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Tutorial 6 :- Machine Learning 1

Q1] a)

$P(H) = \lambda$

$\therefore P(T) = 1 - \lambda$

$P(H$ at $k+1$th toss$) = P(T$ at k toss and H at $k+1$th$)$

$= (1-\lambda)^k \lambda$

b) Let M be no. of tosses to get the first head and let $S = E[M]$

Expectation is additive and tosses are independent.

$$S = (\lambda * 1) + (1-\lambda) \times (S+1)$$

$$= \lambda + S + 1 - \lambda S - \lambda$$

$$S\lambda = 1$$

$$\therefore S = \frac{1}{\lambda}$$

Q2] $X \rightarrow$ random variable

a) Variance of X : $var(X)$
$= E[(x - E[X])^2]$

prove :- $var(x) = E(x^2) - E[x]^2$

Given :-

$Var(x) = E[(x - E[x])^2]$

$= E[x^2 - 2xE[x] + E[x]^2]$

$= E[x^2] - 2E[x]^2 + E[x]^2$

$= E[x^2] - E[x]^2$

b) $E[x] = 0$ and $E[x^2] = 1$

Find :- 1) variance of X
2) if $Y = a + bX$, $var(Y) = ?$

$\rightarrow$ $var(X) = E[X^2] - E[X]^2$
$= 1 - 0^2$

$\therefore var(X) = 1$

2) $Y = a + bX$

$$E[Y^2] = E[(a+bX)^2]$$
$$= E[a^2 + 2abX + b^2X^2]$$
$$= a^2 + 2ab\,E[X] + b^2 E[X^2]$$
$$= a^2 + 2ab(0) + b^2(1)$$

$$\therefore E[Y^2] = a^2 + b^2.$$

$$E[Y] = E[a+bX] = a + bE[X]$$
$$= a + b(0)$$
$$\therefore E[Y] = a$$

$$\mathrm{var}(Y) = E[Y^2] - E[Y]^2 = a^2 + b^2 - a^2$$
$$\mathrm{var}(Y) = b^2$$

Q3) Let $A \rightarrow$ Aku predicting that given horse is winning horse.

Let $\sim A \longrightarrow$ Aku predicting that given horse is not wining horse.

Let B be the event that horse wins and $\sim B$ be the event that given horse does not win.

a) Considering the probability it wins

$P(B) = P(B, A) + P(B, \neg A)$

$= P(B/A)P(A) + P(B/\neg A)\, P(\neg A)$

$= 0.99 \times 10^{-5} + (1 - 0.99999)(1 - 10^{-5})$

$P(B) \approx 1.99 \times 10^{-5}$ ...... (1).

b) Probability that Aku predicts black beauty (winning)

$P(A/B) = \frac{P(A, B)}{P(B)} = \frac{P(A/B)\, P(A)}{P(B)}$

$= \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$ (from I)

$P(A/B) = 0.497$