4. Solving for β

Solve for $\boldsymbol{\beta}$:

$$\mathbf{X}^T\mathbf{X}\boldsymbol{\beta} = \mathbf{X}^T\mathbf{y}$$

Now, assuming that $\mathbf{X}^T\mathbf{X}$ is invertible, we can multiply both sides by $(\mathbf{X}^T\mathbf{X})^{-1}$:

$$\boldsymbol{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

5. Final Formula for Coefficients

Thus, the estimated coefficients $\boldsymbol{\beta}$ are given by the following matrix equation:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

Where:

- $\hat{\boldsymbol{\beta}}$ is the vector of estimated coefficients $\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_p$.
- $\mathbf{X}^T\mathbf{X}$ is a $p + 1 \times p + 1$ matrix.
- $\mathbf{X}^T\mathbf{y}$ is a $p + 1 \times 1$ vector.

This is the matrix form of the solution for **multiple linear regression**.