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COMP90038
Algorithms and Complexity

Lecture 8: Graph Traversal

(with thanks to Harald Søndergaard)

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DMD 8.17 (Level 8, Doug McDonell Bldg)

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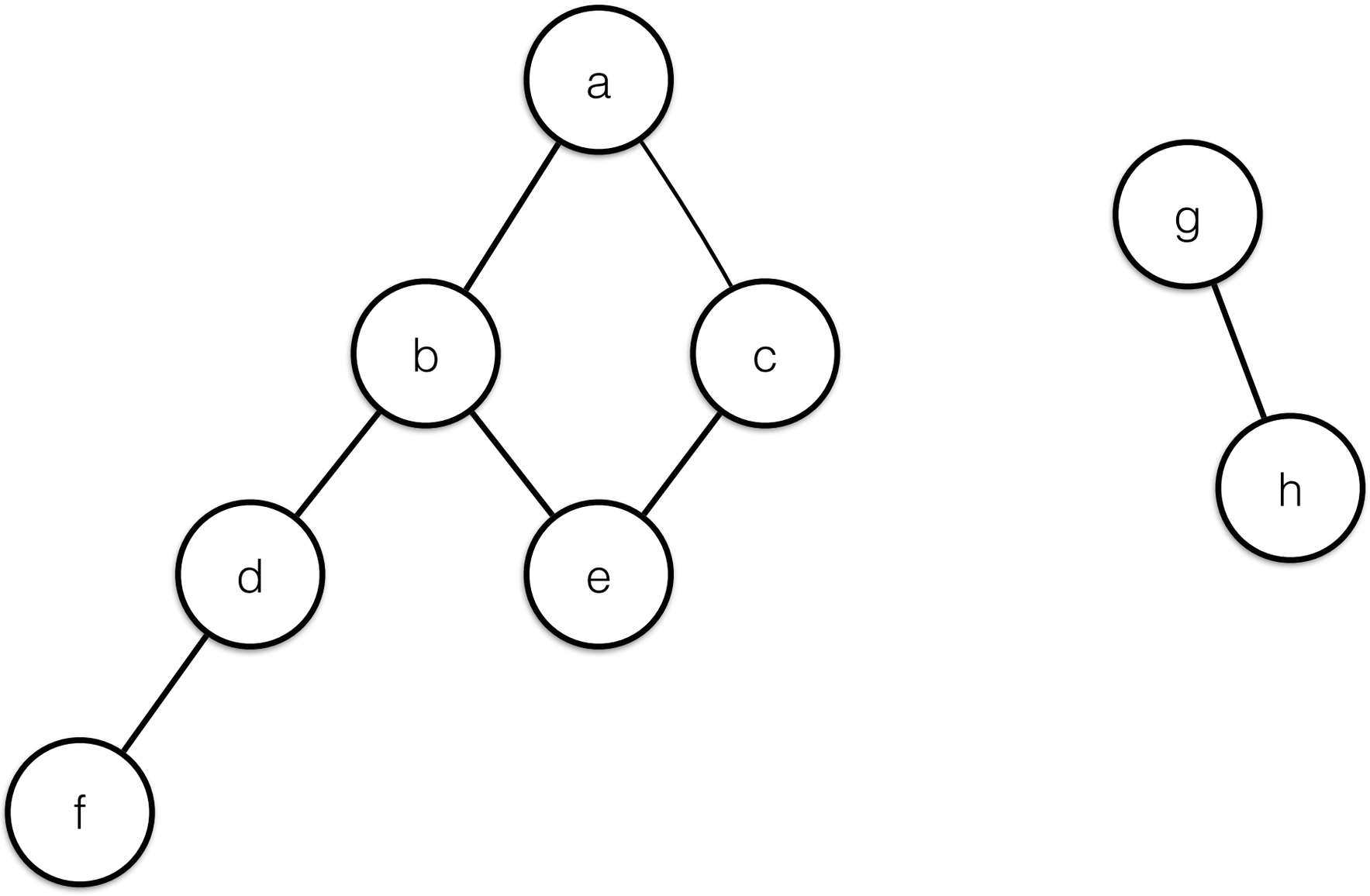
@tobycmurray

Breadth-First and Depth-First Traversal

- Used to **systematically** explore **all** nodes of a graph



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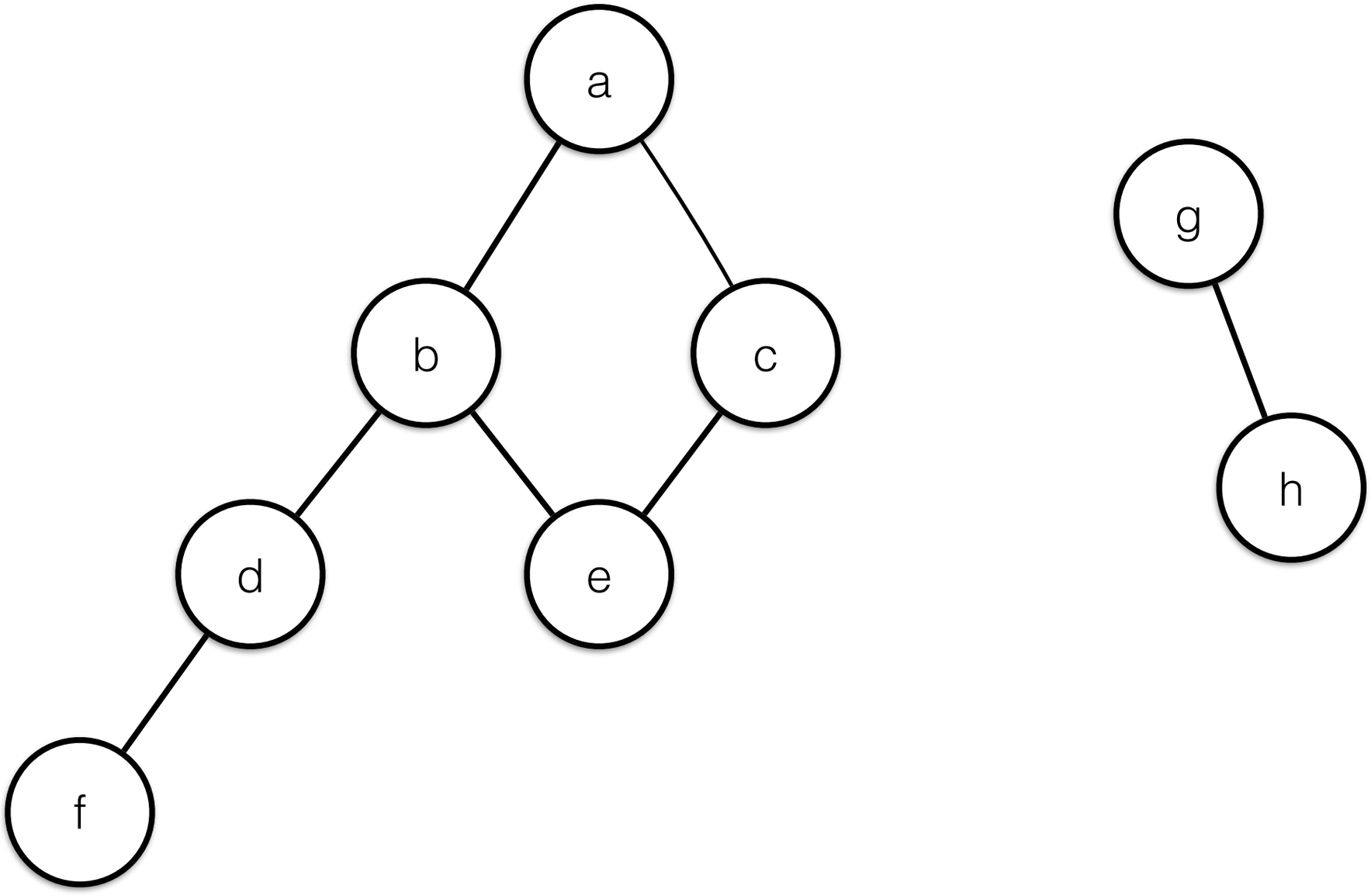
Depth-First Traversal



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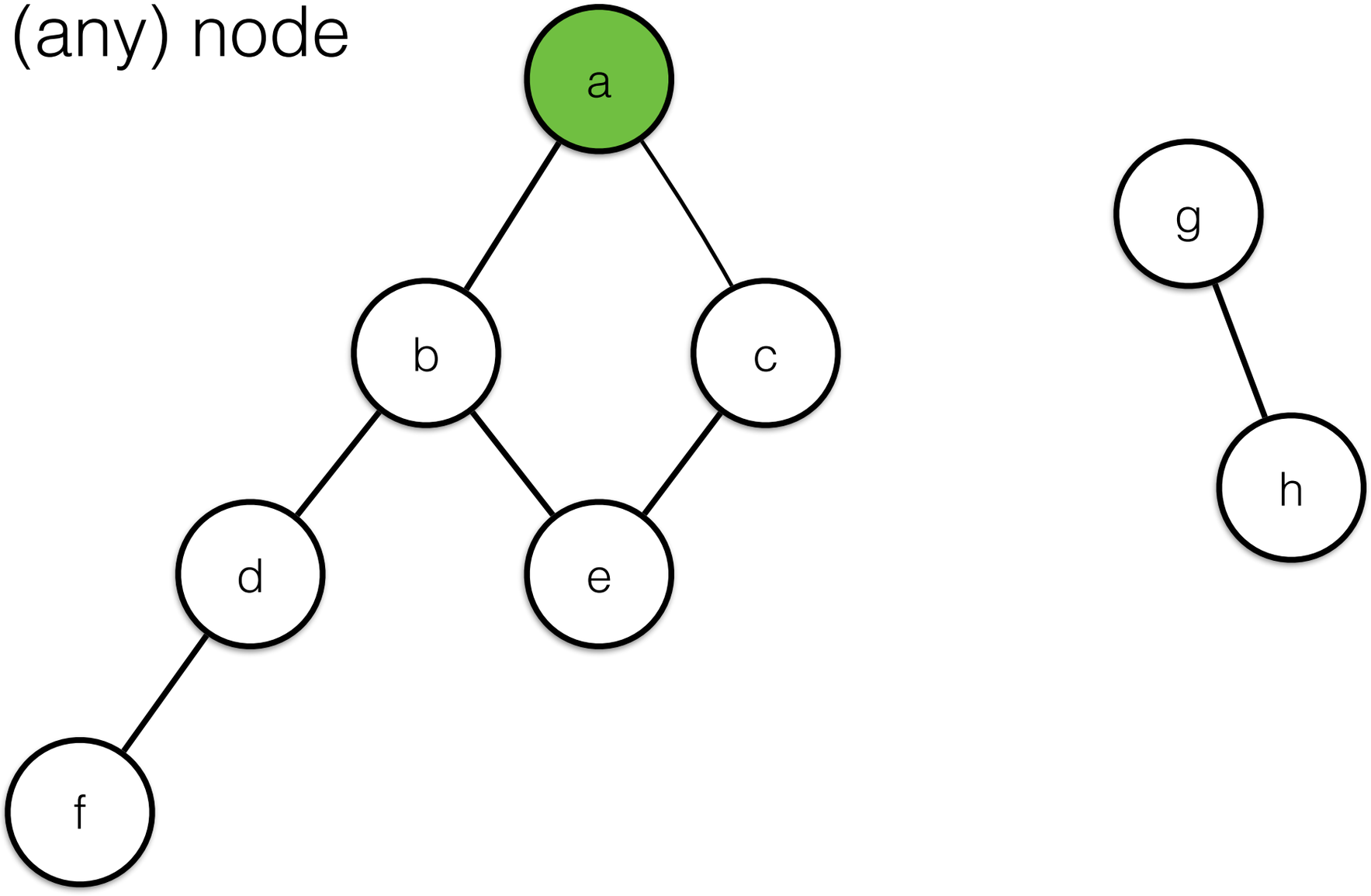
Depth-First Traversal

Nodes visited in this order: a

Start with (any) node



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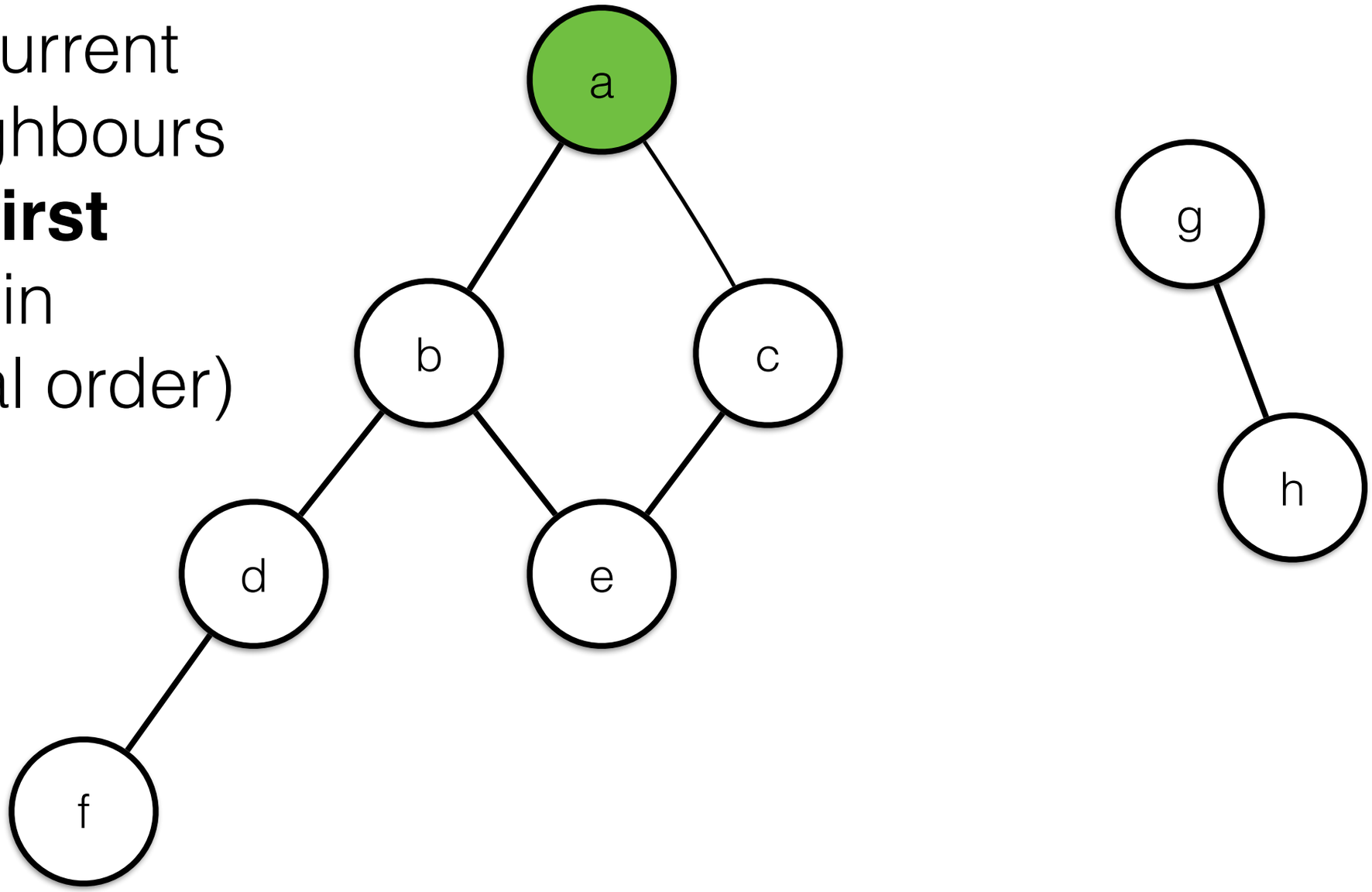
Depth-First Traversal

Nodes visited in this order: a

Visit the current node's neighbours **depth first** (here in alphabetical order)



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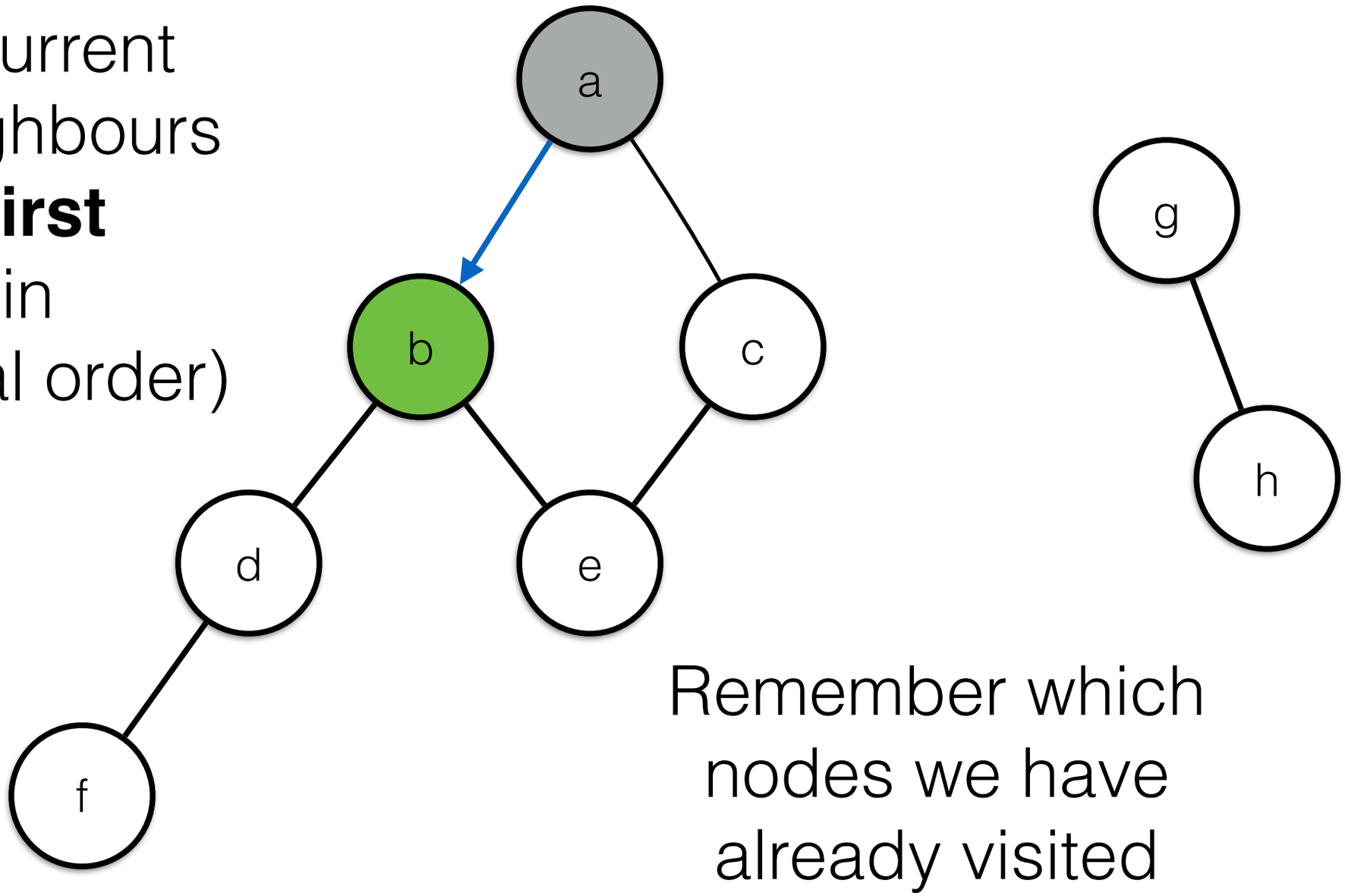
Depth-First Traversal

Nodes visited in this order: a b

Visit the current node's neighbours **depth first** (here in alphabetical order)



Remember which nodes we have already visited to avoid revisiting them

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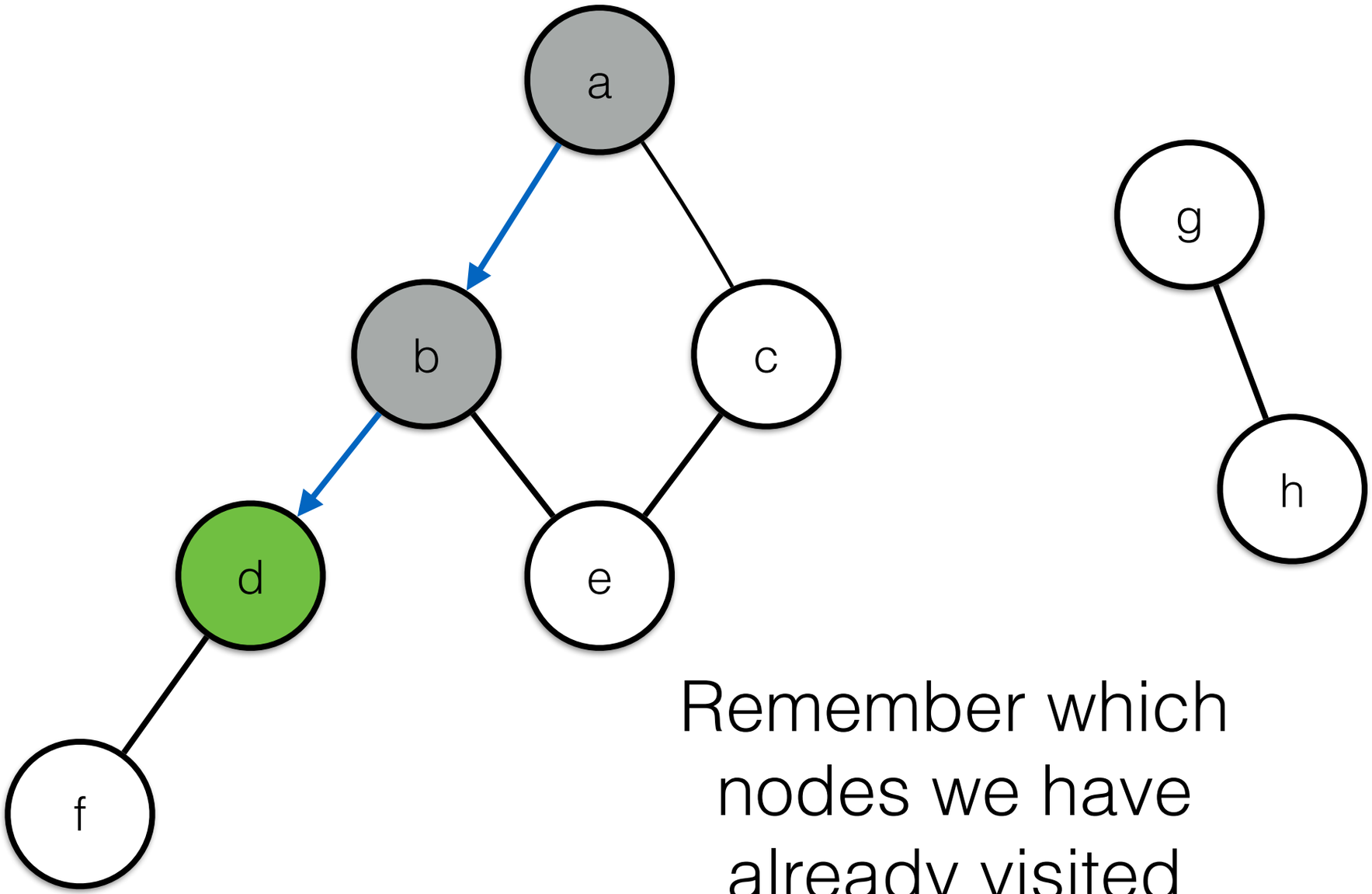
Depth-First Traversal

Nodes visited in this order: a b d



Remember which nodes we have already visited to avoid revisiting them

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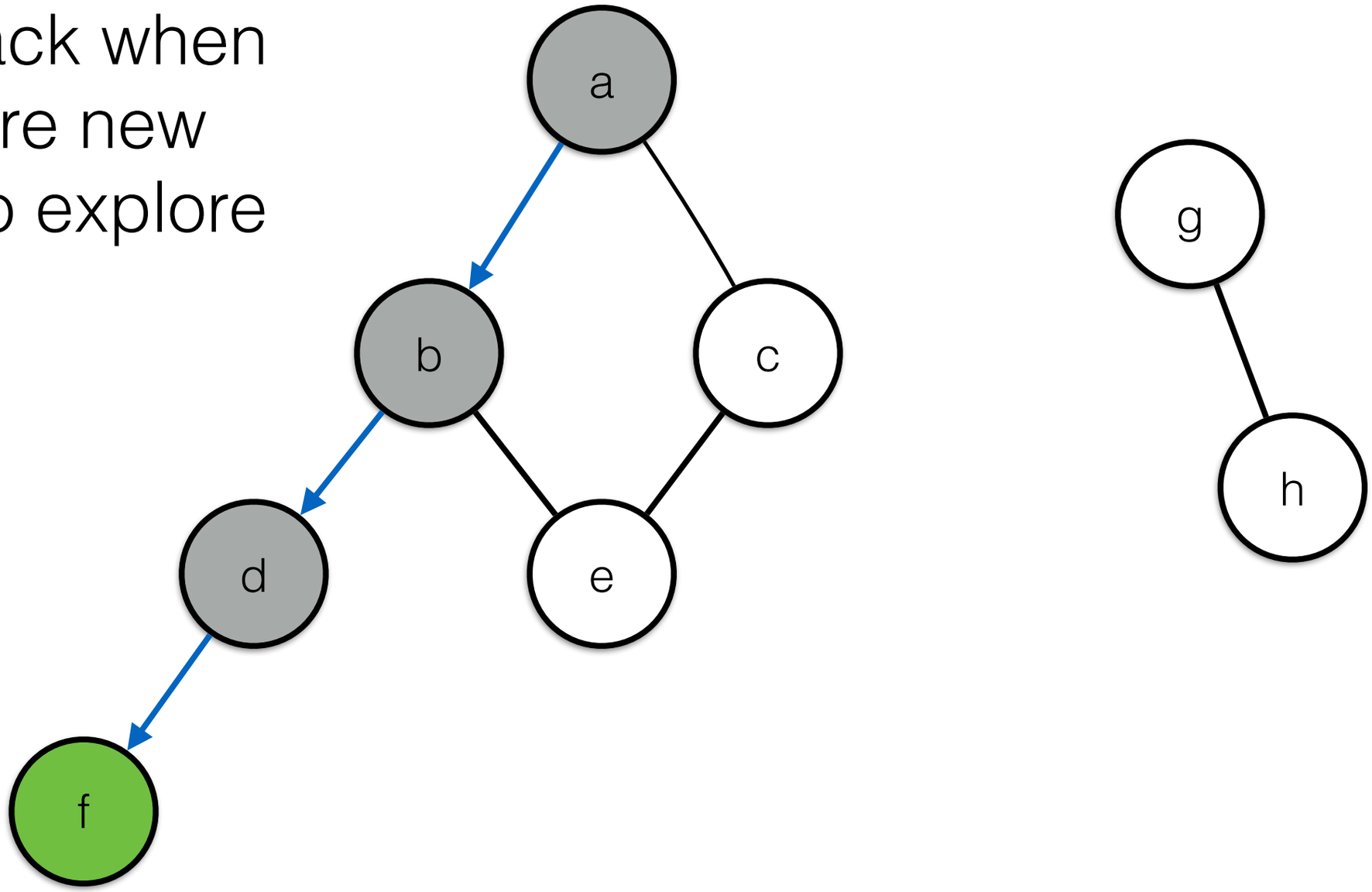
Depth-First Traversal

Nodes visited in this order: a b d f

Back-track when no more new nodes to explore



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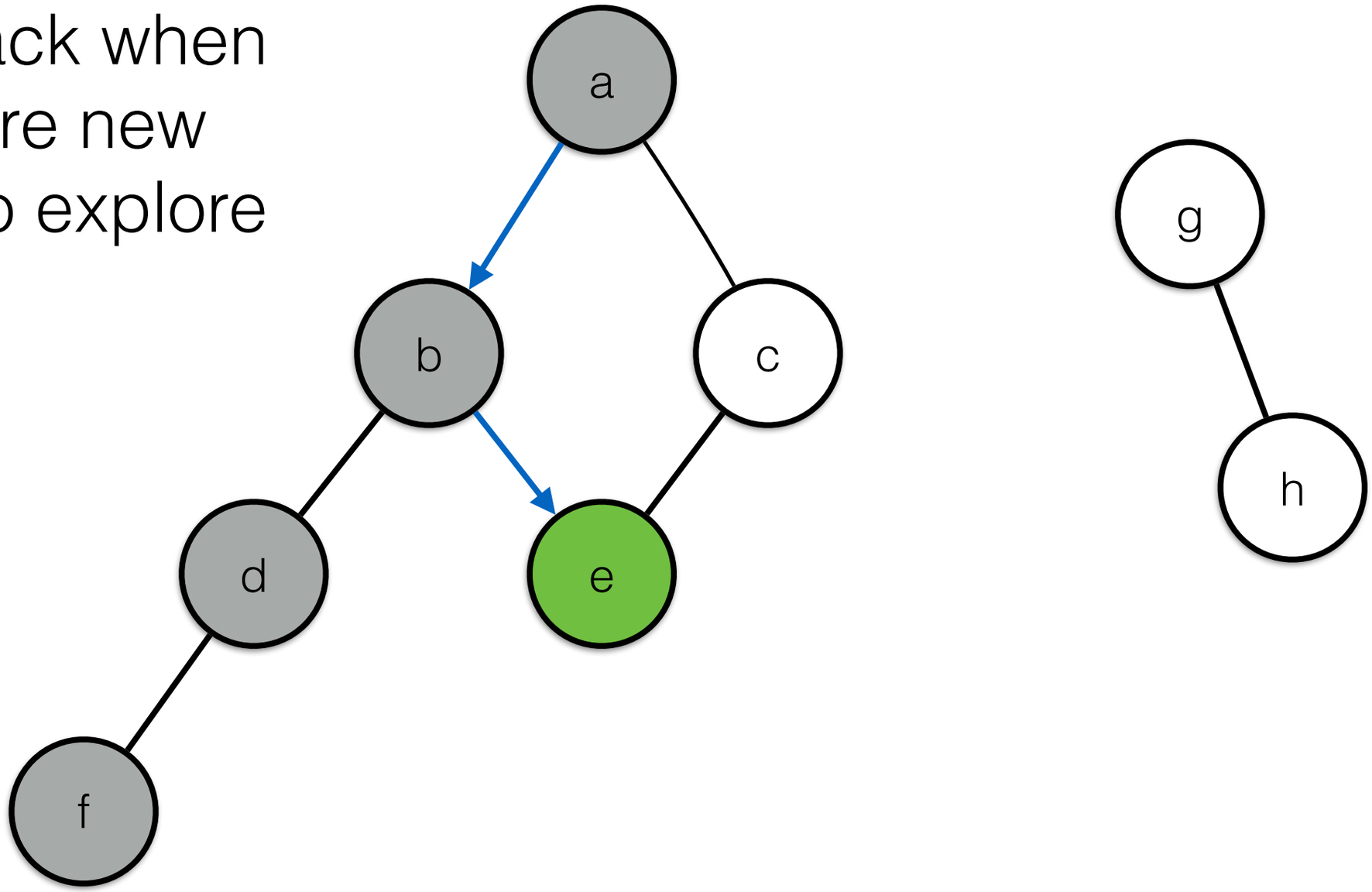
Depth-First Traversal

Nodes visited in this order: a b d f e

Back-track when no more new nodes to explore



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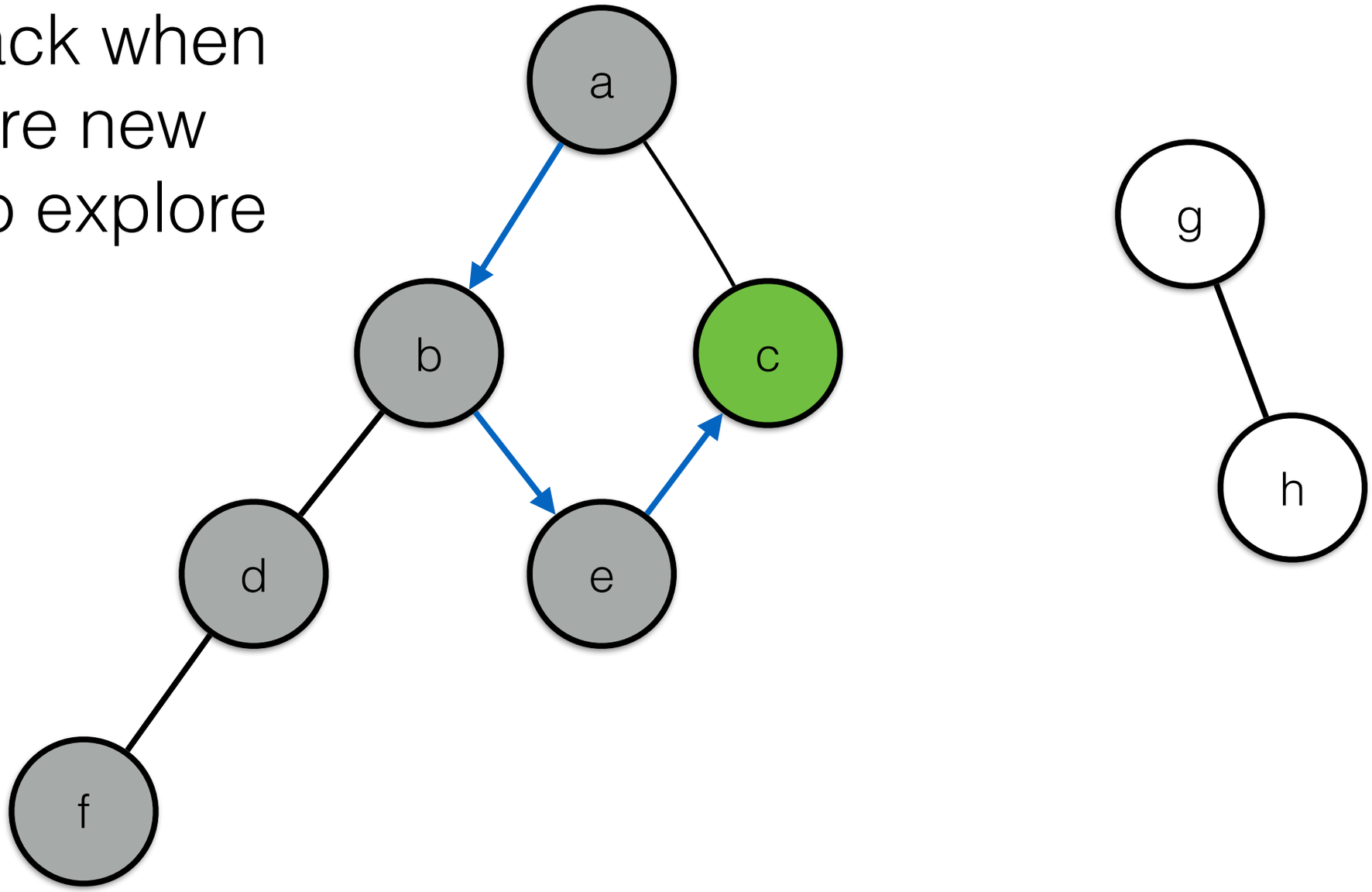
Depth-First Traversal

Nodes visited in this order: a b d f e c

Back-track when no more new nodes to explore



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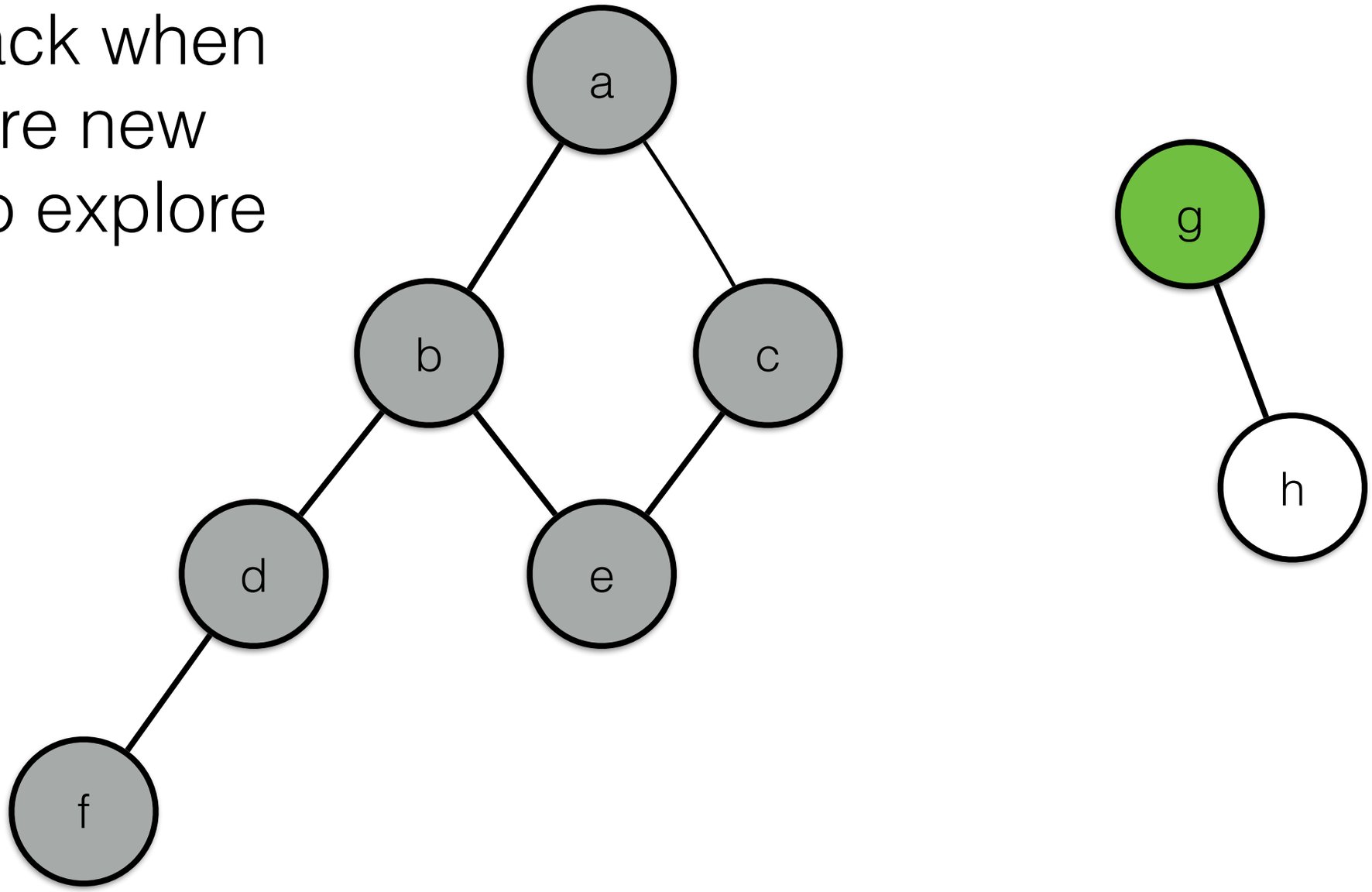
Depth-First Traversal

Nodes visited in this order: a b d f e c g

Back-track when no more new nodes to explore



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Depth-First Traversal

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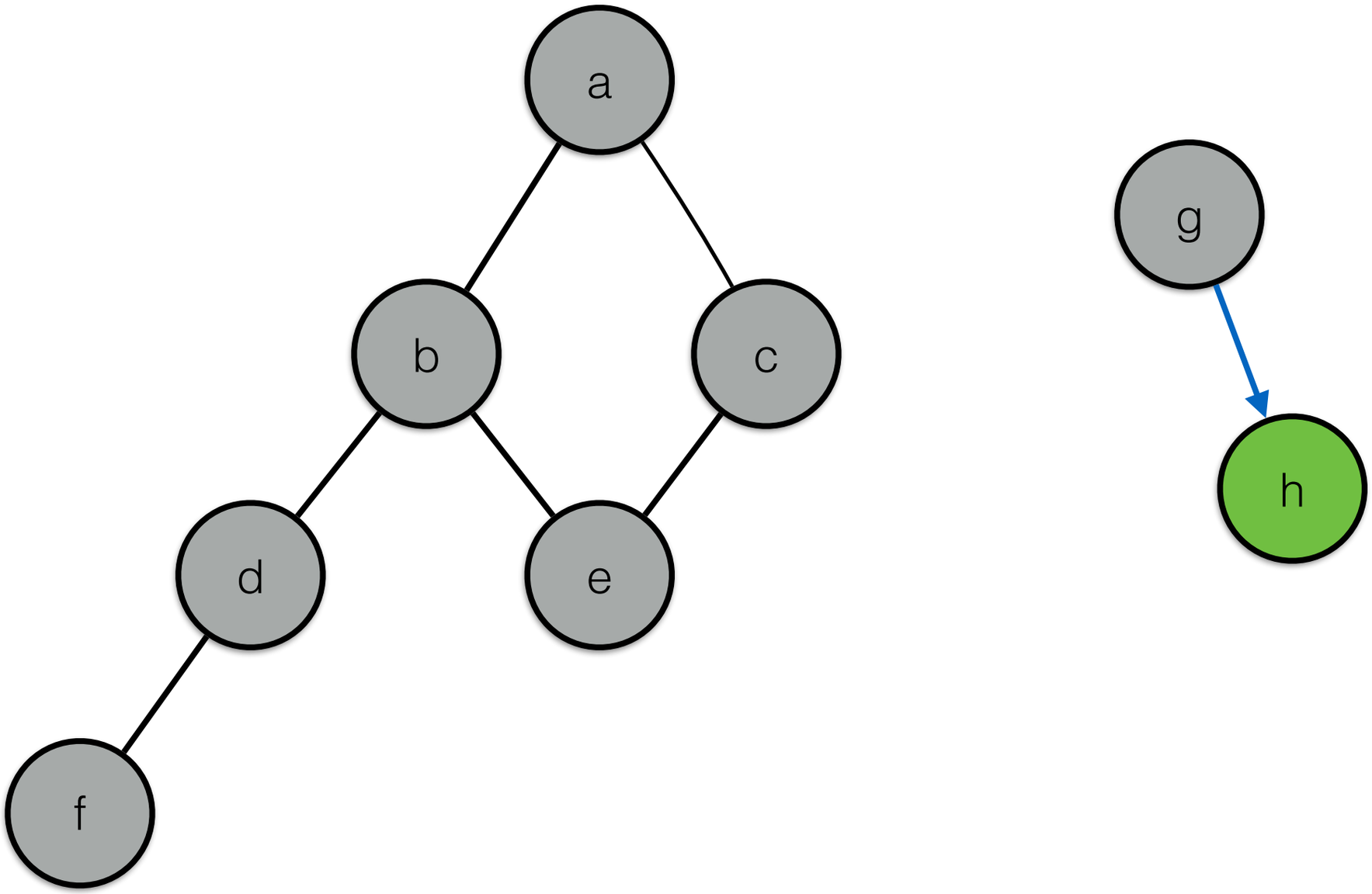
Nodes visited in this order: a b d f e c g h



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Depth-First Traversal: Stack Discipline

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When back-tracking, we go back to the most recently-visited node that still has unvisited neighbours

This is simulated by pushing each node onto a stack as it is visited.

Back-tracking then corresponds to popping the stack.

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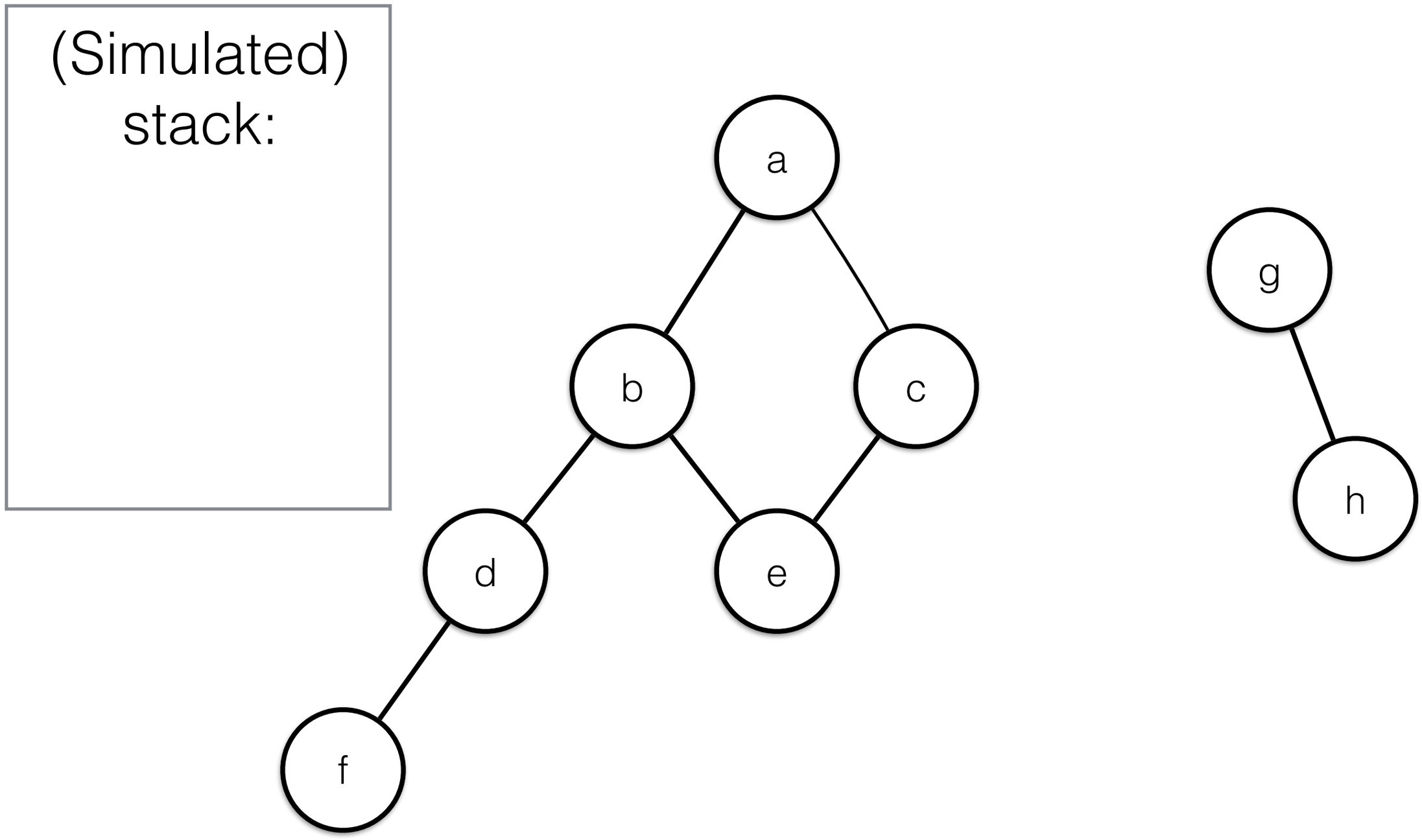
Depth-First Traversal: Stack Discipline



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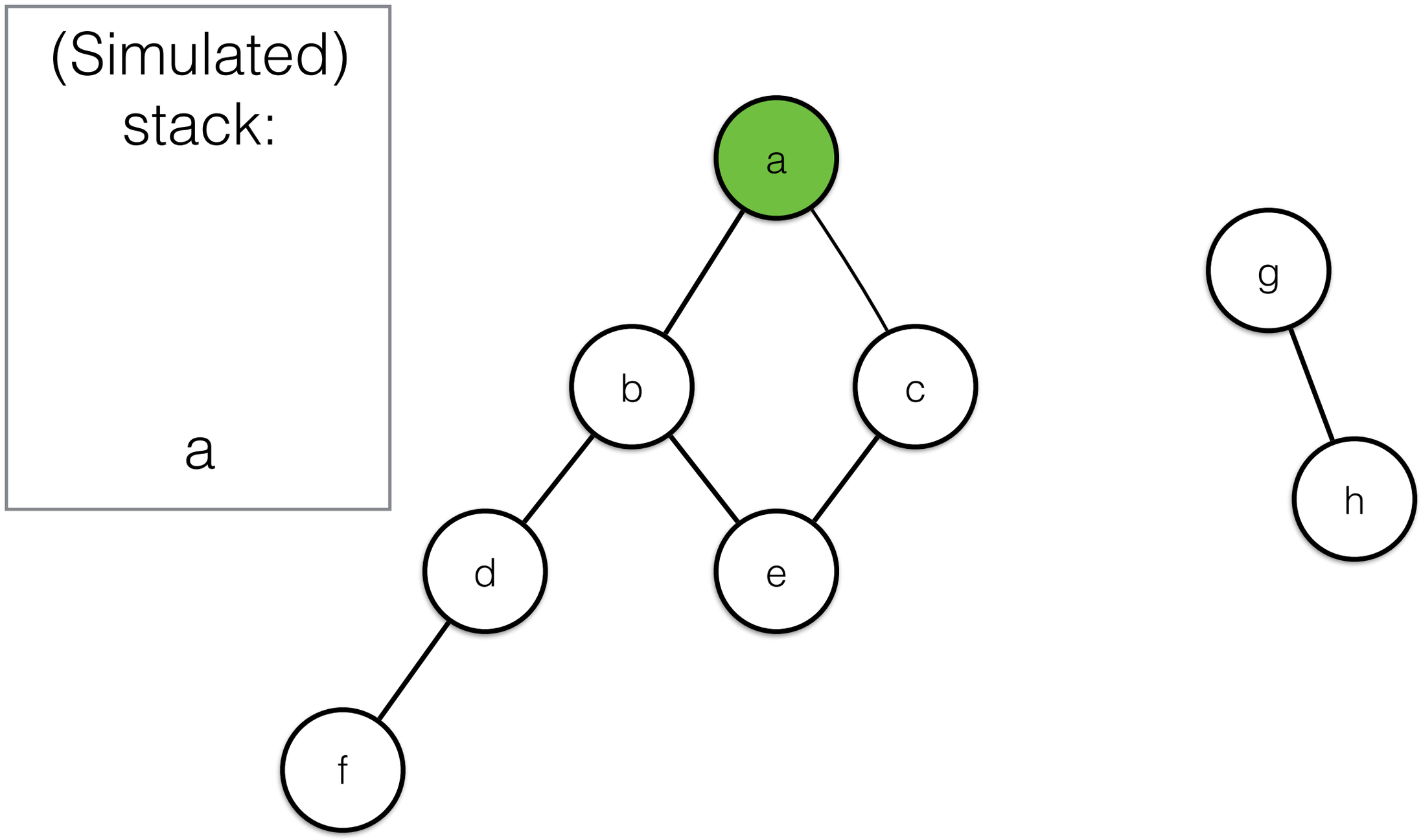

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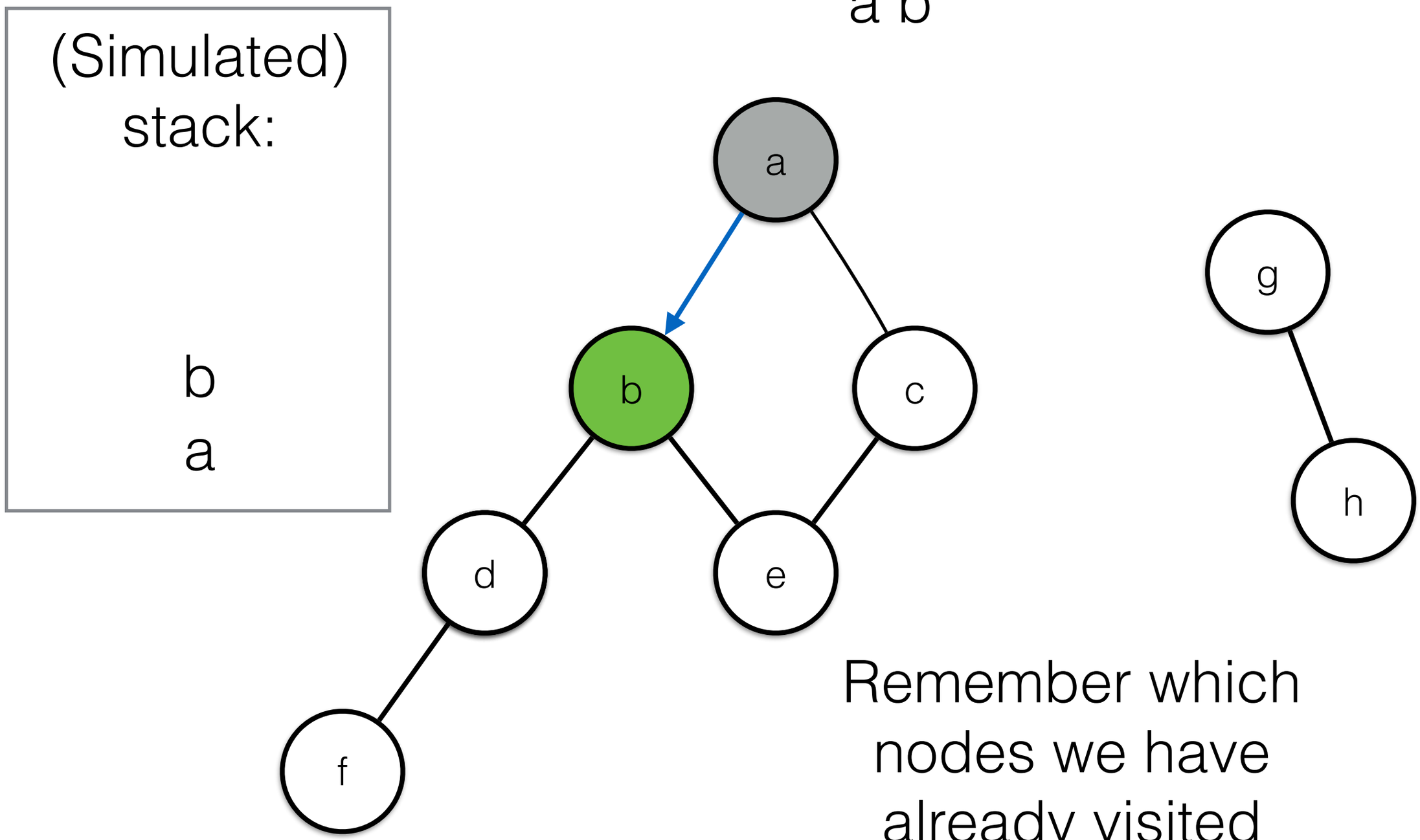
Depth-First Traversal: Stack Discipline



Remember which nodes we have already visited to avoid revisiting them

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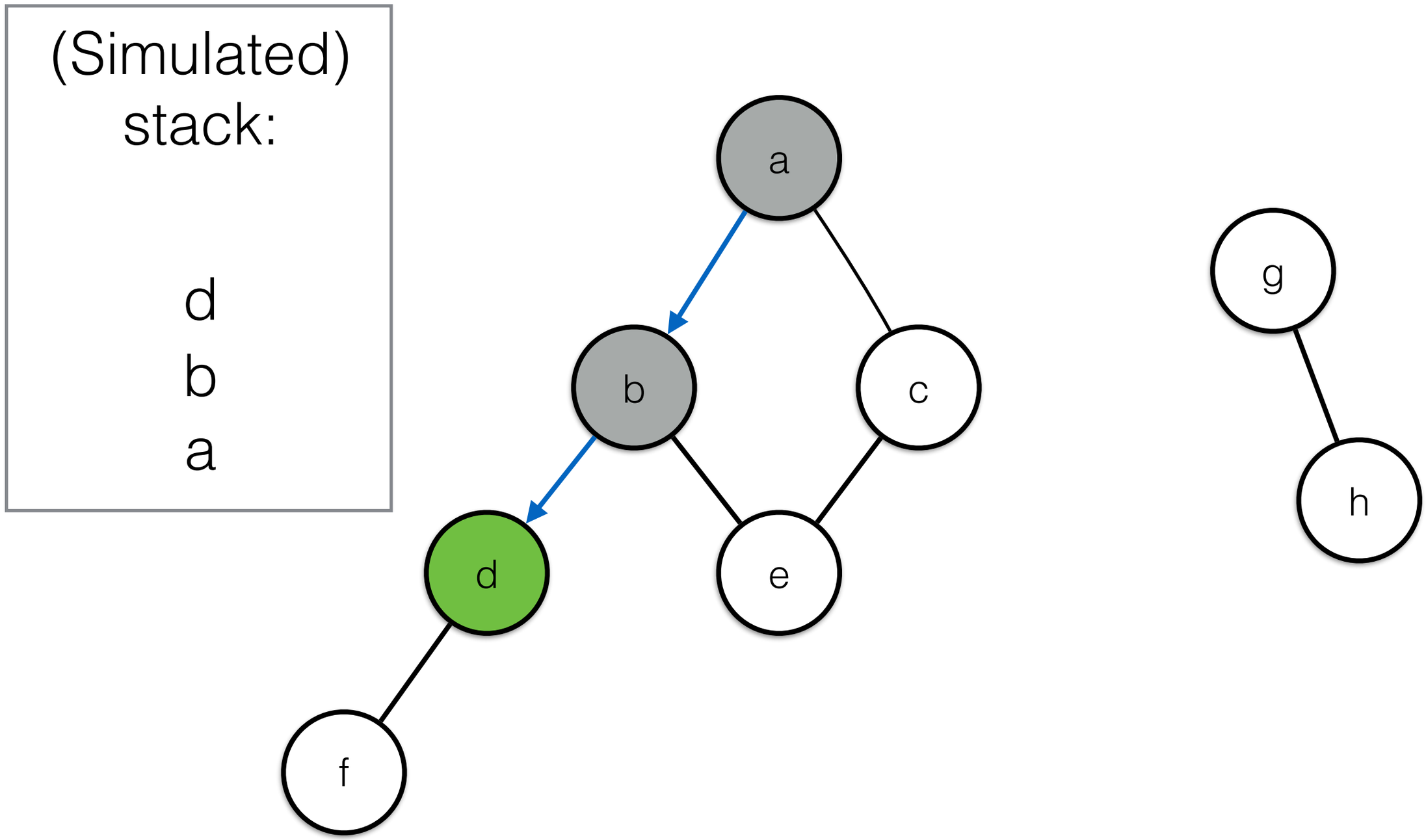
Depth-First Traversal: Stack Discipline



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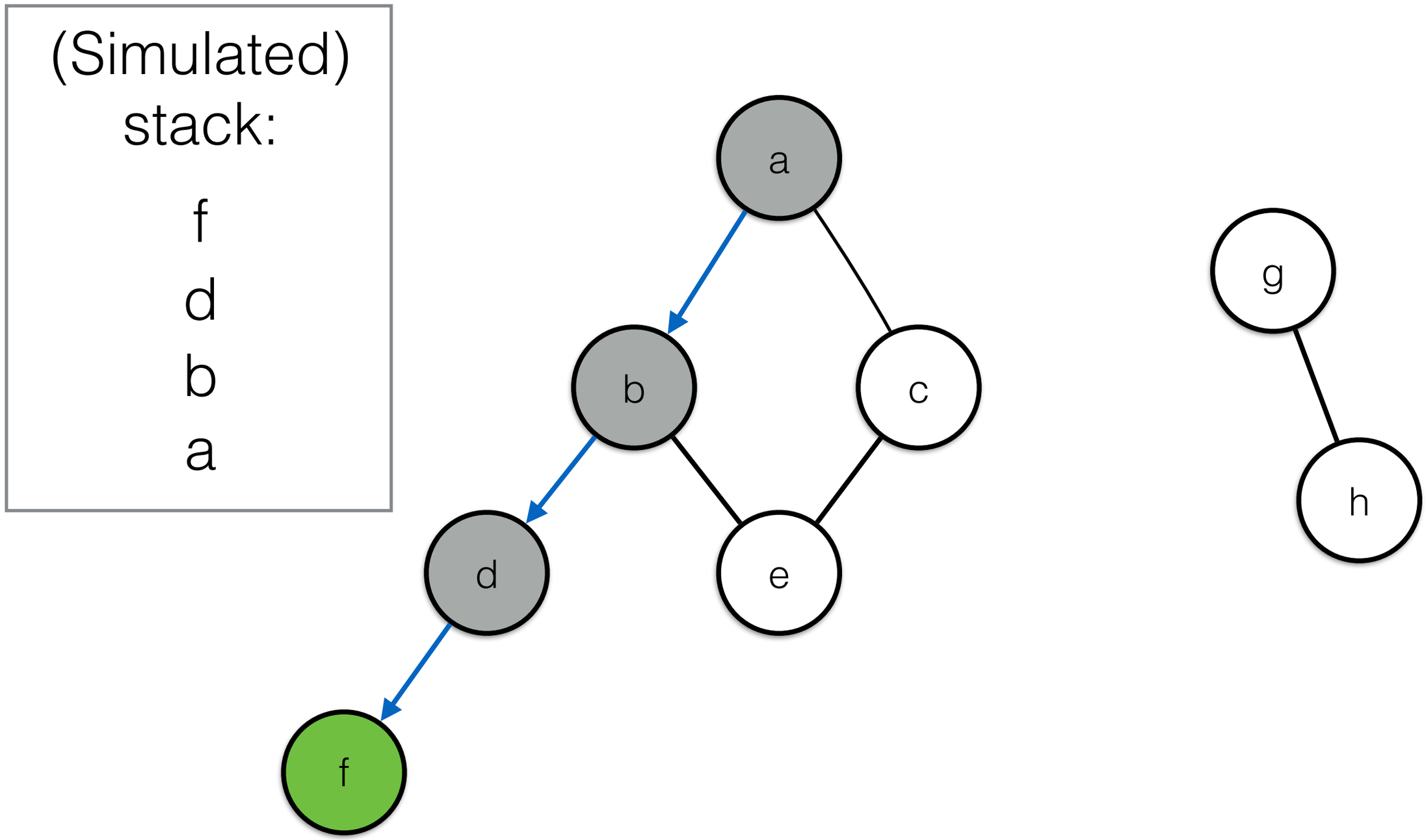

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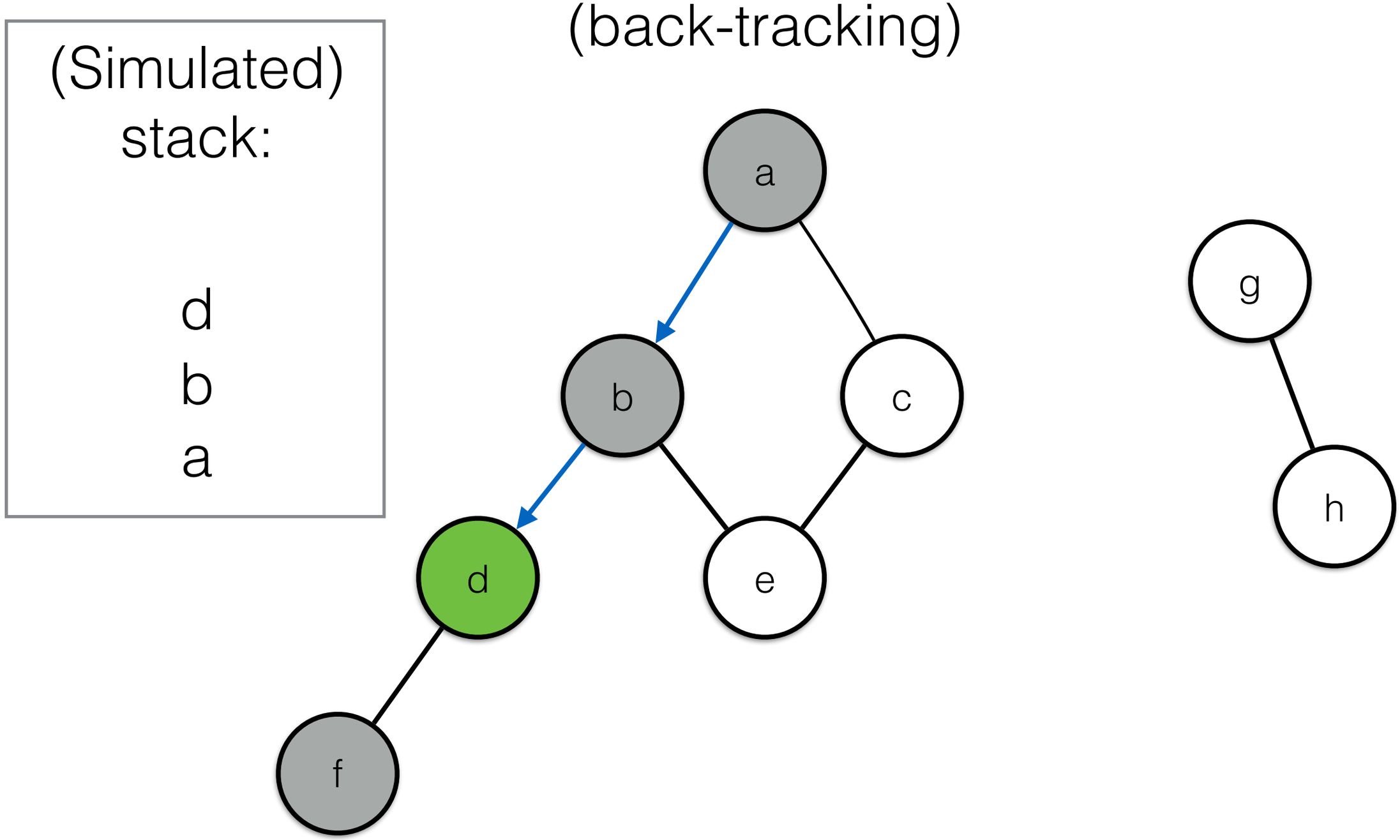
Depth-First Traversal: Stack Discipline



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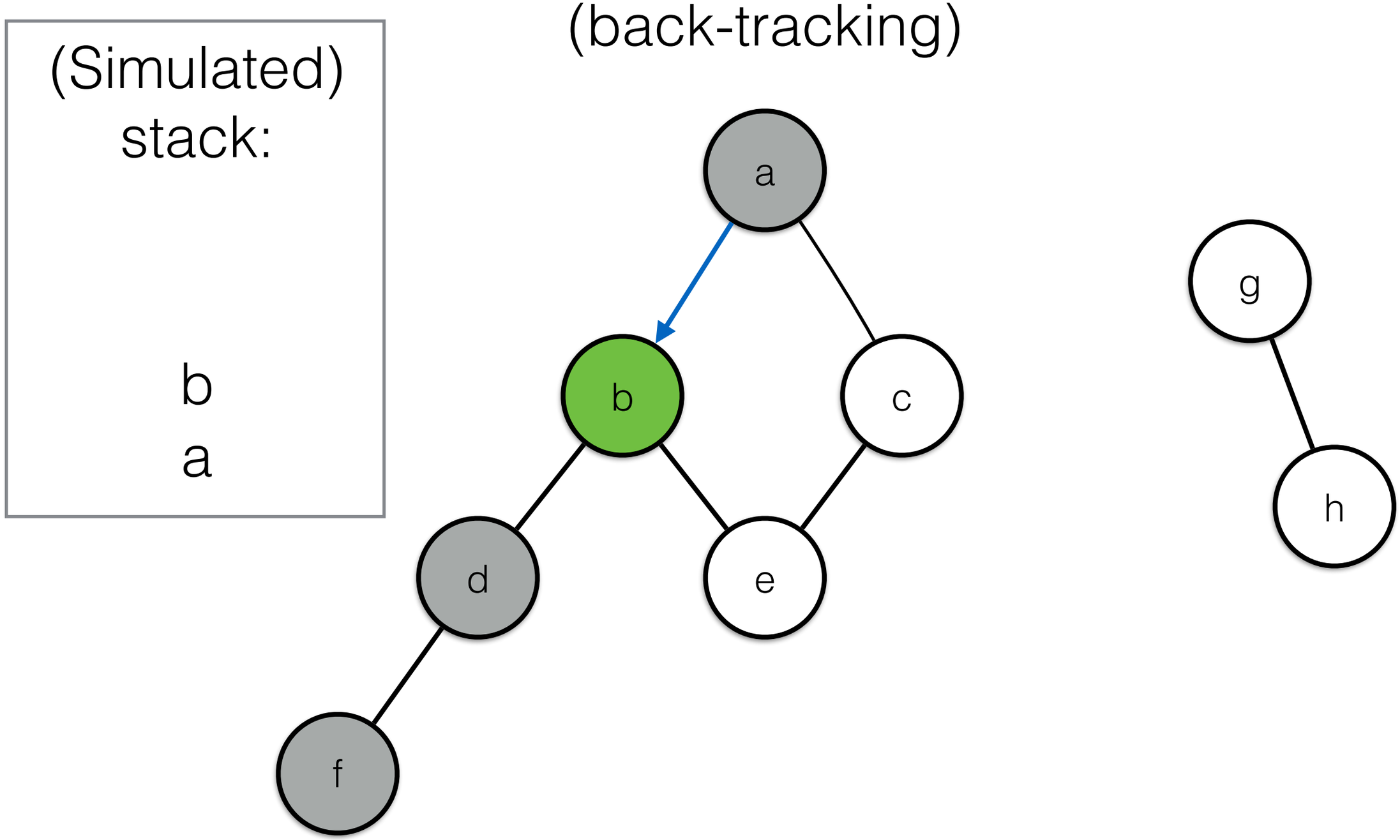
Depth-First Traversal: Stack Discipline



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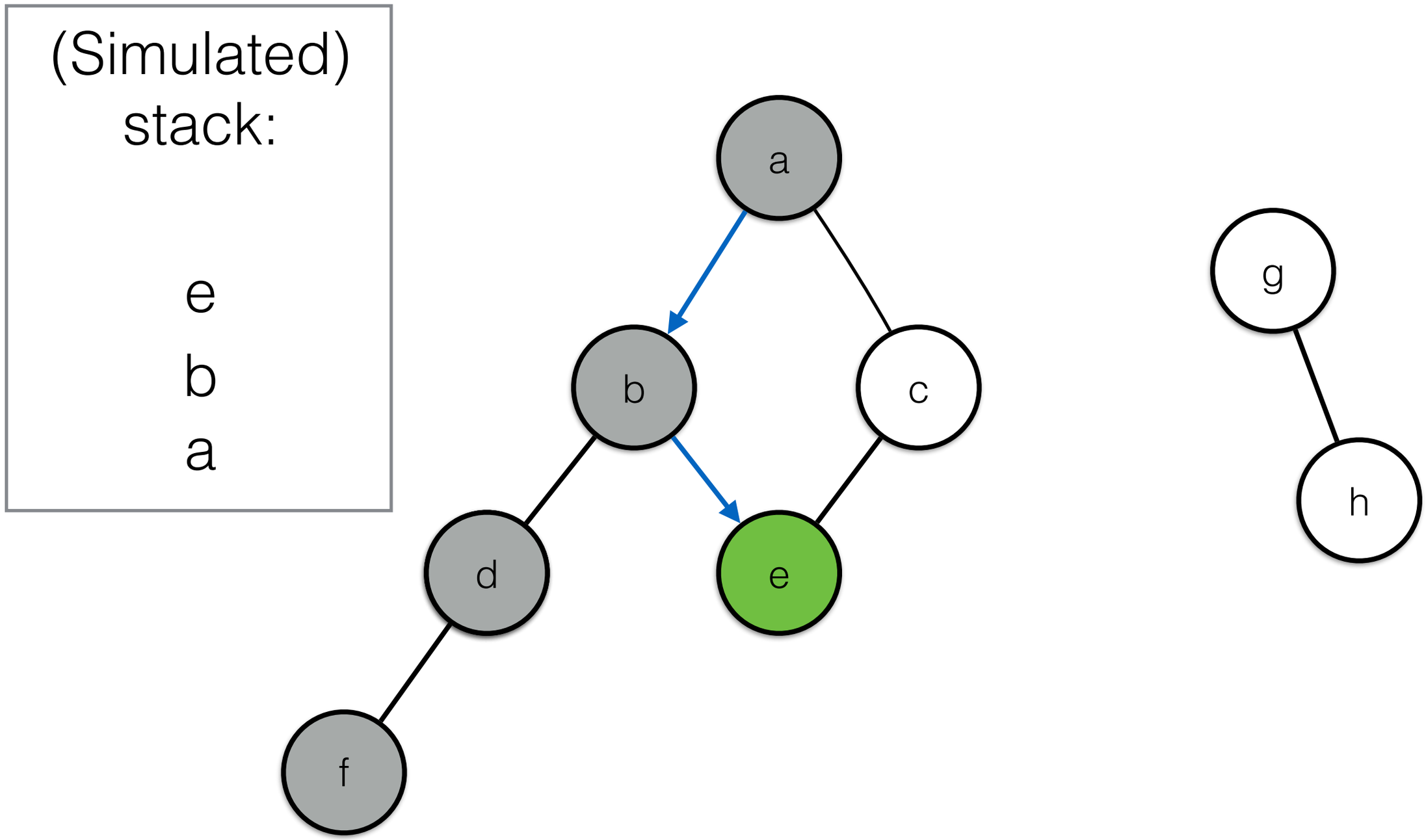
Depth-First Traversal: Stack Discipline



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Depth-First Traversal: Stack Discipline

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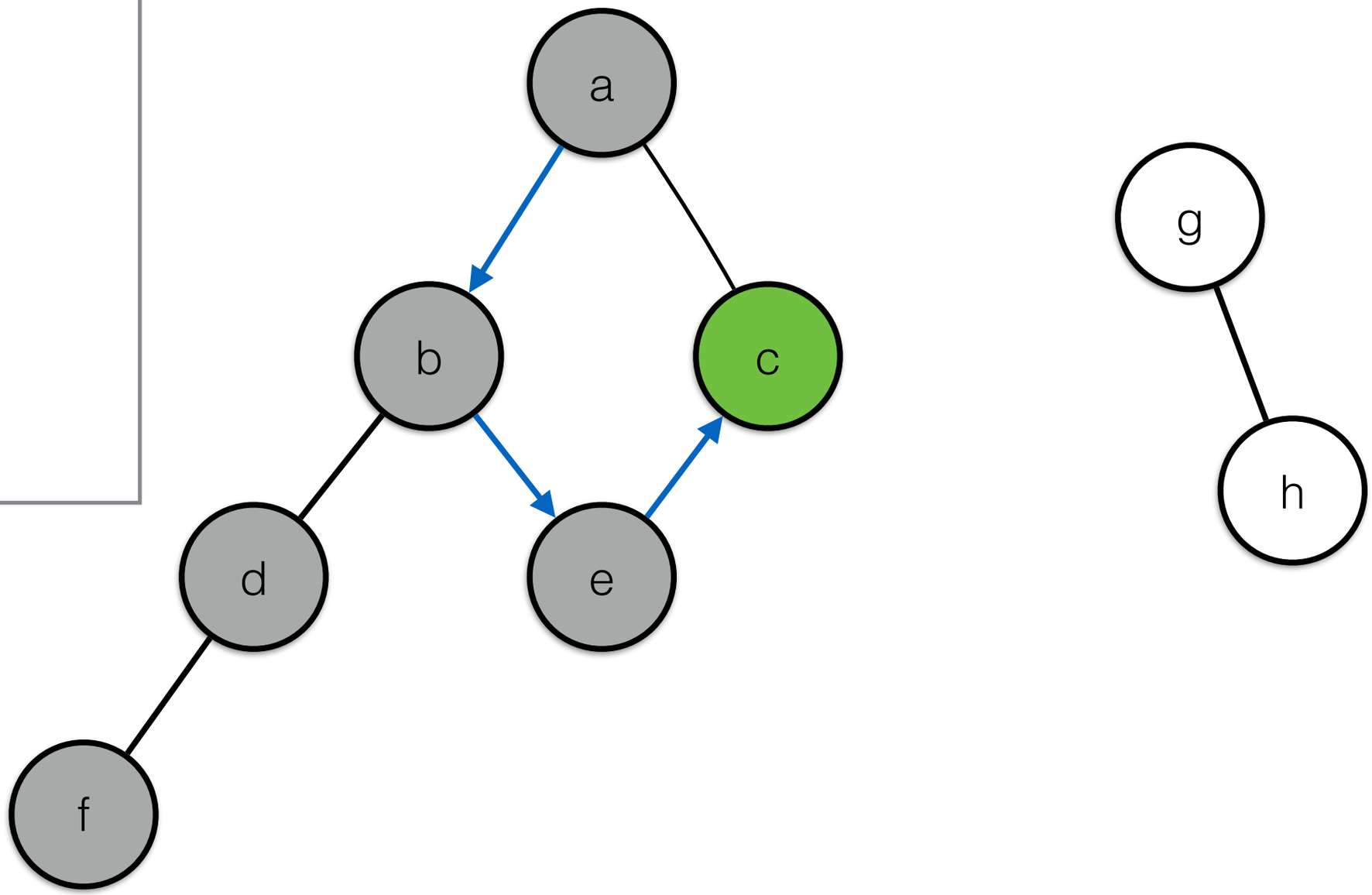
(Simulated) stack:

c
e
b
a



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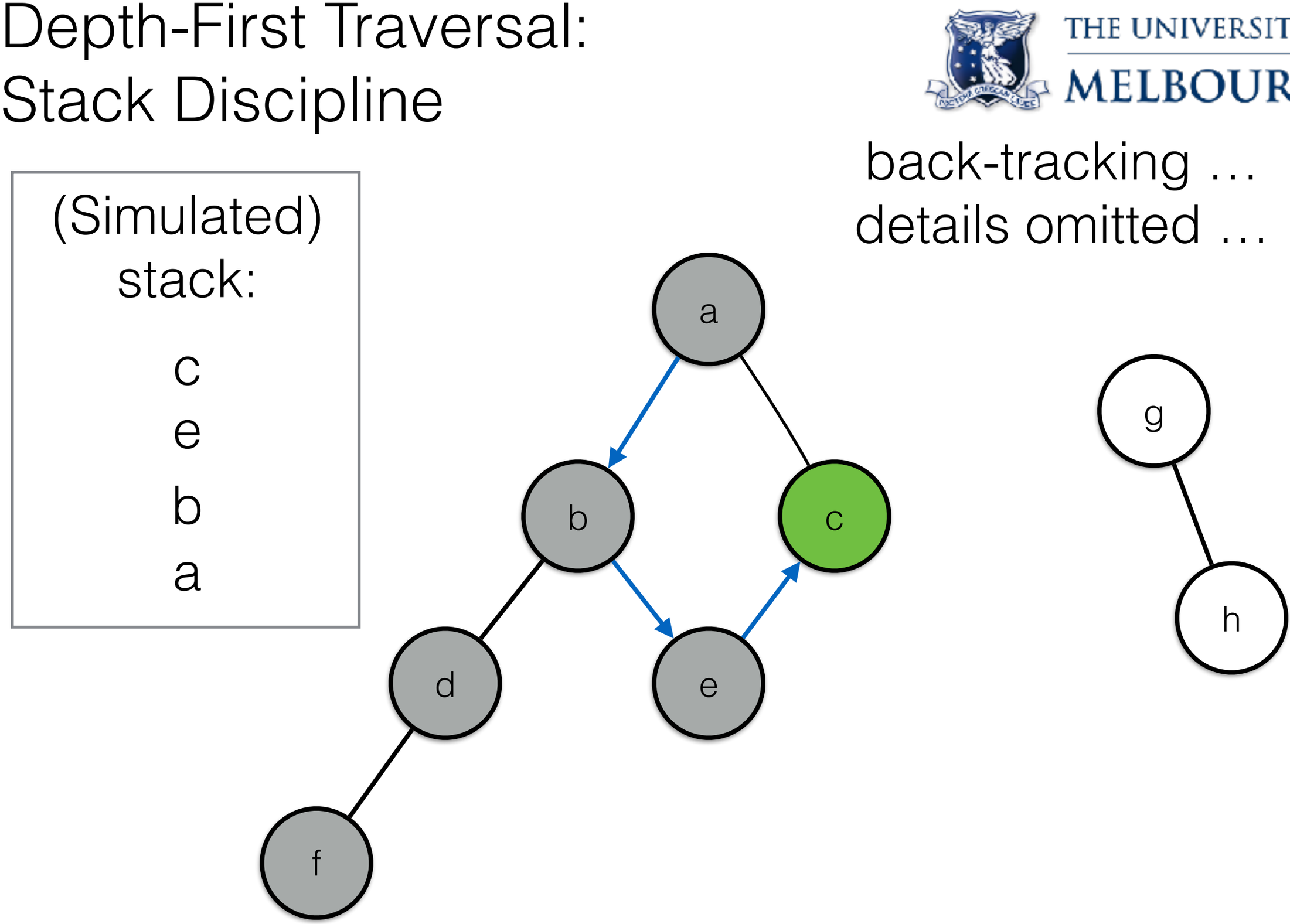
Depth-First Traversal: Stack Discipline



back-tracking …
details omitted …

(Simulated) stack:

c
e
b
a

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Depth-First Traversal: Stack Discipline

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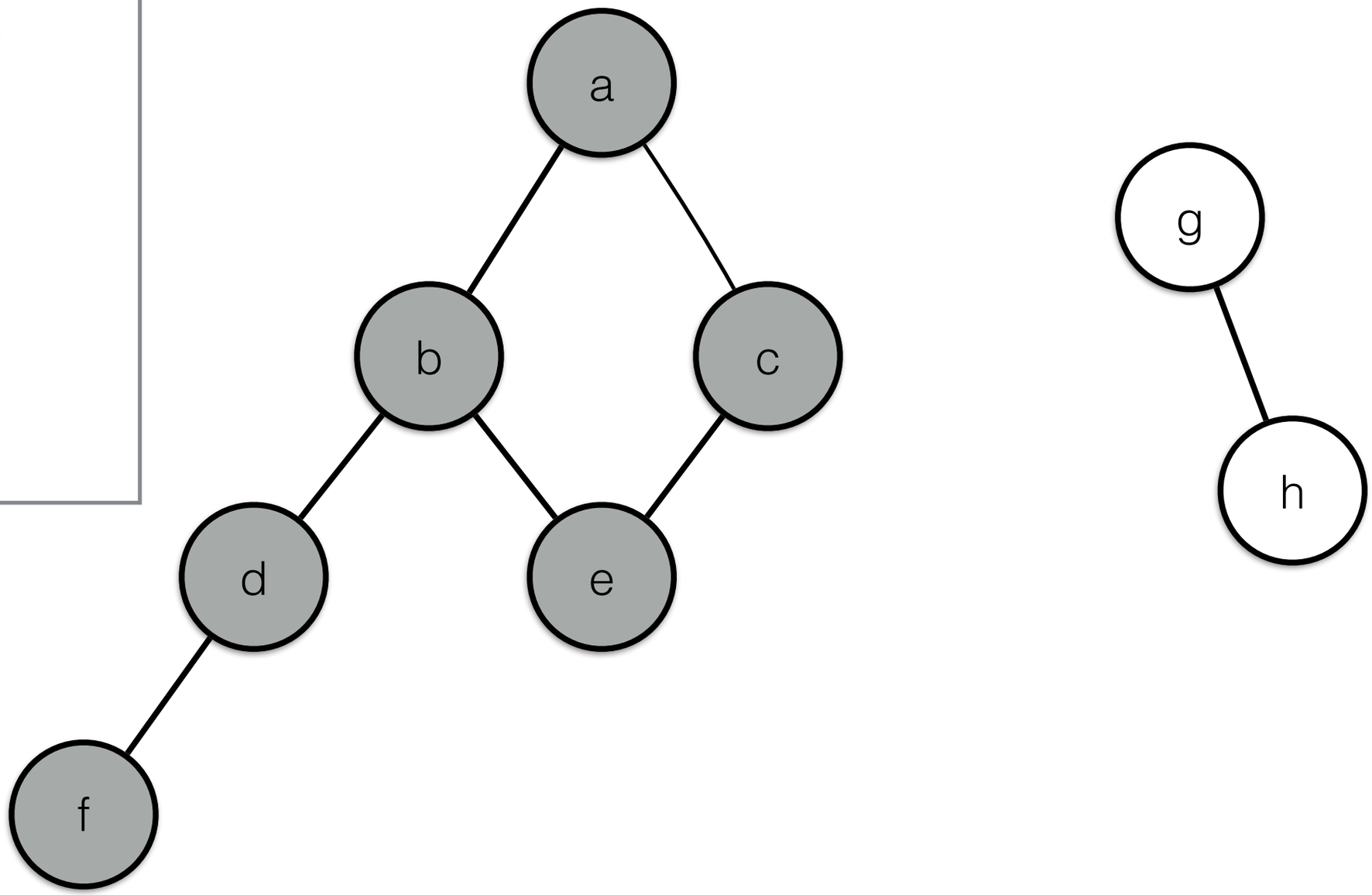
(Simulated) stack:



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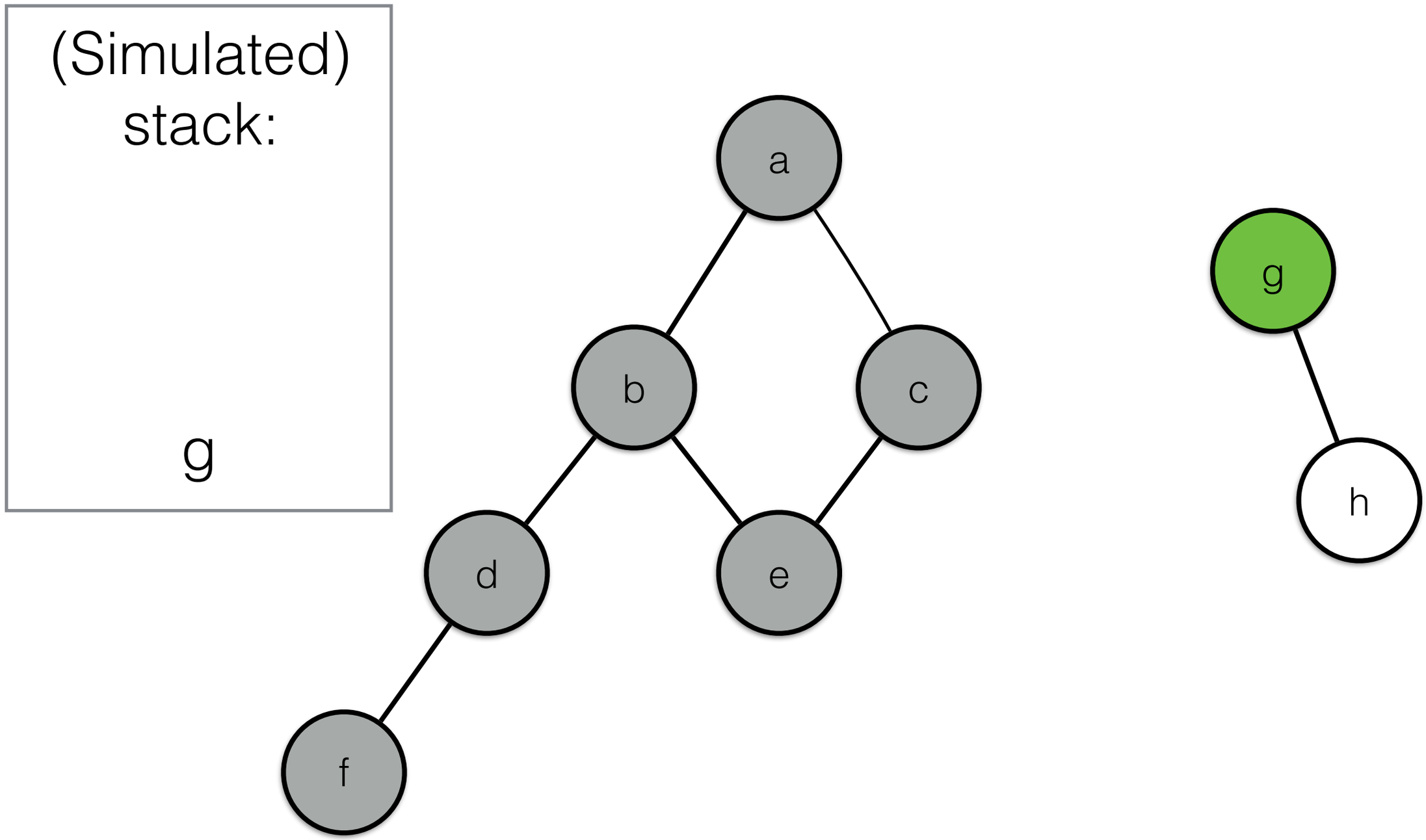
Depth-First Traversal: Stack Discipline



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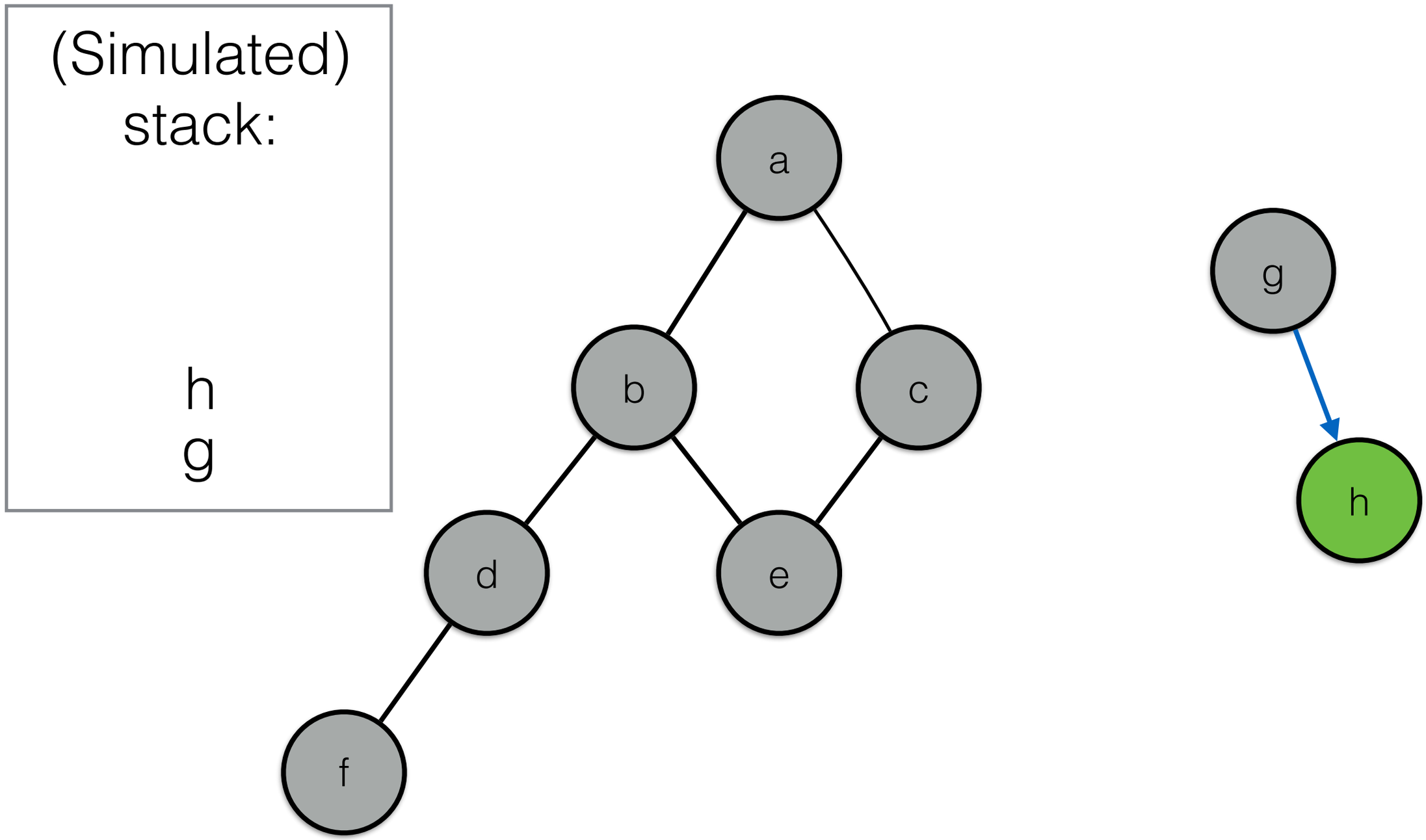
Depth-First Traversal: Stack Discipline



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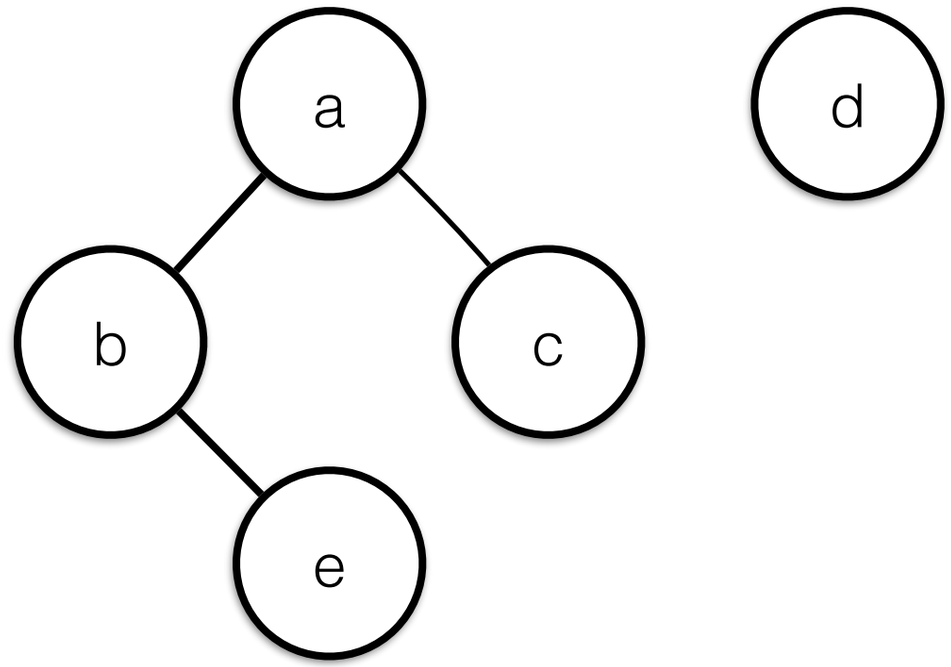
Depth-First Traversal: Recursive Implementation




```
function DFS(⟨V, E⟩)
    mark each node in V with 0
    count ← 0
    for each v in V do
        if v is marked with 0 then
            DfsExplore(v)
```

```
function DfsExplore(v)
    count ← count + 1
    mark v with count
    for each edge (v, w) do
        if w is marked with 0 then
            DfsExplore(w)
```

Call Stack

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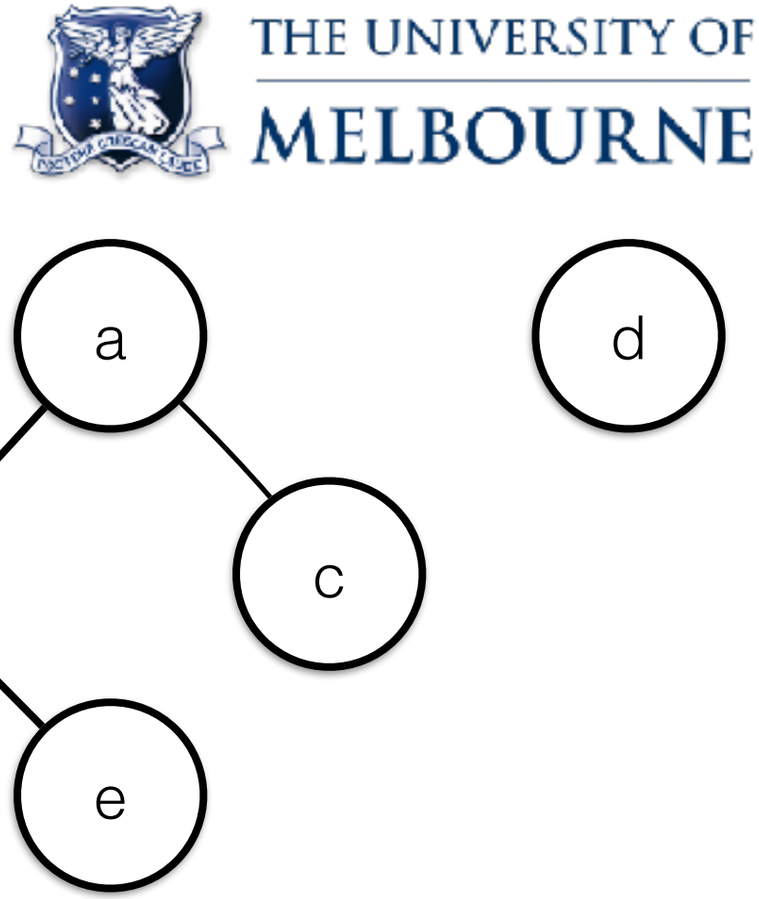
Depth-First Traversal: Recursive Implementation

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```

DFS(⟨V,E⟩)

Call Stack

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Depth-First Traversal: Recursive Implementation

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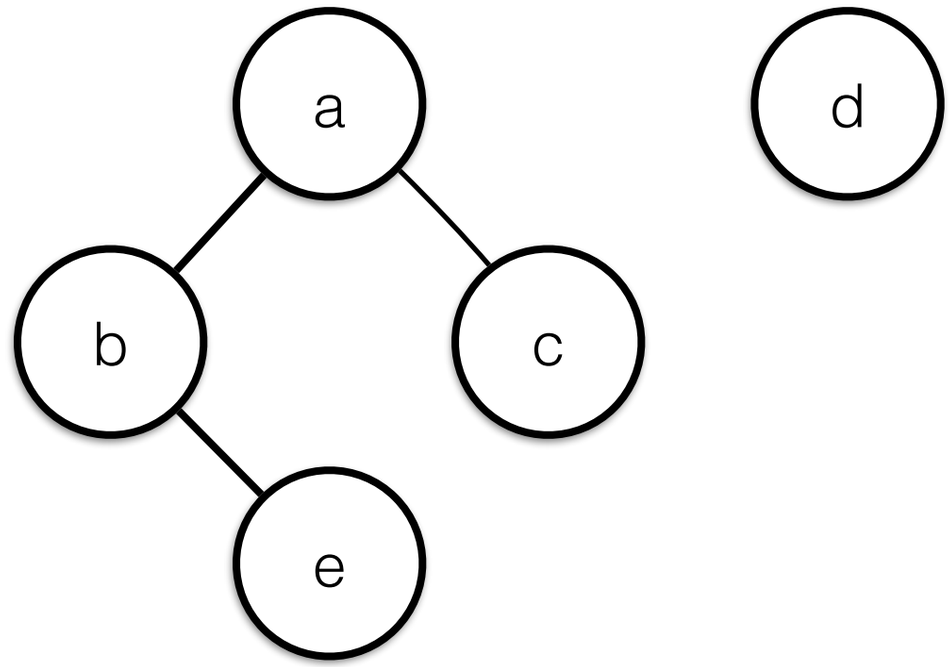


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function DfsExplore(v)
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    mark v with count
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            DfsExplore(w)
```

count:
0

DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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```
function DFS(⟨V, E⟩)
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```

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    mark v with count
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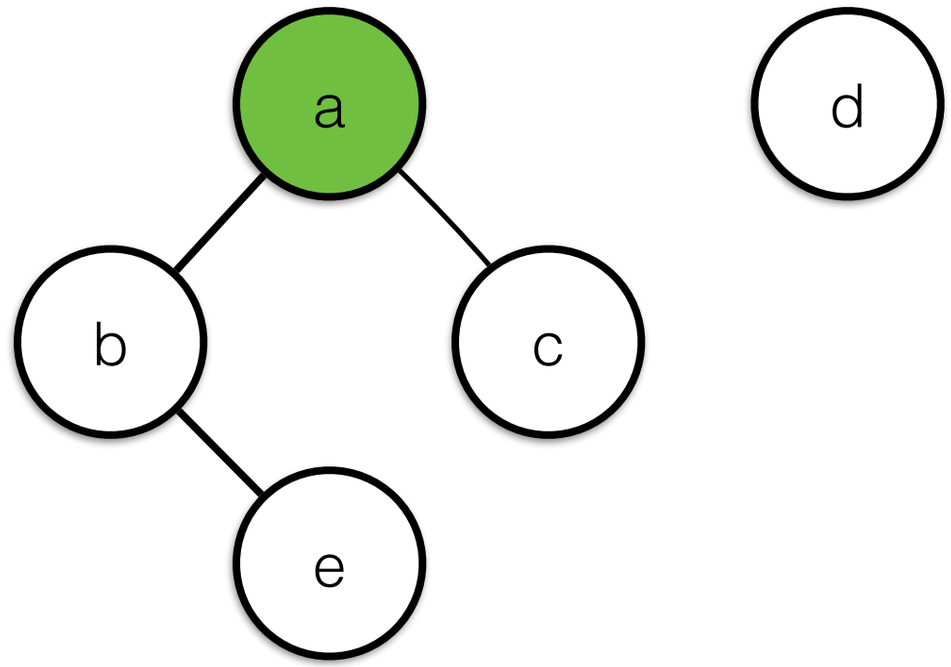
```



count:
0

DfsExplore(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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function DFS($\langle V, E \rangle$)
    mark each node in V with 0
    $count \leftarrow 0$
    for each v in V **do**
        if v is marked with 0 **then**
            DFSEXPLORE(v)

function DFSEXPLORE(v)
    $count \leftarrow count + 1$
    mark v with $count$
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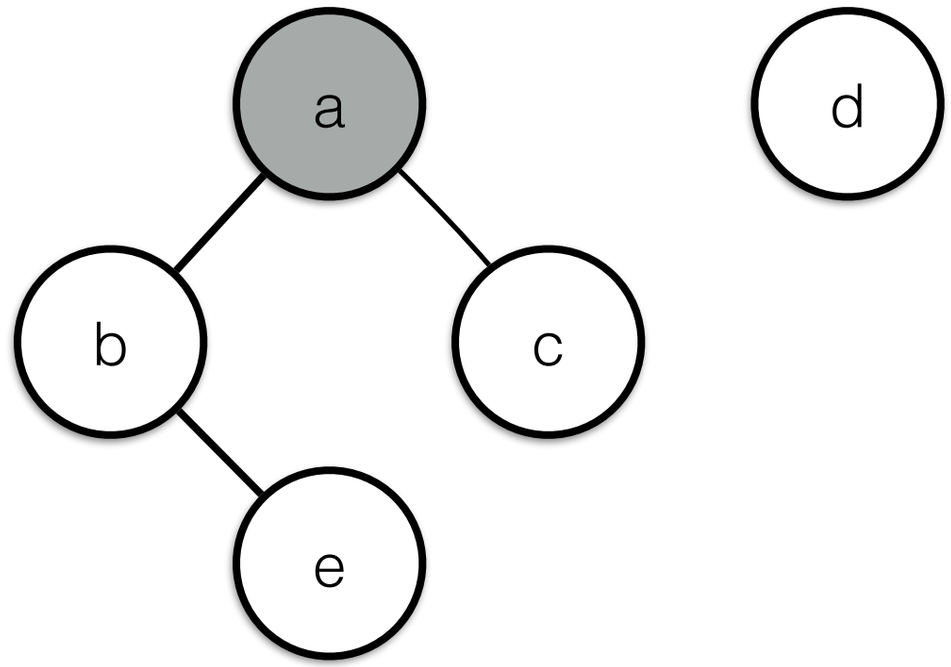

count:
1

DFSEXPLORE(a)
DFS($\langle$V,E$\rangle$)
Call Stack

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Depth-First Traversal: Recursive Implementation

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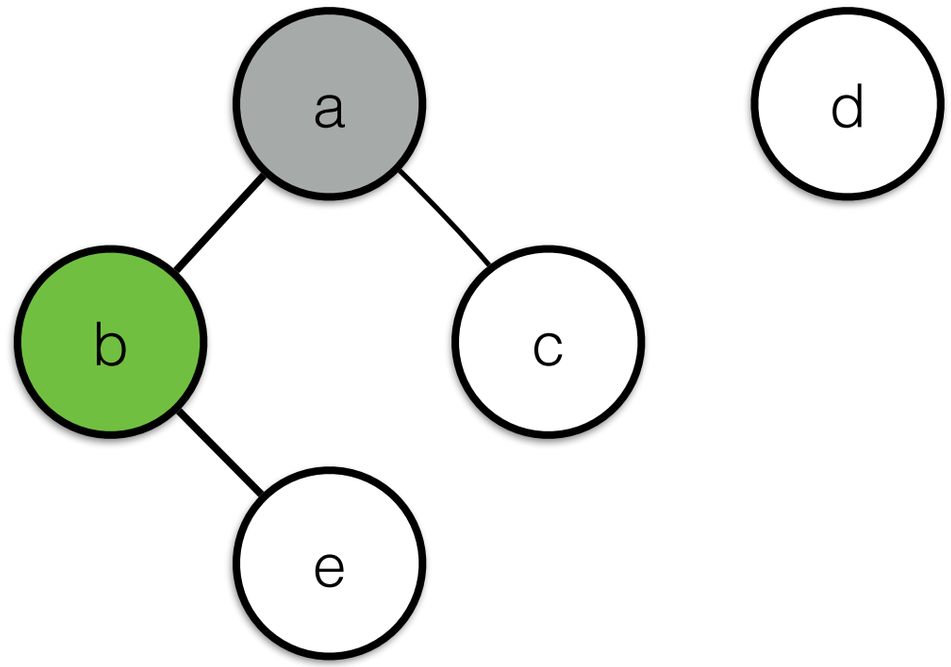

count:
1

DFSEXPLORE(b)
DFSEXPLORE(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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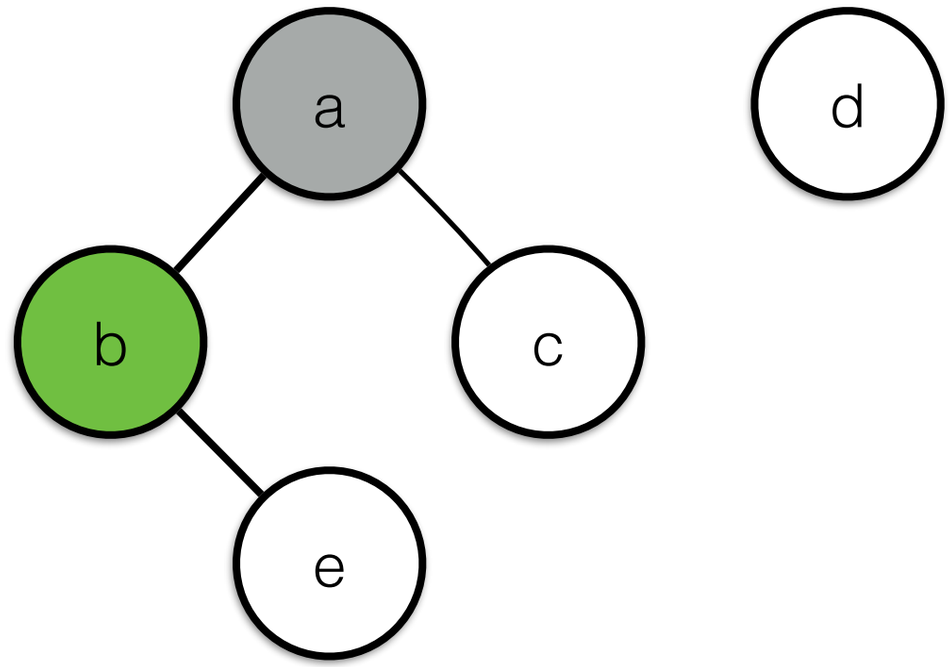

count:
2

DFSEXPLORE(b)
DFSEXPLORE(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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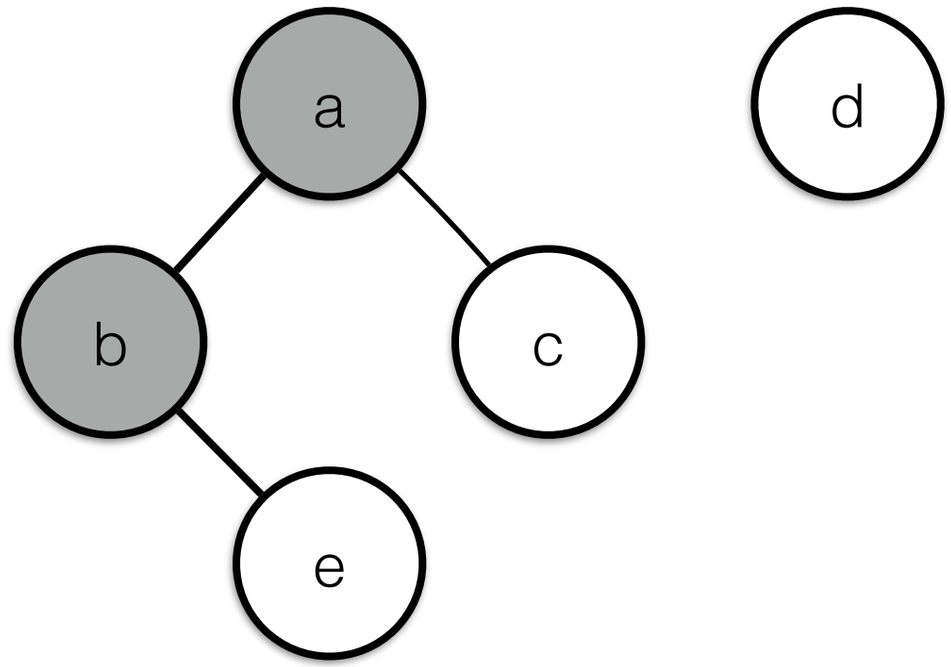

count:
2

DFSEXPLORE(b)
DFSEXPLORE(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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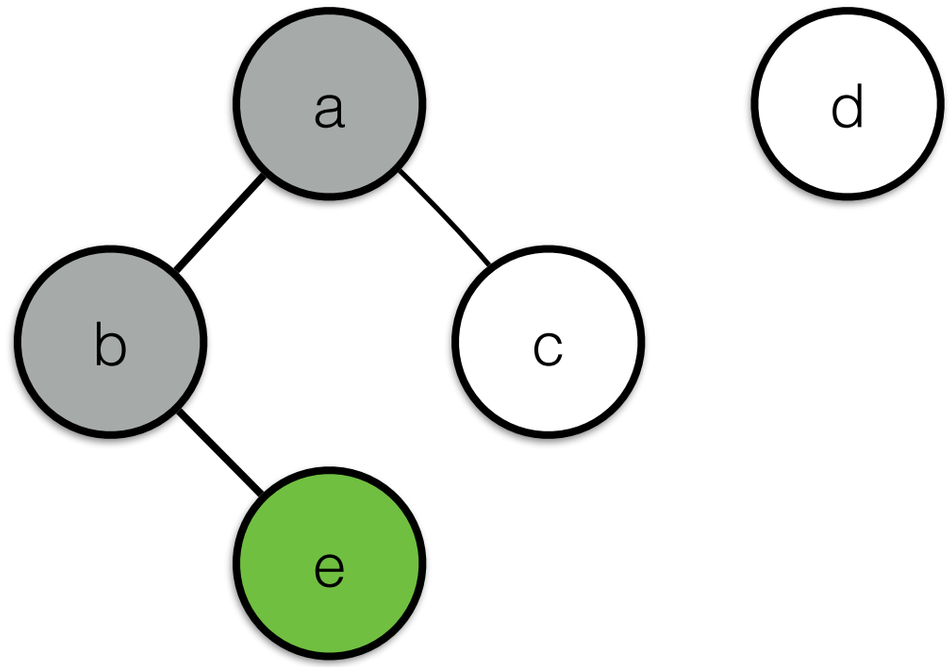

count:
2

DFSEXPLORE(e)
DFSEXPLORE(b)
DFSEXPLORE(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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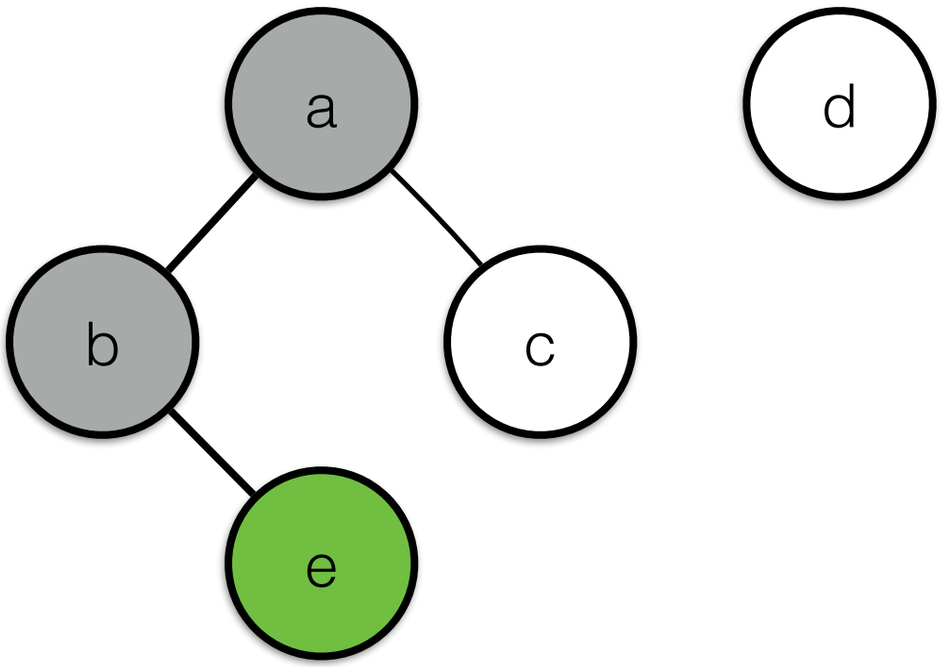

count:
3

DFSEXPLORE(e)
DFSEXPLORE(b)
DFSEXPLORE(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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    for each edge (v, w) do
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            DfsExplore(w)
```

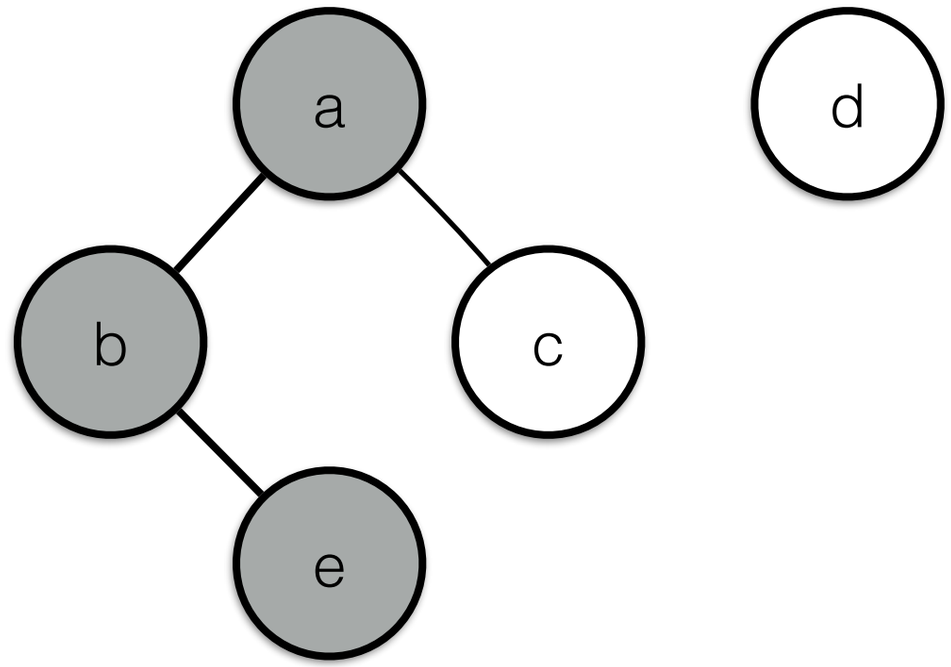

count:
3

DfsExplore(e)
DfsExplore(b)
DfsExplore(a)
DFS(⟨V,E⟩)

Call Stack

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Depth-First Traversal: Recursive Implementation

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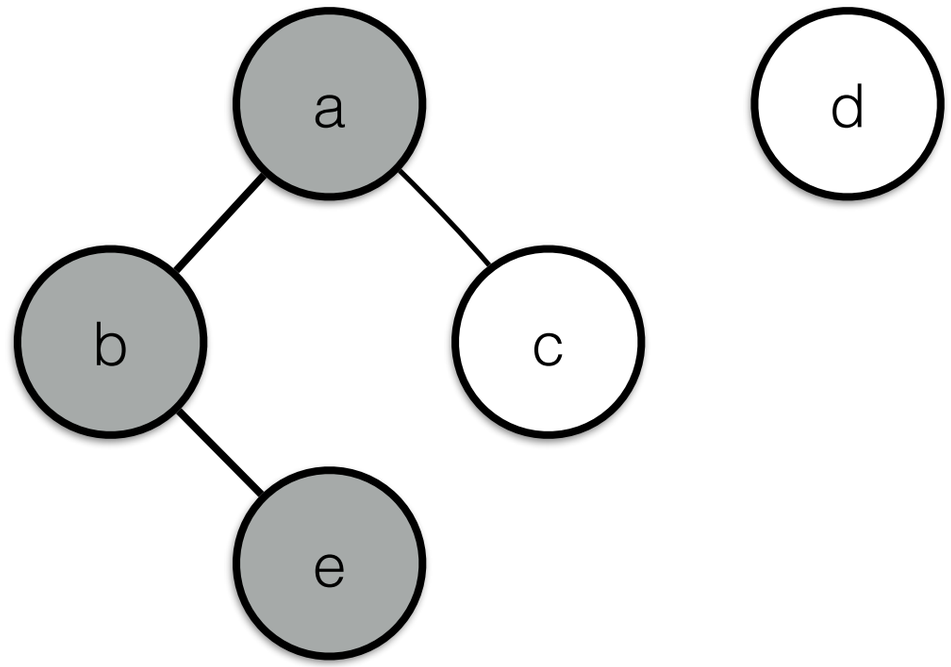
function DFS($\langle V, E \rangle$)
    mark each node in V with 0
    $count \leftarrow 0$
    for each v in V **do**
        if v is marked with 0 **then**
            DFSEXPLORE(v)

function DFSEXPLORE(v)
    $count \leftarrow count + 1$
    mark v with $count$
    for each edge (v, w) **do**
        if w is marked with 0 **then**
            DFSEXPLORE(w)



count:
3

DFSEXPLORE(b)
DFSEXPLORE(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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function DFS($\langle V, E \rangle$)
    mark each node in V with 0
    $count \leftarrow 0$
    for each v in V **do**
        if v is marked with 0 **then**
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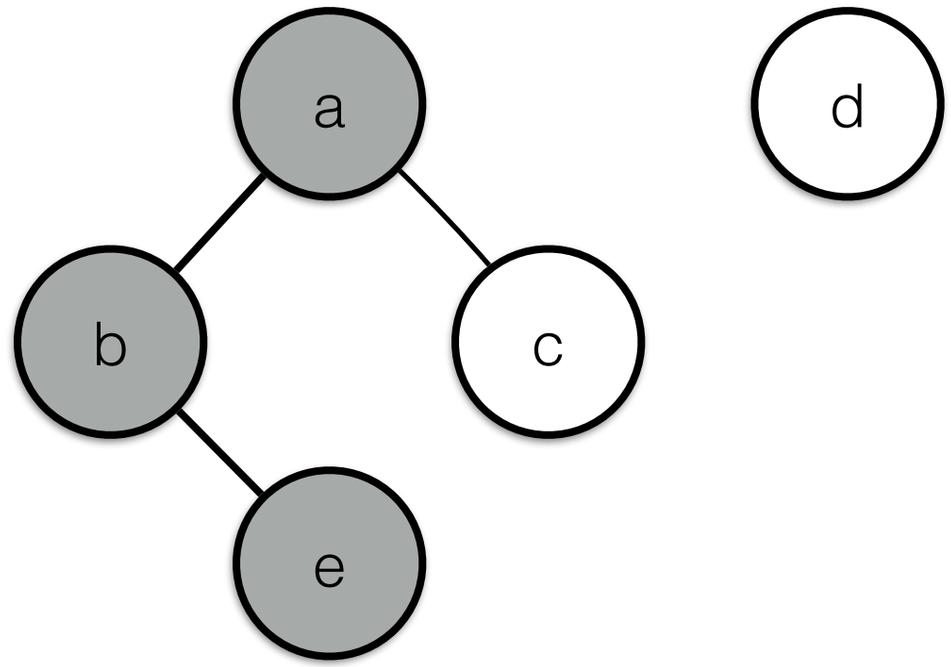
    $count \leftarrow count + 1$
    mark v with $count$
    for each edge (v, w) **do**
        if w is marked with 0 **then**
            DFSEXPLORE(w)



count:
3

DFSEXPLORE(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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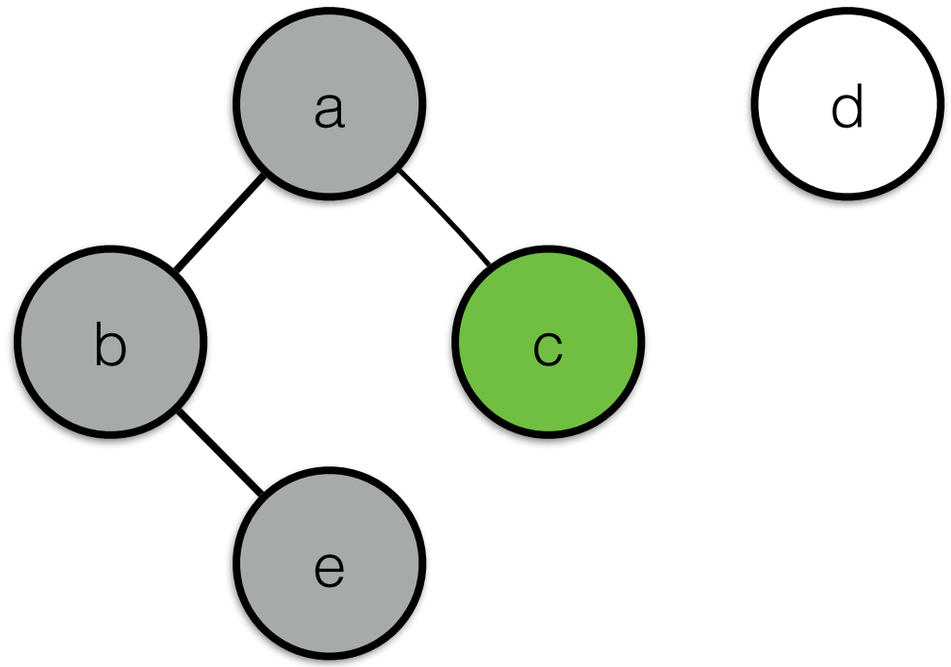
```
function DFS(⟨V, E⟩)
    mark each node in V with 0
    count ← 0
    for each v in V do
        if v is marked with 0 then
            DfsExplore(v)
```

```
function DfsExplore(v)
    count ← count + 1
    mark v with count
    for each edge (v, w) do
        if w is marked with 0 then
            DfsExplore(w)
```



count:
3

DfsExplore(c)
DfsExplore(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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function DFS(⟨V, E⟩)
    mark each node in V with 0
    count ← 0
    for each v in V do
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            DfsExplore(v)
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    for each edge (v, w) do
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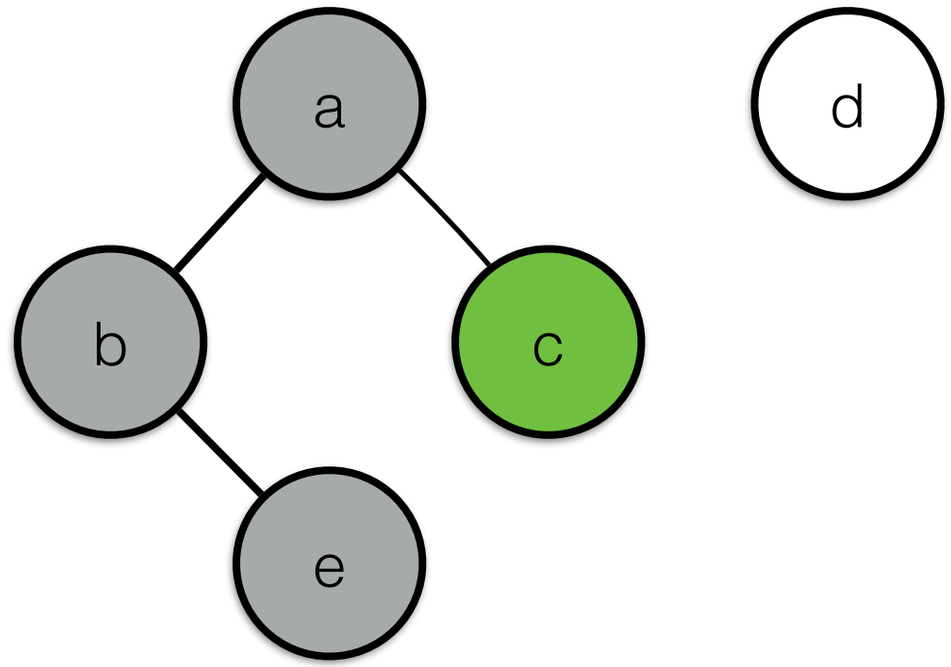

count:
4

DfsExplore(c)
DfsExplore(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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function DFS($\langle V, E \rangle$)
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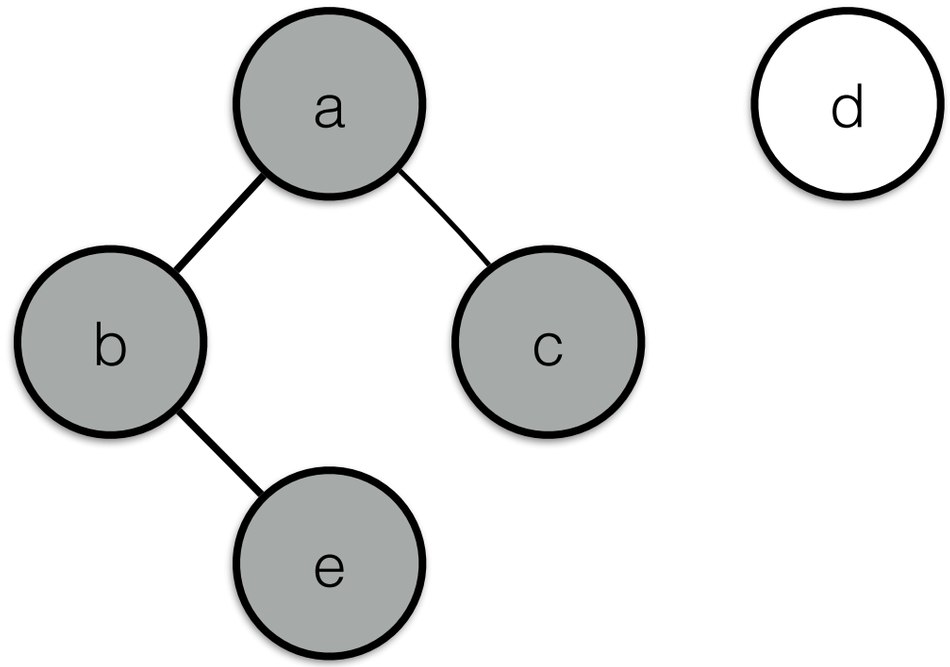

count:
4

DfsExplore(c)
DfsExplore(a)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

THE UNIVERSITY OF MELBOURNE

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    mark v with $count$
    for each edge (v, w) **do**
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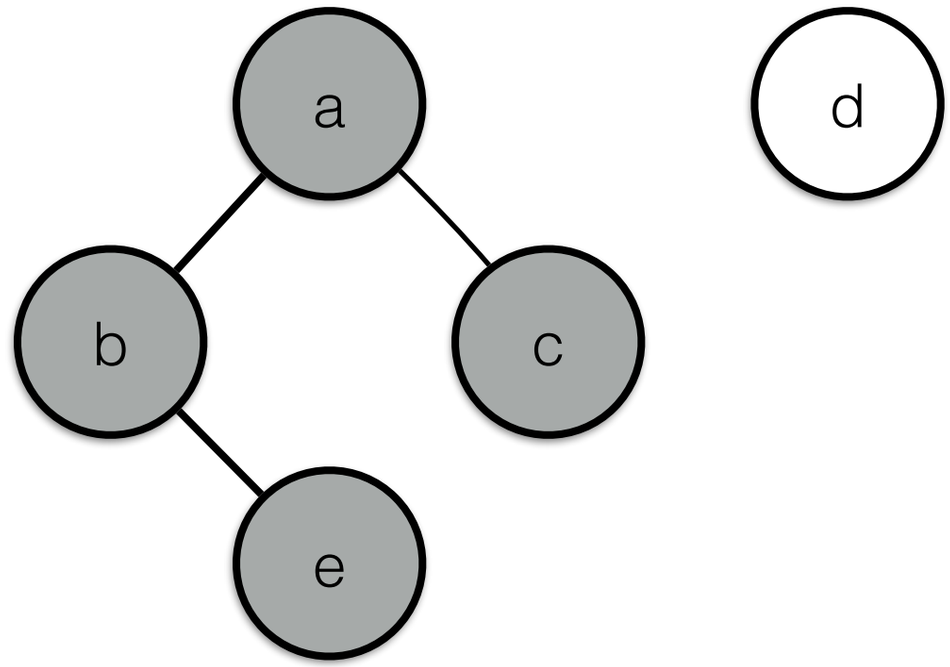
            DFSEXPLORE(w)



count:
4

DFSEXPLORE(a)
DFS(⟨V,E⟩)
Call Stack

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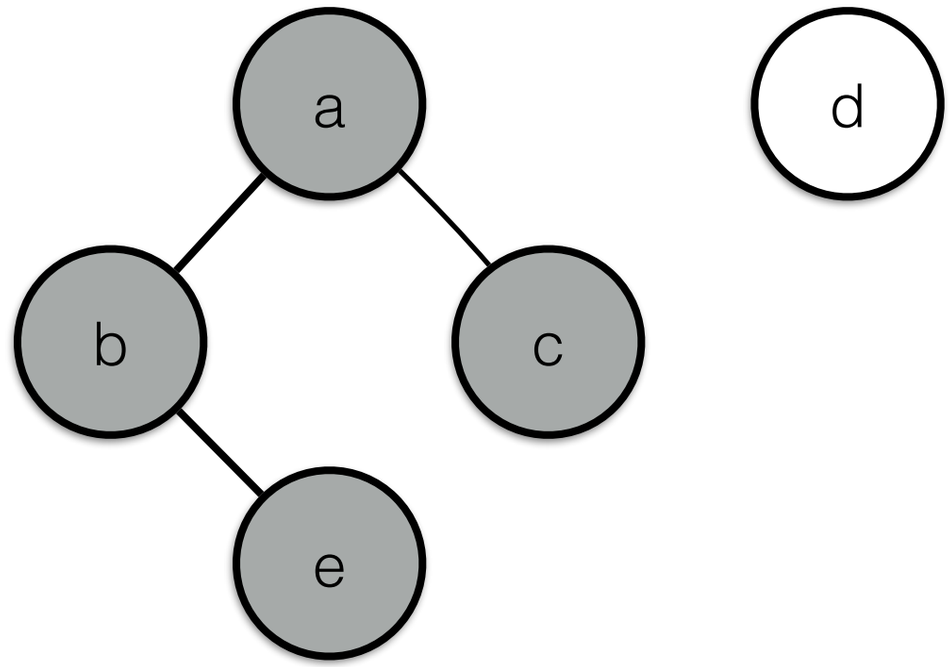
Depth-First Traversal: Recursive Implementation

THE UNIVERSITY OF MELBOURNE

function DFS($\langle V, E \rangle$)
    mark each node in V with 0
    $count \leftarrow 0$
    for each v in V **do**
        if v is marked with 0 **then**
            DfsExplore(v)

function DfsExplore(v)
    $count \leftarrow count + 1$
    mark v with $count$
    for each edge (v, w) **do**
        if w is marked with 0 **then**
            DfsExplore(w)



count:
4

DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

THE UNIVERSITY OF MELBOURNE

function DFS($\langle V, E \rangle$)
    mark each node in V with 0
    $count \leftarrow 0$
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        if v is marked with 0 **then**
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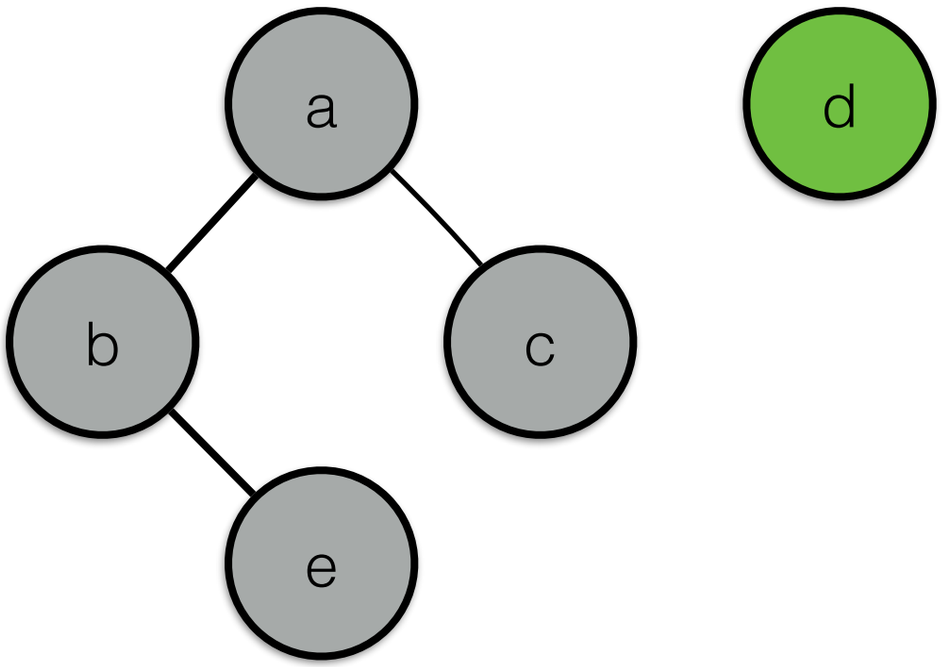
function DFSEXPLORE(v)
    $count \leftarrow count + 1$
    mark v with $count$
    for each edge (v, w) **do**
        if w is marked with 0 **then**
            DFSEXPLORE(w)



count:
4

DFSEXPLORE(d)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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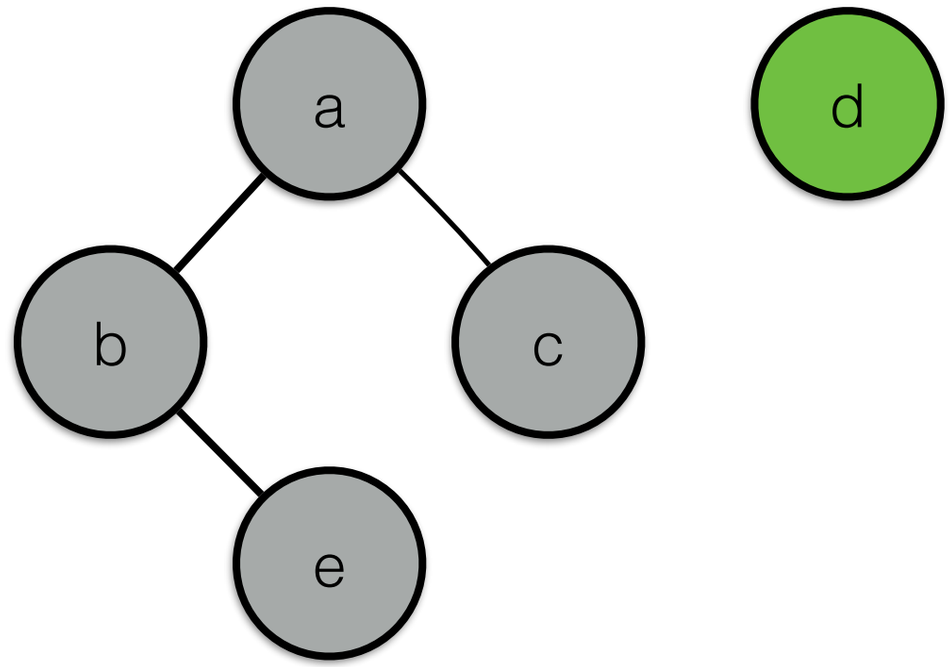
```
function DFS(⟨V, E⟩)
    mark each node in V with 0
    count ← 0
    for each v in V do
        if v is marked with 0 then
            DfsExplore(v)
```

```
function DfsExplore(v)
    count ← count + 1
    mark v with count
    for each edge (v, w) do
        if w is marked with 0 then
            DfsExplore(w)
```



count:
5

DfsExplore(d)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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function DFS($\langle V, E \rangle$)
    mark each node in V with 0
    $count \leftarrow 0$
    for each v in V **do**
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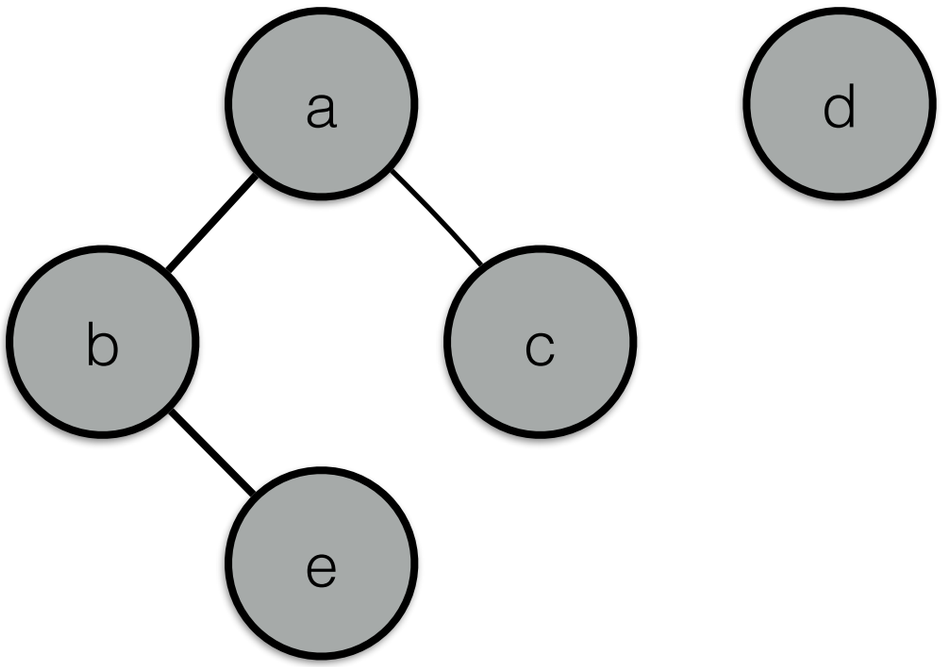
            DFSEXPLORE(v)

function DFSEXPLORE(v)
    $count \leftarrow count + 1$
    mark v with $count$
    for each edge (v, w) **do**
        if w is marked with 0 **then**
            DFSEXPLORE(w)



count:
5

DFSEXPLORE(d)
DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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```
function DFS(⟨V, E⟩)
    mark each node in V with 0
    count ← 0
    for each v in V do
        if v is marked with 0 then
            DfsExplore(v)
```

```
function DfsExplore(v)
    count ← count + 1
    mark v with count
    for each edge (v, w) do
        if w is marked with 0 then
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```

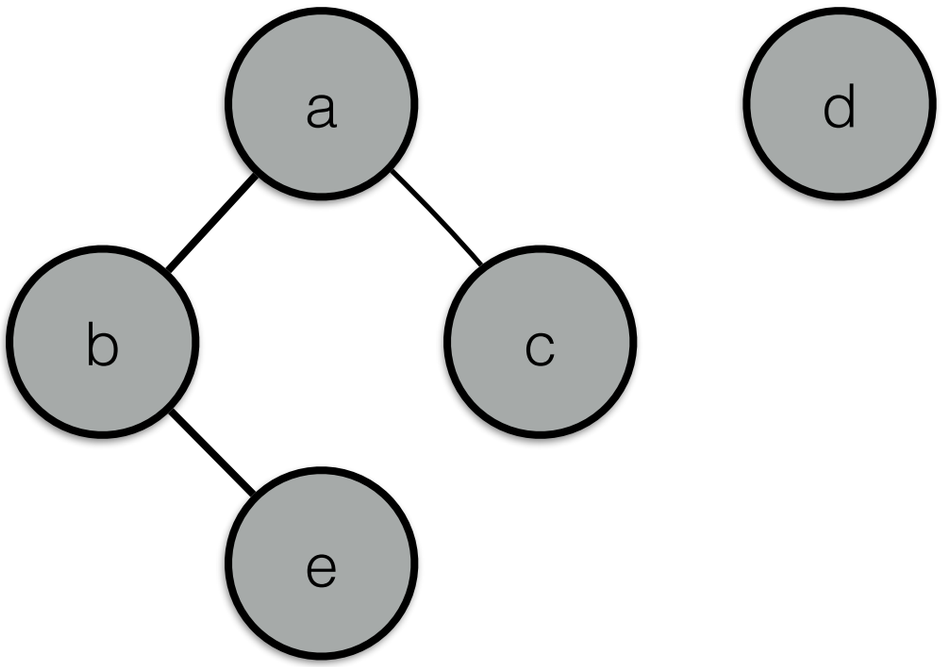

count:
5

DFS(⟨V,E⟩)
Call Stack

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Depth-First Traversal: Recursive Implementation

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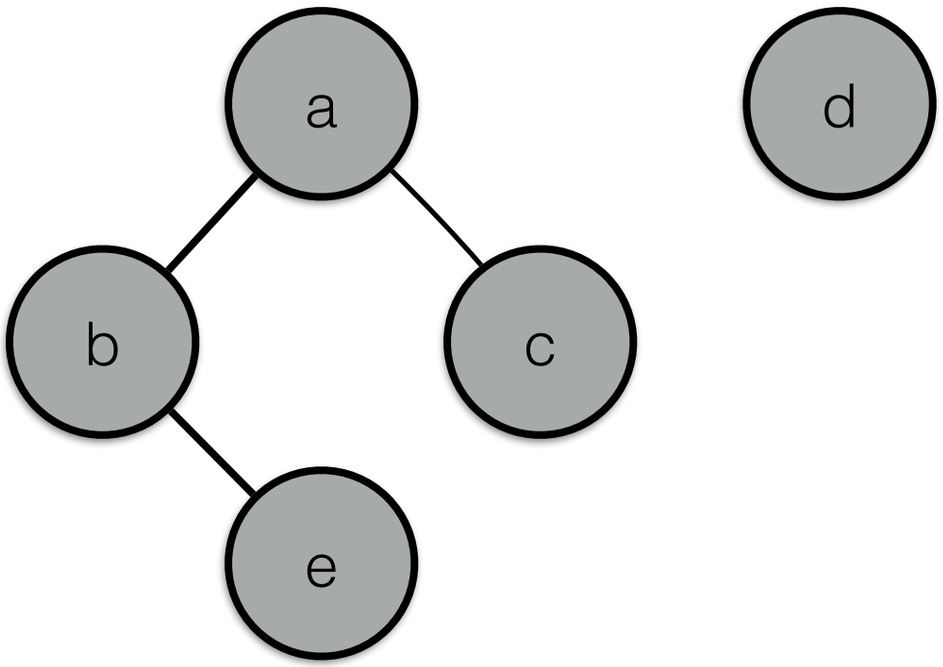
function $\mathrm{DFS}(\langle V, E \rangle)$
    mark each node in V with 0
    $count \leftarrow 0$
    for each v in V **do**
        if v is marked with 0 **then**
            $\textsc{DfsExplore}(v)$

function $\textsc{DfsExplore}(v)$
    $count \leftarrow count + 1$
    mark v with $count$
    for each edge (v, w) **do**
        if w is marked with 0 **then**
            $\textsc{DfsExplore}(w)$



count:
5

Call Stack

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Depth-First Search: Recursive Algorithm Notes

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- Works both for directed and undirected graphs.

- The “marking” of nodes is usually done by maintaining a separate array, **`mark`**, indexed by V .

- For example, when we wrote “mark v with count”, that would be implemented as “**`mark[v] := count`**”.

- How to find the nodes adjacent to v depends on the graph representation used.

- Using an adjacency **matrix** **`adj`**, we need to consider **`adj[v,w]`** for each w in V . Here the complexity of graph traversal is $\Theta(|V|^2)$.

- Using adjacency **lists**, for each v , we traverse the list **`adj[v]`**. In this case, the complexity of traversal is $\Theta(|V| + |E|)$.

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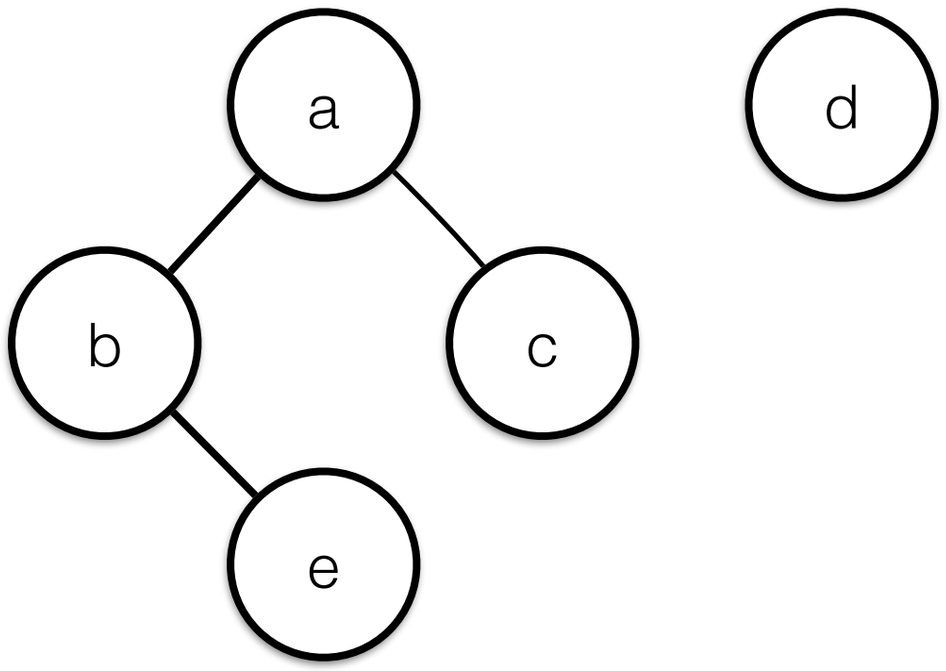
Applications of Depth-First Search (DFS)

- Easy to adapt DFS to decide if a graph is connected.

- How?



function DFS($\langle V, E \rangle$)
  mark each node in V with 0
  $count \leftarrow 0$
  for each v in V **do**
    if v is marked with 0 **then**
      DFSEXPLORE(v)

function DFSEXPLORE(v)
  $count \leftarrow count + 1$
  mark v with $count$
  for each edge (v, w) **do**
    if w is marked with 0 **then**
      DFSEXPLORE(w)

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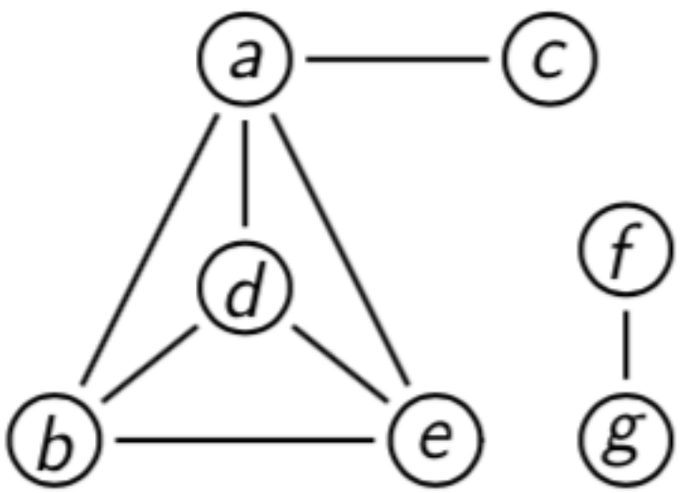
Depth-First Search: Node Orderings

- We can order nodes **either** by the order in which they get **pushed onto** the stack, or by the order in which they are **popped from** the stack



Levitin's compact stack notation

$e_{4,1}$
$d_{3,2}$
$b_{2,3}$ $c_{5,4}$ $g_{7,6}$
$a_{1,5}$ $f_{6,7}$

The first subscripts give the order in which nodes are pushed, the second the order in which they are popped off the stack.

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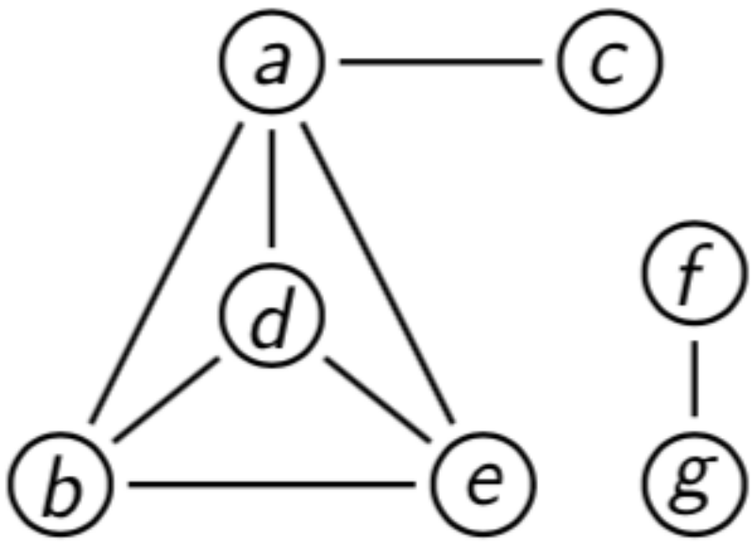
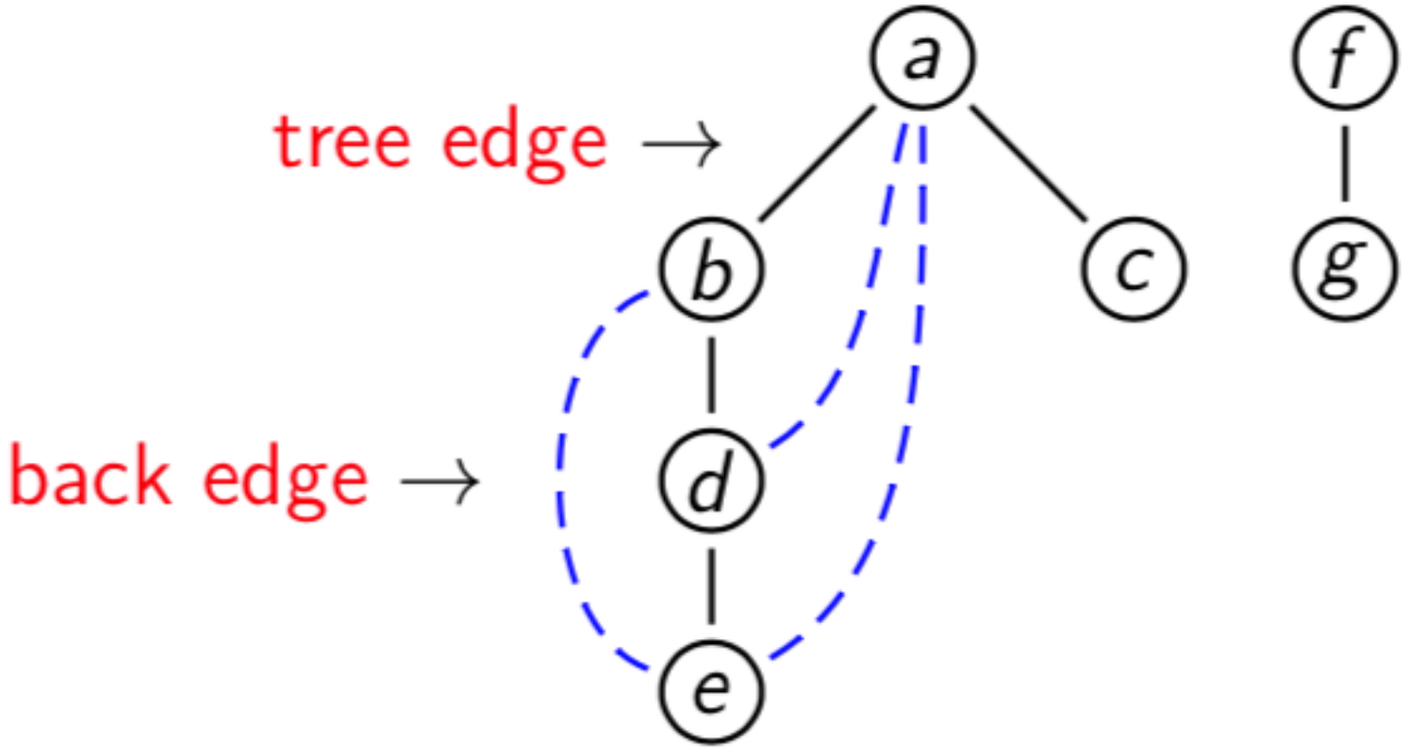
Depth-First Search Forest



DFS can be depicted by the resulting **DFS Forest**
(DFS **Tree** for a connected graph)



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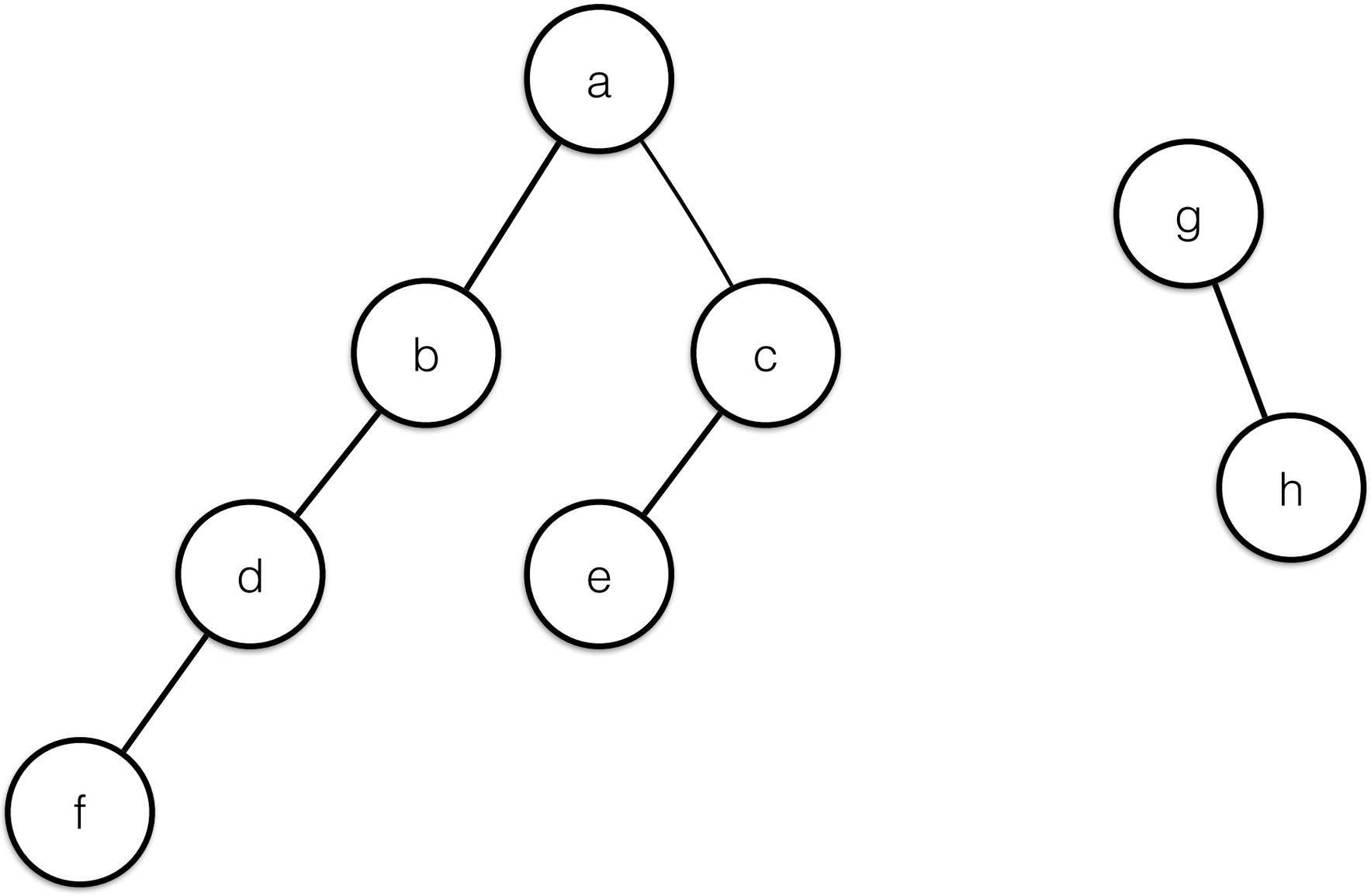
Breadth-First Traversal



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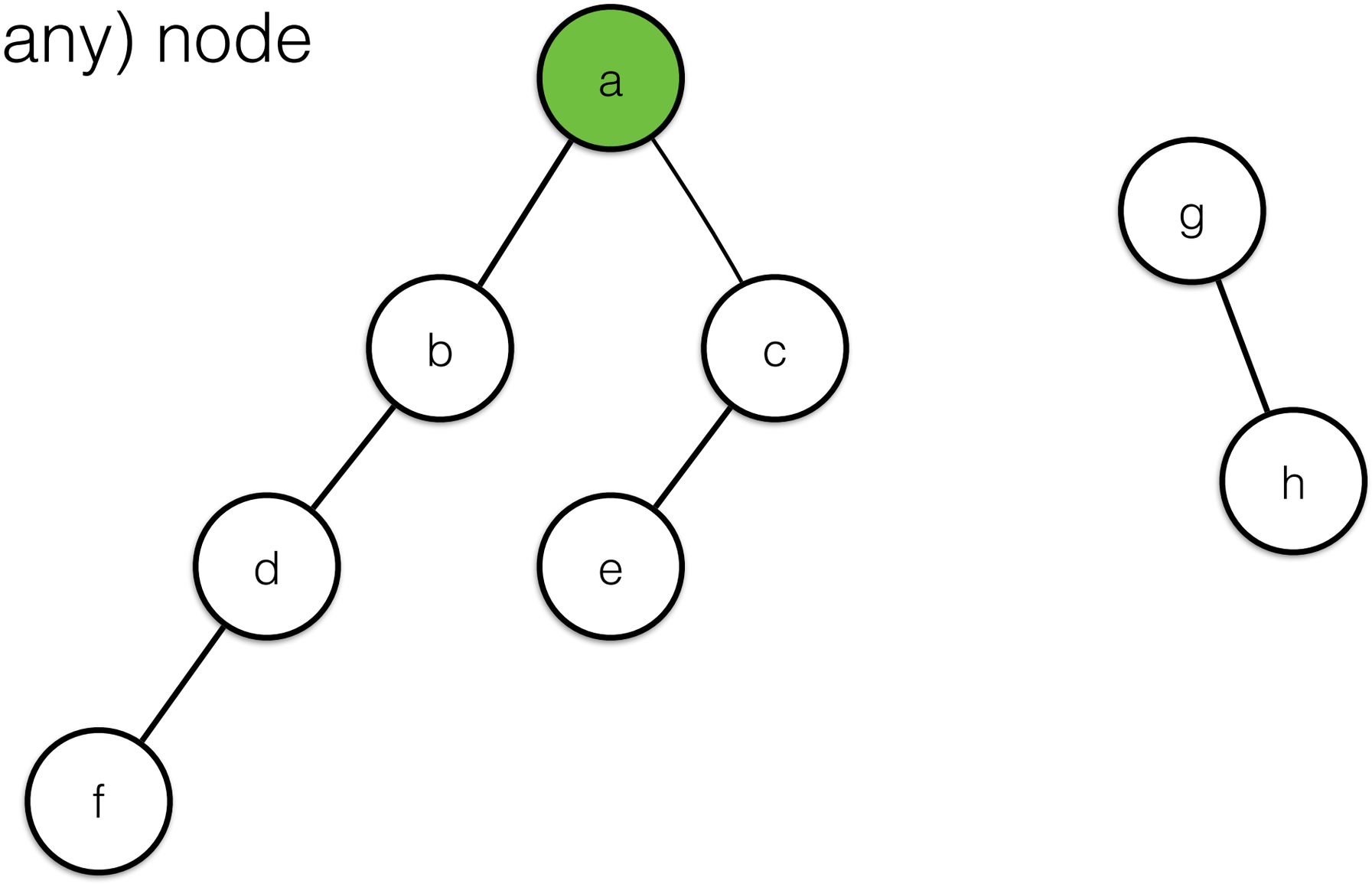
Breadth-First Traversal

Nodes visited in this order: a

Start with (any) node



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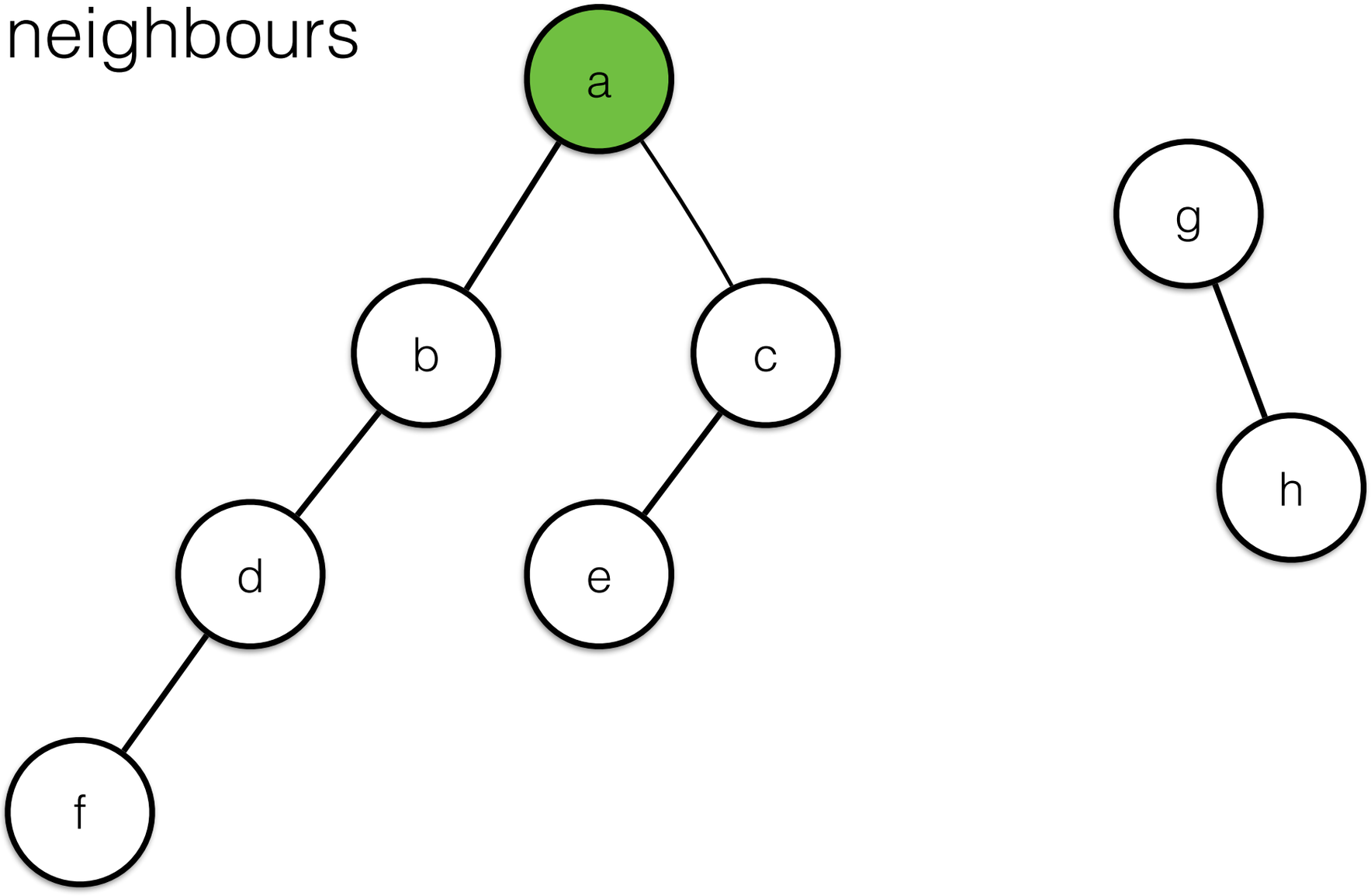
Breadth-First Traversal

Nodes visited in this order: a

Visit all of its neighbours



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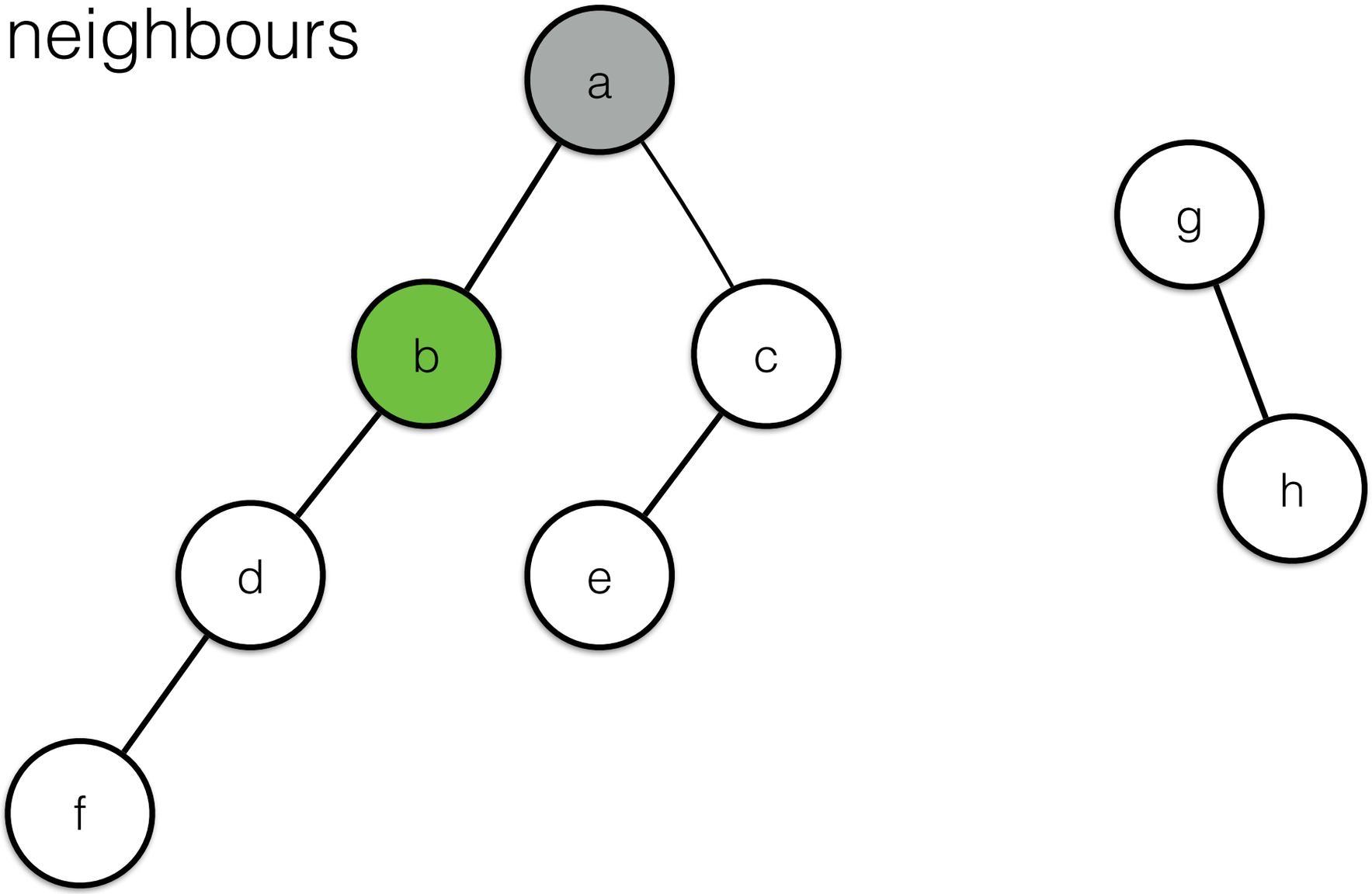
Breadth-First Traversal

Nodes visited in this order: a b

Visit all of its neighbours



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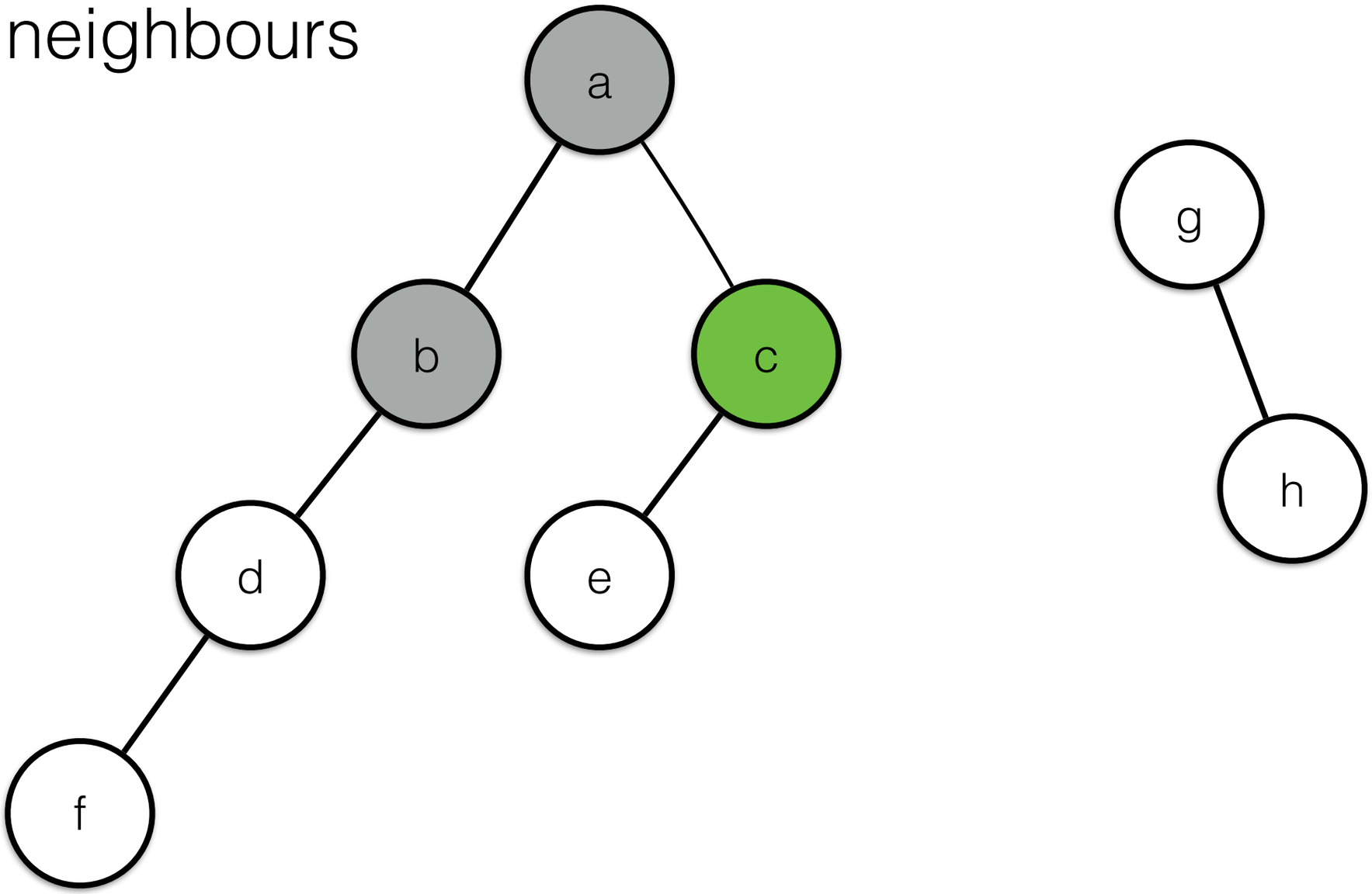
Breadth-First Traversal

Nodes visited in this order: a b c

Visit all of its neighbours



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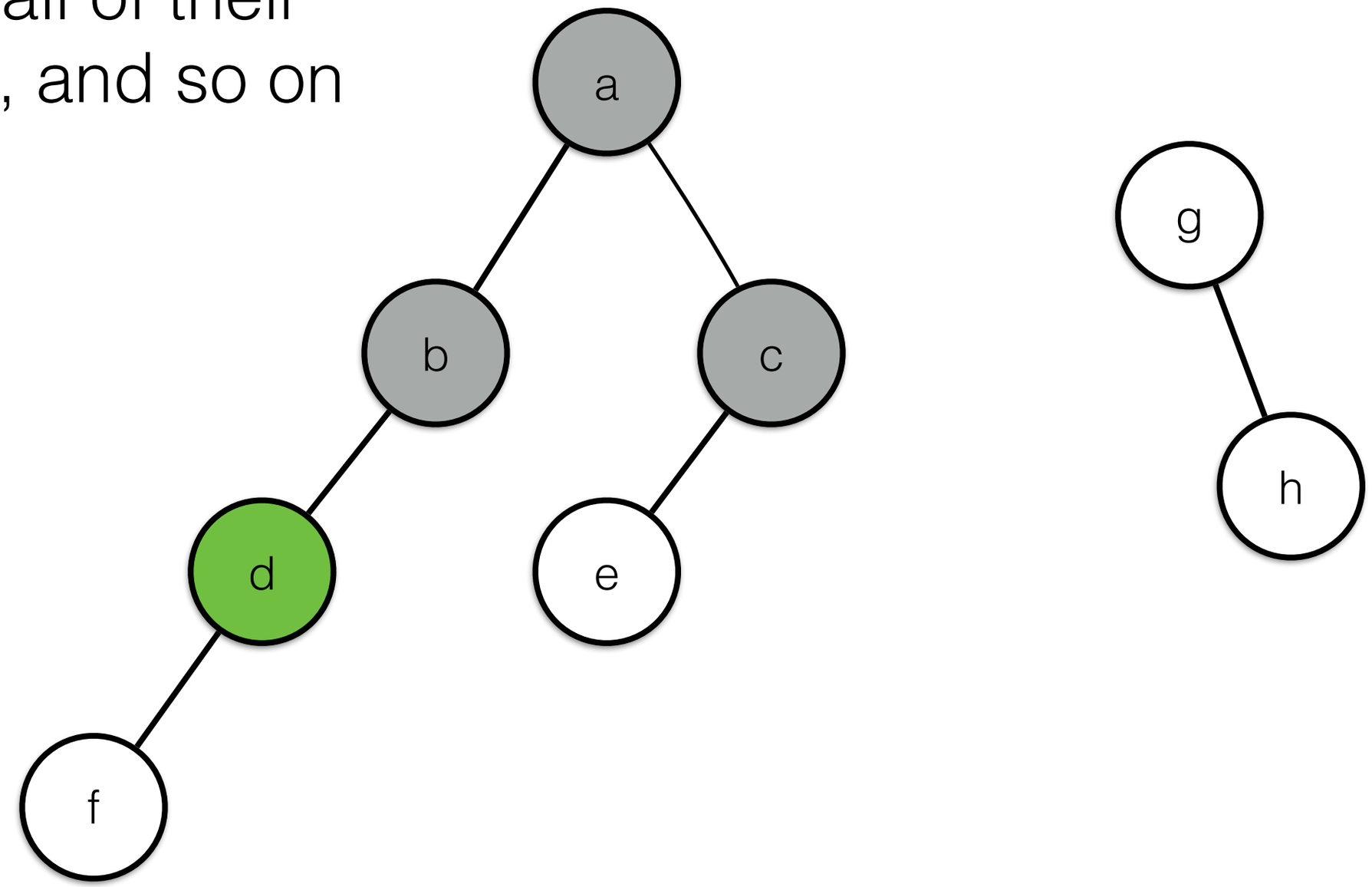
Breadth-First Traversal

Nodes visited in this order: a b c d

Then visit all of their neighbours, and so on



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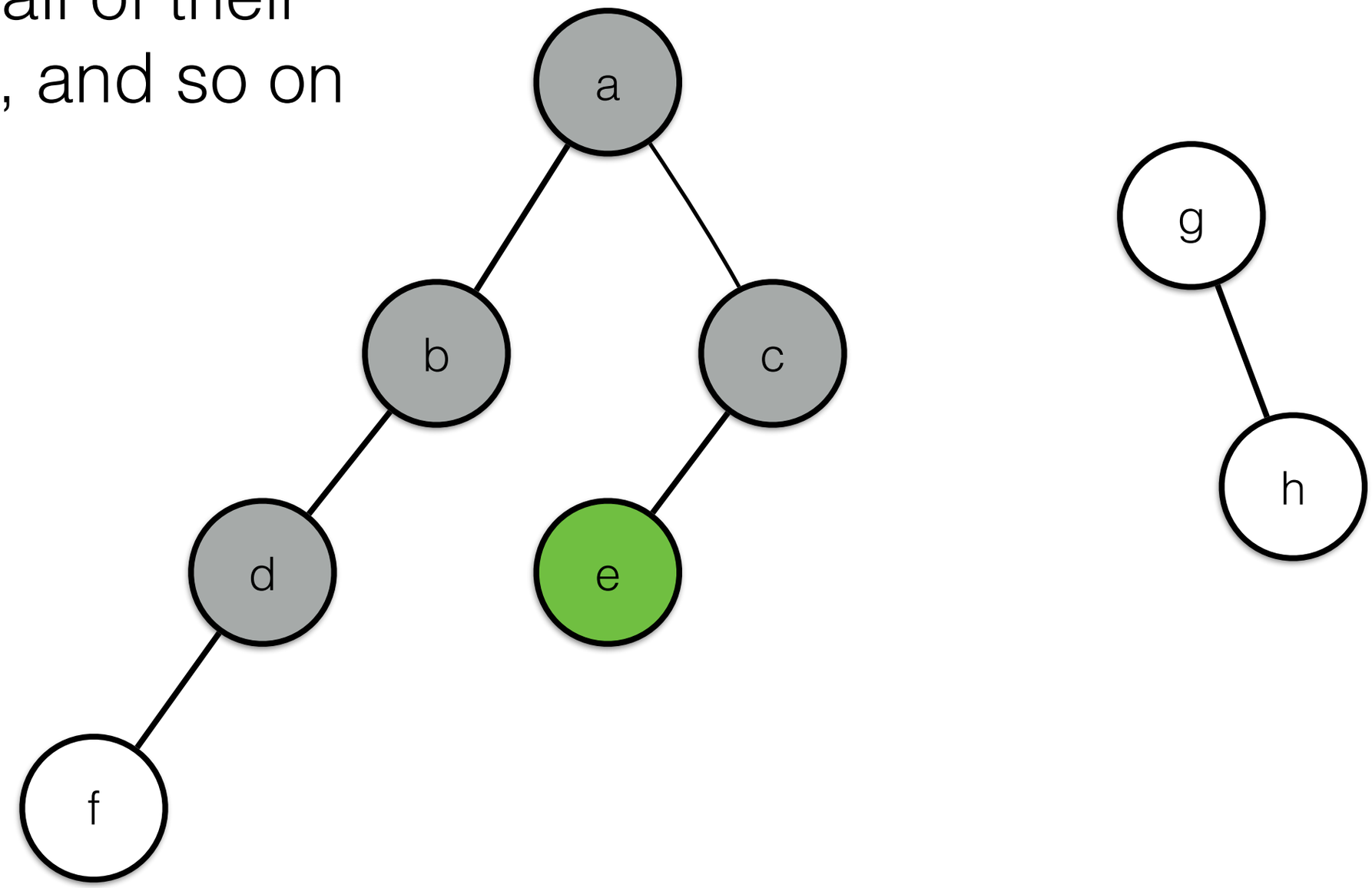
Breadth-First Traversal

Nodes visited in this order: a b c d e

Then visit all of their neighbours, and so on



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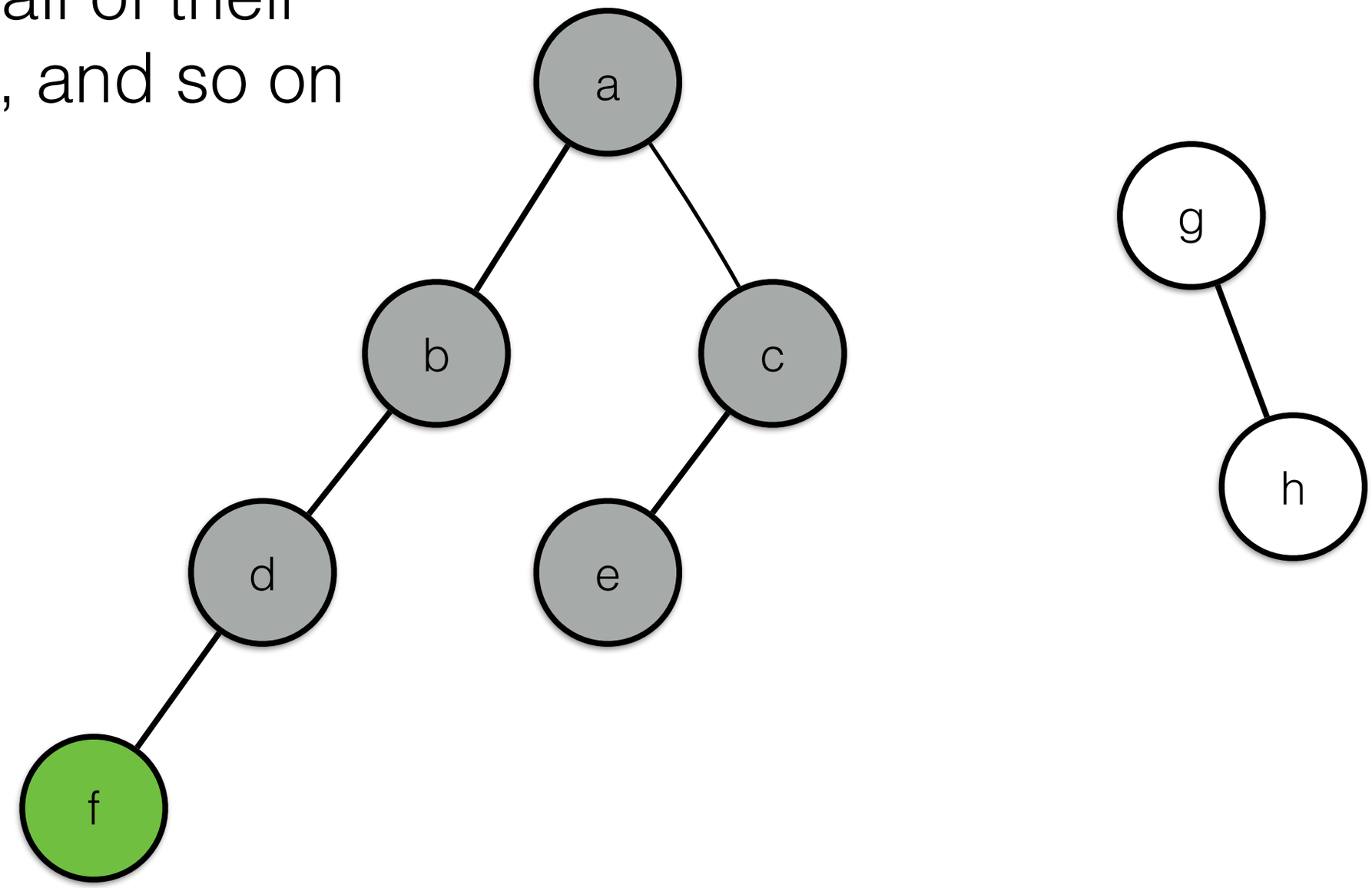
Breadth-First Traversal

Nodes visited in this order: a b c d e f

Then visit all of their neighbours, and so on



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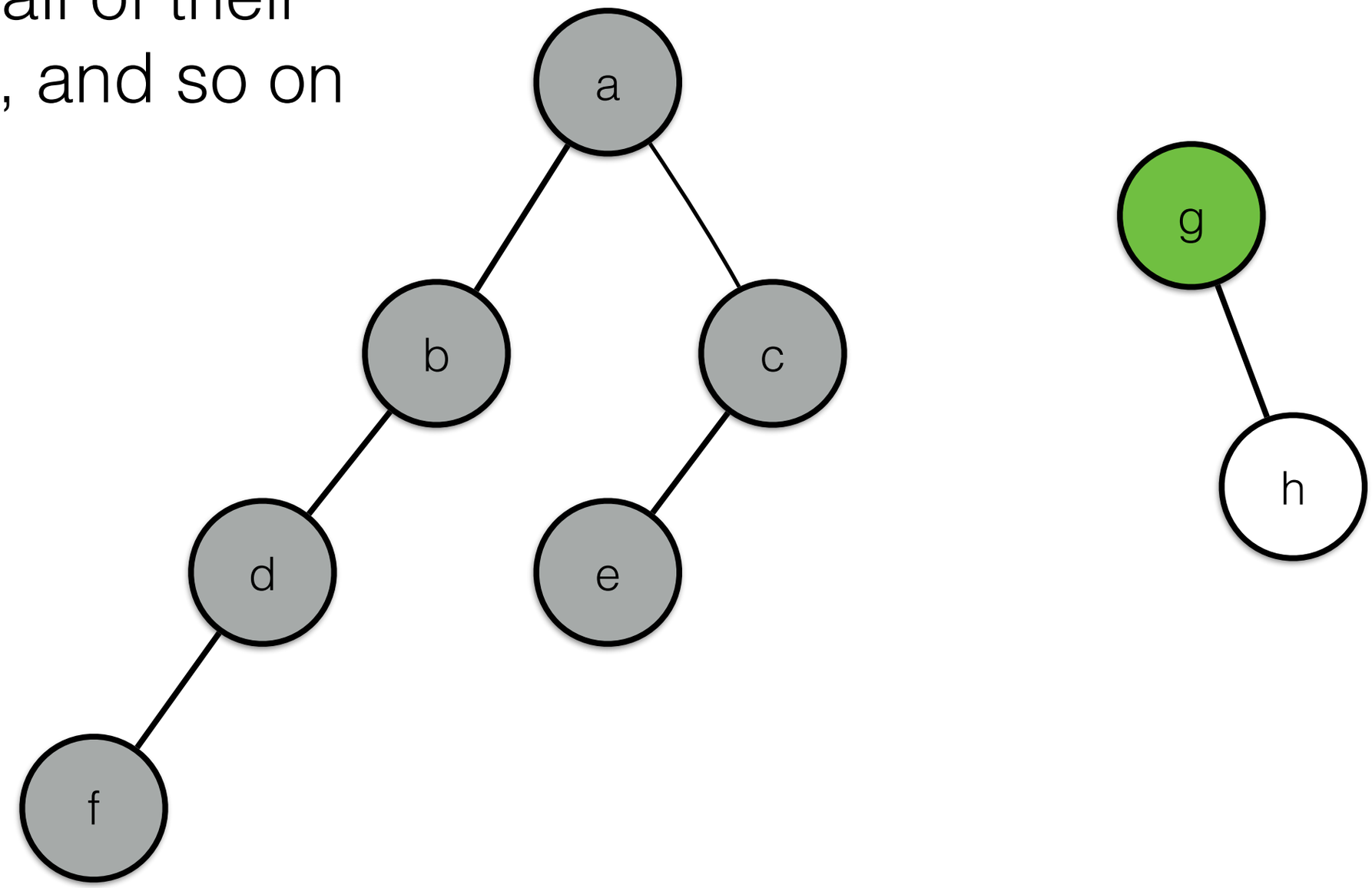
Breadth-First Traversal

Nodes visited in this order: a b c d e f g

Then visit all of their neighbours, and so on



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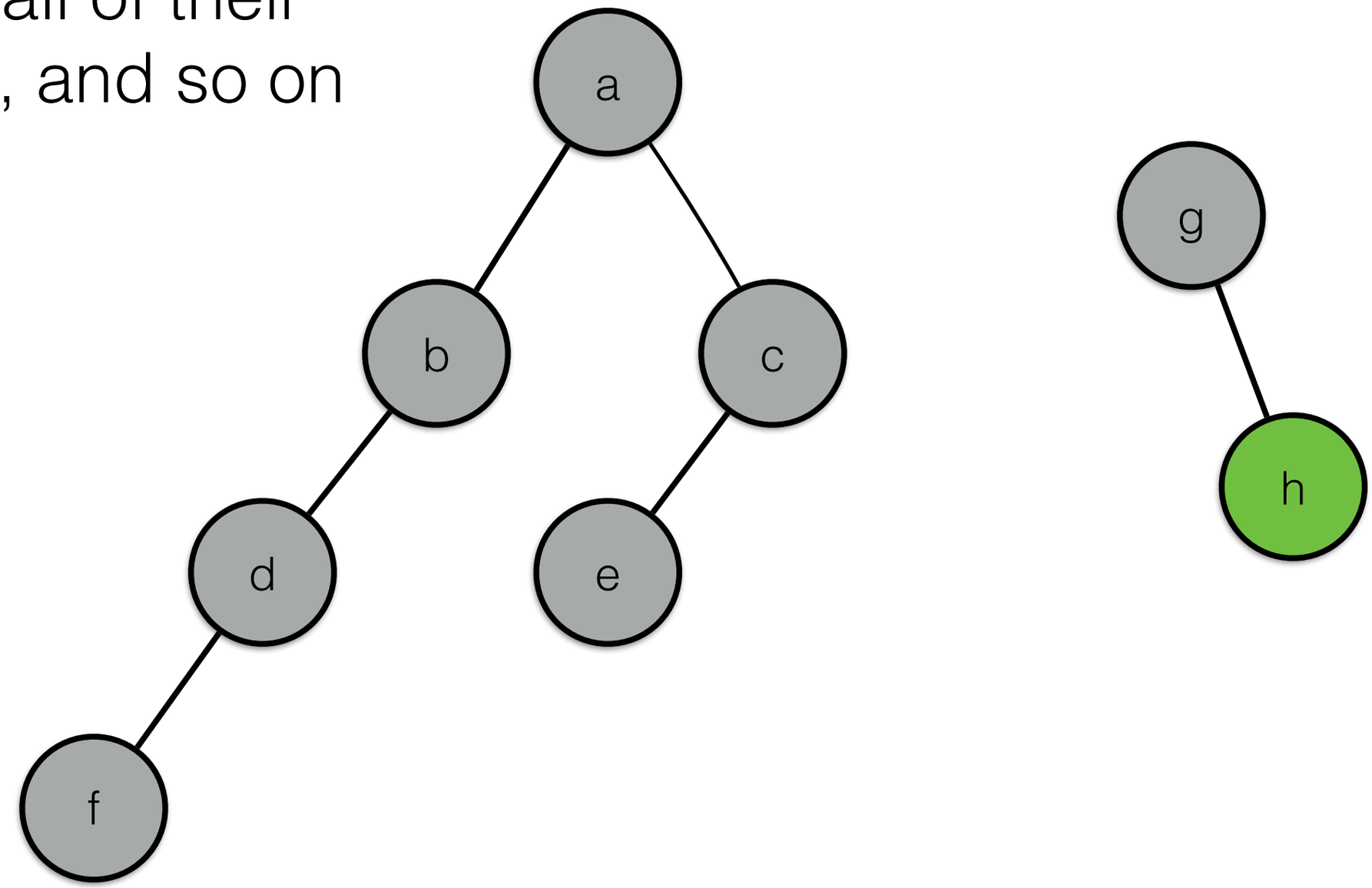
Breadth-First Traversal

Nodes visited in this order: a b c d e f g h

Then visit all of their neighbours, and so on



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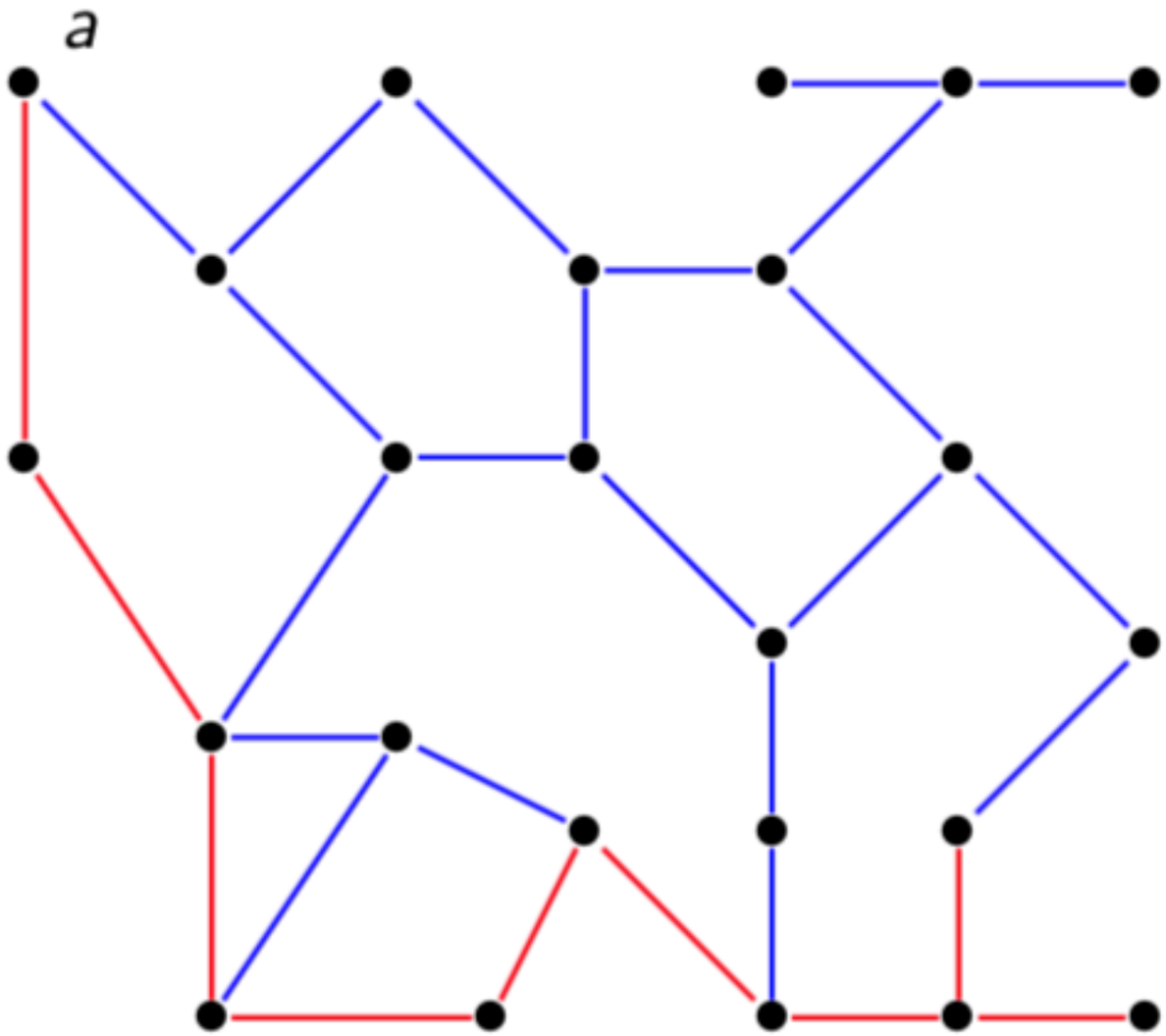
Depth-First vs Breadth-First Search



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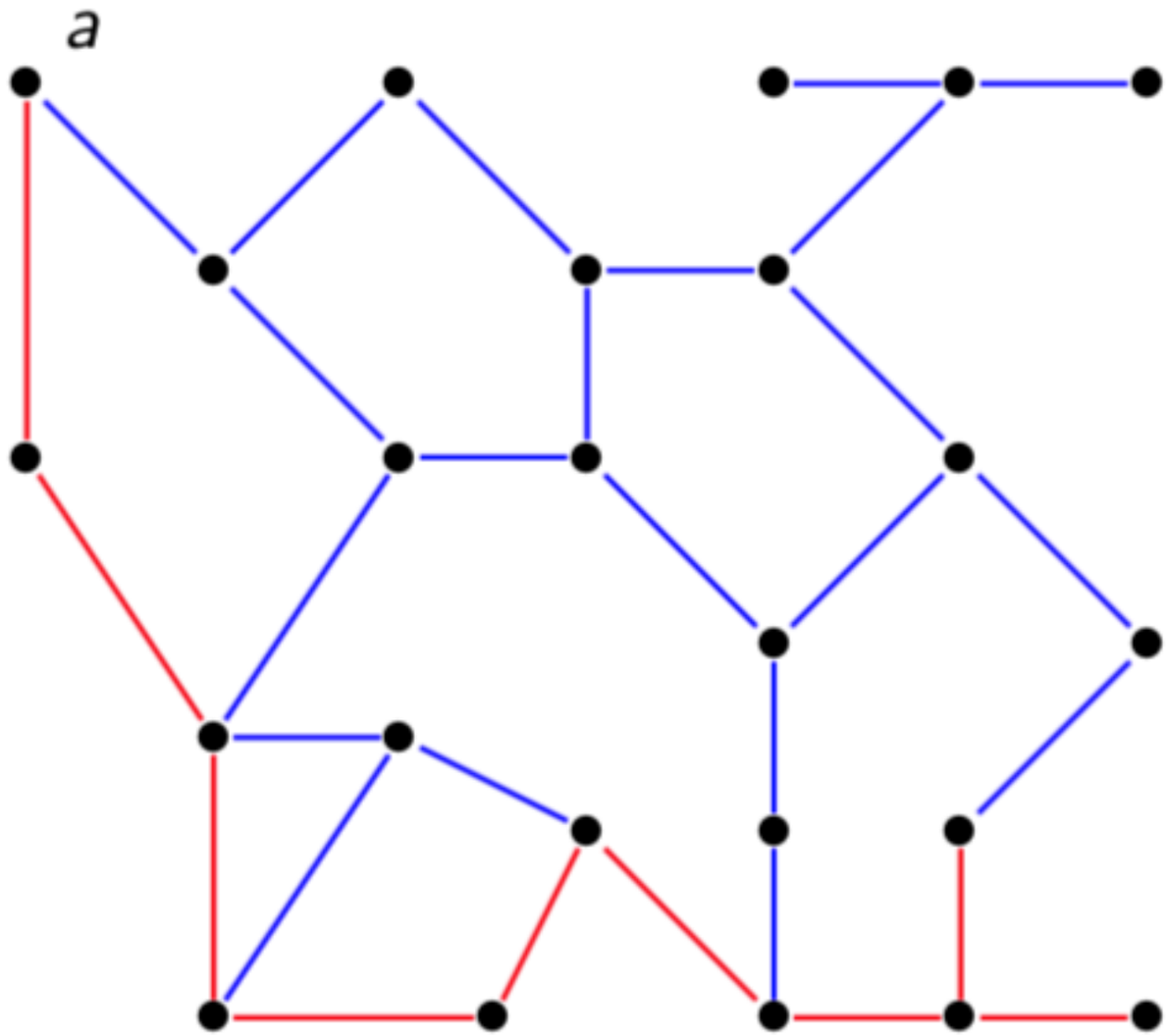
Depth-First vs Breadth-First Search



Typical Depth-First Search

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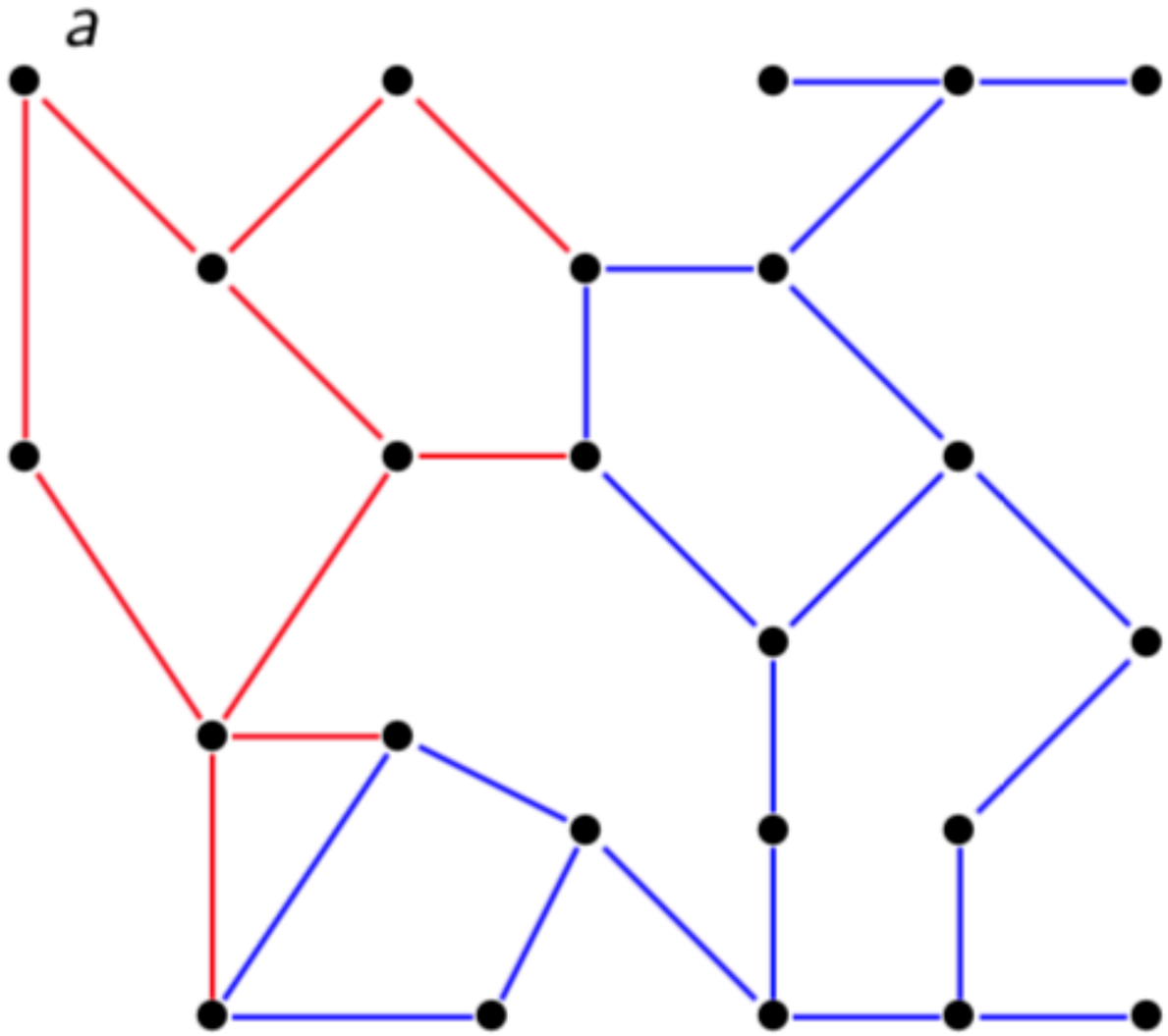
Depth-First vs Breadth-First Search



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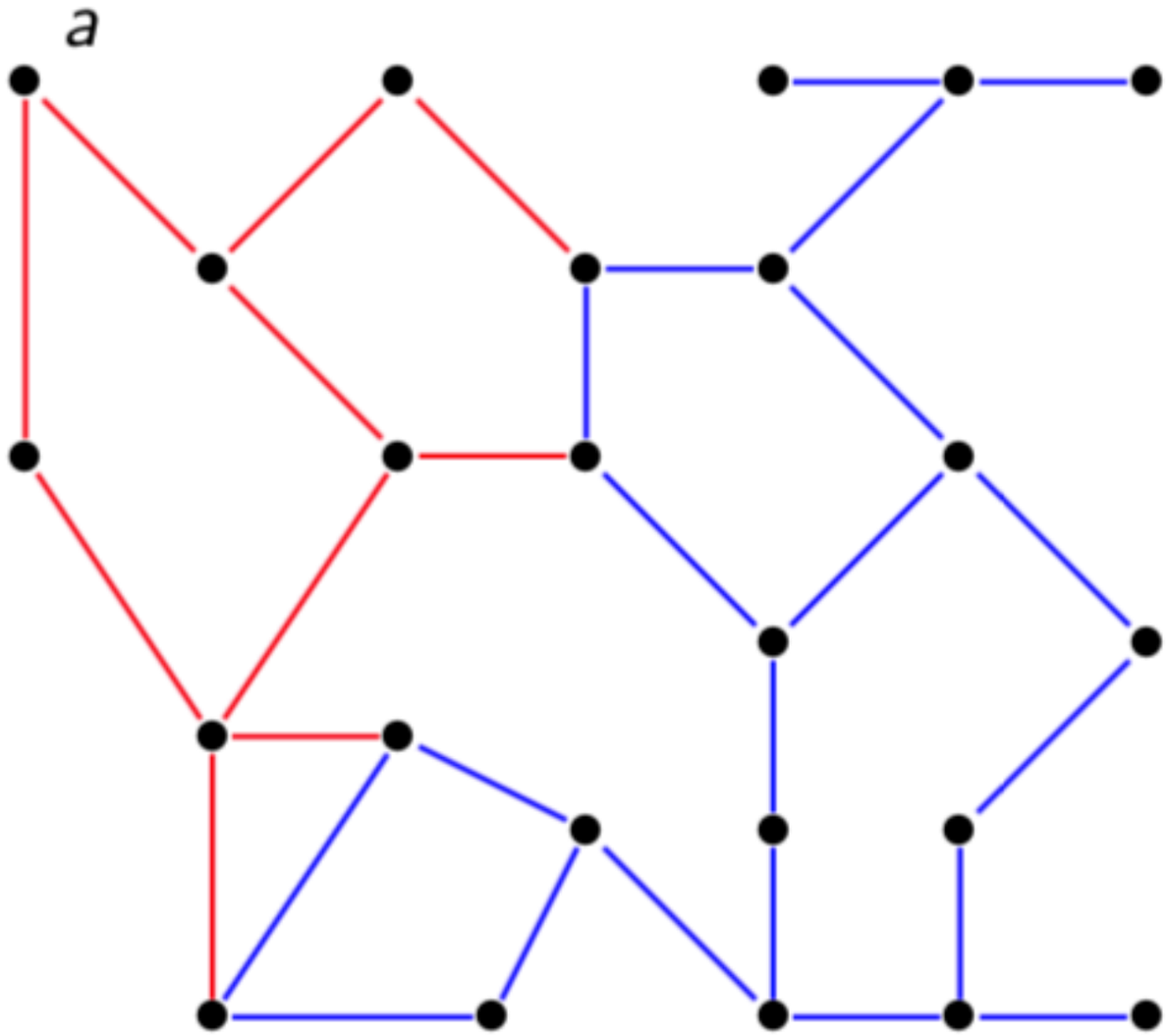
Depth-First vs Breadth-First Search



Typical Breadth-First Search

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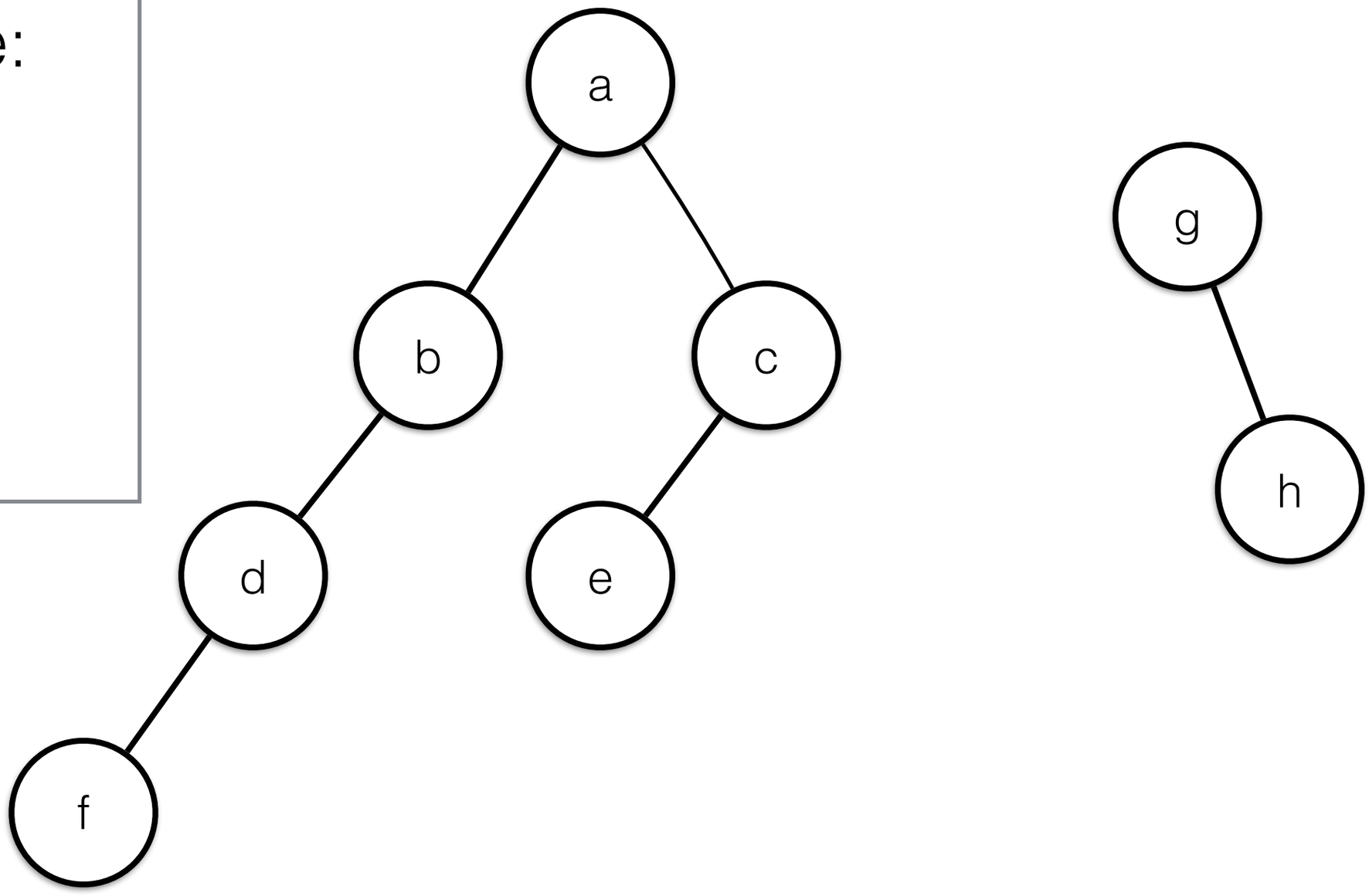
Breadth-First Search: Queue Discipline

THE UNIVERSITY OF MELBOURNE

Traversal Queue:



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Breadth-First Search: Queue Discipline

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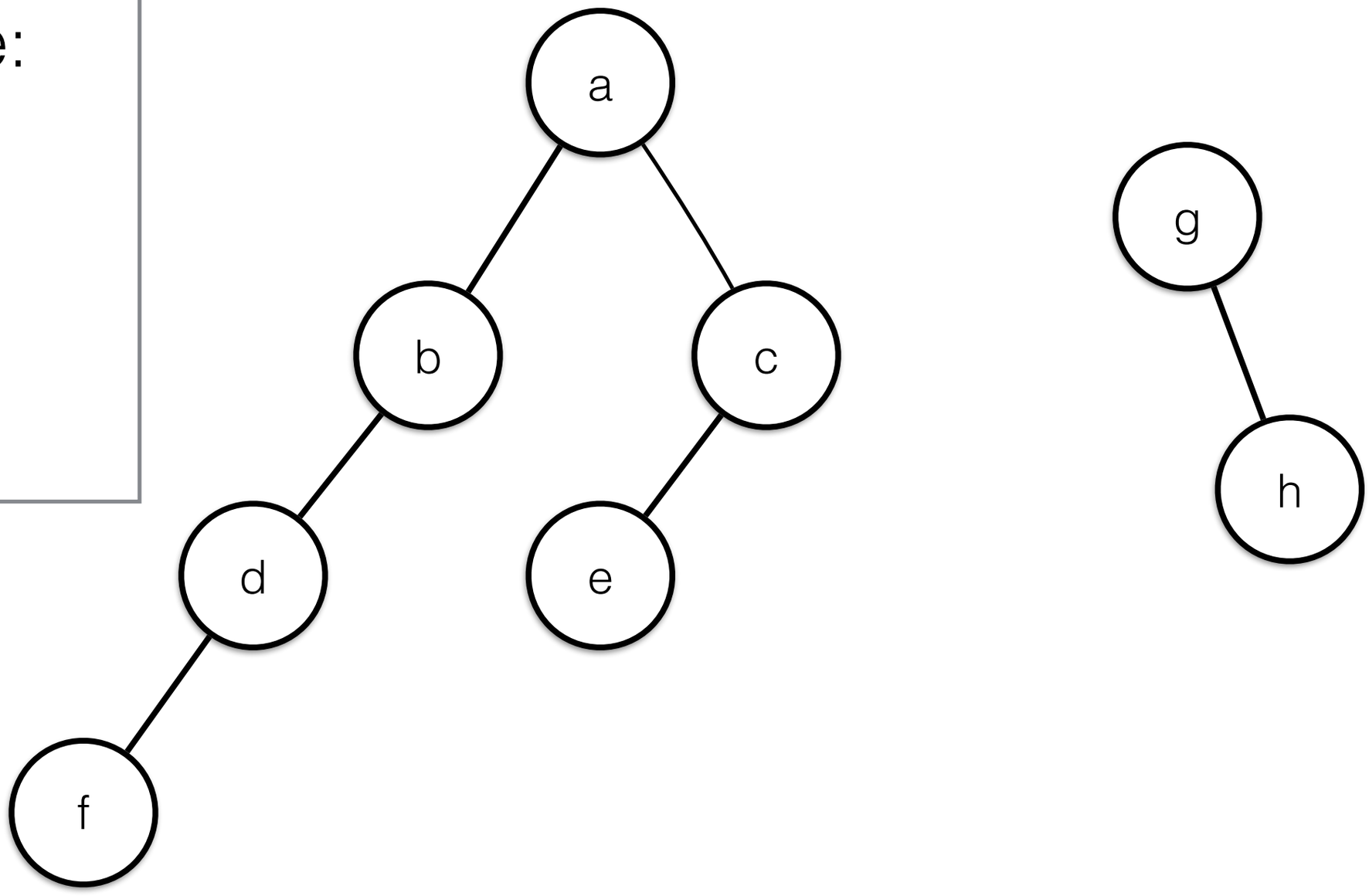
Traversal Queue:

a



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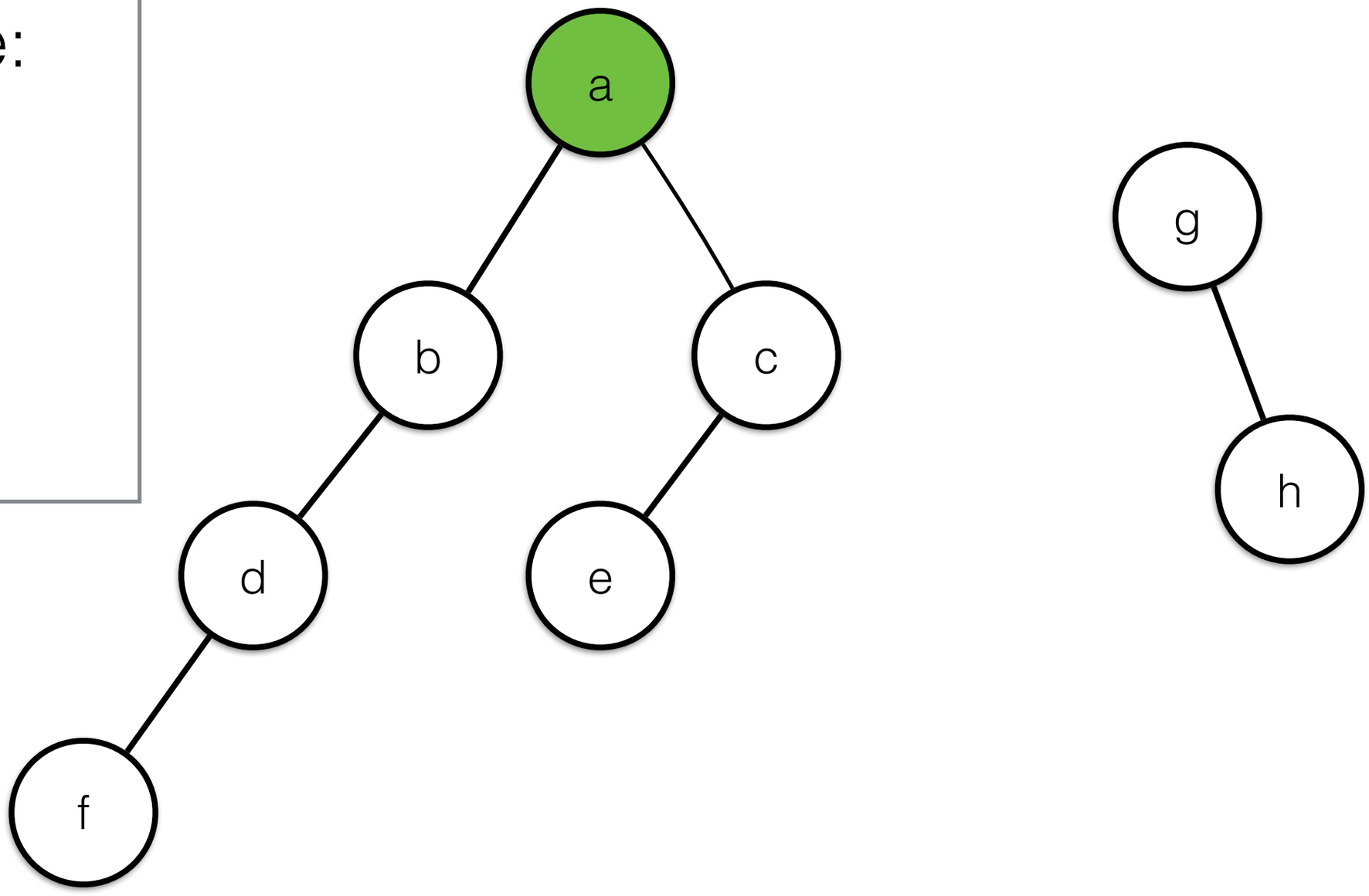
Traversal Queue:



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Breadth-First Search: Queue Discipline

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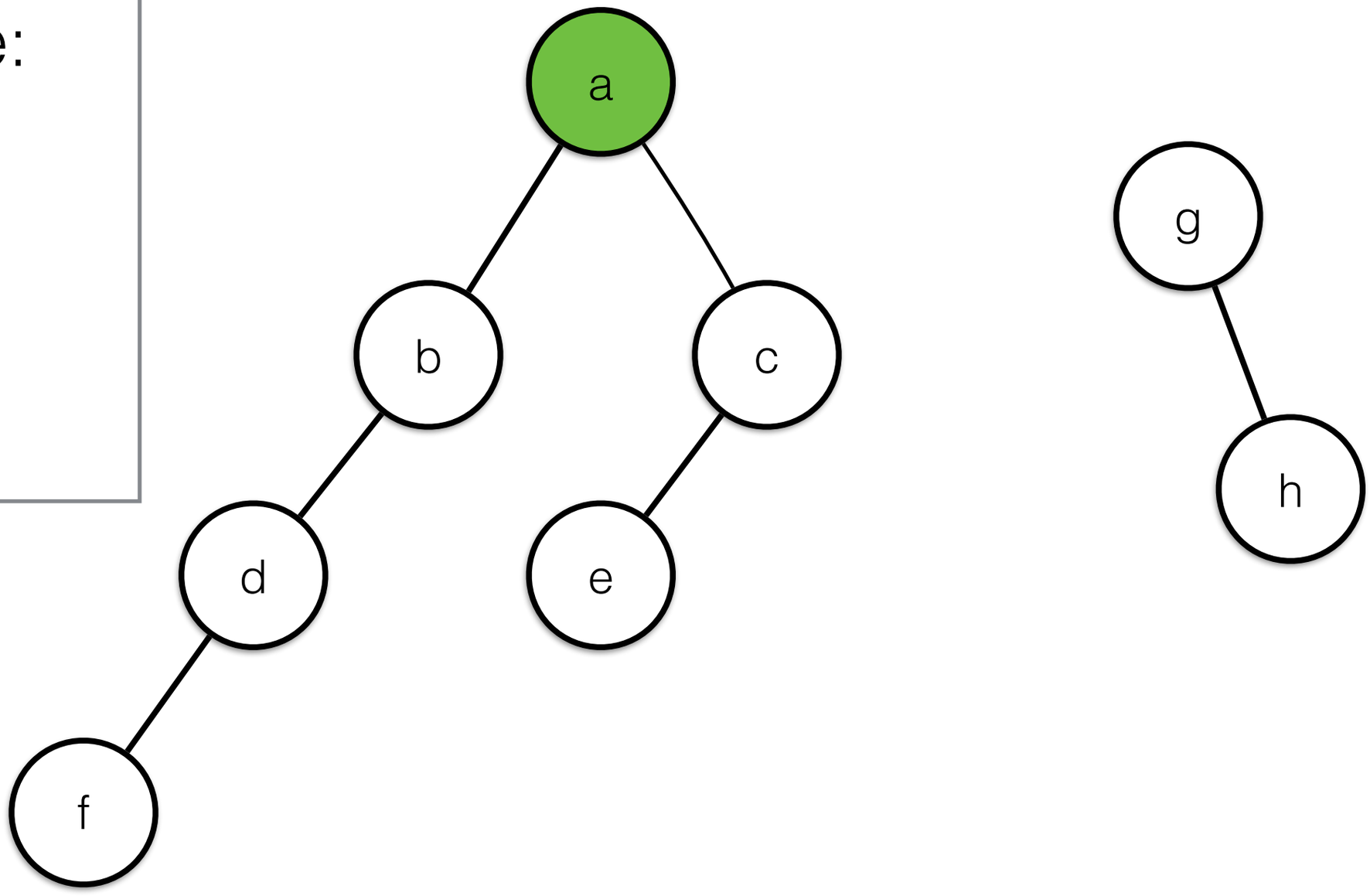
Traversal Queue:

b
c



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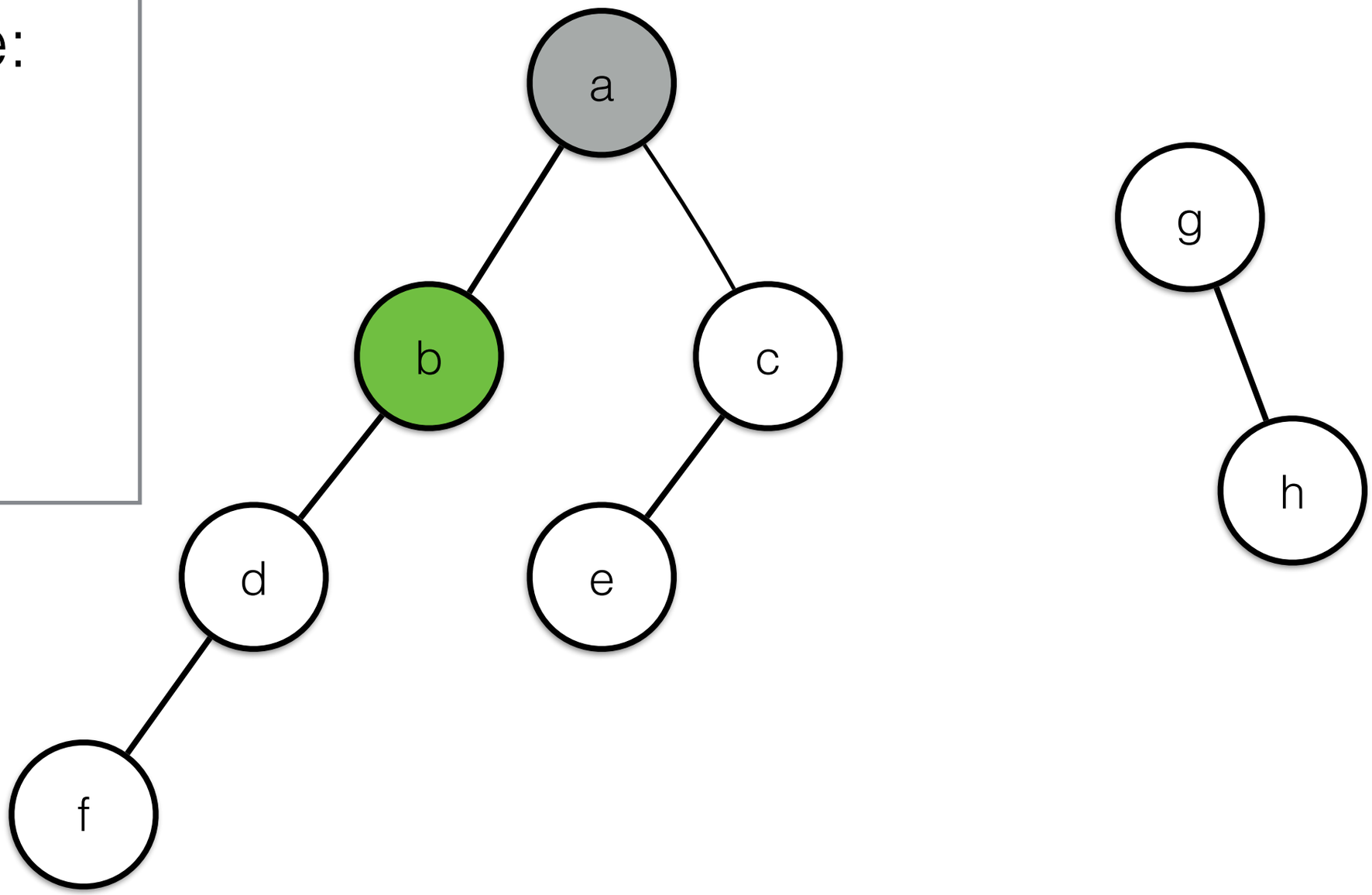
Traversal Queue:

c



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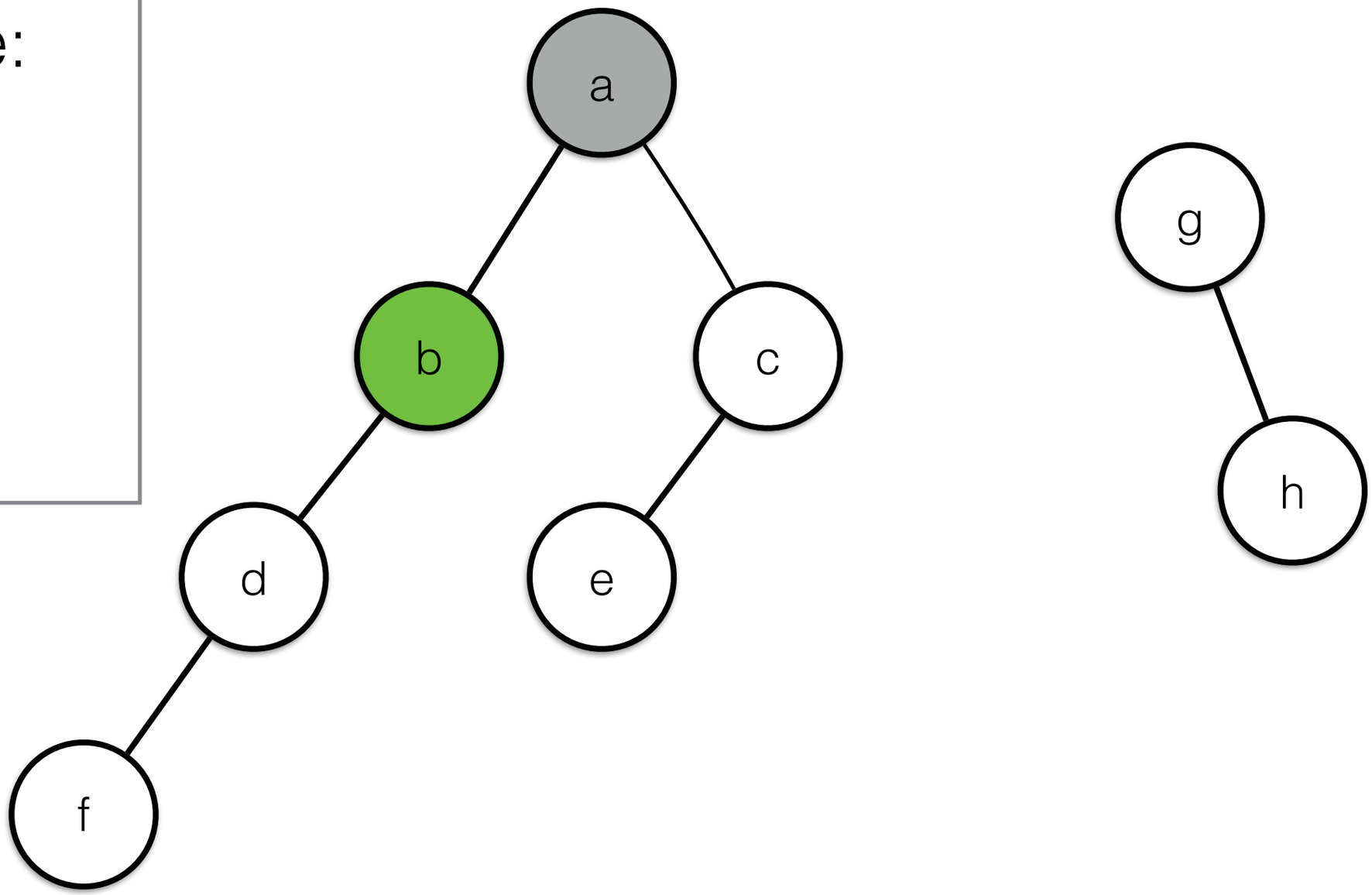
Traversal Queue:

c

d



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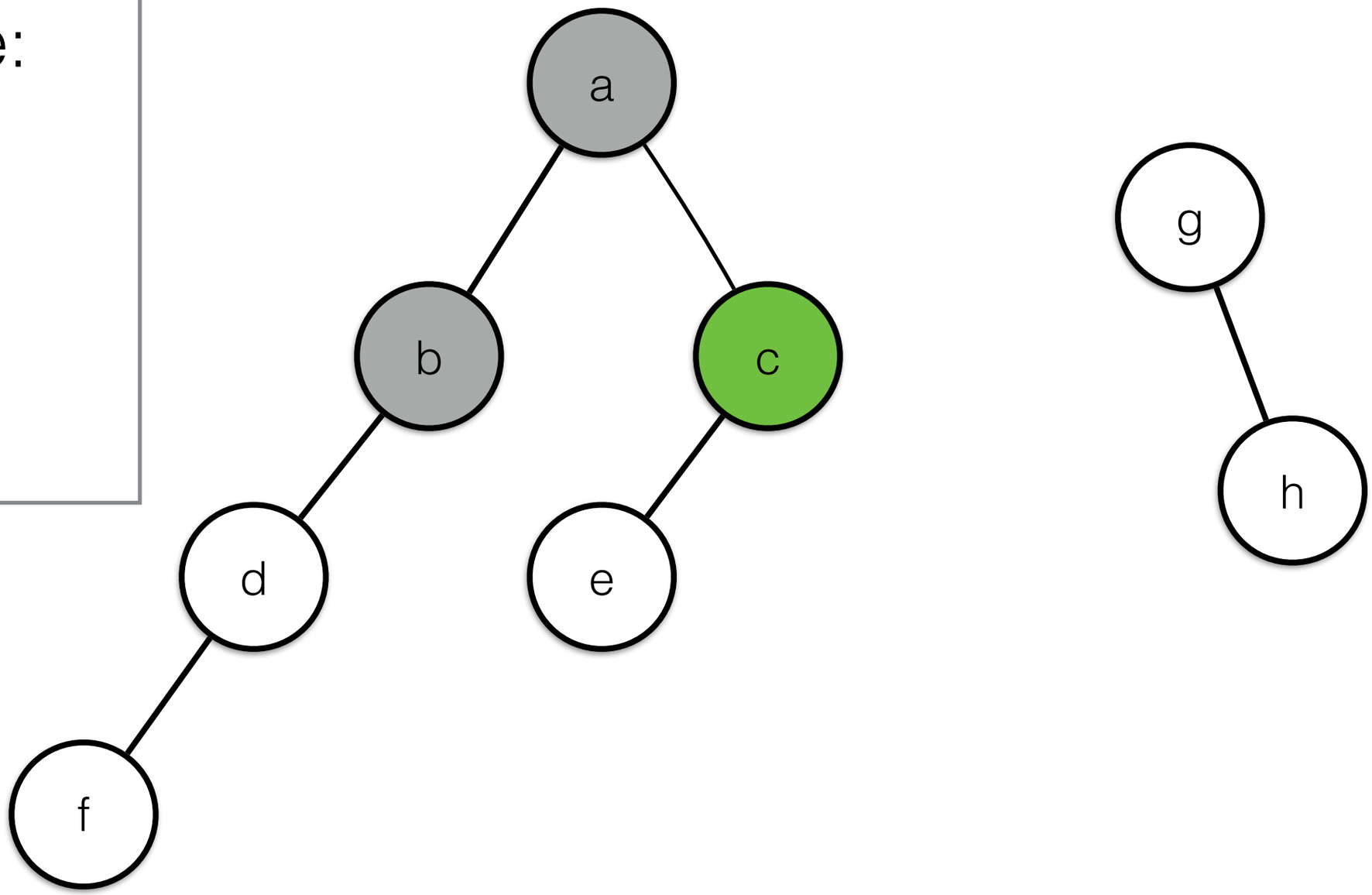
Traversal Queue:

d



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Breadth-First Search: Queue Discipline

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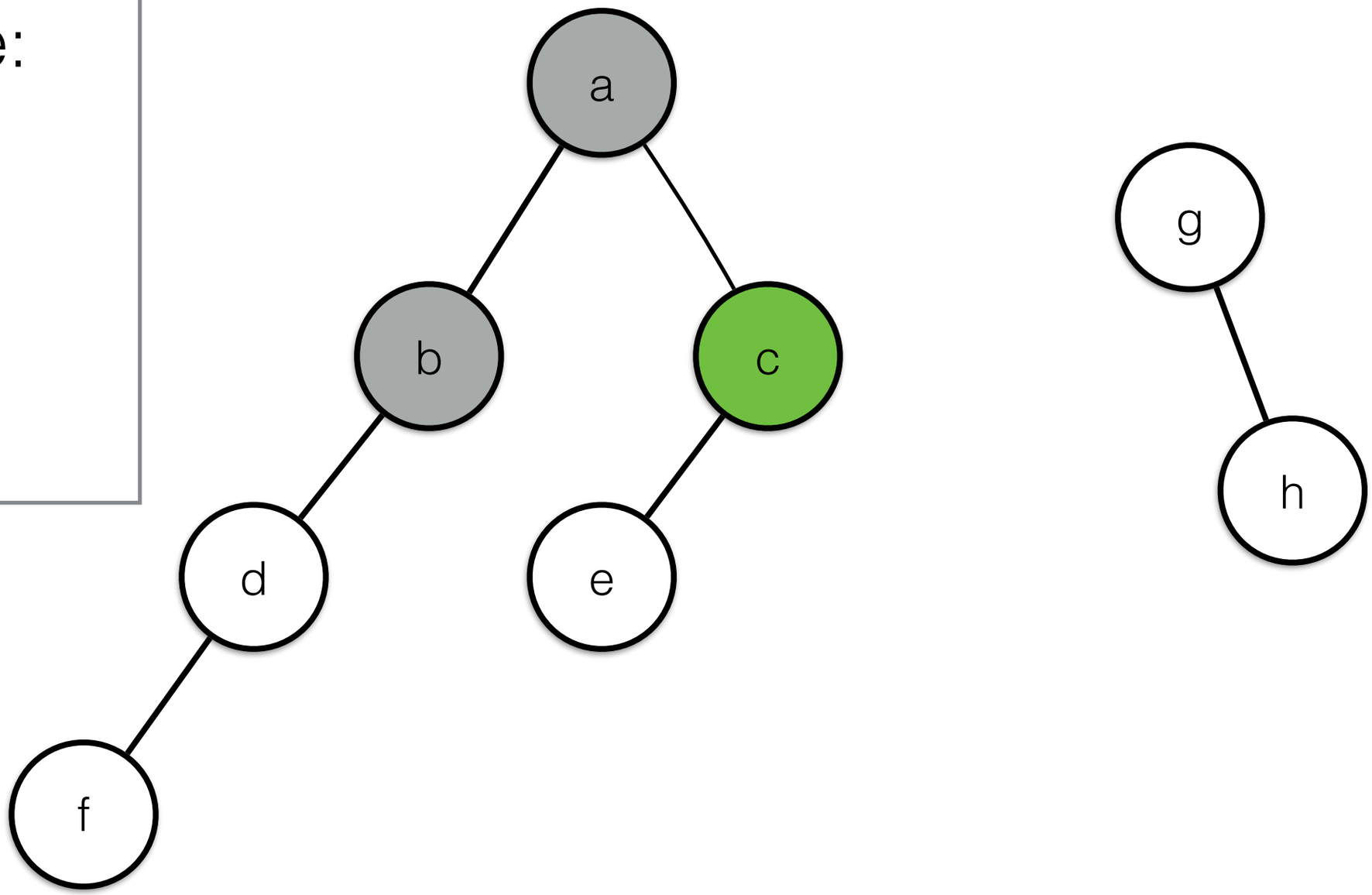
Traversal Queue:

d

e



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Breadth-First Search: Queue Discipline

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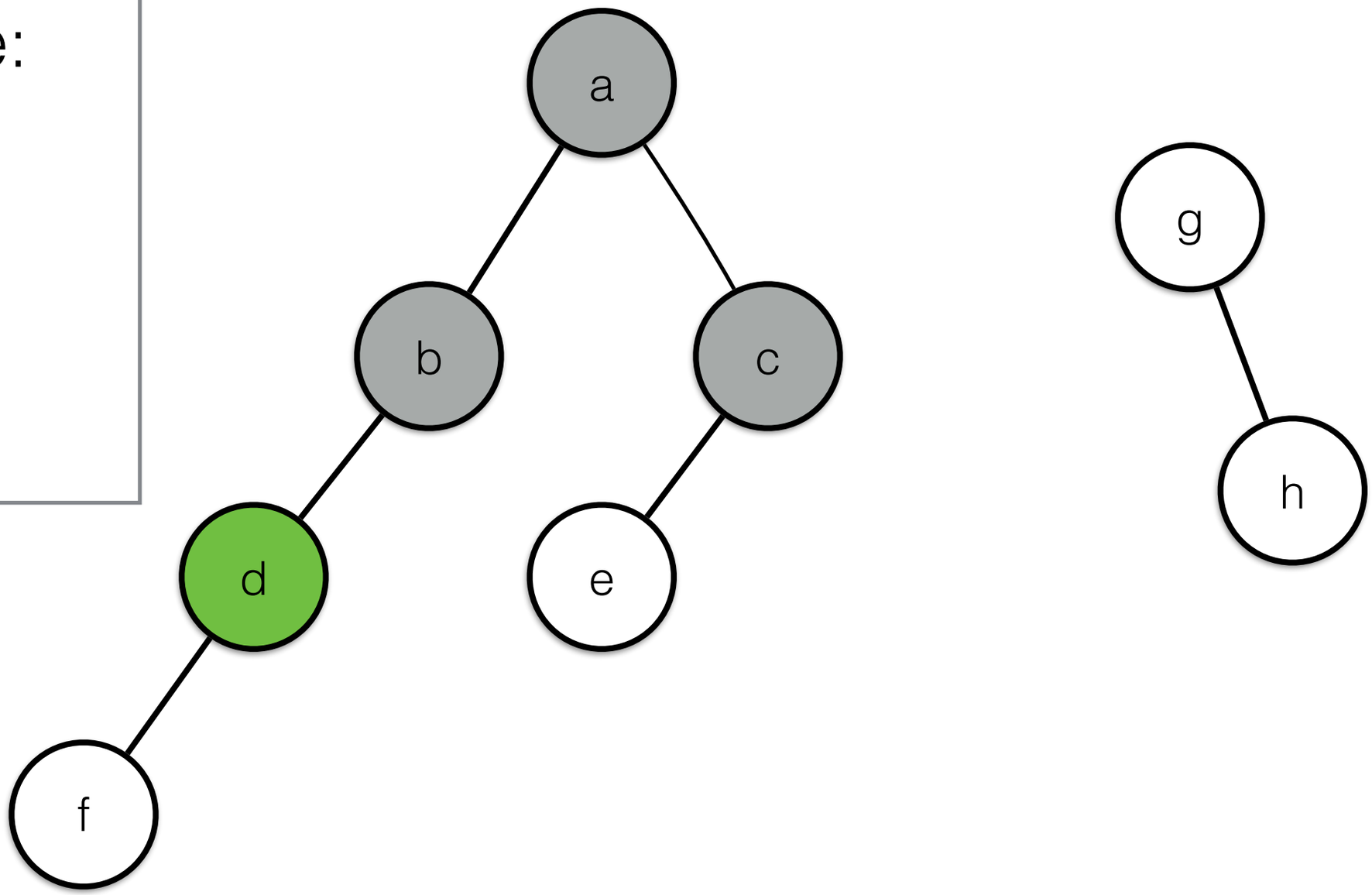
Traversal Queue:

e



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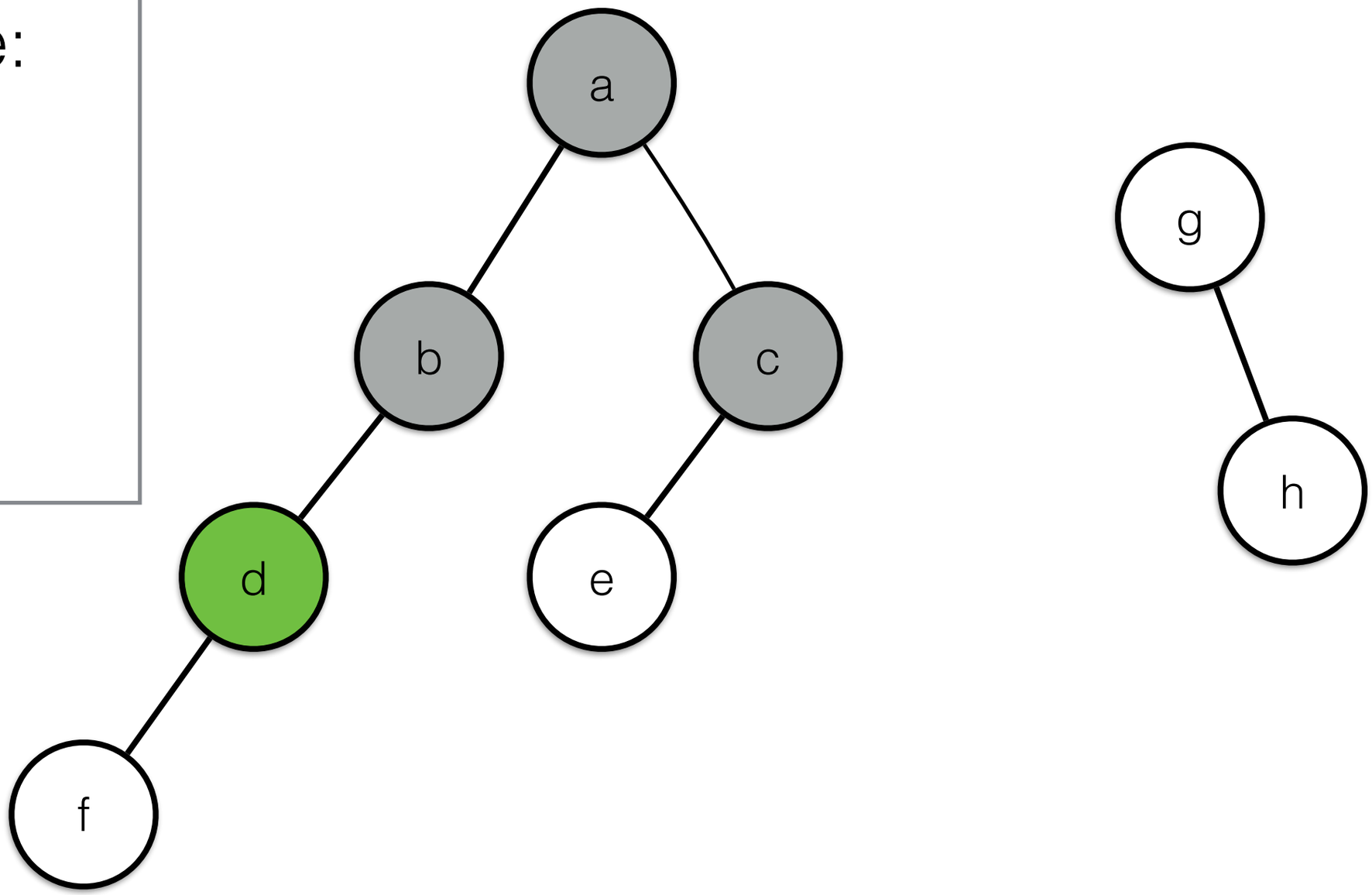
Traversal Queue:

e
f



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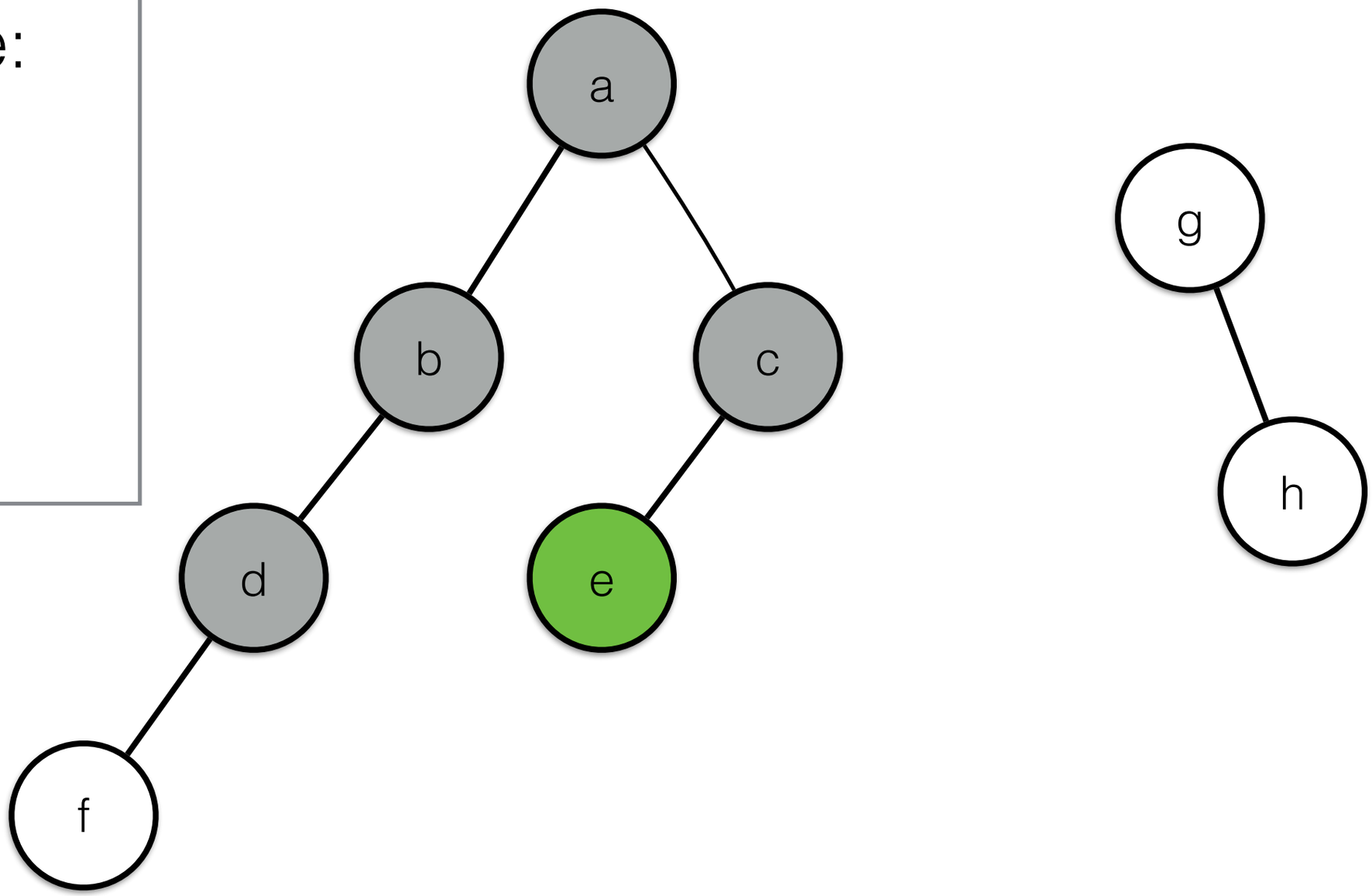
Traversal Queue:

f



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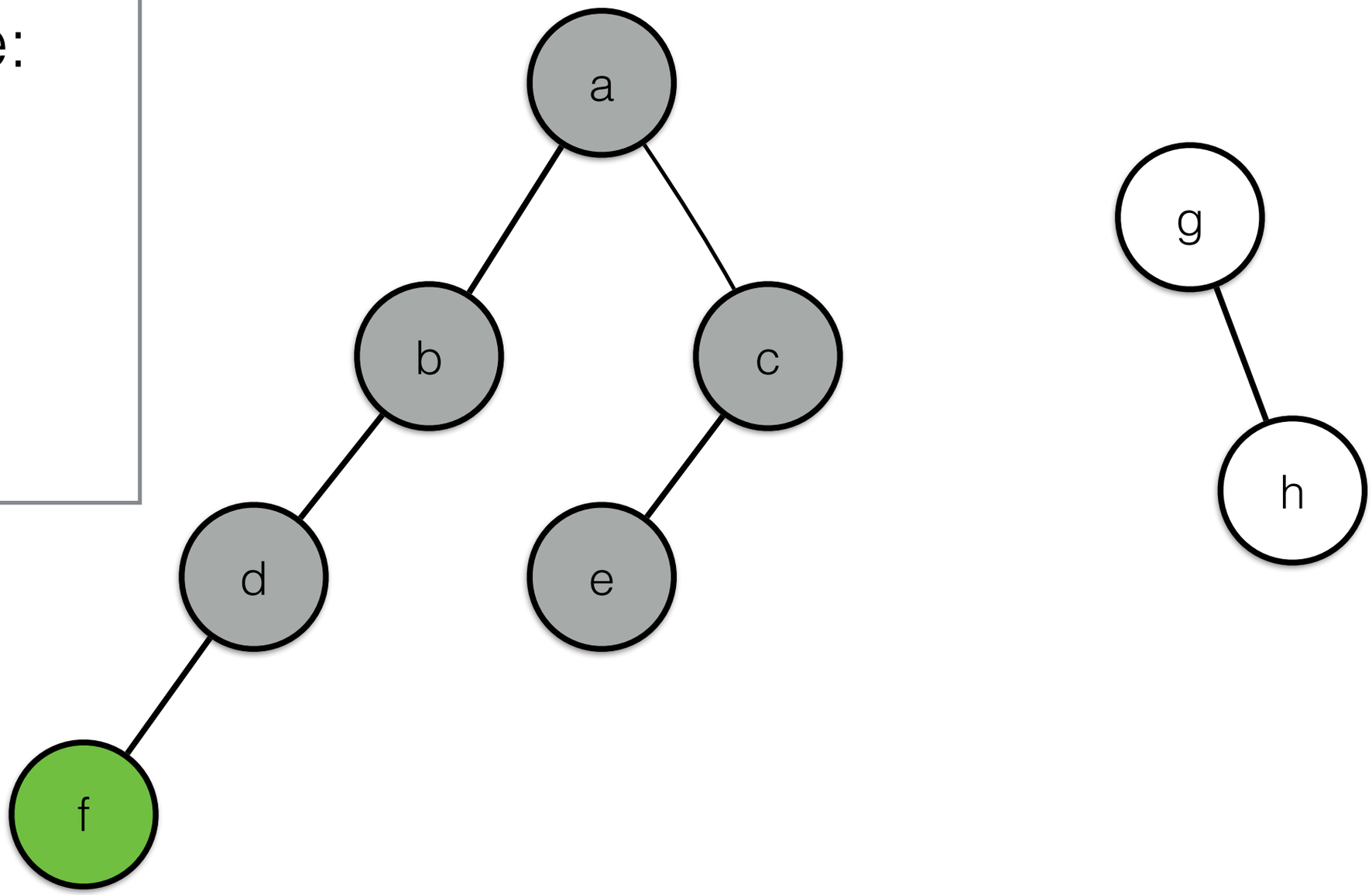
Traversal Queue:



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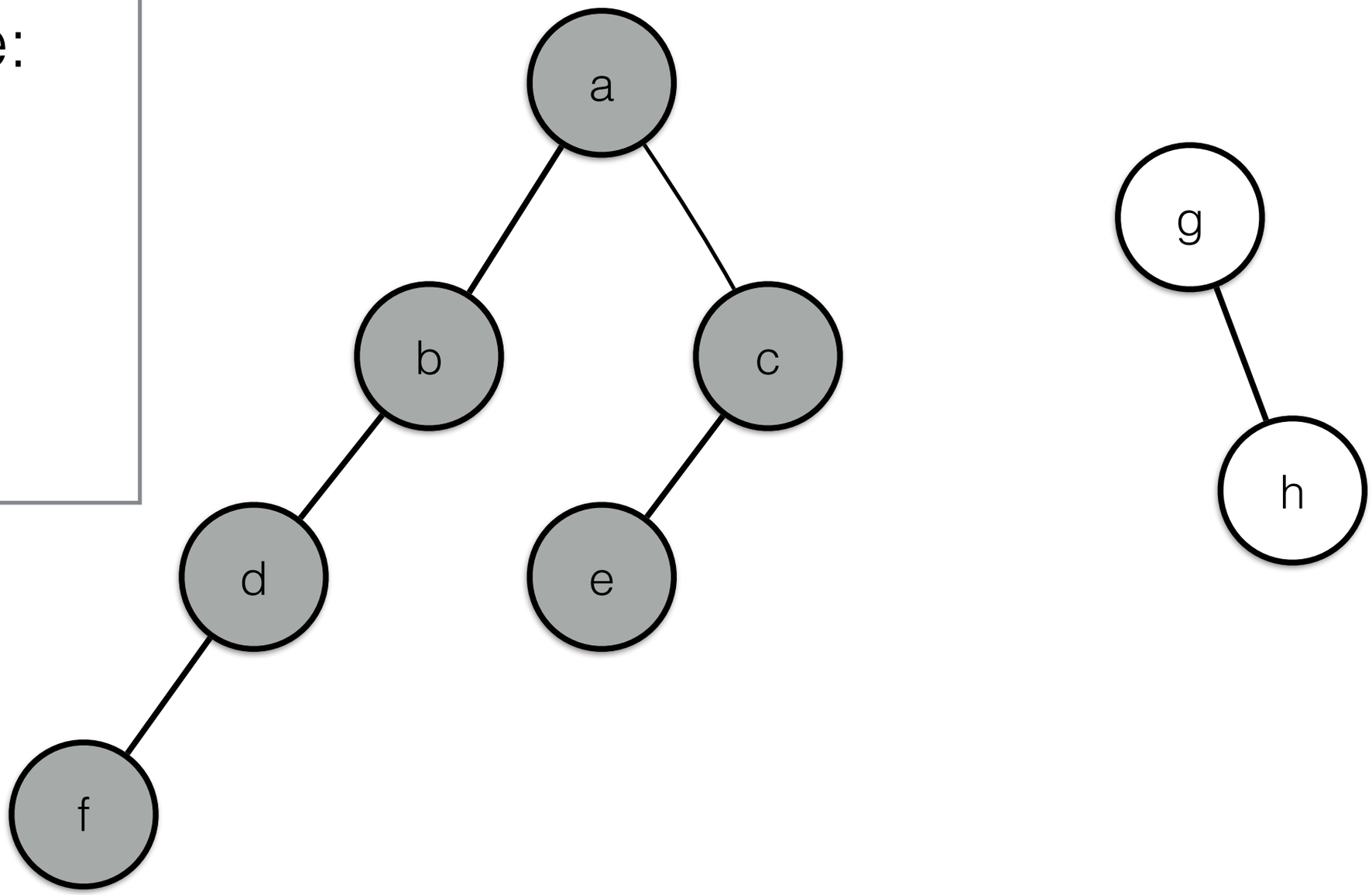
Traversal Queue:



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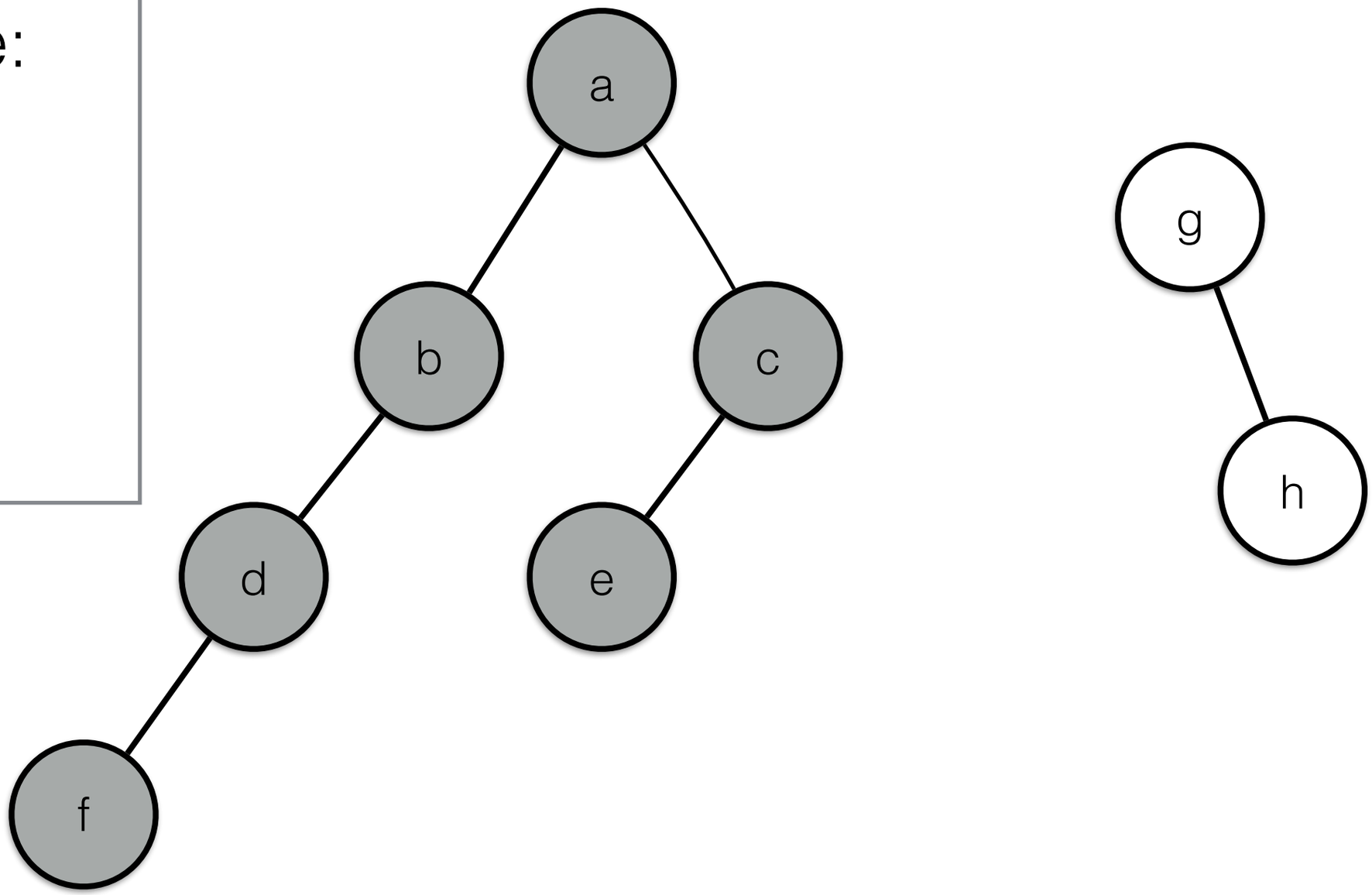
Traversal Queue:

g



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Breadth-First Search: Queue Discipline

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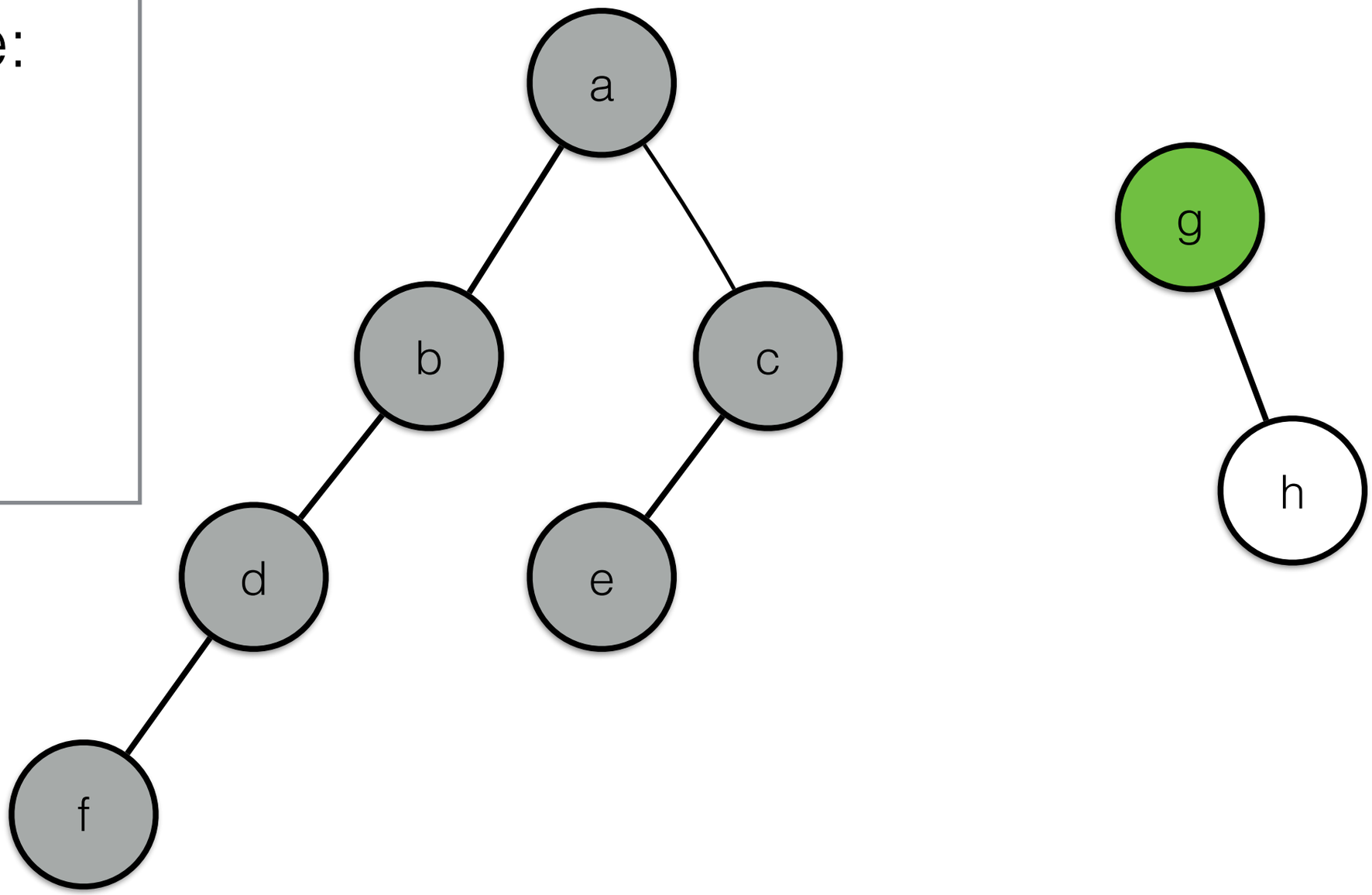
Traversal Queue:



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Breadth-First Search: Queue Discipline

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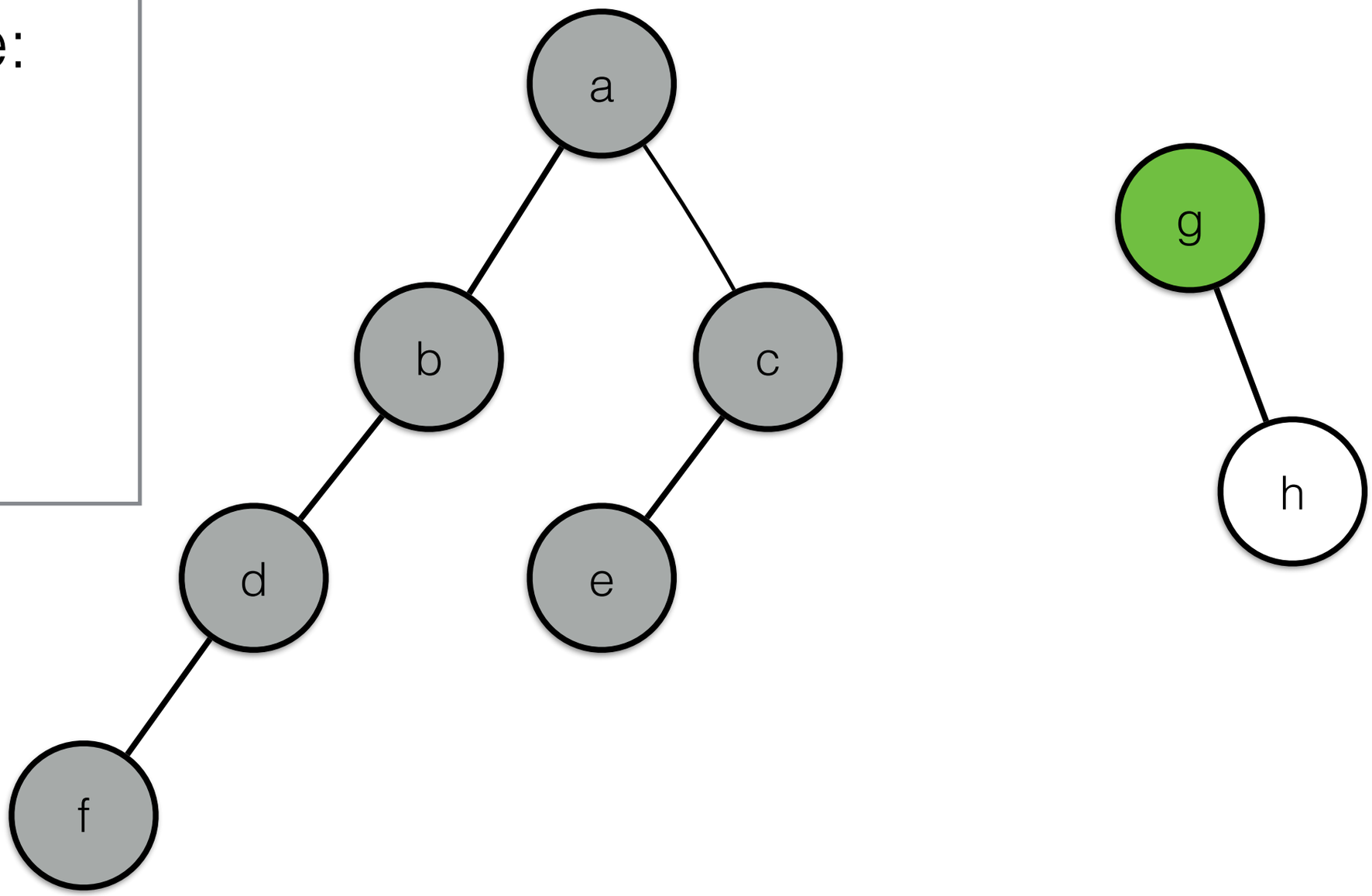
Traversal Queue:
h



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Breadth-First Search: Queue Discipline

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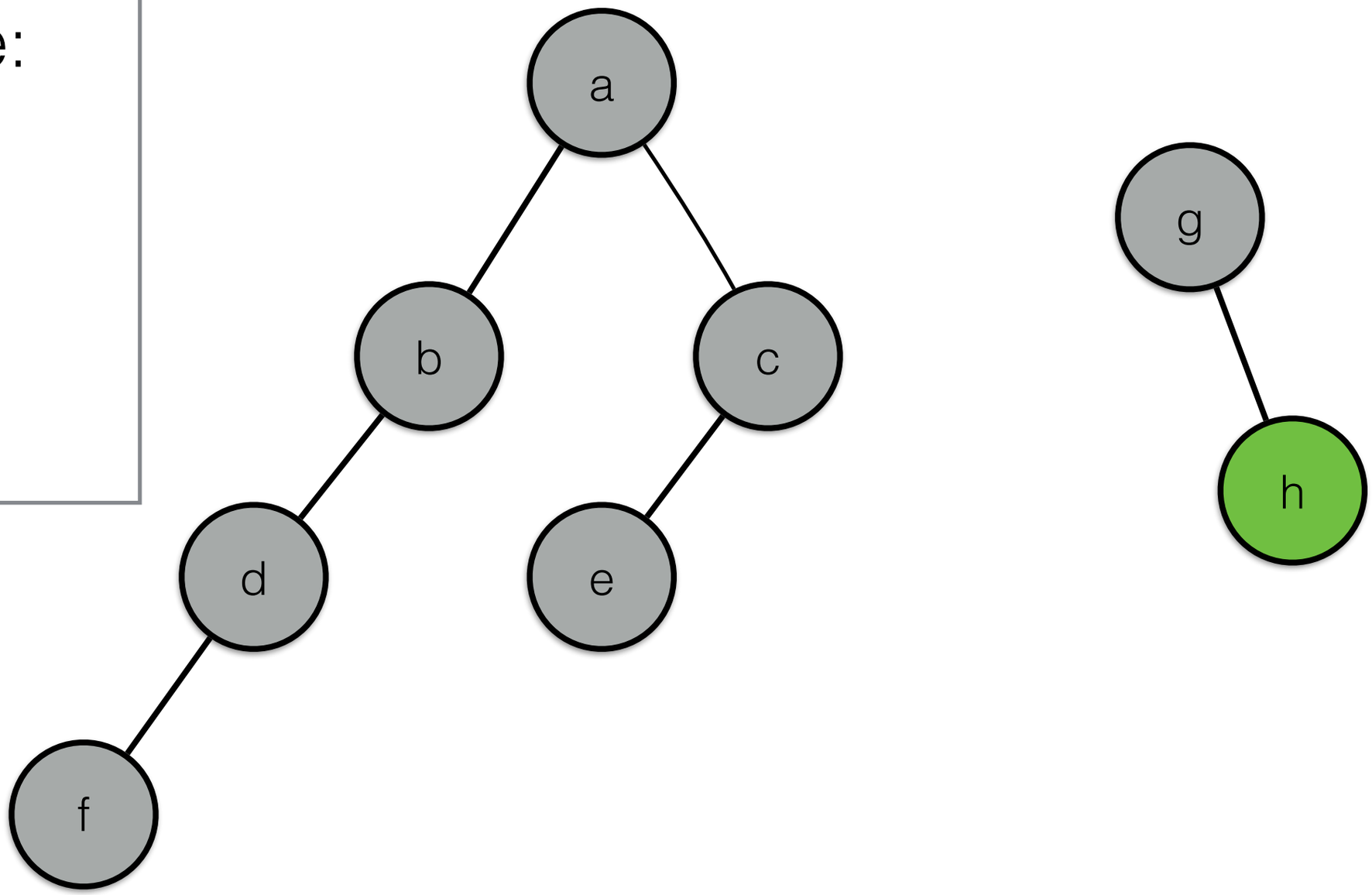
Traversal Queue:



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Breadth-First Search Algorithm

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```
function BFS(⟨V, E⟩)
    mark each node in V with 0
    count ← 0, init(queue)                    ▷ create an empty queue
    for each v in V do
        if v is marked with 0 then
            count ← count + 1
            mark v with count
            inject(queue, v)                  ▷ queue containing just v
            while queue is non-empty do
                u ← eject(queue)              ▷ dequeues u
                for each edge (u, w) do       ▷ w is u's neighbour
                    if w is marked with 0 then
                        count ← count + 1
                        mark w with count
                        inject(queue, w)      ▷ enqueues w
```


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BFS Algorithm Notes

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- BFS has the same complexity as DFS.
- Again, the same algorithm works for directed graphs as well.
- Certain problems are most easily solved by adapting BFS.
- For example, given a graph and two nodes, a and b in the graph, how would you find the length of the shortest path from a to b?

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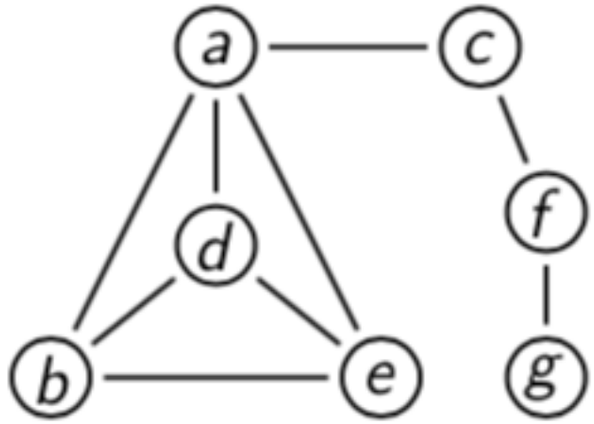
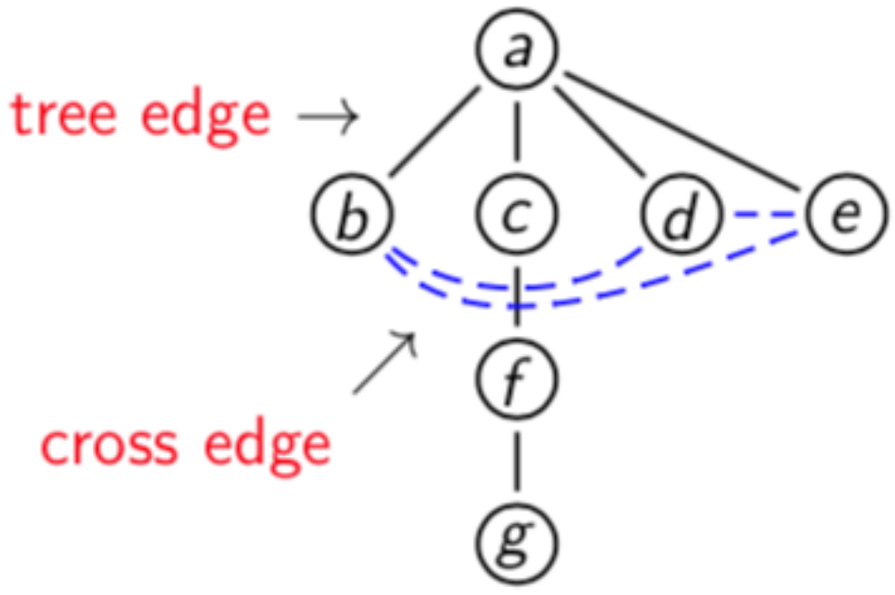
Breadth-First Search Forest



BFS **Tree** for this connected graph:



In general, we may get a BFS **Forest**

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Topological Sorting

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- We mentioned scheduling problems and their representation by directed graphs.
- Assume a directed edge from a to b means that task a must be completed before b can be started.
- The graph must be a dag; otherwise the problem cannot be solved.
- Assume the tasks are carried out by a single person, unable to multi-task.
- Then we should try to **linearize** the graph, that is, order the nodes as a sequence $v_1,v_2,\ldots,v_n$ such that for each edge $(v_i,v_j) \in E$, we have v_i comes before v_j in the sequence (that is, v_i is scheduled to happen before v_j).

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Topological Sorting Example

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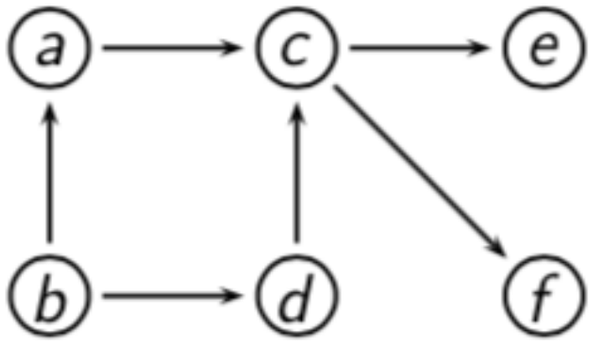
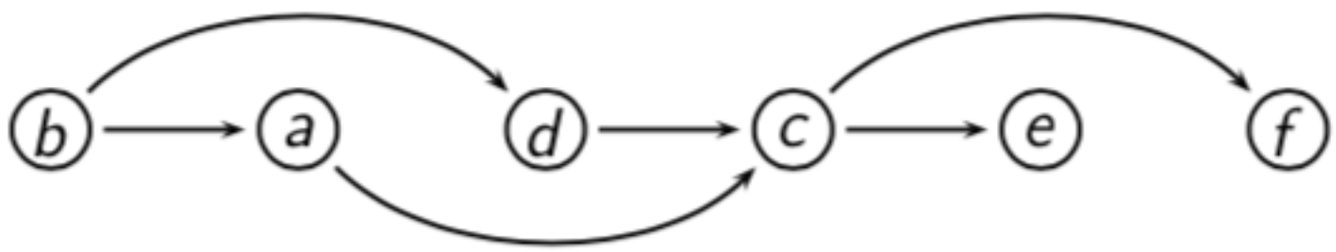
There are 4 ways to linearise the following graph



Here is one:



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Topological Sorting Algorithm 1

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- We can solve the top-sort problem with depth-first search:

  1. Perform DFS and note the order in which nodes are popped off the stack.

  2. List the nodes in the reverse of that order.

- This works because of the stack discipline.

- If (u,v) is an edge then it is possible (given some way of deciding ties) to arrive at a DFS stack with u sitting below v.

- Taking the “reverse popping order” ensures that u is listed before v.

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Topological Sorting Example Again

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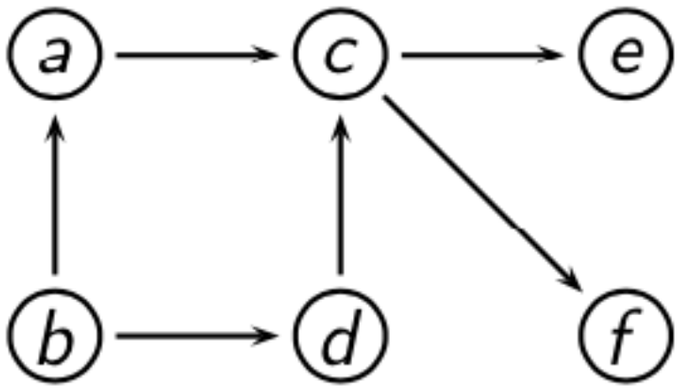
Using the DFS method and resolving ties by using alphabetical order, the graph gives rise to the traversal stack shown on the right (the popping order shown in red):



$e_{3,1}$ $f_{4,2}$

$c_{2,3}$ $d_{6,5}$

$a_{1,4}$ $b_{5,6}$

Taking the nodes in reverse popping order yields b, d, a, c, f , e.

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Topological Sorting Algorithm 2

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- An alternative method would be to repeatedly select a random **source** in the graph (that is, a node with no incoming edges), list it, and remove it from the graph (including removing its outgoing edges).

- This is a very natural approach, but it has the drawback that we repeatedly need to scan the graph for a source.

- However, it exemplifies the general principle of **decrease-and-conque**r.

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Topological Sorting Example Again

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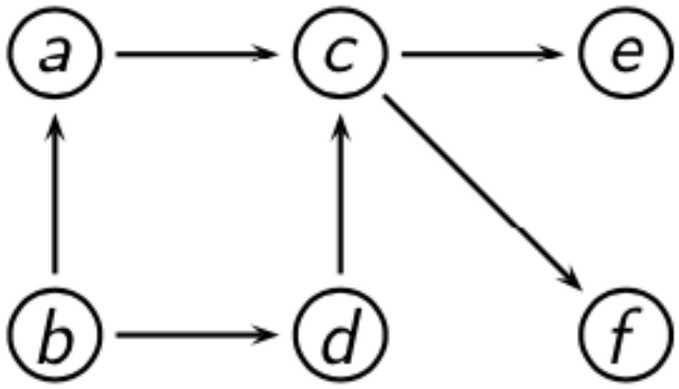
Using the source removal method (and resolving ties alphabetically):



Topological sorted order:

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Topological Sorting Example Again

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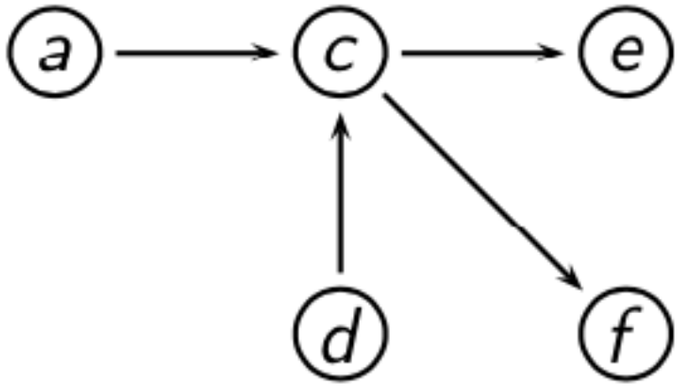
Using the source removal method (and resolving ties alphabetically):



Topological sorted order:
b

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Topological Sorting Example Again

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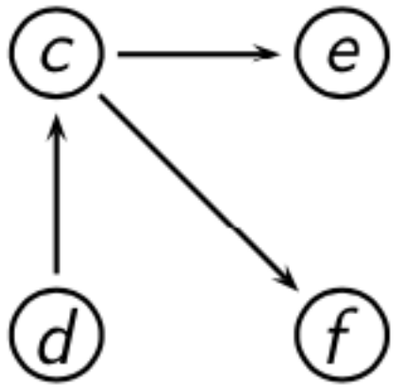
Using the source removal method (and resolving ties alphabetically):



Topological sorted order:
b, a

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Topological Sorting Example Again

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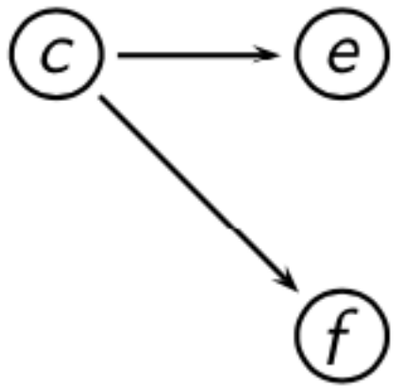
Using the source removal method (and resolving ties alphabetically):



Topological sorted order:
b, a, d

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Topological Sorting Example Again

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Using the source removal method (and resolving ties alphabetically):

Topological sorted order:
b, a, d, c

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Topological Sorting Example Again

Using the source removal method (and resolving ties alphabetically):

Topological sorted order:
b, a, d, c, e

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Topological Sorting Example Again

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Using the source removal method (and resolving ties alphabetically):

Topological sorted order:
b, a, d, c, e, f

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Next time

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- So next we turn our attention to the very useful "decrease and conquer" principle (Levitin Chapter 4).

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