Sakshi Patil - Tutorial 4

Independent Component Analysis

Ex1 :- Mixing statistically independent sources.

Ans:- Variance of mixture can be stated as

$$\begin{aligned}
var(x) &= \langle (x - \langle x \rangle)^2 \rangle \\
&= \langle x^2 \rangle - \langle x \rangle^2 \\
&= \langle (\textstyle\sum_i w_i s_i)^2 \rangle - \langle \textstyle\sum_i w_i s_i \rangle^2 \\
&= \langle (\textstyle\sum_i w_i s_i)^2 \rangle - (\textstyle\sum_i w_i \langle s_i \rangle)^2 \\
&= \langle (\textstyle\sum_i w_i s_i)(\textstyle\sum_j w_j s_j) \rangle - (\textstyle\sum_i w_i \langle s_i \rangle)(\textstyle\sum_j w_j \langle s_j \rangle) \\
&= \langle \textstyle\sum_{ij} w_i w_j s_i s_j \rangle - \textstyle\sum_{ij} w_i w_j \langle s_i \rangle \langle s_j \rangle \\
&= \textstyle\sum_i w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle) + \textstyle\sum_{i,j,i \neq j} w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle)
\end{aligned}$$

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$$= \sum_i w_i^2 \left(\langle s_i s_j \rangle - \langle s_i \rangle^2 \right) + \sum_{i,j \; i \neq j} w_{ij} \left(\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$$

s_i and s_j are statistically independent for $i \neq j \rightarrow \langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle = 0$

also, $var(s_i) = 1$

$$\therefore \; var(x) = \sum_i w_i^2$$

Mixture should have unit variance,

$var(x) = 1 \qquad \sum_i w_i^2 = 1$

The constraint should be imposed on weights w_i to have unit variance.

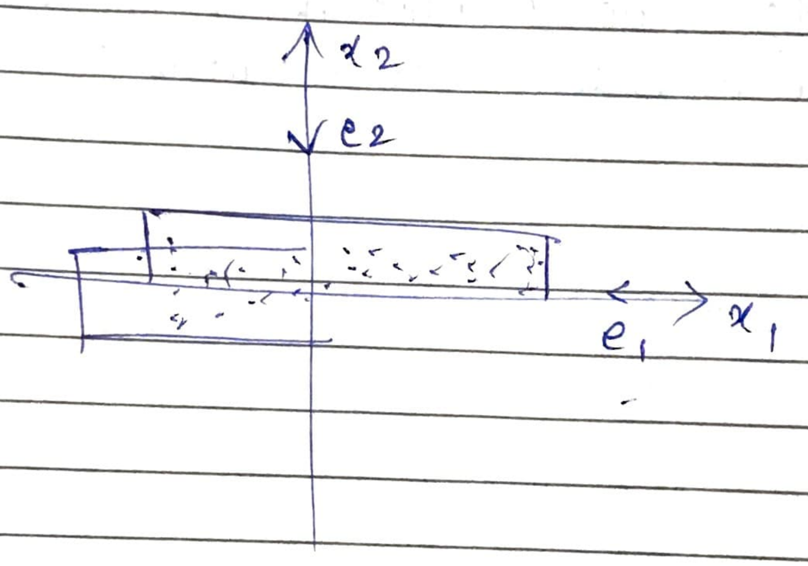
$$\boxed{\sum_i w_i^2 = 1}$$

Ex 2:- Independent components and distribution form data.

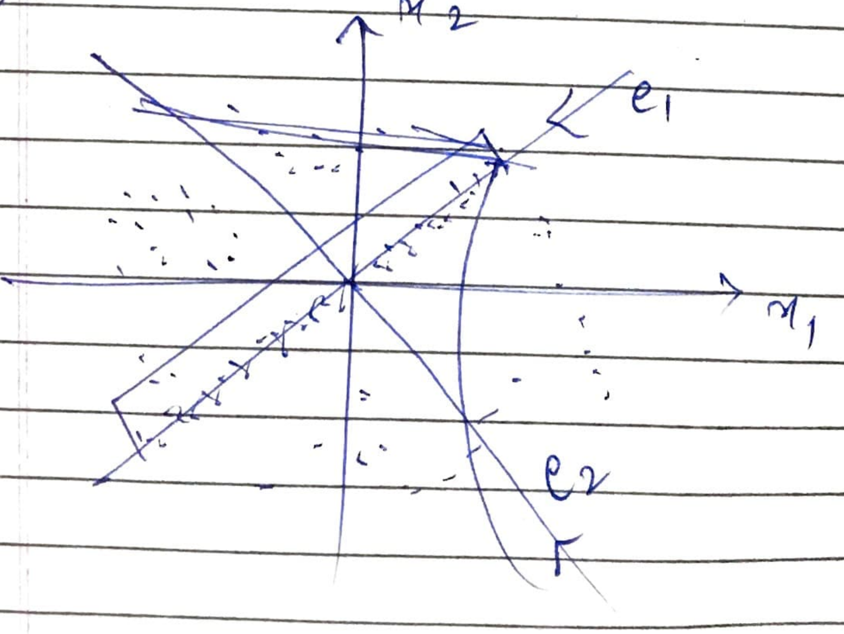
→ Decide whether the distribution can be clearly seperated into independent components.

→

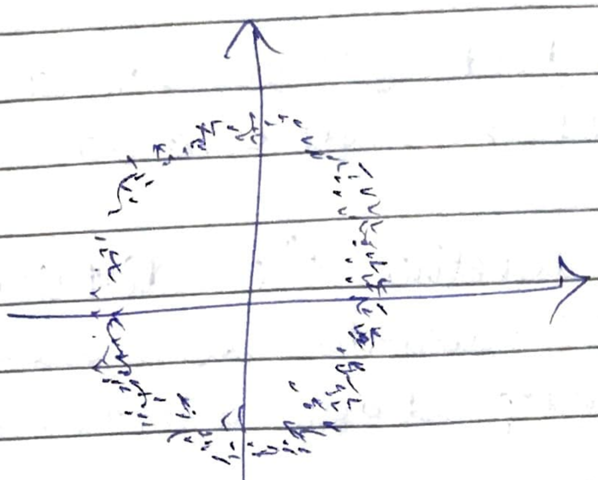
a)



b)



c)



Hence, it cannot be seperated into independent components.