Shri Vile Parle Kelavani Mandal'

Institute Technology , Dhule

Department of Information Technology

Design and Analysis of Algorithms Lab

Name : Sakshi sunil patil

Roll No : 47

PRN : 2254491246048

Class : Second Year (IT)

Modified Warshall (All Pair Shortest Path)

Theory :

The Floyd-Warshall algorithm, named after its creators Robert Floyd and Stephen Warshall, is a fundamental algorithm in computer science and graph theory. It is used to find the shortest paths between all pairs of nodes in a weighted graph. This algorithm is highly efficient and can handle graphs with both positive and negative edge weights, making it a versatile tool for solving a wide range of network and connectivity problems.

Algorithm :

step 1 :

Initialize the solution matrix same as the input graph matrix as a first step.

Step 2 :

Then update the solution matrix by considering all vertices as an intermediate vertex.

Step 3 :

The idea is to pick all vertices one by one and updates all shortest paths which include the picked vertex as an intermediate vertex in the shortest path.

Step 4 :

When we pick vertex number k as an intermediate vertex, we already have considered vertices {0, 1, 2, .. k-1} as intermediate vertices.

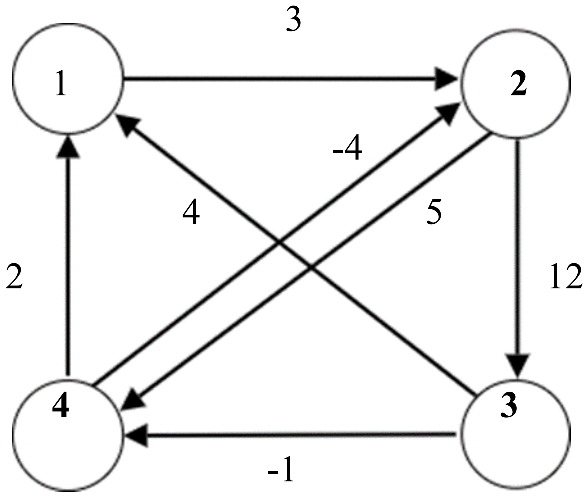
Step 5 :

For every pair (i, j) of the source and destination vertices respectively, there are two possible cases.

1 .k is not an intermediate vertex in shortest path from i to j. We keep the value of dist[i][j] as it is.

2. k is an intermediate vertex in shortest path from i to j. We update the value of dist[i][j] as dist[i][k] + dist[k][j], if dist[i][j] > dist[i][k] + dist[k][j]

Problem : Find the all pair shortest path for the following graph (source vertex is 1)



Solution :

A^0 :

	1	2	3	4
1	0	3	∞	∞
2	∞	0	12	5
3	4	∞	0	-1
4	2	-4	∞	0

Find the matrix for vertex 1

A^1 considering 1 as an intermiadiatary vertex

A^1 :

	1	2	3	4
1	0	3	∞	∞
2	∞	0	**12**	**5**
3	4	**7**	0	**-1**
4	2	**-4**	∞	0

1) $A^0[2,3]$ $\quad A^0[2,1] + A^0[1,3]$
$12 < \infty + \infty$

2) $A^0[2,4]$ $\quad A^0[2,1] + A^0[1,4]$
$5 < \infty + \infty$

3) $A^0[3,2]$ $\quad A^0[3,1] + A^0[1,2]$
$\infty \quad 4 + 3$
$\infty > 7$

4) $A^0[3,4]$ $\quad A^0[3,1] + A^0[1,4]$
$-1 < 4 + \infty$

5) $A^0[4,2]$ $\quad A^0[4,1] + A^0[1,2]$
$-4 \quad 2 + 3$
$-4 < 5$

6) $A^0[4,3]$ $\quad A^0[4,1] + A^0[1,3]$
$\infty = 2 + \infty$

Find the matrix for vertex 2

A^2 considering 2 as an intermiadiatary vertex

A^2 :

	1	2	3	4
1	0	3	∞	∞
2	∞	0	**12**	**5**
3	4	**7**	0	**-1**
4	2	**-4**	∞	0

1) $A^1[1,3]$ $A^1[1,2] + A^1[2,3]$

∞ 3 + 12

∞ > 15

2) $A^1[1,4]$ $A^1[1,2] + A^1[2,4]$

∞ 3 + 5

∞ > 8

3) $A^1[3,1]$ $A^1[3,2] + A^1[2,1]$

4 < 7 + ∞

4) $A^1[3,4]$ $A^1[3,2] + A^1[2,4]$

-1 < 7 + 5

5) $A^1[4,1]$ $A^1[4,2] + A^1[2,1]$

2 < -4 + ∞

6) $A^1[4,3]$ $A^1[4,2] + A^1[2,3]$

∞ -4 + 12

∞ > 8

Find the matrix for matrix of vertex 3

A^3 considering 3 as an intermiadiatary vertex

A^3 :

	1	2	3	4
1	0	**3**	5	**8**
2	**16**	0	12	**5**
3	4	7	0	-1
4	**2**	**-4**	8	0

1) $A^2\,[1,2] \qquad A^2\,[1,3] + A^2\,[3,2]$

$3 \quad < \quad 15 \; + \; 7$

2) $A^2\,[1,4] \qquad A^2\,[1,3] + A^2[\,3,4]$

$8 \quad < \quad 15 \; + \; (-1)$

3) $A^2\,[2,1] \qquad A^2\,[2,3] + A^2\,[3,1]$

$\infty \qquad 12 \; + \; 4$

$\infty \quad > \quad 16$

4) $A^2\,[2,4] \qquad A^2\,[2,3] + A^2\,[3,4]$

$5 \qquad 12 \; + \; (-1)$

$5 \quad > \quad 11$

5) $A^2\,[4,1] \qquad A^2\,[4,3] + A^2\,[3,1]$

$2 \quad < \quad 8 \; + \; 4$

6) $A^2\,[4,3] \qquad A^2\,[4,2] + A^2\,[2,3]$

$-4 \quad < \quad 8 \; + \; 7$

Find the matrix for matrix of vertex 4

A^4 considering 4 as an intermiadiatary vertex

A^4 :

	1	2	3	4
1	0	**3**	**5**	8
2	**14**	0	**12**	5
3	**1**	**3**	0	-1
4	2	-4	8	0

1) A^4 [1,2] A^4 [1,4] + A^4 [4,2]

3 8 + 4

3 < 12

2) A^4 [1,3] A^4 [1,4] + A^4 [4,3]

5 8 + 8

5 < 16

3) A^4 [2,1] A^4 [2,4] + A^4 [4,1]

16 12 + 2

16 > 14

4) A^4 [2,3] A^4 [2,4] + A^4 [4,3]

12 5 + 8

12 < 13

5) A^4 [3,1] A^4 [3,4] + A^4 [4,1]

4 -1 + 2

4 > 1

6) A^4 [3,2] A^4 [3,4] + A^4 [4,2]

7 -1 + 4

7 > 3