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Tutorial 5.

Evaluation and Measurement
- Hypothesis testing

Q1] $H_0 : p = 0.7$

$H_1 : p \neq 0.7$

Level of significance $= \alpha = 0.10$

Test statistic: Binomial variable X
with $P = 0.7$ and value
of $n = 15$

$X = 8$ and $nP_0 = 15 \times 0.7 = 10.5$

$\therefore$

$P = 2 \times P$ where $X \leq 8$ and
$P = 0.7$

$= 2 \sum_{x=0}^{8} b(x, 15, 0.7)$

$= 2 \times 0.1311$ [from binomial Table]

$= \underline{0.2622}$

$P > 0.10$ [which means $P > \alpha$]

$\therefore$ Not rejecting H_0

→ No reason to doubt the claim of builder.

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Q2] $H_0: P = 0.6$
$H_1: P > 0.6$

level of significance $\longrightarrow \alpha = 0.05$

Given: $x = 70$, $n = 100$, $P = 0.6$

$$\therefore Z = \frac{x - nP_0}{\sqrt{nP_0q_0}}$$

$$Z = \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}}$$

$$Z = 2.04$$

$P = P(Z > 2.04)$

$P = 0.0207$ [inferring from Table]

Since $P < \alpha$, we will reject H_0 and hence new drug is considered to be superior.

Q3] Let P_1 be the proportion of Mumbai voters.

$P_2 \longrightarrow$ surrounding area residents

$$\hat{P}_1 = \frac{120}{200} = 0.6 \quad \& \quad \hat{P}_2 = \frac{240}{500} = 0.48$$

$$\hat{P}_p = \frac{120+240}{200+500} = 0.514$$

Hypothesis can be presented as:-

$H_0 : P_1 \leq P_2$

$H_1 : P_1 > P_2$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_p - (1-\hat{P}_p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$Z = \frac{0.6 - 0.48}{\sqrt{0.514(1-0.514)\left(\frac{1}{200} + \frac{1}{500}\right)}}$$

$Z = 2.869$

$P = P(Z > 2.869) = 0.0044$

Since $p < \alpha$, we will reject H_0 and hence can be stated as

proportion of Mumbai voters who are favoring the proposal is higher than the proportion of surrounding area voters.

Q4] a) Null Hypothesis

$H_0: p = 0.20$

Alternative hypothesis

$H_1: p > 0.20$

critical region lies in right tail

b) Null hypothesis

$H_0: \mu = 3$

Alternative hypothesis

$H_1: \mu \neq 3$

critical region lies in both tails.

c) Null hypothesis

$H_0: p = 0.15$

Alternative hypothesis

$H_1: p < 0.15$

critical region lies in left tail.

d) Null hypothesis
$H_0 : \mu = 50$

Alternative hypothesis
$H_1 : \mu > 50$

Critical region lies in right tail.

e) Null hypothesis
$H_0 : \mu = 15$

Alternative hypothesis
$H_1 : \mu \neq 15$

Critical region lies in both tails.

Q5]

Let μ_1 and μ_2 → population mean by company A and company B

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$

Significance level $\alpha = 0.05$

$$\bar{X}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} x_{ij}$$

$$= \frac{9.3 + 8.8 + 6.8 + 8.7 + 8.5 + 6.7 + 8.0 + 6.5 + 9.2 + 7.0}{10}$$

$$\therefore \bar{X}_1 = 7.95$$

$$\bar{X_2} = \frac{1}{n_2}\sum_{i=1}^{n_2} x_{2i}$$

$$= \frac{11.0 + 9.8 + 9.9 + 10.2 + 10.1 + 9.7 + 11.0 + 11.1 + 10.2 + 9.6}{10}$$

$$\bar{X_2} = 10.26$$

$$S_1^2 = \frac{1}{n_1 - 1}\left[\sum_{i=1}^{n_1} x_{1i}^2 - n_1\bar{X_1}^2\right]$$

$$S_1^2 = \frac{10.865}{9} = 1.207$$

$$S_2^2 = \frac{1}{n_2 - 1}\left[\sum_{i=1} x_{2i}^2 - n_2\bar{X_2}^2\right]$$

$$S_2^2 = \frac{2.924}{9} = 0.325$$

As we can see that sample variances are quite different, it cannot be assumed that popular variance equal, so use t-test.

Degree of freedom

$$V = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{S_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{1.207}{10} + \frac{0.325}{10}\right)^2}{\frac{1}{10-1}\left(\frac{1.207}{10}\right)^2 + \frac{1}{10-1}\left(\frac{0.325}{10}\right)^2}$$

$$= 10.30$$

$$\simeq 10$$

Test statistics for testing the hypothesis

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (u_1 - u_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$V = 10$ degrees of freedom, because of null hypothesis $u_1 - u_2 = 0$, so under value of test statistic is:

$$T = \frac{7.095 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.90$$

Two-sided test, then value of test is doubled area under the density curve of t-distribution with 10 degrees of freedom.

$|t| = |-5.90| = 5.90$ i.e. the p-value is

P-value

$= 2 \cdot P(T \geq |t|)$

$= 2 \cdot P(T \geq 5.90)$

$t_{0.0005}(10) = 4.587$ and

since $|t| = 5.90$ is ~~even~~ greater than $P(T \geq 5.90) <$ 0.0005, so p-value < 0.001

As $P < \alpha$, null hypothesis can be rejected in favor of alternate hypothesis and

$\therefore$ mean ~~se~~ ... is not same for the two companies.