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ASSIGNMENT-3

TITLE : TRANSPORTATION PROBLEM

PROBLEM STATEMENT: Obtain the feasible solution for the transportation problem using least cost method & Vogel's approximation.

THEORY :

- TRANSPORTATION PROBLEM:
It is a special type of LPP where objective consists of minimizing transportation cost of given commodity from a number of sources or origins to a number of destinations

- TYPES OF TRANSPORTATION PROBLEM:
1. Balanced problem:
Total supply = total demand

2. Unbalanced problem:
- total supply ≠ total demand
- Add rows if supply is less than demand & add columns if supply is greater

- METHODS TO SOLVE TRANSPORTATION PROBLEM:
1. Least Cost Method
2. North-West Corner Rule
3. Vogel's Approximation Method

- ALGORITHM TO FIND FIRST FEASIBLE SOLUTION:
1. Least Cost Method:
- Balance problem if not already balanced
- Select cell with lowest cost from matrix &

allocate minimum supply of demand

- Remove row / column when supply / demand is fulfilled & cancel them

- Repeat steps (ii) & (iii) on non-cancelled part of matrix to fill all allocations are done.

2. North-West corner Cell Method:
- Balance if not already balanced

- Start allocating from north-west corner

- Remove the row/column where supply / demand is fulfilled

- Repeat steps (ii) & (iii) on other non-cancelled cells.

3. Vogel's Approximation Method:
- Balance the problem if not already balanced

- Find out difference b/w minimum & second minimum value in row / col.

- Select row / col with highest difference (penalty

- Remove row / col for which supply / demand is fulfilled & cancel

- Repeat steps (ii), (iii), (iv) till all allocations are done

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- HOW TO SOLVE TRANSPORTATION PROBLEM:
- Prepare a balanced matrix
- Find basic feasibility method using any of
  - N-W corner method
  - Least cost
  - Vogel's Approximation
- Perform optimality test using either of the following
  - Stepping Stone Method
  - MODI u/v method

- MODI U/V METHOD:

i. Find an initial basic feasible solution

ii. Find u_i, v_j for rows & columns.
To start:
   a. design 0 to any on u_i or v_j on no. of allocation
   b. Calculate other u_i & v_j using $c_{ij} = u_i + v_j$ for all allocated cells.
   c. For all unallocated cells, calculate penalty $C_{ij} = u_i + v_j - c_{ij}$

iii. For all unallocated cells, calculate penalty $C_{ij} = u_i + v_j - c_{ij}$

iv. For all unallocated cells, check the sign of C_{ij}
   a. If $C_{ij} > 0$, then given solution is not optimal & feasible. The process needs to be carried out to obtain optimal solution
   b. If $C_{ij} < 0$, then the solution is optimal, stop the process.

v. Select unallocated cells with largest +ve value of C_{ij} & proceed

vi. Draw a closed path from the unallocated cell. The right angle turn is only allowed at occupied cells & at the original unallocated cells. Mark (+) & (-) sign alternatively at each corner starting from original, unallocated cell.

7. a. Select min. value from cells with (-) sign

b. add this value to other unallocated cells with (+) sign

c. Subtract this value for allocated cells with (-) sign.

Q.1. Solve the following transportation problem & obtain optimal solution

	D_1	D_2	D_3	D_4	Supply
S_1	2	2	2	1	3
S_2	10	8	5	4	7
S_3	7	6	6	8	5
Demand	4	3	4	4	

→ Total supply = 3 + 7 + 5 = 15
Total demand = 4 + 3 + 4 + 4 = 15

Hence the problem is balanced

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	D_1	D_2	D_3	D_4	Supply	P_1	P_2	P_3	P_4	P_5
S_1	2^{3}	2	2	1	~~3~~ 0	1	—	—	—	—
S_2	10	8	5^{3}	4^{4}	~~7~~ ~~3~~ 0	1	1	3	—	—
S_3	7^{1}	6^{3}	6^{1}	8	~~5~~ ~~4~~ ~~1~~ 0	0	0	0	0	0
Demand	~~4~~ ~~1~~ 0	~~3~~ 0	~~4~~ ~~1~~ 0	~~4~~ 0						

P_1	5 ↑	4	3	3
P_2	3	2	1	4 ↑
P_3	3	2	1	—
P_4	7 ↑	6	6	—
P_5	—	6 ↑	6	—

Finally, the initial feasible solution using Vogel's approximation method

	D_1	D_2	D_3	D_4	Supply
S_1	$2^{(3)}$	2	2	1	3
S_2	10	8	$5^{(3)}$	$4^{(4)}$	7
S_3	$7^{(1)}$	$6^{(3)}$	$6^{(1)}$	8	~~5~~ ~~4~~ ~~1~~ 0
Demand	~~4~~ 1	~~3~~ 0	~~4~~ ~~1~~ 0	4	

No. of allocations = 6

$= m + n - 1$

$= 3 + 4 - 1 = 6$

∴ The solution is non-degenerate

$$\text{cost} = (2\times 3) + (5\times 3) + (4\times 4) + (7\times 1) + (6\times 3) + (6\times 1)$$

$$= 68$$

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For optimality
Using MODI method, assume $u_3 = 0$

Similarly, $v_2 = 6$
$v_3 = 6$
$u_1 = -5$
$u_2 = -1$
$v_4 = 5$

Finding $\Delta_{12} = C_{12} - (u_1 + v_2) = 2 - (-5 + 6) = 1$

$\Delta_{13} = 2 - (-5 + 6) = 1$

$\Delta_{14} = 1 - (-5 + 5) = 1$

$\Delta_{21} = 10 - (-1 + 7) = 4$

$\Delta_{22} = 8 - (-1 + 6) = 3$

$\Delta_{34} = 8 - (0 + 5) = 3$

As all $\Delta ij > 0$, sol^n is optimal & unique

$\therefore$ Cost $= 68$

$x_{11} = 3$ $x_{23} = 3$ $x_{24} = 4$

$x_{31} = 1$ $x_{32} = 3$ $x_{33} = 1$

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Q.2. Find optimal solution of given transportation problem

	D_1	D_2	D_3	D_4	Supply
S_1	3	1	7	4	250
S_2	2	6	5	9	350
S_3	8	3	3	2	400
Demand	200	300	350	150	

→ total supply = 250 + 350 + 400 = 1000
total demand = 200 + 300 + 350 + 150 = 10

Hence, problem is balanced

Using N-W corner method

	D_1	D_2	D_3	D_4	
S_1	3 (200)	1 (50)	7	4	~~250~~ ~~50~~ 0
S_2	2	6 (250)	5 (100)	9	~~350~~ ~~100~~ 0
S_3	8	3	3 (250)	2 (150)	~~400~~ ~~150~~ 0
	~~200~~	~~300~~	~~350~~	~~150~~	
	0	~~250~~	~~250~~	0	
		0	0		

no. of allocations = 6 = m + n - 1 = 3 + 4 -

∴ Solution is non-degenerate

Initial feasible solution

	D_1	D_2	D_3	D_4
S_1	3 (200)	1 (50)	7	4
S_2	2	6 (250)	5 (100)	9
S_3	8	3	3 (250)	2 (150)

cost = 3700

Use MODI to check for optimality

	D_1	D_2	D_3	D_4	
S_1	3 (200) +	− 1 (50)	7	4	$u_1 = 0$
S_2	2 −	+ 6 (250)	5 (100)	9	$u_2 = 5$
S_3	8	3	3 (250)	2 (150)	$u_3 = 3$
	v_1	v_2	v_3	v_4	
	3	1	0	−1	

Let $u_1 = 0$ $\therefore$ using $c_{ij} = u_i + v_j$

$u_2 = 5$ $\quad u_3 = 3$

$v_1 = 3$ $\quad v_2 = 1$ $\quad v_3 = 0$ $\quad v_4 = -1$

Finding Δij for unoccupied cells

$\Delta_{13} = C_{13} - (u_1 + v_3) = 7 - 0 = 7$

$\Delta_{14} = C_{14} - (u_1 + v_4) = 4 - (0 + -1) = 5$

$\Delta_{21} = -6$

$\Delta_{31} = 2$

$\Delta_{24} = 5$

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$\Delta_{32} = -1$

As -6 is the most -ve, consider $\oplus$ in Δ_{21}

$x = \min(200, 250)$

$= 200$

Modified sol^n is

3	1 (250)	7	4	$u_1 = -5$
2 (200)	6 (50)	5 (100)	9	$u_2 = 0$
8	3	3 (250)	2	$u_3 = -2$
v_1	v_2	v_3	v_4	
2	6	5	4	

As $\Delta_{32} = -1$, we choose to optimize

$\therefore x = 50$

Modified sol^n is

3	1 (250)	7	4
2 (200)	6	5 (150)	9
8	3 (50)	3 (200)	2 (150)

As all $\Delta ij \geq 0$, this is optimal sol^n

Transportation cost $= 250 + 400 + 750 + 150 + 600 + 300$

$= 2450$

$x_{12} = 250$ $\quad x_{21} = 200$ $\quad x_{23} = 150$

$x_{32} = 50$ $\quad x_{33} = 200$ $\quad x_{34} = 150$

CONCLUSION:
I have understood & implemented feasible solⁿ for transportation problem using least cost & vogel's approximation method.