



Today's agenda

→ Priority Queue

↳ Introduction to Heap/PQ.

↳ K smallest element. ^{±1}

↳ median of an array. ^{leetcode} → Hard



AlgoPrep



// Introduction

insert(n)

getmin()

deletemin()

LinkedList

$O(1) / O(N)$

$O(N)$

$O(N)$

Queue

$O(1)$

$O(N)$

$O(N)$

HashMap

$O(1)$

$O(N)$

$O(N)$

PQ

$O(\log N)$ size of PQ

$O(1)$

$O(\log N)$

min PQ

name

Max PQ

name

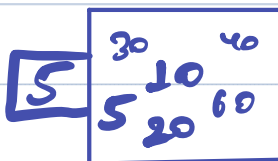
PriorityQueue < Integer > PQ = new

PriorityQueue < > ();

PriorityQueue < Integer > PQ = new

PriorityQueue < > ();

Collections.reverseOrder()



PQ.add(5); → add in PriorityQueue.

PQ.add(10);

PQ.add(20);

PQ.add(2);

PQ.peek(); → get the min element

2

PQ.remove(); → remove and return the min ele

2

PQ.add(5);

PQ.size();

↳ no. of elements in PQ.



Q) Kth Smallest Element

↳ Given n distinct elements, Find k Smallest elements.

Ex: arr[10] = { 8 3 10 4 11 2 7 6 14 1 }

k=4: 1 2 3 4

arr[8] = { -3 6 2 0 8 -3 10 4 }

k=3: -3 -3 0

Idea 1

↳ Sort the array and return the first k elements.

T.C: $O(N \log N + k) \approx O(N \log N)$

Idea 2

↳ Add all the elements to min PQ and get the first elements.

T.C: $O(N \log N + k \log N) \approx O(N \log N)$

S.C: $O(N)$

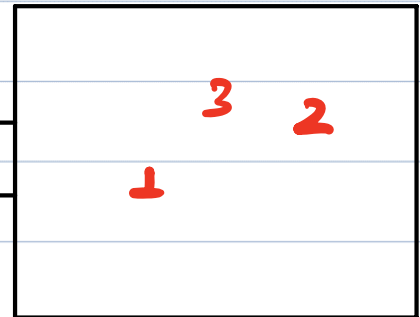


Idea 3

arr[10] = { 8 3 10 4 11 2 7 6 14 1 }
 k=4: 4 3 2 1

logK ←

4



Python code

```
void kthSmallest (int arr[N], int k) {
```

```
    minHeap < Integer > mh;
```

```
    for (int i=0; i<k; i++) {
        mh.add(arr[i]);
```

```
    }
```

T.C: $O(N \log k)$

S.C: $O(k)$

```
    for (int i=k; i<N; i++) {
        if (arr[i] < mh.peek()) {
            mh.remove();
            mh.add(arr[i]);
```

```
        }
```

```
    while (mh.size() > 0) {
```

```
        print(mh.remove());
```

```
    }
```

```
}
```



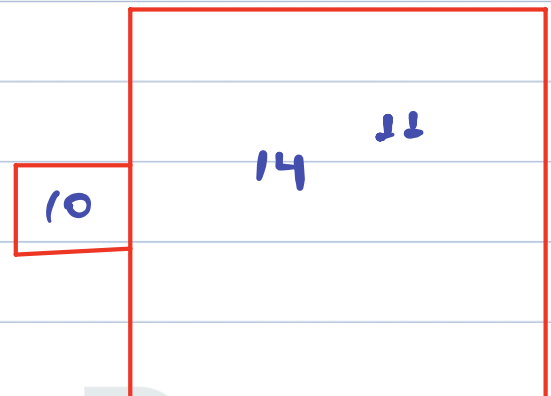
Q) K largest elements

arr[10]: { ⁰8 ¹3 ²10 ³4 ⁴11 ⁵2 ⁶7 ⁷6 ⁸14 ⁹1 }

K:3

↳ 10 11 14

✓ min PQ



AlgoPrep



//median

↳ middle element of sorted number.

arr[3] = { 2 5 3 }

↳ { 2 3 5 } → 3

arr[5] = { 4 3 6 8 5 }

↳ { 3 4 5 6 8 } → 5

arr[6] = { 4 3 9 5 12 2 }

↳ { 2 3 4 5 9 12 } → $\frac{4+5}{2} = 4.5$

arr[4] = { 4 6 10 14 }

↳ $\frac{6+10}{2} = 8$

Break till 10:17 PM



Q) Point median after each insertion.

arr[5]: 9 6 3 10 4
 ↓ ↓ ↓ ↓ ↓
 3 7.5 6 7.5 6

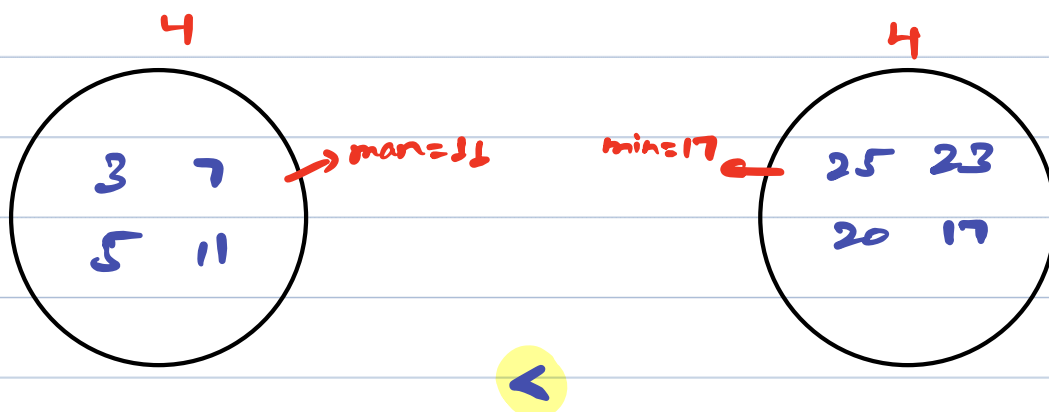
Idea 1

↳ After every insertion, sort the array and return the middle one.

T.C: $O(n \log n * n) \approx O(n^2 \log n)$

Idea 2

even element 3 5 11 23 20 25 17 7

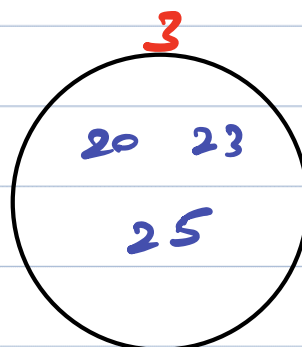
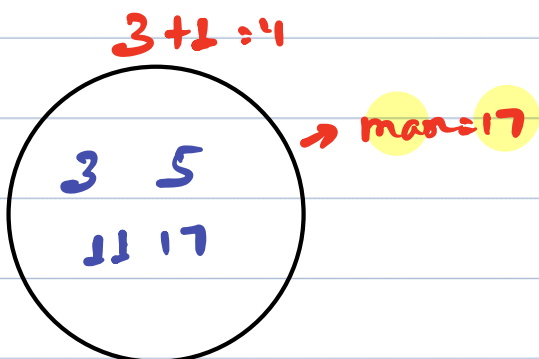


median: $\frac{\text{max of left bucket} + \text{min of right bucket}}{2}$

odd element



3 5 11 23 20 25 17



<

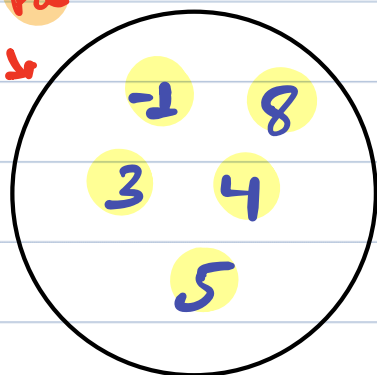
median : max of left

→ 4 9 5 3 12 8 10 8 -1

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

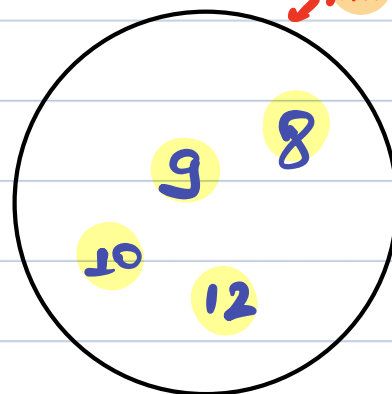
4 6.5 5 4.5 5 6.5 8 8

max PQ



left

min PQ



right

if (left.size() == right.size()) { new element is (odd)th element }



IIPsuedo code

```
Class medianFinder {
```

```
    maxHeap < Integer> left;
```

```
    minHeap < Integer> right;
```

```
    Public medianFinder () {
```

```
        }
```

```
        Public void addnum (int num) {
```

```
            if (left.size() == right.size()) {
```

```
                right.add(num);
```

```
                left.add(right.remove());
```

```
            }
```

```
            else {
```

```
                left.add(num);
```

```
                right.add(left.remove());
```

```
            }
```

```
        }
```

```
        Public double findmedian () {
```

```
            if (left.size() == right.size()) {
```

```
                return (left.peek() + right.peek()) / 2.0;
```

```
            }
```

```
            else {
```

```
                return (left.peek() * 1.0);
```

```
            }
```

```
        }
```

```
}
```

T.C: $O(n \log n)$

S.C: $O(n)$

$3 \log n \leftarrow$

$O(1) \leftarrow$