CSCI-PHYS 3090 - Quantum Computing - Spring 2022
Homework #10

Due Wednesday, April 13, at the start of class

Homework is graded for clarity of explanation as much as for mere "correctness" of the final answer. You will earn partial credit much easier if your writing is legible and organized. Please scan and submit online, making sure that your scan can be easily read!

Problem 1: Discrete Fourier Transform (40 points)

(a) The N-th roots of unity are defined as solutions to the equation: $(\omega_N)^N = 1$. There are exactly N distinct N-th roots of unity.

Let ω be a primitive root of unity, for example $\omega_N = e^{2\pi i/N}$. Show the following:

$$\sum_{k=0}^{N-1} \omega_N^{mk} = \begin{cases} N, & \text{if } N \text{ divides } m \\ 0, & \text{otherwise} \end{cases}$$

(b) For integer $N \geq 2$ let

$$f \equiv \begin{pmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-1) \end{pmatrix}.$$

be a vector function $f : [N] \to \mathcal{C}$. The Discrete Fourier Transform of f is another complex vector function $F : [N] \to \mathcal{C}$ given by

$$F \equiv \begin{pmatrix} F(0) \\ F(1) \\ \vdots \\ F(N-1) \end{pmatrix}.$$

of the same dimension N, with

$$F(k) = \frac{1}{\sqrt{N}} \sum_n \omega_N^{kn} f(n)$$

Thus the Fourier transform is a linear operator represented by the $N \times N$ matrix $A_N = (a_{kn})$ with $a_{kn} = \frac{1}{\sqrt{N}} \omega_N^{kn}$.

(i) Write explicitly the Fourier matrix of order 4 using $\omega_4 = e^{2\pi i/4} = i$.

(ii) Find the Fourier Transform of the vector

$$\begin{pmatrix} 2 \\ 1 \\ -2 \\ 1 \end{pmatrix}$$

using the Fourier matrix from (i).

(iii) Using Part (a), verify that the inverse matrix is

$$A_N^{-1} = \frac{1}{\sqrt{N}}(\omega_N^{-kn})$$

In other words, a vector f can be recovered from its Fourier Transform F by the Fourier Inversion Formula:

$$f(n) = \frac{1}{\sqrt{N}} \sum_k \omega_N^{-nk} F(k)$$

Problem 2: QFT on three qubits (20 points)

In this problem, you will work with the 3-qubit QFT A_8 and its inverse $A_8^\dagger$.

(a) What is ω_8? Simplify as much as possible.

For the next two problems, you may use any resources you would like for the calculations.

(b) Calculate $A_8 |2\rangle$.

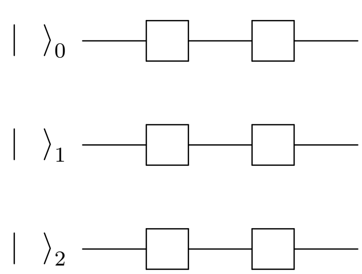
(c) Given

$$\left|\tilde{7}\right\rangle = \begin{bmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{4} - \frac{1}{4}i & -\frac{1}{2\sqrt{2}}i & -\frac{1}{4} - \frac{1}{4}i & -\frac{1}{2\sqrt{2}} & -\frac{1}{4} + \frac{1}{4}i & \frac{1}{2\sqrt{2}}i & \frac{1}{4} + \frac{1}{4}i \end{bmatrix}^T$$

Calculate $A_8^\dagger \left|\tilde{7}\right\rangle$

(d) Fill in the circuit below such that the resulting state will be $\left|\tilde{7}\right\rangle$

[Hint: look at the class exercise for a very similar problem]



Problem 3: QFT Qiskit circuit example (40 points)

Fill out problem 3 on the attached Jupyter notebook.