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B.E. (Computer Science & Engineering (New) / Computer Technology) Third Semester (C.B.S.)

Applied Mathematics

P. Pages : 3 | NJR/KS/18/4372/4377

Time : Three Hours | Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.
10. Use of normal distribution table is permitted.

1. a) Find Laplace Transform of $\frac{\sin^2 t}{t}$ and hence evaluate $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} \cdot dt$. **6**

b) Use convolution theorem to find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ **6**

OR

2. a) Express the function f(t) in terms of unit step function & find its Laplace Transform : **6**

$$f(t)=\begin{cases} t^2, & 0<t<1 \\ 4t, & t>1 \end{cases}$$

b) Solve $\frac{d^2y}{dt^2}+9y=\sin t,\ y(0)=1,\ y\left(\frac{\pi}{2}\right)=-1,$ using Laplace Transform. **6**

3. a) Obtain Fourier series for **6**

$$f(x)=\begin{cases} \pi x, & 0\le x\le 1 \\ \pi(2-x), & 1\le x\le 2 \end{cases}$$

Hence show that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+---$

b) Show that the Fourier sine integral of $f(x)=\begin{cases} \pi/2, & 0<x<\pi \\ 0, & x>\pi \end{cases}$ **6**

is $\int_0^\infty \frac{(1-\cos\pi\lambda)\sin\lambda x}{\lambda}\cdot d\lambda$.

OR

4. a) Obtain half range sine series for $f(x) = \pi x - x^2$ in the interval $(0, \pi)$. **6**

b) Find the Fourier cosine transform of $f(x) = \dfrac{e^{-ax}}{x}$, $a > 0$. **6**

5. a) If $z\{f(n)\} = F(z)$, then prove that **6**

$$z\{f(n+k)\} = z^k \left[F(z) - \sum_{i=0}^{k-1} f(i) \cdot z^{-i} \right]$$

b) Using convolution theorem, find inverse Z-Transform of $\dfrac{z^2}{(z-a)(z-b)}$ **6**

OR

6. a) Find inverse Z-transform of $\dfrac{z^3 - z^2 + z}{(z+2)\left(z^2 - 1\right)}$ **6**

b) Solve the difference equation $y_{n+2} + 5y_{n+1} + 6y_n = 6^n$, $y_0 = 0$, $y_1 = 1$, using Z-Transform. **6**

7. a) Given harmonic function $u = e^{-x}(x \sin y - y \cos y)$. Find ν such that f(z)=u+iν is analytic & express f(z) in terms of z. **7**

b) Expand $f(z) = \left(z^2 + 4z + 3\right)^{-1}$ by Laurent's series valid for : **7**

i) $1 < |z| < 3$ ii) $|z| < 1$ iii) $|z| > 3$

OR

8. a) Find the value of $\oint \dfrac{12z - 7}{(z-1)^2 (2z+3)} \cdot dz$, where C is a circle $|z + i| = \sqrt{3}$. **7**

b) Evaluate $\displaystyle\int_0^{2\pi} \frac{1}{5 - 4\sin\theta} \cdot d\theta$ by contour integration. **7**

9. a) Determine modal matrix for **6**

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

b) Are the following vectors linearly dependent? If so, find the relation between them $X_1 = [1,1,1,3]$, $X_2 = [1,2,3,4]$, $X_3 = [2,3,4,7]$. **6**

c) By using Cayley Hamilton's theorem find A^8, if $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$. **6**

OR

10. a) If $m=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$, find the value of m^2-3m+I and verify the result by Sylvester's theorem. **6**

b) Find the largest eigen value and corresponding eigen vector for the matrix **6**
$A=\begin{bmatrix}-4 & -5\\ 1 & 2\end{bmatrix}$.

c) Solve $\dfrac{d^2x}{dt^2}+4x=0,\ x(0)=1,\ x'(0)=0$ by matrix method. **6**

11. a) Three machines A, B and C produce respectively 60%, 30% and 10% of the total number of times of a factory. The percentages of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. **6**

b) A random variable X has density function **6**

$$f(x)=\begin{cases} kx^2, & 1\le x\le 2\\ kx, & 2<x<3\\ 0, & \text{otherwise}\end{cases}$$

Find k & the distribution function.

OR

12. a) Find mean, variance and moment generating function for exponential distribution **6**

$$f(x)=\begin{cases}\alpha e^{-\alpha x}, & x>0\\ 0, & x\le 0\end{cases}$$

b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. **6**

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Itâ€™s hard to beat a person who never gives up.

~ Babe Ruth