B.E. (Computer Science Engineering / Computer Technology / Computer Engineering / Information Technology) Fourth Semester (C.B.S.)

Discrete Mathematics & Graph Theory

P. Pages : 3 **AHK/KW/19/2137/2142/2147/2152**

Time : Three Hours * 1 1 7 5 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Illustrate your answers whenever necessary with the help of neat sketches.
10. Use of non programmable calculator is permitted.

1. a) Prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$ **5**

b) Prove by Mathematical induction that $n^3 + 2n$ is divisible by 3. **5**

OR

2. a) Check the validity of the following argument If it rains today, then we will not have a party today. It we do not have a party today, then we will have a party tomorrow. Therefore, if it rains today, then we will have a party tomorrow. **5**

b) Prove that $\sqrt{2}$ is an irrational number **5**

3. a) If Z is a set of integers and $a, b \in Z$, a R b if $b = a^r$ for some positive integer r, show that R is a partial order relation. **6**

b) Let A = {1, 2, 3, 4, 5, 6}. Define a relation R on A such that **6**
R = {(x y) / x + y is a divisor of 24}
Find i) Relation matrix R
ii) Relation matrix of $R \circ R$
iii) Draw digraph of R and $R \circ R$

c) If $f : A \to B$ and $g : B \to C$ both f and g are one-one onto then show that $g \circ f$ is also one – one onto and $(g \circ g)^{-1} = f^{-1} \circ g^{-1}$. **6**

OR

4. a) Let $A = R \times R$. A relation R on A is defined as (a, b) R (c, d) iff $a^2 + b^2 = c^2 + d^2$ **6**
Show that R is an equivalence relation.

b) For any integer n, S_n denotes the set of all divisors of n and D denotes the division operator. Draw the Hasse diagram for (i) n= 30 (ii) n = 36. **6**

c) Using characteristic function, show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **6**

5. a) Prove that fourth roots of unity forms a group under multiplication. **6**

b) Prove that the intersection of two subgroups of a group G is a subgroup of G. **6**

OR

6. a) Show that the multiplication group G = {1, i, −1, −i} is isomorphic to the group G^1 = {0, 1, 2, 3} with addition modulo 4 as composition. **6**

b) Show that if f is homomorphism of G into G^1 with kernel K then Kernel K is normal subgroup of G. **6**

7. a) Show that $(I, \oplus, \odot)$ is a commutative ring with identity whose operation $\oplus$ and $\odot$ are defined 'as **6**

for any $a, b \in I$ $a \oplus b = a + b - 1$

and $a \odot b = a + b - ab$

Where I is a set of integer

b) Construct the switching circuit for the Boolean expression. **6**
$(A \cdot B) + C + (A' \cdot C')$
Simplify this and construct an equivalent simplified circuit.

OR

8. a) Define lattice, Draw the Hasse diagram of lattices D_{24} and D_{30}. **6**

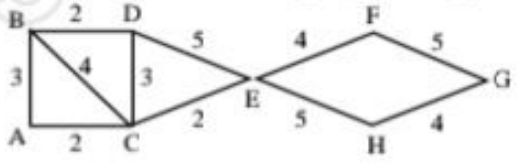
b) Show that the set {0, 1, 2, 3, 4, 5, 6} is a cumulative ring w r to addition module 7 and multiplication module 7 as the composition. **6**

9. a) Draw the digraph corresponding to adjacency matrix A, B, A^T& B^T where. **6**

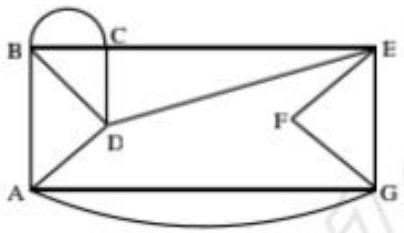
$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Show that the digraph corresponding to A^T& B^T are isomorphic.

b) Find the minimal spanning tree of the following graph using prim's algorithm. **6**



c) Define Eulerian path and Eulerian circuit. Show that the graph giren below is an Eulerian graph and circuit. **6**



OR

10 a) Draw the tree for the algebraic expression. **6**

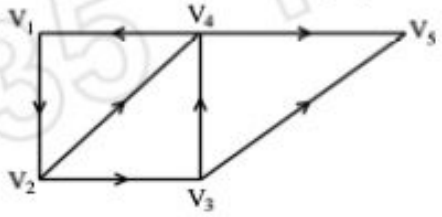
i) $(a+5)*[\{(7+b)+c\}/(g+d)]$

ii) $\left(2x+\left(3-4x\right)\right)+\left(x-\left(3\times 11\right)\right)$

b) Draw the diagraph represented by the given adjacency matrix and interprete **6**

AA^T, A^TA, A^2 Where $A=\begin{bmatrix}0&1&0&1\\1&0&1&0\\0&1&0&1\\1&0&1&0\end{bmatrix}$

c) Find the indegrees and outdegrees of the node of the graph. Give all ellementry cycle of this graph obtain an acyclic digraph by deleting the edge of the given graph. List all the nodes which are reachable from another node of the graph. **6**



11. a) Show that if any five numbers from 1 to 8 are chosen then two of them add up to 9 using pigeon hole principle. **5**

b) Solve the recurrence relation **5**

$2a_n - 7a_{n+1} + 3a_{n-2} = 2^n$

OR

12. a) Find the generating function for the sequence **5**
1, 0, -1, 0, 1, 0, -1, 0, 1, ------

b) How many words of 3 different letter can be formed from the letters of word **5**
i) MAST ii) COMPUTER

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