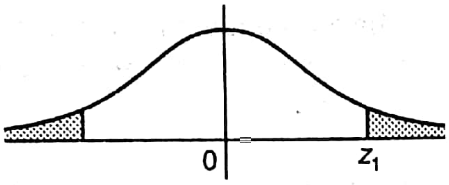
Percentage Points of *t* - distribution



Example

For Φ = 10 d. o. f.

$P(|t| > 1.812) = 0.1$

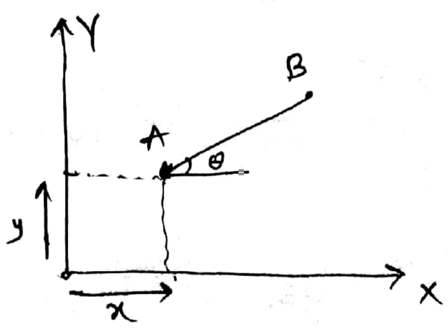
Φ \ P	0.20	0.10	0.05	0.02	0.01
1	3.078	6.314	12.706	31.812	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.287
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.325	2.576

* Student's t-distribution (for small sample) :-
(t-distribution)

— The t-distribution is used when
i) The sample size is 30 or less and
ii) population standard deviation is not known

— Uses of t-distribution :-
i) To estimate the population mean μ from the sample mean $\bar{x}$
ii) To Test the hypothesis that the population mean is μ with the help of the sample mean $\bar{x}$
iii) To Test the hypothesis that two population have same mean with the help of the sample mean.

* Degree of freedom :-
It is defined as the number of independent parameters required to speciffed the location of every link within a mechanism.



degree of freedom = 3

Note that :

* The sample standard deviation is

$$\boxed{s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1}}$$

* The t-distribution formula is

$$\boxed{t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}}$$

here, the degree of freedom = n-1

Example 1. A Random sample of size 16 from a normal population showed a mean of 103.75 cm. and sum of squares of deviation from the mean 843.75 cm^2. Can we say that the population has a mean of 108.75 cm. ?

solution: Given: $n = 16$, $\mu = 108.75$, $\bar{x} = 103.75$

and $\Sigma (x_i - \bar{x})^2 = 843.75$

i) Null hypothesis (H_o) : $\mu = 108.75$

ii) Alternative hypothesis (H_a): $\mu \neq 108.75$

iii) Calculate of test statistic :

* $s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n} = \frac{843.75}{16} = 52.73$

* $t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{103.75 - 108.75}{\sqrt{52.73}/\sqrt{16-1}}$

$$\Rightarrow \quad t = -\frac{5}{1.875} = -2.67$$

$$\Rightarrow \quad |t| = 2.67$$

iv) Level of significance : $\alpha = 0.05$ (5 %)

v) critical value : The value of t_α for 5 % level of significance from the table is 2.131 corresponds to the degree of freedom $= 16 - 1 = 15$

vi) Decision : Note that the computed value $|t| = 2.67$ is greater than the table value $t_\alpha = 2.131$

Hence, the null hypothesis is rejected.

Therefore, we cannot say that the population mean is 108.75

Example 2 : Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches. Discuss the suggestion that the mean height of the universe is 65 inches.

solution: Given: $\mu = 65$ inches

the values of x_i's are

63, 63, 64, 65, 66, 69, 69, 70, 70, 71

$\therefore \quad \overline{x} = \dfrac{\sum x_i}{n} = \dfrac{63+63+64+65+66+69+69+70+70+71}{10}$

$= \dfrac{670}{10} = 67$

$\therefore \quad \overline{x} = 67$

x_i	63	63	64	65	66	69	69	70	70	71
$x_i - \overline{x}$	-4	-4	3	2	1	2	2	3	3	4
$(x_i - \overline{x})^2$	16	16	9	4	1	4	4	9	9	16

$\therefore \quad s^2 = \dfrac{\sum (x_i - \overline{x})^2}{n} = \dfrac{16+16+9+4+1+4+4+9+9+16}{10}$

$= \dfrac{88}{10} = 8.8$

i) Null hypothesis (H_0) : $\mu = 65$

ii) Alternate hypothesis (H_a) : $\mu \neq 65$

iii) Calculation of test statistic :

$t = \dfrac{\overline{x} - \mu}{s/\sqrt{n-1}} = \dfrac{67-65}{\sqrt{8.8}/\sqrt{10-1}} = \dfrac{6}{2.97} = 2.02$

$\therefore \quad |t| = 2.02$

iv) Level of significance : $\alpha = 0.05$ (5%)

v) critical value : The value of t_α at 5% level of significance corresponding to the degree of freedom $10-1 = 9$ is 2.6

vi) Decision: Note that the computed value of t is 2.02 is less than the table value $t_\alpha = 2.6$

Hence, the null hypothesis is accepted.

$\therefore$ The mean height of the universe may be 65 inches

Example 3. Test made on breaking strength of 10 pieces of a metal wire gave the following results

578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 kgs.

Test if the breaking strength of the metal wire can be assumed to be 577 kg ?

Hint.

$\bar{x} = 575.2$, $\mu = 577$

$s^2 = 68.16$

$|t| = 0.65$

Critical value : 2.25 (DoF : 10-1=9)

Dicision : Accepted.