* Normal Distribution :-

let X be the random variable

m be the mean and

σ be the standard deviation

then the contineous random variable X is said to follow normal distribution if its probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

Important Note : * If X is normal variate with parameter m, σ then $\boxed{Z = \frac{X-m}{\sigma}}$ is called Standard Normal variates

* Mean and varience of Normal Distribution :

$\boxed{\text{mean} = m}$ $\boxed{\text{Variance} = Var.(X) = \sigma^2}$

* $\boxed{\text{Mean} = \text{Median} = \text{Mode} = m}$

* Moment Genrating function of Normal distribution

$$\boxed{M_0(t) = e^{\left(mt + \frac{t^2\sigma^2}{2}\right)}}$$

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Note that In standard normal variates,

mean $= m = 0$ and $\sigma = 1$

Hence, generating function of standard normal variates is $\boxed{M_0(t) = e^{\frac{t^2}{2}}}$

* Area property :

let X = Random variates

Z = standard Normal variates (S.N.V)

then the area under the normal curve of X between $X = m$ and $X = x_1$ is equal to area under the standard normal curve between $Z = 0$ to $Z = z_1$

$\Rightarrow$ clearly if $X = m$ then $Z = \frac{X-m}{\sigma} = \frac{m-m}{\sigma} = 0$

and if $X = x_1$ then $Z = \frac{x_1 - m}{\sigma} = z_1$ (say)

Therefore,

$$\boxed{P(m \leq X \leq x_1) = P(0 \leq Z \leq z_1) = \frac{1}{\sqrt{2\pi}} \int_0^{z_1} e^{-\frac{1}{2}x^2} dx}$$

Note that:

* $P(-z_2 \leq Z \leq z_1) = P(0 \leq Z \leq z_2) + P(0 \leq Z \leq z_1)$

Ex ① If mean of normal variate is 2.5 and standard deviation is 3.5 then find the probability that $2 \le X \le 4.5$

→ given that $m = 2.5$

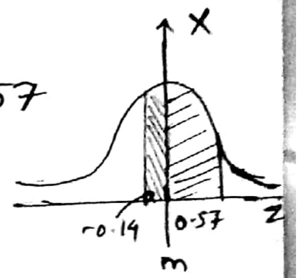
$\sigma = 3.5$ and

$2 \le X \le 4.5$

Note that Standard Normal variates $Z = \frac{X-m}{\sigma}$

$\therefore$ for $X = 2$, $Z = \frac{2-2.5}{3.5} = -0.14$

for $X = 4.5$, $Z = \frac{4.5-2.5}{3.5} = 0.57$



Therefore, the probability is

$$P(2 \le X \le 4.5) = P(-0.14 \le Z \le 0.57)$$

$$= P(0 \le Z \le 0.14) + P(0 \le Z \le 0.57)$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{0.14} e^{-\frac{1}{2}x^2} dx + \frac{1}{\sqrt{2\pi}} \int_0^{0.57} e^{-\frac{1}{2}x^2} dx$$

$$= 0.0557 + 0.2157$$

$$= 0.2714$$

Ex ② If X is a normal variates with mean 10 and the standard deviation 4, find

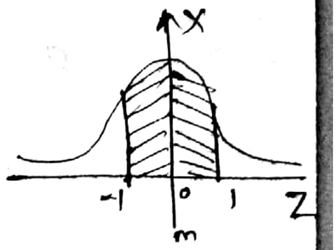
i) $P(|X-14| < 1)$ ii) $P(5 \le X \le 18)$ iii) $P(X \le 12)$

Solution: Note that the standard Normal variates is $Z = \frac{X-m}{\sigma}$

here, $m = 10$ and $\sigma = 4$

i) To find $P(|X-14|<1)$

if $X = 14$, $Z = \frac{14-10}{4} = 1$



$\therefore P(|X-14|<1) = P(|Z| \leq 1)$

$= P(-1 \leq Z < 1)$

$= P(0 \leq Z \leq 1) + P(0 \leq Z \leq 1)$

$= 2\, P(0 \leq Z \leq 1)$

$= 2 \frac{1}{\sqrt{2\pi}} \int_0^1 e^{-\frac{1}{2}x^2} dx$

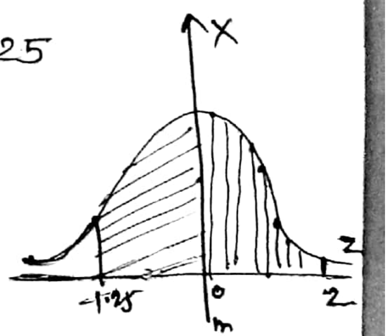
$= 2 \; (0.3413)$

$= 0.6826$

ii) To find $P(5 \leq X < 18)$

if $X = 5$, $Z = \frac{X-m}{\sigma} = \frac{5-10}{4} = -1.25$

if $X = 18$, $Z = \frac{X-m}{\sigma} = \frac{18-10}{4} = 2$



$\therefore P(5 < X < 18) = P(-1.25 \leq Z \leq 2)$

$= P(0 \leq Z \leq 1.25) + P(0 \leq Z \leq 2)$

$= \frac{1}{\sqrt{2\pi}} \int_0^{1.25} e^{-\frac{1}{2}x^2} dx + \frac{1}{\sqrt{2\pi}} \int_0^{2} e^{-\frac{1}{2}x^2} dx$

$= 0.3944 + 0.4772$

$= 0.8716$

iii) To find $P(X \leq 12)$

if $X = 12$, $Z = \frac{X-m}{\sigma} = \frac{12-10}{4} = 0.5$

$$\therefore \; P(X \le 12) = P(Z \le 0.5)$$
$$= P(-\infty \le Z \le 0.5)$$
$$= P(0 \le Z \le \infty) + P(0 \le Z \le 0.5)$$
$$= \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}x^2} dx + \frac{1}{\sqrt{2\pi}} \int_0^{0.5} e^{-\frac{1}{2}x^2} dx$$
$$= 0.5 + 0.1915$$
$$= 0.6915$$

Ex. (3) Monthly salary X in a big organization is normally distributed with mean ₹ 3000 and standard deviation of ₹ 250. what shoud be the minimum salary of a worker in this organisation, so that the probability that he belongs to top 5% workers ?

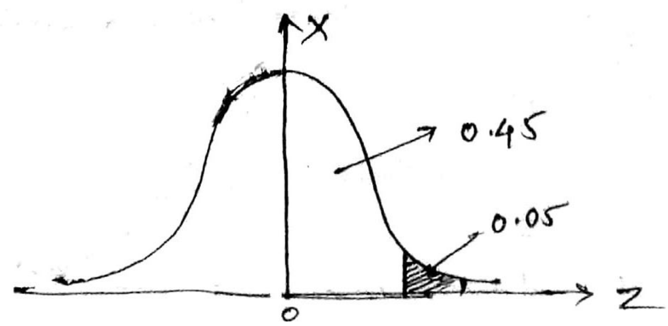
Solution: Note that the standard Normal variates is

$$Z = \frac{X - m}{\sigma}$$

here, mean $= m = 3000$

and standard deviation $= \sigma = 250$

To find z_1 such that $P(Z = z_1) = \frac{5}{100} = 0.05$



$\therefore \quad 0.5 - 0.05 = 0.45$

The corresponding to 0.45 the entry in the area table is 1.64

$$\therefore \quad z_1 = 1.64$$

Now $\quad Z = z_1 = \dfrac{X - m}{\sigma}$

$$\Rightarrow \quad 1.64 = \frac{X - 3000}{250}$$

$$\Rightarrow \quad X = 300 + (250 \times 1.64)$$

$$\Rightarrow \quad \underline{X = ₹\ 3410}$$

$\therefore$ The minimum salary of a worker in this organization is ₹ <u>3410</u>

Ex. 4 The marks obtained by 1000 students in an examination are found to be normally distributed with 70 and stadard deviation 5 Estimate the number of student whose marks will be i> between 60 and 75
ii> more than 75

solution: Note that the standard Normal variates

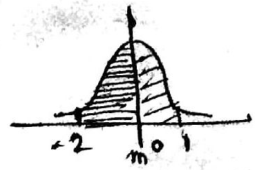
$$Z = \frac{X - m}{\sigma}$$

here, mean $= m = 70$
and standard deviation $= \sigma = 5$

i> if $x = 60$ then $z = \frac{x - m}{\sigma} = \frac{60-70}{5} = -2$

if $x = 75$ then $z = \frac{x-m}{\sigma} = \frac{75-70}{5} = 1$

$P(60 \leq X \leq 75) = P(-2 \leq Z \leq 1)$



$= P(0 \leq Z \leq 2) + P(0 \leq Z \leq 1)$

$= \frac{1}{\sqrt{2\pi}} \int_0^2 e^{-\frac{1}{2}x^2} dx + \frac{1}{\sqrt{2\pi}} \int_0^1 e^{-\frac{1}{2}x^2} dx$

$= 0.4772 + 0.3413$

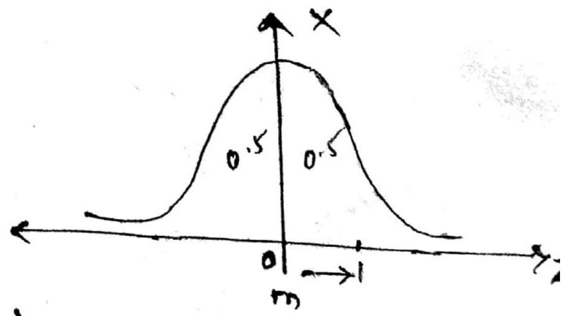
$= 0.8185$

$\therefore$ The number of students getting marks between 60 to 75 $= N \times P$

$= 1000 \times 0.8185$

$= 818$

ii> $P(X \geq 75) = P(Z \geq 1)$



$= 0.5 - P(0 \leq Z \leq 1)$

$= 0.5 - \frac{1}{\sqrt{2\pi}} \int_0^1 e^{-\frac{1}{2}x^2} dx$

$= 0.5 - 0.3413 = 0.1587$

$\therefore$ the number of students getting more than 75 marks $= NP = 1000 \times 0.1587 = \underline{159}$

Ex. 5: Marks obtained by students in an examination follow normal distribution. If 30 % of students got below 35 marks and 10 % got above 60 marks, find the mean and standard deviation.

Solution: Note that mean = m and

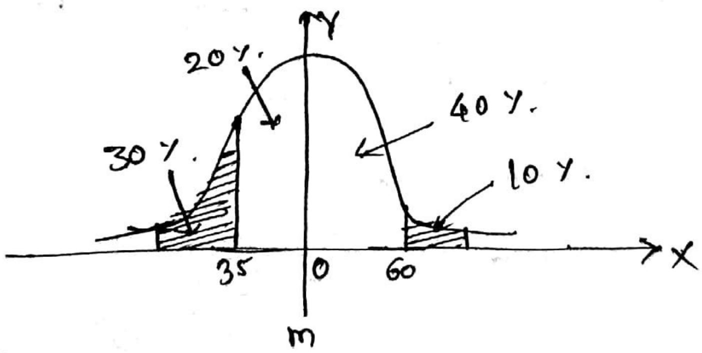
standard deviation = σ

since, 30 % students are below 35

$\Rightarrow$ 20 % students are between 35 and m

since, 10 % students are above 60

$\Rightarrow$ 40 % students are between m and 60



$\therefore$ from the given data (table) (will be provided in Exam)

0.2 area currresponds to $Z = 0.525$

and 0.4 area currresponds to $Z = 1.283$

$$(\text{0.2 area currresponds})\ Z = \frac{35-m}{\sigma}$$

$$\Rightarrow \quad 0.525 = \frac{35-m}{\sigma}$$

$$\Rightarrow \quad m + 0.525\,\sigma = 35 \quad \text{—— (1)}$$

and (0.4 Area corresp) $z = \frac{x - m}{\sigma} = \frac{60 - m}{\sigma}$

$\Rightarrow \quad 1.283 = \frac{60 - m}{\sigma}$

$\Rightarrow \quad m + 1.283\sigma = 60$ —— (2)

solving (1) and (2) we get

$m = 13.83$

$\sigma = 42.26$

(use calci fo system of equation)

$\therefore$ mean $= 13.83$

and standard deviation $= 42.26$.

Homework :

(1) In an Intelligenee test administered to 1000 students, the Avarage was 42 and standasd deviation was 24 find the number of students

i> exceeding the score 50

ii> between 30 and 54

(2) In a distribution exactly normal 7 % of items are under 35 and 89 % are under 63 what is the mean and standard deviation.