B.E. (Computer Science Engineering / Computer Technology / Computer Engineering / Information Technology) Fourth Semester (C.B.S.)

Discrete Mathematics & Graph Theory

P. Pages : 3 NRT/KS/19/3373/3378/3383/3388

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Illustrate your answers whenever necessary with the help of neat sketches.
9. Use of non programmable calculator is permitted.

1. a) For any two non-empty sets A and B, prove that **3**
$A-(B\cup C)=(A-B)\cap(A-C)$

b) Write the inverse, converse and contrapositive of "If x is boss then x is bad". **3**

c) Using principle of mathematical induction, show that for all positive integer 'n' n^3+2n is divisible by 3. **4**

OR

2. a) If my brother stands first in the class then **5**
I will give him a watch.
He stood first or I was out of station.
I did not give him a watch.

Therefore, I was out of station
Check the validity of the above argument

b) If $U=\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, **5**
$A=\{1, 5, 6, 7, 8\}$, $B=\{0, 1, 6, 7\}$, $C=\{1, 2, 3, 5, 8\}$ then verify that
i) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$
ii) $(A\cap B)'=A'\cup B'$

3. a) Let R be the relation defined on the set of ordered pairs of positive integers defined by $(x, y)\ R\ (u, v)$ iff $xv = yu$, then show that 'R' is an equivalence relation. **6**

b) For the relation $R=\{(1,2), (2,3), (3,4), (2,1)\}$. Find the relation matrix of R, draw the graph of R and also find the transitive closure of R. **6**

c) Give $M_R=\begin{bmatrix}1&0&1\\1&1&0\\1&1&1\end{bmatrix}$, $M_S=\begin{bmatrix}1&0&0&1&0\\1&0&1&0&1\\0&1&0&1&0\end{bmatrix}$ **6**

Find R, S, $R\circ S$, $\tilde{R}$, $\tilde{S}$, $S\circ R$ and prove that $M_{\tilde{R}\circ\tilde{S}}=M_{\tilde{S}\circ\tilde{R}}$.

OR

4. a) Let $X = \{\text{ball, bed, dog, egg, let}\}$ and $R = \{(x, y) / x, y \in X, xR_y$ x and y contains some common letter$\}$ 6
Find M_R, draw the graph of R and prove that R is compatible.

b) Using properties of characteristic function prove that 6
i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
ii) $(A')' = A$

c) If $f : X \to Y$ and $g : Y \to Z$ then show that - 6
i) If gof is one-one then f is one-one
ii) If gof is onto then g is onto

5. a) Prove that fourth roots of unity forms an abelian group under multiplication. 6

b) Show that the set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, where $\alpha \in R$, forms a monoid. 6

OR

6. a) Prove that the intersection of two normal subgroups of a group G is a normal subgroup of G. 6

b) A non-empty subset H of a group G is a subgroup of G iff 6
i) $a, b \in H \Rightarrow a \cdot b \in H$
ii) $a \in H \Rightarrow a^{-1} \in H$

7. a) If R is a ring such that $a^2 = a, \forall\, a \in R$, then show that 6
i) $a + a = 0, \forall\, a \in R$
ii) $a + b = 0 \Rightarrow a = b, \forall\, a, b \in R$
iii) R is a commutative ring.

b) Show that the set S of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ where $a, b \in R$ is a field w.r.to matrix addition and multiplication. 6

OR

8. a) Construct switching circuit for the Boolean expression. 6
$A \cdot B + A \cdot B' + A' \cdot B'$ simplify this and construct an equivalent circuit. Verify the equivalent circuit by truth table.

b) Let (L_1, D_6) and $L_2 = (\rho(s), s)$ be two letters, where $D_6 = \{1, 2, 3, 6\}$ and $S = \{a, b\}$. Then show that L_1 and L_2 are isomorphic. 6

9. a) Draw the digraphs corresponding to adjacency matrices. 6

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

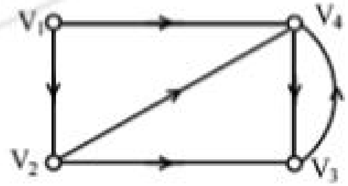
Prove that diagraphs of A and B are isomorphic.

b) Draw a diagraph corresponding to the adjacency matrix $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ and interpret the results AA^T, A^TA, A^2, A^3, A^4. **6**

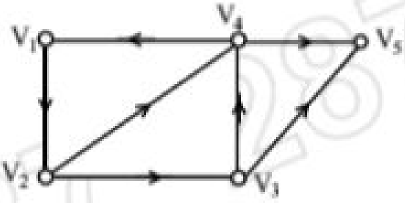
c) Draw a tree for the relation $R = \{(1, 2), (1, 3), (1, 4), (2, 5), (4, 6), (4, 7)\}$ on a set $A = \{1, 2, 3, 4, 5, 6, 7\}$. Also give the corresponding binary tree. **6**

OR

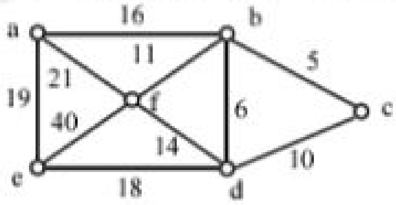
10. a) Give three different elementary paths from V_1 to V_3 for the diagraph given below. What is the shortest distance between V_1 and V_3 ? Is there any cycle in the graph ? **6**



b) Find indegree and outdegree of each node of the graph given below. Give all elementary cycles of this graph list all the nodes which are reachable from other nodes. **6**



c) Using Prim's algorithm, find a minimal spanning tree for the given graph. **6**



11. a) Solve the recurrence relation $a_n = 3a_{n-1} + 2$, $a_0 = 1$. **5**

b) Find the minimum number of students in a class to be sure that four of them are born in the same month. **5**

OR

12. a) Find the generating function of n^2, $n \geq 0$. **5**

b) Prove that $C(n, r) = C(n-1, r-1) + C(n-1, r)$ **5**

****