 7 & 10 \end{vmatrix}$ 4

M_{13} $\begin{vmatrix} 6 & 2 \\ 7 & -1 \end{vmatrix}$ -20

M_{21} $\begin{vmatrix} 3 & 4 \\ -1 & 10 \end{vmatrix}$ 34

M_{22} $\begin{vmatrix} 2 & 4 \\ 7 & 10 \end{vmatrix}$ -8

M_{23} $\begin{vmatrix} 2 & 3 \\ 7 & -1 \end{vmatrix}$ -23

M_{31} $\begin{vmatrix} 3 & 4 \\ 2 & 8 \end{vmatrix}$ 16 M_{32} $\begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix}$ -8 M_{33} $\begin{vmatrix} 2 & 3 \\ 6 & 2 \end{vmatrix}$ -14

COFACTOR

FORMULA : $(-1)^{i+j} \times$ MINOR

where i = No. of rows of the element,
j = No. of column of the element.

$C_{11} = (-1)^{1+1} \times 28 = 28$

$C_{12} = (-1)^{1+2} \times 4 = -4$

$C_{13} = (-1)^{1+3} \times -20 = -20$

$C_{21} = (-1)^{2+1} \times 34 = -34$

$C_{22} = (-1)^{2+2} \times -8 = -8$

$C_{23} = (-1)^{2+3} \times -23 = +23$

$C_{31} = (-1)^{3+1} \times 16 = 16$

$C_{32} = (-1)^{3+2} \times -8 = +8$

$C_{33} = (-1)^{3+3} \times -14 = -14$

COFACTORS

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solution of SIMULTANEOUS LINEAR EQUATIONS by using CRAMER'S RULE

Let us consider following linear equations. When we solve such type of equations firstly we find D by using the copy sets coefficients of unknowns.

$a_1 x + b_1 y + c_1 z = d_1$
$a_2 x + b_2 y + c_2 z = d_2$
$a_3 x + b_3 y + c_3 z = d_3$

(x, y, z) variables or unknowns

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad D_2 = \begin{vmatrix} a_1 & b_1 & c_1 \\ D_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$x = \frac{D1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}$$

D - coefficients of X, Y, Z
D_1 : coefficient of X goes out, coefficient of D_1, D_2, D_3 comes in
D_2 : coefficient of Y goes out, coefficient of $D1, D_2, D_3$ comes in
D_3 = coefficients of z goes out, coefficient of D_1, D_2, D_3 comes in

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Q: solve the following equation by using CRAMER's RULE:

$2x - 3y + 4z = -9$ Coefficients of $x = 2$, $y = -3$
$-3x + 4y + 2z = -12$ $z = 4$
$4x - 2y - 3z = -3$

Ans:

$D = \begin{matrix} 2 & -3 & 4 \\ -3 & 4 & 2 \\ 4 & -2 & -3 \end{matrix}$ $D_1 = \begin{matrix} -9 & -3 & 4 \\ -12 & 4 & 2 \\ 3 & 2 & -3 \end{matrix}$ $D_2 = \begin{matrix} 2 & -9 & 4 \\ -3 & -12 & 2 \\ 4 & 3 & -3 \end{matrix}$

$D_3 = \begin{matrix} 2 & -3 & -9 \\ -3 & 4 & -12 \\ 4 & -2 & 3 \end{matrix}$

$D = 2[-12+4] + 3[9-8] + 4[6-16] = -16 + 3 - 40 = -53$
$D_1 = -9[-12+4] + 3[36+6] + 4[24-12] = 72 + 126 + 144 = 342$
$D_2 = 2[36+6] + 9[9-8] + 4[9+48] = 93 + 228 = 321$
$D_3 = 2[-12-24] + 3[9+48] - 9[6-16] = -72 + 261 = 189$

$x = \frac{D_1}{D} = \frac{342}{-53}$ $y = \frac{D_2}{D} = \frac{321}{-53}$ $z = \frac{D_3}{D} = \frac{189}{-53}$

Q: $5x - 7y + z = 11$ $D = \begin{matrix} 5 & -7 & 1 \\ 6 & -8 & -1 \\ 3 & 2 & -6 \end{matrix}$
$6x - 8y - z = 15$
$3x + 2y - 6z = 7$

$D_1 = \begin{matrix} 11 & -7 & 1 \\ 15 & -8 & -1 \\ 7 & 2 & -6 \end{matrix}$ $D_2 = \begin{matrix} 5 & 11 & 1 \\ 6 & 15 & -1 \\ 3 & 7 & -6 \end{matrix}$

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$$D_3 = \begin{matrix} 5 & -7 & 11 \\ 6 & -8 & 15 \\ 3 & 2 & 7 \end{matrix}$$

(D) $= 5[-8\times-6+2] + 7[-36+3] + 11[12+24]$

$\Rightarrow 250 + (-231) + 36 \Rightarrow 286 - 231 \Rightarrow$ **55**

(D1) $= 11[-48+2] + 7[-90+7] + 1[30+56]$

$\Rightarrow 550 + (-581) + 86 \Rightarrow 636 - 581 \Rightarrow$ **55**

(D2) $\Rightarrow 5[-90+7] - 11[-36+3] + 1[42-45]$

$\Rightarrow -415 + 363 - 3 \Rightarrow -415 + 363 \Rightarrow$ **-55**

(D3) $\Rightarrow 5[-56-30] + 7[42-45] + 11[12+24]$

$\Rightarrow 5[-86] + (-21) + 396 \Rightarrow -430 - 21 + 396 \Rightarrow -451 + 396$

$\Rightarrow$ **-55**

$\underline{X} \Rightarrow \frac{D_1}{D} \Rightarrow \frac{55}{55} \Rightarrow \underline{1}$

$\underline{Y} \Rightarrow \frac{D_2}{D} \Rightarrow \frac{-55}{55} \Rightarrow \underline{-1}$

$\underline{Z} \Rightarrow \frac{D_3}{D} \Rightarrow \frac{-55}{55} \Rightarrow \underline{-1}$

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MATRICES

* Let us consider a set of simultaneous Equation

$3x + y + 4z = 0$
$2x + 3y + 8z = 0$
$5x + 4y + 7z = 0$

• Now, we write the coefficient of x, y, z of the equations and enclose them inside brackets.

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 3 & 8 \\ 5 & 4 & 7 \end{bmatrix}$$

* TYPES of MATRICES :

1) ROW MATRIX : If a Matrix has only 1 row and any no. of columns, it is called a Row Matrix.

$$A = \begin{bmatrix} 2 & 4 & 6 & 7 \end{bmatrix}$$

(1×4) (order of Matrix)

II) COLUMN MATRIX : If a Matrix has only 1 column and any no. of rows, it is called a Column Matrix.

$$A = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

(3×1) (Order of Matrix)

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iii) <u>NULL / ZERO MATRIX</u> : Any Matrix in which all the elements are 0 is a Null Matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2\times3)$$

iv) <u>SQUARE MATRIX</u> : Matrix in which no. of rows is equal to the no. of columns of a Matrix is a square Matrix.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (2\times2)$$

v) <u>DIAGONAL MATRIX</u> : Square Matrix in which all the diagonal elements are non-zero elements.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad (2\times2)$$

* Only Principle diagonals are considered.

vi) <u>UNIT or IDENTITY MATRIX</u> : Square Matrix in which all diagonal elements are 1 and all other elements are 0. It is denoted by I.

$$A = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3\times3)$$

* Only principle diagonals.

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ALGEBRA of MATRICES :

* I) ADDITION of A $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and B $\begin{bmatrix} -7 & 10 \\ 8 & 6 \end{bmatrix}$

Q: Find A+B (Corresponding elements are added)

$$\begin{bmatrix} 2+(-7) & -1+10 \\ 3+8 & 4+6 \end{bmatrix} \Rightarrow \begin{bmatrix} -5 & 9 \\ 11 & 10 \end{bmatrix}$$

* II) SUBTRACTION of A $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and B $\begin{bmatrix} -7 & 10 \\ 8 & 6 \end{bmatrix}$

Q: Find A-B (Corresponding elements are subtracted)

$$\begin{bmatrix} 2-(-7) & -1-10 \\ 3-8 & 4-6 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & -11 \\ -5 & -2 \end{bmatrix}$$

* III) MULTIPLICATION of A $\begin{bmatrix} -1 & 3 \\ 2 & 6 \end{bmatrix}$ and B $\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

The product of 2 Matrices (A and B) is only possible if the no. of columns of A = no. of rows of B.

Q: Find A and B and show that $AB \neq BA$.

Ans: 1st ROW of A and 1st COLUMN of B

$$AB = \begin{bmatrix} -1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

$-1 \times 6 \quad 3 \times 7$

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$$= \begin{array}{c} \begin{array}{cc} \text{1st 1st} & \text{1st 2nd} \end{array} \\ \begin{bmatrix} (-1\times 6)+(3\times 7) & (-1\times 8)+(3\times 9) \\ (2\times 6)+(6\times 7) & (2\times 8)+(6\times 9) \end{bmatrix} \\ \begin{array}{cc} \text{2nd 1st} & \text{2nd 2nd} \end{array} \end{array} = \begin{bmatrix} 15 & 19 \\ 54 & 70 \end{bmatrix} \underline{\underline{AB}}$$

$$\neq$$

$$BA = \begin{array}{c} \begin{array}{cc} \text{1st 1st} & \text{1st 2nd} \end{array} \\ \begin{bmatrix} (6\times -1)+(8\times 2) & (6\times 3)+(6\times 8) \\ (7\times -1)+(9\times 2) & (7\times 3)+(9\times 6) \end{bmatrix} \\ \begin{array}{cc} \text{2nd 1st} & \text{2nd 2nd} \end{array} \end{array} = \begin{bmatrix} 10 & 66 \\ 11 & 75 \end{bmatrix} \underline{\underline{BA}}$$

<u>Q:</u> If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} +1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$

obtain the product AB and explain why BA is not defined.

$$AB = \begin{bmatrix} (0\times 1)+(1\times -1)+(2\times 2) & (0\times -2)+(1\times 0)+(2\times -1) \\ (1\times 1)+(2\times -1)+(3\times 2) & (1\times -2)+(2\times 0)+(3\times -1) \\ (2\times 1)+(3\times -1)+(4\times 2) & (2\times -2)+(3\times 0)+(4\times -1) \end{bmatrix}$$

$$= \begin{bmatrix} 0+(-1)+4 \\ 1+(-2)+6 \\ 2+(-3)+8 \end{bmatrix} \quad \begin{bmatrix} 0+0+(-2) \\ -2+0+(-3) \\ -4+0+(-4) \end{bmatrix}$$

// For more easy understanding
Not a STEP

$$AB = \begin{bmatrix} 3 & -2 \\ 5 & -5 \\ 7 & -8 \end{bmatrix} \checkmark$$

<u>Answer</u>.

the no. of columns in Matrix B is 2 and no. of rows in matrix A is 3.

Therefore, BA is not defined. because no of columns ≠ no. of rows.

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* SCALAR MULTIPLICATION

Q: If $A = \begin{bmatrix} 5 & 10 & 12 \\ -2 & 6 & 13 \\ 3 & 8 & 4 \end{bmatrix}$ than find 5A.

Ans: $5 \times \begin{bmatrix} 5 & 10 & 12 \\ -2 & 6 & 13 \\ 3 & 8 & 4 \end{bmatrix}$ & $\begin{bmatrix} 25 & 50 & 60 \\ -10 & 30 & 65 \\ 15 & 40 & 20 \end{bmatrix}$ = Answer 5A.

Q: If $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ than show that
$A^3 - A^2 - 4A + 4I = 0$

Ans: $A^3 = A \cdot A^2$
$A^2 = A \cdot A$

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ALGEBRA

BASIC FORMULAS :

i) $(a+b)^2 = a^2 + b^2 + 2ab$

ii) $(a-b)^2 = a^2 + b^2 - 2ab$

iii) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

iv) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

v) $a^2 - b^2 = (a+b)(a-b)$

vi) $a^3 + b^3 = (a+b)(a^2+b^2-ab)$

vii) $a^3 - b^3 = (a-b)(a^2+b^2+ab)$

POLYNOMIALS

$ax^2 + bx + c = 0$ (Quadratic Equation) DEGREE 2

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Q: $3x^2 - 9x - 4 = 0$ Find the value of x & solve or factorise the equation.

$a=3$, $b=9$, $c=-4$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(3)(-4)}}{2(3)} \Rightarrow \frac{-9 \pm \sqrt{81+48}}{6}$$

$$\Rightarrow \frac{-9 \pm \sqrt{129}}{6} \Rightarrow$$

1 $\dfrac{-9+\sqrt{129}}{6}$

2 $\dfrac{-9-\sqrt{129}}{6}$

} FINAL VALUES

$* \quad \frac{x^m}{x^n} = x^{m-n}$

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Q: $x^2 - 5x + 6 = 0$ $\Rightarrow a = 1, \; b = -5, \; c = 6$

METHOD ①

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{5 \pm \sqrt{25 + 4(1)(6)}}{2(1)}$$

$$\Rightarrow \frac{5 \pm \sqrt{25-24}}{2} \Rightarrow \frac{5 \pm \sqrt{1}}{2}$$

$$\Rightarrow ① \; \frac{5+\sqrt{1}}{2} \qquad ② \; \frac{5-\sqrt{1}}{2}$$

$$x \Rightarrow \frac{5+1}{2} = 3 \qquad x \Rightarrow \frac{5-1}{2} = 2$$

* By splitting the MID TERM:

METHOD ②

$ax^2 + bx + c = 0$

EXAMPLE:

$$\underbrace{x^2 - 5x + 6}_{a \times c \,=\, 1 \times 6 \,=\, 6} = 0$$

Multiplication of $a \times c$ = Result will be treated as $A \times C$

* Checking 2 numbers that their multiplication is Result and their addition is B.

$-2 + -3 = -5$

$2 \times 3 = 6$

$x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 2x - 3x + 6 = 0$

$\Rightarrow x(x-2) - 3(x-2) = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\downarrow \qquad \downarrow$

$x = 2 \qquad x = 3$

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Q: $x^2 - 2x - 15 = 0$

$5 \times 3 = 15$
$-5 + 3 = -2$

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow x(x-5) + 3(x-5) = 0$

$\Rightarrow (x-5)(x+3) = 0$

$\downarrow \quad \downarrow$

$x = 5 \quad x = -3$

$\Rightarrow (x-5) = 0 \Rightarrow x = 5$

$\Rightarrow (x+3) = 0 \Rightarrow x = -3$

Q: $x^2 + 10x + 25 = 0$

$5 \times 5 = 25$
$5 + 5 = 10$

$\Rightarrow x^2 + 5x + 5x + 25 = 0$

$\Rightarrow x(x+5) + 5(x+5) = 0$

$\Rightarrow (x+5)(x+5) = 0$

$\downarrow \quad \downarrow$

$x = -5 \quad x = -5$

METHOD 1

* $(a+b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow x^2 + 10x + 25 = 0$

$\Rightarrow x^2 + 2(x)(5) + 5^2 = 0$

$\Rightarrow (x+5)^2 = 0$

$\Rightarrow (x+5)(x+5) = 0$

$\downarrow \quad \downarrow$

$-5 \quad -5$

METHOD 2.

Q: $x^2 - 48x + 576 = 0$

$\Rightarrow x^2 - 24x - 24x + 576 = 0$

$\Rightarrow x(x-24) - 24(x-24) = 0$

$\Rightarrow (x-24)(x-24) = 0$

$\downarrow \quad \downarrow$

$24 \quad 24$

METHOD 1

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Q: $x^2 - 49 = 0$

$\Rightarrow (x+7)(x-7) = 0$

$\Rightarrow x = \pm 7$

7 -7

Q: $(676 - 576) = 100$

$\Rightarrow (26+24)(26-24)$

$\Rightarrow (50)(2)$

$\Rightarrow 100$

* $26^2 - 24^2 = 676 - 576 = 100$

Q: $x^3 - 12x^2 + 36x - 32 = 0$ CUBIC POLYNOMIALS

* Remainder theorem:

solution of cubic Polynomial by using Remainder Theorem:

1. CONSTANT TERM $\Rightarrow$ 32 $\Rightarrow$ Factors = $2 \times 2 \times 2 \times 2 \times 2 \times 1$

$x = 1$ $\Rightarrow (1)^3 - 12(1)^2 + 36(1) - 32$

$\Rightarrow 1 - 12 + 36 - 32$

$\Rightarrow 37 - 44 \neq 0$

$x = 2$ $\Rightarrow (2)^3 - 12(2)^2 + 36(2) - 32$

$\Rightarrow 8 - 48 + 72 - 32$

$\Rightarrow 40 + 72 - 32$

✓ $\Rightarrow 0 = 0$

$x = 2$

$\Rightarrow x - 2 = 0$ Factor of the given equation.

a) * DIVISION METHOD

b) * 2^{nd} METHOD

c) *

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a) DIVISION METHOD

$$
\begin{array}{r|l}
x-2 & x^3 - 12x^2 + 36x - 32 \quad | \; x^2 - 10x + 16 \\
 & (-)\ \cancel{x^3} - 2x^2 \\
\hline
 & -\cancel{10x^2} + 36x - 32 \\
 & (-)\ -\cancel{10x^2} + 20x \\
\hline
 & 16x - 32 \\
 & (-)\ 16x - 32 \\
\hline
 & 0
\end{array}
$$

$\Rightarrow (x-2)(x^2-10x+16) = 0$

$\Rightarrow (x-2)\ (x^2-8x-2x+16) = 0$

$\Rightarrow (x-2)\ (x(x-8)-2(x-8)) = 0$

$\Rightarrow (x-2)\ (x-8)(x-2) = 0$

$+2 \quad +8 \quad +2 \quad = 0$

$2, 2, 8$

b) 2nd METHOD

FACTOR written 3 times because degree is 3.

$x^3 - 12x^2 + 36x - 32 = 0$

$x^2(x-2) - 10x(x-2) + 16(x-2) = 0$

add add add

$\Rightarrow x^3 - 2x^2 - 10x^2 + 20x + 16x - 32 = 0$

$\Rightarrow x^3 - 12x^2 + 36x - 32 = 0$

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OR

$\Rightarrow (x-2)(x^2-10x+16)=0$

$\Rightarrow (x^3-2x^2)-(10x^2+20x)+(16x-32)=0$

$\Rightarrow x^3-12x^2+36x-32=0$

iii) 3rd METHOD :

$x^3-12x^2+36x-32=0$

$\Rightarrow \underline{x^3-2x^2}\ \underline{-10x^2+20x}\ \underline{+16x-32}=0$

$\Rightarrow x^2(x-2)-10x(x-2)+16(x-2)=0$

$\Rightarrow (x-2)(x^2-10x+16)=0$

$\Rightarrow (x-2)(x-8)(x-2)=0$

$\downarrow \quad \downarrow \quad \downarrow$

$+2 \quad +8 \quad +2 \quad =0$

2, 2, 8

Q: $x^3+x^2-21x-45=0$

FACTORS of 45 = 0, 1, 3, 5, 9, 15, 45

$\Rightarrow 3\times3\times5\times1 = 45$

$\Rightarrow x=1 \quad \Rightarrow 1^3+1^2-21(1)-45$

$\Rightarrow 1+1-21-45$

$\Rightarrow 1+1-66 \quad \Rightarrow -64 \neq 0$

$x=5 \quad \Rightarrow (5)^3+(5)^2-21(5)-45$

$\Rightarrow 125+25-105-45$

$\Rightarrow 150-150=0$

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$x = -3$

$\Rightarrow (-3)^3 + (-3)^2 - 21(-3) - 45$

$\Rightarrow -27 - 9 + 63 - 45$

$\Rightarrow -36 + 63 - 45 \quad \Rightarrow -72 + 72 = 0$ ✓

$x = -3$

$x + 3 = 0$ FACTOR :

$$
\begin{array}{r|l}
x+3 & x^3 + x^2 - 21x - 45 \quad \Big| \; x^2 - 2x - 15 \\
 & (-)\ x^3 + 3x^2 \\
\hline
 & -2x^2 - 21x - 45 \\
 & (-)\ -2x^2 - 6x \\
\hline
 & -15x - 45 \\
 & -15x - 45 \\
\hline
 & 0
\end{array}
$$

$(x+3)(x^2 - 2x - 15)$

$\Rightarrow (x+3)(x^2 - 5x + 3x - 15)$

$\Rightarrow (x+3)(x(x-5) + 3(x-5))$

$\Rightarrow (x+3)(x+3)(x-5)$

$\downarrow \quad \downarrow \quad \downarrow$

$-3 \quad -3 \quad 5$ ANSWERS

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Q: $x^3 - 6x^2 + 9x - 4 = 0$

* $x = 1$ $\quad (1)^3 - 6(1)^2 + 9(1) - 4$

$= 1 - 6 + 9 - 4$

$= 9 - 9 = 0$ //

$x = 1 \Rightarrow x = -1 \Rightarrow x - 1$

$$
\begin{array}{r|l|l}
(x-1) & x^3 - 6x^2 + 9x - 4 & x^2 - 5x + 4 \\
 & \cancel{x^3} - x^2 & \\
\hline
 & -5x^2 + 9x - 4 & \\
 & -5x^2 + 5x & \\
\hline
 & 4x - 4 & \\
 & 4x - 4 & \\
\hline
 & 0 &
\end{array}
$$

$= (x-1)(x^2 - 5x + 4)$

$= (x-1)(x^2 - 4x - 1x + 4)$ // splitting Mid term

$= (x-1)(x(x-4) - 1(x-4))$

$= (x-1)(x-1)(x-4)$

$\downarrow \quad \downarrow \quad \downarrow$

1 1 4 ANSWERS

Q: $x^3 + x^2 - 21x - 45 = 0$

* $x = -3$ $\quad (-3)^3 + (-3)^2 - 21(-3) - 45$

$= -27 + 9 + 63 - 45$

$= -72 + 72 = 0$ //

$x = -3, \Rightarrow x + 3$

$$
\begin{array}{r|l|l}
x+3 & x^3 + x^2 - 21x - 45 & x^2 - 2x - 15 \\
 & \cancel{x^3} \pm 3x^2 & \\
\hline
 & -2x^2 - 21x \mp 45 & \\
 & -\cancel{2x^2} \mp 6x & \\
\hline
 & -15x - 45 & \\
 & -15x \mp 45 & \\
\hline
 & 0 &
\end{array}
$$

$\Rightarrow (x+3)(x^2-2x-15)$

$= (x+3)(x^2-5x+3x-15)$

$= (x+3)(x(x^2-5)+3(x-5))$

$\Rightarrow (x+3)(x-5)(x+3)$

$\downarrow \quad \downarrow \quad \downarrow$

$-3 \quad +5 \quad -3$ VALUES OF X Answers

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NATURAL NUMBERS

Counting Numbers 1, 2, 3, 4 . . . are called Natural Numbers.

WHOLE NUMBERS

All counting numbers together with as 0 form the set of whole numbers. 0, 1, 2, 3

INTEGERS

All Natural Numbers, 0 and -ve of counting numbers together form the set of Integers. -2, -1, 0, 1, 2, 3 . . .

EVEN NUMBERS

A number divisible by 2 is called an Even Number.

ODD NUMBERS

A number not divisible by 2 is called an Odd Number.

PRIME NUMBERS

A number greater than 1 is called a Prime Number if it has exactly 2 factors namely 1 and the number itself.

COMPOSITE NUMBERS

Numbers greater than 1 which are not prime are known as Composite Numbers.

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NOTE : 1 is neither PRIME nor COMPOSITE.

2 is the only even number which is PRIME.

There are 25 prime numbers between 1 and 100.

* TYPES OF DIVISIBILITY :

I) Divisibility by 2 : A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.

$2 \overline{)4573}$ X NOT DIVISIBLE because UNIT DIGIT is 3

$2 \overline{)458}$ ✓ DIVISIBLE as UNIT DIGIT is 8.

II) Divisibility by 3 :

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Q: Find the LCM of 64, 128, 512, 624.

2	64, 128, 512, 624
2	32, 64, 256, 312
2	16, 32, 128, 156
2	8, 16, 64, 78
2	4, 8, 32, 39
2	2, 4, 16, 39
2	1, 2, 8, 39
4	1, 1, 4, 39
3	1, 1, 1, 39
13	1, 1, 1, 13
	1, 1, 1, 1

$\Rightarrow 2^4 * 2^3 * 4 * 3 * 13 =$

$\Rightarrow 16 * 8 * 156 \Rightarrow 128 \times 156 \Rightarrow \underline{19968}$

Q: Simplify $= 896 * 896 - 204 * 204$

$= (896)^2 - (204)^2 \Rightarrow (896 + 204)(896 - 204)$

$\Rightarrow 1100 * 692 = \underline{761200}$

Q: simplify $= 387 * 387 + 114 * 114 + 2 * 387 * 114$

$a^2 + b^2 + 2ab$

$\Rightarrow (387 + 114)^2$

$\Rightarrow (501)^2 \Rightarrow \underline{251001}$

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INVERSE of A MATRIX

FORMULA :

$$\frac{adj.\ A}{|A|}, \quad |A| \neq 0$$

Q: $A = \begin{bmatrix} 2 & \cancel{6}\,-3 & 4 \\ 6 & 2 & 8 \\ 7 & -1 & 10 \end{bmatrix}$

$\Rightarrow 2[20+8] - 3[60-56] + 4[-6-14]$

$\Rightarrow 56 - 12 - 80 \Rightarrow -36 \neq 0$

} CHECKING $|A| \neq 0$

$$\begin{bmatrix} 28 & -4 & -20 \\ -34 & -8 & 23 \\ 16 & 8 & -14 \end{bmatrix}$$

MATRIX OBTAINED BY COFACTORS OF A.

↓ MAKING TRANSPOSE

ROWS TO COLUMNS

$$Adj.A = \begin{bmatrix} 28 & -34 & 16 \\ -4 & -8 & 8 \\ -20 & 23 & -14 \end{bmatrix}$$

// ADJOINT OF A

$$\frac{\begin{bmatrix} 28 & -34 & 16 \\ -4 & -8 & 8 \\ -20 & 23 & -14 \end{bmatrix}}{-36}$$

OR

$$-\frac{1}{36}\begin{bmatrix} 28 & -34 & 16 \\ -4 & -8 & 8 \\ -20 & 23 & -14 \end{bmatrix}$$

17
09
19

ARITHMATIC PROGRESSION

A.P.

2, 4, 6, 8

4 - 2 = 2 , 6 - 4 = 2 , 8 - 6 = 2

These type of series are known as Arithmatic Progression

* $T_n = a + (n-1)d$
* SUM OF SERIES : $\frac{n}{2}[2a + (n-1)d]$

Q: How many numbers b/w 11 and 90 are divisible by 9.

Ans: 18, 27, 36 - - - - 81

T_n term = 81 , diff = 9

a = 18

• $T_n = a + (n-1)d$

$81 = 18 + (n-1)9 \Rightarrow 81 = 18 + 9n - 9$

$\Rightarrow 9n = 81 + 18 - 9 \Rightarrow 9n = 72$

$n = \frac{72}{9} \Rightarrow 8$

Q: Find the sum of all odd Numbers upto 100.

$T_n = 99$, $a = 1$, $d = 2$

$T_n = a + (n-1)d \Rightarrow 99 = 1 + (n-1)2$

$\Rightarrow 99 = 1 + 2n - 2 \Rightarrow 99 = -1 + 2n$

$-2n = -1 - 99 \Rightarrow -2n = -100 \Rightarrow n = \frac{-100}{-2} = 50$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \frac{50}{2}[2(1) + (n-1)2] \Rightarrow \cancel{25[2 + 2n - 2]}$

$\Rightarrow 25[2 + [50-1]2]$

$\Rightarrow 25[100] \Rightarrow 2500$

Q: Find the sum of all 2 digit numbers which are divisible by 3.

$T_n = a + (n-1)3$

$99 \Rightarrow 12 + (n-1)3$

$99 \Rightarrow 12 + 3n - 3 \Rightarrow 99 = 10 + 3n$

$\Rightarrow -3n = 09 - 99$

$-3n \Rightarrow -90$

$\Rightarrow n = \frac{-90}{-3} = 30$

$S_n = \frac{30}{2}[2(12) + (30-1)3]$

$\Rightarrow 15[24 + 87] \Rightarrow 111 \Rightarrow 1665$

$111 \times 15 = 1665$

$87 + 24 = 111$

Q: How many numbers b/w 200 and 600 are divisible by 4, 5, 6.

204, , , , 596.

- $596 = 204 + (n-1)4$

$\Rightarrow 204 + 4n - 4 \Rightarrow 596 = 200 + 4n \Rightarrow -4n = 200 - 596$

$\Rightarrow -4n = -396 \Rightarrow n = \frac{-396}{-4}$