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ASSIGNMENT - 2

TITLE: DUALITY THEORY

PROBLEM STATEMENT: Convert a given primal optimization problem into its dual using duality principle.

THEORY:

- DUALITY:

- In mathematical optimization, duality principle is the principle that optimization problems can be viewed from ~~two~~ perspectives, the primal problem & the dual problem.

- It is a unifying theory that develops the relationships b/w a given ~~rotour~~ linear problem & other related linear problems stated in terms of variables with optimal simplex multiplier interpretation.

Primal	Dual
maximize $z = \sum_{j=1}^{n} c_j x_j$	minimize $w = \sum_{i=1}^{n} b_i y_i$
subject to	subject to
$\sum_{i=1}^{m} a_{ij} x_j \leq b_j \quad (i=1,\ldots,m)$	$\sum_{i=1}^{n} a_{ij} y_i \geq c_i \quad (j=1,\ldots,n)$
$x_j \geq 0 \quad (j=1,\ldots n)$	$y_i \geq 0 \quad (i=1,\ldots,m)$

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Generalization:

Primal (Maximize)	Dual (Minimize)
i^{th} constraint ≤ 0	i^{th} variable ≥ 0
i^{th} constraint $\geq$	i^{th} variable ≤ 0
i^{th} constraint $=$	i^{th} variable unrestricted
j^{th} variable $\geq$	j^{th} constraint $\geq$
j^{th} variable $\leq$	j^{th} constraint $\leq$
j^{th} variable unrestricted	j^{th} constraint $=$

Optimal Property:

If $\hat{x}_j$, $j = 1, 2, \ldots, n$

is a feasible solution to the primal problem &

$\hat{y}_j$, $j = 1, 2, \ldots, m$

is a feasible solution to the dual problem

then

$$\sum_{j=1}^{n} c_j \hat{x}_j = \sum_{i=1}^{m} b_i \hat{y}_i$$

They are both optimal for their problems.

Q.1. Maximize : $Z = 3x_1 + 5x_2$

subject to : $6x_1 + 6x_2 \leq 55$

$8x_1 + 2x_2 = 30$

$5x_1 - 3x_2 \geq 18$; $x_1, x_2 \geq 0$

$x_3 =$ unrestricted

For unrestricted variable $x_3 = x_3' - x_3''$

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Primal	Dual
Min.	Max. ~~Max~~
3 variables	3 constraints
3 constraints	3 variables
1 '=' type	3rd variable unrestricted
1 unrestricted variable	3rd constraint '=' type

$\therefore Z = 3x_1 - 2x_2 + x_3' - x_3''$

Subject to :
$$-2x_1 + 3x_2 - x_3' + x_3'' \geq -5$$
$$4x_1 - 2x_2 + 0x_3' + 0x_3'' \geq 9$$
$$-8x_1 + 4x_2 + 3x_3' - 3x_3'' \geq 8$$
$$8x_1 - 4x_2 - 3x_3' + 3x_3'' \geq -8$$

$\therefore$ Dual will be

Maximize $W = -5y_1 + 9y_2 + 8y_3' - 8y_3''$

Subject to :
$$-2y_1 + 4y_2 - 8y_3' + 8y_3'' \leq 3$$
$$3y_1 - 2y_2 + 4y_3' - 4y_3'' \leq -2$$
$$-y_1 + 3y_3' - 3y_3'' \leq 1$$
$$y_1 - 3y_3' + 3y_3'' \leq -1$$

Replace $y_3' - y_3'' \Rightarrow y_3$

$\therefore$ Maximize $W = -5y_1 + 9y_2 + 8y_3$

Subject to :
$$-2y_1 + 4y_2 - 8y_3 \leq 3$$
$$3y_1 - 2y_2 + 4y_3 \leq -2$$
$$-y_1 + 3y_3 \leq 1 \; ; y_1, y_2 \geq 0$$
$y_3 \Rightarrow$ unrest.

CONCLUSION:

Thus we have learnt principle of duality.