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give the fourier coefficients of a polynomial

$ax^6 + bx^5 + cx^4 + dx^3 + gx^2 + hx + k$  with the limit of  $-s, s$  considering even or odd give directly the fourier coefficients



To find the Fourier coefficients of the polynomial  $f(x) = ax^6 + bx^5 + cx^4 + dx^3 + gx^2 + hx + k$  over the interval  $[-s, s]$ , we will derive the coefficients based on whether the polynomial is even or odd.

## Definitions

### 1. Fourier coefficients:

- The Fourier coefficients  $a_0, a_n$ , and  $b_n$  for the function  $f(x)$  over the interval  $[-s, s]$  are given by:

$$a_0 = \frac{1}{2s} \int_{-s}^s f(x) dx$$

$$a_n = \frac{1}{s} \int_{-s}^s f(x) \cos\left(\frac{n\pi x}{s}\right) dx \quad (n \geq 1)$$

$$b_n = \frac{1}{s} \int_{-s}^s f(x) \sin\left(\frac{n\pi x}{s}\right) dx \quad (n \geq 1)$$

## Even and Odd Functions

- Even Function:**  $f(x)$  is even if  $f(-x) = f(x)$ . In this case, all  $b_n$  coefficients will be zero.
- Odd Function:**  $f(x)$  is odd if  $f(-x) = -f(x)$ . In this case, all  $a_n$  coefficients will be zero.

## Polynomial Analysis

- Even part of  $f(x)$ :** Contains even powers.
- Odd part of  $f(x)$ :** Contains odd powers.

Thus, we can break down the polynomial:

- Even part:  $f_e(x) = ax^6 + cx^4 + gx^2 + k$
- Odd part:  $f_o(x) = bx^5 + dx^3 + hx$

## Fourier Coefficients

- For the even part  $f_e(x)$ :**
  - $a_0 = \frac{1}{2s} \int_{-s}^s (ax^6 + cx^4 + gx^2 + k) dx$
  - $a_n = \frac{1}{s} \int_{-s}^s (ax^6 + cx^4 + gx^2 + k) \cos\left(\frac{n\pi x}{s}\right) dx$
  - $b_n = 0$
- For the odd part  $f_o(x)$ :**
  - $a_n = 0$
  - $b_n = \frac{1}{s} \int_{-s}^s (bx^5 + dx^3 + hx) \sin\left(\frac{n\pi x}{s}\right) dx$

## Final Results

- If  $a, c, g, k$  are non-zero (even part):
  - $a_0 = \frac{1}{2s} \left( \frac{as^7}{7} + \frac{cs^5}{5} + \frac{gs^3}{3} + k(2s) \right)$
  - $b_n = 0$

- $a_n$  would require integration for specific values.
2. If  $b, d, h$  are non-zero (odd part):

- $a_n = 0$
- $b_n = \frac{1}{s} \left( \frac{bs^6}{6} + \frac{ds^4}{4} + \frac{hs^2}{2} \right)$

## Conclusion

The Fourier coefficients depend on whether the polynomial has more even or odd terms. The coefficients can be computed specifically if the values of  $a, b, c, d, g, h, k$  and the interval  $s$  are known.



give in python code



Here's a Python code snippet that calculates the Fourier coefficients for the polynomial  $f(x) = ax^6 + bx^5 + cx^4 + dx^3 + gx^2 + hx + k$  over the interval  $[-s, s]$ . The code uses `scipy.integrate` to perform the necessary integrations.

python

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```
import numpy as np from scipy.integrate import quad def fourier_coefficients(a, b, c,
d, g, h, k, s): # Define the even part of the polynomial def f_even(x): return a*x**6 +
c*x**4 + g*x**2 + k # Define the odd part of the polynomial def f_odd(x): return b*x**5
+ d*x**3 + h*x # Calculate a_0 a_0, _ = quad(lambda x: a*x**6 + b*x**5 + c*x**4 + d*x**3
+ g*x**2 + h*x + k, -s, s) a_0 /= (2 * s) # Calculate a_n def a_n_func(n): return
quad(lambda x: a*x**6 + c*x**4 + g*x**2 + k * np.cos(n * np.pi * x / s), -s, s)[0] / s #
Calculate b_n def b_n_func(n): return quad(lambda x: b*x**5 + d*x**3 + h*x * np.sin(n *
np.pi * x / s), -s, s)[0] / s # Coefficients coefficients = { 'a_0': a_0, 'a_n':
[a_n_func(n) for n in range(1, 6)], # Calculating a_n for n = 1 to 5 'b_n': [b_n_func(n)
for n in range(1, 6)] # Calculating b_n for n = 1 to 5 } return coefficients # Example
usage a, b, c, d, g, h, k = 1, 1, 1, 1, 1, 1, 1 # Coefficients for the polynomial s = 1
# Limit for the integration coeffs = fourier_coefficients(a, b, c, d, g, h, k, s)
print("Fourier Coefficients:") print(f"a_0: {coeffs['a_0']}") print(f"a_n:
{coeffs['a_n']}") print(f"b_n: {coeffs['b_n']}")
```

## Explanation

- The code defines a function `fourier_coefficients` that takes the coefficients of the polynomial and the limit  $s$ .
- It computes  $a_0$ ,  $a_n$ , and  $b_n$  using numerical integration.
- You can change the coefficients and the value of  $s$  in the example usage section to see how the Fourier coefficients change.