

## 2. Greedy (Solution) Method

JMS  
SM = TM

- Greedy method is a straight forward technique if a problem has  $n$  inputs and require us to obtain a subset that satisfies some constraints any subset that satisfy these constraints is called a feasible solution.
- We need to find a feasible solution that either maximiz or minimiz a given Objective function.
- A feasible solution that does this is called an optimal solution
- The greedy method such as that one can device an algorithm that works in stages considering one input at a time
- At each stage decision is made regarding whether a particular input is in a an optimal solution .

Greedy (a,n)

{

solution =  $\emptyset$

for  $i = 1$  to  $n$  do

{

$w = \text{select}(a);$

if feasible (solution,  $w$ ) then

solution = Union (solution,  $w$ );

}

return solution;

}

In feasible solutions, maximum feasible soln is an optimal solution.

## Method

### Knapsack Problem:

~~2mp  
SPN 7M~~

$$n=7 \quad m=15$$

Object	1	2	3	4	5	6	7
Profit	10	5	15	7	6	18	3
Weight	2	3	5	7	1	4	1

$$\begin{array}{l} \cdot P \\ \cdot w \end{array} \begin{array}{l} 5 \\ \frac{5}{2} \end{array} \begin{array}{l} \frac{10}{2} \\ \frac{1}{6} \end{array} \begin{array}{l} \frac{5}{4} \\ \frac{3}{4} \end{array} \begin{array}{l} \frac{7}{7} \\ \frac{1}{7} \end{array} \begin{array}{l} \frac{6}{1} \\ \frac{6}{1} \end{array} \begin{array}{l} \frac{18}{3} \\ \frac{4.5}{3} \end{array} \begin{array}{l} \frac{1}{5} \\ \frac{3}{5} \end{array}$$

2/3	2-2=0	object 2	1	2	3	4	5	6	7
	3-1=2	object 7							
	8-5=3	object 4							
	12-4=8	object 6	2x={	1	2/3	1	0	1	1
	14-2=12	object 1	1	0	1	1	1	1	3
	15-1=14	object 5							

15

$$\cdot \sum x_i w_i = 1 \times 2 + 2/3 \times 3 + 1 \times 5 + 0 \times 7 + 1 \times 1 + 4 \times 1 + 1 \times 1$$

$$\sum x_i w_i = 2 + 2 + 5 + 1 + 4 + 1$$

$$\sum x_i w_i = 15$$

$$\cdot \sum x_i p_i = 1 \times 10 + 2/3 \times 5 + 1 \times 15 + 0 \times 7 + 1 \times 6 + 1 \times 18 + 1 \times 3$$

$$\sum x_i p_i = 10 + 3.3 + 15 + 6 + 18 + 3$$

$$\therefore \sum x_i p_i = 55.3$$

### Algorithm:

Greedy knapsack(m,n)

$p[1:n]$  and  $w[1:n]$  are profit, weight respectively n objects such that

$$p[i] \geq p[i+1]$$

$$w[i] \leq w[i+1]$$

m is knapsack size

for

for  $i=1$  to  $n$  do  $x[i]=0$ ;

```

U = m;
for i = 1 to n do
{
    if (w[i] > U) then break;
    x[i] = 1;
    U = U - w[i];
}
if (i <= n) then x[i] = U/w[i];
}

```

We are given  $n$  objects and a knapsack. Object  $i$  has weight  $w_i$  and knapsack has capacity  $m$ . If a fraction  $x_i$  of object  $i$  is placed into the knapsack then profit of  $p_i x_i$  is earned. The objective is to obtain a filling of knapsack that maximizes the total profit earned. Since the knapsack capacity is  $m$  we require the total weight of all chosen objects to be at most  $m$ .

$$\text{maximize } \sum_{i \leq 1 \leq n} p_i x_i$$

$$\text{Subject to } \sum_{i \leq 1 \leq n} w_i x_i \leq m \\ \text{and}$$

$$0 \leq x_i \leq 1, 1 \leq i \leq n$$

Q. Find an optimal solution to the knapsack instance  $n=7, m=15$  pr

Object	1	2	3	4	5	6	7
profit	15	10	7	18	3	4	2
weight	2	3	5	7	1	4	1

•  $\frac{P}{W}$   $\frac{7.5}{1}$   $\frac{3.3}{2}$   $\frac{1.4}{6}$   $\frac{2.5}{4}$   $\frac{3}{3}$   $\frac{1}{7}$   $\frac{2}{5}$

$y_5$	$1-1=0$	Object 3
	$2-1=1$	Object 7
	$3-1=2$	Object 4
	$10-1=9$	Object 5
	$13-3=10$	Object 2 $x = \{1, 1, y_5, 1, 1, 0, 1\}$
15	$15-2=13$	Object 1

$$\begin{aligned} \cdot \sum x_i w_i &= 1 \times 2 + 1 \times 3 + y_5 \times 5 + 1 \times 7 + 2 \times 1 + 0 + 1 \times 1 \\ &= 2 + 3 + 7 + 1 + 1 + 1 \\ \therefore \sum x_i w_i &= 15 \end{aligned}$$

$$\begin{aligned} \cdot \sum x_i p_i &= 1 \times 15 + 1 \times 10 + y_5 \times 7 + 1 \times 18 + 1 \times 3 + 0 + 1 \times 2 \\ &= 15 + 10 + 1.4 + 18 + 3 + 2 \\ \therefore \sum x_i p_i &= 49.4 \end{aligned}$$

#### \* Job sequencing with deadlines.

We are given set of  $n$  jobs associated with job  $i$  is an integer deadline,  $d_i \geq 0$  and profit  $p_i > 0$  for any job  $i$ ; the profit  $p_i$  is earned iff the job is completed by its deadline. To complete a job one has to process the job on a machine for 1 unit of time. Only one machine is available for processing job. Feasible solution for these problem is a subset  $j$  of jobs such that each job in the subset can be completed by its deadline. The value of feasible solution  $j$  is the sum of profits of jobs in  $j$ .

and optimal solution is feasible solution with maximum value

e.g.  $(P_1, P_2, P_3, P_4, P_5)$

Algorithm

Greedy Job ( $d, J, n$ )

{

$J = \sum J_i$ ;

for  $i = 2$  to  $n$  do

{

if All jobs in  $J \cup \sum J_i$  can be completed by their deadlines then  $J = J \cup \{J_i\}$ ;

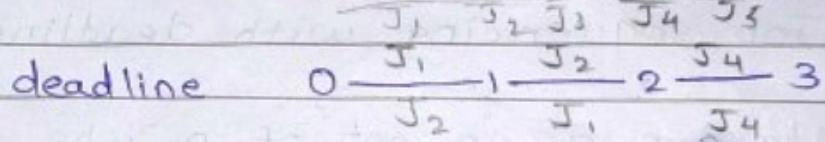
}

}

jobs < deadline  
profit

e.g.  $(P_1, P_2, P_3, P_4, P_5) = (20, 15, 10, 5, 1)$

$(d_1, d_2, d_3, d_4, d_5) = (2, 2, 1, 3, 3)$



Job consider      slot assign      Solution      Profit

-	-	-	0
$J_1$	$[1, 2]$	$J_1$	20
$J_2$	$[0, 1] [1, 2]$	$J_1, J_2$	20 + 15
$J_3$	$[0, 1] [1, 2]$	$J_1, J_2$	20 + 15
$J_4$	$[0, 1] [1, 2] [2, 3]$	$J_1, J_2, J_3$	20 + 15 + 5
$J_5$	$[0, 1] [1, 2] [2, 3]$	$J_1, J_2, J_4$	<u>40</u>

Q.	Job	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$	$J_6$	$J_7$
	profit	35	30	25	20	15	12	5
	deadline	3	4	4	2	3	1	2

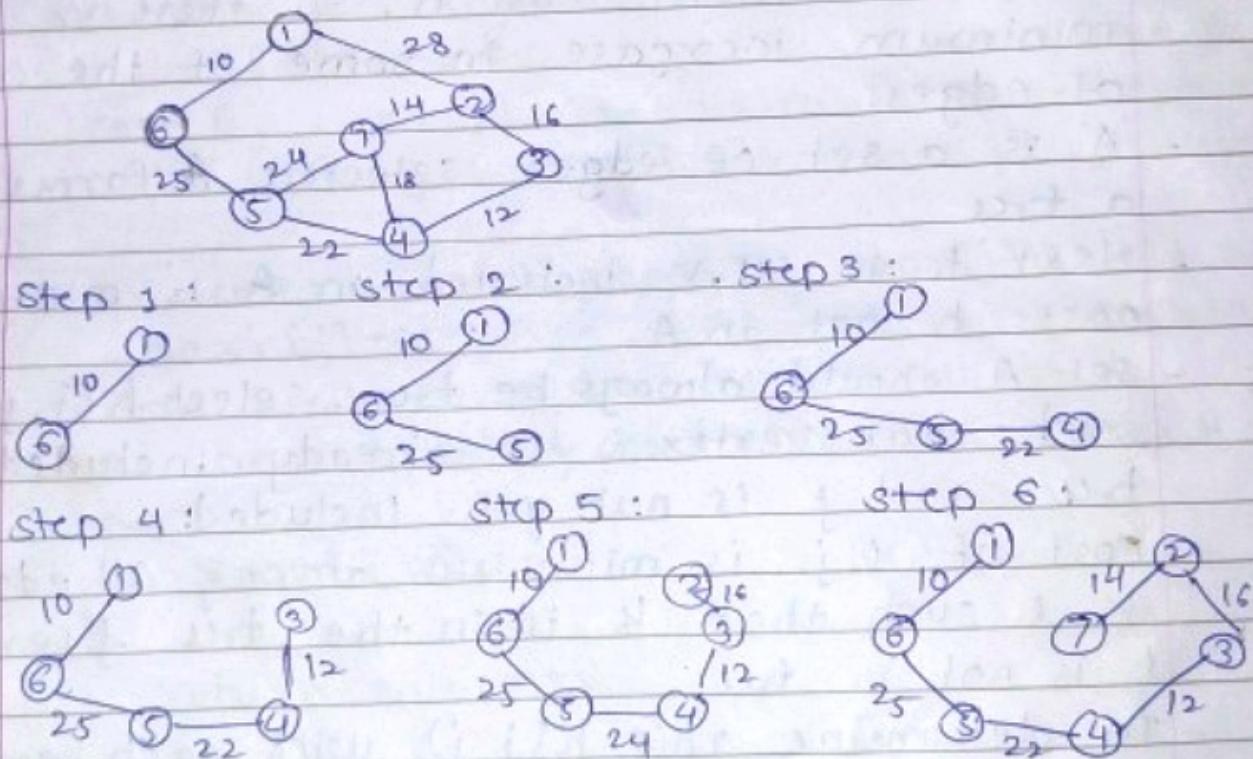
deadline

0 J<sub>4</sub> J<sub>3</sub> J<sub>2</sub> J<sub>1</sub> J<sub>3</sub> J<sub>2</sub> 4

job consider	slot assign	solution	Profit
-	-	-	0
J <sub>1</sub>	[2, 3]	J <sub>1</sub>	35
J <sub>2</sub>	[2, 3] [3, 4]	J <sub>1</sub> , J <sub>2</sub>	35 + 30
J <sub>3</sub>	[2, 3] [3, 4] [1, 2]	J <sub>1</sub> , J <sub>2</sub> , J <sub>3</sub>	35 + 30 + 25
J <sub>4</sub>	[0, 1] [1, 2] [2, 3] [3, 4]	J <sub>4</sub> [3, 5] [1, 2]	35 + 30 + 25 + 20
J <sub>5</sub>	[0, 1] [1, 2] [2, 3] [3, 4]	J <sub>4</sub> [3, 5] [1, 2]	35 + 30 + 25 + 20
J <sub>6</sub>	[0, 1] [1, 2] [2, 3] [3, 4]	J <sub>4</sub> [3, 5] [1, 2]	35 + 30 + 25 + 20
J <sub>7</sub>	[0, 1] [1, 2] [2, 3] [3, 4]	J <sub>4</sub> [3, 5] [1, 2]	35 + 30 + 25 + 20

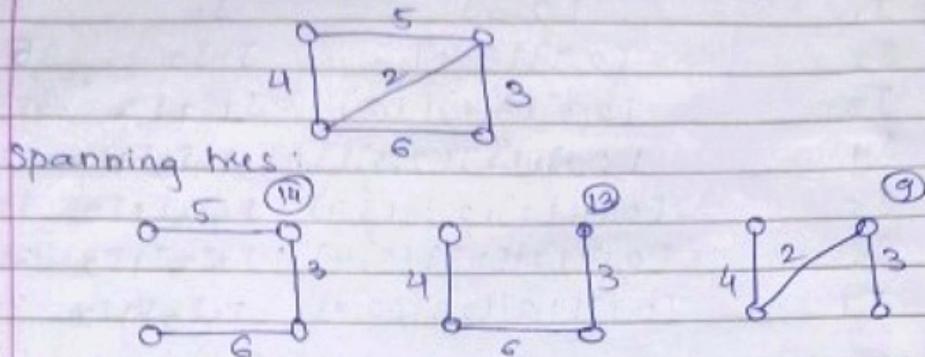
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## \* minimum cost spanning Tree :



- Let  $G = V, E$  be undirected connected graph. subgraph  $t = V, E'$  of graph  $G$  is a spanning tree of graph  $G$  iff  $t$  is a tree.

- Its spanning tree number of vertices are set & connected. with pure no. of edges. 'n' vertices atleast 'n-1' edges & connected graph is a p'tree i.e. no cycle in graph.



#### \* Prims Algorithm:

- Select minimum cost h, so. there is minimum increase in some of the cost of edges.
- 'A' is a set of edges selected. A forms a tree.
- Next h is u,v included in A is minimum cost h not in A.
- set A should always be tree. select h(i,j) such that vertex i is already included in tree and j is not yet included.
- cost of i,j is minimum among all edges k,l such that k is in the tree & vertex l is not in tree.
- To determine this h(i,j) with each vertex j not get included in the tree. A value near [j] is minimum among all choices.

### Algorithm

Prims(E, cost, n, t)

E is set of edges

{

let  $(k, l)$  be an edge of minimum cost in E,

$\text{mincost} = \text{cost}[k, l]$

$t[i, 1] = k, t[i, 2] = l$       value to assign to adjacent

for  $i = 1$  to  $n$  do

if  $(\text{cost}[i, l] < \text{cost}[i, k])$  then

$\text{near}[i] = l$  else

$\text{near}[i] = k$ ;

for  $i = 2$  to  $n - 1$  do

{

let  $j$  be an index such that

$\text{near}[j] \neq 0$       next vertex f

$\text{cost}[j, \text{near}[j]]$  is minimum their edges are

$t[i, 1] = j, t[i, 2] = \text{near}[j], j \neq \text{near}[j]$

$\text{mincost} = \text{mincost} + \text{cost}[j, \text{near}[j]]$ ; add of

$\text{near}[j] = 0$ ;      weights

for  $k = 1$  to  $n$  do

if  $(\text{near}[k] \neq 0) \ \& \ \text{cost}[k, \text{near}[k]] > \text{cost}[k, j]$

then

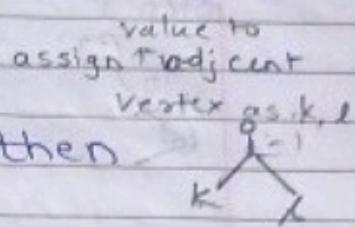
$\text{near}[k] = j$ ;

}

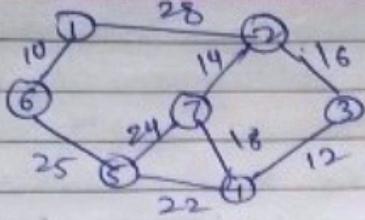
return mincost;

}

The graph should be tree in all stages.

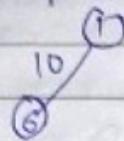


\* Kruskal's method:

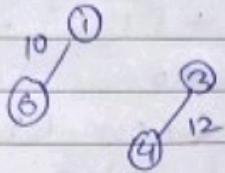


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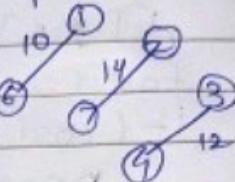
Step 1 :



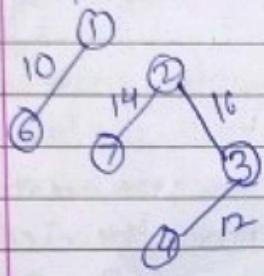
Step 2 :



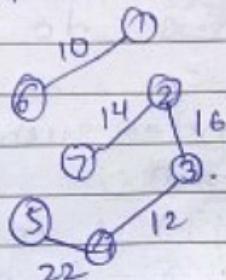
Step 3 :



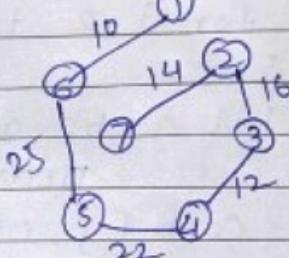
Step 4 :



Step 5 :



Step 6 :



$t = \emptyset$ ;

while ( $t$  has less than  $n-1$  edges) & ( $E \neq \emptyset$ ). do

3

choose an edge  $(v, w)$  from  $E$  of lowest cost;

Delete  $(v, w)$  from  $E$ ;

if  $(v, w)$  does not create a cycle in  $t$

then add  $(v, w)$  to  $t$ ,

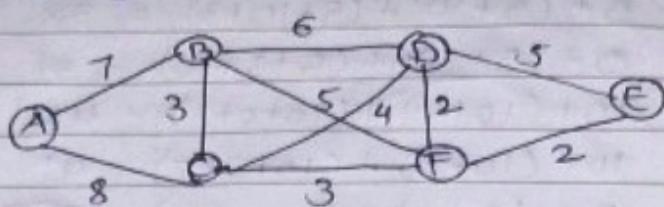
else discard  $(v, w)$ ;

3

- A set of edges so far selected for the spanning tree we such that it is possible to complete  $t$  into a tree.  $t$  is may not be tree in all stages in algorithm.

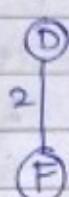
It only being a forest since a set of edges  
it can be completed in root tree iff there  
are no cycle in tree.

eg ①

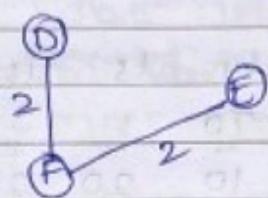


→ Prim's method:

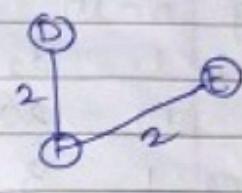
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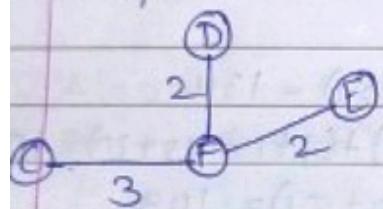
Step 2:



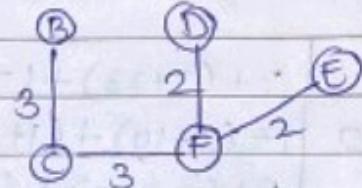
Step 3:



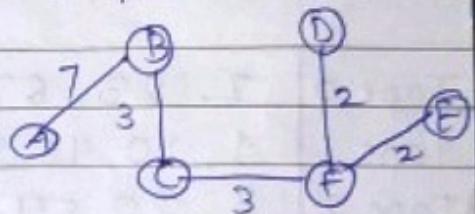
Step 4:



Step 5:

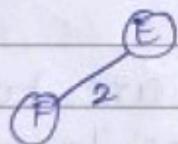


Step 6:

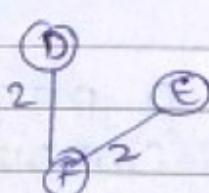


→ Kruskal's method:

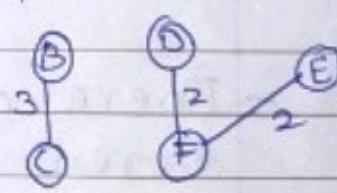
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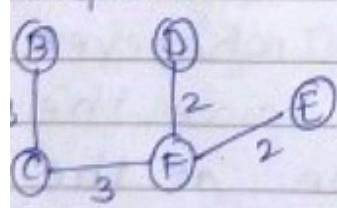
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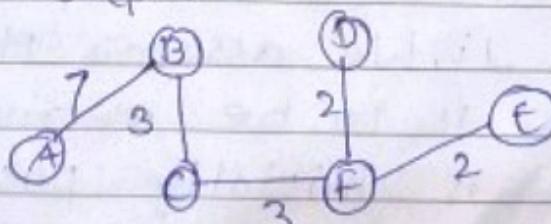
Step 3:



Step 4:



Step 5:



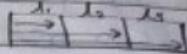
Index mod no. of tapes

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\* Optimal storage on tape:

$n=3$



$$(l_1, l_2, l_3) = (5, 10, 3)$$

$$(1, 2, 3) = 5 + (5+10) + (5+10+3) = 38$$

$$(1, 3, 2) = 5 + (5+3) + (5+3+10) = 31$$

$$(2, 1, 3) = 10 + (10+5) + (10+5+3) = 43$$

$$(2, 3, 1) = 10 + (10+3) + (10+3+5) = 41$$

$$(3, 1, 2) = 3 + (3+5) + (3+5+10) = \textcircled{29}$$

$$(3, 2, 1) = 3 + (3+10) + (3+10+5) = 34$$

e.g.  $n=10$

	$l_1$	$l_2$	$l_3$	$l_4$	$l_5$	$l_6$	$l_7$	$l_8$	$l_9$	$l_{10}$
Asending	10	20	45	70	1	3	7	54	23	67
mod.	1	3	7	10	20	23	45	54	67	70

$$\text{Tape 0 } 7, 23, 67 \quad | \quad 7 + (7+23) + (7+23+67) = 134$$

$$\text{Tape 1 } 1, 10, 45, 70 \quad | \quad 1 + (1+10) + (1+10+45) + (1+10+45+70) = 194$$

$$\text{Tape 2 } 3, 20, 54 \quad | \quad 3 + (3+20) + (3+20+54) = 103$$

$$\therefore \text{Mean retrieval time} = \frac{134 + 194 + 103}{3} = \frac{431}{3} = 143.6$$

- There are  $n$  programs that are to be stored on computer tape of length  $l$ . Associated with each program  $i$  is a length  $l_i$ . We assume that whenever a program is to be retrieved from the store to tape is initially positioned at the front. Hence, if the programs are stored in order,

$$I = i_1, i_2, i_3, \dots, i_n$$

The time  $t_j$  needed to retrieve program  $j$  is proportional to  $\sum_{1 \leq k \leq j} t_k$

- If all programs are retrieved equally often then expected mean retrieval time (MRT) is

$$(1/n) \sum_{1 \leq j \leq n} t_j$$

- In an optimal storage on tape problem we are required to find a permutation for  $n$  programs so that when they are stored on the tape in this order the MRT is minimized.

- Minimizing the MRT is equivalent to minimizing

$$d(I) = \sum_{1 \leq j \leq n} \sum_{1 \leq k \leq j} t_k$$

#### Algorithm:

Store (n, m)

// n is no. of programs

// m is no. of tapes

3

j = 0; // next tape to store on  
for i = 1 to n do

{

    write("Append program", i, "to permutation",  
        "for tape", j);

    j = (j + 1) mod m;

}

3 . . .

3

Q. Find an optimal placement for 13 programs on 3 tapes to, t<sub>1</sub>, t<sub>2</sub> where programs are of lengths

1.	10	16	14	15	10	17	10	19	110	16	14
12	5	8	132	7	5	18	26	4	3	11	10
1	2	3	4	5	6	7	8	10	11	12	13
Assending	3	4	5	5	6	7	8	10	11	12	18
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
Mod	1	2	0	1	2	0	1	2	0	1	

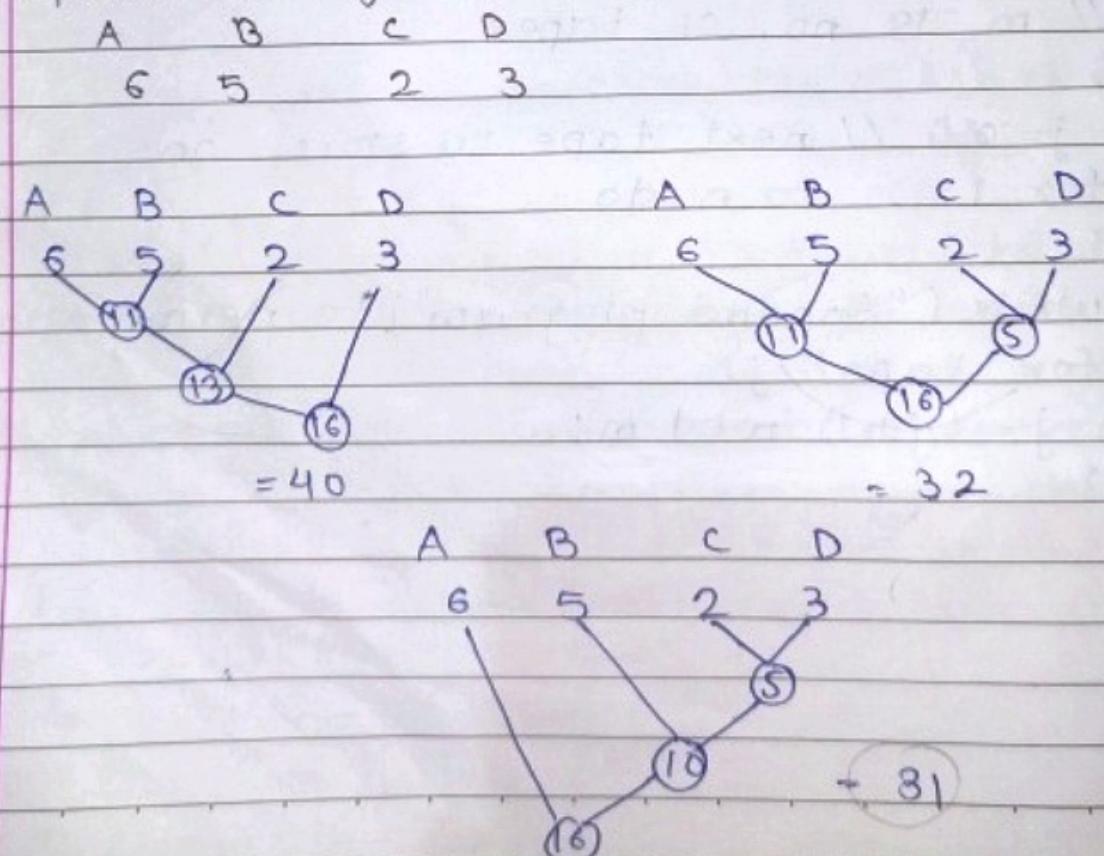
Tape 0	5, 7, 11, 26	$(5) + (5+7) + (5+7+11) + (5+7+21+26) = 89$
Tape 1	3, 5, 8, 12, 32	$3 + (3+5) + (3+5+8) + (3+5+8+12) + (3+5+8+12+32) = 115$
Tape 2	4, 6, 10, 18	$4 + (4+6) + (4+6+10) + (4+6+10+18) = 72$

$$\text{Tape } 0 = 89$$

$$\text{Tape } 1 = 115$$

$$\text{Tape } 2 = 72$$

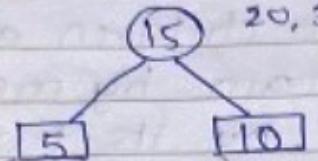
\* Optimal merge pattern



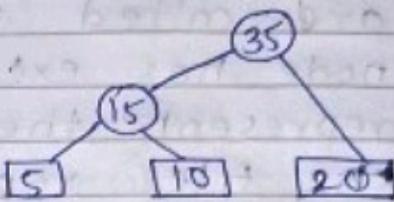
List	211	212	213	214	215
Sizes	20	30	10	5	30
Asc.	5	10	20	30	30
	214	213	211	212	215

Steps

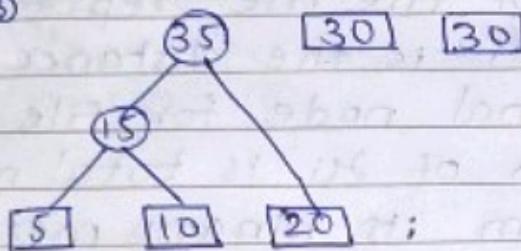
①



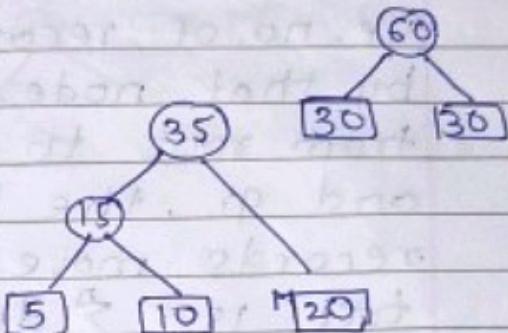
②



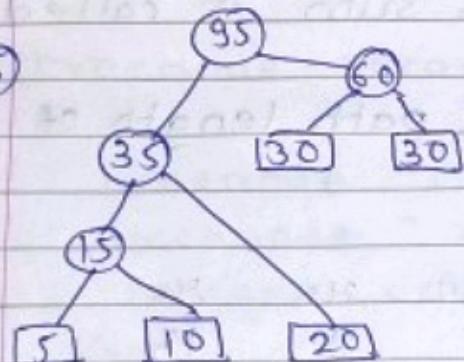
③



④



⑤



= 205

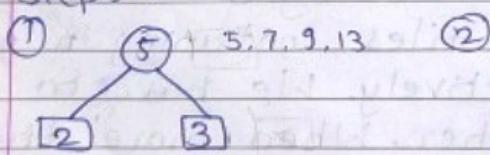
$$5 \times 3 + 10 \times 3 + 20 \times 2 + 30 \times 2 + 30 \times 2 = 205$$

- We have to sorted files containing  $n_1, n_2, n_3, \dots, n_m$  records respectively. We have to merge them together. When more than two sorted files are to merge together the merge can be accomplished by repeatedly merging sorted file in pairs. Thus if file  $21_1, 21_2, 21_3, \dots, 21_m$  are to be merged we put<sup>①</sup> merge  $21_1$  &  $21_2$  to get file by 2. Then we could merge it.

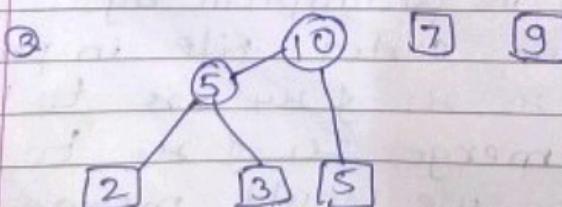
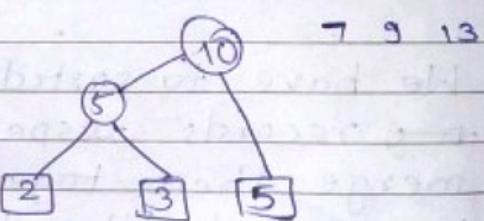
$y_1 \neq y_2$  to get  $y_2$  so on the h-leaf nodes are drawn as squares, and represent the given files. The nodes are called external nodes. The remaining nodes are drawn as circles, are called internal nodes. Each internal node has exactly two children and it represents the file obtained by merging the files represented by its two children. The number in each node is the length i.e. no. of records of the file represented by that node. If  $d_i$  is the distance from root to external node for file  $x_i$  and  $q_i$ , the length of  $x_i$  is total no. of records move from its binary merge tree is  $\sum_{i=1}^n d_i q_i$ . This sum is called the weighted external path length of the tree.

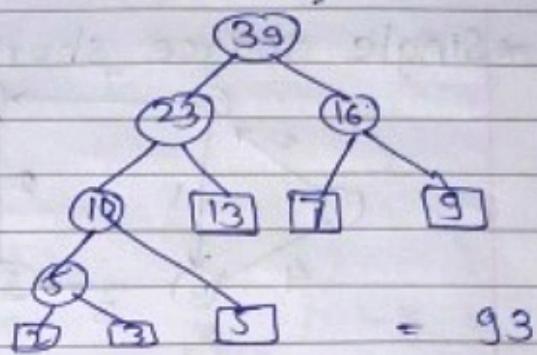
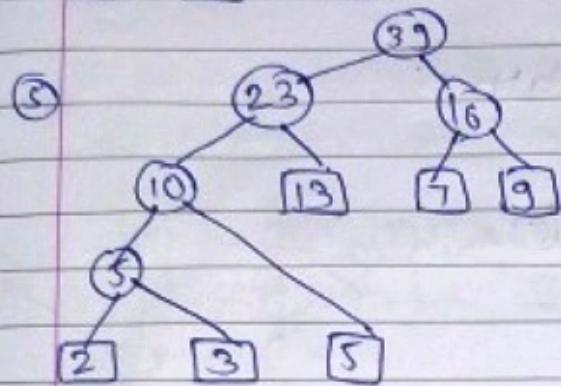
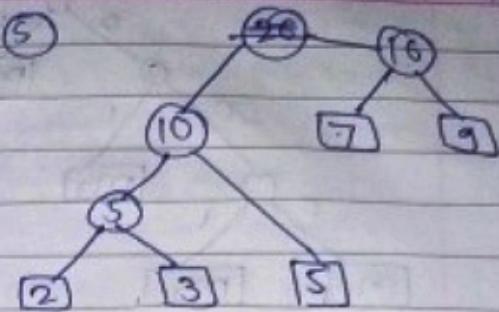
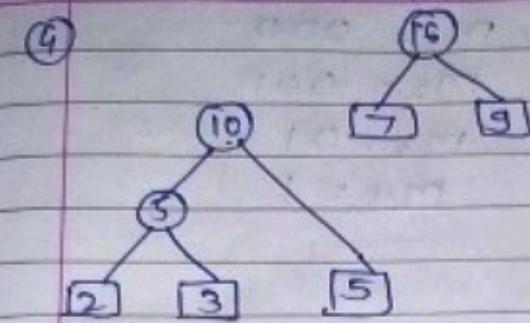
eg. list  $x_1 x_2 x_3 x_4 x_5 x_6$   
sizes 2 3 5 7 9 13  
Asc.

Steps:



②





Algorithm:

treenode = record

{

    treenode \* lchild;

    treenode \* rchild;

    integer weight;

}

Algorithm Tree(n)

{

    for i=1 to n-1 do

{

        pt = new treenode;

        (pt->lchild) = least(l1st)

        (pt->rchild) = least (l1st)

        (pt->weight) = ((pt->lchild) -> weight)  
                         + (pt->rchild) -> weight)

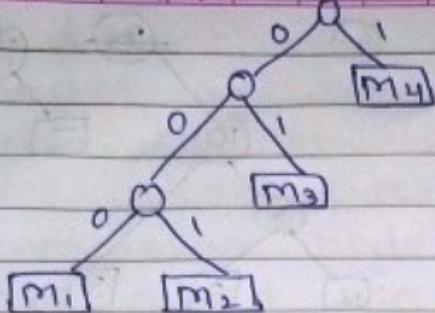
        insert (l1st, pt);

    }

    return least (l1st);

}

\* Huffman's code :



$$M_1 = 000$$

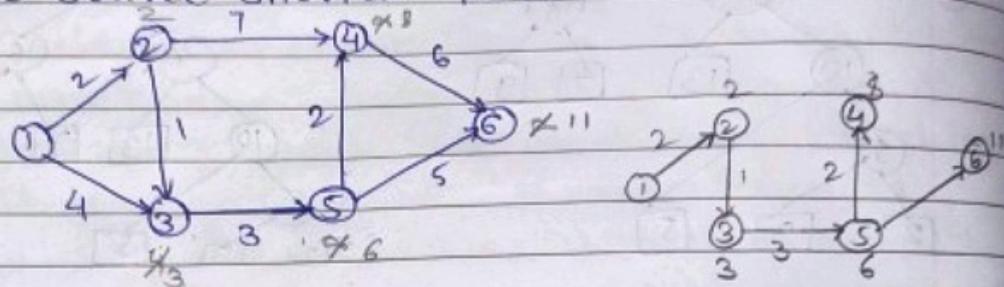
$$M_2 = 001$$

$$M_3 = 01$$

$$M_4 = 1$$

Q.2

\* Single source shortest path:



$$\textcircled{1} \quad d(2) + c(2,3) < d(3)$$

$$2 + 1 < 4$$

$$3 < 4$$

$$d(v) + c(v,w) < d[w]$$

$$d[w] = d(v) + c(v,w)$$

$$\textcircled{2} \quad d(3) + c(3,5) < d(5)$$

$$3 + 3 < \infty$$

$$6 < \infty$$

$$\textcircled{3} \quad d(5) + c(5,4) < d(4)$$

$$6 + 2 < d(4)$$

$$8 < d(4) \infty$$

$$\textcircled{4} \quad d(4) + c(4,6) < d(6)$$

$$8 + 6 < \infty$$

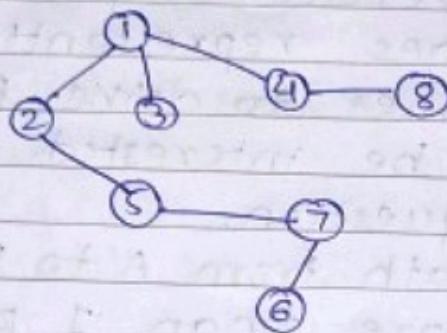
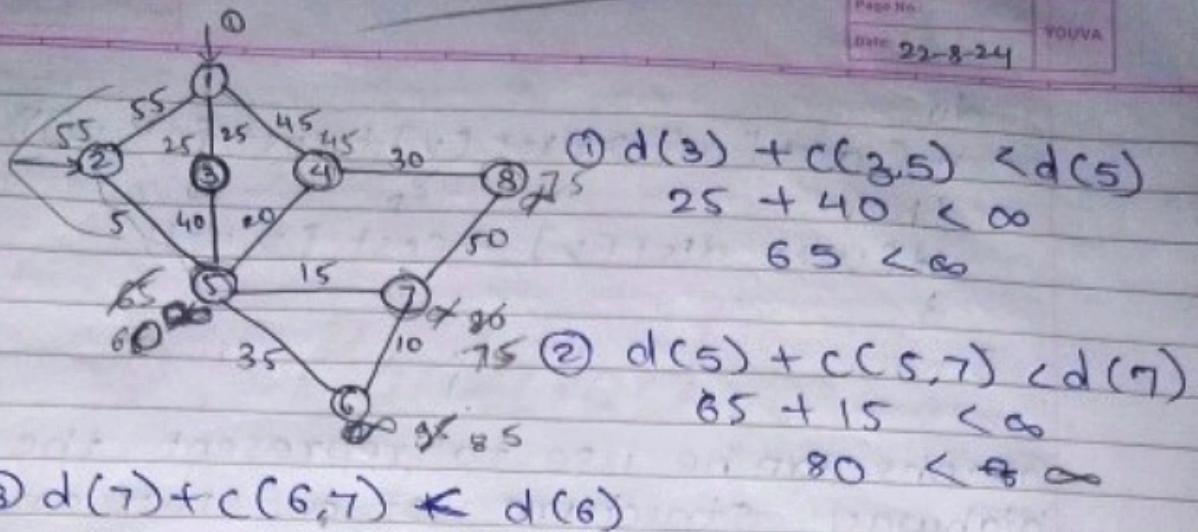
$$14 < \infty$$

a-2

$$d(u) + c(u,w) < d(w)$$

M	T	T	F	S
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Algorithm:

Shortestpaths(v, cost, dist, n)

dist[j] - set to length of shortest path  
from v to j

dist[v] = 0

{

for i = 1 to n do

{

s[i] = false, dist[i] = cost[v, i];

}

s[v] = true ; dist[v] = 0.0;

for num = 2 to n do

{

s[v] = true

for (each w adjust to u with-  
s[w] = false)

do

if ( $\text{dist}[w] > \text{dist}[v] + \text{cost}[v, w]$ )

then

$d[w] = \text{dist}[v] + \text{cost}[v, w];$

3

3

- Graphs can be used to represent the highway structure of a state or country with vertices representing cities and edges representing highway. A voteriest wishes to drive from city A to B would be interested in answers to following question.

(1) Is there a path from A to B

(2) Is there more than 1 path from A to B which is the smallest path.

- The problem define is called shortest path problem. The length of path is defined by <sup>sum</sup> of the bits of the edges on path

- The starting vertex is referred as source, last vertex is referred as destination.

- Graphs are bigra di graphs in problem we consider we are given directed graph  $G = V, E$ .

- A waiting function cost for edge of cost graph and to source vertex  $v_0$

- Problem is to determine shortest path from  $v_0$  to all remaining vertices of graph

- It is assume that all the weights are positive.

Shortest path bet<sup>n</sup> v<sub>0</sub> & some other  
is an ordering amount of among sum  
of edges.

### 3. Backtracking

Algorithm Backtrack(k)

Method

Find all possible sol<sup>n</sup>  
use the one you want

- Backtracking represents one of most unique technique many problems which deal with searching set of solution or which ask for optimal solution satisfying some constraint can be solved using backtracking formulation.
- In many application of backtrack method the desire solution is express as n tuple  $x_1, x_2, \dots, x_n$  where  $x_i$  are chosen for some finite set  $S_i$ . The problem to be solved for finding one vector that maximize or minimize a criterian function  $P(x_1, \dots, x_n)$ . Sometimes it all vectors that satisfy  $P$ .
- Suppose  $m_i$  is size of set  $S_i$  then there are  $m = m_1, m_2, \dots, m_n$  n tuple that are possible candidates for satisfying  $\exists^n P$ . The two-force approach could be form n tuples evaluate each one with  $P$  and save those which yet optimum.
- The backtrack algorithm has ability to answer with far fewer with n trials.

M T W T F S  
Page No.:  
Date 23-8-24 YODUVA

- The basic idea is to build solution vector one component at a time and to use modified criterion function called bounding function to test whether the vector form has any chance of success.

• Backtrack(k)

//  $x[1], x[2], \dots, x[k-1]$  of solution vector

3

for (each  $x[k] \in T(x[1], \dots, x[k-1])$ ) do

{

  IF ( $B_k(x[1], x[2], \dots, x[k]) \neq 0$ ) then

{

    IF ( $x[1], x[2], \dots, x[k]$  is path to answer node)

      then write  $x[1-k]$ ;

    IF ( $k < n$ )

      then

        Backtracking (k+1)

3

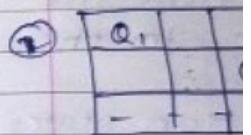
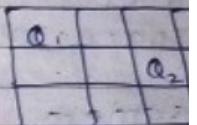
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- let  $T$  of  $x_1, x_2$  be set of all possible values for  $x_{i+1}$  such that  $x_1, x_2, \dots, x_{i+1}$  is also a path to problem state

-  $T(x[1], \dots, x[n]) = \text{Null}$ , we assume the existence of bounding function  $B_i$  such that if  $B_i(f_i) = [x_1, x_2, \dots, x_{i+1}]$  is false for a path  $(x_1, x_2, \dots, x_{i+1})$  from root node to app problem state  $B_i$ . Then path cannot be extended to reach an answer node. Thus the candidates for

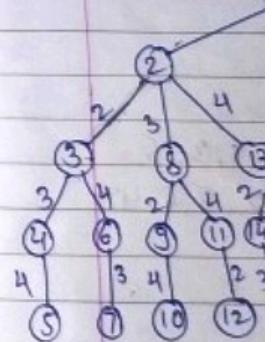
position 1-  
are those  
by T and

~~Input~~ + N queens  
(Q1, Q2, Q3)

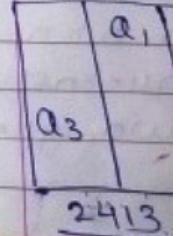


\* State sp

1m  
1mp



Mirror



position  $i+1$  of the solution vector  $x_1, x_2, \dots, x_n$   
are those values which are generated  
by  $T$  and satisfy  $B(x+i)$

Imp + N queens problem:

①  $Q_1, Q_2, Q_3, Q_4$

Q <sub>1</sub>			
	Q <sub>2</sub>		
-	-	-	-

row  
column  
diagonal

②

Q <sub>1</sub>			
	Q <sub>2</sub>		
-	-	-	-

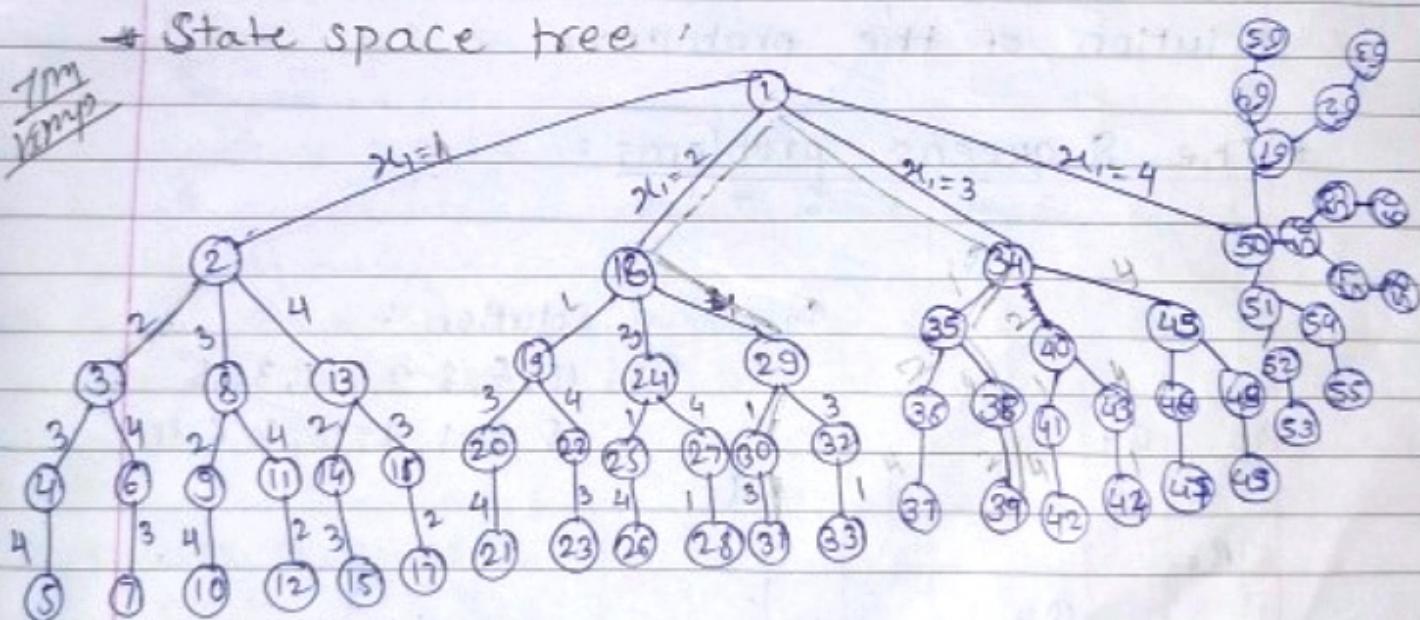
③

Q <sub>1</sub>			
		Q <sub>2</sub>	
-	-	-	-

④

Q <sub>1</sub>			
			Q <sub>2</sub>
-	-	-	-

\* State space tree



Mirror image:

	Q <sub>1</sub>		Q <sub>2</sub>
Q <sub>3</sub>			
	Q <sub>4</sub>		

2 4 1 3

	Q <sub>1</sub>		Q <sub>2</sub>
		Q <sub>4</sub>	
			Q <sub>3</sub>

3 1 4 2

- Backtracking algorithm determine problem solution by systematically searching solution space for the given problem instant these search is facilitated by using tree organization for solution space.
- Each node in these tree determines problem state all paths from root to other nodes define the state space of the problem.
- Solution states are those problem states 's' for which the path, from the root to 's' defines a tuple in the solution space.
- Answer states are those solution states 's' for which the path from root to 's' defines a tuple i.e. member of the set of solution of the problem.

### \* The 8 queens problems:

	1	2	3	4	5	6	7	8
1								Q <sub>1</sub>
2								Q <sub>2</sub>
3								
4								Q <sub>4</sub>
5								Q <sub>5</sub>
6								Q <sub>6</sub>
7								Q <sub>7</sub>
8								Q <sub>8</sub>

. Solution :

Q<sub>3</sub> 4, 6, 8, 2, 7, 1, 3, 5  
5, 3, 1, 7, 2, 8, 6, 4

- N queens are to be placed on  $n \times n$  chessboard such that no two queens are attacking each other i.e. no two queens

- are on the same row, column or diagonal
- The chessboards square be numbered as the indices of two dimensional array.  $a[1:n, 1:n]$  then observe that every element on the same diagonal that runs from upper left to lower right has same row, column value.
- Suppose two queens are to placed at position  $i, j$  and  $k, l$  then by the above they are on same diagonal only if  $i-j = k-l$  or  $i+j = k+l$
- First eq<sup>n</sup> implies  $j-l = i-k$  and 2<sup>nd</sup> eq<sup>n</sup> implies  $j-l = k-i$

Algorithm Nqueens ( $K, n$ )

{  
for  $i = 1$  to  $n$  do

    IF place ( $K, i$ ) then

$x[K] = i;$

        if ( $K = n$ ) then write ( $x[1:n]$ );

        else Nqueens ( $K+1, n$ )

09-24

\* Sum of subset:

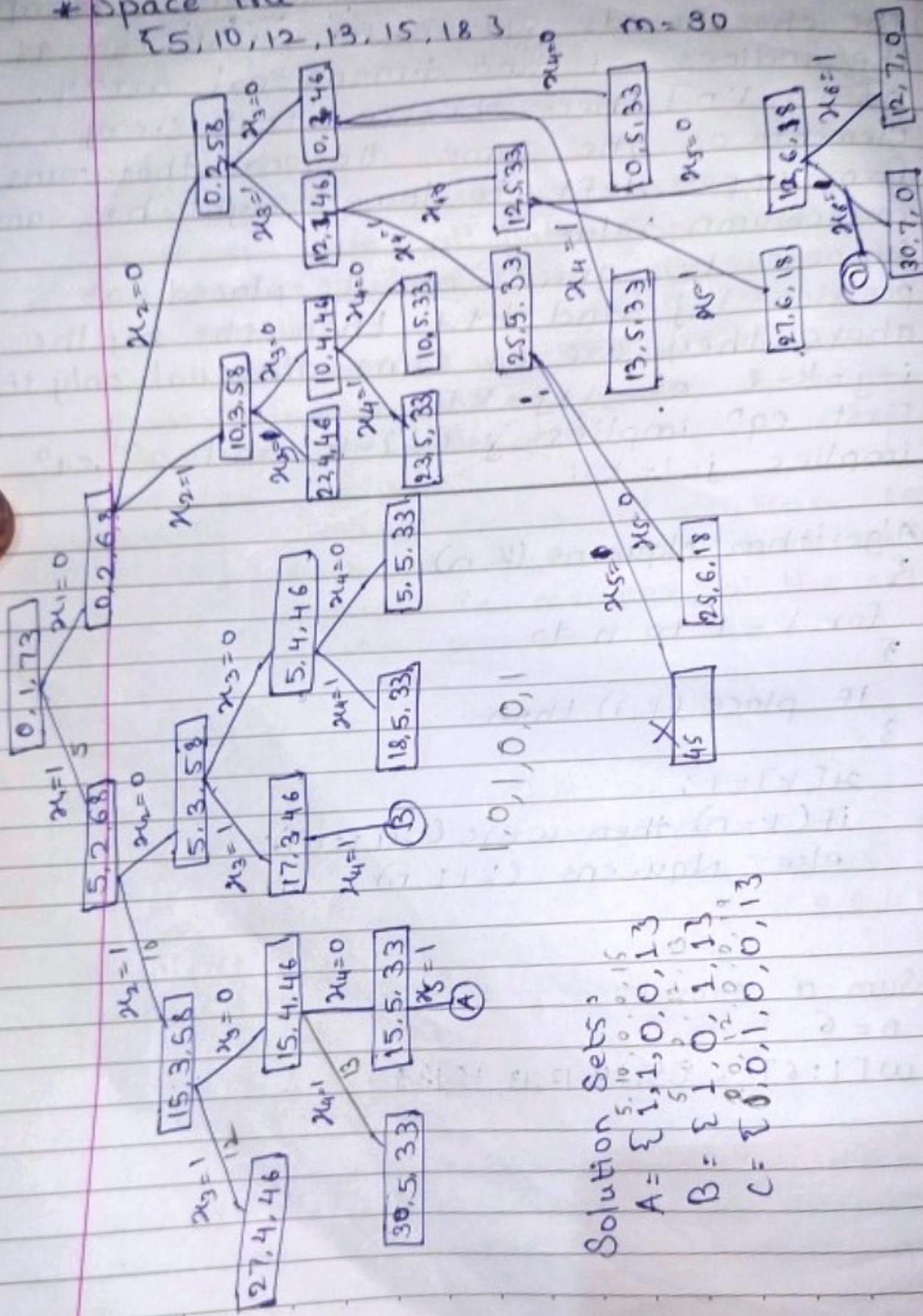
$n = 6$

$w[1:6] = \{5, 10, 12, 13, 15, 18\}$

\* Space tree :

5, 10, 12, 13, 15, 18 3

m = 30



Solution Sets:

$$A = \{1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0\}$$

$$B = \{1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1\}$$

$$C = \{0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1\}$$

- Suppose, we are given  $n$  distinct positive numbers usually called dates, we desire to find all combination of these numbers whose sum are  $m$ . This is called sum of subset problem.
- The child of any node are easily generated for node at level  $i$ , the left child corresponds to  $x_i = 1$  & the right child  $x_i = 0$ . The simple choice for bounding function is true. If

① Sum of weights + included so far  
 $\sum_{i=1}^k w_i x_i + w_{k+1}$  weight for next object  $\leq m$

②  $\sum_{i=1}^k w_i x_i + \sum_{i=k+1}^n w_i \geq m$

Sum of weight + remaining weight  
 included so far to be included.

### \*Sum of Sub (S, k, r)

Generate left child

$S + w[k] \leq m$  since  $B_{k-1}$  is true.

$x[k] = 1$

if ( $S + w[k] = m$ ) then write ( $x[1:k]$ );

else if ( $S + w[k] + w[k+1] \leq m$ )

then sumofsub ( $S + w[k], k+1, r - w[k]$ )

if ( $S + r - w[k] > m$ ) & ( $S + w[k+1] \leq m$ )

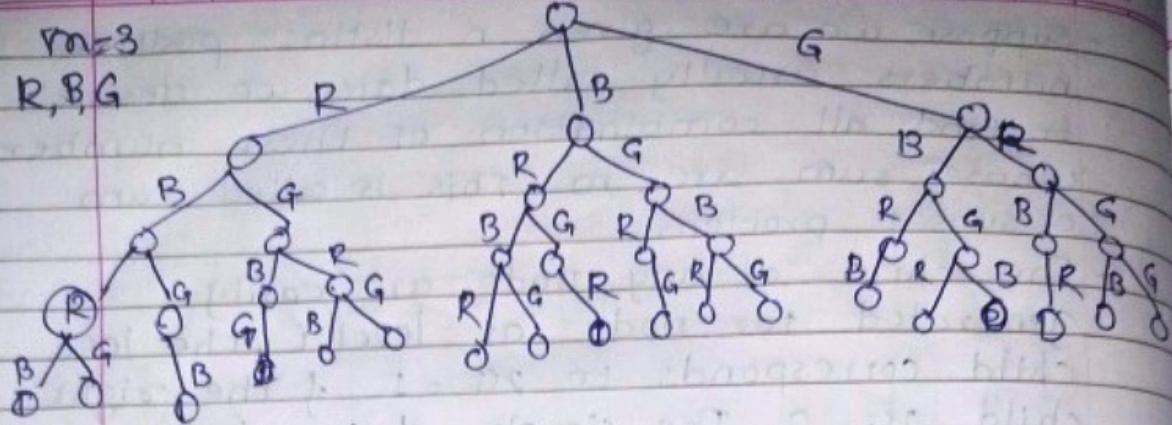
then

$x[k] = 0$ ;

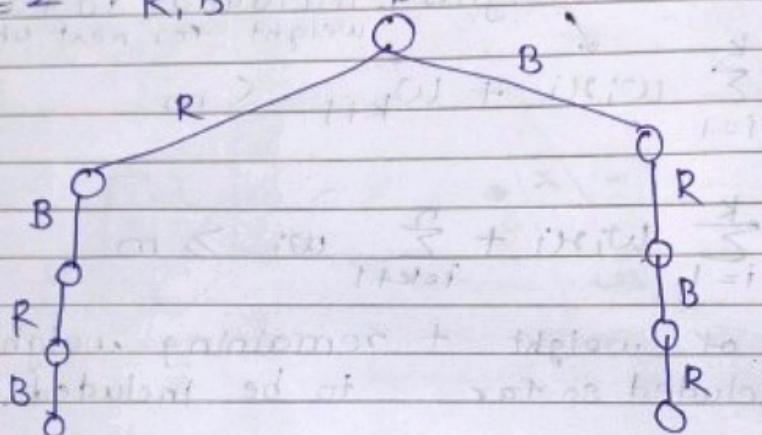
sum of subset ( $S, k+1, r - w[k]$ )

3

3



②  $m=2$  R, B



Algorithm mcoloring( $k$ )

```

repeat
    NextValue( $k$ );
    if ( $x[k]=0$ ) then return;
    if ( $k=n$ ) then write ( $x[1:n]$ );
    else mcoloring ( $k+1$ );
until (false);
    
```