* Testing the Difference between means :-

Case 1 If samples are independent

* formulae for standard deviation :

- Genral :

$$S_p = \sqrt{\frac{\Sigma (x_{i1} - \bar{x}_1)^2 + \Sigma (x_{i2} - \bar{x}_2)^2}{n_1 + n_2 - 2}}$$

- for unbiased standard deviation :

$$S_p = \sqrt{\frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{n_1 + n_2 - 2}}$$

- for standard deviation :

$$S_p = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$$

* Standard error (S.E) :

$$\boxed{S.E = S_p \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

if standard deviation of populations σ_1, σ_2 are given then

$$S.E = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

* t - distribution formula:

$$\boxed{t = \frac{\overline{X_1} - \overline{X_2}}{S.E.}}$$

Example 1. If two independent random samples of sizes 15 and 8 have respectively the following mean and population standard deviation,

$\overline{X_1} = 980 \qquad \overline{X_2} = 1012$

$\sigma_1 = 75 \qquad \sigma_2 = 80$

Test the hypothesis that $\mu_1 = \mu_2$ at 5% level of significance

solution:- Given: $\overline{X_1} = 980, \quad \overline{X_2} = 1012$

$\sigma_1 = 75, \quad \sigma_2 = 80$

i) Null hypothesis H_0 : $\mu_1 = \mu_2$

ii) Alternate hypothesis : $\mu_1 \neq \mu_2$

iii) Calculation of test statistic :

$$S.E = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{(75)^2}{15} + \frac{(80)^2}{8}}$$

$$= \sqrt{375 + 800} = 34.28$$

Now, $$t = \frac{\overline{X_1} - \overline{X_2}}{S.E.} = \frac{980 - 1012}{34.28} = -0.93$$

$\therefore \quad |t| = 0.93$

iv) Level of significance : $\alpha = 0.05$ (5%)

v) critical value : The table value of t at 5% level of significance is 1.96

vi) Decision : Note that the computed value $|t| = 0.93$ is less than the table value 1.96

Hence, the Null hypothesis is accepted.

$\therefore$ The population means are equal $\mu_1 = \mu_2$

Example 2. The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively. The sum of the squares of the deviations from the means are 26.94 and 18.73 respectively. Can the samples be considered to have been drawn from the same population ?

Solution: Given: $n_1 = 9$, $n_2 = 7$

$\bar{X}_1 = 196.42$, $\bar{X}_2 = 198.82$

and $\Sigma(X_{i1} - \bar{X}_1)^2 = 26.94$, $\Sigma(X_{i2} - \bar{X}_2)^2 = 18.73$

i) Null Hypothesis H_0 : $\mu_1 = \mu_2$

Alternative Hypothesis H_a : $\mu_1 \neq \mu_2$

iii) Calculation of test statistic :

The samples standard deviation is

$$S_p = \sqrt{\frac{\Sigma(x_{i1}-\bar{x}_1)^2 + \Sigma(x_{i2}-\bar{x}_2)^2}{n_1+n_2-2}} = \sqrt{\frac{26.94+18.73}{9+7-2}}$$

$$\therefore \quad S_p = 1.81$$

Now the standard error is

$$S.E = S_p \times \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = (1.81)\sqrt{\frac{1}{9}+\frac{1}{7}}$$

$$= 0.91$$

$$\text{Now, } t = \frac{\bar{x}_1 - \bar{x}_2}{S.E.} = \frac{196.42-198.82}{0.91} = -2.64$$

$$\therefore \quad |t| = 2.64$$

iv) Level of significance : $\alpha = 0.05$

v) Critical value : The table value of t at 5% level of significance corresponding to the degree of freedom $9+7-2=14$ is 2.145

vi) Decision: Note that the computed value $|t|=2.64$ is greater than the table value 2.145

Hence, The Null hypothesis is rejected.

$\therefore$ The samples cannot be considered to have been drawn from the same population

Example 3 Two independent samples of sizes 8 and 7 gave the following results.

sample 1 :	19	17	15	21	16	18	16	14
sample 2 :	15	14	15	19	15	18	16	

Is the difference between sample mean significant

Hint:

* find $\overline{x}_1$ from sample 1 and $\overline{x}_2$ from sample 2 $\left(\overline{x}_1 = 17,\ \overline{x}_2 = 16\right)$
* find s_1 using sample 1 and s_2 using sample 2 $\left(s_1 = 2.12,\ s_2 = 1.69\right)$
* find S_p and S.E (S.E = 1.073)
* 't'

$(t = 0.93)$

(Decision: Accepted)

Case 2: If samples are not independent

* formulae :

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n-1}}$$

Example ① A certain injection administered to 12 patients resulted in the following changes of blood pressure : 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4

Can it concluded that the injection will be in genral accompanied by an increases in blood pressure ?

solution: Given : $n = 12$ and the values of x_i's are 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4

$$\therefore \bar{x} = \frac{\sum x_i}{n} = \frac{5+2+8+(-1)+3+0+6+(-2)+1+5+0+4}{12}$$

$$= 2.58$$

x	5	2	8	-1	3	0	6	-2	1	5	0	4
$x_i - \bar{x}$	2.42	0.58	5.42	-3.58	0.42	-2.58	3.42	-4.58	-1.58	2.42	-2.58	1.42
$(x_i - \bar{x})^2$	5.86	0.34	29.38	12.82	0.18	6.66	11.70	20.98	2.50	5.86	6.66	2.02

Now, $s^2 = \frac{\sum (x_i - \bar{x})^2}{n} = \frac{104.92}{12} = 8.74$

i) Null hypothesis : H_0 : $\mu = 0$

ii) Alternate hypothesis H_a : $\mu \neq 0$

iii) Calculation of statistic :

The t - distribution is

$$t = \frac{\bar{x} - \bar{u}}{s/\sqrt{n-1}} = \frac{2.58 - 0}{\sqrt{8.74}/\sqrt{12-1}} = 2.89$$

$\therefore \quad |t| = 2.89$

iv) Level of significance : $\alpha = 0.05$

v) Critical value : The value of t_α at 5% level of significance corresponds to the degree of freedom $12-1 = 11$ is 2.201

vi) Decision : Note that the computed value $|t| = 2.83$ is greater that critical value 2.201

Hence, Null hypothesis is rejected.

$\therefore$ There is rise in blood pressure.