Chapter 4
Scalar Diffraction

Perhaps the most fundamental task associated with Fourier optics is describing the evolution of an optical field as it propagates from one location to another. The phenomenon of *diffraction* underlies the behavior of propagating waves. Extensive theory developed for diffraction provides the basis for modeling optical propagation on the computer. This chapter is essentially a summary of scalar diffraction theory with a listing of the expressions commonly used today to describe optical diffraction of monochromatic light. The presentation closely follows the diffraction development in Goodman.[1] More details can be found in that reference, as well as others.[2-4] This chapter sets the stage for the computer methods of simulating of optical propagation described in Chapter 5.

4.1 Scalar Diffraction

Diffraction refers to the behavior of an optical wave when its lateral extent is confined, for example, by an aperture. It accounts for the fact that light rays do not follow strictly rectilinear paths when the wave is disturbed on its boundaries. In our everyday experience we rarely notice diffractive effects with light. The effects of reflection, from a mirror, or refraction due to a lens are much more obvious. In fact, the effects of diffraction become most apparent when the confinement size is on the order of the wavelength of the radiation. Nevertheless, diffraction plays a role in many optical applications and is a critical consideration for applications involving high resolution, such as astronomical imaging, or long propagation distances, such as laser radar, and in applications involving small structures such as photolithographic processes.

The propagation behavior of an optical wave is fundamentally governed by Maxwell's equations. In general, coupling exists between the wave's electric field $\vec{E}$ with components (E_x, E_y, E_z) and its magnetic field $\vec{H}$ with components (H_x, H_y, H_z). There is also coupling between the individual components of the electric field as well as between the magnetic components. However, consider a wave that is propagating in a dielectric medium that is *linear* (field quantities from separate sources can be summed), *isotroptic* (independent of the wave polarization, i.e. the directions of $\vec{E}$ and $\vec{H}$), *homogeneous* (permittivity of the medium is independent of position), *nondispersive* (permittivity is independent of wavelength), and *nonmagnetic* (magnetic permeability is equal to the vacuum permeability). In this case, Maxwell's vector expressions become decoupled and the behavior of each component of the electric or magnetic fields can be expressed independently from the other components. Scalar diffraction refers to the propagation behavior of light under this ideal situation.

The long list of assumptions for the medium suggests a rather limited application regime for scalar diffraction theory. However, scalar diffraction can clearly be used for describing free space optical (FSO) propagation, which refers to transmission through space or the atmosphere and encompasses a huge number of interesting applications such as lidar, imaging, and laser communications. Furthermore, for many problems involving less benign propagation media, scalar solutions can provide a reasonable approximation of the principle effects of the propagation and establish a basis for comparison with full vector results. All of the developments and applications in this book assume scalar diffraction.

4.2 Monochromatic Fields and Irradiance

Some terminology and definitions related to optical fields are needed at this point. A *monochromatic* (single frequency) scalar field propagating in an isotropic medium can be expressed as

$$u(P,t) = A(P)\cos\left[2\pi\nu t - \phi(P)\right], \tag{4-1}$$

where $A(P)$ is the amplitude and $\phi(P)$ is the phase at a position P in space (x, y, z coordinates) and ν is the temporal frequency. This expression models a propagating transverse optical (electric) field of a single polarization.

Monochromatic light provides the basis for our analytic and computer simulation approaches to diffraction theory. Monochromatic light is also *coherent*. Coherence refers to the correlation of the optical field phase at two different points in the field separated by time and/or space and enables the formation of interference in a time-averaged sense. Although some lasers can produce near-monochromatic radiation, true monochromatic light is unachievable. But as will be discussed in Chapters 7 and 9, the extension of monochromatic results to polychromatic radiation, as well as partially coherent and incoherent radiation, can be straightforward in many useful cases (…fortunately!).

To give an example, a specific form of (4-1) corresponding to a plane wave propagating in the z-direction would be

$$u(z,t) = A\cos\left[2\pi\nu t - kz\right], \tag{4-2}$$

where the wavenumber k is defined as

$$k = \frac{2\pi}{\lambda}. \tag{4-3}$$

and where λ is the vacuum wavelength. Also $\nu = c/\lambda$ where c is the speed of light in vacuum. This wave has no dependence on x and y and therefore is interpreted as extending infinitely in these directions.

If the field in (4-1) is propagating in a linear medium (assumed for scalar diffraction) the temporal frequency of the resulting field will remain unchanged, so it is not necessary to explicitly carry the temporal term. Furthermore, substituting a complex *phasor* form for the cosine function provides a valid propagation result and aids in mathematical manipulation. These changes lead to a function that simply describes the spatial distribution of the field

$$U(P) = A(P)\exp\left[j\phi(P)\right]. \tag{4-4}$$

This complex phasor form of the optical field will be used extensively in our analytic and simulation developments. As an example, the phasor form of (4-2) is

$$U(z) = A\exp\left(jkz\right). \tag{4-5}$$

The descriptions in (4-1) and (4-4) are related by

$$u(P,t) = \mathrm{Re}\left\{U(P)\exp\left(-j2\pi\nu t\right)\right\}, \tag{4-6}$$

where the complex phasor $\exp(-j2\pi\nu t)$ is introduced for the temporal component of the field. To further refine (4-4), the explicit dependence on the z position can be removed where z is assumed to be the fundamental propagation direction. Thus

$$U_1(x,y) = A_1(x,y)\exp\left[j\phi_1\left(x,y\right)\right], \tag{4-7}$$

indicates the field in the x-y plane located at some position "1" on the z axis.

Detectors do not currently exist that can follow the extremely high frequency oscillations (> 10^{14} Hz) of the optical electric field. Instead optical detectors respond to the time-averaged squared-magnitude of the field. So a quantity of considerable interest is the *irradiance,* which is defined here as

$$I_1(x,y) = U_1\left(x,y\right)U_1\left(x,y\right)^* = \left|U_1(x,y)\right|^2. \tag{4-8}$$

Irradiance is a radiometric term for the flux (watts) per unit area falling on the observation plane. It is a power density quantity that in other laser and Fourier optics references is often called "intensity". The expression (4-8) actually represents a shortcut for determining the time-averaged square-magnitude of the field and is valid when the field is modeled by a complex phasor.

On a bookkeeping note, since $A_1(x,y)$ is the electric field amplitude, with typical units of (volt/m), then to yield the corresponding irradiance value with units of (watt/m^2) the right side of (4-8) needs to be multiplied by the constant $1/(2\eta)$ where η is the characteristic impedance of the medium (η = 377 Ω for vacuum). Since we are most interested in the relative spatial form of the field, this constant is usually dropped in our discussions.

4.3 Optical Path Length and Field Phase Representation

The refractive index n of a medium is the ratio of the speed of light in vacuum to the speed in the medium. For example, a typical glass used for visible light might have an index of about 1.6. For light propagating a distance d in a medium of index n, the *optical path length* (OPL) is defined as

$$\mathrm{OPL} = nd \; . \tag{4-9}$$

The OPL multiplied by the wavenumber k shows up in the phase of the complex exponential used to model the optical field. Think of k as the "converter" between the distance spanned by one wavelength and 2π radians of phase. For example, in the plane-wave expression of (4-2), z is the OPL where the propagation is assumed to be in vacuum so n = 1. The term kz gives the number of radians the sinusoid phase of the field has progressed over this distance. Sinusoids or complex exponentials are modulo 2π entities, so only the relative phase between 0 and 2π has meaning. If the plane wave propagates a distance d through a piece of glass with index n then the OPL is as indicated in (4-9) and the field phasor representation is

$$U(d) = A\exp\left(jknd\right). \tag{4-10}$$

In effect the wavelength shortens by λ/n in the glass. There are other variations of this theme, for example exp(jkr) where r is a radial distance in vacuum.

Phasor forms associated with the optical field can also be a function of transverse position x and y, for example

$$\exp\left[-j\frac{k}{2f}\left(x^2 + y^2\right)\right]. \tag{4-11}$$

This is known as a "chirp" term and indicates a phase delay in the field as a function of transverse position. However, from the definition of (4-6), phase in general becomes more *negative* as time progresses. Thus (4-11) indicates that the phase in the center of the function *lags* the rest of the phase. The further away from the center, the more the phase *leads*. This interpretation would be reversed if the negative sign in front of j were removed. This type of term appears in a variety of situations to model a contracting or expanding optical field.

4.4 Analytic Diffraction Solutions

4.4.1 Rayleigh-Sommerfeld Solution I

Consider the propagation of monochromatic light from a 2-dimensional plane (source plane) indicated by the coordinate variables x and y (Fig. 4.1). At the source plane, an area Σ defines the extent of a source or an illuminated aperture. The field distribution in the source plane is given by $U_1(x, y)$ and the field $U_2(x, y)$ in a distant observation plane can be predicted using the first *Rayleigh-Sommerfeld solution*

$$U_2(x,y) = \frac{z}{j\lambda} \iint_{\Sigma} U_1(\xi,\eta) \frac{\exp(jkr_{12})}{r_{12}^2} d\xi d\eta \ . \tag{4-12}$$

Here, λ is the optical wavelength, k is the wavenumber, which is equal to $2\pi/\lambda$ for free space, z is the distance between the centers of the source and observation coordinate systems and r_{12} is the distance between a position on the source plane and a position in the observation plane. ξ and η are variables of integration and the integral limits correspond to the area of the source Σ. With the source and observation positions defined on parallel planes, the distance r_{12} is

$$r_{12} = \sqrt{z^2 + (x-\xi)^2 + (y-\eta)^2} \ . \tag{4-13}$$

The expression in (4-12) is a statement of the Huygens-Fresnel principle. This principle supposes the source acts as an infinite collection of fictitious point sources, each producing a spherical wave associated with the actual source field at any position (ξ, η). The contributions of these spherical waves are summed at the observation position (x, y), allowing for interference. The extension of (4-12) and (4-13) to non-planar geometries is straightforward, for example, involving a more complicated function for r, but the planar geometry is more commonly encountered and is our focus here.

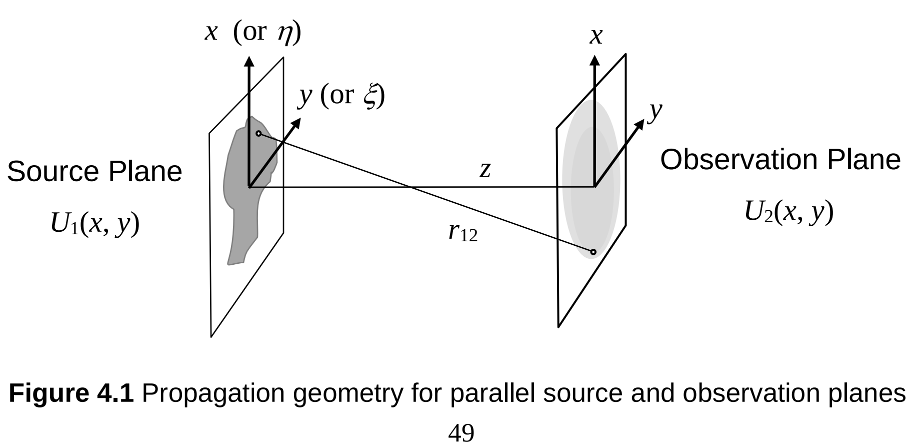


Figure 4.1 Propagation geometry for parallel source and observation planes.

The expression in (4-12) is, in general, a superposition integral but with the transmit and receive areas defined on parallel planes it becomes a convolution integral, which can be written

$$U_2(x,y) = \iint U_1(\xi,\eta) h(x-\xi, y-\eta) d\xi d\eta \,, \tag{4-14}$$

where the Rayleigh-Sommerfeld impulse response is

$$h(x,y) = \frac{z}{j\lambda} \frac{\exp(jkr)}{r^2} \,, \tag{4-15}$$

and

$$r = \sqrt{z^2 + x^2 + y^2} \,. \tag{4-16}$$

The Fourier convolution theorem is applied to write (4-14) as

$$U_2(x,y) = \Im^{-1}\{\Im\{U_1(x,y)\}\Im\{h(x,y)\}\} \,, \tag{4-17}$$

or equivalently,

$$U_2(x,y) = \Im^{-1}\{\Im\{U_1(x,y)\} H(f_X, f_Y)\} \,, \tag{4-18}$$

where *H* is the Rayleigh-Sommerfeld transfer function given by

$$H(f_X, f_Y) = \exp\left(jkz\sqrt{1-(\lambda f_X)^2 - (\lambda f_Y)^2} \right). \tag{4-19}$$

Strictly speaking, $\sqrt{f_X^2 + f_Y^2} < 1/\lambda$ must be satisfied for propagating field components. An angular spectrum analysis is often used to derive (4-19).

The Rayleigh-Sommerfeld expression is the most accurate diffraction solution considered in this book. Other than the assumption of scalar diffraction, this solution only requires that $r >> \lambda$, the distance between the source and the observation position be much greater than a wavelength.

4.4.2 Fresnel Approximation

The square root in the distance terms of (4-13) or (4-16) can make analytic manipulations of the Rayleigh-Sommerfeld solution difficult and can add execution time to a digital simulation. By introducing approximations for these terms, a more convenient scalar diffraction form is developed. By considering a binomial expansion

$$\sqrt{1+b} = 1 + \frac{1}{2}b - \frac{1}{8}b^2 + \ldots, \tag{4-20}$$

where b is a number less than one then expanding (4-13) and keeping the first two terms yields

$$r_{12} \approx z\left[1 + \frac{1}{2}\left(\frac{x-\xi}{z}\right)^2 + \frac{1}{2}\left(\frac{y-\eta}{z}\right)^2\right]. \tag{4-21}$$

This approximation is applied to the distance term in the phase of the exponential in (4-12), which amounts to assuming a parabolic radiation wave rather than a spherical wave for the fictitious point sources. Furthermore, using the approximation $r_{12} \approx z$ in the denominator of (4-12), we arrive at the *Fresnel diffraction* expression[1]

$$U_2(x,y) = \frac{e^{jkz}}{j\lambda z}\iint U_1(\xi,\eta)\exp\left\{j\frac{k}{2z}\left[(x-\xi)^2 + (y-\eta)^2\right]\right\}d\xi d\eta\,. \tag{4-22}$$

This expression is also a convolution of the form in (4-14) where the impulse response is

$$h(x,y) = \frac{e^{jkz}}{j\lambda z}\exp\left[\frac{jk}{2z}\left(x^2 + y^2\right)\right], \tag{4-23}$$

and the transfer function is

$$H(f_X, f_Y) = e^{jkz}\exp\left[j\pi\lambda z\left(f_X^2 + f_Y^2\right)\right]. \tag{4-24}$$

The expressions in (4-14) and (4-18) are again applicable in this case for computing diffraction results.

Another useful form of the Fresnel diffraction expression is obtained by moving the quadratic phase term that is a function of x and y outside the integrals

$$U_2(x,y) = \frac{\exp(jkz)}{j\lambda z}\exp\left[j\frac{k}{2z}\left(x^2 + y^2\right)\right]$$
$$\cdot\iint\left\{U_1(\xi,\eta)\exp\left[j\frac{k}{2z}\left(\xi^2 + \eta^2\right)\right]\right\}\exp\left[-j\frac{2\pi}{\lambda z}\left(x\xi + y\mu\right)\right]d\xi d\eta\,. \tag{4-25}$$

Along with the amplitude and chirp multiplicative factors out front, this expression is recognized as a Fourier transform of the source field times a chirp function where the following variable substitutions are used for the transform

$$f_X \to \frac{x}{\lambda z}, \qquad f_Y \to \frac{y}{\lambda z} . \tag{4-26}$$

The accuracy of the Fresnel expression when modeling scalar diffraction at close ranges suffers as a consequence of the approximations involved. By allowing a 1 radian maximum phase change (due to dropping the $b^2/8$ term in the series of (4-20)), the following condition is derived

$$z^3 >> \left(\frac{\pi}{4\lambda} \left[(x-\xi)^2 + (y-\eta)^2 \right]^2 \right)_{\max} , \tag{4-27}$$

where the "max" notation indicates the maximum value that is of interest for a given source and observation plane geometry.

The criterion of (4-27) provides a well-defined condition where the Fresnel approximation can be applied with little loss in accuracy. However, for fields in the source plane with little spatial variation, such as a simple aperture back-illuminated by a plane wave, the Fresnel approximation can provide high accuracy even when (4-27) is violated. A looser criterion is the Fresnel number, which is commonly used for determining when the Fresnel expression can be applied. The Fresnel number is given by

$$N_F = \frac{w^2}{\lambda z}, \tag{4-28}$$

where w is the half-width of a square aperture in the transmit plane, or the radius of a circular aperture, and z is the distance to the receive plane. If N_F is *less than* $\approx$ *1* for a given scenario, then it is commonly accepted that the observation plane is in the *Fresnel region* where the Fresnel approximations typically lead to useful results. However, for relatively "smooth" fields over the source aperture, the Fresnel expression can be applicable up to Fresnel numbers of even 20 or 30. In a geometrical optics context, the Fresnel expression describes diffraction under the paraxial assumption, where only rays that make a small angle (< ~0.1 rad) relative to the optical axis are considered.

4.4.3 Fraunhofer Approximation

Fraunhofer diffraction, which refers to diffraction patterns in a regime that is commonly known as the "far-field", is arrived at mathematically by approximating the chirp term multiplying the initial field within the integrals of (4-25) as unity. The assumption involved is

$$z >> \left(\frac{k(\xi^2 + \eta^2)}{2} \right)_{\max} , \tag{4-29}$$

and the resulting *Fraunhofer diffraction* expression is

$$U_2(x,y)=\frac{\exp(jkz)}{j\lambda z}\exp\left[j\frac{k}{2z}\left(x^2+y^2\right)\right]\iint U_1(\xi,\eta)\exp\left[-j\frac{2\pi}{\lambda z}\left(x\xi+y\eta\right)\right]d\xi d\eta. \tag{4-30}$$

Although the condition of (4-29) typically requires very long distances relative to the source field size, the Fraunhofer diffraction expression is a powerful tool and finds use in many applications. For example, spectroscopy can involve propagation distances that are much larger than the size of the structures of the diffraction grating used to separate the wavelengths. A form of the Fraunhofer pattern also appears in the propagation analysis involving lenses.

Along with multiplicative factors out front, the Fraunhofer expression can be recognized simply as a Fourier transform of the source field with the variable substitutions

$$f_X \rightarrow \frac{x}{\lambda z}, \qquad f_Y \rightarrow \frac{y}{\lambda z}. \tag{4-31}$$

The Fraunhofer expression cannot be written as a convolution integral, so there is no impulse response or transfer function. But since it is a scaled version of the Fourier transform of the initial field, it is easy to calculate, and as with the Fresnel expression, the Fraunhofer approximation is often used with success in situations where (4-29) is not satisfied. For simple transmitting structures such as a plane wave illuminated aperture, the Fraunhofer result can be useful even when (4-29) is violated by more than a factor of 10, particularly if the main quantity of interest is the irradiance pattern at the receiving plane. Using the Fresnel number N_F, the *commonly accepted requirement for the Fraunhofer region is* $N_F << 1$.

4.4 Fraunhofer Diffraction Example

It is extremely difficult (impossible?) to find closed-form diffraction solutions using the Rayleigh-Sommerfeld expression. The Fresnel expression is more tractable but solutions are still complicated even for simple cases such as a rectangular aperture illuminated by a plane wave.[1,2] So Fresnel or Rayleigh-Sommerfeld calculations are left for the computer in the next chapter. Analytic Fraunhofer diffraction analysis is easier and, for our purposes, serves as a check on some of the computer results.

Consider a circular aperture illuminated by a unit amplitude plane wave. The complex field immediately beyond the aperture plane is

$$U_1(x,y) = \text{circ}\left(\frac{\sqrt{x^2+y^2}}{w}\right). \tag{4-32}$$

To find the Fraunhofer diffraction field, the Fourier transform is taken

$$\Im\{U_1(x,y)\} = w^2 \frac{\mathrm{J}_1\left(2\pi w\sqrt{f_X{}^2 + f_Y{}^2}\right)}{w\sqrt{f_X{}^2 + f_Y{}^2}}. \tag{4-33}$$

Then with the substitutions in (4-31) and applying the leading amplitude and phase terms of (4-30), the field is found

$$U_2(x,y) = \frac{\exp(jkz)}{j\lambda z}\exp\left(j\frac{k}{2z}\left(x^2+y^2\right)\right) \cdot w^2 \frac{\mathrm{J}_1\left(2\pi \frac{w}{\lambda z}\sqrt{x^2+y^2}\right)}{\frac{w}{\lambda z}\sqrt{x^2+y^2}}. \tag{4-34}$$

The irradiance, using (4-8), is

$$I_2(x,y) = \left(\frac{w^2}{\lambda z}\right)^2 \left[\frac{\mathrm{J}_1\left(2\pi \frac{w}{\lambda z}\sqrt{x^2+y^2}\right)}{\frac{w}{\lambda z}\sqrt{x^2+y^2}}\right]^2. \tag{4-35}$$

Some of the $w/\lambda z$ terms could be canceled, but the symmetry of this form is helpful for programming and spotting errors.

Let's exercise MATLAB® to display this irradiance pattern. Suppose w = 1 mm and λ = 0.633 μm (HeNe laser wavelength). The Fresnel number constraint requires $w^2/\lambda z < 0.1$ or $z > 10w^2/\lambda$, which leads to z > 15.8 m. We'll use z = 50 m.

Now to choose some mesh parameters. The displayed function will probably look fine if the array side length is perhaps 5 times wider than the pattern's central lobe. The Bessel function J_1 has a first zero when the argument is equal to 1.22π. If y = 0, then the first zero in the pattern occurs when

$$2\pi\frac{w}{\lambda z}x = 1.22\pi, \tag{4-36}$$

Solve for x to get half the center lobe width and doubling this result gives the full width

$$1.22\frac{\lambda z}{w}. \tag{4-37}$$

So choose $L = 5{\times}1.22\lambda z/w \approx 0.2$ m.

Now for some code. It is helpful to make a function that handles the Bessel function ratio. In a new M-file (name **jinc**) enter the following:

```
1  %
2  % jinc function
3  %
4  % J1(2*pi*x)/x
5  % divide by zero fix
6  %
7  function[out]=jinc(x);
8  % locate non-zero elements of x
9  mask=(x~=0);
10 % initialize output with pi (value for x=0)
11 out=pi*ones(size(x));
12 % compute output values for all other x
13 out(mask)=besselj(1,2*pi*x(mask))./(x(mask));
14 end
```

This function evaluates $J_1(2\pi x)/x$. A masking approach is used to avoid the divide-by-zero condition when $x = 0$. The code may appear to be a roundabout way of doing things, but it allows the input x to be a vector or an array. In line 9, the array `mask` picks up the dimension of x and takes on a value of 1 for any element where x is nonzero (`~=` means $\neq$). In line 11, `out` is initialized with the dimension of x, `ones` fills the array with 1's and π is the value of the function for $x = 0$. Then logical indexing, `out(mask)` and `x(mask)`, is applied to evaluate the function for all elements where `mask` is one. This leaves the value of π for $x = 0$. The MATLAB® call `besselj(1,…)` is the Bessel function of the first kind, order 1.

As with sinc functions, there are several definitions in the literature for "jinc" functions - and I may be the only one that uses this particular variation. So beware, not all jinc functions are the same. Now for the Fraunhofer pattern. Name this file **fraun_circ**:

```
%fraun_circ - Fraunhofer irradiance plot

L=0.2;          %side length (m)
M=250;          %# samples
dx=L/M;         %sample interval
x=-L/2:dx:L/2-dx; y=x;  %coords
```

```
7  [X,Y]=meshgrid(x,y);
8
9  w=1e-3;          %x half-width
10 lambda=0.633e-6;%wavelength
11 z=50;            %prop distance
12 k=2*pi/lambda;  %wavenumber
13 lz=lambda*z;
14
15 %irradiance
16 I2=(w^2/lz)^2.*(jinc(w/lz*sqrt(X.^2+Y.^2))).^2;
17
18 figure(1)   %irradiance image
19 imagesc(x,y,nthroot(I2,3));
20 xlabel('x (m)'); ylabel('y (m)');
21 colormap('gray');
22 axis square
23 set(gca,'YDir','normal');
24
25 figure(2)   %x-axis profile
26 plot(x,I2(M/2+1,:));
27 xlabel('x(m)'); ylabel('Irradiance');
```

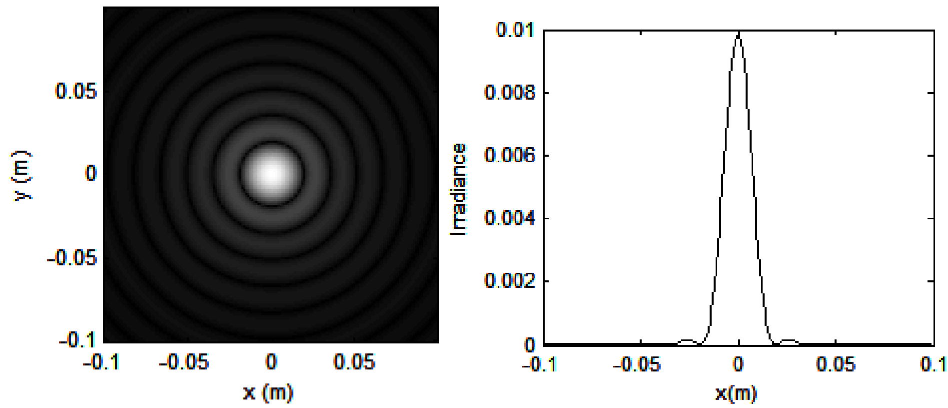
Here are a few comments on the routine with associated line numbers:

[9] Scientific notation can be done several ways: `e-3` and `10^-3` mean the same thing. Don't use the ^ symbol in the exponential `e` notation!

[16] `jinc` function is called.

[19] 3rd root is used to bring out the "rings" in the image display.

Running the script produces the results in Fig. 4.2. The Fraunhofer pattern of a circular aperture is commonly known as the *Airy pattern*. The central core of this pattern, whose width is given in (4-37), is known as the *Airy disk*.



(a) (b)

Figure 4.2 Fraunhofer irradiance (a) image pattern and (b) x-axis profile for a circular aperture. This is known as the Airy pattern.

References

1. Goodman, J. W., *Introduction to Fourier Optics*, 3rd ed., Roberts & Company Publishers (2005).
2. Gaskill, J. D., *Linear Systems, Fourier Transforms, and Optics*, Wiley-Interscience (1978).
3. Ersoy, O. K., *Diffraction, Fourier Optics and Imaging*, Wiley-Interscience (2006).
4. Hecht, E., *Optics*, 4th ed., Addison-Wesley (2002).

Exercises

4.1 Consider a plane wave of wavelength λ incident on two pieces of glass of different thicknesses and refractive indices as shown in the figure. Find an expression for the optical path (length) difference (OPD) between planes *a* and *b*.

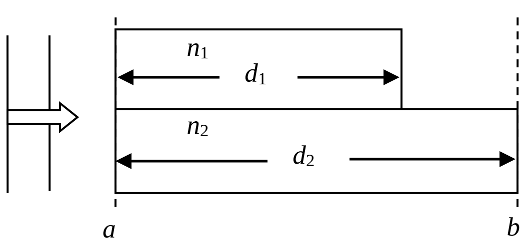


Figure 4.3

4.2 For plane-wave illumination (λ = 0.5 μm) of an aperture of *diameter* 1 mm, determine the range of propagation distances that are adequate for the Rayleigh-Sommerfeld, Fresnel, and Fraunhofer diffraction regimes.

4.3 Verify that the Fresnel transfer function is the Fourier transform of the Fresnel impulse response.

4.4 Derive expressions for the Fraunhofer field and irradiance patterns for the following apertures illuminated with a unit amplitude plane-wave.

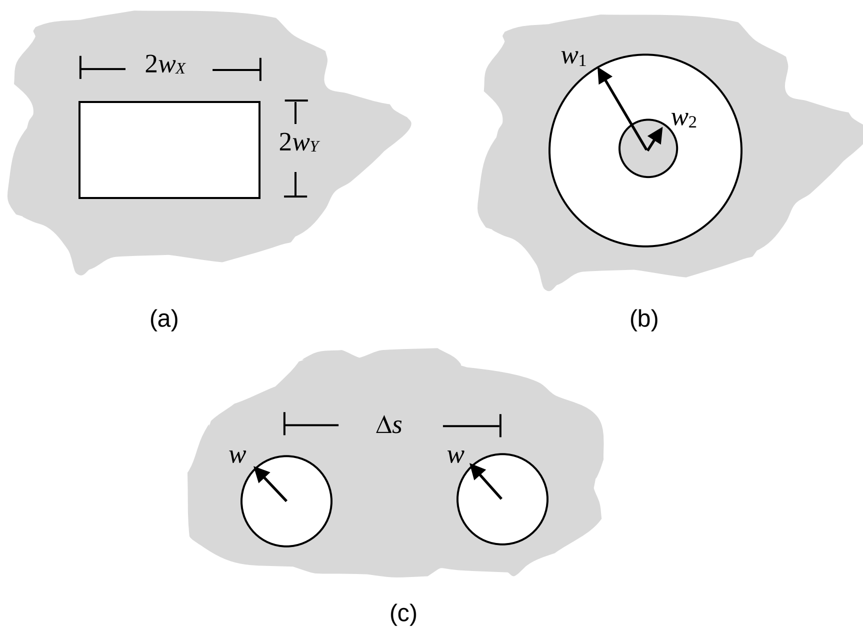


Figure 4.4

Plot the Fraunhofer irradiance pattern images and profiles for the above apertures on the computer. Choose suitable propagation distances z and side lengths L in the observation plane. Use λ = 0.633 μm and the following parameters:

(a) w_X = 0.1 mm, w_Y = 0.05 mm

(b) w_1 = 1 mm, w_2 = 0.2 mm.

(c) w = 1 mm, Δs = 4 mm

4.5 Suppose the plane-wave field incident on the aperture of 4.4(c) is attenuated by different amounts in passing through each hole. The field exiting one hole has a magnitude of A_1 and the field exiting the other hole has a magnitude of

A_2. Find an analytic expression for the Fraunhofer irradiance for this aperture. Plot the irradiance pattern and profile (along the x-axis) for $A_1 = 1$ and $A_2 = 0.4$. [Hint: take squared-magnitude before combining complex exponentials into cosine term.]