B.E. Third Semester (Computer Engineering / Information Technology) (C.B.S.)

Applied Mathematics - III

P. Pages : 4 **NKT/KS/17/7242/7247**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

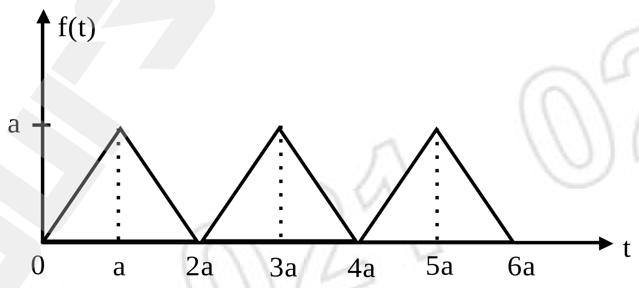
1. a) If $L[f(t)] = \overline{f}(s)$ then $L\left[\int_0^t f(u)\,du\right] = \frac{\overline{f}(s)}{s}$ **7**

Hence find $L\left[\int_0^t \sin u.du\right]$.

b) Find $L^{-1}\left[\log\left(1+\frac{1}{S^2}\right)\right]$, and hence show that $L^{-1}\left[\frac{1}{S}\log\left(1+\frac{1}{S^2}\right)\right] = \int_0^t \frac{2}{x}(1-\cos x)\,dx$ **7**

OR

2. a) Find Laplace Transform of periodic function with period '2a' shown in following fig. **7**



b) Solve $\frac{d^2y}{dt^2} + y = 1$, given $y(0) = 1$ $y\left(\frac{\pi}{2}\right) = 0$ by using Laplace Transform. **7**

3. a) Find the Fourier transform of **7**

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

Hence find $\int_0^\infty \frac{\sin x}{x}\,dx$.

OR

4. a) Express $f(x)=\begin{cases}1, & 0\le x\le \pi\\ 0, & x>\pi\end{cases}$ **7**

as a Fourier sine integral and hence evaluate $\int_0^{\infty}\frac{1-\cos\pi\lambda}{\lambda}\sin x\lambda\, d\lambda$

5. a) Find $Z\left[n\,a^n\right]$ and hence find $Z\left[n^2 a^n\right]$. **7**

b) If $Z[f(n)]=\overline{f}(z)$, then show that $Z\left[\frac{f(n)}{n+p}\right]=Z^p\int_Z^{\infty}\frac{\bar{f}(Z)}{Z^{p+1}}dZ$ **7**

Hence find $Z\left[\frac{1}{n+1}\right]$.

OR

6. a) Use convolution theorem and find $Z^{-1}\left[\frac{Z^2}{(Z-1)(Z-3)}\right]$. **7**

b) Solve $y_{n+2}+3y_{n+1}+2y_n=\mu_n$ **7**

subject to $y_0=1, y_n=0, n<0$

where $\mu_n=\begin{cases}0, & n<0\\ 1 & n\ge 0\end{cases}$ by Z-transform method.

7. a) Investigate the linear dependence or independence of vectors. **6**

$X_1=(1,2,4),\ X_2=(2,-1,3),\ X_3=(0,1,2)\ X_4=(-3,7,2)$

b) Find the modal matrix B corresponding to matrix $A=\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}$ and verify $B^{-1}AB$ is a diagonal form. **6**

c) Find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ where **6**

$$A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$$

by Caylay Hamilton's theorem.

OR

8. a) Using Sylvester's theorem, verify $\log_e e^A = A$ **6**

where $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

b) Reduce the quadratic form $3x^2 + 3y^2 + 3z^2 + 2xy + 2xz - 2yz$ to the canonical form by orthogonal transformation. **6**

c) Solve the differential equation **6**

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0, \quad x(0) = 2$$

$$x'(0) = 0$$

by matrix method.

9. a) The content of urn I, II, III are as follows : 2 white, 2 black, 3 red, 2 white, 1 black, 1 red and 4 white, 5 black, 3 red balls respectively. One urn is chosen at random and two balls drawn, they happen tobe white and red. What is the probability that they come from urn I? **7**

b) A random variable X has the density function **7**

$$f(x) = \begin{cases} cx^2, & 1 \le x \le 2 \\ cx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) constant C (ii) $P(X > 2)$ (iii) $P\left(\frac{1}{2} < x < \frac{3}{2}\right)$.

OR

10. a) The joint probability function of X and Y is given by **7**

$$f(x, y) = \begin{cases} c(2x + y), & x = 0,1,2 \\ & y = 0,1,2,3 \\ 0, & \text{Otherwise} \end{cases}$$

Find (i) constant C (ii) $P(x \ge 1, y \le 2)$ (iii) The marginal probability function of X and Y.

b) Find the conditional density function of (i) X given Y (ii) Y given X for the distribution function. **7**

$$f(x, y) = \begin{cases} \frac{3(x^2 + y^2)}{2}, & 0 \le x \le 1, \; 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

11. a) A random variable X is expected value of $E\left[(X-1)^2\right] = 10$ and $E\left[(X-2)^2\right] = 6$ find (i) E(X) (ii) Var (X) (iii) σ_x S.D. of x. **6**

b) Find moment generating function of the random variable. **7**

$$X = \begin{cases} \frac{1}{2}, & \text{Probability } \frac{1}{2} \\ -\frac{1}{2}, & \text{Probability } \frac{1}{2} \end{cases}$$

OR

12. a) Let X and Y be joint density function **7**

$$f(x, y) = \begin{cases} x + y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

find (i) $E(x + y)$

(ii) The conditional expectation of X given Y and Y given X.

(iii) Conditional Variance of Y given X = 0.5

b) Suppose that the customers are arriving at a ticket counter according to a Poisson process with a mean rate of 2 per minutes. Then in an arrival of 5 minutes, find the probability that the number of customers arriving is (i) Exactly 5 (ii) Less than 4 (iii) greater than 3. **6**

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