jitter
(random
phase shift)

Tutorial Sheet 9

1) Cosine:

$$P(f) = \begin{cases} \frac{1}{2B_0} & 0 < |f| < f_1 \\ \frac{1}{4B_0}\left[1+\cos\left(\frac{\pi(|f|-f_1)}{2(B_0-f_1)}\right)\right] & f_1 \le |f| < 2B_0-f_1 \\ 0 & f > 2B_0-f_1 \end{cases}$$

To get normalized eqn

let $f_1 = (1-\alpha)B_0$

and multiply $P(f)$ with $2B_0$

2)

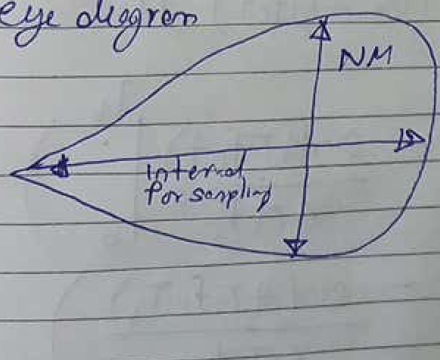
This results in

$$P_N(f) = \begin{cases} 1 & f < (1-\alpha)B_0 \quad \cancel{f \ldots (1-\alpha)} \\ \frac{1}{2}\left[1 + \cos\left(\frac{\pi\left(|f| - (1-\alpha)B_0\right)}{2\left(B_0 - (1-\alpha)B_0\right)}\right)\right] & (1-\alpha)B_0 < f < 2B_0 - 2(1-\alpha)B_0 \\ 0 & f > 2B_0 - (1-\alpha)B_0 \end{cases}$$

$$= \begin{cases} 1 & f < (1-\alpha)B_0 \\ \frac{1}{2}\left[1 + \cos\left(\frac{\pi\left(\frac{|f|}{B_0} - 1 + \alpha\right)}{2\alpha}\right)\right] & (1-\alpha)B_0 < |f| < 2\alpha B_0 \\ 0 & f > 2\alpha B_0 \end{cases}$$

Use same for trapezoidal

2) eye diagram



NM = 0.6V

Interval for sampling = 0.7 μs

$< |f| < f_1$

$f_1 \le |f| < 2B_0 - f$

$f > 2B_0 - f_1$

3)

Avg value of waveform $= \frac{A_c^2}{2} T_b \times$ Duty cycle $\times$ prob(

$= \frac{\cancel{4}^{2}}{\cancel{2}} \times \frac{1}{10\times10^3} \times 0.8 \times 0.6$

$= \frac{2\times0.48}{10^4}$

$= 0.96\times10^{-4}\ V^2/s$

4)

5)

$s_1(t) = A_c \sin(2\pi f_c t)$

$E_b = \int_0^{T_b} s_1^2(t)\,dt$

$= A_c^2 \int_0^{T_b} \sin^2(2\pi f_c t)\,dt$

$\frac{A_c^2}{2} \int_0^{T_b} 1-\cos(4\pi f_c t)\,dt$

$\frac{A_c^2}{2}\left(T_b - \frac{\sin(4\pi f_c t)}{4\pi f_c}\Big|_0^{T_b}\right)$

$\frac{A_c^2}{2}\left(T_b - \frac{\sin(4\pi f_c T_b)}{4\pi f_c}\right)$

Consider $f_c T_b = m \Rightarrow$ integer

$\sin(4\pi m) = 0$

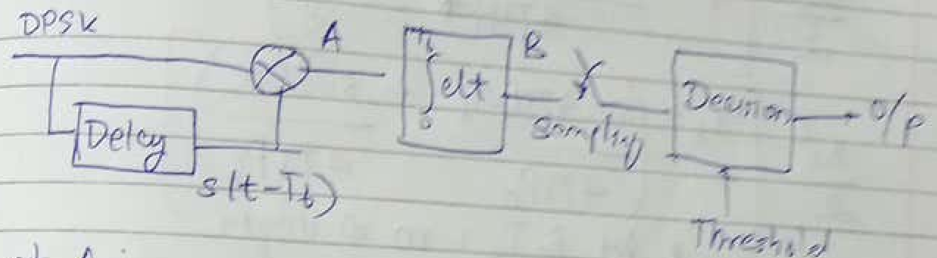
$\therefore E_b = \frac{A_c^2}{2}(T_b - 0) = \frac{A_c^2 T_b}{2}$

Hence Proved

y cycle × prob(1)

ger

6) $s(t) = A_c \cos(2\pi f_c t + \theta)$



at A :

$y(t) = A_c \cos(2\pi f_c t + \theta) \cdot A_c \cos(2\pi f_c t \underbrace{- 2\pi f_c T_b + \theta}_{\theta'})$

at B

$y(t) = A_c^2 \int_0^{T_b} \cos(2\pi f_c t + \theta) \cos(2\pi f_c t + \theta') dt$ $\boxed{\theta' = \theta - 2\pi f_c T_b}$

$A_c^2 \int_0^{T_b} [\cos(2\pi f_c t)\cos\theta - \sin(2\pi f_c t)\sin\theta][\cos(2\pi f_c t)\cos\theta' - \sin(2\pi f_c t)\sin\theta'] dt$

$\cancel{\cos\theta} = \theta$

$\cos\theta' = \cos(\theta - 2\pi f_c T_b)$

$= \cos\theta \cos 2\pi f_c T_b + \sin\theta \sin 2\pi f_c T_b$

consider $f_c T_b = m$ = integer

$\therefore \cos\theta' = \cos\theta$

$\sin\theta' = \sin(\theta - 2\pi f_c T_b)$

$= \sin\theta \cos 2\pi f_c T_b - \sin 2\pi f_c T_b \cos\theta$

$= \sin\theta$

$y(t) = A_c^2 \int^{T_b} (\cos(2\pi f_c t)\cos\theta - \sin(2\pi f_c t)\sin\theta)^2 dt$

$= A_c^2 \int_0^{T_b} \cos^2(2\pi f_c t + \theta) dt = A_c^2 \int_0^{T_b} \frac{1 + \cos(4\pi f_c t + 2\theta)}{2}$

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$$\frac{A_c^2}{2}\left[T_b + \frac{\sin(4\pi f_c t + 2\theta)}{4\pi f_c}\Bigg|_0^{T_b}\right]$$

$$\frac{A_c^2}{2}\left[T_b + \left(\frac{\sin 4\pi f_c T_b}{4\pi f_c} - \frac{\sin 2\theta}{4\pi f_c}\right)\right]$$

& let $f_c T_b = m \Rightarrow$ integer

$\therefore \sin 4\pi f_c T_b = 0$

$$E_b = \frac{A_c^2}{2}\left(T_b - \frac{\sin 2\theta}{4\pi f_c}\right)$$

7) $P_e \simeq 2\left(1 - \frac{1}{\sqrt{4}}\right) \operatorname{erfc}\left(\sqrt{\frac{\cancel{2}E_b \cancel{(4-1)} \times \cancel{3}}{\cancel{3} \times \cancel{2} N_0 \cancel{(4-1)}}}\right)$

$(M = 4)$

$$P_e = 2\left(1 - \frac{1}{2}\right) \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$= \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

8)