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B.E. (Computer Science & Engineering (New) / Computer Technology) Third Semester (C.B.S.)

Applied Mathematics

P. Pages : 3 NRJ/KW/17/4372/4377

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = F(s)$ then show that **6**

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\,ds$$

hence find $L\left\{\frac{\sin t}{t}\right\}$.

b) Find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ by using convolution theorem. **6**

OR

2. a) Express $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$ **6**

in terms of unit step function and find Laplace transform.

b) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$ **6**

given $y(0) = 0,\ y'(0) = 1$

by using Laplace transform method.

3. a) Find the Fourier series to represent **6**

$f(x) = x^2 - 2,\ -2 \le x \le 2$.

b) Find Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$. **6**

OR

4. a) Using the Fourier Cosine integral show that **6**

$$\int_0^\infty \frac{\cos \lambda x}{1+\lambda^2}\,d\lambda = \frac{\pi}{2}e^{-x}$$

b) Find the half range cosine series for sin x when $0 < x < \pi$, hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+.......=\frac{\pi}{4}$. **6**

5. a) If $z\{f(n)\}=F(z)$ then show that **6**

$$z\left\{\frac{f(n)}{n+k}\right\}=z^k\int_z^\infty \frac{F(z)}{z^{k+1}}dz$$

hence find $z\left\{\frac{1}{n+1}\right\}$.

b) Prove that $\frac{1}{n!}*\frac{1}{n!}=\frac{2^n}{n!}$ **6**

where $*$ is a convolution operation.

OR

6. a) Find Z-Transform of $\frac{(n+1)(n+2)}{2!}a^n$. **6**

b) Solve $y_{n+2}-2\cos\alpha . y_{n+1}+y_n=0$ given $y_0=0, y_1=1$ by using Z-Transform. **6**

7. a) If $f(z)$ is analytic function with constant modulus. Show that $f(z)$ is constant. **7**

b) Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}dz$ where C is a circle $|z-i|=2$ by Cauchy Integral formula. **7**

OR

8. a) Evaluate $\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}d\theta$ by using Contour Integration. **7**

b) Expand in Taylor's series $f(z)=\frac{z}{(z+1)(z+2)}$ about Z = 2. Also find the region of convergence. **7**

9. a) Investigate the linear dependence of vectors **6**
$X_1=(2, -1, 3, 2), X_2=(1, 3, 4, 2), X_3=(3, -5, 2, 2)$
and if so find the relation.

b) Find the modal matrix B corresponding to matrix $A=\begin{bmatrix}1 & 2\\3 & 2\end{bmatrix}$ and verify that $B^{-1}AB$ is diagonal form. **6**

c) By using Cayley Hamilton's theorem find A^8 if $A=\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$. **6**

OR

10. a) If $A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$ **6**

verify $2\sin A = (\sin 2)A$

by Sylvester's theorem.

b) Find the largest eigen value and corresponding eigen vector for the matrix **6**

$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ by iteration method.

c) Solve $\frac{d^2x}{dt^2} + 4x = 0,\ x(0) = 1,\ x'(0) = 0$ by matrix method. **6**

11. a) Each of the three identical Jewellery boxes has two drawers. In each drawer of the first box there is a gold watch. In each drawer of the second box there is a silver watch. In one of the drawer of the third box there is a gold watch while in the other there is silver watch. If we select a box at random, open one of the drawer and find it to contain a silver watch. What is the probability that the other drawer has gold watch. **6**

b) The distribution function of a random variable X is given by **6**

$$F(x) = \begin{cases} cx^3 \ , & 0 \le x < 3 \\ 1 \ , & x \ge 3 \\ 0 \ , & x < 0 \end{cases}$$

Find :

i) Probability density function

ii) C

iii) $p(x > 1)$

OR

12. a) A random variable X can assume the value 1 and –1 with probability $\frac{1}{2}$ each. **6**

Find (i) moment generating function (ii) first two moments about origin and about mean.

b) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused. **6**

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The secret of getting ahead is getting started.

~ Mark Twain