

TITLE: Random Variable

g) Find the mean and variance for the following

(a)	X	-1	0	1	2
	P(X)	0.1	0.2	0.3	0.4

Solution:

X	P(X)	X · P(X)	$E(X)^2$	$[E(X)]^2$
-1	0.1	-0.1	0.1	0.01
0	0.2	0	0	0
1	0.3	0.3	0.3	0.09
2	0.4	0.8	0.16	0.64
TOTAL	$\sum = 1$	$\sum = 1$	$\sum E(X)^2 = 0.20$	$\sum [E(X)]^2 = 0.74$

$$\therefore \text{Mean} = E(X) = \sum x_i \cdot P(X) = 1$$

$$\begin{aligned} \therefore \text{Variance} &= V(X) = \sum E(X)^2 - \sum [E(X)]^2 \\ &= 0.20 - 0.74 \\ &= 1.24 \end{aligned}$$

$$\therefore \text{Mean } E(X) = 1 \text{ & variance } V(X) = 1.24.$$

(b)	X	-1	0	1	2
	$P(X)$	$1/8$	$1/8$	$1/4$	$1/2$

Solution :

X	$P(X)$	$X \cdot P(X)$	$E(X)^2$	$[E(X)]^2$
-1	$1/8$	$-1/8$	$1/8$	$1/64$
0	$1/8$	0	0	0
1	$1/4$	$1/4$	$1/4$	$1/16$
2	$1/2$	1	$4/2$	$16/4$
TOTAL	$\Sigma = 1$	$\Sigma = 9/8$	$\Sigma = 19/8$	$\Sigma = 69/64$

$$\text{Mean } E(X) = \sum X \cdot P(X) = 9/8$$

$$\text{Variance } V(X) = \sum E(X)^2 - \sum (E(X))^2$$

$$= \frac{19}{8} - \frac{69}{64}$$

$$= \frac{152 - 69}{64}$$

$$= \frac{83}{64}$$

$$\therefore \text{Mean } E(X) = 9/8 \text{ & variance } V(X) = 83/64.$$

(c)	X	-3	10	15	0	1	4
	$P(X)$	0.4	0.35	0.25	0.1	0.05	0.05

Solution:

X	$P(X)$	$X \cdot P(X)$	$E(X)^2$	$[E(X)]^2$
-3	0.4	-1.2	3.6	1.44
10	0.35	3.5	35	12.25
15	0.25	3.75	56.25	14.0625
TOTAL	$\Sigma = 1$	$\Sigma = 6.05$	$\Sigma = 94.85$	$\Sigma = 27.7525$

$$\therefore \text{Mean} = E(X) = \sum X \cdot P(X) = 6.05$$

$$\therefore \text{Variance} = V(X) = \sum E(X)^2 - [E(X)]^2$$

$$= 94.85 - 27.7525$$

$$= 67.095$$

Mean $E(X) = 6.05$ & variance $V(X) = 67.095$.

Q2: If $P(X)$ is pmf of a random variable X , if $P(X)$ represents pmf for r.v. X , find value of K . Then evaluate mean & variance.

Solution: As $P(X_i)$ is a pmf it should satisfy the properties of pmf which are
 (a) $P(X_i) \geq 0$ for all sample space.
 (b) $\sum P(X_i) = 1$

X	-1	0	1	2	01	8-	X
$P(X)$	$K+1/13$	$K/13$	$1/13$	$K-4/13$		$4/9$	$(X)9$

$$\therefore \sum P(X_i) = 1 = \frac{K+1}{13} + \frac{K}{13} + \frac{1}{13} + \frac{K-4}{13}$$

$$1 = \frac{K+1+K+1+K-4}{13}$$

$$4+1$$

$$13$$

$$13 = 3K - 2$$

$$15 = 3K$$

$$K = 5$$

X	$P(X)$	$X \cdot P(X)$	$E(X)^2$	$[E(X)]^2$
-1	$6/13$	$-6/13$	$6/13$	$36/189$
0	$5/13$	0	0	0
1	$1/13$	$1/13$	$1/13$	$1/189$
2	$1/13$	$2/13$	$4/13$	$4/189$
TOTAL	$\sum = 1$	$\sum = -3/13$	$\sum = 11/13$	$\sum = 41/189$

$$\text{Mean} = E(X) = \sum X \cdot P(X) = -3/13$$

$$\therefore \text{Variance} = V(X) = \sum E(X)^2 - \sum [E(X)]^2$$

$$= \frac{11}{13} - \frac{41}{189}$$

$$= \frac{143 - 41}{169}$$

$$= \frac{102}{169}$$

$$\therefore \text{Mean} = -3/13 \text{ f variance } \frac{102}{169} = 102/169.$$

Q3. The pmf of random variable X is given by

X	-3	-1	0	1	2	3	5	8
$P(X)$	0.1	0.2	0.15	0.2	0.1	0.15	0.05	0.05

Obtain cdf. Find ① $P(-1 \leq X \leq 2)$, ② $P(1 \leq X \leq 5)$,
 ③ $P(X \leq 2)$, ④ $P(X \geq 0)$.

Solution:

X	-3	-1	0	1	2	3	5	8
$P(X)$	0.1	0.2	0.15	0.2	0.1	0.15	0.05	0.05
$F(X)$	0.1	0.35	0.45	0.65	0.75	0.90	0.95	1.0

$$\begin{aligned}
 \textcircled{1} \quad P(-1 \leq X \leq 2) &= P(X \leq 2) - P(X \leq -1) + P(X = -1) \\
 &= F(x_b) - F(x_a) + p(a) \\
 &= F(2) - F(-1) + P(-1) \\
 &= 0.75 - 0.3 + 0.1 \\
 &= 0.25
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad P(1 \leq X \leq 5) &= F(x_b) - F(x_a) + p(a) \\
 &= F(5) - F(-1) + P(1) \\
 &= 0.95 - 0.65 + 0.2 \\
 &= 0.15
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad P(X \leq 2) &= P(X = 1) + P(X = -1) + P(X = 3) + P(X = 0) + P(X = 2) \\
 &= 0.1 + 0.2 + 0.15 + 0.2 + 0.1 \\
 &= 0.75
 \end{aligned}$$

$$(3) P(n \geq 0) = 1 - P(0) + P(0)$$

$$= 1 - 0.45 + 0.15$$

$$= 0.40$$

(4) Let f be continuous random variable with pdf

$$\therefore f(n) = \frac{n+1}{2}; -1 < n < 1$$

Obtain cdf of n . Find mean.

Solution: By definition of cdf, we have:

$$\begin{aligned} F(x) &= \int_{-1}^{x+1} f(t) dt \\ &= \int_{-1}^x \frac{t+1}{2} dt \\ &= \frac{1}{2} \left(\frac{1}{2}n^2 + n \right) \text{ for } -1 < n < 1 \end{aligned}$$

Hence the cdf is

$$\begin{aligned} F(x) &= 0 \quad \text{for } n \leq -1. \\ &= \frac{1}{2}n^2 + \frac{1}{2}n \quad \text{for } -1 < n \leq 1 \\ &= 1 \quad ; \quad n \geq 1 \end{aligned}$$

5. Let f be c.r.v. with pdf

$$\therefore f(x) = \begin{cases} \frac{n+2}{18}, & -2 \leq n \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

calculate cdf.

Solution: By definition of cdf, we have,

$$\begin{aligned} F(x) &= \int_{-2}^x t \cdot dt \\ &= \int_{-2}^x \frac{n+2}{18} \cdot dn \\ &= \frac{1}{18} \left(\frac{1}{2} n^2 + 2n \right) \text{ for } -2 \leq n \leq 4 \end{aligned}$$

Hence cdf is

$$\begin{aligned} F(x) &= 0 \text{ for } n < -2 \\ &= \frac{1}{18} \left(\frac{1}{2} n^2 + 2n \right), \text{ for } -2 \leq n \leq 4 \\ &= 0, \text{ for } n > 4 \end{aligned}$$

Note

PRACTICAL-02

TITLE: Binomial Distribution

- Q1. An unbiased coin is tossed 4 times calculate probability of obtaining no head, at least one head & more than one tail.

NO HEAD :

> dbinom(0, 4, 0.5)

[1] 0.0625

AT LEAST ONE HEAD :

> 1 - dbinom(0, 4, 0.5)

[1] 0.9375

MORE THAN ONE TAIL :

> pbinom(1, 4, 0.5, lower.tail = F)

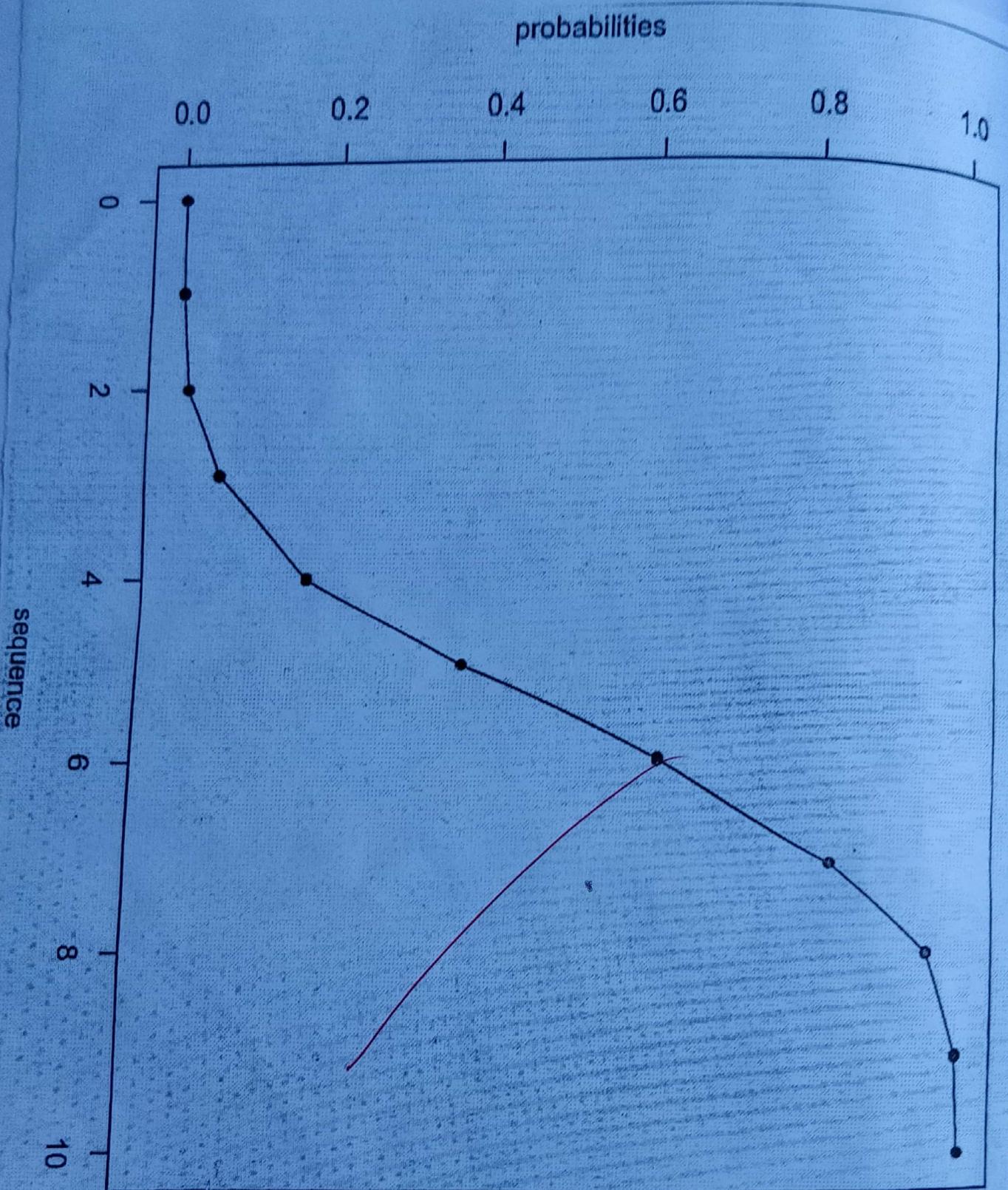
[1] 0.9375

- Q2. If probability of student being accepted is 0.3, 5 students apply, what's the probability of almost 2 are accepted :

> pbinom(2, 5, 0.3)

[1] 0.83692

- Q3. An unbiased coin is tossed 6 times, probability of head at any toss = 0.3. Let 'n' be no. of head that comes up. Calculate $P(X=2)$, $P(X=3)$, $P(1 < n < 5)$



```

> dbinom(2, 6, 0.3) (0.0, 0.1, 0.2)
[1] 0.324135 (0.0, 0.1, 0.2)
> dbinom(3, 6, 0.3) (0.0, 0.1, 0.2)
[1] 0.18522 (0.0, 0.1, 0.2)
> dbinom(2, 6, 0.3) + dbinom(3, 6, 0.3) +
  dbinom(4, 6, 0.3) (0.0, 0.1, 0.2)
[1] 0.74373 (0.0, 0.1, 0.2)

```

For $n=10$, $p=0.6$, evaluate binomial probabilities and plot graphs of pmf & cdf.

```

> n = seq(0, 10)
> y = dbinom(n, 10, 0.6)
> y
[1] 0.0001048576 0.0015728640 0.0106168320
0.0424673280 0.1114767360 0.2006581248
0.0403107840 0.0060466176 0.1209323520

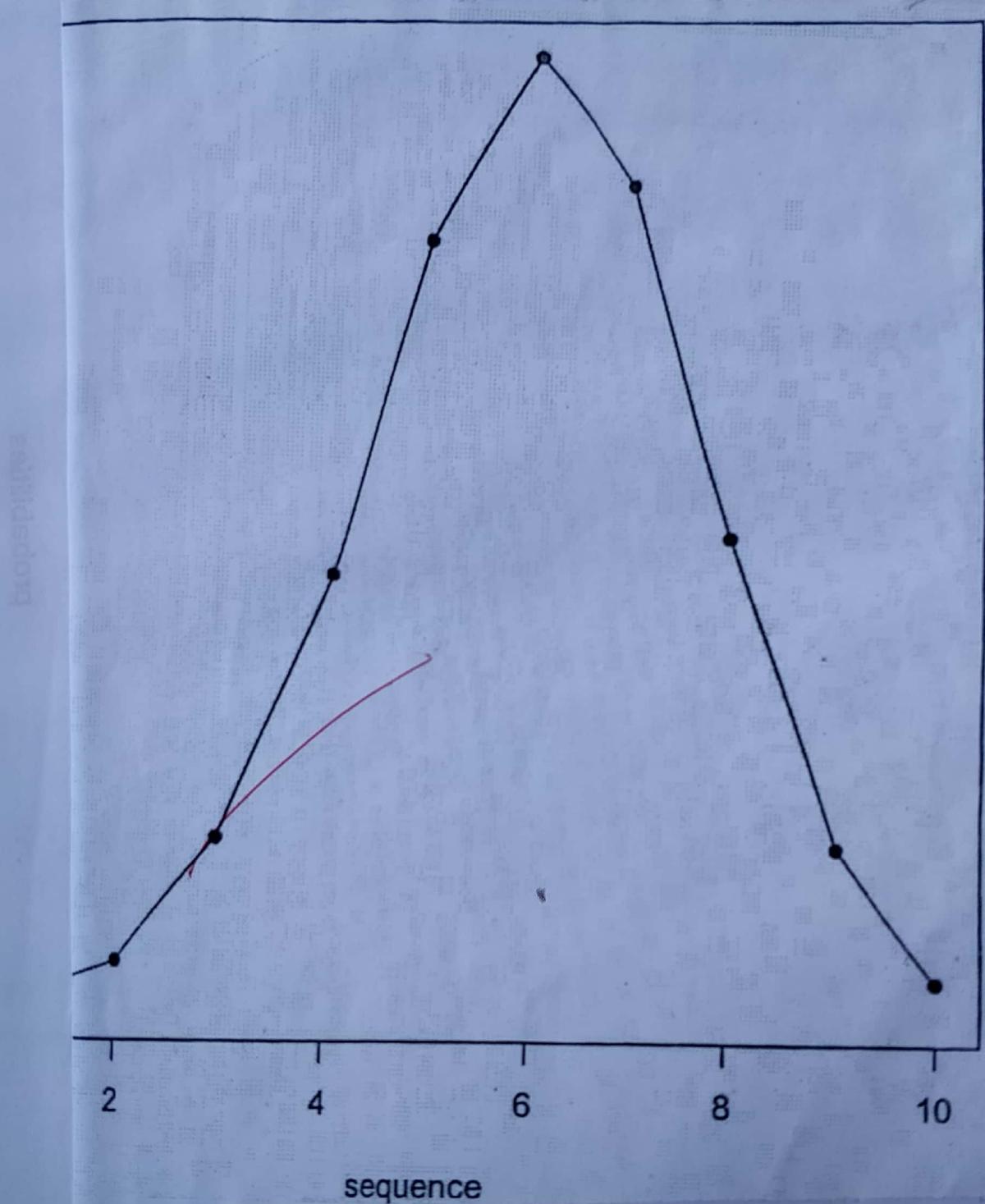
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> plot(n, y, xlab = "sequence", ylab = "probabilities",
  "0", pch = 16)

```

(25.0, 0.0, 1) margin 1
 8.835518e-01 [0]



Q7. Bits are sent for communication channel in packet of 12. If probability of bit being corrupted is 0.1. What is probability of no more than 2 bits are corrupted in a packet?

~~$\geq \text{pbinom}(2, 12, 0.1, \text{lowertail} = \text{F}) + \text{dbinom}(2, 12, 0.1)$~~
~~[1] 0.3409977~~

~~(21, 04, 06) WRONG
 NPT 1222000000~~

~~(21, 04, 07) WRONG - (21, 04, 07) WRONG
 PPA1FFFF1001~~

~~(21, 04, 08) WRONG - (21, 04, 08) WRONG
 102E018-0 011~~

~~(21, 04, 09) WRONG - 18
 111101000001~~

This answer is wrong because it is not in the question. The question asks for the probability of no more than 2 bits being corrupted, while the answer given is for 18 bits.

~~(21, 02, 07) WRONG
 PPA1FFFF1001~~

TITLE: Normal Distribution

A normal distribution of 100 students with mean = 40, SD = 15. Find no. of students whose marks are
 ① $P(X < 30)$,
 ② $P(40 \leq X < 70)$, ③ $P(25 < X < 35)$, ④ $P(X \geq 60)$

> pnorm(30, 40, 15)
 [1] 0.2524925

> pnorm(70, 40, 15) - pnorm(40, 40, 15)
 [1] 0.4772499

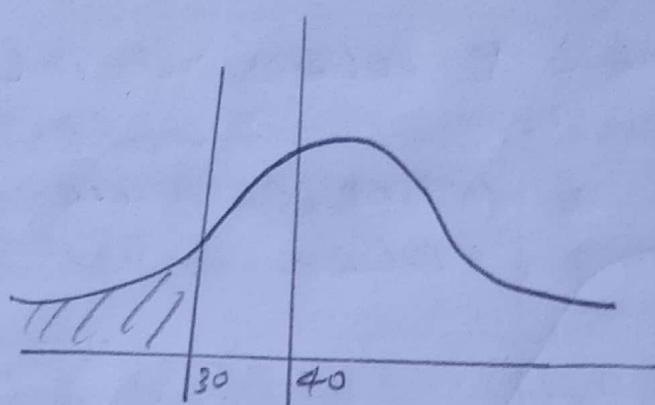
> pnorm(35, 40, 15) - pnorm(25, 40, 15)
 [1] 0.2107861

> 1 - pnorm(60, 40, 15)
 [1] 0.09121122

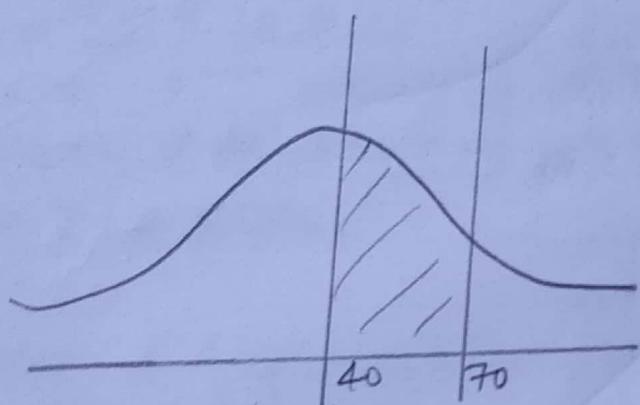
Q2. If random variable n follows normal distⁿ with mean = 50, $\sigma = 10$. Find
 ① $P(n \leq 70)$, ② $P(n > 65)$, ③ $P(n < 32)$, ④ $P(35 < n < 60)$
 ⑤ $P(20 < n = 30)$

> pnorm(70, 50, 10)
 [1] 0.9772499

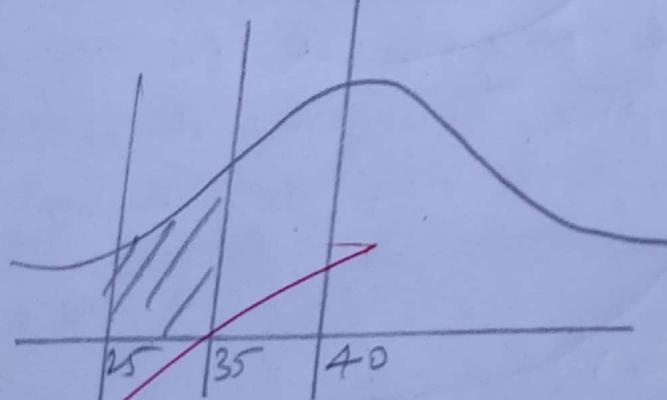
(a)



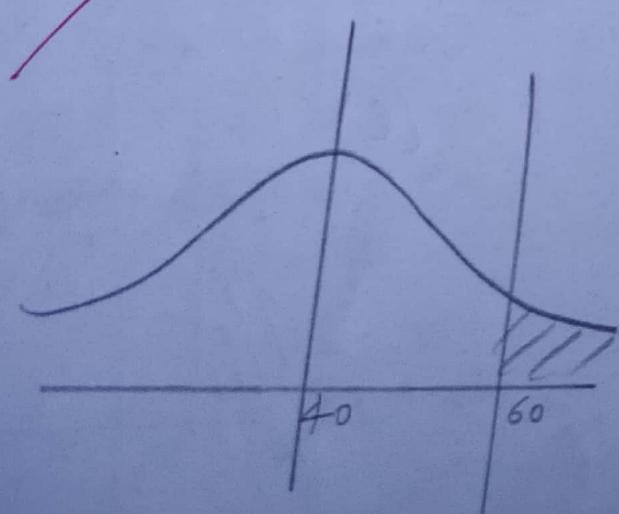
(b)



(c)



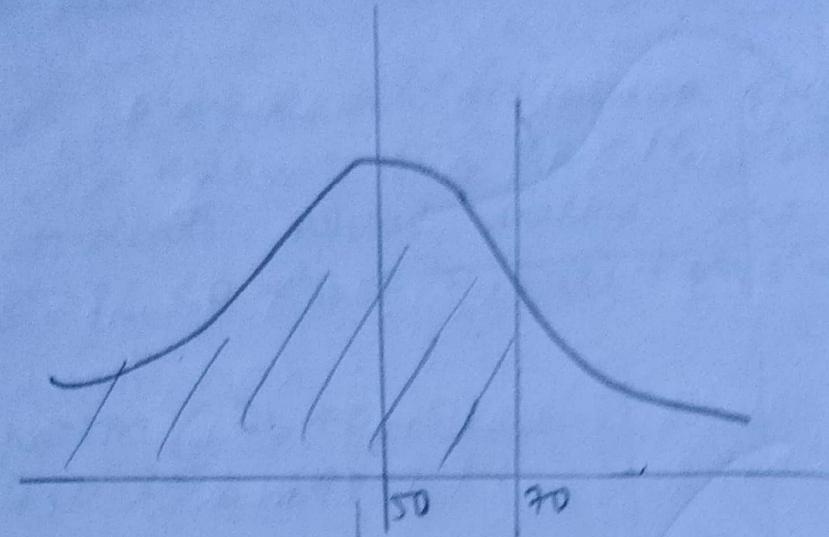
(d)



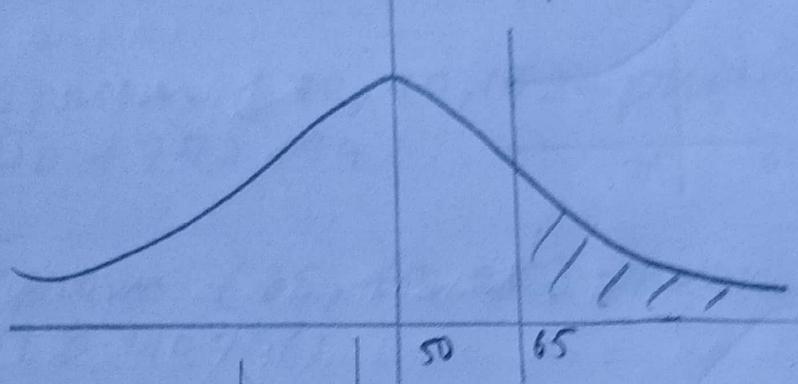
Q2

•

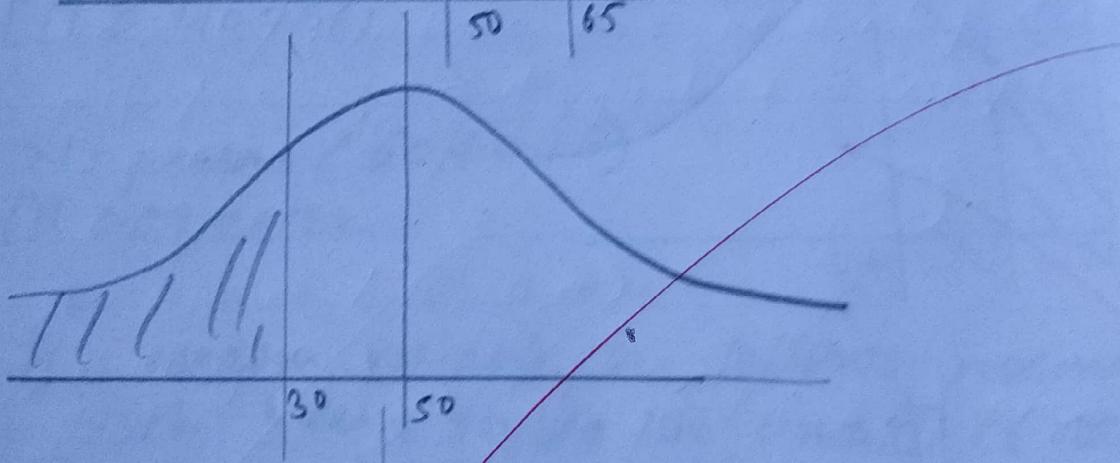
(a)



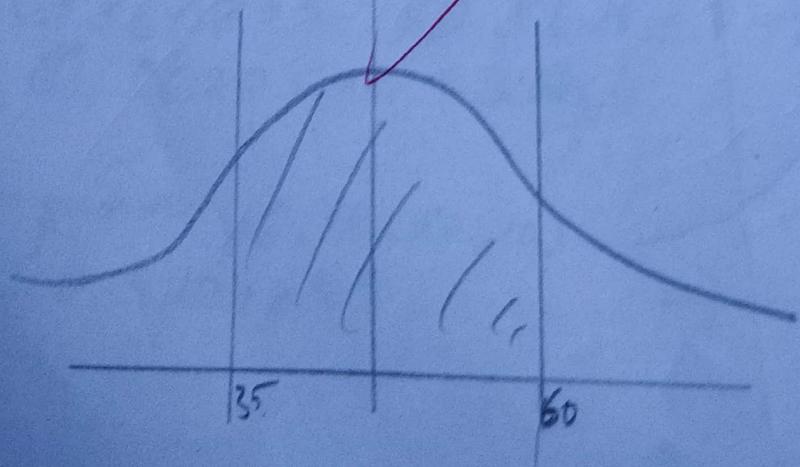
(b)



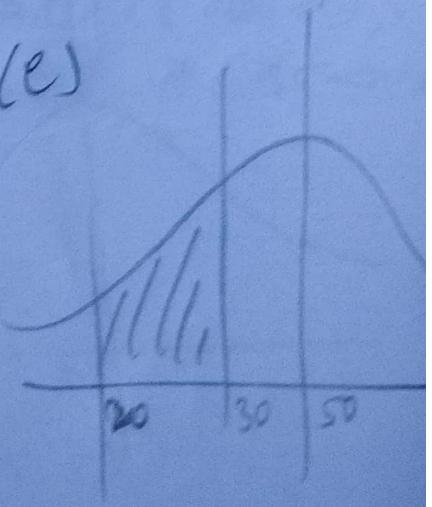
(c)



(d)



(e)



> rnorm (65, 50, 10)
 [1] 0.0668072

> rnorm (32, 50, 10)
 [1] 0.03593032

> rnorm (60, 50, 10) - rnorm (35, 50, 10)
 [1] 0.7745375.

> rnorm (30, 50, 10) - rnorm (20, 50, 10)
 [1] 0.02140023.

Q3. Let $x \sim N(160, 400)$ find k_1 & k_2 such
 that $P(X < k_1) = 0.6$ & $P(X > k_2) = 0.4$

> qnorm (0.6, 160, 20)
 [1] 165.0669

> qnorm (0.6, 160, 20)
 [1] 176.8324

Q4. A random variable X follows normal distribution with $\mu=10$, $\sigma=2$. Generate 100 observations & evaluate its mean & variance, median.

$> x = rnorm(100, 10, 2)$

$> summary(x)$

	Min	1st Q	Median	Mean	3rd Q	Max
[1]	5.713	8.444	9.723	9.914	11.325	14.238

$> var(x)$

[1] 3.648924

Q5. Write a command to generate 10 random variable/ numbers for normally distribution with $\mu=50$, $\sigma=4$, find the sample mean & median.

$> x = rnorm(10, 50, 4)$

$> summary(x)$

	Min	1st Q	Median	Mean	3rd Q	Max
[1]	44.73	50.46	52.01	52.35	54.39	58.85

Ans

PRACTICAL-04TOPIC : TESTING OF HYPOTHESIS.

case I : Sample mean and standard deviation given single population.

Q1. Suppose the food label on cookies bag states that it has atmost 2g of saturated fats in a single cookie. In sample of 55 cookies, it was found that mean and of saturated fat per cookies is 2.1g. Assume that sample std. significance can at 15%. Level of significance can be rejected the claims of food.

$$\rightarrow \sigma = 0.3, \bar{n} = 2.1$$

$$n = 35, \mu = 2$$

$$H_0 \text{ (null hypothesis)} = \mu \leq 2$$

$$H_1 \text{ (alt. hypothesis)} = \mu > 2.$$

$$Z = \frac{\bar{n} - \mu}{\sigma / \sqrt{n}} = \frac{2.1 - 2}{0.3 / \sqrt{3}}$$

$$= \boxed{1.972027}$$

$$\text{pvalue} = 1 - \text{pnorm}(z)$$

$$= 0.0243$$

\therefore Reject the null hypothesis.

$$\therefore \text{pvalue} < 0.05$$

Accept alternate hypothesis.

Q2. A sample of 100 customers was randomly selected. It was found that avg. spending was 275/- . The S.D. = 30. Using 0.05 level of significance could you conclude that the amt. is 250/- whereas restaurant claims that it is not 250/-.

$$\bar{x} = 275 ; \mu = 250$$

$$H_0 : \mu = 250$$

$$H_1 : \mu > 250$$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{275 - 250}{30/\sqrt{100}}$$

$$= 8.333$$

$$\text{pt. } (2.99, \text{ one-tail}) = P$$

$$\therefore \text{pvalue} = 2.305736e-15$$

Reject the null hypothesis.

$\therefore p\text{ value} < 0.05$.

\therefore Accept alternative hypothesis.

$$\{\because \mu > 250\}$$

- Q3. A quality control engineer finds that sample of 100 bulbs have avg. life of 470 hours. Assuming population test whether the population mean is 480 hours v/s population mean < 480 . at 1%

$$\rightarrow \bar{x} = 100, \mu < 480, \sigma = 25, H_0 = 480, \\ n = 470.$$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = -4$$

~~$$\text{P} + (Z, 99, \text{lower tail} = T) \\ = [6.112576e-05]$$~~

\therefore Reject the null hypothesis

$$\therefore p < 0.05$$

∴ Accept the alternative hypothesis
 $(\mu < 100)$

Q4. A principal at school claims that IQ is 100 of students. A random sample of 30 students whose IQ was found to be 112. The SD. of population is 15. Test the claim of principal.

→ Method - I :

$$H_0 = \mu = 100$$

$$H_1 = \mu > 100.$$

$$\bar{x} = 112, SD = 15, \mu = 100, n = 30.$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{112 - 100}{15/\sqrt{30}}$$

$$= \boxed{+3.8178}$$

$$\therefore p\text{value} = \boxed{5.88567 \times 10^{-6}}$$

∴ Reject the null hypothesis - claims of principal [$\mu = 100$].

Method - 2 :

$$H_0: \mu = 100$$

$$H_1: \mu \neq 100$$

$$\therefore P\text{ value} = 2 \cdot \text{program}(\text{abs}(z)) \\ = 2 \cdot [1.1771345e-05]$$

\therefore Reject the null hypothesis.

$$P\text{ value} < 0.05.$$

* Single Population Hypothesis Testing

Pt is believed that coin is fair.
 The coin is tossed 40 times, 25 times head. Indicate whether the coin is fair or not at 95% L.O.C.

$$\rightarrow Z = \frac{p - p_0}{\sqrt{\frac{p_0 q_0}{n}}}.$$

$$p_0 = 0.5, q_0 = 1 - 0.5 = 0.5$$

$$p = \frac{28}{40} = 0.7, n = 40.$$

$$\therefore z = \frac{0.7 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{40}}}$$

$$n_0 = N = 0.5$$

$$n_1 = N \neq 0.5$$

$$\text{pvalue} = 2 \times (1 - \text{pnorm}(\text{abs}(z)))$$

$$\therefore \text{pvalue} = 0.01141209.$$

→ Reject the null hypothesis $\therefore p < 0.05$.
 Accept the alternate hypothesis.

Q. In an hospital 480 females & 520 males are born. To confirm that male & female are equal in -

~~$$Z = \frac{p - p_0}{\sqrt{\frac{p_0 q_0}{n}}}; p = \frac{520}{1000} = 0.52, p_0 = 0.5, q_0 = 0.5$$~~

~~$$n = 1000, n_0 = [p = p_0], n_1 = [p \neq p_0]$$~~

~~$$z = (p - p_0) / \text{sqrt}(\text{abs}(z))$$~~

$$z = 1.2645.$$

~~$$\text{pvalue} = 2 \times [1 - \text{pnorm}(\text{abs}(z))]$$~~

$$\text{pvalue} = 0.2060506$$

\therefore Reject null hypothesis.

In a big city, 325 men out of 600 men are found to be self employed. Conclusion is that maximum men in city are self employed.

$$\rightarrow Z = \frac{P - P_0}{\sqrt{\frac{P_0 q_0}{n}}} ; P \rightarrow \frac{325}{600} = 0.541607$$

$$P_0 = 0.5, q_0 = 0.5, n = 600, M_0 = [P_0 - P]$$

$$M_1 = [P \neq P_0]$$

$$Z = (0.5416 - 0.5) / (\text{sqrt}(0.5 * 0.5 / 600))$$

$$Z = 2.037975$$

~~pvalue = 0.04155239.~~

~~Reject the null hypothesis :: pvalue < 0.5.~~

- Q. Experience shows that 20% of manufactured products are of top quality. In 1 day product of 400 articles only 50 are of top quality. Test hypothesis that experience product is wrong?

$$z = \frac{p - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \quad p = 0.125 (50/400), p_0 = 0.2, q_0 = 0.8$$

$$n = 400$$

$$H_0 = [p = 0.2]$$

$$H_1 = [p \neq 0.2]$$

$$z = (0.125 - 0.2) / \sqrt{(0.2 * 0.8) / 400}$$

$$z = -3.75$$

$$\text{pvalue} = 2 \times (1 - \text{pnorm}(\text{abs}(z)))$$

$$\text{pvalue} = 0.001768346.$$

\therefore Reject the null hypothesis $\because \text{pvalue} < 0.2$.
 Accept null hypothesis $i.e. p \neq 0.2$.

~~Q1.~~ In an election campaign, a telephone of 800 registered voters shows favour 460. Second note opinion 520 of 100 registered voters favoured the condition of 0.5%. QOC is there sufficient evidence that popularity has decreased.

$$\rightarrow n = 800, p_1 = 460/800 = 0.575, m = 100, p_2 = 520/100 = 0.52$$

$$p = (0.575 * 800 + 0.52 * 100) / (180) = 0.52$$

$$p = 0.54444.$$

$$z = \text{sgn}((0.544 * 0.46) + 1/8)$$

$$z = 0.001121394,$$

$$H_0: p = 0.544$$

$$H_1: p < 0.544$$

$$\text{pvalue} = 2 \times (1 - \text{pnorm}(\text{abs}(z)))$$

$$\therefore \text{pvalue} = 0.9991053$$

∴ Accept the null hypothesis ∵ pvalue > 0.5.

$$\therefore \text{Accept } p = 0.544.$$

Q2. From a consignment A, 100 articles are drawn & 44 were found defective from consignment B, 200, samples are drawn out of which 30 are defective. Test whether proportion of defective items in 2 consignments are significantly different.

$$\rightarrow H_0: p_1 = p_2$$

$$H_1: p_1 \neq p_2$$

$$p_2 = 30/200 = 0.15$$

$$\phi = \left(\frac{p_1 n + p_2 m}{n + m} \right)$$

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$$p = (0.22 * 200 + 0.15 * 200) / 400$$
$$p = 0.125$$

$$z = (\text{sqrt} (0.185 * 0.815)) * (2/200)$$
$$z = 0.003882976.$$

$$\text{pvalue} = 2 * (1 - \text{pnorm}(\text{abs}(z)))$$

$$\therefore \text{pvalue} = 0.9969018.$$

$$\text{pvalue} \geq p.$$

∴ Accept the null hypothesis $1 - c - p_1 = p_2$

Next

PRACTICAL-05CHI SQUARE

Q1. Use the foll^g. data to test whether the attribute conditions of Home & child are independent

		condition of homes	
		clean	dirty
condition of a child	clean	70	50
	of clean	80	20
	dirty	35	45

H_0 = Both are independent, H_1 = Both are dependent

$$n = c(70, 80, 35)$$

$$y = c(50, 20, 45)$$

$z = \text{data.frame}(n, y)$

		n	y
		70	50
z	1	80	20
	2	35	45

> chisq.test(z)

Pearson's chi squared test

data : z

$$\chi^2 = 25.646, \text{ df} = 2, \text{ pvalue} = 2.6$$

\therefore Reject null hypothesis.
Both are dependent.

Q2. A dice is tossed 120 times & following results are obtained:

No. of terms	Freq.
1	36
2	25
3	18
4	10
5	22
6	15

Test - ~~the~~ the hypothesis that dice is unbiased

$\therefore H_0$ = dice is unbiased, H_1 = dice is biased.

$$\geq \text{obs} = c(30, 25, 18, 10, 22, 15)$$

$$\geq \text{exp} = \text{sum}(\text{obs}) / \text{length}(\text{obs})$$

$\geq \text{ex p}$

[1] 20.

$\text{z-sum} \cdot (\text{obs-exp})^2 / \text{exp}$
 $\rightarrow \text{pctsq } (2, df = \text{length(obs)} - 1)$
 [1] 0.956659.

\therefore Accept the null hypothesis.
 Dice is unbiased.

\S IQ test was conducted if the students were observed before & after training the result are following :

Before	After
110	120
120	118
123	125
132	136
125	121

~~test whether there is change in the IQ test after training.~~

H_1 = no change in IQ

H_0 = IQ increased after training.

- > $a = c(120, 118, 125, 136, 121)$
- > $b = c(110, 120, 123, 123, 132, 125)$
- > $z = \text{sum}[(b-a) \geq 0] / a$
- > $\text{pnisq}[z, df = \text{length}(b) - 1]$

[1] 0.1135950.

Accept the null hypothesis

There is change in IQ after training

84. Online face
to face Graduate

Is there any association between student's preferences for type of education & method.

$\therefore H_0$ = Independent, H_1 = Dependent

> z = matrix(n, nrow=2)

? chisq.test(2)

Pearson's chi-squared test with fatality continuity correction.

data = z

χ^2 -squared = 18.05, df = 1, p-values = 2.15×10^{-5}

- ∴ Reject null hypothesis.
- ∴ Both are dependent.

Q5. A dice is tossed 180 times

Use of terms	frequency
1	20
2	30
3	35
4	40
5	12
6	43

Test the hypothesis that dice is unbiased

H_0 = dice is biased

H_1 = dice is unbiased.

> n = c(20, 30, 35, 40, 12, 43)

> chisq.test(n)

Chi-squared test for given probability.

data = z

χ^2 -squared = 23.933, df = 5, pvalue ~~<0.000~~
 $= 0.000223$.

∴ Reject null hypothesis

Dice is unbiased

reject

Topic - T-test

Q1 Let $n = 3366, 3337, 3361, 3410, 3316, 3357, 3348, 3356, 3376, 3382, 3377, 3355, 3408, 3401, 3398, 3424, 3388, 3374, 3384, 3374$.

Write test hypothesis.

$$\textcircled{1} H_0: \mu = 3400, H_1: \mu \neq 3400$$

$$\textcircled{2} H_0: \mu = 3400, H_1: \mu > 3400$$

$$\textcircled{3} H_0: \mu = 3400, H_1: \mu < 3400.$$

at 95% level of confidence. Also, check at 97% level of confidence.

$$\rightarrow \textcircled{1} H_0: \mu = 3400$$

$$H_1: \mu \neq 3400$$

~~> $n = (3366, 3337, 3361, 3410, 3316, 3357, 3348, 3356, 3376, 3382, 3377, 3355, 3408, 3401, 3398, 3424, 3383, 3374, 3384, 3374)$.~~

~~> t-test ($n, \mu = 3400$, alter \rightarrow "two-sided", confe. level = 0.95).~~

data : n

$$t = -4.4865, df = 19, p\text{ value} = 0.0002528$$

alternative hypothesis : True mean is not equal to 3400 ..

95% confidence level:

$$3361.797 \quad 3386.103$$

Sample estimates:

mean of n :

$$3373.95$$

\therefore Reject H_0 .

\therefore Accept H_1 .

t -test ($n, \mu = 3400$, alter = "two-sided",
conf. level = 0.97)

One sample test

data: n

$$t = -4.4865, df = 19, P\text{-value} = 0.00028$$

Sample estimates:

mean of x :

$$3373.95$$

\therefore Reject H_0 .

\therefore Accept H_1 .

$$\rightarrow ① H_0: \mu = 3400$$

$$H_1: \mu > 3400.$$

t -test ($n, \mu = 3400$, alter = "greater",
conf. level = 0.95)

One sample t-test:

data : x

$t = -4.4865$, $df = 19$, $P\text{-value} = 0.9999$

alternative hypothesis : true mean is greater than 3400, $z_{\text{avg}} = 3363.71$ Tnf.

sample estimates :

mean of x :

3373.75, $\therefore \text{Accept } H_0$.

③ $H_0: \mu = 3400$

$H_1: \mu < 3400$

$t\text{-test}$ ($n, \text{mu}, \text{alt} = \text{"less"}$; $\text{conf.level} = 0.95$)
one-sided $t\text{-test}$.

data : x

$t = -4.4865$, $df = 19$, $p\text{value} = 0.0001264$

alternative hypothesis : true mean is less than 3400.

$-z_{\text{avg}} = 3383.99$.

~~Sample estimates :~~

mean of x :

3373.75:

$\therefore \text{Reject } H_0$

$\therefore \text{Accept } H_1$,

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7 t-test (α , $\mu_1 = 3400$, alter = "less", conf. level 0.99)

one sample t-test.

data: n

$t = -4.4865$ df = 19, p-value = 0.0001264

alternative hypothesis: true-mean is less than 3400.

93% level of confidence

-3385.563

sample estimates:

mean of x .

3378.95

∴ Reject H_0

∴ Accept H_1 ,

Q2. Below all the data of gain in weights.
diets A and B.

Diet A: 25, 32, 30, 43, 24, 14, 32, 24, 31, 31, 32, 25

Diet B: 44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18

→ $\therefore H_0 = a - b = 0$

$\therefore H_1 = a - b \neq 0$

$\geq a = c(25, 32, 30, 43, 24, 14, 32, 24, 31, 31, 32, 25, 18, 21)$

$\geq b = c(44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18, 21)$

t-test (a, b , paired = 7, alter = "two-sided",
conf. level = 0.95).

Paired t-test.

data : a and b

$t = -0.62782$, $df = 11$, $p\text{-value} = 0.5429$.

alternative hypothesis : the difference in means
is equal to 0.

confidence interval

-14.267330 7.933997.

sample estimates :

mean of differences

-3.110667

\therefore Accept H_0 .

There is no difference in ~~congrat~~. weights.

Q3. Eleven student gave the test after 1 month they again gave the last after tuitions, do the marks gives evidence that students have benefitted by coaching.

E1: 23, 20, 19, 21, 18, 20, 18, 17, 23, 16, 19.

E2: 24, 19, 22, 18, 20, 22, 20, 20, 23, 20, 27.

test at 99 level of confidence.

$\rightarrow \therefore H_0: e_1 = e_2$

$\therefore H_1: e_1 < e_2$.

t-test (e_1, e_2 , paired = T, alter = "less",
conf. level = 0.99).

Paired t-test.

data : d_1 and d_2

$t = -1.4832$, $df = 10$, $p\text{-value} = 0.08441$
 alternative hypothesis: true difference in
 mean is less than 0.

99 % confidence interval :

$$-7.9 < \mu < 0.863333$$

sample estimates :

mean of differences

$$-1$$

\therefore Accept H_0 .

Q4: Two drugs for BP was given and data was collected

$$D1: 0.7, -1.6, 0.2, -1.2, -0.1, 2.4, 3.7, 0.8, 0.1$$

$$D2: 1.4, 0.8, 1.1, 0.1, 0.1, 4.4, 5.5, 1.6, 4.6, 3.4$$

The 2 drugs have same effect, check whether 2 drugs have same ~~less~~ effect on patient or not.

\rightarrow Paired t-test (d_1, d_2 , alter = "two-sided", paired = T, conf.level = 0.95).

Paired t-test.

data : d_1 and d_2

$t = -4.0621$, $df = 9$, $p\text{-values} = 0.002833$

\therefore Reject H_0

\therefore Accept H_1 .

If there is difference in salaries for the same job in 2 different countries.

CA : 53000, 49958, 41974, 44366, 40470, 36963 :

CB : 62990, 58850, 49475, 52263, 47674, 43552 .

$$\rightarrow H_0: \bar{x}_1 = \bar{x}_2$$

$$H_1: \bar{x}_1 \neq \bar{x}_2$$

$\bar{x}_{CA} = \bar{C}(53000, 49958, 41974, 44366, 40470, 36963)$

$\bar{x}_{CB} = \bar{C}(62990, 58850, 47674, 52263, 43552)$

t -test (CA, CB, paired = T, alter = "two-sided", conf.level = 0.95)

paired t -test .

data : CA and CB .

$t = -4.4569$, df = 5, p-values = 0.00666,
alternative hypothesis = true difference in
means is not equal to 0 .

95% confidence level :

$$-10404.821 - 2792.846$$

Sample estimates :

mean of differences :

$$-6598.833$$

∴ Reject H_0 .

∴ Accept H_1 .

PRACTICAL - 07

Topic: F-test .

- Q1. Life expectancy in 10 regions of India in 1990 and 2000 are given below. Test whether variance at the two times are the same.

1990: 37, 39, 36, 42, 45, 44, 46, 49, 50, 51

2000: 44, 45, 47, 43, 42, 49, 50, 41, 48, 52, 58, 48.

$\rightarrow x = c(37, 39, 36, 42, 45, 44, 46, 49, 50, 51)$

$\rightarrow y = c(44, 45, 47, 43, 42, 49, 50, 41, 48, 52, 58, 48)$

$\rightarrow \text{var.test}(x, y)$

F test to compare two variances:

data : x and y

$F = 1.1449$, num df = 9, denom df = 11, pvalue = 0

alternative hypothesis: true ratio of variances is not equal to 1

95% confidence interval:

0.3191005 4.4789350

Sample ~~differences~~ estimates:

ratio of variances:

1.1449

Q2. For following data, test hypothesis for -

- ① Equality of two population mean (\rightarrow T-test)
- ② Equality of two proportion variance (\rightarrow F-test)

Sample 1: 175, 168, 145, 190, 181, 185, 175, 200

Sample 2: 180, 170, 153, 180, 179, 183, 187, 205.

$\rightarrow \bar{x} = c(175, 168, 145, 190, 181, 185, 175, 200)$

$\rightarrow y = c(180, 170, 153, 180, 179, 183, 187, 205)$.

$\rightarrow \text{var.test}(x, y)$

F test to compare two variances.

Data: x and y .

$F = 1.25$, num df = 7, denom df = 7, p-value 0.759
alternative hypothesis: true ratio of variances is not equal to 1.

0.2502589 0.2437398

sample estimates:

ratio of variances

1.250021.

~~t-test~~ (n)

One sample t-test

Data: 0

Sample estimates:

mean of x :

177.375.

Q3.

The following are prices of commodities in sample of shop selected at random from different cities.

C-A: 74.10, 77.70, 75.35, 74, 73.80, 79.30, 75.80, 76.80
C-B: 70.80, 74.90, 76.20, 72.80, 78.10, 77.10, 76.40,
14.70, 69.80, 81.20

$$\rightarrow \bar{a} = C(74.10, 77.70, 75.35, 74, 73.80, 79.30, 75.80, 76.80, 77.10, 76.40)$$
$$\rightarrow \bar{b} = C(70.80, 74.90, 76.20, 72.80, 78.10, 74.40, 69.80, 81.20)$$

> var.test(a, b)

F-test to compare two variances.

data: a and b.

F = 0.22579, num df = 9, denom df = 7,
pvalue = 0.04249.

alternative hypothesis: true ratio of variance
is not equal to 1

95% confidence level:

sample estimates:

ratio of variances:

0.225791.

Q4. Prepare csv file in excel, import file in R & apply the test to check equality of variance of 2 datas.

Obs. 1 → 10, 15, 17, 11, 16, 20

Obs 2 → 15, 14, 16, 11, 12, 19.

> Data = read.csv(file choose(), header = TRUE)

> Data

	Obs. 1	Obs 2
1	10	15
2	15	14
3	17	16
4	11	11
5	16	12
6	20	19

> attach(Data).

> var.test(Obs.1, Obs.2).

F test to compare two variances

data: Obs.1 and Obs.2

F = 1.7068, num df = 5, denom df = 5, p-value = 0.5717.

alternative hypothesis: true ratio of variances is not equal to 1

95 % confidence interval:

0.238838 12.197640

sample estimates:

85.

ratio of variances is
1.706827.

now

3210 1230

21 01 P

21 21

21 51

11 11 p

21 21

P 05 3

(std) ditha

(std) net PIVS

residuals of squares of $\chi^2 + 7$

squares 1.630 0.63

values of $\chi^2 = 8805 - 1 = 7$

+ 152 0

values of $\chi^2 = 8805 - 1 = 7$

1st 1st 1st

1st 1st 1st

1st 1st 1st

PRACTICAL-08SIGN TEST.

Q1. The time of failures (in hours) of 10 randomly selected 9-volt battery of a certain company is as follows -

28.9, 15.2, 28.7, 72.5, 48.6, 52.4, 37.6, 49.5, 62.1, 54.5.

Test hypothesis that population median is 63 against alternative that is less than 63 at 5% level of significance.

$\rightarrow x = c(28.9, 15.2, 28.7, 72.5, 48.6, 52.4, 37.6, 49.5, 62.1, 54.5)$

$$H_0: \text{median} = 63$$

$$H_1: \text{median} < 63.$$

$$y = \text{which}(x > 63)$$

[1] 4

~~$$z = \text{which}(x < 63)$$~~

~~$\# length(z)$~~

[1] 6

$n \rightarrow$ Total negative and positive signs.

$$n = y + z$$

$$> qbinom(0.5, 10, 0.5)$$

[1] 2

$\therefore qbinom < 2$, Accept null hypothesis.

Q2. Following data gives weight of 40 students in random sample -

46, 49, 57, 64, 46, 67, 54, 48, 69, 61, 57, 54, 50,
 48, 65, 61, 66, 54, 50, 48, 49, 62, 47, 49, 47, 51,
 59, 63, 53, 56, 67, 49, 60, 64, 53, 50, 48, 51, 52, 54,

Use sign test to test whether mean weight of population is 50 kg against alternative that it is greater than 50 kg.

→ $n = c(46, 49, 57, 64, 46, 67, 54, 48, 69, 61, 57, 54, 50,$
 $48, 65, 61, 66, 54, 50, 48, 49, 62, 47, 49, 47, 51,$
 $59, 63, 53, 56, 67, 49, 60, 64, 53, 50, 48, 51, 52, 54)$

H_0 : mean weight ≤ 50

H_1 : mean weight > 50

y = which ($x > 50$)

length(y)

[1] 25

z = which ($x < 50$)

length(z)

[1] 12

$qbinom(0.05, 40, 0.5)$

[1] 15

∴ Reject null hypothesis

Q: Median age of tourists is visiting a certain place is claimed to be 41 years. A random sample of 17 tourists have age -

48, 52, 25, 29, 57, 39, 45, 36, 30, 49, 28, 39, 44, 63, 32, 65, 42.

Use sign test to check claim.

$\rightarrow x = c(48, 52, 25, 29, 57, 39, 45, 36, 30, 49, 28, 39, 44, 63, 32, 65, 42)$

H_0 : age = 41 years

H_1 : age \neq 41 years.

$z = \text{which}(x > 41)$

length(z)

[1] 8

$j = \text{which}(x < 41)$

length(j)

[1] 8

$\text{q binom}(0.05, 17, 0.5)$

[1] 5.

\therefore Accept null hypothesis.

Q4. The time in minutes that a patient has to wait for consultation is recorded as follows.

15, 17, 24, 25, 20, 21, 32, 28, 12, 25, 24, 26.

Use Wilcoxon sign test to check whether the median in time is greater than 20 at 5% level of significance.

→ H_0 : minutes > 20
 H_1 : minutes < 20 .

$x = c(15, 17, 24, 25, 20, 21, 32, 28, 12, 25, 24, 26)$

$z = \text{which}(x > 20)$

$\text{length}(z)$

[1] 8

$j = \text{which}(x < 20)$

$\text{length}(j)$

[1] 3

wilcox.test(x, alternative = "greater")

[1]

Wilcoxon signed rank test with

continuity correction.

data : x

V = 78, p-value = 0.001253

alternative hypothesis : true location is greater than 0

∴ Accept null hypothesis.

- Q5. The weight in kgs of the person before and after stop smoking is -
- | Before | After |
|--------------------|--------------------|
| 65, 75, 75, 72, 62 | 72, 72, 72, 66, 82 |

Use wilcoxon test to check weight of a person is less after smoking at 5% level of significance.

$$\rightarrow x = c(65, 75, 75, 72, 62)$$

$$y = c(72, 72, 72, 62, 82)$$

$$z = x - y$$

\sim

$$[1] -7 \quad 0 \quad 3 \quad -7 \quad -4$$

wilcox.test(z, mu=0, alternative = "less")

[1]

wilcoxon signed rank test with continuity correction

Data: z

V = 1, p-value = 0.09873

alternative hypothesis : true location is less than 0.

∴ Reject null hypothesis

Neat

* Topic :

Q1. The following data gives effect of 3 treatments

	T_1	T_2	T_3
1	2	10	10
2	3	8	13
3	7	7	14
4	2	5	13
5	6	10	15

Test the hypothesis of all the treatments
are equally effective.

$$t_1 = c(2, 3, 7, 2, 6)$$

$$t_2 = c(10, 8, 7, 5, 10)$$

$$t_3 = c(10, 13, 14, 13, 15)$$

data = data.frame(t_1, t_2, t_3)

data.

$t_1 \quad t_2 \quad t_3$

1	2	10	10
2	3	8	13
3	7	7	14
4	2	5	13
5	6	10	15

c = stack(data)

c

	values	ind
1	2	t1
2	3	t1
3	7	t1
4	2	t1
5	6	t1
6	10	t2
7	8	t2
8	7	t2
9	5	t2
10	10	t2
11	10	t3
12	13	t3
13	14	t3
14	13	t3
15	15	t3

aov (values ~ ind, e)

Call:

aov (formula = values ~ ind, e)

Terms :

	ind	Residuals
Sum of squares	203.3333	54.0000
Deg. of Freedom	2	12

Residual standard error : 2.12132

Estimated effects may be unbalanced.

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oneway.test (values, ~ ind, e)

One-way analysis of means (not assume equal variance)

data: values and ind

 $F = 21.537$, num df = 2.0000, denom df = 7.9314

p-value = 0.0006232

∴ Reject NULL HYPOTHESIS.

Q2. The following gives life of types of 4 brands

A,B,C,D;

	A	B	C	D
20	22	19	16	
23	24	17	20	
18	17	17	16	

	A	B	C	D
21	19			
22	17			
20				

Test that average life of all brands are same.

 $\rightarrow a = c(20, 23, 18, 17, 22, 24)$ $b = c(19, 15, 17, 20, 16, 17)$ $c = c(21, 19, 22, 17, 20)$

data = data.frame(a, b, c)

data

	a	b	c	d
1	20	19	21	15
2	22	16	19	19
3	23	17	17	16
4	24	17	22	18
	18	20	20	16
	17	16		14

e = list (a1 = a, b1 = b, c1 = c, d1 = d)

a
b
c

aov (values ~ ind, f)

call :

aov (formula = values ~ ind, data = f)

Terms :

	ind	Residuals
sum of squares	97.46812	82.96667
Deg. of Freedom	3	19

Residual standard error: 2.059657
 Estimated effects may be damaged.

One way . test (values ~ ind, data = f)

p-value = 0.006291

∴ Reject NULL HYPOTHESIS.

93.

58:

3 types of war is applied for protection of cars and number of days of protection per noted. Test whether those are equally effective.

A	B	C
46, 46, 44, 45, 48, 49	40, 42, 51, 52, 95	50, 53, 58, 59.

$$\rightarrow a = B(44, 45, 46, 47, 48, 49)$$

$$b = C(40, 42, 51, 52, 55)$$

$$c = C(50, 53, 58, 59)$$

$$e = \text{list}(a1 = a, b1 = b, c1 = c)$$

$$d = \text{stack}(e)$$

$$\text{aov}(\text{values} \sim \text{ind}, \text{data} = d)$$

call:

$$\text{aov}(\text{formula} = \text{values}, \text{~} \text{ind} = d)$$

Terms:

	ind	Residuals.
Sum of squares	182.4333	245.5000
Deg. of freedom	2	12

Residual standard error: 4.523069

Estimated effect may be unbalanced.

one way test (values ~ ind, data = d)

One-way analysis of mean

data : values and ind

F = 6.2398, num df = 2.0000, denom df = 5.4

p-value = 0.03822

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∴ Reject NULL HYPOTHESIS.

Q4. An experiment was performed on 8 people & observations were noted. Test hypothesis for these three groups.

G1	G2	G3
26, 23, 51, 48, 58, 37, 29, 44	22, 27, 29, 39, 46, 48, 49, 65	59, 66, 38, 49, 56, 60, 56, 62.

Test hypothesis for all groups have equal results on their health.

a = c(23, 26, 51, 48, 58, 37, 39, 44)

b = c(27, 22, 28, 39, 46, 48, 49, 65)

c = c(59, 66, 38, 49, 56, 60, 56, 62)

d = data.frame(a, b, c)

e = stack(d)

f

~~one way~~ aov(values ~ ind, data = d)
call:

aov(formula = values ~ ind, f)

Terms:

Sum of squares

Deg. of Freedom

ind	Residuals
172.4009	267.0691
2	12

one way test (values ~ ind, data = e)

one way analysis of means

data : values and ind .

F = 5.3711 , numdf = 2.00 , denomdf = 13.41 ,
pvalue = 0.01935

∴ Reject NULL HYPOTHESIS.

Next