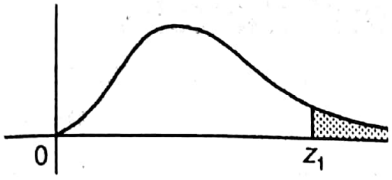
Percentage Points of χ^2 - Distribution



Example

For $\Phi = 10$ d. o. f.

$P(\chi^2 > 15.99) = 0.10$

Φ \ P	0 = .99	0.95	0.50	0.10	0.05	0.02	0.01
1	.000157	.00393	.455	2.706	3.841	5.214	6.635
2	.0201	.103	1.386	4.605	5.991	7.824	9.210
3	.115	.352	2.366	6.251	7.815	9.837	11.341
4	.297	.711	3.357	7.779	9.488	11.668	13.277
5	.554	1.145	4.351	9.236	11.070	13.388	15.086
6	.872	1.635	5.348	10.645	12.592	15.033	16.812
7	1.339	2.167	6.346	12.017	14.067	16.622	18.475
8	1.646	2.733	7.344	13.362	15.507	18.168	20.090
9	2.088	3.325	8.343	14.684	16.919	19.679	21.666
10	2.558	3.940	9.340	15.987	18.307	21.161	23.209
11	3.053	4.575	10.341	17.275	19.675	22.618	24.725
12	3 571	5.226	11.340	18.549	21.026	24.054	26.217
13	4.107	5.892	12.340	19.812	22.362	25.472	27.688
14	4.660	6.571	13.339	21.064	23.685	26.873	29.141
15	4.229	7.261	14.339	22.307	24.996	28.259	30.578
16	5.812	7.962	15.338	23.542	26.296	29.633	32.000
17	6.408	8.672	16.338	24.769	27.587	30.995	33.409
18	7.015	9.390	17.338	25.989	28.869	32.346	34.805
19	7.633	10.117	18.338	27.204	30.144	33.687	36.191
20	8.260	10.851	19.337	28.412	31.410	35.020	37.566
21	8.897	11.591	20.337	29.615	32.671	36.349	38.932
22	9.542	12.338	21.337	30.813	33.924	37.659	40.289
23	10.196	13.091	22.337	32.007	35.172	38.968	41.638
24	10.856	13.848	23.337	32.196	36.415	40.270	42 980
25	11.524	14.611	24.337	34.382	37.652	41.566	44.314
26	12.198	15.379	25.336	35.363	38.885	41.856	45.642
27	12.879	16.151	26.336	36.741	40.113	44.140	46.963
28	13.565	16.928	27.336	37.916	41.337	45.419	48.278
29	14.256	17.708	28.336	39.087	42.557	46.693	49.588
30	14.953	18 493	29.336	40.256	43.773	47.962	50.892

* Chi-square Test: (χ^2 - Distribution)

1) To Test independent of attributes:

— χ^2 - test is widely used to test whether there is association between two or more attributes.

In this case the Null hypothesis is like "there is no association between the attributes".

for ex. It can be used to determine whether there is association between the colour of mother's eye and daughter's eye between inoculation and prevention of a diseases.

2) To Test the Goodness to fit:

χ^2 - test is very commonly known as χ^2 - test of goodness to fit because it enables us to ascertain how well the theoretical distribution such as poeston or Normal fit the observed frequencies. In this case the Null hypothesis is like "The theory supports the observations.

* Examples on Independence of attributes

Ex 1. A sample of 400 students of under-graduate and 400 students of post graduate classes was taken to know their opinion about atonomous colleges. 290 of the undergraduate and 310 of the post graduate students favoured the autonomous status. present these facts in the form of a table and test at 5 % level, that the opinion regarding autonomous status of colleges in independent of the level of classes of students.

solution:

opinion about autonomous colleges

	Favoured	Not-Favoured	Total
Under-graduate	290	110	400
post-graduate	310	90	400
Total	600	200	800

i) Null hypothesis H_0 : There is no association between the classes and the opinion.

Alternative hypothesis H_a : There is association

ii) Calculation of test statistic:

on the basis of hypothesis,

The Number in first cell $= \frac{A \times B}{N}$

where, A = Number of under-graduate students

B = Number who favoured

N = Total Number of students.

∴ The Number in first cell = $\frac{400 \times 600}{800}$ = 300

opinion about the autonomous colleges.

	Favoured	Not-favoured	Total
Under-graduate	300	100	400
Post-graduate	300	100	400
	600	200	800

Calculation of $(O-E)^2/E$

O	E	$(O-E)^2$	$(O-E)^2/E$
290	300	100	0.33
310	300	100	0.33
110	100	100	1.00
90	100	100	1.00
		Total. (χ^2)	2.66

iii) level of significance : $\alpha = 0.05$ (5%.)

Degree of freedom = $(r-1)(c-1) = (2-1)(2-1) = 1$

iv) Critical value : The Table value at 5% level of significance corresponds to 1 DOF is

$\chi^2 = 3.84$

v) Decision: clearly, the calculated value of $\chi^2 = 2.66$ is less than the table value of $\chi^2 = 3.84$

Hence, the Null hypothesis is accepted.

∴ There is no association between the opinion and the level of classes.

Ex 2. To test the effect of a new drug, a controlled experiment was conducted. 300 patients were given the new drug while 200 patients were given no drug. On the basis of examination of these person, the following result were obtained.

	cured	condition worsoned	No effect	Total
Given the new drug	200	40	60	300
Not given drug	120	30	50	200
Total	320	70	110	500

Use χ^2 test to find effect of new drug.

Solution: i) Null Hypothesis H_0: The drug is not effective.

Alternative Hypothesis H_a: The drug is effective

ii) Calculation of test statistic:

The Number in the first cell $= \frac{A \times B}{N}$

where, A = total in the first column

B = total in first Row

N = Total number of observations

$\therefore$ Number in first cell $= \frac{320 \times 300}{500} = 192$

Ily, The Number in second cell $= \frac{70 \times 300}{500} = 42$

$\therefore$ Table of calculated frequencies.

	Cured	Condition worsoned	No effect	Total
Given the new doug	192	42	66	300
Not given the doug	128	28	44	200
Total	320	70	110	500

Calculation of $(O-E)^2/E$

O	E	O - E	$(O-E)^2$	$(O-E)^2/E$
200	192	8	64	0.333
40	42	-2	4	0.095
60	66	-6	36	0.545
120	128	-8	64	0.500
30	28	2	4	0.143
50	44	6	36	0.818
			Total (χ^2)	2.434

iii) Level of significance; $\alpha = 0.05$

Degree of freedom = $(r-1)(c-1) = (2-1)(3-1) = 2$

iv) Critical value: The table value at 5% level of significance corresponds to 2 DOF is $\chi^2 = 5.991$

v) Decision: Note that the calculated value of $\chi^2 = 2.435$ is less than the table value $\chi^2 = 5.991$

Hence, the Null hypothesis is Accepted.

$\therefore$ The New drug is not effective.

* Examples on Goodness of fit :-

Ex ① The following table gives the number of accidents in a city during a week. find whether the accidents are uniformly distributed over a week

Day :	Sun	Mon	Tue	Wed	Thu	fri	Sat	Total
No. of accidents :	13	15	9	11	12	10	14	84

Solution: i) Null Hypothesis H_0 : Accidents are equally distributed over all the days of a week

ii) Alternative hypothesis H_a : Accidents do not occur equally.

iii) Calculation of test statistic :

$$E = \frac{\text{Total Number of Accidents}}{\text{Total days}} = \frac{84}{7} = 12$$

days.	O	E	$(O-E)^2$	$(O-E)^2/E$
Sun	13	12	1	$\frac{1}{12}$
Mon	15	12	9	$\frac{3}{4}$
Tue	9	12	9	$\frac{3}{4}$
Wed	11	12	1	$\frac{1}{12}$
Thu	12	12	0	0
fri	10	12	4	$\frac{1}{3}$
Sat.	14	12	4	$\frac{1}{3}$
			Total (χ^2)	2.33

$\therefore \chi^2 = 2.33$

iv) level of significance : $\alpha = 0.05$

Degree of freedom : $n - 1 = 7 - 1 = 6$

v) Critical Value : The table value of χ^2 at 5 % level of significance corresponds to 6 DOF is 12.59

vi) Decision: Note that the calculated value of $\chi^2 = 2.33$ is less than the table value $\chi^2 = 12.59$

Hence, The Null hypothesis is Accepted.

∴ The accidents occur equally on all working days.

Ex. 2. A die was thrown 132 times and the following frequencies were observed.

No. obtained :	1	2	3	4	5	6	Total
frequency :	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased.

Solution: i) Null Hypothesis H_0 : The die is unbiased.

ii) Alternative Hypothesis H_a : The die is not unbiased.

iii) Calculation of test statistic :

$$E = \frac{\text{Total frequency}}{\text{Total Number obtained}} = \frac{132}{6} = 22$$

$$\therefore \quad \underline{E = 22}$$

No.	O	E	$(O-E)^2$	$(O-E)^2/E$
1	15	22	49	2.227
2	20	22	4	0.182
3	25	22	9	0.409
4	15	22	49	2.227
5	29	22	49	2.227
6	28	22	36	1.636
			Total (χ^2)	8.91

$\therefore \chi^2 = 8.91$

iv) level of significance : $\alpha = 0.05$

Degree of freedom : $n-1 = 6-1 = 5$

v) critical Value : The table value of χ^2 at 5 % level of significance curresponds to 5 DOF is 11.07

vi) Decision; Note that the calculated value $\chi^2 = 8.91$ is less than the critical value $\chi^2 = 11.07$

$\therefore$ The Null Hypothesis is accepted.

$\therefore$ The die is unbiased.

Ex,3. Theory predicts that the proportion of beans in the four groups A, B, C. D should be 9:3:3:1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental results support the theory ?

solution: i) Null Hypothesis H_0: The proportion of the beans in the four groups A, B, C, D is given proportion 9 : 3 : 3 : 1

ii) Alternative Hypothesis H_a: The proportion is not as given above

iii) Calculation of test statistic:

Note that the sum $9+3+3+1=16$

∴ The number of beans in the four groups will be

$$A = \frac{9}{16} \times 1600 = 900 \quad , \quad B = \frac{3}{16} \times 1600 = 300$$

$$C = \frac{3}{16} \times 1600 = 300 \quad , \quad D = \frac{1}{16} \times 1600 = 100$$

O	E	$(O-E)^2$	$(O-E)^2/E$
882	900	324	0.0378
313	300	169	0.5633
287	300	169	0.5633
118	100	324	3.24
		Total (χ^2)	4.72

∴ $\chi^2 = 4.72$

iv) level of significance : $\alpha = 0.05$

degree of freedom = $n-1 = 4-1 = 3$

v) critical value: The table value of χ^2 at 5 % level of significance corresponds to 3 DoF is 7.81

vi) Decision: Note that the calculated value $\chi^2 = 4.72$ is less than the table value $\chi^2 = 7.81$

Hence, The Null Hypothesis is accepted.

∴ The proportion 9:3:3:1 is correct.

Ex 4. In an experiment on pea breeding the following frequencies were obtained.

Round and Yellow	wrinkled and yellow	Round and green	wrinkled and green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportion of 9:3:3:1

Examine the correspondence between theory and experiment using chi-square test.