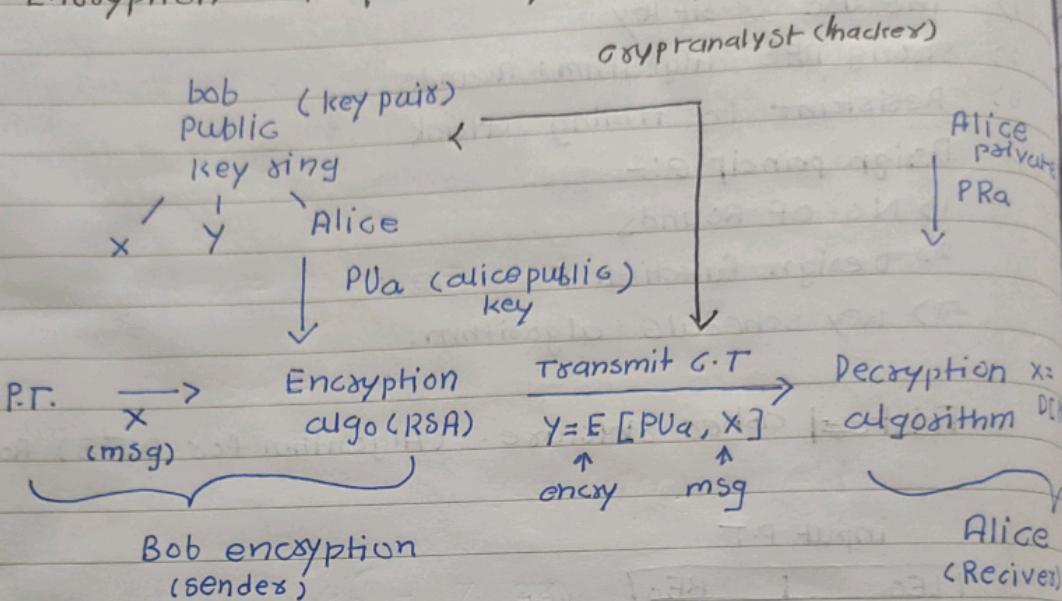
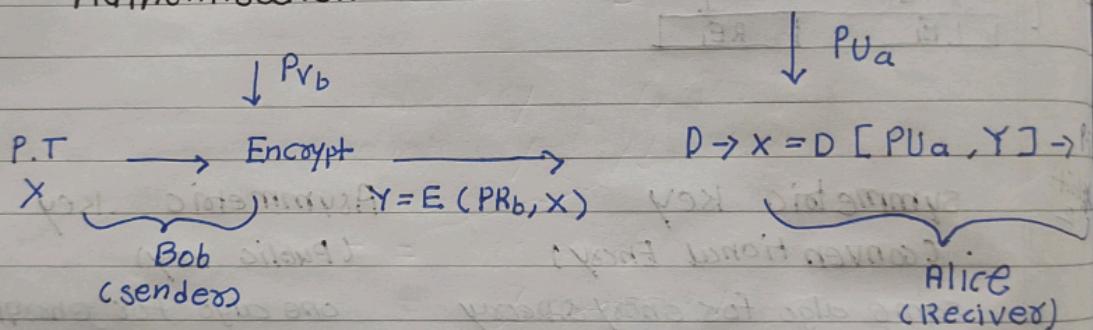


Unit-2 Principles of public key cryptosystems.

Encryption with public key (secrecy)



Authentication:-



RSA Algorithm :

1) Problem

Publickey cryptography

Encryption :-

$$C \cdot T = M^e \mod n \quad (\text{Sender})$$

$$M = C^d \mod n \quad (\text{Receiver})$$

1) Select 2 prime no $p & q$ resp. $p \neq q$

2) $n = p * q$

3) $\phi(n) = (p-1)(q-1)$ start

4) $ed = 1 + k\phi(n)$ $k=0$

Calculate e, d key pair $\{e, n\}$

$\{d, n\}$ Receiver

Analysis of S.I.P.

Lexical Rules:-

~~RSA algorithm~~ RSA is a block cipher in which the P.T. & C.T. are represented as integers both a and $n-1$ for some value of n . The larger msg broke down into number of blocks. each block would have to be represented by an integer.

$$ed = 1 + k\phi(n)$$

$\left\{ \begin{array}{l} \text{select } e \text{ (integer)} \\ \text{calculated} \end{array} \right.$

$$\gcd(\phi(n), e) = 1; 1 < e < \phi(n)$$

$$d = e^{-1} \pmod{\phi(n)}$$

e.g. :-

$$\text{Value of } e \ P = 3, q = 5, M = 4 \ (\text{P.T.})$$

$$P = 3, q = 5 \Rightarrow n = 3 \times 5 = 15$$

$$\phi(n) = (p-1)(q-1) = 2 \times 4 = 8$$

$$\text{For } k = 0, k = 1$$

$$ed = 1 + k\phi(n)$$

$$= 1 + 1 \cdot 8 = 9 \quad \gcd(\phi(n), e) = 1;$$

$$ed = 9^2 = 81 \quad 1 < e < \phi(n)$$

$$\text{Suppose } e = 3, d = 3$$

$$\text{Public Key} = \{3, 15\}$$

$$\text{Private Key} = \{3, 15\}$$

$$\text{Now Cipher Text} \quad C = M^e \pmod{n}$$

$$= 4^3 \pmod{15}$$

$$= 64 \pmod{15}$$

$$= 4 \quad (\text{Remainder})$$

Plain text

$$C^d \pmod{n}$$

$$= 4^3$$

$$= 64 \pmod{15}$$

$$64 \pmod{15} \Rightarrow \text{Ans} - 4 \Rightarrow \text{Ans} \times 15$$

$$4.0077$$

$$= 6$$

2) $P=13, q=17 \quad P=17, q=11 \quad M=88$

$$n = P \times q$$

$$= 17 \times 11$$

$$n = 187 \Rightarrow \phi(n) = 160$$

$$\phi(n) = 16 \times 10 = 160$$

$$ed = 1 + 160 \times 5 = P \times q = 187$$

$$d = 1 = 161 = (1-P)(q-1) = (q-1)$$

$$\Rightarrow \gcd(\frac{\phi(n)}{160}, e) = 1$$

$$ed = 7, 23 + 1 =$$

$$e=7, d=23 = 160$$

public key { 7, 160 }

private key { 23, 160 }

$$G \cdot T = M^e \bmod n$$

$$= 88^7 \bmod 187$$

$$C \cdot T = 11$$

$P=3, q=11, M=5$

$$n = P \times q$$

$$= 11 \times 3 = 33$$

$$\phi(n) = 10 \times 2 = 20$$

$$ed = 1 + 20 \\ = 21$$

$$e = 3$$

$$d = 7$$

$$G \cdot T = M^e \bmod n \\ = 5^3 \bmod 33 \\ = 26$$

The purpose of Diffie-Hellman algorithm is to enable two users to securely exchange a key that can be used for subsequent encryption of msg.

Diffie-Hellman key exchange procedure:-
to exchange key b/w two (S & R) to achieve asymmetric encryption

Alice q prime number (consider)
 $\alpha \neq q$ and α is primitive root of
 User Key Generation
 select private x_A $x_A < q$

calculate public y_A $y_A = \alpha^{x_A} \pmod{q}$

User B Key Generation

x_B $x_B < q$

$y_B = \alpha^{x_B} \pmod{q}$

calculation of secret key by User A

$$K = (y_B)^{x_A} \pmod{q}$$

$$K = (y_A)^{x_B} \pmod{q} \quad \text{for user B}$$

$$K_A = (y_B)^{x_A} \pmod{q}$$

$$K_A = (\alpha^{x_B} \pmod{q})^{x_A} \pmod{q}$$

$$K_A = (\alpha^{x_B})^{x_A} \pmod{q}$$

$$K_A = (\alpha^{x_B x_A} \pmod{q})$$

$$K_A = (\alpha^{x_A})^{x_B} \pmod{q}$$

$$K_A = (\alpha^{x_A} \pmod{q})^{x_B} \pmod{q}$$

$$K_B = (y_A)^{x_B} \pmod{q}$$

Primitive Root:- eg: 5 and 7

5 is primitive of 7
 3 - " - 7

let p as a prime number
 then a is primitive root of p if the powers
 of a modulo p all integers from 1 to p-1 in

Some permutation

p-1

$\alpha \mod p, \alpha^2 \mod p, \dots, \alpha^{p-1} \mod p$

Eg: $p = 7$ then primitive root is 3 cause powers of $3^i \mod 7$ generates all the integers from $i=0$ to $p-1$

$$\begin{aligned} 3^0 &\equiv 3 \pmod{7} \\ 3^1 &\equiv 9 \pmod{7} \equiv 2 \\ 3^2 &\equiv 27 \pmod{7} \equiv 6 \\ 3^3 &\equiv 81 \pmod{7} \equiv 4 \\ 3^4 &\equiv 243 \pmod{7} \equiv 5 \\ 3^5 &\equiv 729 \pmod{7} \equiv 1 \\ 3^6 &\equiv 2187 \pmod{7} \equiv 3 \end{aligned}$$

example :-

1)

$$q = 23$$

$\alpha = 5$ primitive root of q

Alice selects private key $x_A = 6$
Alice computes

$$Y_A = \alpha^{x_A} \mod q$$

$$Y_A = 5^6 \mod 23$$

$$Y_A = 8 \quad (\text{Alice public key})$$

Bob private key $x_B = 15$

$$Y_B = \alpha^{x_B} \mod q$$

$$= 5^{15} \mod 23$$

$$= 19 \quad (\text{Bob public key})$$

Alice Y_A to Bob send Y_B to Alice

Alice compute key $k = (Y_B)^{x_A} \bmod q$
 $= (19)^6 \bmod 23$
 $k = 2$

Bob compute key
 $k = (Y_A)^{x_B} \bmod q$
 $= (8)^{15} \bmod 23$
 $k = 2$

2) $q = 19, \alpha = 10, x_A = 7, x_B = 8$

Given $x_A = 7$

alice $Y_A = \alpha^{x_A} \bmod q$
 $= 10^7 \bmod 19$
 $= 15$

107,0/019 - Ans $\times 9$
 $= \text{Ans}$

bob publickey $Y_B = \alpha^{x_B} \bmod q$
 $= 10^8 \bmod 19$
 $= 17$

Y_A to bob send Y_B to alice

$K_B = (Y_B)^{x_A} \bmod q$
 $= (17)^7 \bmod 19$
 $= 5$

alice calculate shared secret key
 $K_A = (Y_A)^{x_B} \bmod q$
 $= (15)^8 \bmod 19$
 $= 5$

IMP

Message Authentication: Hash function (not use key)
use for data integrity

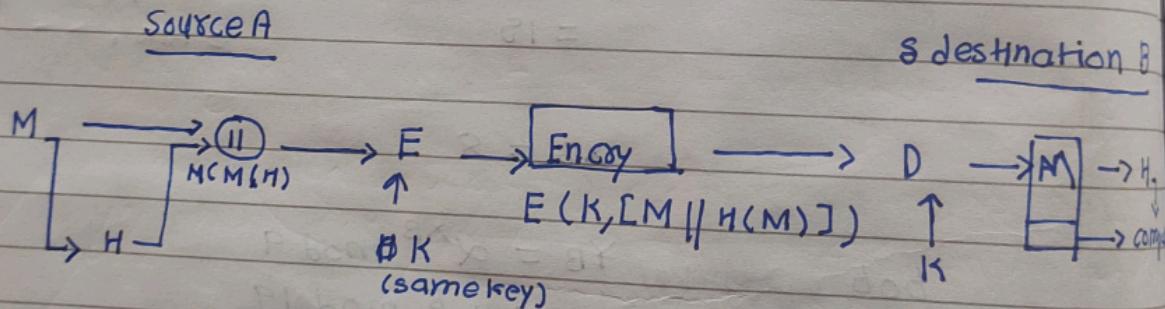
L bits \rightarrow Hash function \rightarrow hash value
Msg M H $h = H(M)$

Hash fun's use for data integrity.

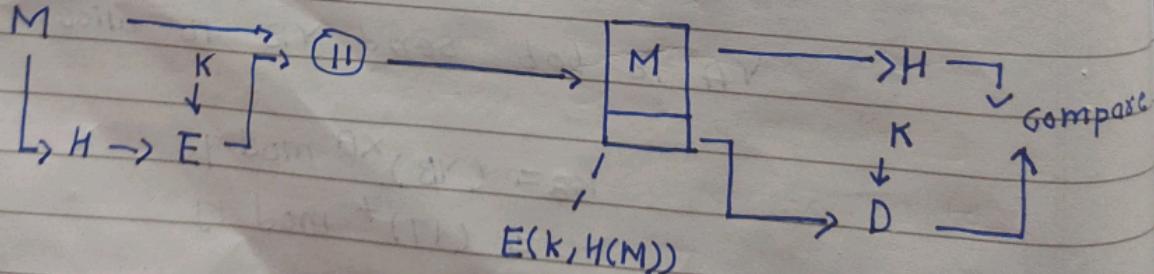
h value $\xrightarrow{\text{Hash fun}^n}$ h it check
 arrive at Receiver \rightarrow this h value
 if same data integrity should be there.

How Authentication is achieve (way's)

A)



B)



Msg Authentication (MAC) Msg Authentication code
(use key)

- uses a shared secret key to generate a MAC (cryptographic checksum)
- MAC append to the Msg
- A → B uses same secret key

calculate the MAC

$$MAC = G(K, M)$$

- A sends $(MAC + M) \rightarrow B$
- B performs the same $(M + MAC)$ using the secret key it generates New MAC'
- Received MAC compared with new MAC'. If Same then data is received successfully.

eg:

Msg - Hello, K

$$\begin{aligned} MAC &= G(K, \text{Hello}) \\ &= \text{pqst} \end{aligned}$$

$$MAC = G = \underline{\text{Hello}} + \underline{\text{pqst}}$$

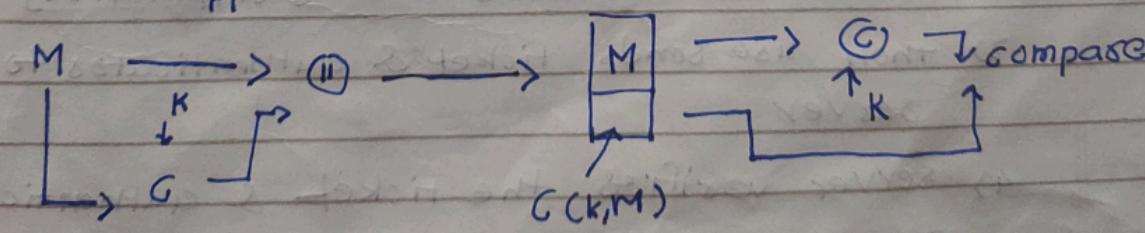
\uparrow compare

$$MAC' = (K, \text{Hello}) = \text{pqst}$$

A

A

B



B

①

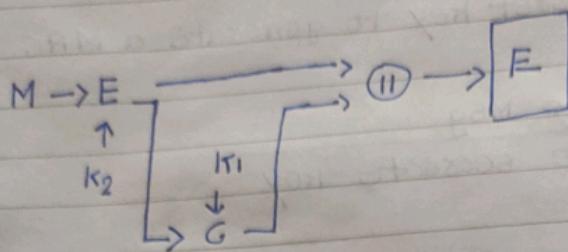
②

③

④

authentication tied to plain text

Authentication tied with ciphertext



Possible approaches to attacking the RSA algorithm

- 1) Brute force
- 2) Mathematical attack
- 3) Timing attacks
- 4) H/W fault-based attack
- 5) chosen ciphertext attacks.

Authentication Key

→ Kerberos

1. user login and request to host for ticket-granting service

Authentication Server (Ac) verifies user access in DB and then send Encrypted response granting ticket and session key user decrypted using user key

- 2) then user send ticket to ticket-granting server (TGS)
the ticket contain username & N/w address
- Now TGS server verifies ticket and then create ticket for requesting service (Application)
- 3) then user sends ticket & authenticator to server
- 4) server verifies the ticket & authenticator and then generate access to the service.
then user can access Appln service.