A*X^2+B*X+C=0 SOLVE BY FINDING SOLUTION METHOD

To solve the quadratic equation of the form:

$$A \cdot X^2 + B \cdot X + C = 0$$

We use the Quadratic Formula method. Here's how:

Step-by-Step Solution Using the Quadratic Formula

The general quadratic equation is:

$$AX^2 + BX + C = 0$$

Step 1: Identify coefficients

From the equation, identify:

- $A = \text{coefficient of } X^2$
- B = coefficient of X
- C = constant term

✓ Step 2: Use the Quadratic Formula

The formula to find the solutions is:

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Where:

- $\sqrt{B^2 4AC}$ is called the **discriminant**.
- Step 3: Analyze the discriminant
- If $B^2 4AC > 0$: Two real and distinct solutions.

- If $B^2 4AC = 0$: One real and repeated solution.
- If $B^2 4AC < 0$: Two **complex** solutions (no real roots).

Example

Let's solve:

$$2X^2 + 3X - 5 = 0$$

Step 1:

$$A = 2$$
, $B = 3$, $C = -5$

Step 2: Discriminant

$$\Delta = B^2 - 4AC = 3^2 - 4(2)(-5) = 9 + 40 = 49$$

Step 3: Apply the formula

$$X = \frac{-3 \pm \sqrt{49}}{2 \cdot 2} = \frac{-3 \pm 7}{4}$$

Step 4: Solutions

$$X_1 = \frac{-3+7}{4} = \frac{4}{4} = 1$$

$$X_2 = \frac{-3-7}{4} = \frac{-10}{4} = -2.5$$

✓ Final Answer:

$$X = 1$$
 or $X = -2.5$

Would you like to solve a specific equation? If so, give me the values of A, B, and C.



SYMBOLIC

To solve the quadratic equation symbolically:

$$AX^2 + BX + C = 0$$

We use the quadratic formula symbolically:

Symbolic Solution

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

∧ Notes:

- The expression under the square root, $\Delta = B^2 4AC$, is the **discriminant**.
- The two solutions are:

$$X_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A}, \quad X_2 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

These are **symbolic expressions**—no specific numbers substituted. This form is used when solving algebraically or when A, B, and C are unknown or general parameters.

Would you like this in LaTeX, Python/SymPy, or another format?

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AX 2 +BX+C=0

We use the quadratic formula symbolically: FINDING GENERALISED SOLUTION TYPE

Great — you're asking for a generalized symbolic solution to the quadratic equation:

$$AX^2 + BX + C = 0$$

✓ Generalized Solution Type (Symbolic)

We solve it using the quadratic formula:

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

This formula always gives a **general symbolic solution**, and depending on the value of the **discriminant**:

$$\Lambda = R^2 - 4AC$$

We classify the type of solution as follows:

Types of Solutions Based on Discriminant

Discriminant $\Delta = B^2 - 4AC$	Nature of Roots	Type of Solution
$\Delta > 0$	Two distinct real roots	$X=rac{-B\pm\sqrt{\Delta}}{2A}$, real and unequal
$\Delta = 0$	One real repeated root	$X = \frac{-B}{2A}$, real and equal
$\Delta < 0$	Two complex conjugate roots	$(X = \frac{-B \pm i}{sqrt}$



The general symbolic solution of:

$$AX^2 + BX + C = 0$$

is always:

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

This is the **generalized solution type**, valid for any real or complex coefficients A, B, C, as long as A = 0.

Would you like this expressed in a computer algebra system format (e.g., Python/SymPy or Mathematica)?



NO TAKING COMMON ROOTS SQUARE AND

Ah! It sounds like you're asking to solve the quadratic equation symbolically by factoring — that is, taking common terms, completing the square, or expressing the solution without directly using the quadratic formula.

Let's walk through the **symbolic method using completing the square**, which leads to the general solution:



$$AX^2 + BX + C = 0$$

We want to solve it symbolically without directly applying the quadratic formula — by completing the square.



Step 1: Divide both sides by A (assuming A = 0):

$$X^2 + \frac{B}{A}X + \frac{C}{A} = 0$$

Step 2: Move the constant to the other side:

$$X^2 + \frac{B}{A}X = -\frac{C}{A}$$

Step 3: Complete the square

To complete the square, add and subtract $\left(\frac{B}{2A}\right)^2$ to the left-hand side:

$$X^{2} + \frac{B}{A}X + \left(\frac{B}{2A}\right)^{2} = -\frac{C}{A} + \left(\frac{B}{2A}\right)^{2}$$
$$\left(X + \frac{B}{2A}\right)^{2} = \frac{B^{2} - 4AC}{4A^{2}}$$

Step 4: Take square roots on both sides:

$$X + \frac{B}{2A} = \pm \frac{\sqrt{B^2 - 4AC}}{2A}$$

Step 5: Solve for X:

Quadratic Equation Solution

$$X = -\frac{B}{2A} \pm \frac{\sqrt{B^2 - 4AC}}{2A}$$
$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

✓ Final Generalized Symbolic Solution (By Completing the Square):

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

This **re-derives the quadratic formula** by algebraically manipulating and completing the square — **not just plugging into a memorized formula**.

Let me know if you'd like this same process for a specific numerical equation!



FOR CUBIC THAT WAY

Great! You're asking for the **general symbolic solution for a cubic equation**, similar to how we completed the square for quadratics. This is much more complex, but yes — we can symbolically solve a **general cubic equation** without jumping straight to the cubic formula, using **algebraic manipulation**.

General Cubic Equation:

$$AX^3 + BX^2 + CX + D = 0$$

We want a **symbolic solution** using a structured method (like completing the square for quadratics). Here's a high-level breakdown of the **classical method**, known as **Cardano's Method**.

Step-by-Step: Solving the Cubic Symbolically (Depressed Cubic + Substitution)



 $lue{\mathsf{Y}}$ Step 1: Divide by A to normalize the equation:

$$X^{3} + \frac{B}{A}X^{2} + \frac{C}{A}X + \frac{D}{A} = 0$$

Let:

$$x = X$$
, $b = \frac{B}{A}$, $c = \frac{C}{A}$, $d = \frac{D}{A}$

So we rewrite the equation as:

$$x^3 + bx^2 + cx + d = 0$$

Step 2: Remove the quadratic term (depress the cubic)

We perform the substitution:

$$x = y - \frac{b}{3}$$

This transforms the cubic into a simpler form called the **depressed cubic**:

$$y^3 + py + q = 0$$

Where:

$$p = c - \frac{b^2}{3}$$
, $q = \frac{2b^3}{27} - \frac{bc}{3} + d$



The Step 3: Solve the depressed cubic using Cardano's formula

Let the depressed cubic be:

$$v^3 + pv + q = 0$$

Then the solution is:

$$y = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

\checkmark Final Step: Recover x

$$x = y - \frac{b}{3}$$

Summary: Symbolic General Solution (Cardano's Method)

Given:

$$AX^3 + BX^2 + CX + D = 0$$

- 1. Divide by A to normalize.
- 2. Substitute $x = y \frac{B}{3A}$ to eliminate the x^2 term.
- 3. You now have $y^3 + py + q = 0$
- 4. Solve using:

$$y = \sqrt[3]{-\frac{q}{2} + \sqrt{(\frac{q}{2})^2 + (\frac{p}{3})^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{(\frac{q}{2})^2 + (\frac{p}{3})^3}}$$

5. Substitute back: $x = y - \frac{B}{3A}$

∧ Note:

The cubic has:

• 1 real root if the discriminant $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 > 0$

• 3 real roots if $\Delta \leq 0$ (involving complex cube roots, handled with trigonometric or complex methods)

Would you like to see this done **symbolically** for a specific cubic equation, or explore the **trigonometric solution** for 3 real roots (casus irreducibilis)?



FOR 4 TH ORDER

You're asking for the **general symbolic solution of a 4th-degree polynomial equation** (also called a **quartic equation**) — similar to how we completed the square or used Cardano's method for lower-degree equations.

This gets into Ferrari's method, the classical way to symbolically solve quartics.

General Quartic Equation

$$AX^4 + BX^3 + CX^2 + DX + E = 0$$

We aim to solve this **symbolically**, without simply applying a memorized formula. Let's walk through the **Ferrari method** step by step.

Step 1: Normalize the equation

Divide through by A (assuming A = 0):

$$X^4 + aX^3 + bX^2 + cX + d = 0$$

Where:

$$a = \frac{B}{A}, \quad b = \frac{C}{A}, \quad c = \frac{D}{A}, \quad d = \frac{E}{A}$$

Step 2: Depress the quartic (remove the cubic term)

Substitute:

$$X = x - \frac{a}{4}$$

This transforms the equation into a **depressed quartic** of the form:

$$x^4 + px^2 + qx + r = 0$$

The coefficients p, q, r are calculated in terms of a, b, c, d, and are tedious but manageable.

Step 3: Ferrari's Trick — Add and subtract a square

We want to complete a perfect square, similar to how we completed the square for quadratics.

Assume:

$$x^4 + px^2 + qx + r = (x^2 + mx + n)^2 - (mx^2 + nx + m^2 - p)^2$$

Instead, Ferrari's idea is to introduce an auxiliary variable y and rewrite:

$$x^4 + px^2 + qx + r = (x^2 + \frac{p}{2} + y)^2 - [(2y)x^2 + \dots]$$

Through a process of matching terms, we solve a **resolvent cubic** in y, just like how the cubic is reduced to a depressed cubic in Cardano's method.

Step 4: Solve the resolvent cubic

You get a **cubic equation** in y, which you solve using **Cardano's method**. Once you find a suitable real root y, the depressed quartic can be factored as a product of two quadratics:

$$x^4 + px^2 + qx + r = (x^2 + \alpha x + \beta)(x^2 - \alpha x + \gamma)$$

These can then be solved using the quadratic formula.

Final Step: Solve the quadratics, then reverse the substitution:

Solve the two quadratics for x, and recall $X = x - \frac{a}{4}$, so the final solutions are:

$$X = x_i - \frac{a}{4}$$
, for each root x_i

Summary: General Symbolic Solution of a Quartic (Ferrari's Method)

- 1. Normalize: divide by A
- 2. Depress the quartic (remove cubic term)
- 3. Use Ferrari's method:
 - Add an auxiliary square
 - Solve the resolvent cubic
- 4. Factor the quartic into two quadratics
- 5. Solve each using the quadratic formula
- 6. Undo substitution

• The general quartic can always be solved algebraically, unlike the general quintic (5th degree), which cannot be solved in radicals in general (Abel–Ruffini theorem).

• Ferrari's method is exact but very algebraically heavy for hand calculation.

Would you like me to work through an actual symbolic example of a depressed quartic, or derive the resolvent cubic step-by-step?



FOR 5 TH ORDER

Excellent question — now we're stepping into deep territory. You're asking for a **symbolic** solution to a general 5th-degree polynomial (quintic):

$$AX^5 + BX^4 + CX^3 + DX^2 + EX + F = 0$$

Short Answer:

There is no general symbolic solution in radicals for the quintic (5th-degree) equation.



This is the result of the Abel–Ruffini Theorem, which states:

"There is no general solution in radicals to polynomial equations of degree five or higher."

In other words, unlike quadratics (solvable via the quadratic formula), cubics (Cardano's method), and quartics (Ferrari's method), no such formula using only addition, multiplication, and roots exists for the general quintic.



What Does "No General Solution in Radicals" Mean?

It means:

- For all quintic equations? X No.
- For **some specific quintics?** Yes.

Some quintic equations can be solved by radicals if they have special structure (e.g., solvable Galois group). For example:

$$X^5 - 1 = 0$$

can be solved exactly using roots of unity.

But a general quintic like:

$$X^5 + X + 1 = 0$$

has no radical solution. You need numerical methods, or express the solution using elliptic functions or Bring radicals (advanced transcendental objects).

So How Are Quintics Solved?

- 1. Numerical methods (e.g., Newton-Raphson) Used to approximate roots.
- 2. Special functions

Solutions may be expressed using:

- Elliptic functions (Jacobi, Weierstrass)
- **Bring radical** (for depressed quintic)
- 3. Galois theory

Determines if a specific quintic is solvable by radicals.

Summary

Degree	Always Solvable in Radicals?	Method
2	✓ Yes	Quadratic formula
3	✓ Yes	Cardano's method
4	✓ Yes	Ferrari's method
5+	💢 No (in general)	Only special cases solvable in radicals

// Want an example?

If you have a specific 5th-degree polynomial, I can:

- Check if it's solvable by radicals
- Attempt to factor it
- Solve it numerically or symbolically (with special functions)

Let me know!

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OK BY 6 TH ORDER

Great — let's now talk about the 6th-degree polynomial (sextic equation):

$$AX^{6} + BX^{5} + CX^{4} + DX^{3} + EX^{2} + FX + G = 0$$

You're asking whether there's a **symbolic**, **generalized method** (like we used for quadratics, cubics, quartics) to solve this.

Short Answer (Like the Quintic):

There is no general solution in radicals for 6th-degree equations.

This is a direct consequence of **Galois Theory**. The **Abel–Ruffini Theorem** applies to **all** degrees \geq 5:

Polynomials of degree 5 or higher cannot generally be solved by radicals.

Deeper Explanation

- Some **specific 6th-degree equations** *can* be solved in radicals if they have a special structure.
- But **there is no universal formula** for solving **all** 6th-degree polynomials symbolically with radicals.

✓ When Can a 6th-Degree Polynomial Be Solved?

- 1. If it factors into lower-degree polynomials (e.g. two cubics, three quadratics, etc.)
- 2. If it has symmetry or a solvable Galois group
- 3. If it's a biquadratic in form, like:

$$X^6 + aX^3 + b = 0$$

- then make substitution: $Y = X^3$, and solve the resulting quadratic in Y
- 4. Cyclotomic or special symmetric equations, like:

$$X^6 - 1 = 0$$

— solvable using roots of unity.



1. Try to Factor

If the polynomial can be factored (even partially), solve each piece.

2. Numerical Roots

Use algorithms like:

- Newton-Raphson
- Durand-Kerner
- Bairstow's method

3. Computer Algebra Systems

Tools like WolframAlpha, SymPy, or Mathematica can sometimes return symbolic or numeric roots depending on structure.

Example (Solvable Case)

$$X^6 - 9X^3 + 8 = 0$$

Let $Y = X^3$, then:

$$Y^2 - 9Y + 8 = 0 \Rightarrow Y = 1 \text{ or } 8 \Rightarrow X^3 = 1 \text{ or } 8 \Rightarrow X = 1, \omega, \omega^2 \text{ or } 2, 2\omega, 2\omega^2$$

(with ω = cube roots of unity)

This one is solvable exactly!



Degree	General Symbolic Solution in Radicals?	Notes
2–4	✓ Yes	Always solvable with formulas

Degree	General Symbolic Solution in Radicals?	Notes
5+	X No (in general)	Only special cases solvable
6	X No (in general)	Try factoring or substitution

Would you like to test a **specific 6th-degree polynomial** to see if it's solvable by factoring or substitution? I'm happy to try!



WHAT ABOUT NTHJ DEGREE

Great — now you're asking the big question:



Let's break it down clearly and definitively.

\checkmark The General Polynomial (Degree n):

$$P(X) = a_n X^n + a_{n-1} X^{n-1} + \dots + a_1 X + a_0 = 0$$

X General Solution in Radicals?

Degree n	General Solution by Radicals?	Method
1	✓ Yes	Linear solution

Degree <i>n</i>	General Solution by Radicals?	Method
2	✓ Yes	Quadratic formula
3	✓ Yes	Cardano's method
4	✓ Yes	Ferrari's method
5 or more	X No (in general)	Proven by Abel–Ruffini Theorem



This is where Galois theory comes in:

- For degree $n \ge 5$, most polynomials have a Galois group that is not solvable.
- That means the roots cannot be expressed using only:
 - Arithmetic operations (+, -, ×, ÷)
 - Root extractions (like $\sqrt{1}$, $\sqrt[3]{1}$, etc.)

What Can Be Done Instead?

Even though there's no general solution in radicals, we have **other powerful tools**:

✓ 1. Special Cases

Some higher-degree polynomials can be solved exactly if they have special symmetry or structure.

Example:

$$X^{2n} - 1 = 0$$
 (Roots of unity)

2. Numerical Solutions

All real and complex roots of any polynomial can be found **numerically**, to any desired precision.

Methods include:

- Newton-Raphson
- Durand–Kerner
- Jenkins–Traub

3. Symbolic but Non-Radical Solutions

For some cases, solutions can be expressed using:

- Elliptic functions
- Modular functions
- Power series expansions
- **Special functions** (e.g., Bring radical)

These are **beyond radicals** but still analytical.

Final Answer:

A general symbolic formula using only radicals does NOT exist for polynomials of degree 5 or higher. This is a fundamental result of modern algebra.

But:

- Many specific high-degree polynomials can be solved.
- Numerical methods can solve any polynomial.
- Galois theory can **tell us** whether a specific equation is solvable by radicals.

Would you like to:

• See how to check if a specific polynomial is solvable by radicals?

- Factor or solve a specific n-degree polynomial numerically or symbolically?
- Learn how Galois theory determines solvability?

Let me know!