



## Today's agenda

- ↳ Binary number system
- ↳ Operators
- ↳ Problems
- ↳ Constraints



# AlgoPrep



↳ decimal no. system

↳ {0 - 9} System

0

10

20

90

1

11

21

91

2

.

:

92

3

:

:

:

4

:

:

:

.

:

:

:

:

:

:

:

g

19

29

99

↳ binary no. system

↳ {0 1}

↓ 00      ↓ 10  
01      11

↓ 100  
101  
110  
111



## In Conversion

In convert decimal to binary :-

ex: 30

2	30	— 0	↑
2	15	— 1	
2	7	— 1	→ 1 1 1 1 0
2	3	— 1	
2	1	— 1	
	0		

Quiz:

2	45	1	↑
2	22	0	
2	11	1	1 0 1 1 0 1
2	5	1	
2	2	0	
2	1	1	
	0		



↳ binary no. to decimal no. → bit index

ex:  $(10101)_2$

Bit Indexes: 4 3 2 1 0  
 $2^4 \times 1$      $2^3 \times 0$      $2^2 \times 1$      $2^1 \times 0$      $2^0 \times 1$

0: unset bit / off bit  
1: set bit / on bit

$$\rightarrow 2^4 \times 1 + 2^3 \times 0 + 2^2 \times 1 + 2^1 \times 0 + 2^0 \times 1 \\ 16 + 0 + 4 + 0 + 1 = 21$$

Quiz:

6 5 4 3 2 1 0  
1 0 1 1 0 1 0

↳  $2^6 + 2^3 + 2^4 + 2^6 = 90$   
 $64 + 8 + 16 + 64 = 90$

→  $(101020)_2$  : invalid input



## Add binary numbers

3 6 8

4 5 4

8 2 2

$$0+0 \rightarrow 0$$

$$0+1 \rightarrow 1$$

$$1+0 \rightarrow 1$$

$$1+1 \rightarrow 10$$

$$1+1+1 \rightarrow 11$$

Q1

$$\begin{array}{r} +1 +1 +1 +1 \\ 0 1 0 1 1 1 \\ \hline 0 1 1 1 1 0 \\ \hline 1 1 0 1 0 1 \end{array}$$

Quiz

$$\begin{array}{r} +1 +1 +1 +1 +1 \\ 0 1 0 1 0 1 1 \\ \hline 0 1 0 0 1 1 1 \\ \hline 1 0 1 0 0 1 0 \end{array}$$

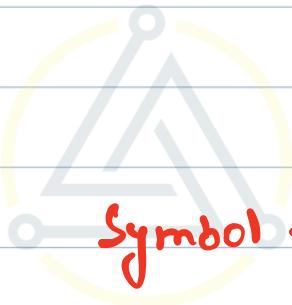
\* int  $\rightarrow$  32 bits

long  $\rightarrow$  64 bits



Bitwise operators: { and, or, xor, leftshift, rightshift }

A	B	$A \& B$	$A   B$	$A ^ B$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0



Symbol →  $\&&$

relational op.

$\&$  → Bitwise operators  
 $|$  → O dominant

false  
 if ( $n > 2 == 0$   $\&&$   $n > 3 == 0$ ) {

}

$==$  relational op.  
 $!=$  true do mirror



①  $23 \wedge 10 = 2$

$$23: \begin{array}{r} 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{array}$$

$$10: \begin{array}{r} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{array}$$

$$\frac{0 \ 1 \ 0 \ 1 \ 0}{0 \ 0 \ 0 \ 1 \ 0} \rightarrow 2^0 = 2$$

Quiz:

$$20: \begin{array}{r} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{array}$$

$$10: \begin{array}{r} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{array}$$

$$\rightarrow 0$$

②  $23 \mid 10$

$$23: \begin{array}{r} 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{array}$$

$$10: \begin{array}{r} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{array}$$

$$\frac{0 \ 1 \ 0 \ 1 \ 0}{1 \ 1 \ 1 \ 1 \ 1} \rightarrow 2^0 + 2^1 + 2^2 + 2^3 + 2^4 : 2^5 - 1 = 31$$

Quiz:

$$20: \begin{array}{r} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{array}$$

$$10: \begin{array}{r} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{array}$$

$$\frac{0 \ 1 \ 0 \ 1 \ 0}{1 \ 1 \ 1 \ 1 \ 0} \rightarrow 2^0 + 2^1 + 2^3 + 2^4 = 30$$

③  $23^{\wedge}10$

$$23: \begin{array}{r} 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{array}$$

$$10: \begin{array}{r} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{array}$$

$$\frac{0 \ 1 \ 0 \ 1 \ 0}{1 \ 1 \ 1 \ 0 \ 1} \rightarrow 29$$

Quiz:

$$20: \begin{array}{r} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{array}$$

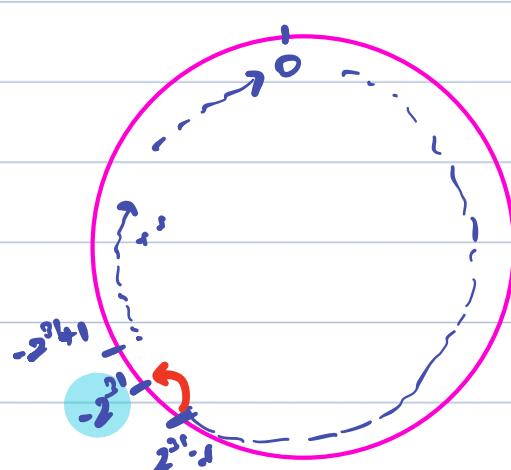
$$15: \begin{array}{r} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}$$

$$\frac{0 \ 1 \ 1 \ 1 \ 1}{1 \ 1 \ 0 \ 1 \ 1} \rightarrow 2^0 + 2^1 + 2^3 + 2^4 = 27$$



int → 32 bits

$-2^{31}$  to  $2^{31}-1$



$$2^{31} - 1 = 2^{31} - 2^{31} + 1 = 2^{31}(2^1 - 1) = 3 \times 2^{31}$$

$$2^{31}-1+1 = -2^{31}$$

$$\frac{1}{2^{31}}$$



Break till 10:40 pm



Q) you have been given a positive no., identify whether the number is even or odd.

Ex:  $N=8 \rightarrow$  even

$N=7 \rightarrow$  odd

Note: Use of  $+$ ,  $-$ ,  $*$ ,  $/$  or  $\%$  is not allowed.

$N=10:$      $\begin{array}{r} 1010 \\ 0001 \end{array}$

$N=11:$      $\begin{array}{r} 1011 \\ 0001 \end{array}$

$N=12:$      $\begin{array}{r} 1100 \\ 0001 \end{array}$

$N=13:$      $\begin{array}{r} 1101 \\ \quad \quad \quad \end{array}$

The diagram illustrates the binary representation of the first four positive integers. Each integer is represented by a 4-bit binary number. The 0th bit index (the most significant bit) is highlighted in yellow in all four cases. A pink arrow points from the 0th bit index of 10 to the 0th bit index of 11, then to 12, and finally to 13, indicating a pattern or rule being demonstrated.

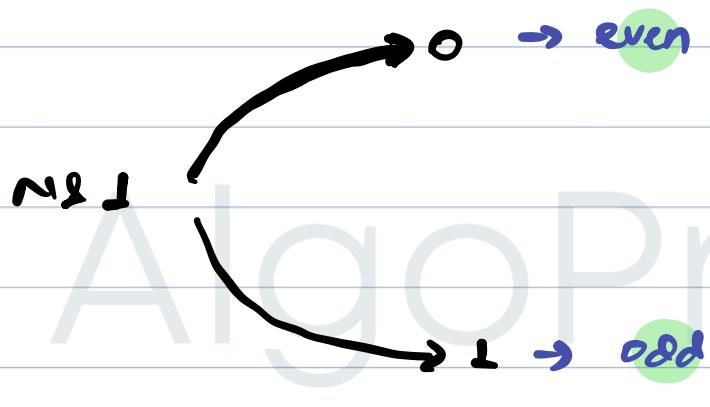
- ↳ 0th bit index for even no. will be 0.
- ↳ 0th bit index for odd no. will be 1.



↳ How to check last bit?

$$\text{int } N = 10: \quad \begin{array}{r} 3 \ 2 \ 1 \ 0 \\ 1 \ 0 \ 1 \ 0 \\ \times \ 1 \\ \hline 0 \ 0 \ 0 \ 1 \\ \hline 0 \ 0 \ 0 \ 0 \end{array}$$

$$N = 11: \quad \begin{array}{r} 3 \ 2 \ 1 \ 0 \\ 1 \ 0 \ 1 \ 1 \\ \times \ 1 \\ \hline 0 \ 0 \ 0 \ 1 \\ \hline 0 \ 0 \ 0 \ 1 \end{array}$$



void checker (int n) {

    if ( $(n \& 1) == 0$ ) {  
        s.o.p ("Even");  
    }

    else {  
        s.o.p ("Odd");  
    }



## 11 Properties

### 4. ① Commutative Property

$$a \& b = b \& a$$

$$a \mid b = b \mid a$$

$$a \wedge b = b \wedge a$$

### ② Associative Property

$$a \& b \& c \rightarrow (a \& b) \& c = a \& (b \& c)$$

$$a \mid b \mid c \rightarrow (a \mid b) \mid c = a \mid (b \mid c)$$



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③ ④  $n \& n = n$

$$\begin{array}{r} n \rightarrow 101 \\ \underline{\&} \\ n \rightarrow 101 \\ \hline 101 \end{array}$$

$$\begin{array}{r} n \rightarrow 101 \\ \underline{\&} \\ 0 \rightarrow 000 \\ \hline 000 \end{array}$$

⑤  $n \mid n = 0$

$$\begin{array}{r} n \rightarrow 101 \\ | \\ 0 \rightarrow 000 \\ \hline 101 \end{array}$$

$$\begin{array}{r} n \rightarrow 101 \\ | \\ n \rightarrow 101 \\ \hline 101 \end{array}$$

⑥  $n \mid n = n$

⑦  $n^0 = n$

$$\begin{array}{r} n \rightarrow 101 \\ ^\wedge \\ 0 \rightarrow 000 \\ \hline 101 \end{array}$$

$$\begin{array}{r} n \rightarrow 101 \\ ^\wedge \\ n \rightarrow 101 \\ \hline 000 \end{array}$$

⑧  $n^n = 0$



Q) Given  $\text{arr}[n]$ , every element appears twice except for one element which appears once, find that unique element.

Ex:  $\text{arr}[7]: \{ 6 \ 8 \ 8 \ 7 \ 7 \ 10 \ 6 \} \rightarrow 10$

$\text{arr}[5]: \{ 2 \ 1 \ 9 \ 2 \ 9 \} \rightarrow 1$

Idea 1

↳ nested loop.

$\text{arr}[7]: \{ 6 \ 8 \ 8 \ 7 \ 7 \ 10 \ 6 \}$

T.C:  $O(n^2)$

Idea 2

$\text{arr}[7]: \{ 6 \ 8 \ 8 \ 7 \ 7 \ 10 \ 6 \}$

inc. order

$\{ 6 \ 6 \ 7 \ 7 \ 8 \ 8 \ 10 \}$

↳ you don't actually need to rearrange.



$\text{ans}[7]: \{$

6: 0110

14: 1110

8:  $\frac{1000}{1110} = 5$

6:  $\frac{0110}{1000} = 8$

8: 1000

15: 1111

7:  $\frac{0111}{1111} \rightarrow 15$

8:  $\frac{1000}{0111} \rightarrow 7$

7: 0111

13: 1101

10:  $\frac{1010}{1101} \rightarrow 13$

7:  $\frac{0111}{1010} \rightarrow 10$

## 11 Pseudo code

int unique\_element (int arr[n]) {

    int ans = 0;

    for (int i=0; i<n; i++) {  
        ans = ans ^ arr[i];

}

    return ans;

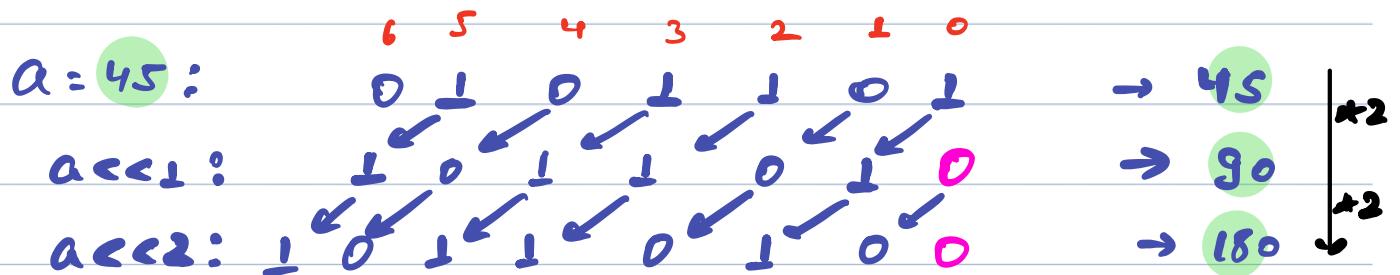
}

T.C: O(n)

S.C: O(1)



↳ left shift ( $<<$ ) <sup>→ 0(1)</sup>



$$a << 1 \rightarrow a \times 2$$

$$a << 2 \rightarrow a \times 2 \times 2$$

$$a << 3 \rightarrow a \times 2 \times 2 \times 2$$

$$a << n \rightarrow a \times 2^n$$

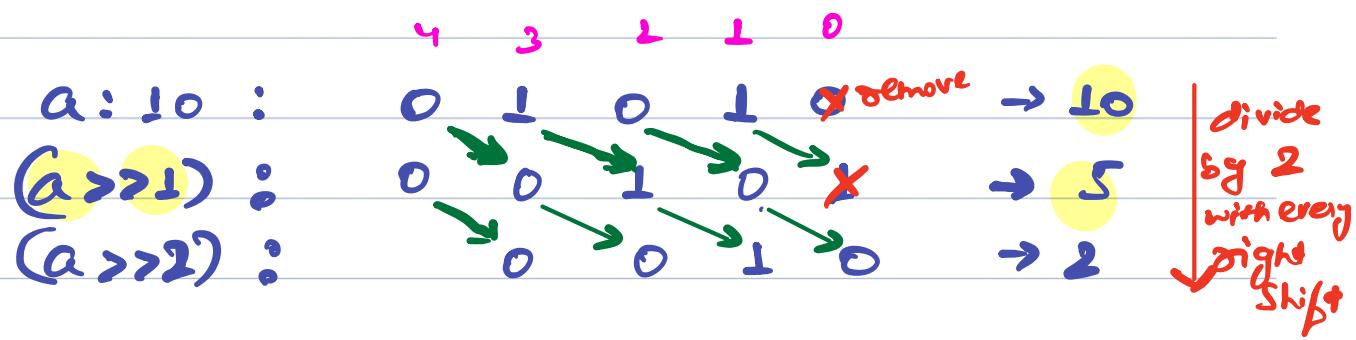
Qn2:

$$a = 1$$

$$1 << n \rightarrow 1 \times 2^n = 2^n$$



Use right shift ( $>>$ )  $\rightarrow O(1)$



$$A \gg n = \frac{A}{2^n}$$



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Constraints

1 sec =  $10^8$  iterations

↳ array length =  $10^5$

↳  $O(N^2) \rightarrow (10^5)^2 = 10^{10}$  iterations/loop

↳  $O(N\sqrt{N}) \rightarrow 10^5 * \sqrt{10^5} \Rightarrow 10^5 * 10^{2.5} = 10^{7.5}$



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