

Problems 5 (c) and 5 (d)

```
In[ ]:= v1 = {Cos[ $\frac{\theta_1}{2}$ ], e(I* $\phi_1$ ) Sin[ $\frac{\theta_1}{2}$ ]};
```

```
v2 = {Cos[ $\frac{\theta_2}{2}$ ], e(I* $\phi_2$ ) Sin[ $\frac{\theta_2}{2}$ ]};
```

```
 $\psi$  = ArrayFlatten[TensorProduct[v1, v2], 1];
```

```
MatrixForm[ $\psi$ ]
```

Out[]//MatrixForm=

$$\begin{pmatrix} \cos\left[\frac{\theta_1}{2}\right] \cos\left[\frac{\theta_2}{2}\right] & e^{i\phi_2} \cos\left[\frac{\theta_1}{2}\right] \sin\left[\frac{\theta_2}{2}\right] \\ e^{i\phi_1} \cos\left[\frac{\theta_2}{2}\right] \sin\left[\frac{\theta_1}{2}\right] & e^{i\phi_1+i\phi_2} \sin\left[\frac{\theta_1}{2}\right] \sin\left[\frac{\theta_2}{2}\right] \end{pmatrix}$$

In[]:=

```
iswap = {{1, 0, 0, 0}, {0, 0, I, 0}, {0, I, 0, 0}, {0, 0, 0, 1}};
```

```
MatrixForm[iswap]
```

Out[]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[]:=

```
swap = {{1, 0, 0, 0}, {0, 0, 1, 0}, {0, 1, 0, 0}, {0, 0, 0, 1}};
```

```
MatrixForm[swap]
```

Out[]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[]:=

```
res1 = iswap.ψ; res2 = swap.ψ;
MatrixForm[res1]
```

Out[]//MatrixForm=

$$\begin{pmatrix} \cos\left[\frac{\theta_1}{2}\right] \cos\left[\frac{\theta_2}{2}\right] \\ i e^{i\phi_1} \cos\left[\frac{\theta_2}{2}\right] \sin\left[\frac{\theta_1}{2}\right] \\ i e^{i\phi_2} \cos\left[\frac{\theta_1}{2}\right] \sin\left[\frac{\theta_2}{2}\right] \\ e^{i\phi_1+i\phi_2} \sin\left[\frac{\theta_1}{2}\right] \sin\left[\frac{\theta_2}{2}\right] \end{pmatrix}$$

In[]:= MatrixForm[res2]

Out[]//MatrixForm=

$$\begin{pmatrix} \cos\left[\frac{\theta_1}{2}\right] \cos\left[\frac{\theta_2}{2}\right] \\ e^{i\phi_1} \cos\left[\frac{\theta_2}{2}\right] \sin\left[\frac{\theta_1}{2}\right] \\ e^{i\phi_2} \cos\left[\frac{\theta_1}{2}\right] \sin\left[\frac{\theta_2}{2}\right] \\ e^{i\phi_1+i\phi_2} \sin\left[\frac{\theta_1}{2}\right] \sin\left[\frac{\theta_2}{2}\right] \end{pmatrix}$$

In[]:= iswapconc =

```
FullSimplify[(2 * Abs[Part[res1, 1] * Part[res1, 4] - Part[res1, 2] * Part[res1, 3]])]
```

Out[]:= $e^{-i\phi_1-i\phi_2} \text{Abs}[\sin[\theta_1] \sin[\theta_2]]$

In[]:=

```
FullSimplify[iswapconc, Assumptions -> {φ1, φ2, θ1, θ2} ∈ Reals]
```

Out[]:= $\text{Abs}[\sin[\theta_1] \sin[\theta_2]]$

In[]:= swapconc =

```
FullSimplify[(2 * Abs[Part[res2, 1] * Part[res2, 4] - Part[res2, 2] * Part[res2, 3]])]
```

Out[]:= 0

In[]:=

$$\text{iswapep} = \left(\text{Integrate}\left[\frac{(\sin[\theta_1] * \sin[\theta_2])^3}{2}, \right. \right. \\ \left. \left. \{\theta_1, 0, \pi\}, \{\theta_2, 0, \pi\}, \{\phi_1, 0, 2 * \pi\}, \{\phi_2, 0, 2 * \pi\} \right] \right) * \frac{1}{(4 * \pi)^2}$$

Out[]:= $\frac{2}{9}$