# COMP90038
# Algorithms and Complexity

## Lecture 14: Transform-and-Conquer
## (with thanks to Harald Søndergaard & Michael Kirley)

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# Review from Lecture 13

- We saw priority queues, heaps and heapsort.
- A priority queue is a set of elements, each containing a priority (key) value. Elements with higher priorities are ejected first.
- A heap is a a complete binary tree that satisfies the condition:
  - Each child has a priority (key) which is not greater than its parents.
- Heapsort is a sorting algorithm that repeatedly ejects the higher priority element, then turns the remaining binary tree into a Heap.

# Transform and Conquer

- Transform-and-Conquer is a group of design techniques that:
  - *Transform*: Modify the problem to a more amenable form, and then
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  - Representational change
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  - Instance simplification
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# Instance Simplification

- General principle: Try to make the problem easier through some sort of pre-processing typically sorting.

- We can pre-sort input to speed up, for example:

  – Finding the median

  – Uniqueness checking

  – Finding the mode

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# Uniqueness Checking, Brute-Force

- The problem:
  Given an unsorted array $A[0, \cdots, n-1]$, is $A[i] \neq A[j]$ whenever $i \neq j$?

- The obvious approach is brute-force:

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for i ← 0 to n − 2 do
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|---|---|
| $n$ | 8 |
| $i$ | 0 |
| $j$ | |
| $A[i]$ | 2 |
| $A[j]$ | |
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- What is the complexity of this? $O(n^2)$

# Uniqueness Checking, with Presorting

- Sorting makes the problem easier:

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$\textsc{Sort}(A[0..n-1])$
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- What is the complexity of this? $O(n \log n) + O(n) = O(n \log n)$

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# Exercise: Computing the Mode

- A mode is a list of array elements which occurs most frequently in the list/array.

- For example, in:

42, 78, 13, 57, 42, 57, 78, 42

the element 42 is the mode.

# Exercise: Computing the Mode

- A mode is a list of array elements which occurs most frequently in the list/array.
- For example, in:

  42, 78, 13, 57, 42, 57, 78, 42

  the element 42 is the mode.
- The problem:

  Given an array $A$, find a mode.
- Discuss a brute-force approach vs a pre-sorting approach.

# Mode Finding with Presorting

```
SORT(A[0..n − 1])
i ← 0
maxfreq ← 0
while i < n do
    runlength ← 1
    while i + runlength < n and A[i + runlength] = A[i] do
        runlength ← runlength + 1
    if runlength > maxfreq then
        maxfreq ← runlength
        mode ← A[i]
    i ← i + runlength
return mode
```

# Mode Finding with Presorting

$$A[0, \cdots, n-1] = [42, 78, 13, 57, 42, 57, 78, 42]$$

Sort the array: [13, 42, 42, 42, 57, 57, 78, 78]

$\textsc{Sort}(A[0..n-1])$
$i \leftarrow 0$
$maxfreq \leftarrow 0$
**while** $i < n$ **do**
    $runlength \leftarrow 1$
    **while** $i + runlength < n$ **and** $A[i + runlength] = A[i]$ **do**
        $runlength \leftarrow runlength + 1$
    **if** $runlength > maxfreq$ **then**
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    $i \leftarrow i + runlength$
**return** $mode$

# Mode Finding with Presorting

$$A[0, \cdots, n-1] = [42, 78, 13, 57, 42, 57, 78, 42]$$

$\textsc{Sort}(A[0..n-1])$
$i \leftarrow 0$
$maxfreq \leftarrow 0$ ← Frequency of the most common element so far
**while** $i < n$ **do**
  $runlength \leftarrow 1$
  **while** $i + runlength < n$ **and** $A[i + runlength] = A[i]$ **do**
    $runlength \leftarrow runlength + 1$
  **if** $runlength > maxfreq$ **then**
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    if runlength > maxfreq then
        maxfreq ← runlength
        mode ← A[i]
    i ← i + runlength
return mode
```

This counter keeps track of sequences of equal numbers

# Mode Finding with Presorting

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**return** $mode$

While we do not overflow and the sequence continues

# Mode Finding with Presorting

$$A[0, \cdots, n-1] = [42, 78, 13, 57, 42, 57, 78, 42]$$

$\text{SORT}(A[0..n-1])$
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$\mathit{maxfreq} \leftarrow 0$
**while** $i < n$ **do**
  $\mathit{runlength} \leftarrow 1$
  **while** $i + \mathit{runlength} < n$ **and** $A[i + \mathit{runlength}] = A[i]$ **do**
    $\mathit{runlength} \leftarrow \mathit{runlength} + 1$ ← Increase the sequence counter
  **if** $\mathit{runlength} > \mathit{maxfreq}$ **then**
    $\mathit{maxfreq} \leftarrow \mathit{runlength}$
    $\mathit{mode} \leftarrow A[i]$
  $i \leftarrow i + \mathit{runlength}$
**return** $\mathit{mode}$

# Mode Finding with Presorting

$$A[0, \cdots, n-1] = [42, 78, 13, 57, 42, 57, 78, 42]$$

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SORT(A[0..n − 1])
i ← 0
maxfreq ← 0
while i < n do
    runlength ← 1
    while i + runlength < n and A[i + runlength] = A[i] do
        runlength ← runlength + 1
    if runlength > maxfreq then        ← If the sequence is the largest so far
        maxfreq ← runlength
        mode ← A[i]
    i ← i + runlength
return mode
```

# Mode Finding with Presorting

$$A[0, \cdots, n-1] = [42, 78, 13, 57, 42, 57, 78, 42]$$

$\textsc{Sort}(A[0..n-1])$
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    $i \leftarrow i + runlength$
**return** $mode$

Update both the frequency and mode variables

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  **if** $runlength > maxfreq$ **then**
    $maxfreq \leftarrow runlength$
    $mode \leftarrow A[i]$
  $i \leftarrow i + runlength$ ← Skip the complete sequence of equal numbers
**return** $mode$

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```

- Again, after sorting, the rest takes linear time.

# Searching with Presorting

- The problem:

  Given unsorted array $A$, find item $x$ (or determine that it is absent).

- Compare these two approaches:
  - Perform a sequential search
  - Sort, then perform binary search
- What are the complexities of these approaches?

# Searching with Presorting

- What if we need to search for $m$ items?
- Let us do a back-of-the envelope calculation (consider worst-cases for simplicity):
  - Take $\mathrm{n} = 1024$ and $m = 32$.
  - Sequential search: $\mathrm{m} \times \mathrm{n} = 32,768$
  - Sorting + binsearch: $n\, log_2 n + \mathrm{m} \times log_2 n = 10{,}240 + 320 = 10{,}560$.
  - Average-case analysis will look somewhat better for sequential search, but pre-sorting will still win.

# Exercise: Finding Anagrams

- An anagram of a word *w* is a word which uses the same letter as *w* but in a different order.
  - Example: 'ate', 'tea' and 'eat' are anagrams.
  - Example: 'post', 'spot', 'pots' and 'tops' are anagrams.
  - Example: 'garner' and 'ranger' are anagrams.
- You are given a very long list of words in lexicographic order.
- Device an algorithm to find all anagrams in the list.

# Exercise: Finding Anagrams

```
words = ['bat', 'rats', 'god', 'dog', 'cat', 'arts', 'star']
sort_words = {}
for word in words:
    sort_words[word] = ''.join(sorted(word))

print sort_words
anagrams = []
for i in range(len(words)):
    ana = [words[i]]
    for j in range(i + 1, len(words)):
        if sort_words[words[i]] == sort_words[words[j]]:
            ana.append(words[j])
    if len(ana) != 1:
        anagrams.append(ana)

print anagrams
```

- Finding anagrams from a words list:
- Time complexity?
  - Sorting words
    - $O(n \times m \log m)$
    - $n$ words of length $m$.
  - Test all combinations
    - $O(n^2)$
  - Can we do better?
    - Yes! Sort words as well!
    - Apply “mode idea”

# Binary Search Trees

- A binary search tree, or BST, is a binary tree that stores elements in all internal nodes, with each sin-tree satisfying the BST property:

  Let the root be $r$; then each element in the left subtree is smaller that $r$ and each element in the right sub-tree is larger than $r$. (For simplicity, we assume that all keys are different.)

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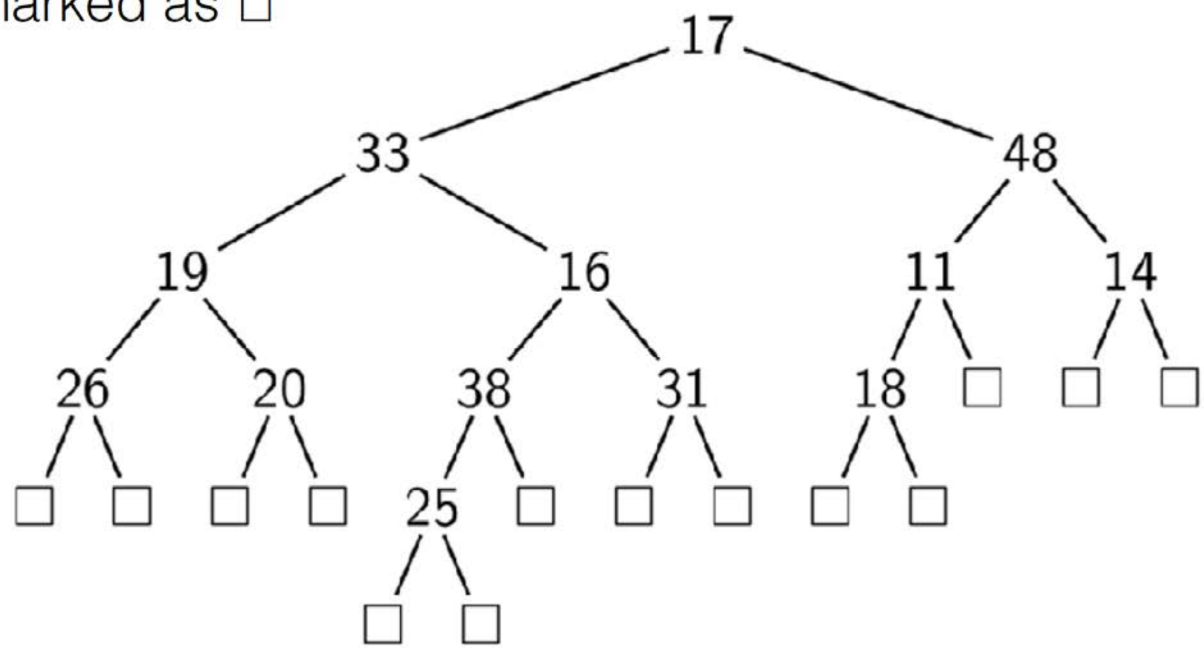
- First a review of binary trees from Lecture 12:

# Transform and Conquer

## Binary Trees

- An example of a **binary tree**, with empty subtrees marked as □




- This tree has **height** 4, the empty tree having height -1

# Review from Lecture 12

## Binary Tree Concepts

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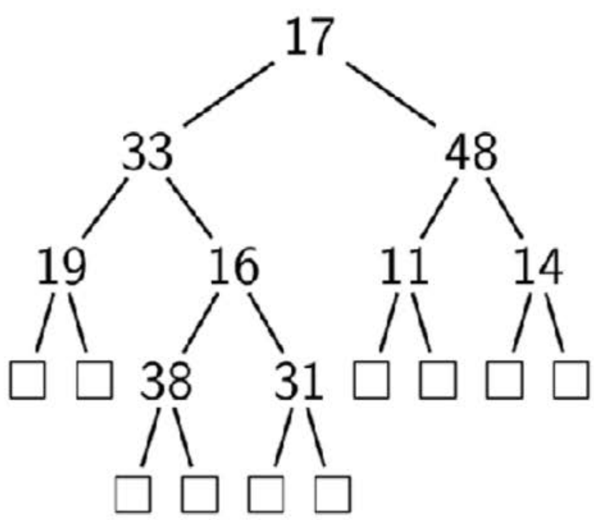
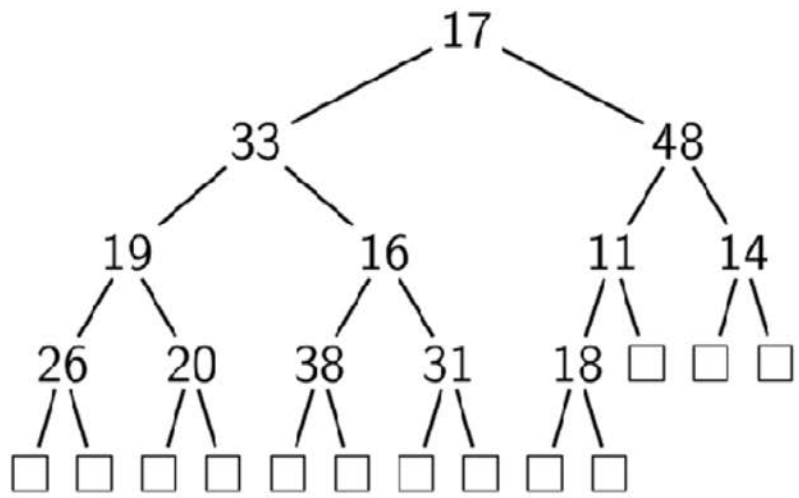
- Special trees have their **external nodes** □ only at level $h$ and $h$+1 for some $h$:




A **full** binary tree:
Each node has 0 or 2 (non-empty) children.




A **complete** tree: Each level filled left to right.
(Every level except perhaps the last is completely filled.)

# Review from Lecture 12

## Calculating the Height

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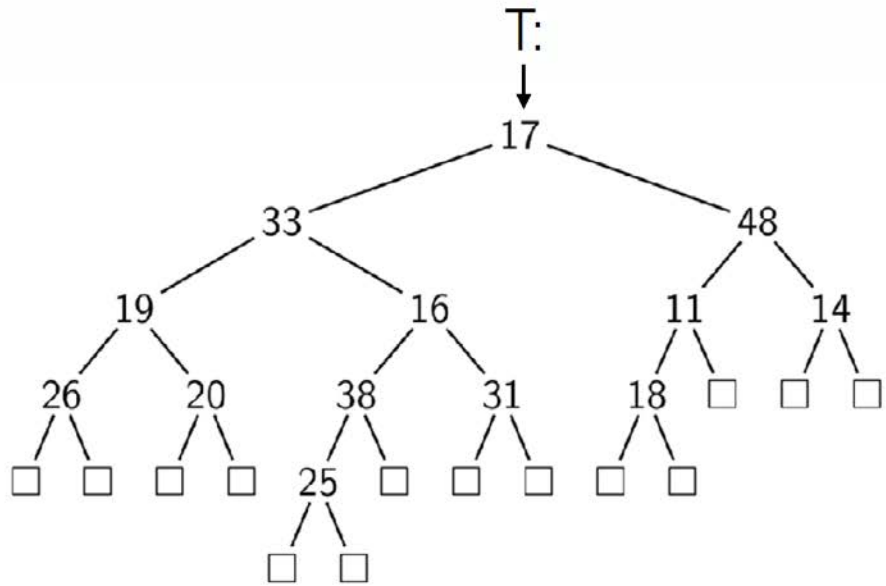
- Recursion is the natural way of calculating the **height**:




**function** HEIGHT($T$)
  **if** $T = null$ **then**
    **return** $-1$
  **else**
    **return** $max(\text{HEIGHT}(T.left), \text{HEIGHT}(T.right)) + 1$

# Review from Lecture 12

## Binary Tree Traversal

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- **Preorder** traversal visits the root, then the left subtree, and finally the right subtree.
- **Inorder** traversal visits the left subtree, then the root, and finally the right subtree.
- **Postorder** traversal visits the left subtree, the right subtree, and finally the root.
- **Level-order** traversal visits the nodes, level by level, starting from the root.

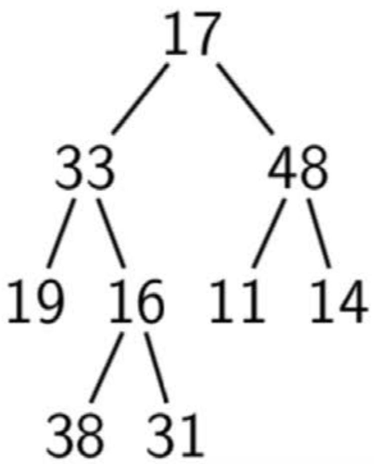
 

# Review from Lecture 12

## Inorder Traversal

Visit order: 19 33 38 16 31 17 11 48 14

**procedure** INORDERTRAVERSE($T$)
  **if** $T \neq null$ **then**
    INORDERTRAVERSE($T.left$)
    visit $T.root$
    INORDERTRAVERSE($T.right$)




Call Stack

# Build a Binary Search Tree Example

- Let's attempt to build a BST by inserting $[15, 8, 20, 5, 9, 17, 25, 29, 2, 6, 12, 10]$ one at a time.
- Remember: Let the root be $r$; then each element in the left subtree is smaller that $r$ and each element in the right sub-tree is larger than $r$. (For simplicity, we assume that all keys are different.)

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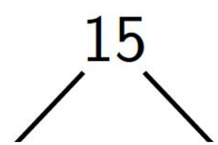
- Keep in mind we really mean:

$$root.val \quad = 15$$
$$root.left \quad = null$$
$$root.right \ = null$$



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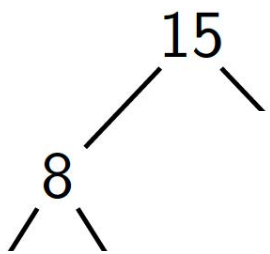
- Keep in mind we really mean:

$$root.val \quad = 8$$
$$root.left \quad = null$$
$$root.right \quad = null$$



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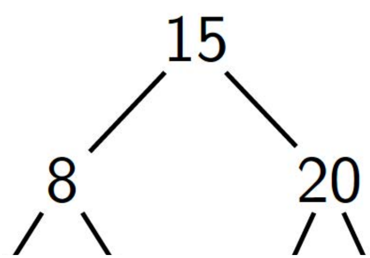

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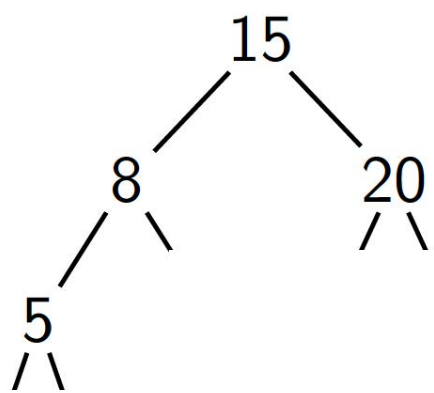

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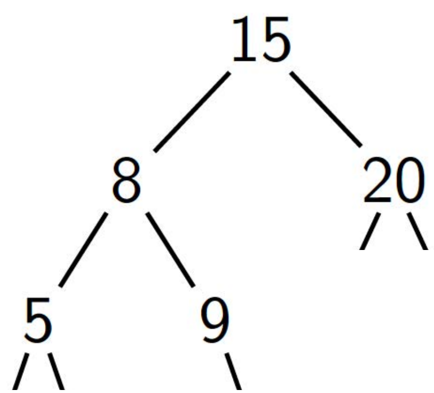

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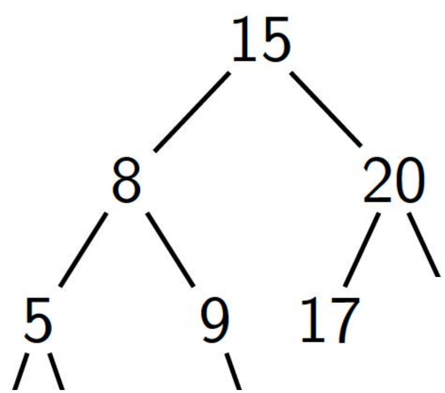

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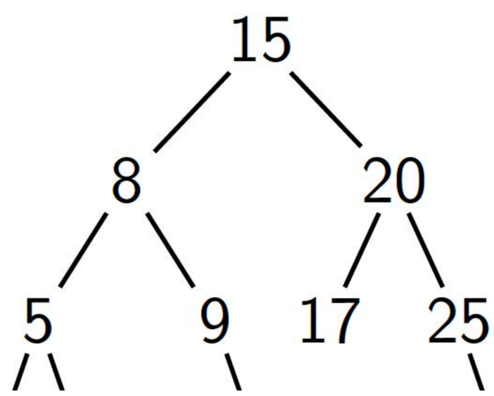

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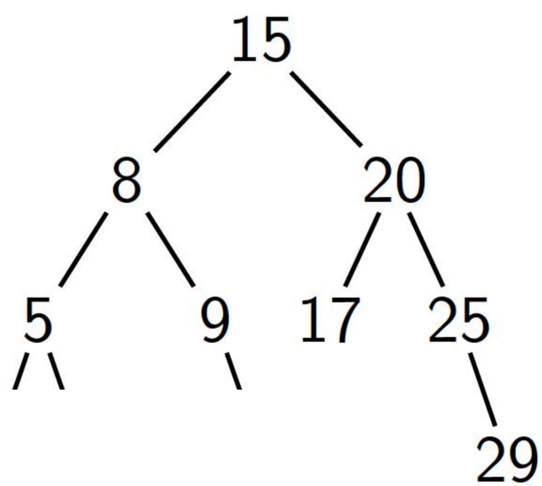

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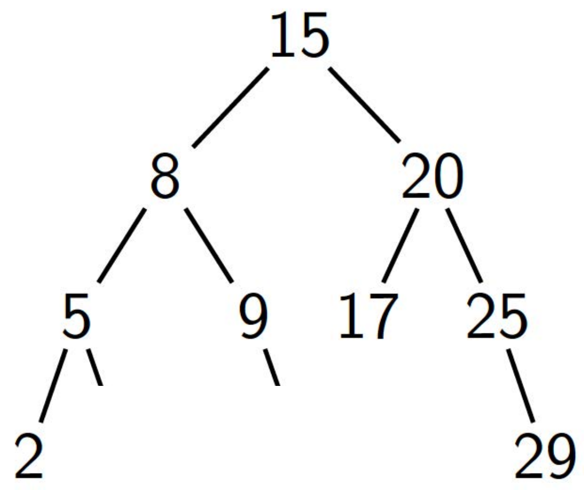

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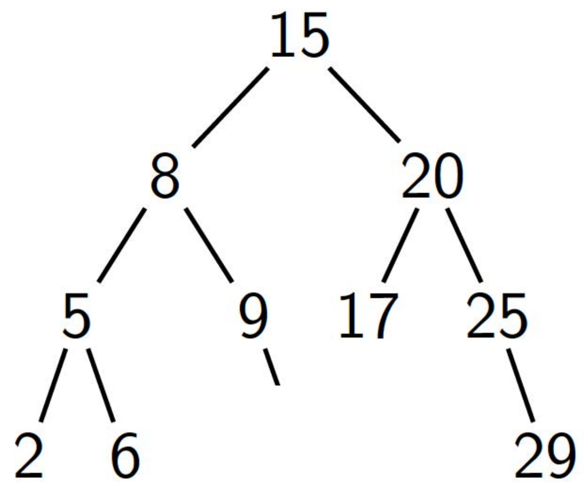

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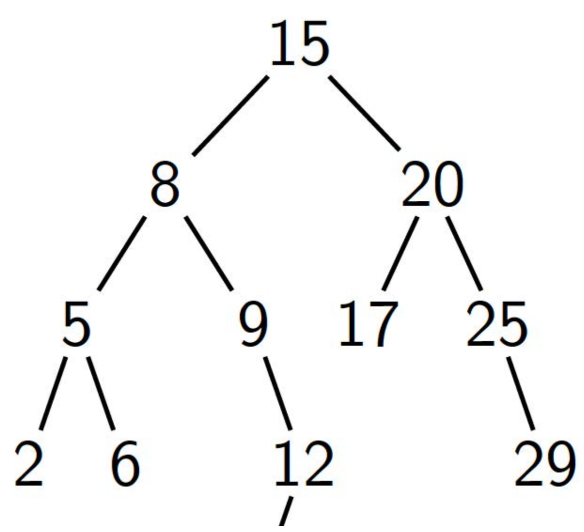

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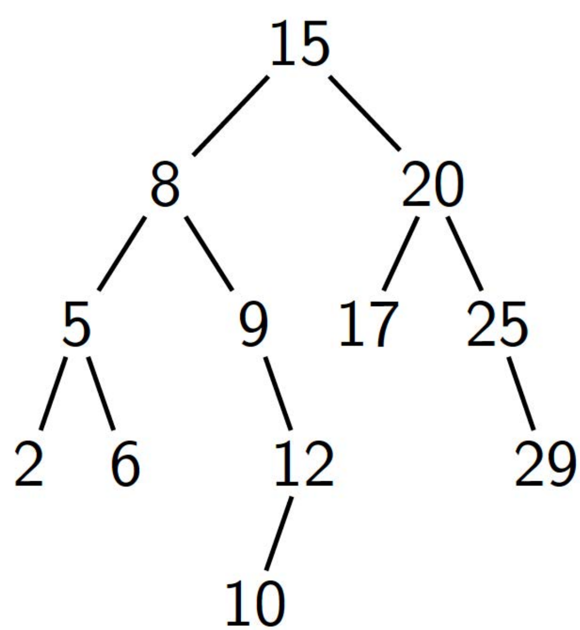
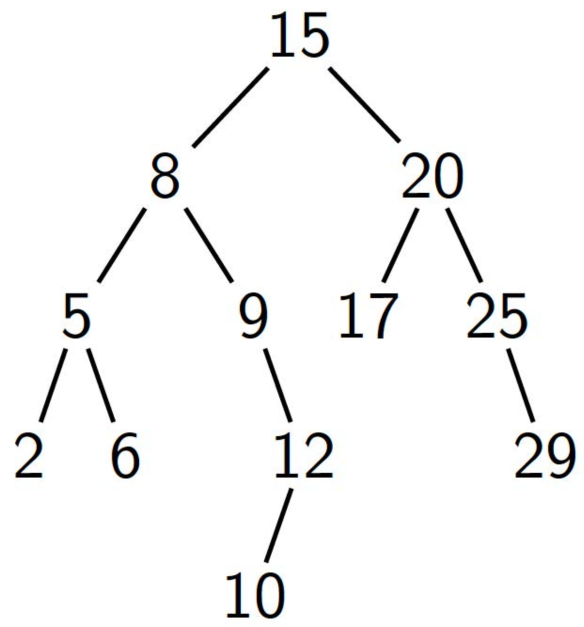


# Binary Search Trees

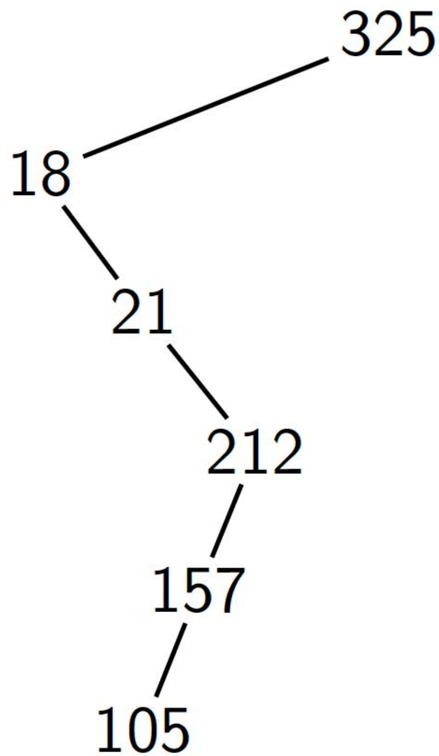
- BSTs are useful for search applications. To search for $k$ in a BST, compare against its root r. If $\mathrm{r} = k$, we are done; otherwise search in the left or right sub-tree, according to $\mathrm{k} < \mathrm{r}$ or $\mathrm{k} > \mathrm{r}$.

- If a BST with n elements is "reasonably" balanced, search involves in the worst case, $\Theta(\log n)$ comparisons.



# Binary Search Trees

- If the BST is not well balanced, search performance degrades, and may be as bad as linear search:



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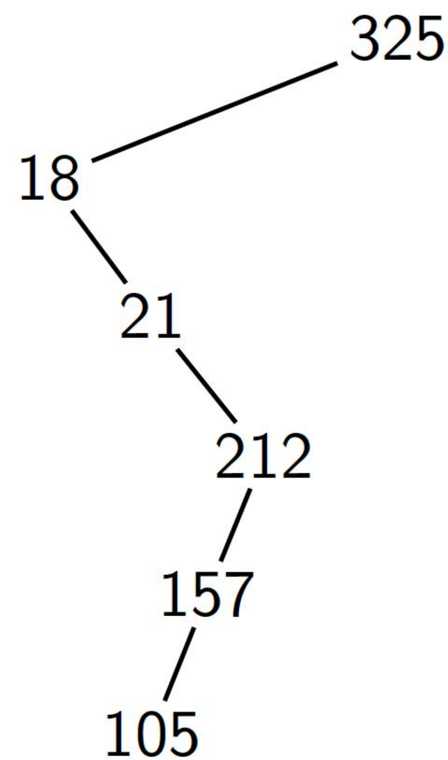
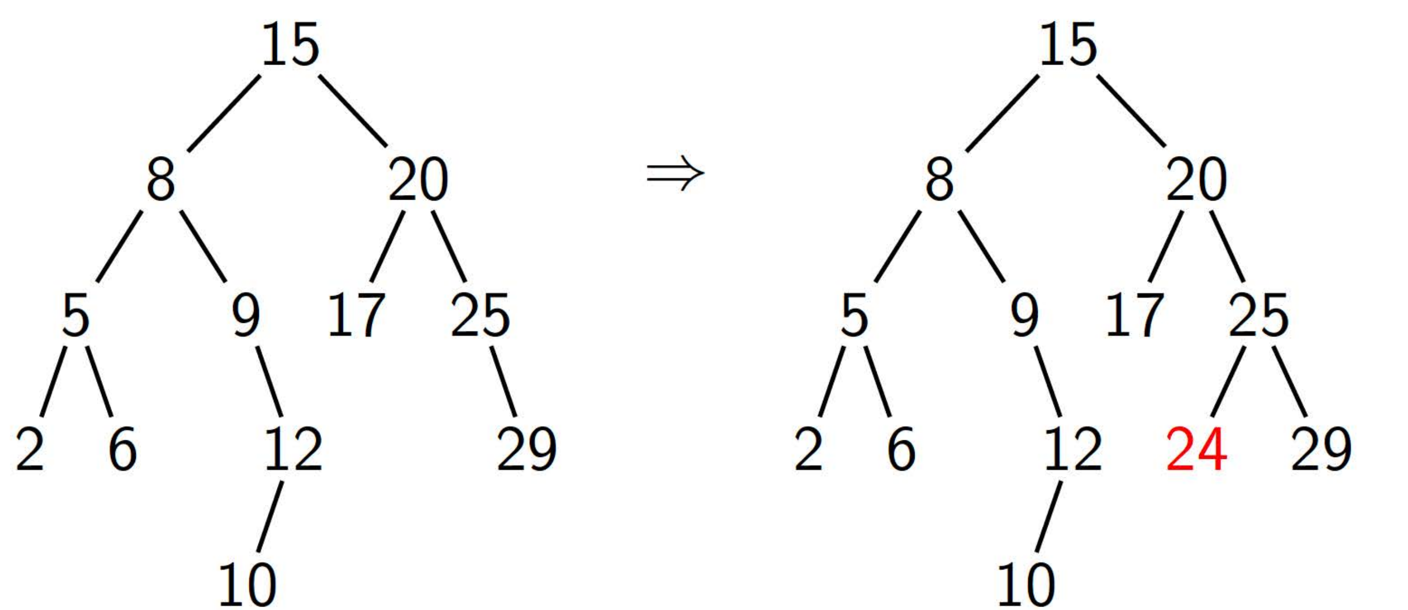



- Is this a valid BST?
  – We'll come back to this at the end

# Insertion in Binary Search Trees

- To insert a new element $k$ into a BST, we pretend to search for $k$.
- When the search has taken us to the fringe of the BST (we find an empty sub-tree), we insert $k$ where we would expect to find it.
- For example, inserting 24:



# BST Traversal Quiz

- Performing ……………...... traversal of a BST will produce its elements in sorted order.

# Review from Lecture 12

## Inorder Traversal

Visit order: 19 33 38 16 31 17 11 48 14

```
procedure InorderTraverse(T)
    if T ≠ null then
        InorderTraverse(T.left)
        visit T.root
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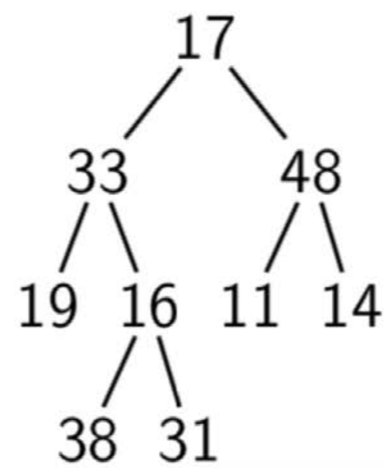


We go as far down the left as possible, visit the left subtree, visit the root, then finally visit the right subtree.

Call Stack

 

# BST Traversal Quiz

- Performing ……………...... traversal of a BST will produce its elements in sorted order.
- Example: build a BST by inserting $[8, 10, 5, 3, 13, 6]$ one at a time.

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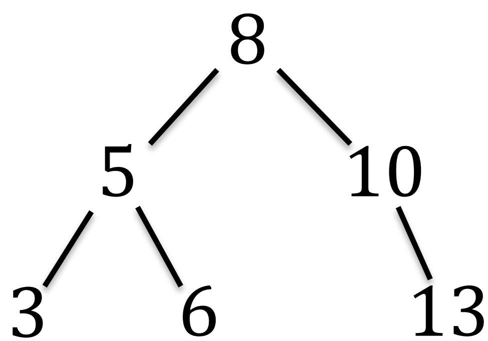

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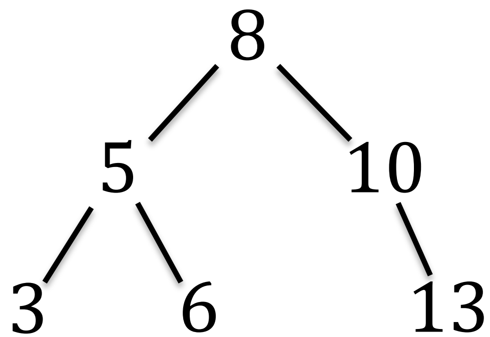
- Performing ……………....... traversal of a BST will produce its elements in sorted order.
- Example: build a BST by inserting [8, 10, 5, 3, 13, 6] one at a time.
- Now look at an Inorder Traversal for this BST



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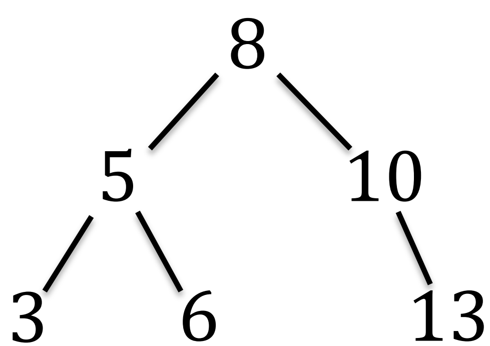


We go as far down the left as possible, visit the left subtree, visit the root, then finally visit the right subtree.

[3, 5, 6, 8, 10, 13]

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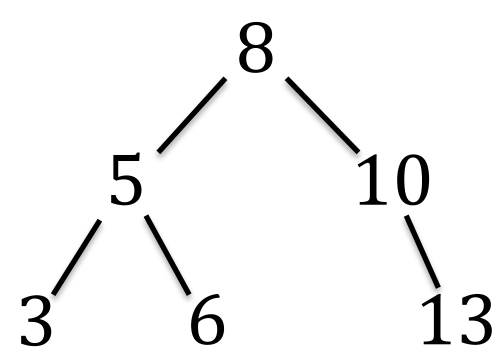


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# Binary Search Trees

- Try to build a BST by inserting $[325, 18, 21, 212, 157, 105]$ one at a time.

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325

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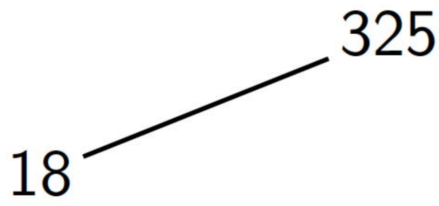

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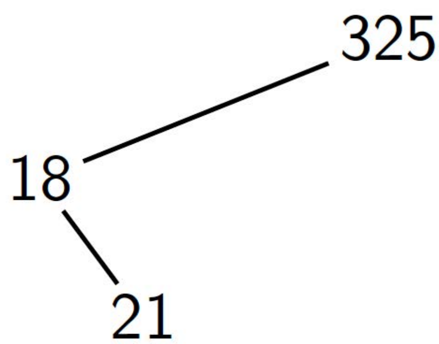

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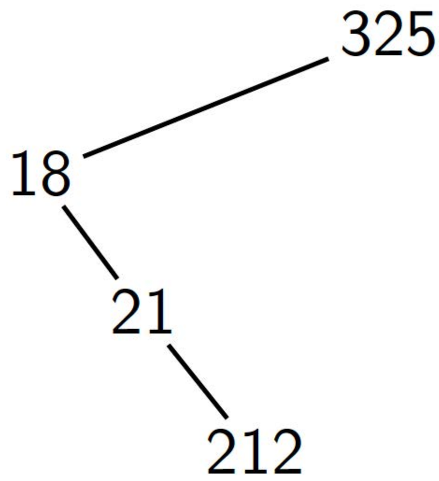

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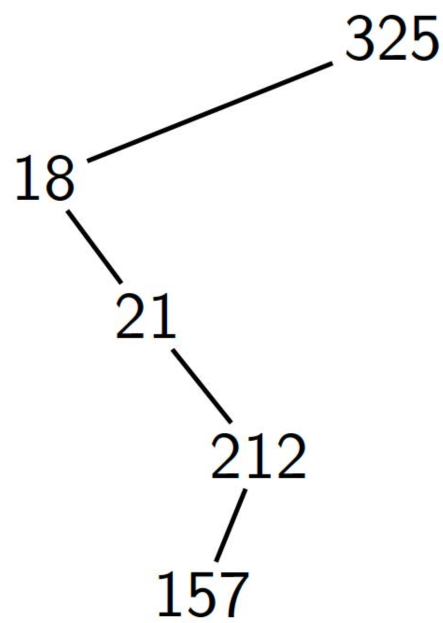

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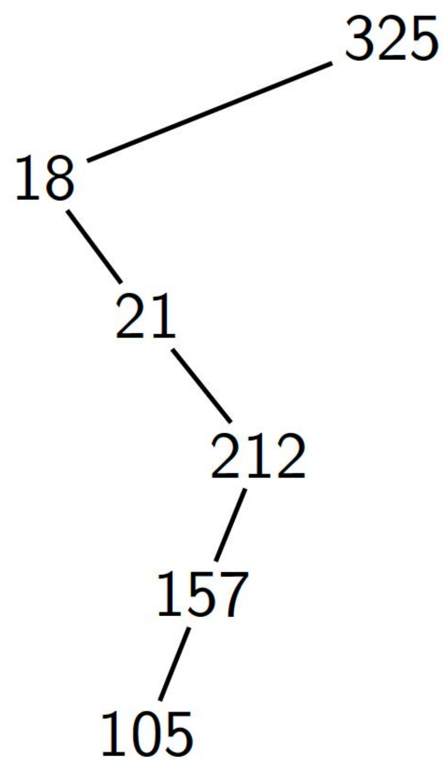

# Coming Up Next

- To optimise the performance of BST search, it is important to keep trees (reasonably) balanced.

- Next we will look at AVL trees and 2-3 trees (Levitin Section 6.3).