To find the coefficients in partial fraction decomposition quickly, especially for complex rational functions, you can use the following numerical technique. This method simplifies finding coefficients $A, B, C, D, E, \ldots$ without needing to derive a system of equations explicitly.

Steps for Numerical Coefficient Extraction

1. **Identify the Function**: Consider a rational function $R(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials.
2. **Perform Long Division**: If the degree of $P(x)$ is greater than or equal to the degree of $Q(x)$, perform polynomial long division first to separate the polynomial part from the proper fraction.
3. **Factor the Denominator**: Factor $Q(x)$ into its linear or irreducible quadratic factors:

$$Q(x) = (x - r_1)(x - r_2)(x^2 + ax + b)$$

4. **Set Up the Partial Fraction Form**: Write the partial fraction decomposition in the form:

$$R(x) = \frac{A_1}{(x - r_1)^{n_1}} + \frac{A_2}{(x - r_2)^{n_2}} + \frac{Bx + C}{(x^2 + ax + b)^m} + \ldots$$

5. **Use Residue Method for Coefficients**:
   - To find the coefficient A_1 corresponding to the factor $(x - r_1)^{n_1}$:

$$A_1 = \lim_{x \to r_1} (x - r_1)^{n_1} R(x)$$

   - Repeat for each A_2, B, C, etc., using their respective roots or factors.
6. **Solve Using the Method of Residues**: For complex roots or higher-order factors, if r is a root of multiplicity n:

$$A = \frac{1}{(n-1)!} \lim_{x \to r} \frac{d^{n-1}}{dx^{n-1}} \left[(x - r)^n R(x)\right]$$

   Here, you differentiate $n - 1$ times and then evaluate at $x = r$.
7. **Numerical Evaluation**: For numerical approximation, you can use:
   - **Direct Substitution**: Substitute values close to the roots into $R(x)$ and calculate the limits to find coefficients.
   - **Software Tools**: Use computational tools (e.g., Python, MATLAB) to evaluate limits numerically, especially for complex rational functions.

Example

For example, if you have $R(x) = \frac{2x^2+3}{(x-1)(x+2)}$, you'd identify:

1. **Setup**:

$$R(x) = \frac{A}{x-1} + \frac{B}{x+2}$$

2. **Find Coefficients**:
   - For A:

$$A = \lim_{x \to 1}(x-1)R(x) = \lim_{x \to 1}(x-1)\frac{2x^2+3}{(x-1)(x+2)} = \frac{2(1)^2+3}{1+2} = \frac{5}{3}$$

   - For B:

$$B = \lim_{x \to -2}(x+2)R(x) = \frac{2(-2)^2+3}{(-2-1)} = \frac{8+3}{-3} = -\frac{11}{3}$$

Conclusion

This numerical approach using limits and residues allows for a quick extraction of coefficients for partial fraction decomposition. It minimizes the algebra involved in solving systems of equations and can be particularly useful for complex or higher-order functions.