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A DYNAMIC ABSORBER

⊙ PROBLEM STATEMENT:

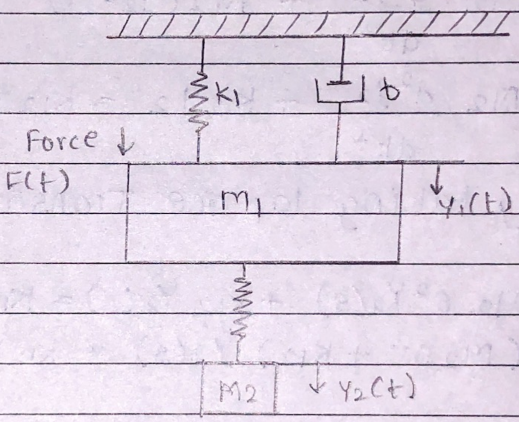
A dynamic vibration absorber is shown in fig. This system is representative of many situations involving the vibration of machines containing unbalanced components. The parameters M2 and K12 may be chosen so that the main mass M1 does not vibrate in the steady state when F(t) = 2*sin(10*t).

Obtain the differential equation describing sys.

M1 = 100

K1 = 50

b = 50



⊙ GIVEN DATA:

F(t) = 2*sin(10*t)

M1 = 100

K1 = 50

b = 50

a = 2

ω_0 = 10

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DERIVATION:

Applying Newton's second law of motion on mass m_1,

$$M_1 \frac{d^2y_1}{dt^2} = F - k_1y_1 - b\frac{dy_1}{dt} - K_{12}(y_1 - y_2)$$

$$m_1\frac{d^2y_1}{dt^2} + b\frac{dy_1}{dt} + k_1y_1 + k_{12}(y_1 - y_2) = F \quad \text{(I)}$$

Applying Newton's second law of motion on Mass m_2,

$$m_2\frac{d^2y_2}{dt^2} + k_{12}(y_2 - y_1) = 0$$

$$m_2\frac{d^2y_2}{dt^2} + k_{12}y_2 = k_{12}y_1 \quad \text{(II)}$$

By taking laplace Transform,

$$m_2 s^2 Y_2(s) + k_{12} Y_2(s) = k_{12}Y_1(s)$$

$$(m_2s^2 + k_{12})Y_2(s) = k_{12}Y_1(s)$$

$$Y_2(s) = \frac{k_{12}}{m_2s^2 + k_{12}} Y_1(s) \quad \text{(III)}$$

The force, $f(t) = a\sin\omega_0 t$

Take laplace transform of the force function,

$$F(s) = \frac{a\omega_0}{s^2 + \omega_0^2}$$

For the mass m_1 not to vibrate under steady state, the forces,

$$y_1(t) = 0$$

$$Y_1(s) = 0$$

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From equation (III)

$$Y_1(s) = \left[\frac{M_2 s^2 + k_{12}}{k_{12}}\right] Y_2(s)$$

$$0 = \left[\frac{M_2 s^2 + k_{12}}{k_{12}}\right] Y_2(s)$$

$$M_2 s^2 + k_{12} = 0$$

$$k_{12} = -s^2 M_2$$

put $s = j\omega_0$

$$k_{12} = -(j\omega_0)^2 M_2$$

$$k_{12} = \omega_0^2 M_2 \qquad \therefore k_{12} = 100 M_2$$

From equation (I)

$$M_1 \frac{d^2 y_1}{dt^2} + b\frac{dy_1}{dt} + k_1 y_1 + k_{12}(y_1 - y_2) = F$$

Substitute $k_{12} = \omega_0^2 M_2$ and $f(t) = a \sin \omega_0 t$

$$M_1 \frac{d^2 y_1}{dt^2} + b\frac{dy_1}{dt} + k_1 y_1 + \omega_0^2 M_2 (y_1 - y_2) = a \sin \omega_0 t$$

$$M_1 \frac{d^2 y_1}{dt^2} + b\frac{dy_1}{dt} + k_1 y_1 + M_2 \omega_0^2 y_1 = a \sin \omega_0 t + M_2 \omega_0^2 y_2$$

From equation (II)

$$M_2 \frac{d^2 y_2}{dt^2} + k_{12} y_2 = k_{12} y_1$$

Substituting $k_{12} = \omega_0^2 M_2$

$$M_2 \frac{d^2 y_2}{dt^2} + \omega_0^2 M_2 y_2 = \omega_0^2 M_2 y_1$$

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$$\frac{d^2 y_2}{dt^2} + \omega_0^2 y_2 = \omega_0^2 y_1 \qquad \text{IV}$$

Differential equations after substituting given values,

$$M_1 \frac{d^2 y_1}{dt^2} + b \frac{dy_1}{dt} + K_1 y_1 + M_2 \omega_0^2 y_1 = a \sin \omega_0 t + M_2 \omega_0^2 y_2$$

$$100 \frac{d^2 y_1}{dt^2} + 50 \frac{dy_1}{dt} + 50 y_1 + M_2 * 100 y_1 = 2 \sin 10t + M_2 * 100 y_2$$

In eq^n IV

$$\frac{d^2 y_2}{dt^2} + \omega_0^2 y_2 = \omega_0^2 y_1$$

$$\therefore \frac{d^2 y_2}{dt^2} + 100 y_2 = 100 y_1$$