* Important Formulae :

1] Arithmetic Mean ($\bar{x}$)

i) For ungrouped data : $\bar{x} = \dfrac{\Sigma fx}{N}$

ii) For grouped data : $\bar{x} = A + h\dfrac{\Sigma fu}{N}$

2] Standard Deviation : $\sigma = h\sqrt{\dfrac{1}{N}\Sigma fu^2 - \left(\dfrac{\Sigma fu}{N}\right)^2}$

3] Coefficient of Variation (C.V.) = $\dfrac{\sigma}{\bar{x}} \times 100$

4] Moments :

$$\mu_0 = 1$$
$$\mu_1 = 0$$
$$\mu_2 = \mu_2' - (\mu_1')^2$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 2(\mu_1')^4$$

i) Coefficient of Skewness = $\beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$

ii) Coefficient of Kurtosis = $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$

5] Coefficient of Correlation (r) :

$r = \dfrac{cov(x,y)}{\sigma_x \sigma_y}$, where :

$$\operatorname{cov}(x,y) = \frac{1}{N}\Sigma xy - (\bar{x}\bar{y})$$

$$\sigma_x^2 = \frac{1}{N}\Sigma x^2 - (\bar{x})^2$$

$$\sigma_y^2 = \frac{1}{N}\Sigma y^2 - (\bar{y})^2$$

$$\bar{x} = \frac{1}{N}\Sigma x \qquad \bar{y} = \frac{1}{N}\Sigma y$$

6] Regression:

Regression line of y on x	Regression line of x on y
$(y-\bar{y}) = b_{yx}(x-\bar{x})$	$(x-\bar{x}) = b_{xy}(y-\bar{y})$
$(y-\bar{y}) = r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$	$(x-\bar{x}) = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$

$$r = \pm\sqrt{b_{xy}b_{yx}}$$

Note: If $b_{xy} > 0$ and $b_{yx} > 0$, then $r > 0 \rightarrow r$ +ve
If $b_{xy} < 0$ and $b_{yx} < 0$, then $r < 0 \rightarrow r$ -ve

7] Curve Fitting:

(i) For $y = ax + b$
i) $\Sigma y = a\Sigma x + nb$
ii) $\Sigma xy = a\Sigma x^2 + b\Sigma x$

Page No.
Date

(ii) For $y = ax^2 + bx + c$

i] $\sum y = a\sum x^2 + b\sum x + nc$

ii] $\sum xy = a\sum x^3 + b\sum x^2 + c\sum x$

iii] $\sum x^2 y = a\sum x^4 + b\sum x^3 + c\sum x^2$

May - 2022 [5869] - 286

Page No.
Date

Q1]

a) $\bar{x} = 2$, $\bar{y} = -3$, $\beta_{xy} = -0.11$

Formula : $(x - \bar{x}) = \beta_{xy}(y - \bar{y})$

$\therefore (x - 2) = -0.11(y - (-3))$

$\therefore x - 2 = -0.11y - 0.33$

$\therefore x = -0.11y + 1.67$

Probable value of x when y = 0

$\therefore x = -0.11(0) + 1.67$

$\boxed{\therefore x = 1.67}$

$\boxed{\text{iv) } 1.77}$

e) Given : $\mu_1 = 0$, $\mu_2 = 16$, $\mu_3 = -64$, $\mu_4 = 162$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{162}{16^2} = 0.6328$

$\boxed{\text{ii) } 0.6328}$

Q2]

a) Let $A = 52$, $u = \frac{x-A}{h} = x-52$
$h = 1$

x	f	fx	u	u^2	fu	fu^2
46	1	46	-6	36	-6	36
34	1	34	-18	324	-18	324
52	1	52	0	0	0	0
78	1	78	26	676	26	676
65	1	65	13	169	13	169
	$N=5$	$\Sigma fx = 275$			$\Sigma fu = 15$	$\Sigma fu^2 = 1205$

To find Standard Deviation (σ)

$$SD = \sigma = h\sqrt{\frac{1}{N}\Sigma fu^2 - \left(\frac{\Sigma fu}{N}\right)^2}$$

$$= 1\sqrt{\frac{1}{5}(1205) - \left(\frac{15}{5}\right)^2}$$

$$\boxed{\therefore \sigma = 15.2315}$$

To find Arithmetic Mean ($\bar{x}$):

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{275}{5}$$

$$\boxed{\therefore \bar{x} = 55}$$

b)

Date

b)

p	100	120	140	160	180	200
y	0.9	1.1	1.2	1.4	1.6	1.7

$y = ap + b$ ---- (1)

$n = 6$

p	y	py	p^2
100	0.9	90	10000
120	1.1	132	14400
140	1.2	168	19600
160	1.4	224	25600
180	1.6	288	32400
200	1.7	340	40000
$\Sigma p = 900$	$\Sigma y = 7.9$	$\Sigma py = 1242$	$\Sigma p^2 = 142000$

Using least square approximation on eq^n (1) :

i] $\Sigma y = a\Sigma p + nb$

$\therefore 7.9 = 900a + 6b \rightarrow$ (I)

ii] $\Sigma py = a\Sigma p^2 + b\Sigma p$

$1242 = 142000a + 900b \rightarrow$ (II)

Solving I & II, we get :

$\therefore a = 0.00814, \; b = 0.0952$

$$\boxed{\therefore y = 0.00814p + 0.0952}$$

Page No. | Date

c] $9x + y - \lambda = 0 \rightarrow (1)$

$4x + y = \mu \rightarrow (2)$

Since regression lines pass through the point of means $(\bar{x}, \bar{y}) = (2, -3)$, substitute these values into both equations to find λ and μ.

$\therefore 9(2) + (-3) = \lambda$

$\boxed{\therefore \lambda = 15}$

$\therefore 4(2) + (-3) = \mu$

$\boxed{\therefore \mu = 5}$

Rewriting eq^n (1) & (2) in slope-intercept form:

$y = -9x + \lambda$

$\therefore y = -9x + 15 \rightarrow (3)$

$x = \dfrac{\mu - y}{4}$

$\therefore x = -\dfrac{1}{4}y + 5 \rightarrow (4)$

From eq^n (3) & (4):

$b_{yx} = -9, \quad b_{xy} = -\dfrac{1}{4}$

Page No.
Date

Correlation Coefficient $(r) = \pm\sqrt{b_{yx} \cdot b_{xy}}$

$\therefore r = \pm\sqrt{-9 \times -\frac{1}{4}}$

$\therefore r = \pm \frac{3}{2}$

since $b_{yx} < 0$ and $b_{xy} < 0$, then $r < 0$

$$\boxed{\therefore r = -\frac{3}{2}}$$

Q3]

a) Given: $A = 5$, $\mu_1' = 2$, $\mu_2' = 20$, $\mu_3' = 40$
$\mu_4' = 50$.

1] $\boxed{\mu_1 = 0}$

2] $\mu_2 = \mu_2' - (\mu_1')^2$

$= 20 - (2)^2 = 20 - 4$

$$\boxed{\therefore \mu_2 = 16}$$

3] $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= 40 - 3(20)(2) + 2(2)^3$

$$\boxed{\therefore \mu_3 = -64}$$

4] $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 50 - 4(40)(2) + 6(20)(2)^2 - 3(2)^4$

$\boxed{\therefore \mu_4 = 162}$

To find β_1, β_2:

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(-64)^2}{16^3}$

$\boxed{\therefore \beta_1 = 1}$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{162}{16^2} = 0.6328$

$\boxed{\therefore \beta_2 = 0.6328}$

b)

x	0	1	2	3
y	2	2	4	8

$y = ax^2 + bx + c \longrightarrow$ (1)

$n = 4$

~~Using Least Square Method on eq^n (1):~~

i] $\Sigma y = a\Sigma x^2 + b\Sigma x + nc$

$\therefore$

x	y	xy	x^2	x^2y	x^3	x^4
0	2	0	0	0	0	0
1	2	2	1	2	1	1
2	4	8	4	16	8	16
3	8	24	9	72	27	81
$\Sigma x = 6$	$\Sigma y = 16$	$\Sigma xy = 34$	$\Sigma x^2 = 14$	$\Sigma x^2y = 90$	$\Sigma x^3 = 36$	$\Sigma x^4 = 98$

Using Least Square Method on eq^n (1):

i] $\Sigma y = a\Sigma x^2 + b\Sigma x + nc$

$\therefore \quad 16 = 14a + 6b + 4c \quad \rightarrow \quad$ (I)

ii] $\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$

$\therefore \quad 34 = 36a + 14b + 6c \quad \rightarrow \quad$ (II)

iii] $\Sigma x^2y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$

$\therefore \quad 90 = 98a + 36b + 14c \quad \rightarrow \quad$ (III)

Solving (I), (II) & (III) we get:

$a = 1, \quad b = -1, \quad c = 2$

Substituting above values in eq^n (1):

$$\boxed{\therefore \; y = x^2 - x + 2}$$

Page No.
Date

c) Given: $n = 10$, $\Sigma x = 40$, $\Sigma x^2 = 190$, $\Sigma y^2 = 200$, $\Sigma xy = 150$, $\Sigma y = 40$.

(i) $\bar{x} = \frac{1}{N} \Sigma x = \frac{40}{10} = 4$

(ii) $\bar{y} = \frac{1}{N} \Sigma y = \frac{40}{10} = 4$

(iii) $\text{cov}(x, y) = \frac{1}{N} \Sigma xy - (\bar{x}\bar{y})$

$= \frac{150}{10} - (4 \times 4)$

$= -1$

(iv) $\sigma_x^2 = \frac{1}{N} \Sigma x^2 - (\bar{x})^2 = \frac{190}{10} - 4^2$

$= 3$

(v) $\sigma_y^2 = \frac{1}{N} \Sigma y^2 - (\bar{y})^2 = \frac{200}{10} - 4^2$

$= 4$

(vi) $r = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y} = \frac{-1}{\sqrt{3}\sqrt{4}}$

$\boxed{\therefore r = -0.2886}$

November - 2022 [5925] - 260

Page No.
Date

Q1] Given :

y :	1	2	3
x :	1	5	9

$$x = ay + b \rightarrow (1)$$

y	x	yx	y^2
1	1	1	1
2	5	10	4
3	9	27	9
$\Sigma y = 6$	$\Sigma x = 15$	$\Sigma yx = 38$	$\Sigma y^2 = 14$

Using least square fit in eq^n (1) :

i] $\Sigma x = a\Sigma y + nb$

$\therefore 15 = 6a + 3b \rightarrow$ (I)

ii] $\Sigma yx = a\Sigma y^2 + b\Sigma y$

$\therefore 38 = 14a + 6b \rightarrow$ (II)

Solving (I) & (II), we get :

$\therefore a = 4 \quad , \quad b = -3$

Substituting in eq^n (1) :

$\therefore x = 4y - 3$

$\boxed{d)\ x = 4y - 3}$

v) Given: $\Sigma xy = 2638$, $\bar{x} = 14$, $\bar{y} = 17$, $n = 10$

$$cov(x,y) = \frac{1}{N}\Sigma xy - (\bar{x}\bar{y})$$

$$= \frac{2638}{10} - (14 \times 17)$$

$$= 25.8$$

$\boxed{a)\ 25.8}$

Q2]

a) Given: $A = 4$, $\mu_1' = -1.4$, $\mu_2' = 17$, $\mu_3' = -30$
$\mu_4' = 108$

1] $\boxed{\mu_1 = 0}$

2] $\mu_2 = \mu_2' - (\mu_1')^2$

$$= 17 - (-1.4)^2$$

$$\boxed{\therefore \mu_2 = 15.04}$$

3] $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$$= -30 - 3(17)(-1.4) + 2(-1.4)^3$$

$$\boxed{\therefore \mu_3 = 35.912}$$

4] $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 108 - 4(-30)(-1.4) + 6(17)(-1.4)^2 - 3(-1.4)^4$

$\boxed{\therefore \mu_4 = 151.4448}$

Coefficient of ~~Kurtosis~~ Skewness (β_1) :

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(35.912)^2}{(15.04)^3}$$

$\boxed{\therefore \beta_1 = 0.379}$

Coefficient of Kurtosis (β_2) :

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{151.4448}{(15.04)^2}$$

$\boxed{\therefore \beta_2 = 0.6695}$

b) Fit a linear curve of the type $y = ax + b$:

x	10	15	20	25	30
y	0.75	0.935	1.1	1.2	1.3

$y = ax + b$ ... (1)

$n = 5$

x	y	xy	x^2
10	0.75	7.5	100
15	0.935	14.025	225
20	1.1	22	400
25	1.2	30	625
30	1.3	39	900
$\Sigma x = 100$	$\Sigma y = 5.285$	$\Sigma xy = 112.525$	$\Sigma x^2 = 2250$

Using least square method on eq^n 1 :

i] $\Sigma y = a\Sigma x + nb$

$\therefore \ 5.285 = 100a + 5b \longrightarrow \text{I}$

ii] $\Sigma xy = a\Sigma x^2 + b\Sigma x$

$\therefore \ 112.525 = 2250a + 100b \longrightarrow \text{II}$

Solving I & II, we get:

$a = 0.0273 \quad , \quad b = 0.511$

Substituting in eq^n 1 :

$$\boxed{\therefore \ y = 0.0273x + 0.511}$$

c]

Population Density	200	500	400	700	800
Death Rate	12	18	16	21	10

$N = 5$, ~~ADD~~

x	y	xy	x^2	y^2
200	12	2400	40000	144
500	18	9000	250000	324
400	16	6400	160000	256
700	21	14700	490000	441
800	10	8000	640000	100
$\Sigma x = 2600$	$\Sigma y = 77$	$\Sigma xy = 40500$	$\Sigma x^2 = 1580000$	$\Sigma y^2 = 1265$

(i) $\bar{x} = \frac{1}{N} \Sigma x = \frac{2600}{5}$

$\therefore \bar{x} = 520$

(ii) $\bar{y} = \frac{1}{N} \Sigma y = \frac{77}{5} = 15.4$

(iii) $cov(x,y) = \frac{1}{N} \Sigma xy - (\bar{x}\,\bar{y})$

$= \frac{40500}{5} - (520 \times 15.4) = 92$

(iv) $\sigma_x^2 = \frac{1}{N} \Sigma x^2 - (\bar{x})^2 = \frac{1580000}{5} - (520)^2$

$= 45600$

(v) $\sigma_y^2 = \frac{1}{N} \Sigma y^2 - (\bar{y})^2 = \frac{1265}{5} - (15.4)^2$

$= 15.84$

Coefficient of Correlation $(r) = \dfrac{Cov(x, y)}{\sigma_x \sigma_y}$

$$\therefore r = \frac{92}{\sqrt{45600}\sqrt{15.84}}$$

$$\boxed{\therefore r \doteq 0.1082}$$

Q3]

a) Find coefficient of variability for following data

CI	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Freq. (F)	4	7	8	12	25	18	10

Let $A = 35$ and $h = 10$

$$\therefore u = \frac{x - A}{h} = \frac{x - 35}{10}$$

CI	f	x	u	u^2	fu	fu^2
0-10	4	5	-3	9	-12	36
10-20	7	15	-2	4	-14	28
20-30	8	25	-1	1	-8	8
30-40	12	35	0	0	0	0
40-50	25	45	1	1	25	25
50-60	18	55	2	4	36	72
60-70	10	65	3	9	30	90
	N = 84				$\Sigma fu = 57$	$\Sigma fu^2 = 259$

To find Standard Deviation (σ):

Page No.
Date

$$\sigma = h\sqrt{\frac{1}{N}\sum fu^2 - \left(\frac{\sum fu}{N}\right)^2}$$

$$= 10\sqrt{\frac{1}{84}(259) - \left(\frac{57}{84}\right)^2}$$

$$\sigma = 16.1953$$

To find arithmetic mean $(\bar{x})$:

$$\therefore \bar{x} = A + h\frac{\sum fu}{N}$$

$$= 35 + 10\left(\frac{57}{84}\right)$$

$$\therefore \bar{x} = 41.7857$$

To find coefficient of variability (CV):

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{16.1953}{41.7857} \times 100$$

$$\boxed{\therefore C.V. = 38.758\ \%}$$

b) Same solution Q2] (b) May 2022 Paper

c) The regression equations are $8x - 10y + 66 = 0$ and $40x - 18y = 214$. The value of variance of x is 9. Find

i] The mean values of x and y:

Since regression lines must pass through the point of means $(\bar{x}, \bar{y})$, substitute these values into both equations. Therefore,

$$8\bar{x} - 10\bar{y} + 66 = 0$$

$$\therefore \; 8\bar{x} - 10\bar{y} = -66 \rightarrow (1)$$

$$40\bar{x} - 18\bar{y} = 214 \rightarrow (2)$$

On solving (1) & (2), we get:

$$\boxed{\therefore \; \bar{x} = 13, \quad \bar{y} = 17}$$

ii] The correlation x and y

$$r = \sqrt{b_{xy} \cdot b_{yx}}$$

Given: $8x - 10y + 66 = 0 \rightarrow (3)$

$40x - 18y - 214 = 0 \rightarrow (4)$

Rewriting equations in slope intercept form:

$$\therefore \; y = \frac{8x + 66}{10}$$

$\therefore \quad y = 0.8x + 6.6 \rightarrow (5)$

From eq^n (4):

$$x = \frac{18y + 214}{40}$$

$\therefore \quad x = 0.45y + 5.35 \rightarrow (6)$

From eq^n (5) & (6):

$b_{yx} = 0.8$ and $b_{xy} = 0.45$

Now, $r = \pm\sqrt{b_{yx} \cdot b_{xy}}$

$= \pm\sqrt{0.8 \times 0.45}$

$= \pm 0.6$

since, $b_{yx} > 0$ and $b_{xy} > 0$, then $r > 0$

$$\boxed{\therefore \quad r = 0.6}$$

iii] the standard deviation of $y(\sigma_y)$:

Given: $\sigma_x^2 = 9 \quad \therefore \sigma_x = 3$

$$b_{yx} = \frac{r\sigma_y}{\sigma_x}$$

$$\therefore \quad 0.8 = 0.6 \times \frac{\sigma_y}{3}$$

$$\therefore \sigma_y = \frac{0.8 \times 3}{0.6}$$

$$\boxed{\therefore \sigma_y = 4}$$

May - 2023   [6002]-160

Q1]

a) Given: $b_{xy} = -\frac{5}{6}$, $b_{yx} = -\frac{8}{15}$

$r = \pm\sqrt{b_{xy} \cdot b_{yx}}$

$\therefore r = \pm\sqrt{\frac{-5}{6} \times \frac{-8}{15}} = \pm\frac{2}{3}$

Since $b_{xy} < 0$ and $b_{yx} < 0$, then $r < 0$

$\boxed{\therefore \; r = -\frac{2}{3}}$

b] $r = 1 \rightarrow$ Perfect Positive Correlation
$r = -1 \rightarrow$ Perfect Negative Correlation
$r = 0 \rightarrow$ No linear correlation

$\boxed{\text{iii]} \quad -1 \le r \le 1}$

Q2]

a) Same Solution Q2]a November - 2022 Paper

b)

x	0	1	2	3	4
y	-2	1	4	7	10

$y = a + bx$ ... (1)

$N = 5$

x	y	xy	x^2
0	-2	0	0
1	1	1	1
2	4	8	4
3	7	21	9
4	10	40	16
$\Sigma x = 10$	$\Sigma y = 20$	$\Sigma xy = 70$	$\Sigma x^2 = 30$

Using least square method on eqⁿ ①:

i] $\Sigma y = na + b\Sigma x$ ... (I)

$\therefore 20 = 5a + 10b$

ii] $\Sigma xy = a\Sigma x + b\Sigma x^2$

$\therefore 70 = 10a + 30b$ ... (II)

Solving (I) & (II), we get:

$a = -2$, $b = 3$

Substituting in eqⁿ ①, we get:

$$\boxed{y = -2 + 3x}$$

c] Given: $3x + 2y = 26 \rightarrow ①$
$6x + y = 31 \rightarrow ②$

Since regression lines pass through the point of means $(x, y) = (\bar{x}, \bar{y})$.

$\therefore$ Solving eq^n ① & ②, we get :

$\boxed{\bar{x} = 4}$, $\boxed{\bar{y} = 7}$

© To find the correlation coefficient between x and y (r) :

$$r = \pm\sqrt{b_{yx} \cdot b_{xy}} \longrightarrow ③$$

Rewriting eq^n ① & ② in slope-intercept form :

$$\therefore 3x + 2y = 26 \rightarrow ①$$

$$\therefore y = \frac{26 - 3x}{2}$$

$$\therefore y = -1.5x + 13$$

$$\therefore b_{yx} = -1.5$$

$$\therefore 6x + y = 31 \longrightarrow ②$$

$$\therefore x = \frac{31 - y}{6}$$

$$\therefore x = \frac{-1}{6}y + \frac{31}{6}$$

$$\therefore b_{xy} = \frac{-1}{6}$$

Substituting values of b_{yx} & b_{xy} in eq^n ③ :

$$\therefore \quad r = \pm\sqrt{-1.5 \times -\frac{1}{6}}$$

$$\therefore \quad r = \pm\frac{1}{2}$$

Since $b_{xy} < 0$, $b_{yx} < 0$ then $r < 0$

$$\boxed{\therefore \quad r = -\frac{1}{2}}$$

Q3]

a) Calculate coefficient of variation of given data : 36, 15, 25, 10, 14.

Let $A = 25$, $h = 1$, $u = \frac{x - A}{h}$

$$\therefore \quad u = x - 25$$

x	f	fx	u	fu	fu^2
36	1	36	11	11	121
15	1	15	-10	-10	100
25	1	25	0	0	0
10	1	10	-15	-15	225
14	1	14	-11	-11	121
	$N = 5$	$\Sigma fx = 100$		$\Sigma fu = -25$	$\Sigma fu^2 = 567$

To find Standard Deviation (σ):

$$\therefore \quad \sigma = h\sqrt{\frac{1}{N}\Sigma fu^2 - \left(\frac{\Sigma fu}{N}\right)^2}$$

$$= 1\sqrt{\frac{1}{5}(567) - \left(\frac{-25}{5}\right)^2}$$

$$\therefore \quad \sigma = 9.4$$

To find arithmetic mean ($\bar{x}$):

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{100}{5}$$

$$\therefore \bar{x} = 20$$

To find coefficient of variation (C.V.) :

$$\therefore \quad C.V. = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{9.4}{20} \times 100$$

$$\boxed{\therefore \quad C.V. = 47\%}$$

b)

x	0	1	2	3
y	2	1	6	17

$y = a + bx + cx^2 \quad \dots \; (1)$

Page No.
Date

$n = 4$

x	y	xy	x^2	x^2y	x^3	x^4
0	2	0	0	0	0	0
1	1	1	1	1	1	1
2	6	12	4	24	8	16
3	17	51	9	153	27	81
$\Sigma x = 6$	$\Sigma y = 26$	$\Sigma xy = 64$	$\Sigma x^2 = 14$	$\Sigma x^2y = 178$	$\Sigma x^3 = 36$	$\Sigma x^4 = 98$

Using Least Square Method on eq^n 1 :

i] $\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\therefore \; 26 = 4a + 6b + 14c \longrightarrow$ I

ii] $\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\therefore \; 64 = 6a + 14b + 36c \longrightarrow$ II

iii] $\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

$\therefore \; 178 = 14a + 36b + 98c \longrightarrow$ III

Solving I, II & III, we get:

$a = 2 \;,\; b = -4 \;,\; c = 3$

Substituting above values in eq^n 1

$$\boxed{\therefore \; y = 2 - 4x + 3x^2}$$

c] Same Solution Q2] c November 2022 Paper c] Same Solution Q2] c November 2022 P

Q1] a) Given : $\bar{x} = 2$, $\bar{y} = -3$, $b_{yx} = -4$

Regression Equation of y on x :

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$\therefore y - (-3) = -4(x-2)$$

$$\therefore y + 3 = -4x + 8$$

$$\therefore y = -4x + 5$$

when $x = 1$

$$\therefore y = -4(1) + 5$$

$$\therefore y = 1$$

$$\boxed{\text{ii) } 1}$$

e) Given : $\mu_1 = 0$, $\mu_2 = 0.453$, $\mu_3 = 0.06$, $\mu_4 = 0.502$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{0.502}{(0.453)^2}$$

$$\boxed{\therefore \beta_2 = 2.4463}$$

Q2] a) Same Solution May 2022 - Q.3 (a)

b)

x	-2	1	3	6	8	9
y	17	14	12	9	7	6

$$y = a + bx \quad \ldots (1)$$

$n = 6$

x	y	xy	x^2
-2	17	-34	4
1	14	14	1
3	12	36	9
6	9	54	36
8	7	56	64
9	6	54	81
$\Sigma x = 25$	$\Sigma y = 65$	$\Sigma xy = 180$	$\Sigma x^2 = 195$

Using Least Square Method on eq^n (1):

i] $\Sigma y = na + b\Sigma x$

$65 = 6a + 25b$

ii] $\Sigma xy = a\Sigma x + b\Sigma x^2$

$\therefore 180 = 25a + 195b$

Solving (I) & (II), we get:

$a = 15, \; b = -1$

$$\boxed{\therefore y = 15 - x}$$

c) Same Solution : November 2022 – Q3 (c)

Q3)

a)

Score by (x) Batsman A	38	47	34	18	33
Score by (y) Batsman B	37	35	41	27	35

For Batsman A : $u = \frac{x - A}{h}$

Let $A = 34$, $h = 1$, $u = x - 34$

x	f	fx	u	fu	fu^2
38	1	38	4	4	16
47	1	47	13	13	169
34	1	34	0	0	0
18	1	18	-16	-16	256
33	1	33	-1	-1	1
	$N = 5$	$\Sigma fu = 170$		$\Sigma fu = 0$	$\Sigma fu^2 = 442$

To find Arithmetic mean $(\bar{x}_A)$:

$$\therefore \bar{x}_A = \frac{\Sigma fx}{N} = \frac{170}{5} = 34$$

To find standard Deviation (σ_A)

$$\therefore \sigma_A = h\sqrt{\frac{1}{N}\Sigma fu^2 - \left(\frac{\Sigma fu}{N}\right)^2}$$

$\therefore \sigma_A = 1\sqrt{\frac{442}{5} - 0} = 9.4021$

To find coefficient of variance (CV_A):

$\therefore CV_A = \frac{\sigma_A}{\overline{x}_A} \times 100$

$= \frac{9.4021}{34} \times 100$

$\therefore \cancel{CV_A = 27}$

$$\boxed{\therefore CV_A = 27.653\,\%}$$

* For Batsman B : $u = \frac{x - A}{h}$

Let $A = 41$, $h = 1$, $u = x - 41$

y	f	fy	u	fu	fu^2
37	1	37	−4	−4	16
35	1	35	−6	−6	36
41	1	41	0	0	0
27	1	27	−14	−14	196
35	1	35	−6	−6	36
	N=5	$\Sigma fy = 175$		$\Sigma fu = -30$	$\Sigma fu^2 = 284$

To find Arithmetic Mean ($\bar{y}$):

$\therefore \bar{y} = \frac{\Sigma fy}{N} = \frac{175}{5} = 35$

Page No.
Date

To find Standard Deviation (σ_B):

$$\therefore \sigma_B = h\sqrt{\frac{1}{N}\Sigma fu^2 - \left(\frac{\Sigma fu}{N}\right)^2}$$

$$= 1\sqrt{\frac{1}{5}(284) - \left(\frac{-30}{5}\right)^2}$$

$$= 4.5607$$

To find coefficient of variance (CV_B):

$$CV_B = \frac{\sigma_B}{\bar{y}} \times 100 = \frac{4.5607}{35} \times 100$$

$$\boxed{CV_B = 13.03\,\%}$$

Since $CV_A > CV_B$, the batsman B is more consistent in scoring runs.

As $\bar{x} < \bar{y}$, the batsman B has performed better.

b)

x	-2	-1	0	1	2
y	-2	5	8	7	2

$$y = a + bx + cx^2 \quad \cdots (1)$$

$$n = 5$$

x	y	xy	x^2	x^2y	x^3	x^4
-2	-2	4	4	-8	-8	16
-1	5	-5	1	5	-1	1
0	8	0	0	0	0	0
1	7	7	1	7	1	1
2	2	4	4	8	8	16
$\Sigma x = 0$	$\Sigma y = 20$	$\Sigma xy = 10$	$\Sigma x^2 = 10$	$\Sigma x^2y = 12$	$\Sigma x^3 = 0$	$\Sigma x^4 = 34$

Using least square method on eq^n (1):

i] $\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\therefore \quad 20 = 5a + 10c \quad \ldots \text{(I)}$

ii] $\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\therefore \quad 10 = 10b \quad \ldots \text{(II)}$

iii] $\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

$\therefore \quad 12 = 10a + 34c \quad \ldots \text{(III)}$

On solving (I), (II) & (III), we get:

$\therefore \quad a = 8, \quad b = 1, \quad c = -2$

Substituting above values in eq^n (1):

$$\boxed{\therefore \; y = 8 + x - 2x^2}$$

c)

x	43	22	25	42	58
y	99	65	79	75	87

Here, $n = 5$

x	y	xy	x^2	y^2
43	99	4257	1849	9801
22	65	1430	484	4225
25	79	1975	625	6241
42	75	3150	1764	5625
58	87	5046	3364	7569
$\Sigma x = 190$	$\Sigma y = 405$	$\Sigma xy = 15858$	$\Sigma x^2 = 8086$	$\Sigma y^2 = 33461$

To find coefficient of correlation (r), we need:

(i) $\bar{x} = \frac{1}{N} \Sigma x = \frac{190}{5} = 38$

(ii) $\bar{y} = \frac{1}{N} \Sigma y = \frac{405}{5} = 81$

(iii) $cov(x,y) = \frac{1}{N} \Sigma xy - (\bar{x}\bar{y}) = \frac{1}{5}(15858) - (38 \times 81)$

$= 93.6$

(iv) $\sigma_x^2 = \frac{1}{N} \Sigma x^2 - (\bar{x})^2 = \frac{1}{5}(8086) - (38^2)$

$= 173.2$

(v) $\sigma_y^2 = \frac{1}{N} \Sigma y^2 - (\bar{y})^2 = \frac{1}{5}(33461) - (81^2)$

$= 131.2$

(vi) $r = \dfrac{\operatorname{cov}(x,y)}{\sigma_x \sigma_y} = \dfrac{93.6}{\sqrt{173.2}\sqrt{131.2}}$

$$\boxed{\therefore r = 0.6209}$$

May - 2024 [6261] - 35

Page No.

Date

Q1]

a) Given : $\mu_1' = 2$, $\mu_2' = 20$, $\mu_3' = 40$

To find μ_3 :

$$\mu_3 = \mu_3' - 3(\mu_2')(\mu_1') + 2(\mu_1')^3$$

$$= 40 - 3(20)(2) + 2(2)^3 = -64$$

$$\boxed{\therefore \mu_3 = -64}$$

B) $\boxed{\text{ii) } \frac{\sigma}{\bar{x}} \times 100}$

Q2]

a)

x	1	2	3	4
f	9	6	5	3

Let $A = 2$, $h = 1$, $u = \frac{x-A}{h} = x - 2$

x	f	fx	u	fu	fu^2
1	9	9	-1	-9	9
2	6	12	0	0	0
3	5	15	1	5	5
4	3	12	2	6	12
	$N = 23$	$\Sigma fx = 48$		$\Sigma fu = 2$	$\Sigma fu^2 = 26$

* To find arithmetic mean $(\bar{x})$:

$$\therefore \bar{x} = \frac{\Sigma fx}{N} = \frac{48}{23}$$

$$\boxed{\therefore \bar{x} = 2.087}$$

c) * To find coefficient of variation for x (CV):

(i) To find Standard Deviation (σ):

$$\sigma = h \sqrt{\frac{1}{N} \Sigma fu^2 - \left(\frac{\Sigma fu}{N}\right)^2}$$

$$= 1 \sqrt{\frac{1}{23}(26) - \left(\frac{2}{23}\right)^2}$$

$$\therefore \sigma = 1.0596$$

$$C.V. = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.0596}{2.087} \times 100$$

$$\boxed{\therefore C.V. = 50.77 \%}$$

b)

x	-2	-1	0	1	2
y	5	3	1	-1	-3

$$y = ax + b \quad \ldots (1)$$

$$n = 5$$

x	y	xy	x^2
-2	5	-10	4
-1	3	-3	1
0	1	0	0
1	-1	-1	1
2	-3	-6	4
$\Sigma x = 0$	$\Sigma y = 5$	$\Sigma xy = -20$	$\Sigma x^2 = 10$

Using least square method on eq^n 1, we have:

i] $\Sigma y = a\Sigma x + nb$

$\therefore \quad 5 = 5b \longrightarrow$ I

ii] $\Sigma xy = a\Sigma x^2 + b\Sigma x$

$\therefore \quad -20 = 10a \longrightarrow$ II

On solving I & II, we get:

$a = -2, \quad b = 1$

Substituting above values in eq^n 1:

$$\boxed{\therefore \quad y = -2x + 1}$$

c) Given: $\bar{x} = 8.2, \ \bar{y} = 12.4, \ \sigma_x = 6.2, \ \sigma_y = 20$
$r(x, y) = 0.9$

Line of Regression of x on y is given as:

$$x-\bar{x} = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

$$\therefore x-8.2 = 0.9 \times \frac{6.2}{20}(y-12.4)$$

$$\therefore x = 0.279y - 3.4596 + 8.2$$

$$\therefore x = \cancel{0.279} \ 0.279y + 4.7404$$

For $y = 10$:

$$\therefore x = 0.279(10) + 4.7404$$

$$\boxed{\therefore x = 7.5304}$$

3]

a) Given: $A=2$, $\mu_1' = 2$, $\mu_2' = 10$, $\mu_3' = 20$, $\mu_4' = 25$

* To find first four moments:

(i) $\mu_1 = 0$

(ii) $\mu_2 = \mu_2' - (\mu_1')^2$

$$= 10 - (2)^2 = 10-4$$

$$\therefore \mu_2 = 6$$

(iii) $\mu_3 = \mu_3' - 3(\mu_2')(\mu_1') + 2(\mu_1')^3$

$= 20 - 3(10)(2) + 2(2)^3$

$\therefore \mu_3 = -24$

(iv) $\mu_4 = \mu_4' - 4(\mu_3')(\mu_1') + 6(\mu_2')(\mu_1')^2 - 3(\mu_1')^4$

$= 25 - 4(20)(2) + 6(10)(2)^2 - 3(2)^4$

$\therefore \mu_4 = 57$

To find coefficient of skewness (β_1) and kurtosis (β_2):

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(-24)^2}{6^3}$$

$$\boxed{\therefore \beta_1 = 2.67}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{57}{6^2}$$

$$\boxed{\therefore \beta_2 = 1.583}$$

b)

x	-1	0	1	2
y	3	1	3	9

$y = ax^2 + bx + c \quad \ldots (1)$

Page No. Date

x	y	xy	x^2	x^2y	x^3	x^4
-1	3	-3	1	3	-1	1
0	1	0	0	0	0	0
1	3	3	1	3	1	1
2	9	18	4	36	8	16
$\Sigma x = 2$	$\Sigma y = 16$	$\Sigma xy = 18$	$\Sigma x^2 = 6$	$\Sigma x^2y = 42$	$\Sigma x^3 = 8$	$\Sigma x^4 = 18$

Using Least Square Method on eq^n (1):

i.] $\Sigma y = a\Sigma x^2 + b\Sigma x + nc$

$\therefore \; 16 = 6a + 2b + 4c \longrightarrow$ (I)

ii] $\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$

$\therefore \; 18 = 8a + 6b + 2c \longrightarrow$ (II)

iii] $\Sigma x^2y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$

$\therefore \; 42 = 18a + 8b + 6c \longrightarrow$ (III)

On solving (I), (II) & (III) we get:

$$a = 2, \quad b = 0, \quad c = 1$$

Substituting above values in eq^n (1), we get:

$$\boxed{\therefore \; y = 2x^2 + 1}$$

Date

c) * To find coefficient of correlation (r):

x	y	xy	x^2	y^2
5	9	45	25	81
7	6	42	49	36
9	12	108	81	144
11	3	33	121	9
13	15	195	169	225
$\Sigma x = 45$	$\Sigma y = 45$	$\Sigma xy = 423$	$\Sigma x^2 = 445$	$\Sigma y^2 = 495$

(i) $\bar{x} = \frac{1}{N} \Sigma x = \frac{45}{5} = 9$

(ii) $\bar{y} = \frac{1}{N} \Sigma y = \frac{45}{5} = 9$

(iii) $cov(x,y) = \frac{1}{N} \Sigma xy - (\bar{x}\bar{y}) = \frac{423}{5} - (9 \times 9)$

$= 3.6$

(iv) $\sigma_x^2 = \frac{1}{N} \Sigma x^2 - (\bar{x})^2 = \frac{445}{5} - (9)^2$

$= 8$

(v) $\sigma_y^2 = \frac{1}{N} \Sigma y^2 - (\bar{y})^2 = \frac{495}{5} - (9)^2$

$= 18$

Now, to find r: $r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y}$

$$\therefore \gamma = \frac{3.6}{\sqrt{8}\,\sqrt{18}}$$

$$\boxed{\therefore \gamma = 0.3}$$

November - 2024 [6352] - 35

]

a) Given: $\mu_1 = 0$, $\mu_2 = 2.5$, $\mu_3 = 0.7$, $\mu_4 = 18.75$

To find Coefficient of Kurtosis (β_2):

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{18.75}{2.5^2} = 3$$

$$\boxed{\text{iv) } \beta_2 = 3}$$

e) $$\boxed{\text{ii) } \sqrt{\frac{1}{N} \Sigma f(x-\bar{x})^2}}$$

Q2]

a) Same Solution: May - 2022 - Q3 (a)

b)

x	6	2	10	4	8
y	9	11	5	8	7

Here, $n = 5$

x	y	xy	x^2	y^2
6	9	54	36	81
2	11	22	4	121
10	5	50	100	25
4	8	32	16	64
8	7	56	64	49
$\Sigma x = 30$	$\Sigma y = 40$	$\Sigma xy = 214$	$\Sigma x^2 = 220$	$\Sigma y^2 = 340$

(i) $\bar{x} = \frac{1}{N}\Sigma x = \frac{30}{5} = 6$

(ii) $\bar{y} = \frac{1}{N}\Sigma y = \frac{40}{5} = 8$

(iii) $cov(x,y) = \frac{1}{N}\Sigma xy - (\bar{x}\bar{y}) = \frac{214}{5} - (6\times 8)$

$= -5.2$

(iv) $\sigma_x^2 = \frac{1}{N}\Sigma x^2 - (\bar{x})^2 = \frac{220}{5} - 6^2$

$= 8$

(v) $\sigma_y^2 = \frac{1}{N}\Sigma y^2 - (\bar{y})^2 = \frac{340}{5} - 8^2$

$= 4$

(vi) To find r: $r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} = \frac{-5.2}{\sqrt{8}\sqrt{4}}$

$\therefore r = -0.9192$

To find Regression line of x on y:

$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$\therefore x - 6 = -0.9192 \times \frac{\sqrt{8}}{\sqrt{4}}(y - 8)$

Page No.
Date

$\therefore x = -1.299y + 10.399 + 6$

$\boxed{\therefore x = -1.299y + 16.399}$

x	0	1	2	3	4	5	6	7
y	-5	-3	-1	1	3	5	7	9

$y = ax + b \quad \cdots (1)$

$n = 8$

x	y	xy	x^2
0	-5	0	0
1	-3	-3	1
2	-1	-2	4
3	1	3	9
4	3	12	16
5	5	25	25
6	7	42	36
7	9	63	49
$\Sigma x = 28$	$\Sigma y = 16$	$\Sigma xy = 140$	$\Sigma x^2 = 140$

Using Least Square Method on eq^n (1) :

i] $\Sigma y = a\Sigma x + nb$

$\therefore 16 = 28a + 8b \quad \cdots (\mathrm{I})$

ii] $\Sigma xy = a\Sigma x^2 + b\Sigma x$

$\therefore 140 = 140a + 28b \quad \cdots (\mathrm{II})$

On solving (I) & (II), we get:

$a = 2$; $b = -5$

Substituting above values in eq^n (1):

$$\boxed{\therefore \quad y = 2x - 5}$$

Q3]

a) Same Solution: May 2022 - Q3 (c)

b)

x	2000	3000	4000	5000	6000
y	15	15.5	16	17	18

$y = ax^b \quad \cdots (1)$

Taking $\log_{10}$ on both sides:

$\log_{10} y = \log_{10}(ax^b)$

$\log_{10} y = \log a + \log_{10} x^b$

$\therefore \log_{10} y = \log a + b \log x$

Let $\log_{10} y = Y$, $\log_{10} a = A$, $\log_{10} x = X$

$\therefore \quad Y = A + bX \quad \cdots (2)$

$n = 5$

x	y	$X = \log_{10} x$	$Y = \log_{10} y$	X^2	XY
2000	15	3.30103	1.17609	10.8967	3.88498
3000	15.5	3.47712	1.19033	12.09037	4.13892
4000	16	3.60206	1.20412	12.97483	4.33731
5000	17	3.69897	1.23044	13.682379	4.55139
6000	18	3.77815	1.2552	14.27442	4.7426
		$\Sigma X = 17.85$	$\Sigma Y = 6.056$	$\Sigma X^2 = 63.9188$	$\Sigma XY = 21.652$

Using least square method on eq^n (2), we get:

i] $\Sigma Y = NA + b\Sigma x$

$\therefore \; 6.056 = 5a + 17.85b \quad \text{—— (I)}$

ii] $\Sigma XY = A\Sigma X + b\Sigma x^2$

$\therefore \; 21.652 = 17.85A + 63.9188B \quad \text{—— (II)}$

On solving (I) & (II), we get:

$A = 0.6217, \quad b = 0.1651$

As $A = \log_{10} a$

$\therefore \; a = 10^A$

$\therefore \; a = 10^{0.6217} = 4.185$

Similarly, $b = 0.1651$

Substitution of above values in eq^n (1) gives:

Page No.
Date

$$\therefore \boxed{y = 4.185\, x^{0.1651}}$$

c] Given: $x = 19.13 - 0.87y \rightarrow ①$
$y = 11.64 - 0.50x \rightarrow ②$

Since regression lines must pass through the point of means $(\bar{x}, \bar{y})$. $(\bar{x}, \bar{y}) = (x, y)$.

Eqⁿs ① & ② can be written as:

$\bar{x} + 0.87\bar{y} = 19.13 \rightarrow \text{I}$
$0.50\bar{x} + \bar{y} = 11.64 \rightarrow \text{II}$

Solving eqⁿ I & II, we get:

$$\boxed{\bar{x} = 15.9348, \quad \bar{y} = 3.6725}$$

* To find coefficient of correlation (r):

$r = \pm\sqrt{b_{yx} \cdot b_{xy}} \rightarrow ③$

We know: $x = -0.87y + 19.13$

$\therefore b_{xy} = -0.87$

And, $y = -0.50x + 11.64$

$\therefore b_{yx} = -0.50$

Substituting above values in eqⁿ ③, we get:

$$\therefore \gamma = \pm\sqrt{-0.50 \times -0.87}$$

$$\therefore \gamma = \pm 0.6595$$

since $b_{xy} < 0$, $b_{yx} < 0$, then $\gamma < 0$.

$$\boxed{\therefore \gamma = -0.6595}$$