To find the Fourier coefficients of the polynomial $f(x) = ax^6 + bx^5 + cx^4 + dx^3 + gx^2 + hx + k$ over the interval $[-s, s]$, we will derive the coefficients based on whether the polynomial is even or odd.

Definitions

1. **Fourier coefficients**:
   - The Fourier coefficients a_0, a_n, and b_n for the function $f(x)$ over the interval $[-s, s]$ are given by:

$$a_0 = \frac{1}{2s} \int_{-s}^{s} f(x)\, dx$$

$$a_n = \frac{1}{s} \int_{-s}^{s} f(x) \cos\left(\frac{n\pi x}{s}\right)\, dx \quad (n \geq 1)$$

$$b_n = \frac{1}{s} \int_{-s}^{s} f(x) \sin\left(\frac{n\pi x}{s}\right)\, dx \quad (n \geq 1)$$

Even and Odd Functions

- **Even Function**: $f(x)$ is even if $f(-x) = f(x)$. In this case, all b_n coefficients will be zero.
- **Odd Function**: $f(x)$ is odd if $f(-x) = -f(x)$. In this case, all a_n coefficients will be zero.

Polynomial Analysis

1. **Even part of $f(x)$**: Contains even powers.
2. **Odd part of $f(x)$**: Contains odd powers.

Thus, we can break down the polynomial:

- Even part: $f_e(x) = ax^6 + cx^4 + gx^2 + k$
- Odd part: $f_o(x) = bx^5 + dx^3 + hx$

Fourier Coefficients

- **For the even part $f_e(x)$**:
  - $a_0 = \frac{1}{2s} \int_{-s}^{s} (ax^6 + cx^4 + gx^2 + k)\, dx$
  - $a_n = \frac{1}{s} \int_{-s}^{s} (ax^6 + cx^4 + gx^2 + k) \cos\left(\frac{n\pi x}{s}\right)\, dx$
  - $b_n = 0$
- **For the odd part $f_o(x)$**:
  - $a_n = 0$

- $b_n = \frac{1}{s} \int_{-s}^{s} (bx^5 + dx^3 + hx) \sin\left(\frac{n\pi x}{s}\right) \, dx$

Final Results

1. **If a, c, g, k are non-zero** (even part):
   - $a_0 = \frac{1}{2s} \left(\frac{as^7}{7} + \frac{cs^5}{5} + \frac{gs^3}{3} + k(2s)\right)$
   - $b_n = 0$
   - a_n would require integration for specific values.
2. **If b, d, h are non-zero** (odd part):
   - $a_n = 0$
   - $b_n = \frac{1}{s} \left(\frac{bs^6}{6} + \frac{ds^4}{4} + \frac{hs^2}{2}\right)$

Conclusion

The Fourier coefficients depend on whether the polynomial has more even or odd terms. The coefficients can be computed specifically if the values of a, b, c, d, g, h, k and the interval s are known.