B.E. (Computer Engineering / Information Technology) Third Semester (C.B.S.)

Applied Mathematics - III

P. Pages : 3 | **NRJ/KW/17/4382/4387**

Time : Three Hours | Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_S^\infty \bar{f}(s)\,ds$, Hence find $L\left\{\frac{\sin 2t}{t}\right\}$. **7**

b) Find $L^{-1}\left\{\frac{1}{(s-2)(s-3)^2}\right\}$ using convolution theorem. **7**

OR

2. a) Express $f(t) = \begin{cases} e^{-t} & , \quad 0 < t < 3 \\ 0 & , \quad t > 3 \end{cases}$ in terms of unit step function and find its laplace transform. **7**

b) Solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t$ where $x(0) = 0$ and $x'(0) = 1$. **7**

3. Find Fourier transform of f(x), **7**

$$f(x) = \begin{cases} 1 - x^2 & , \quad |x| \le 1 \\ 0 & , \quad |x| > 1 \end{cases}$$

Hence find $\int_0^\infty \frac{\sin x - x\cos x}{x^3} \cdot \cos\frac{x}{2}\,dx$

OR

4. Solve the integral equation **7**

$$\int_0^\infty f(x)\cos\alpha x\,dx = \begin{cases} 1-\alpha & , \quad 0 \le \alpha \le 1 \\ 0 & , \quad \alpha > 1 \end{cases}$$

and hence evaluate $\int_0^\infty \frac{\sin^2 t}{t^2}dt$

5. a) Find Z-transform of cos nθ and using this find Z transform of $a^n \cos n\theta$. **7**

b) Solve the difference equation **7**

$u_{n+2} + 4u_{n+1} + 3u_n = 2^n$, $u_0 = 0$, $u_1 = 1$

OR

6. a) Find inverse Z-transform of $\left\{\frac{Z^2}{(Z-1)(Z-3)}\right\}$ using convolution theo. **7**

b) Prove that $Z\{n^P\} = -Z\frac{d}{dz}z\{n^{p-1}\}$ where P is any positive integer and hence deduce **7**

$Z\{n\} = \frac{Z}{(Z-1)^2}$

7. a) Are the following vectors linearly dependent, if so find relation between them. **6**

$X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$, $X_4 = (-3, 7, 2)$

b) Find eigen value, eigen vector and model matrix for $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **6**

c) Use Sylvester's theorem to prove that $\sin^2 A + \cos^2 A = I$ where $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ **6**

OR

8. a) Verify Caley Hamilton theorem for matrix **6**

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find A^{-1}

b) Solve $\frac{d^2y}{dx^2} + 4y = 0$ given that y(0) = 8 $y'(0) = 0$ by matrix method. **6**

c) Find largest eigen value and corresponding eigen vector for matrix $A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ **6**

9. a) Three machines A, B, C produce respectively 60% 30% and 10% of the total no. of items in a factory. The percentage of defective out put of these. Machines are respectively 2% 3% and 4% An item is selected at random and is found defective. Find the probability that the item was produced by Machine C. **7**

b) The distribution function of random variable X is $F(x) = \begin{cases} 1-e^{-\alpha x} & x \ge 0 \\ 0 & x < 0 \end{cases}$ **7**

Find :

i) Density fun. f(x) ii) $P(X > 2)$

iii) $P(-3 < x < 4)$

OR

10. a) An electric device consist of two component. Let X and Y be times of failure of first and second component respectively. Assume that X and Y has the density function. **7**

$$f(x,\ y) = \begin{cases} 4e^{-2(x+y)} & , \quad x \ge 0,\ y \ge 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

i) Are X and Y independent

ii) What is probability that the first component will have lifetime of 2 years or longer.

b) Let X and Y be two random variables with mass function **7**

$$f(x,\ y) = \begin{cases} \dfrac{x+2y}{27} & , \quad x = 0, 1, 2,\ y = 0, 1, 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find ;

i) Find marginal probability function of x & y

ii) Conditional probability function y given x and x given y.

11. a) A density function of random variable X is $f(x) = \begin{cases} 2e^{-2x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$ **7**

Find :

i) E(X) ii) Var(X)

iii) σ_x iv) $E\left[(x-1)^2\right]$

b) Find : **6**

i) Moment generating function and find first two moment about origin.

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

OR

12. a) The joint density function of two random variables X and Y is - **7**

$$f(x,\ y) = \begin{cases} x+y & , \quad 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find :

i) Conditional expectation of X given Y

and ii) Conditional variance of X given Y.

b) State the postulates of Poisson process and prove that a Poisson process follows a Poisson distribution. **6**

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