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ASSIGNMENT-1

TITLE : SIMPLEX

PROBLEM STATEMENT : Solve the given LPP using simplex method.

THEORY :

- SIMPLEX :

- Simplex method is used to solve any linear model for which the solⁿ exists

- It works in a manner such that the value of objective function in each iteration of the process is less in a minimization problem & more in a maximization problem.

- ALGORITHM :

i. Build a matrix containing coefficients of the constraints & slack variables

ii. Choose a variable v in the objective function with a +ve coefficient to increase

iii. Select the entering variables:
   - Maximization problems ⇒ max $(c_j - z_j)$
   - Minimization problems ⇒ min $(c_j - z_j)$

iv. Select the leaving variables:
   - It is the min (b_i / a_{ik})

v. Find the pivot element

vi. In the next iteration, decide the row of pivot by pivot & continue the process.

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vii. The stopping condition for maximization problem is all $C_j - Z_j \le 0$ & for minimization problem is $C_j - Z_j \ge 0$

Q.1. Minimize ; $Z = 12x_1 + 20x_2$

Subject to ; $6x_1 + 8x_2 \ge 100$

$7x_1 + 12x_2 \ge 120$; $x_1, x_2 \ge 0$

$\rightarrow$ $6x_1 + 8x_2 - S_1 + A_1 = 100$

$7x_1 + 12x_2 - S_2 + A_2 = 120$

$Z = 12x_1 + 20x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

	C_j	12	20	0	0	M	M		
C_B	Basis	x_1	x_2	S_1	S_2	A_1	A_2	b	θ
M	A_1	6	8	-1	0	1	0	100	25/2
M	A_2	7	$\boxed{12}$	0	-1	0	1	120	$\boxed{10}$
	Z_j	13M	20M	-M	-M	M	M	$Z_{min} =$ 220M	
	$C_j - Z_j$	12-13M	$\boxed{20-20M}$	M	M	0	0		

most -ve value

least +ve value

Pivot = 12 Entering variable = x_2

C_B	Basis	x_1	x_2	S_1	S_2	A_1	A_2	b	θ
M	A_1	$\boxed{4/3}$	0	-1	2/3	1	-2/3	20	$\boxed{15}$
20	x_2	7/12	1	0	-1/12	0	1/12	10	120/7
	Z_j	(4+35)/3	20	-M	(2M-5)/3	M	(-2M+5)/3	Z_{min} = 20M + 200	
	$C_j - Z_j$	$\boxed{(1-4M)/3}$	0	M	(5-2M)/3	0	(5M-5)/3		

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Pivot = 4/3 Entering variable = x_1

C_B	Basis	x_1	x_2	S_1	S_2	A_1	A_2	b	θ
12	x_1	1	0	$-3/4$	$1/2$	$3/4$	$-1/2$	15	
20	x_2	0	1	$7/16$	$-3/8$	$-7/16$	$3/8$	$5/4$	
	z_j	12	20	$-1/4$	$-3/2$	$1/4$	$3/2$	$Z_{min} = 205$	
	$c_j - z_j$	0	0	$1/4$	$3/2$	$M-3/4$	$M-3/2$		

Stopping condition reached

$Z_{min} = 205$

$x_1 = 15$

$x_2 = 5/4$

Q.2. Maximize : $Z = 2x_1 + 3x_2 + 4x_3$

Subject to : $3x_1 + x_2 + 4x_3 \leq 600$

$2x_1 + 4x_2 + 2x_3 \geq 480$

$2x_1 + 3x_2 + 3x_3 = 540$

$x_1, x_2, x_3 \geq 0$

→ $3x_1 + x_2 + 4x_3 + S_1 = 600$

$2x_1 + 4x_2 + 2x_3 - S_2 + A_1 = 480$

$2x_1 + 3x_2 + 3x_3 + A_2 = 540$

$Z = 2x_1 + 3x_2 + 4x_3 + 0S_1 + 0S_2 - MA_1 - MA_2$

	c_j	2	3	4	0	0	$-M$	$-M$		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	A_1	A_2	b	θ
0	S_1	3	1	4	1	0	0	0	600	600
$-M$	A_1	2	$\boxed{4}$	2	0	-1	1	0	480	$\boxed{120}$
$-M$	A_2	2	3	3	0	0	0	1	540	180
	z_j	$-4M$	$-7M$	$-5M$	0	M	$-M$	$-M$	$Z_{max} = -1020M$	
	$c_j - z_j$	$2+4M$	$\boxed{3+7M}$	$4+5M$	0	$-M$	0	0		

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Pivot = 4 Entering variable = x_2

	Basis	x_1	x_2	x_3	S_1	S_2	A_1	A_2	b	θ
C_B	S_1	$5/2$	0	$7/2$	1	$1/4$	$-1/4$	0	430	$900/7$
0	x_2	$1/2$	1	$1/2$	0	$-1/4$	$1/4$	0	120	240
3	A_2	$1/2$	0	$\boxed{3/2}$	0	$3/4$	$-3/4$	1	180	$\boxed{120}$
$-M$	Z_j	$(3-M)/2$	3	$(3-3M)/2$	0	$(3-3M)/4$	$(3-3M)/4$	$-M$	$Z_{max} = 360 -$	
	$C_j - Z_j$	$(1+M)/2$	0	$(5+3M)/2$	0	$(3+3M)/4$	$-(3+M)/4$	0		$120M$

Pivot = $3/2$ Entering variable = x_3

	C_j	2	3	4	0	0	$-M$	$-M$		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	A_1	A_2	b	θ
0	S_1	$4/3$	0	0	1	$-3/2$	$3/2$	$-7/3$	60	
3	x_2	$1/3$	1	0	0	$-1/2$	$1/2$	$-1/3$	60	
4	x_3	$1/3$	0	1	0	$1/2$	$-1/2$	$2/3$	120	
	Z_j	$7/3$	3	4	0	$1/2$	$-1/2$	$5/3$	$Z_{max} = 660$	
	$C_j - Z_j$	$-1/3$	0	0	0	$-1/2$	$-M+1/2$	$-M-5/3$		

Stopping condition is reached

$Z_{max} = 660$

at $x_1 = 0$

$x_2 = 60$

$x_3 = 120$

CONCLUSION:

Thus, I have studied the simplex algorithm for solving LPP & implemented it.