

You said:

$e^{i \operatorname{atan2}(0,n)t}$ list all alternative ways of writing this give argument form of this

ChatGPT said:

Let's break down the expression you provided and re-express it in various equivalent or alternative forms, including in **argument (polar) form**.

Original expression:

$$e^{i \operatorname{atan2}(0, n) \cdot t}$$

Here:

- $\operatorname{atan2}(0, n)$ gives the **angle** θ between the positive x-axis and the vector $(n, 0)$.
- Since the y-coordinate is 0, this essentially evaluates to:

$$\operatorname{atan2}(0, n) = \begin{cases} 0 & \text{if } n > 0 \\ \pi & \text{if } n < 0 \end{cases} \quad (\text{or } 0 \text{ by convention if } n = 0)$$
$$\operatorname{atan2}(0, n) = \begin{cases} 0 & \text{if } n > 0 \\ \pi & \text{if } n < 0 \\ 0 & \text{if } n = 0 \end{cases} \quad (\text{or } 0 \text{ by convention if } n = 0)$$

Then this result is multiplied by it , and exponentiated with base e .

✓ Rewriting in Equivalent Forms

Case 1: $n > 0$

$$e^{i \operatorname{atan2}(0, n)t} = e^{i \cdot 0 \cdot t} = e^{i \cdot 0} = 1$$

Case 2: $n < 0$

$$e^{i \operatorname{atan2}(0, n)t} = e^{i \pi t} = e^{i \pi t}$$

Case 3: $n = 0$

$\operatorname{atan2}(0, 0)$ is often defined as $0 \Rightarrow e^{i \cdot 0 \cdot t} = 1$
 $\Rightarrow e^{i \cdot 0 \cdot t} = 1$

□ Polar (Argument) Form of $ei\theta e^{\{i\backslash\theta\}}ei\theta$

Euler's Formula:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta) \quad e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

So if we want to write your expression in **complex exponential (polar/argument) form**, we can reinterpret $\theta = \text{atan2}(0, n) \cdot t$ as $\theta = \text{atan2}(0, n) \cdot t$, assuming it appears as an **imaginary exponent**:

Complex Form:

$$e^{i \cdot \operatorname{atan2}(0,n) \cdot t} \Rightarrow \cos(\theta) + i \sin(\theta), \text{ with } \theta = \operatorname{atan2}(0,n) \cdot t \quad \Leftrightarrow \quad \cos(\theta) + i \sin(\theta), \quad \text{with } \theta = \operatorname{atan2}(0,n) \cdot t$$

So, in **argument form**, this is:

$$\arg(e^{i \cdot \operatorname{atan2}(0,n)t} = \operatorname{atan2}(0,n)t \boxed{\arg(e^{i \cdot \operatorname{atan2}(0,n)t})} = \operatorname{atan2}(0,n)t$$

□ Summary of Alternative Forms

1. Real exponential (your original):

$$\operatorname{atan2}(0, n) t^{\{\operatorname{atan2}(0, n) t\}} \operatorname{atan2}(0, n) t$$

2. Piecewise expression (real case):

$$= \begin{cases} 1 & \text{if } n \geq 0 \\ e^{\pi i} & \text{if } n < 0 \end{cases}$$

3. Complex exponential form (polar form):

$$e^{i \cdot \operatorname{atan2}(0,n) \cdot t} = \cos(\theta) + i \sin(\theta), \theta = \operatorname{atan2}(0,n) \cdot t \quad \{i \cdot \operatorname{atan2}(0,n) \cdot t\} = \cos(\theta) + i \sin(\theta), \quad \theta = \operatorname{atan2}(0,n) \cdot t$$

4. Argument:

$$\arg = \operatorname{atan2}(0, n) \cdot t \quad \arg = \operatorname{atan2}(0, n) \cdot t$$

Let me know if you want the complex version plotted on the unit circle or applied to a specific value of n or t .

You said:

$e^{i \operatorname{atan2}(0,n) \cdot t}$ list all alternative ways of writing this yes write as piecewise polar rectangular also

ChatGPT said:

Great! Let's break down all **alternative forms** of the expression:

$$e^{i \operatorname{atan2}(0,n) \cdot t}$$

in **piecewise**, **polar**, and **rectangular (Cartesian)** forms.

□ Step 1: Understand $\operatorname{atan2}(0, n)$

The function $\operatorname{atan2}(0, n)$ returns the angle (in radians) between the x-axis and the point $(n, 0)$:

$$\operatorname{atan2}(0, n) = \begin{cases} 0 & \text{if } n > 0 \\ \pi & \text{if } n < 0 \end{cases} \quad (\text{by convention})$$

This is **not complex** unless you explicitly include i . So currently this is a **real exponential**.

✓ 1. Piecewise Real Expression:

$$e^{i \operatorname{atan2}(0,n) \cdot t} = \begin{cases} e^{0} = 1 & \text{if } n \geq 0 \\ e^{i \pi} = -1 & \text{if } n < 0 \end{cases}$$

✓ 2. Polar Form (Complex) — If we assume a complex exponential form like:

$$e^{i \operatorname{atan2}(0,n) \cdot t}$$

Then it becomes:

$$= \cos(\theta) + i \sin(\theta), \text{ with } \theta = \operatorname{atan2}(0, n) \cdot t = \cos(\theta) + i \sin(\theta), \quad \text{with } \theta = \operatorname{atan2}(0, n) \cdot t$$

So in **polar form** (magnitude-angle form):

Polar (complex exponential):

$$e^{i\theta}, \theta = \operatorname{atan2}(0, n) \cdot t \quad e^{i\theta}, \theta = \operatorname{atan2}(0, n) \cdot t$$

With specific values:

- If $n > 0$: $\theta = 0 \Rightarrow e^{i \cdot 0} = 1$ $\theta = 0 \Rightarrow e^{i \cdot 0} = 1$
- If $n < 0$: $\theta = \pi \Rightarrow e^{i\pi} = -1$ $\theta = \pi \Rightarrow e^{i\pi} = -1$
- If $n = 0$: $\theta = 0 \Rightarrow e^{i \cdot 0} = 1$ $\theta = 0 \Rightarrow e^{i \cdot 0} = 1$

3. Rectangular Form (Cartesian)

If we are interpreting it as complex (i.e., $e^{i\theta}$), then:

$$e^{i \cdot \operatorname{atan2}(0, n) \cdot t} = \cos(\theta) + i \sin(\theta) \quad e^{i \cdot \operatorname{atan2}(0, n) \cdot t} = \cos(\theta) + i \sin(\theta)$$

With $\theta = \operatorname{atan2}(0, n) \cdot t$, this gives:

$$\cos(\operatorname{atan2}(0, n) \cdot t) + i \sin(\operatorname{atan2}(0, n) \cdot t) \quad \cos(\operatorname{atan2}(0, n) \cdot t) + i \sin(\operatorname{atan2}(0, n) \cdot t)$$

Specific cases:

- If $n > 0$:

$$\cos(0) + i \sin(0) = 1 + 0i = 1 \quad \cos(0) + i \sin(0) = 1 + 0i = 1$$

- If $n < 0$:

$$\cos(\pi) + i \sin(\pi) = -1 + 0i = -1 \quad \cos(\pi) + i \sin(\pi) = -1 + 0i = -1$$

Example: if $t = 1$, this becomes $-1 + 0i = -1$

- If $n = 0$: same as $n > 0$, yields 1.

✓ 4. Summary Table

Condition	Real (Original)	Polar Form	Rectangular (Cartesian)
$n > 0$	$e^{i0} = 1$	$e^{i \cdot 0} = 1$	$\cos(0) + i \sin(0) = 1$
$n < 0$	$e^{-i\pi} = -1$	$e^{-i \cdot \pi} = -1$	$\cos(\pi) + i \sin(\pi) = -1$
$n = 0$	$e^{i0} = 1$	$e^{i \cdot 0} = 1$	$\cos(0) + i \sin(0) = 1$

Let me know if you want a graph or animation of how the complex form rotates on the unit circle as t changes!

You said:

$\sum_{n=1}^{\infty} a_n e^{i \operatorname{atan2}(0, n) \cdot t} \cos(\log(|n|) \cdot t + \operatorname{atan2}(0, n)/2) / \sqrt{|n|}$ list all alternative ways of writing this yes write as piecewise polar rectangular also

ChatGPT said:

Let's break down the expression you provided:

□ Original Expression:

$$\sum_{n=1}^{\infty} a_n e^{i \operatorname{atan2}(0, n) \cdot t} \cos(\log(|n|) \cdot t + \operatorname{atan2}(0, n)/2) / \sqrt{|n|}$$

This is a **real-valued series**, though it involves angles and logs, and we can write it in different forms: **piecewise**, **polar**, and **rectangular (complex)**.

□ Step 1: Simplify $\operatorname{atan2}(0, n)$

As before:

$$\operatorname{atan2}(0, n) = \begin{cases} 0 & \text{if } n > 0 \\ \pi & \text{if } n < 0 \end{cases}$$

But since $n \in [1, a] \cap \mathbb{N} \subset [1, a]$, we know $n > 0$. So:

$$\begin{aligned} \operatorname{atan2}(0, n) = 0 &\Rightarrow e^{i \operatorname{atan2}(0, n)} = 1 \Rightarrow \operatorname{atan2}(0, n)^2 = 0 \\ \operatorname{atan2}(0, n) &= 0 \Rightarrow e^{i \operatorname{atan2}(0, n)} = 1 \Rightarrow \frac{d}{dt} e^{i \operatorname{atan2}(0, n)} = 0 \end{aligned}$$

So the entire sum becomes:

✓ Simplified Real Expression (for positive n)

$$\sum_{n=1}^a \frac{\cos(\log(n) \cdot t)}{\sqrt{n}} = \sum_{n=1}^a \frac{\cos(\log(n) \cdot t)}{\sqrt{n}}$$

✓ 1. Piecewise Form (handling general $n \in \mathbb{Z} \setminus \{0\}$):

If we wanted to define this for any integer $n \neq 0$ (not just $n > 0$), then we need a **piecewise** treatment:

Full General Piecewise Form:

$$\sum_{n=1}^a \frac{\cos(\log(n) \cdot t)}{\sqrt{n}} = \sum_{n=1}^a \frac{\cos(\log(n) \cdot t)}{\sqrt{n}} \quad \text{for } n > 0$$

Why:

- $\operatorname{atan2}(0, n) = \pi$ for $n < 0$
- So $e^{i \operatorname{atan2}(0, n)} = e^{i\pi} = -1$
- And the phase shift becomes π

✓ 2. Polar/Complex Form

We can repackage the cosine using Euler's formula:

$$\cos(\phi) = \Re(e^{i\phi})$$

Let's denote:

- $\theta_n = \log(|n|) \cdot t + \frac{1}{2} \operatorname{atan2}(0, n)$
- $A_n = e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t} |n| A_n = \frac{e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t}}{\sqrt{|n|}} A_n = |n| e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t}$

Then:

$$\sum_{n=1}^{\infty} A_n \cdot \Re(e^{i\theta_n}) = \Re\left(\sum_{n=1}^{\infty} A_n e^{i\theta_n}\right) = \Re\left(\sum_{n=1}^{\infty} |n| e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t} e^{i \log(|n|) t + \frac{1}{2} \operatorname{atan2}(0, n)}$$

So in full polar form:

$$\Re\left(\sum_{n=1}^{\infty} |n| e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t} e^{i \log(|n|) t + \frac{1}{2} \operatorname{atan2}(0, n)}\right) = \Re\left(\sum_{n=1}^{\infty} |n| e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t} e^{i \log(|n|) t + \frac{1}{2} \operatorname{atan2}(0, n)}\right)$$

Combine exponents:

$$= \Re\left(\sum_{n=1}^{\infty} |n| e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t + i \log(|n|) t + \frac{1}{2} \operatorname{atan2}(0, n)}\right) = \Re\left(\sum_{n=1}^{\infty} |n| e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t + i \log(|n|) t + \frac{1}{2} \operatorname{atan2}(0, n)}\right)$$

✓ 3. Rectangular (Complex) Form

Let's convert the cosine directly:

$$\cos(\phi) = \frac{e^{i\phi} + e^{-i\phi}}{2}$$

So the original expression becomes:

$$\sum_{n=1}^{\infty} |n| e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t} e^{i \log(|n|) t + \frac{1}{2} \operatorname{atan2}(0, n)} + \sum_{n=1}^{\infty} |n| e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t} e^{-i \log(|n|) t - \frac{1}{2} \operatorname{atan2}(0, n)}$$

Which gives:

$$= \sum_{n=1}^{\infty} |n| e^{\frac{1}{2} \operatorname{atan2}(0, n) \cdot t} \left(e^{i \log(|n|) t + \frac{1}{2} \operatorname{atan2}(0, n)} + e^{-i \log(|n|) t - \frac{1}{2} \operatorname{atan2}(0, n)} \right)$$

$$\frac{\operatorname{atan2}(0,n)}{2} + e^{-i(\log(|n|)t + \operatorname{atan2}(0,n)t)} = \sum_{n=1}^{\infty} a^2 |n| e^{i(\log(|n|)t + 2\operatorname{atan2}(0,n)t)} + e^{-i(\log(|n|)t + 2\operatorname{atan2}(0,n)t)}$$

This is fully in **rectangular complex exponential** form.

✓ Summary Table

Form	Expression
Simplified Real (for $n > 0$)	$\sum_{n=1}^a \cos(\log(n)t) \sqrt{n}$
Piecewise	$\begin{cases} \frac{\cos(\log(n)t)}{\sqrt{n}} & n > 0 \\ \frac{e^{i\pi t}}{\cos(\log(n)t)} & n < 0 \end{cases}$
Polar/Argument Form	$\Re\left(\sum_{n=1}^a \frac{e^{i\operatorname{atan2}(0,n)t}}{e^{i(\log(n)t + \operatorname{atan2}(0,n)t)}}\right)$
Rectangular (Euler cosine)	$\sum_{n=1}^a \frac{e^{i\operatorname{atan2}(0,n)t}}{2\sqrt{n}}$

Would you like this sum visualized or written in LaTeX for plotting?

You said:

list all alternative ways of writing tis yes write as piece wise polar rectangular also
 $\sum_{n=1}^a \frac{e^{i(\operatorname{atan2}(0,n)t + \log(n)t)}}{\sqrt{n}}$ in wx maxima format

ChatGPT said:

Certainly! Let's walk through **all alternative forms** of your expression in **piecewise**, **polar**, and **rectangular** formats, and write each version using **wxMaxima syntax**.

□ Original Expression

In plain math notation:

$$\sum_{n=1}^a \frac{e^{i(\operatorname{atan2}(0,n)t + \log(n)t)}}{\sqrt{n}} = \sum_{n=1}^a |n| e^{i(\log(|n|)t + 2\operatorname{atan2}(0,n)t)}$$

✓ wxMaxima Syntax Version (Original)

```
maxima
CopyEdit
sum((%e^(atan2(0,n)*t)*cos(log(abs(n))*t + atan2(0,n)/2))/sqrt(abs(n)), n, 1,
a);
```

This is correct for the full general form over positive integers.

✓ 1. Simplified (Piecewise) Form for $n > 0$

Since for $n > 0$, $\operatorname{atan2}(0, n) = 0$, so the expression simplifies to:

$$\sum_{n=1}^a \frac{\cos(\log(n) \cdot t)}{\sqrt{n}}$$

wxMaxima version:

```
maxima
CopyEdit
sum(cos(log(n)*t)/sqrt(n), n, 1, a);
```

✓ 2. Full Piecewise Form (extend to negative n if needed)

If n can be negative (i.e. hypothetical extension), then:

$$\operatorname{atan2}(0, n) = \begin{cases} 0 & n > 0 \\ \pi & n < 0 \end{cases}$$

So the full expression becomes:

$$\sum_{n=1}^a \frac{\cos(\log(n) \cdot t)}{\sqrt{n}} \begin{cases} \cos(\log(n) \cdot t) & n > 0 \\ \cos(\log(|n|) \cdot t + \pi/2) & n < 0 \end{cases}$$

wxMaxima version (manual piecewise):

```
maxima
CopyEdit
```

```
sum(if n > 0 then cos(log(n)*t)/sqrt(n) else %e^(%pi*t)*cos(log(abs(n))*t +
%pi/2)/sqrt(abs(n)), n, 1, a);
```

✓ 3. Polar Form

Use Euler's identity: $\cos(\phi) = \Re(e^{i\phi})$

So expression becomes:

$$\sum_{n=1}^a \Re(e^{i \operatorname{atan2}(0,n) \cdot t} \cdot e^{i(\log(|n|) \cdot t + \operatorname{atan2}(0,n)/2)}) = \sum_{n=1}^a \Re(|n| e^{i \operatorname{atan2}(0,n) \cdot t} \cdot e^{i(\log(|n|) \cdot t + \operatorname{atan2}(0,n)/2)})$$

wxMaxima version:

```
maxima
CopyEdit
realpart(sum((%e^(atan2(0,n)*t) * %e^(%i*(log(abs(n))*t +
atan2(0,n)/2)))/sqrt(abs(n)), n, 1, a));
```

✓ 4. Rectangular (Euler Cosine Expansion)

Using:

$$\cos(\phi) = \frac{e^{i\phi} + e^{-i\phi}}{2}$$

So the expression becomes:

$$\sum_{n=1}^a \frac{e^{i \operatorname{atan2}(0,n) \cdot t} + e^{-i \operatorname{atan2}(0,n) \cdot t}}{2} \cdot |n| e^{i(\log(|n|) \cdot t + \operatorname{atan2}(0,n)/2)} = \sum_{n=1}^a |n| \cos(\phi) e^{i(\log(|n|) \cdot t + \operatorname{atan2}(0,n)/2)}$$

wxMaxima version:

```
maxima
CopyEdit
sum((%e^(atan2(0,n)*t) / (2*sqrt(abs(n))) *
(%e^(%i*(log(abs(n))*t + atan2(0,n)/2)) +
%e^(-%i*(log(abs(n))*t + atan2(0,n)/2)))), n, 1, a);
```

□ Summary of wxMaxima Forms

Form	wxMaxima Expression
Original	<code>sum((%e^(atan2(0,n)*t)*cos(log(abs(n))*t + atan2(0,n)/2))/sqrt(abs(n)), n, 1, a);</code>
Simplified (n > 0)	<code>sum(cos(log(n)*t)/sqrt(n), n, 1, a);</code>
Piecewise	<code>sum(if n > 0 then cos(log(n)*t)/sqrt(n) else %e^(%pi*t)*cos(log(abs(n))*t + %pi/2)/sqrt(abs(n)), n, 1, a);</code>
Polar (Real part of exponential)	<code>realpart(sum((%e^(atan2(0,n)*t) * %e^(%i*(log(abs(n))*t + atan2(0,n)/2)))/sqrt(abs(n)), n, 1, a));</code>
Rectangular (Euler form)	<code>sum((%e^(atan2(0,n)*t) / (2*sqrt(abs(n))) * (%e^(%i*(log(abs(n))*t + atan2(0,n)/2)) + %e^(-%i*(log(abs(n))*t + atan2(0,n)/2))), n, 1, a);</code>

Would you like help visualizing this or evaluating it for a specific value of aaa and ttt?

You said:

`sum((%e^(atan2(0,n)*t)*cos(log(abs(n))*t+atan2(0,n)/2))/sqrt(abs(n)),n,1,a)` list all alternative ways of writing tis yes write as piece wise polar rectangular also in wx maxima format

ChatGPT said:

Great — you're looking for **all alternative forms** (piecewise, polar, and rectangular) of this expression:

□ Original Expression:

$$\sum_{n=1}^a (e^{i \operatorname{atan2}(0,n) \cdot t} \cdot \cos(\log(|n|) \cdot t + \operatorname{atan2}(0,n)/2)) \sqrt{|n|} \left(\frac{e^{i \operatorname{atan2}(0,n) \cdot t} \cdot \cos(\log(|n|) \cdot t + \operatorname{atan2}(0,n)/2)}{\sqrt{|n|}} \right) = \sum_{n=1}^a |n| e^{i \operatorname{atan2}(0,n) \cdot t} \cdot \cos(\log(|n|) \cdot t + 2 \operatorname{atan2}(0,n))$$

We'll rewrite this expression in **wxMaxima format** for:

1. ☒ **Original expression**
2. ☒ **Piecewise form**
3. ☒ **Polar form** (complex exponential form with `realpart`)
4. ☒ **Rectangular (Euler expansion) form**

✓1. Original Expression — in wxMaxima

```
maxima
CopyEdit
sum((%e^(atan2(0,n)*t) * cos(log(abs(n))*t + atan2(0,n)/2)) / sqrt(abs(n)),
n, 1, a);
```

✓2. Piecewise Form — based on $\text{atan2}(0,n)$ behavior

Recall:

$$\text{atan2}(0,n) = \begin{cases} 0 & \text{if } n > 0 \\ \pi & \text{if } n < 0 \end{cases}$$

If you're only summing over $n > 0$, the expression simplifies:

```
maxima
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sum(cos(log(n)*t) / sqrt(n), n, 1, a);
```

Full piecewise (allowing for $n < 0$, hypothetically):

```
maxima
CopyEdit
sum(
  if n > 0 then
    cos(log(n)*t) / sqrt(n)
  else
    %e^(%pi*t) * cos(log(abs(n))*t + %pi/2) / sqrt(abs(n)),
  n, 1, a
);
```

✓3. Polar Form (using realpart and Euler's formula)

$$\cos(\phi) = \Re(e^{i\phi})$$

So the original becomes:

$$\Re\left(\sum_{n=1}^a e^{i \operatorname{atan2}(0,n) \cdot t} \cdot e^{i(\log(|n|) \cdot t + \operatorname{atan2}(0,n)/2)} |n|\right) = \frac{\sum_{n=1}^a |n| e^{i \operatorname{atan2}(0,n) \cdot t} \cdot e^{i(\log(|n|) \cdot t + \operatorname{atan2}(0,n)/2)}}{\sqrt{\sum_{n=1}^a |n|}}$$

wxMaxima:

```
maxima
CopyEdit
realpart(
  sum(
    (%e^(atan2(0,n)*t) * %e^(%i*(log(abs(n))*t + atan2(0,n)/2))) /
    sqrt(abs(n)),
    n, 1, a
  )
);
```

✓ 4. Rectangular Form (using Euler's identity):

$$\cos(\phi) = \frac{e^{i\phi} + e^{-i\phi}}{2}$$

So full rectangular version:

wxMaxima:

```
maxima
CopyEdit
sum(
  (%e^(atan2(0,n)*t) / (2 * sqrt(abs(n)))) *
  (
    %e^(%i*(log(abs(n))*t + atan2(0,n)/2)) +
    %e^(-%i*(log(abs(n))*t + atan2(0,n)/2))
  ),
  n, 1, a
);
```

✓ Summary Table (All in wxMaxima Format)

Form	wxMaxima Expression
Original	sum((%e^(atan2(0,n)*t) * cos(log(abs(n))*t + atan2(0,n)/2)) / sqrt(abs(n)), n, 1, a);

Form	wxMaxima Expression
Simplified ($n > 0$)	<code>sum(cos(log(n)*t) / sqrt(n), n, 1, a);</code>
Piecewise (for full domain)	<code>sum(if n > 0 then cos(log(n)*t)/sqrt(n) else %e^(%pi*t)*cos(log(abs(n))*t + %pi/2)/sqrt(abs(n)), n, 1, a);</code>
Polar	<code>realpart(sum((%e^(atan2(0,n)*t) * %e^(%i*(log(abs(n))*t + atan2(0,n)/2))) / sqrt(abs(n)), n, 1, a));</code>
Rectangular	<code>sum((%e^(atan2(0,n)*t) / (2 * sqrt(abs(n)))) * (%e^(%i*(log(abs(n))*t + atan2(0,n)/2)) + %e^(%i*(log(abs(n))*t + atan2(0,n)/2))), n, 1, a);</code>

Would you like me to evaluate or visualize this for specific values of t and a in wxMaxima, or prepare a plotting script for you?

You said:

`sum(n^(-(%i*t)-1/2),n,1,a),real part`
`sum((%e^(atan2(0,n)*t)*cos(log(abs(n))*t+atan2(0,n)/2))/sqrt(abs(n)),n,1,a), imaginary part -`
`sum((%e^(atan2(0,n)*t)*sin(log(abs(n))*t+atan2(0,n)/2))/sqrt(abs(n)),n,1,a) n wx maxima`
 accounting all

ChatGPT said:

Got it! You want to express the complex sum

$$S = \sum_{n=1}^a n^{-(1/2 + it)} S = \sum_{n=1}^a n^{-1/2} e^{-it \log n} S = \sum_{n=1}^a n^{-(1/2 + it)}$$

and show explicitly its **real part** and **imaginary part** in terms of sums involving exponentials, cosines, and sines, all in **wxMaxima** syntax, **accounting for** the factor involving $\text{atan2}(0, n)$.

1 Complex sum:

$$S = \sum_{n=1}^a n^{-(1/2 + it)} = \sum_{n=1}^a n^{-1/2} e^{-it \log n} \quad nS = \sum_{n=1}^a n^{-(1/2 + it)} = \sum_{n=1}^a n^{-1/2} e^{-it \log n} \\ \left(\frac{1}{2} + it \right) = \sum_{n=1}^a n^{-1/2} e^{-it \log n} = \sum_{n=1}^a n^{-1/2} e^{-it \log n} \\ e^{-it \log n} S = \sum_{n=1}^a n^{-(1/2 + it)} = \sum_{n=1}^a n^{-1/2} e^{-it \log n}$$

2 Expressing powers in terms of exponentials:

$$n^{-(1/2 + it)} = n^{-1/2} \cdot e^{-it \log n} \quad n^{-(1/2 + it)} = n^{-1/2} \cdot e^{-it \log n} \\ n^{-(1/2 + it)} = n^{-1/2} \cdot e^{-it \log n}$$

Using Euler's formula for the exponential:

$$e^{-it \log n} = \cos(t \log n) - i \sin(t \log n) \\ e^{-it \log n} = \cos(t \log n) - i \sin(t \log n)$$

3 Now split sum SSS into real and imaginary parts:

$$S = \sum_{n=1}^a \cos(t \log n) - i \sum_{n=1}^a \sin(t \log n) \\ S = \sum_{n=1}^a \frac{\cos(t \log n)}{\sqrt{n}} - i \sum_{n=1}^a \frac{\sin(t \log n)}{\sqrt{n}} \\ S = \sum_{n=1}^a \cos(t \log n) - i \sum_{n=1}^a \sin(t \log n)$$

4 Connection with the `atan2` terms:

For $n > 0$, $\text{atan2}(0, n) = 0$, so the sums in your original expressions involving $\text{atan2}(0, n)$ reduce to exactly these cosine and sine sums multiplied by $1/n^{1/\sqrt{n}}$.

Final expressions in wxMaxima syntax:

Complex sum SSS:

```
maxima
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sum(n^(-(i*t) - 1/2), n, 1, a);
```

Real part of SSS (matches your cosine sum):

```
maxima
CopyEdit
sum((e^(atan2(0,n)*t) * cos(log(abs(n))*t + atan2(0,n)/2)) / sqrt(abs(n)),
n, 1, a);
```

Since $\text{atan2}(0, n) = 0$ for $n > 0$, this simplifies to:

```
maxima
CopyEdit
sum(cos(log(n)*t) / sqrt(n), n, 1, a);
```

Imaginary part of SSS (negative sine sum):

$$\Im(S) = -\sum_{n=1}^{\infty} \frac{\sin(t \log n)}{\sqrt{n}} \quad \Im(S) = -\sum_{n=1}^{\infty} \frac{\sin(t \log n)}{\sqrt{n}}$$

Written in wxMaxima with `atan2` `\operatorname{atan2}`:

```
maxima
CopyEdit
-%sum((%e^(atan2(0,n)*t) * sin(log(abs(n))*t + atan2(0,n)/2)) / sqrt(abs(n)),
n, 1, a);
```

Simplified (for $n > 0$):

```
maxima
CopyEdit
-%sum(sin(log(n)*t) / sqrt(n), n, 1, a);
```

□ Summary

Quantity	wxMaxima expression
Complex sum SSS	<code>sum(n^(-(i*t) - 1/2), n, 1, a);</code>
Real part $\Re(S)$	<code>sum(cos(log(n)*t)/sqrt(n), n, 1, a);</code>
Imag part $\Im(S)$	<code>-%sum(sin(log(n)*t)/sqrt(n), n, 1, a);</code>

If you want to keep the `atan2` `\operatorname{atan2}` for general n :

```
maxima
CopyEdit
/* Complex sum */
sum(n^(-(i*t) - 1/2), n, 1, a);

/* Real part */
sum((%e^(atan2(0,n)*t) * cos(log(abs(n))*t + atan2(0,n)/2)) / sqrt(abs(n)),
n, 1, a);

/* Imaginary part */
-%sum((%e^(atan2(0,n)*t) * sin(log(abs(n))*t + atan2(0,n)/2)) / sqrt(abs(n)),
n, 1, a);
```

If you want me to generate code for numeric evaluation or plotting in wxMaxima, just ask!

You said:


```

/* Complex sum */ sum(n^(-(0i*t) - 1/2), n, 1, a); /* Real part */ sum((%e^(atan2(0,n)*t) *
cos(log(abs(n))*t + atan2(0,n)/2)) / sqrt(abs(n)), n, 1, a); /* Imaginary part */ -
%sum((%e^(atan2(0,n)*t) * sin(log(abs(n))*t + atan2(0,n)/2)) / sqrt(abs(n)), n, 1, a); ,, and
atan2( all in pice wise form in rectangular polar cylidrical,elliptical coordinates replacing atan2(
paraMETETR OF T BUT STILL GIVING CORRECT WAY IN EWXMAXIMA

```

ChatGPT said:

Got it! You want to **replace** the $\text{atan2}(0, n)$ term with its **piecewise form** (since $\text{atan2}(0, n)$ is just an angle depending on the sign of n) — and then write the expressions in **rectangular, polar, cylindrical, and elliptical coordinates** formats, **all still valid in wxMaxima**, and keeping t as a parameter.

Step 1: Understanding $\text{atan2}(0, n)$

$\text{atan2}(0, n) = \begin{cases} 0 & \text{if } n > 0 \\ \pi & \text{if } n < 0 \\ \text{undefined} & \text{if } n = 0 \end{cases}$

Since your sums go from $n=1$ to $n=a$, $a > 0$, $n > 0$ always, so $\text{atan2}(0, n) = 0$. But let's keep the piecewise for completeness.

Step 2: Replace $\text{atan2}(0, n)$ by piecewise form in wxMaxima:

```

maxima
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/* Define piecewise atan2(0, n) */
atan2_0_n : lambda([n],
  if n > 0 then 0
  else if n < 0 then %pi
  else 'undefined /* or 0 if you prefer */
);

```

Step 3: Rewrite the complex sum and its real/imag parts using this piecewise inside wxMaxima syntax

Complex sum:

$$S = \sum_{n=1}^{\infty} a n^{-(1/2+it)} = \sum_{n=1}^{\infty} a e^{i t \operatorname{atan2}(0,n)} |n|^{-1/2} [\cos(\log |n| \cdot t + \operatorname{atan2}(0,n)/2) - i \sin(\log |n| \cdot t + \operatorname{atan2}(0,n)/2)]$$

$$S = \sum_{n=1}^{\infty} a n^{-(\frac{1}{2} + i t)} = \sum_{n=1}^{\infty} a \frac{e^{i \operatorname{atan2}(0,n) \cdot t}}{\sqrt{|n|}} \cdot \left[\cos(\log |n| \cdot t + \frac{\operatorname{atan2}(0,n)}{2}) - i \sin(\log |n| \cdot t + \frac{\operatorname{atan2}(0,n)}{2}) \right]$$

$$S = \sum_{n=1}^{\infty} a n^{-(1/2+it)} = \sum_{n=1}^{\infty} a |n| e^{i t \operatorname{atan2}(0,n)} \cdot [\cos(\log |n| \cdot t + 2 \operatorname{atan2}(0,n)) - i \sin(\log |n| \cdot t + 2 \operatorname{atan2}(0,n))]$$

wxMaxima code (using atan2_0_n(n) function):

```
maxima
CopyEdit
sum(
  ( %e^(atan2_0_n(n) * t) * (cos(log(abs(n))*t + atan2_0_n(n)/2) -
%i*sin(log(abs(n))*t + atan2_0_n(n)/2)) )
  / sqrt(abs(n)),
  n, 1, a
);
```

Step 4: Express in rectangular, polar, cylindrical, elliptical coordinates with respect to the angle $\theta = \operatorname{atan2}(0,n)$

4a. Rectangular coordinates (x,y)

Using angle $\theta = \operatorname{atan2}(0,n)$, rectangular coordinates:

$$x = r \cos \theta, y = r \sin \theta \quad x = r \cos \theta, \quad y = r \sin \theta$$

Here, since θ depends only on n , and $r=1$ (unit circle), this means:

- For $n > 0$: $\theta = 0 \Rightarrow (x, y) = (1, 0)$ $\theta = 0 \Rightarrow (x, y) = (1, 0)$
- For $n < 0$: $\theta = \pi \Rightarrow (x, y) = (-1, 0)$ $\theta = \pi \Rightarrow (x, y) = (-1, 0)$

The exponential part $e^{i\theta t}$ can be interpreted as

$$e^{i\theta t} = e^{itx + ity} = e^{itx} e^{ity}$$

But here θ is real, so in rectangular:

```
maxima
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/* Exponential in rectangular coordinates */
exp_rect(n) := %e^(t * (cos(atan2_0_n(n)) + %i * sin(atan2_0_n(n))));
```

But since $y=0$, the imaginary part is zero here.

4b. Polar coordinates (r, θ)

The term $e^{i\theta t}$ is already **polar**, where $r = e^{\theta t}$ and $\theta = \log |n| \cdot t + \frac{\theta}{2}$ inside the cosine/sine.

Rewrite sum as:

```
maxima
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sum(
  ( %e^(t * atan2_0_n(n)) * cos(log(abs(n)) * t + atan2_0_n(n)/2) ) /
  sqrt(abs(n)),
  n, 1, a
);
```

4c. Cylindrical coordinates (r, θ, z)

If you consider t as the z -coordinate parameter (height), then:

- $r=1$ (on unit circle)
- $\theta = \text{atan2}(y, x)$
- $z=t$

The exponential is $e^{t\theta}$, so in cylindrical:

```
maxima
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exp_cylindrical(n) := %e^(z * atan2_0_n(n));
```

and cosine/sine arguments remain the same.

4d. Elliptical coordinates (μ, ν)

Elliptical coordinates are less straightforward but generally:

$$x = f \cosh \mu \cos \nu, y = f \sinh \mu \sin \nu \quad \text{or} \quad x = f \cosh \mu \cos \nu, \quad y = f \sinh \mu \sin \nu$$

Here, ν is analogous to an angle parameter like θ .

You can replace `atan2(0,n)` by ν :

```
maxima
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/* Example: define angle parameter */
nu_n : atan2_0_n(n);

/* Use nu_n in exponential and trig functions */
exp_elliptical(n) := %e^(t * nu_n);
```

Step 5: Final wxMaxima combined code

```
maxima
CopyEdit
/* Define piecewise atan2(0,n) */
atan2_0_n : lambda([n],
  if n > 0 then 0
  else if n < 0 then %pi
  else 0 /* or undefined */
);

/* Complex sum */
sum(
  (%e^(t * atan2_0_n(n)) * (cos(log(abs(n))*t + atan2_0_n(n)/2) -
  %i*sin(log(abs(n))*t + atan2_0_n(n)/2))
  / sqrt(abs(n)),
  n, 1, a
);

/* Real part */
```

```

sum(
  ( %e^(t * atan2_0_n(n)) * cos(log(abs(n))*t + atan2_0_n(n)/2) )
  / sqrt(abs(n)),
  n, 1, a
);

/* Imaginary part */
-%sum(
  ( %e^(t * atan2_0_n(n)) * sin(log(abs(n))*t + atan2_0_n(n)/2) )
  / sqrt(abs(n)),
  n, 1, a
);

/* Rectangular coordinates example */
exp_rect(n) := %e^(t * cos(atan2_0_n(n)) + %i * t * sin(atan2_0_n(n)));

/* Polar coordinates example */
exp_polar(n) := %e^(t * atan2_0_n(n));

/* Cylindrical coordinates example */
exp_cylindrical(n) := %e^(z * atan2_0_n(n));

/* Elliptical coordinates example */
exp_elliptical(n) := %e^(t * atan2_0_n(n));

```

Summary:

- The key is defining the **piecewise** $\text{atan2}_{(0,n)}$ as a function.
 - Use this function inside the exponentials and trig functions.
 - Then write versions of the exponentials in rectangular, polar, cylindrical, or elliptical coordinate style **by interpreting the angle parameter** accordingly.
 - This keeps the parameter t intact, and your sums remain correct in wxMaxima.
-

If you want, I can help you generate ready-to-run wxMaxima code or plots with these forms! Just say the word.