NagpurStudents

www.nagpurstudents.org

B.E. (Computer Science & Engineering (New) / Computer Technology) Third Semester (C.B.S.)

Applied Mathematics - III

P. Pages : 3 **NIR/KW/18/3317/3322**

Time : Three Hours * 1 8 6 0 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Illustrate your answers whenever necessary with the help of neat sketches.
10. Use of non programmable calculator is permitted.
11. Use of normal distribution table is permitted.

1. a) If $L\{f(t)\} = F(s)$, then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_S^\infty \bar{f}(s)\,dS$ and hence Find $L\left\{\frac{1-\cos t}{t}\right\}$ **6**

b) Find $L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$ by convolution theorem. **6**

OR

2. a) Express $f(t) = \begin{cases} t-1, & 1<t<2 \\ 3-t, & 2<t<3 \end{cases}$, in terms of unit step function and hence find its Laplace transform. **6**

b) Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$, given $y(0) = 1$ by using Laplace Transform. **6**

3. a) Find the Fourier series with period 2 represent $f(x) = 2x - x^2$ in the range (0,2) **6**

b) Using the Fourier integral, show that **6**

$$\int_0^\infty \frac{w\sin(xw)}{1+w^2}\,dw = \frac{\pi}{2}e^{-x},\ x > 0$$

OR

4. a) Find the Fourier Transform of $e^{-|x|}$, hence show that $\int_0^\infty \frac{\cos \lambda x}{1+\lambda^2}\,d\lambda = \frac{\pi}{2}e^{-|x|}$. **6**

b) Obtain half range sine series for $f(x) = \pi x - x^2$ in the interval $(0,\pi)$. **6**

5. a) If $Z\{f(n)\} = F(z)$, then prove that $Z\left\{\frac{f(n)}{n+k}\right\} = z^k \int_z^\infty \frac{F(z)}{z^{k+1}}\,dz$. **6**

Also find $Z\left\{\frac{1}{n+1}\right\}$

b) Find $Z^{-1}\left\{\frac{z^2}{(z-1)(z-3)}\right\}$ by convolution theorem. **6**

OR

6. a) Find Z-transform of $a^n \cos n\theta$ and $a^n \sin n\theta$ **6**

b) Solve the difference equation by Z-transform. **6**
$y_{n+2} + 5y_{n+1} + 6y_n = 6^n,\ y_0 = 0,\ y_1 = 1$

7. a) If $u + v = e^x [\cos y + \sin y]$, find the analytic function f(z) in term of z. **7**
Also find u and v.

b) If $f(a) = \oint_C \frac{3z^2 + 7z + 1}{(z-a)}dz$ where C is a circle |z| = 2. Find the value of **7**

i) $f(3)$ ii) $f'(1-i)$ iii) $f''(1-i)$

OR

8. a) Expand $f(z) = \left(z^2 + 4z + 3\right)^{-1}$ by Laurentz series valid for **7**

a) $1 < |z| < 3$ b) $0 < |1 + z| < 2$
c) $|z| < 1$ d) $|z| > 3$

b) Evaluate $\int_0^\pi \frac{d\theta}{3 + 2\cos\theta}$ by contour integration. **7**

9. a) Are the following vectors are linearly dependent? If so find the relation between thm **6**
$x_1 = (1,2,1),\ x_2 = (2,1,4)\ x_3 = (4,5,6)\ x_4 = (1,8,-3)$

b) Find the modal matrix B corresponding to matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ and verify $B^{-1}AB$ is a diagonal form. **6**

c) If $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ then prove that $\text{Sec}^2 A - \tan^2 A = I$, by using Sylvester's theorem. **6**

OR

10. a) Solve $\frac{d^2x}{dt^2} + x = 0$, given $x(0) = 1,\ x'(0) = 1$ by matrix method. **6**

b) Determine largest eigen value and eigen vector of the matrix **6**

$$A = \begin{bmatrix} -4 & -5 \\ 1 & 2 \end{bmatrix}$$

c) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and hence find A^{-1}. **6**

11. a) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck driver. The respective probabilities of an accident are 0.01, 0.03 and 0.15. One of ensured person meet an accident. What is the probability that he is scooter driver, car driver or truck driver? **6**

b) Can the function $F(x) = \begin{cases} C\left(1 - x^2\right), & 0 \le x \le 1 \\ 0 \quad , & \text{otherwise} \end{cases}$ be a distribution function? Explain. **6**

OR

12. a) Find the moment generating function for the random variable X having density function **6**

$$f(x) = \begin{cases} e^{-x} & , x \ge 0 \\ 0 & , x < 0 \end{cases}$$

also find first four moments about origin.

b) A random variable X has density function $f(x) = \frac{C}{x^2 + 1}, -\infty < x < \infty$ **6**

Find i) The constant C

ii) $p\left(1/3 \le X^2 \le 1\right)$

iii) The distribution function.

****

NagpurStudents

www.nagpurstudents.org

The best time to plant a tree was 20 years ago. The second best time is now.

~ Chinese Proverb