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P4 HIGH PASS FILTER DESIGN

PROBLEM STATEMENT:

Using the bilinear transformation, design a highpass filter, monotonic in passband with cutoff frequency of 1000 Hz and down 10 dB at 350 Hz. The sampling frequency is 5000 Hz. Implement using basic building blocks. Show the derivation for this filter. Demonstrate the filter's output for 5 different frequencies ranging from 100 Hz to 10,000 Hz.

GIVEN DATA:

$\alpha_p = 3\,dB$

$\omega_c = \omega_p = 2 \times \pi \times 1000 = 2000\pi$ rad/sec

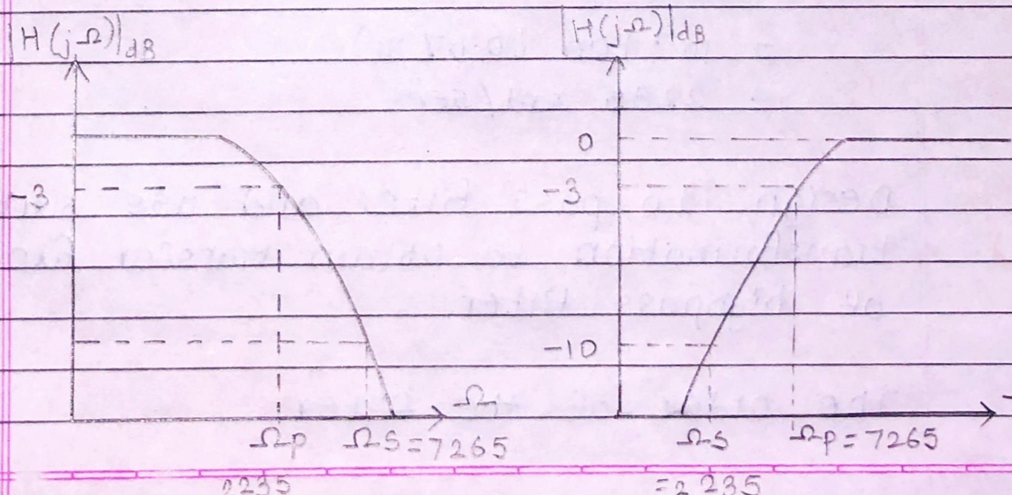
$\alpha_s = 10\,dB$

$\omega_s = 2 \times \pi \times 350 = 700\pi$ rad/sec

$f_s = 5000$ Hz

$$T = \frac{1}{f_s} = \frac{1}{5000} = 2 \times 10^{-4} \text{ sec}$$

FREQUENCY RESPONSE:



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DERIVATION:

- Passband frequency of high pass filter is stopband frequency of low pass filter.
- Also, stopband frequency of high pass filter is passband frequency of low pass filter.
- To meet charcteristics of monotonic in passband and stopband, the filter is Butterworth filter.
- Prewrapping the digital frequencies we have, Ωp, Ωs,

$$\Omega p = \frac{2}{T} \tan \frac{\omega pT}{2}$$

$$= \frac{2}{2 \times 10^{-4}} \tan \frac{(2000\pi \times 2 \times 10^{-4})}{2}$$

$$= 10^4 \tan (0.2\pi)$$

$$= 7265 \text{ rad/sec.}$$

$$\Omega s = \frac{2}{T} \tan \frac{\omega sT}{2}$$

$$= \frac{2}{2 \times 10^{-4}} \tan \frac{(700\pi \times 2 \times 10^{-4})}{2}$$

$$= 10^4 \tan (0.07\pi)$$

$$= 2235 \text{ rad/sec.}$$

Design low pass filter and use suitable transformation to obtain transfer function of highpass filter.

The order of the filter.

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$$N = \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log \frac{\Omega_s}{\Omega_p}} = \frac{\log \sqrt{\frac{10^{0.1(10)}-1}{10^{0.1(3)}-1}}}{\log \frac{7265}{2235}}$$

$$= \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118}$$

$$= 0.932.$$

Take $N = 1$;

The first order Butterworth filter for $\Omega_c = 1$ rad/sec is,

$$H(s) = \frac{1}{1+s}$$

The highpass filter for $\Omega_c = \Omega_p = 7265$ rad/sec can be obtained by using the transformation.

$$s \rightarrow \frac{\Omega_c}{s} \quad \text{i.e.} \quad s \rightarrow \frac{7265}{s}$$

The transfer function of highpass filter,

$$H(s) = \frac{1}{1+s}\bigg|_{s = \frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

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Using Bilinear transformation,

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{s}{s+7265}\Big|_{s=\frac{2}{2\times10^{-4}}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{10000\left(1-z^{-1}/1+z^{-1}\right)}{10000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+7265}$$

$$H(z) = \frac{0.5792\,(1-z^{-1})}{1-0.1584z^{-1}}$$

$$= \frac{0.5792\,(1-z^{-1})}{1-0.1584z^{-1}}$$