* poisson Distribution :-

let n be the number of trials

p be the probability of success in each trials

and np be the avarage success say 'm'

i.e. $\boxed{m = np}$

Then a random variable X is said to follow possion distribution if the probability of x is given by

$$\boxed{P(X = x) = \frac{e^{-m} m^{x}}{x!}} \quad , \quad x = 0, 1, 2, \ldots$$

Note that : * n is infinitely large i.e $n \to \infty$

* p is always constant and infinitely small i.e $p \to 0$

* m is finite and $\boxed{m = np}$

* Expected value :-

The sum of product of values and their probability is called as Expected value & it is denoted as $E(X)$

i.e. $E(X) = P_1 x_1 + P_2 x_2 + P_3 x_3 + \ldots$

Note that : $\boxed{E(X) = m}$

* Note that

— If X and Y are two variates.

then 1> $E(X+Y) = E(X)+E(Y)$

2> $E(X-Y) = E(X)-E(Y)$

— If X and Y are two independent variates

then $E(XY) = E(X) \cdot E(Y)$

— If X is a variates and 'a', 'b' are any constants

then $E(aX+b) = a\,E(X)+b$

* Moment Generating function :- (m.g.f)

The moment genrating function (m.g.f) of a random variate X is denoted by $M_0(t)$ and is defined by

$$M_0(t) = E(e^{tx}) \qquad \text{(about origin)}$$

Ex. 1 Derive the moments of the posson's Distributions:

Solution: here we find below the first two moments about the origin

1. $\mu_1' = E(X) = \sum p_i x_i$

$$= \sum_{x=0} \frac{e^{-m} m^x}{x!} \cdot x = \sum_{x=1} \frac{e^{-m} m^x}{(x-1)!}$$

$$= \sum_{x=1} \frac{e^{-m} \cdot m \cdot m^{x-1}}{(x-1)!} = e^{-m} \cdot m \sum_{x=1} \frac{m^{x-1}}{(x-1)!}$$

$$= m e^{-m}\left[1 + m + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots\right]$$

$$= m e^{-m} \cdot e^{m}$$

$$= m$$

Hence, $\boxed{\text{mean} = m}$

② $$u_2' = E(x^2) = \sum P_i x_i^2 = \sum_{x=0} \frac{e^{-m} \cdot m^x}{x!} \cdot x^2$$

$$= \sum_{x=0} \frac{e^{-m} m^x}{x!}(x + x^2 - x)$$

$$= \sum_{x=0} \frac{e^{-m} \cdot m^x}{x!}[x + x(x-1)]$$

$$= \sum_{x=0} \frac{e^{-m} \cdot m^x \cdot x}{x!} + \sum_{x=0} \frac{e^{-m} m^x}{x!} x(x-1)$$

$$= e^{-m} \cdot m \sum_{x=1} \frac{m^{x-1}}{(x-1)!} + e^{-m} m^2 \sum_{x=2} \frac{m^{x-2}}{(x-2)!}$$

$$= e^{-m} \cdot m\left[1 + m + \frac{m^2}{2!} + \cdots\right] + e^{-m} \cdot m^2\left[1 + m + \frac{m^2}{2!} + \cdots\right]$$

$$= e^{-m} \cdot m \cdot e^{m} + e^{-m} \cdot m^2 \cdot e^{m}$$

$$= m + m^2$$

$$\therefore \mu_2 = \mu_2' - \mu_1'^2 = m + m^2 - m^2 = m$$

$\therefore$ $\boxed{\text{variance} = m}$

Therefore, the mean and varience of the posson's distribution are both equal to 'm'

Ex ② obtain the moment Generating function of poisson's distribution.

Solution: Note that the moment generating function about origin is

$$M_0(t) = E(e^{tx})$$

$$= \sum p(x) e^{tx}$$

$$= \sum_{x=0} \frac{e^{-m} \cdot m^x}{x!} \cdot e^{tx}$$

$$= e^{-m} \sum_{x=0} \frac{m^x \cdot (e^t)^x}{x!}$$

$$= e^{-m} \cdot \sum_{x=0} \frac{(me^t)^x}{x!}$$

$$= e^{-m} \left[1 + me^t + \frac{(me^t)^2}{2!} + \frac{(me^t)^3}{3!} + \cdots \right]$$

$$= e^{-m} \cdot e^{me^t}$$

$$= e^{-m + me^t}$$

$$= e^{m(e^t - 1)}$$

$$\therefore \boxed{M_0(t) = e^{m(e^t - 1)}}$$

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Ex ③ If the variance of a poisson distribution is 2, find the probabilities of $r = 1, 2, 3, 4$ from the recurrence relation of poisson distribution.

$\longrightarrow$ Note that $p(X = x) = \dfrac{e^{-m} \cdot m^x}{x!}$

given that Varience $= m = 2$

$\therefore$ for $x = 0$, $p(X=0) = \dfrac{e^{-2} \cdot 2^0}{0!} = e^{-2}$

i.e. $p(0) = e^{-2}$

Now, the recurrence relation is $p(x+1) = \dfrac{m}{x+1} p(x)$

$\therefore$ put $x = 0$, $p(0+1) = p(1) = \dfrac{m}{0+1} p(0) = \dfrac{2}{1} e^{-2} = 2e^{-2}$

put $x = 1$, $p(1+1) = p(2) = \dfrac{2}{1+1} p(1) = \dfrac{2}{2} 2e^{-2} = 2e^{-2}$

put $x = 2$, $p(2+1) = p(3) = \dfrac{2}{2+1} p(2) = \dfrac{2}{3}(2e^{-2}) = \dfrac{4}{3} e^{-2}$

put $x = 3$, $p(3+1) = p(4) = \dfrac{2}{3+1} p(3) = \dfrac{2}{4} \dfrac{4}{3} e^{-2} = \dfrac{2}{3} e^{-2}$

Therefore, $p(1) = 2e^{-2}$, $p(2) = 2e^{-2}$, $p(3) = \dfrac{4}{3} e^{-2}$, $p(4) = \dfrac{2}{3} e^{-2}$

Ex ④ If a random variable X follows poisson distribution such that $P(X=1) = 2P(X=2)$, find the mean and the variance of the distribution. Also find $P(X=3)$

$\longrightarrow$ Note that $P(X = x) = \dfrac{e^{-m} \cdot m^x}{x!}$

given that $P(X=1) = 2P(X=2)$

$\Rightarrow \dfrac{e^{-m} \cdot m^1}{1!} = 2 \dfrac{e^{-m} m^2}{2!}$

$$\Rightarrow \quad m e^{-m} = m^2 e^{-m}$$

$$\Rightarrow \quad m = m^2$$

$$\Rightarrow \quad m = 1$$

$\therefore$ The mean = variance = $m = 1$

Now $$P(X=3) = \frac{e^{-1} \cdot (1)^3}{3!} = \frac{e^{-1}}{1 \times 2 \times 3} = \underline{\underline{0.0613}}$$

Ex. 5 A hospital switch board receives an avarage of 4 emergency calls in a 10 minutes interval. what is the probability that

i) there are atmost 2 emergency calls

ii) there are exactly 3 emergency call in an interval of 10 minutes

iii) more than 2 emergency calls

$\longrightarrow$ Note that $$P(X=x) = \frac{e^{-m} \cdot m^x}{x!}$$

here, $m = 4$

i) $$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$
$$= \frac{e^{-4} \cdot 4^0}{0!} + \frac{e^{-4} \cdot 4^1}{1!} + \frac{e^{-4} \cdot 4^2}{2!}$$
$$= e^{-4}(1 + 4 + 8) = \underline{\underline{0.238}}$$

ii) $$P(X=3) = \frac{e^{-4} \, 4^3}{3!} = \underline{\underline{0.195}}$$

iii) $$P(X>2) = P(X=3) + P(X=4) + P(X=5) + \cdots$$
$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$
$$= 1 - \left[\frac{e^{-4} \cdot 4^0}{0!} + \frac{e^{-4} \cdot 4^1}{1!} + \frac{e^{-4} \cdot 4^2}{2!}\right]$$
$$= 1 - 0.238$$
$$= 0.762$$