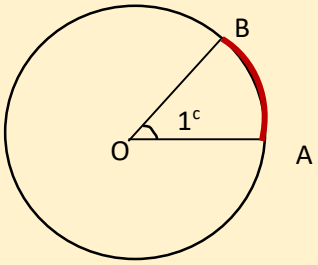
1) Circular Measure (Radian Measure):



A unit measure of angles. On radian is the angle made at the center of a circle by arc whose length is equal to the radius of the circle. It is written as 1^c.

In given figure radius OA = arc AB. = 1^c

The measure of $1^c = 57.2727^0$ [Nearly]

- 'c' used in the notation of radian is the first letter of the word circular system.

⇒ Prove that 1^c is a constant angle.

We know that,

$2\pi r = 360^0$

or $1 = \frac{360^0}{2\pi r}$

or $r = \frac{360^0}{2\pi r} \times r$

$\therefore r = \frac{180^0}{\pi}$

i.e. $1^c = \frac{180^0}{\pi} = 57.2727^0 \quad [\because \pi = \frac{22}{7}]$

Hence, 1^c is constant. Proved

⇒ Relation among degree, grade and radian measure.

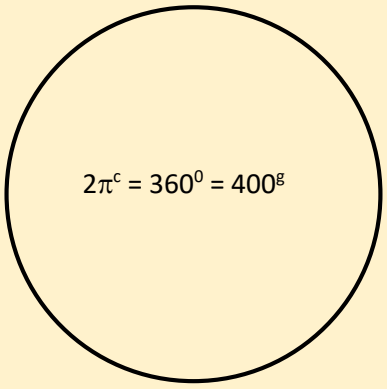
We know that,

$90^0 = 100^g$

Now,

$1^c = \frac{180^0}{\pi}$

or $\pi^c = \frac{180^0}{\pi} \times \pi$



$\therefore \pi^c = 180^0 = 200^g \quad [\because 2\times 90^0 = 2\times 100^g, \text{ i.e. } 180^0 = 200^g]$

⇒ Relation among sexagesimal, centesimal and circular measure.

Degree and grade	Degree and radian	Grade and radian
$1^0 = \left(\frac{10}{9}\right)^g$	$1^0 = \left(\frac{\pi}{180}\right)^c$	$1^g = \left(\frac{\pi}{200}\right)^c$
$1^g = \left(\frac{9}{10}\right)^0$	$1^c = \frac{180^0}{\pi}$	$1^c = \left(\frac{200}{\pi}\right)^g$

⇒ Convert 1^c to sexagesimal measure.

We know that,

$$\pi^c = 180^0$$

or $$1^c = \frac{180^0}{\pi} = \frac{180^0}{\frac{22}{7}} = 57.2727^0$$

$$= 57^0 + 0.2727^0$$

$$= 57^0 + (0.2727 \times 60)'$$

$$= 57^0 + 16.362'$$

$$= 57^0 + 16' + (0.362 \times 60)''$$

$$= 57^0 + 16' + 22''$$

$\therefore$ $$1^c = \frac{180^0}{\pi} = 57.2727^0 = 57^0\ 16'\ 22'' \quad //$$

Example 1: Express 45^0 into radian.

Solution:

Here,

$$1^c = \left(\frac{\pi}{180}\right)^c$$

Now,

$$45^0 = \left(\frac{\pi}{180}\right)^c \times 45$$

$$= \left(\frac{\pi}{4}\right)^c$$

$\therefore$ $$45^0 = \left(\frac{\pi}{4}\right)^c \quad //$$

Example 2: Express 80^g into radian.

Solution:

Here,

$$1^g = \left(\frac{\pi}{200}\right)^c$$

Now,

$$80^g = \left(\frac{\pi}{200}\right)^c \times 80$$

$$= \left(\frac{2\pi}{5}\right)^c$$

$\therefore$ $$80^g = \left(\frac{2\pi}{5}\right)^c \quad //$$

Example 3: Express $\left(\frac{3\pi}{8}\right)^c$ into degree.

Solution:

Here,

$$1^c = \frac{180^0}{\pi}$$

Now,

$$\left(\frac{3\pi}{8}\right)^c = \frac{180^0}{\pi} \times \left(\frac{3\pi}{8}\right)^c$$

$$= 67.5^0$$

Example 4: Express 80^0 into grade.

Solution:

Here,

$$1^c = \left(\frac{200}{\pi}\right)^g$$

Now,

$$\left(\frac{2\pi}{5}\right)^c = \left(\frac{200}{\pi}\right)^g \times \left(\frac{2\pi}{5}\right)$$

$$= 80^g$$

$\therefore$ $$\left(\frac{2\pi}{5}\right)^c = 80^g \quad //$$

$\therefore \quad \left(\frac{3\pi}{8}\right)^{c} = 67.5^{0}$ //