



Today's agenda

↳ No. of iterations

↳ Big O Notations



AlgoPrep



Quiz

↳ How many numbers are in range $[3, 10]$ (corners included)?

$\{3, 4, 5, 6, 7, 8, 9, 10\} \rightarrow 8$

$$\overset{\text{inc}}{\downarrow} [a, b] = b - a + 1 \overset{\text{inc}}{\downarrow}$$

$$\overset{\text{inc}}{\downarrow} [a, b) = b - a \overset{\text{exc.}}{\downarrow}$$

$$\overset{\text{exc.}}{\downarrow} (a, b) = b - a - 1 \overset{\text{exc.}}{\downarrow}$$

//log basics

$$n * 2 = 10$$

$$\rightarrow n = 10/2$$

$$n + 2 = 10$$

$$\rightarrow n = 10 - 2$$

$$n^2 = 27$$

$$\rightarrow n = \sqrt[3]{27}$$

$$n^2 = 16$$

$$\rightarrow n = \sqrt{16}$$

$$n^2 = 16$$

$$\rightarrow 2 = \log_n 16$$



→ $\log_a b$

↑ number
↑ base of log

ex: $2^{\text{ans}} = 10 \rightarrow \text{ans} = \log_2 10$

↓
 $\text{ans} = 3.33 \dots$

(a & b are nos)

* $\log_b a = \text{ans} \Rightarrow a = b^{\text{ans}}$
(hit and trial for ans)

(i) $\log_2 64 = \text{ans}$

$64 = 2^{\text{ans}}$
↳ $\text{ans} = 6$

(ii) $\log_3 343 = \text{ans}$

$343 = 3^{\text{ans}}$
↳ $\text{ans} = 5 \dots$



Properties:

$$\hookrightarrow \textcircled{1} \log_a a^n = n$$

$$\textcircled{II} \log_c (a * b) = \log_c a + \log_c b$$

$$\text{ex: } \log_2 10 = \log_2 (2 * 5) = \log_2 2 + \log_2 5$$



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Quiz

↳ No. of steps for $N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \dots \rightarrow 1$

$$\left(\left(N * \frac{1}{2} \right) * \frac{1}{2} \right) * \frac{1}{2} \dots = 1$$

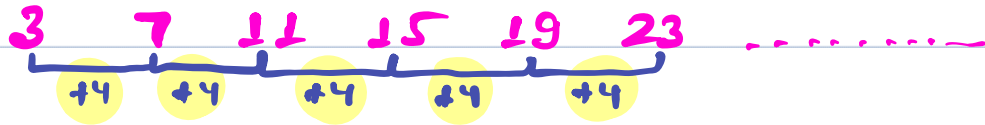
$$N * \frac{1}{2^{\text{no. of steps}}} = 1$$

$$N = 2^{\text{no. of steps}}$$

$$\log_2 N = \text{no. of steps}$$



A.P → Arithmetic Progression

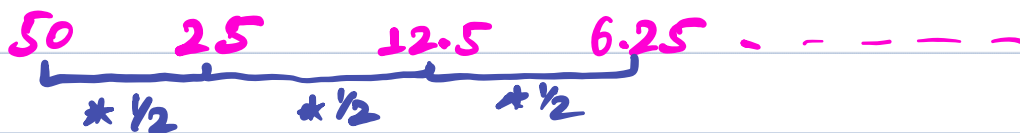
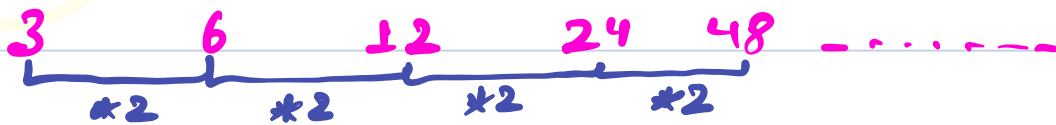


first term = a = Starting no. $\rightarrow a = 3$
Common difference = d = diff. betⁿ cons. nos $\rightarrow d = 4$

$$\begin{matrix} 1 & 2 & 3 & 4 & & n\text{th term} \\ (a+0d) & (a+d) & (a+2d) & (a+3d) & \dots & a+(n-1)d \end{matrix}$$

∴ Sum of first n terms of A.P. = $\frac{n}{2} * [2a + (n-1)d]$

G.P → Geometric Progression



first term = a = Starting no. $\rightarrow a = 3$
Common ratio = r = multiplier to get next no. $\rightarrow r = 2$

$$\begin{matrix} 1 & 2 & 3 & 4 & & n\text{th} \\ a & ar & ar^2 & ar^3 & \dots & ar^{n-1} \end{matrix}$$

∴ Sum of first n terms of G.P. = $a * \frac{r^n - 1}{r - 1}$



Quiz

```
int Sum = 0;
for (int i = 1; i <= N; i++) {
    Sum = Sum + 1;
}
```

$\rightarrow [1, N] \rightarrow N - 1 + 1$
 N iterations
 \downarrow
 $O(N)$

Quiz

```
void func (int N, int m) {
```

```
    for (int i = 1; i <= N; i++) {
        Print(i);
    }
```

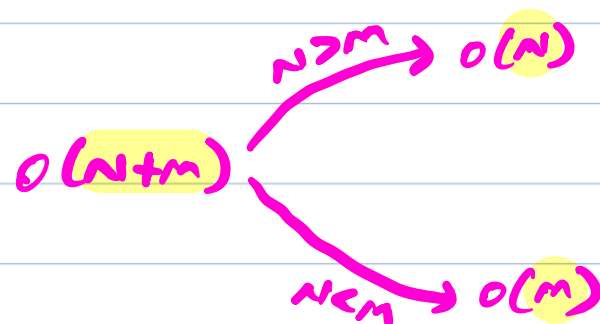
$\rightarrow [1, N] = N - 1 + 1$
 N iterations

```
    for (int i = 1; i <= m; i++) {
        Print(i);
    }
```

$\rightarrow [1, m] = m - 1 + 1$
 m iterations

Total: $N + m$ iterations

}





Quiz

```
int fun (int n){  
    int s = 0;  
    for (int i = 0; i <= 100; i++){  
        s = s + i2;  
    }  
    return s;  
}
```

$\rightarrow [0, 100] \rightarrow 100 - 0 + 1$
↓ iterations
↓
 $O(1)$

Quiz

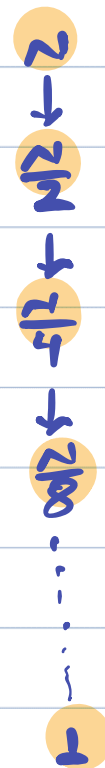
```
void fun (int n){  
    int s = 0;  
    for (int i = 1; i <= n; i++){  
        s = s + i2;  
    }  
    return s;  
}
```

$\rightarrow [1, \sqrt{n}] \rightarrow \sqrt{n} - 1 + 1$
↓ iterations
↓
 $O(\sqrt{n})$



Quiz

```
void fun (int n) {  
    int i = n;  
    while (i >= 1) {  
        i = i / 2;  
    }  
}
```



→ $\log_2 n$ iterations
 \downarrow
 $O(\log n)$

Quiz

```
void fun (int n) {
```

```
    int s = 0;
```

```
    for (int i = 0; i <= n; i = i * 2) {
```

```
        s = s + i;
```

→ [0, 0, 0, ..., ∞]
infinite

```
}
```



Quiz

```
void fun (int n) {
```

```
    int S = 0;
```

```
    for (int i = 1; i <= n; i = i * 2) {
```

```
        S = S + i;
```

```
    }
```

```
}
```

$\log_2 n$ iterations

$\log_2 n$

$\log_2 n$

$\log_2 n$

n

\downarrow

$n/2$

\downarrow

$n/4$

\vdots

\vdots

8

\downarrow

4

\downarrow

2

\downarrow

1

1

\downarrow

2

\downarrow

4

\downarrow

8

\vdots

\vdots

$n/2$

\downarrow

$n/4$

\downarrow

$n/8$

\vdots

\vdots

1

\downarrow

1

\downarrow

1

\downarrow

1

P1

P2

Break till 10:38 PM

$1 + 2 + 3 + \dots + n$

$1 + 2 + 3 + \dots + n$

$\rightarrow 20$



Nested loops

Quiz

```
void fun (int n) {  
    int S = 0;  
    for (int i = 1; i <= 10; i++) {  
        for (int j = 1; j <= n; j++) {  
            S = S + 10;  
        }  
    }  
}
```

i	j	Count
1	[1, n]	n
2	[1, n]	n
3	[1, n]	n
⋮		⋮
10	[1, n]	n
$O(n)$		$10 \times n$ iterations

Quiz

```
void fun (int n) {  
    int S = 0;  
    for (int i = 1; i <= n; i++) {  
        for (int j = 1; j <= n; j++) {  
            S = S + 10;  
        }  
    }  
}
```

i	j	Count
1	[1, n]	n
2	[1, n]	n
⋮		⋮
n	[1, n]	n
$O(n^2)$		$n \times n$ iterations



Quiz

```

void fun (int n) {
    int s = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= i; j++) {
            s = s + 10;
        }
    }
}

```

i	j	Count
1	[1, 1]	1
2	[1, 2]	2
3	[1, 3]	3
⋮	⋮	⋮
N	[1, N]	N

$$\frac{N * (N+1)}{2} \text{ iterations}$$

$$\frac{N^2 + N}{2} = \frac{N^2}{2} + \frac{N}{2} = O(N^2) \leftarrow$$

Quiz

```

void fun (int n) {

```

```

    for (int i = 1; i <= 2^n; i++) {
        print(i);
    }
}

```

→ [1, 2ⁿ] → 2ⁿ iterations

$$O(2^n)$$



Ques 2

```
void fun (int n) {
    int s = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= 2^i; j++) {
            s = s + 10;
        }
    }
}
```

i	j	Count
1	[1, 2 ¹]	2 ¹
2	[1, 2 ²]	2 ²
3	[1, 2 ³]	2 ³
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮
N	[1, 2 ^N]	2 ^N

3

$$2^1 + 2^2 + 2^3 + 2^4 + \dots + 2^N$$

$$a = 2$$

$$r = 2$$

no. of terms $\rightarrow N$

$$\text{Sum of first } N \text{ terms of G.P} = a * \frac{r^N - 1}{r - 1}$$

$$= 2 * \frac{2^N - 1}{2 - 1}$$

$$= 2 * (2^N - 1) \text{ iterations}$$

$$\cancel{2 * 2^N} - \cancel{2} = O(2^N)$$



Comparison of iteration $\rightarrow N = 10^5$

$$1 < \log N < \sqrt{N} < N < N \log N < N\sqrt{N} < N^2 < 2^N$$

Time Complexity \rightarrow Approximate iteration Count
 \hookrightarrow Big O Notation

- (i) Calculate iteration Count
- (ii) around + count, neglect lower order term. \rightarrow keep highest order term
- (iii) neglect constants

ex: iteration count: $\cancel{10}N^2 + \cancel{20}N + \cancel{30}$
 \downarrow
 $O(N^2)$

ex: $\cancel{X}N^2 + \cancel{15N} \log N + \cancel{20}N$
 \downarrow
 $O(N^2)$



$$\text{en: } \cancel{n \log n} + \cancel{3n} + \cancel{10} \\ \downarrow \\ O(n \log n)$$

↳ The magic you are looking for is in the work you are avoiding.



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