ECEN 4005 - Homework 5

Problem 1(a)

In[1]:=
```
V1 = Cos[ω01 * t] * ℏ * Ω01;
FullSimplify[InverseFourierTransform[V1, t, ω, FourierParameters → {-1, 1}]]
```

Out[2]= $\pi\,\Omega01\,\hbar\,\mathrm{DiracDelta}[\omega - \omega01] + \pi\,\Omega01\,\hbar\,\mathrm{DiracDelta}[\omega + \omega01]$

In[3]:=
$$V2 = \mathrm{Cos}[\omega01 * t] * \hbar * \Omega01 * e^{\left(\frac{-t^2}{2*\mathrm{tgate}^2}\right)};$$
```
FullSimplify[InverseFourierTransform[V2, t, ω, FourierParameters → {-1, 1}]]
```

Out[4]= $$e^{-\frac{1}{2}\,\mathrm{tgate}^2\,(\omega+\omega01)^2}\left(1 + e^{2\,\mathrm{tgate}^2\,\omega\,\omega01}\right)\sqrt{\frac{\pi}{2}}\,\sqrt{\mathrm{tgate}^2}\,\Omega01\,\hbar$$

In[5]:= $$\mathrm{FullSimplify}\left[\mathrm{Expand}\left[e^{-\frac{1}{2}\,\mathrm{tgate}^2\,(\omega+\omega01)^2}\left(1 + e^{2\,\mathrm{tgate}^2\,\omega\,\omega01}\right)\right]\right]$$

Out[5]= $$e^{-\frac{1}{2}\,\mathrm{tgate}^2\,(\omega+\omega01)^2}\left(1 + e^{2\,\mathrm{tgate}^2\,\omega\,\omega01}\right)$$

Problem 1(b)

In[6]:=

$$\mathrm{hmat} = \left(\frac{\hbar}{2} * \Omega01\right) * \{\{0, 1, 0\}, \{1, 0, \lambda\}, \{0, \lambda, 0\}\};\ \mathrm{MatrixForm}[\mathrm{hmat}]$$

Out[6]//MatrixForm=
$$\begin{pmatrix} 0 & \frac{\Omega01\,\hbar}{2} & 0 \\ \frac{\Omega01\,\hbar}{2} & 0 & \frac{\lambda\,\Omega01\,\hbar}{2} \\ 0 & \frac{\lambda\,\Omega01\,\hbar}{2} & 0 \end{pmatrix}$$

In[7]:=

```
Eigenvalues[hmat]
```

Out[7]= $$\left\{0, -\frac{1}{2}\sqrt{1+\lambda^2}\,\Omega01\,\hbar, \frac{1}{2}\sqrt{1+\lambda^2}\,\Omega01\,\hbar\right\}$$

In[8]:=

```
Eigenvectors[hmat]
```

Out[8]= $$\left\{\{-\lambda, 0, 1\}, \left\{\frac{1}{\lambda}, -\frac{\sqrt{1+\lambda^2}}{\lambda}, 1\right\}, \left\{\frac{1}{\lambda}, \frac{\sqrt{1+\lambda^2}}{\lambda}, 1\right\}\right\}$$

Problem 1 (c)

In[9]:=

$$\text{FullSimplify}\left[\frac{\sqrt{1+\lambda^2}}{\lambda * \left(\frac{2}{\lambda}+2*\lambda\right)} * \left(e^{-I*\left(\frac{1}{2}\sqrt{1+\lambda^2}\,\Omega01\,\hbar\right)*\frac{t}{\hbar}} - e^{-I*\left(-\frac{1}{2}\sqrt{1+\lambda^2}\,\Omega01\,\hbar\right)*\frac{t}{\hbar}}\right)\right]$$

Out[9]=

$$-\frac{i\,\text{Sin}\left[\frac{1}{2}\,t\,\sqrt{1+\lambda^2}\,\Omega01\right]}{\sqrt{1+\lambda^2}}$$

In[10]:=

$$p[t\_] = \frac{\text{Sin}\left[\frac{1}{2}\,t\,\sqrt{1+\lambda^2}\,\Omega01\right]^2}{1+\lambda^2}$$

Out[10]=

$$\frac{\text{Sin}\left[\frac{1}{2}\,t\,\sqrt{1+\lambda^2}\,\Omega01\right]^2}{1+\lambda^2}$$

In[11]:= FullSimplify[D[p[t], t] == 0]

Out[11]=

$$\frac{\Omega01\,\text{Sin}\left[t\,\sqrt{1+\lambda^2}\,\Omega01\right]}{\sqrt{1+\lambda^2}} == 0$$

In[12]:=

$$\text{Solve}\left[t\,\sqrt{1+\lambda^2}\,\Omega01 == \frac{\pi}{2}, t\right]$$

Out[12]=

$$\left\{\left\{t \to \frac{\pi}{2\,\sqrt{1+\lambda^2}\,\Omega01}\right\}\right\}$$

In[13]:=

$$p\left[\frac{\pi}{2*\Omega01*\sqrt{1+\lambda^2}}\right]$$

Out[13]=

$$\frac{1}{2\,\left(1+\lambda^2\right)}$$

Problem 1 (d)

In[14]:=

$$\text{p1new}\,[t\_] = \frac{e^{\frac{-t}{T1}}}{2*\left(1+\left(e^{-\left(\frac{\text{Ecover}\hbar*t}{\sqrt{2}}\right)^2}\right)\right)};$$

In[15]:=
```
Ecoverℏ = 2 * π * 0.2 * 10^9;
T1 = 10^-6;
Plot[p1new[t], {t, 0, 50 * 10^-9}, PlotRange → Full, AxesLabel → {tgate, P1}]
```

Out[17]=