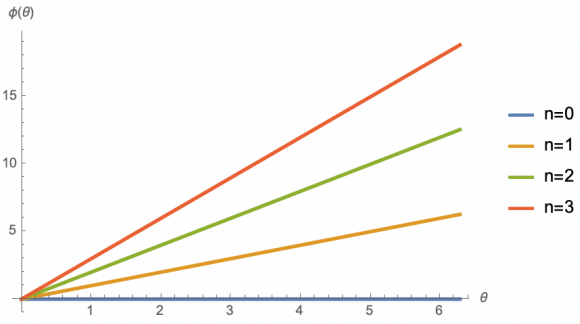
Problems 1(a) and 1(b)

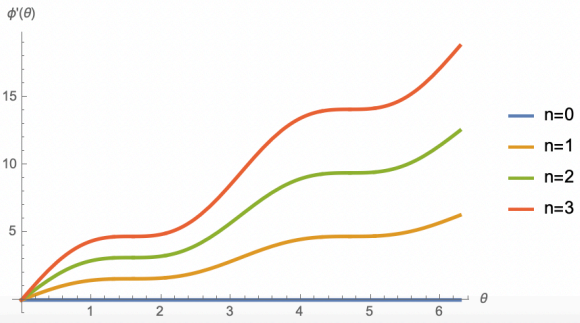
In[3]:=
```
ϕ1[n_] = n * θ;
Plot[{ϕ1[0], ϕ1[1], ϕ1[2], ϕ1[3]}, {θ, 0, 2 * π}, PlotLegends → {"n=0", "n=1", "n=2", "n=3"}, PlotStyle → Thickness[0.008], AxesLabel → {"θ", "ϕ(θ)"}]
```

Out[4]=



In[5]:=
```
ϕ2[n_] = n * (θ + Cos[θ] * Sin[θ]);
Plot[{ϕ2[0], ϕ2[1], ϕ2[2], ϕ2[3]}, {θ, 0, 2 * π}, PlotLegends → {"n=0", "n=1", "n=2", "n=3"}, PlotStyle → Thickness[0.008], AxesLabel → {"θ", "ϕ'(θ)"}]
```

Out[6]=

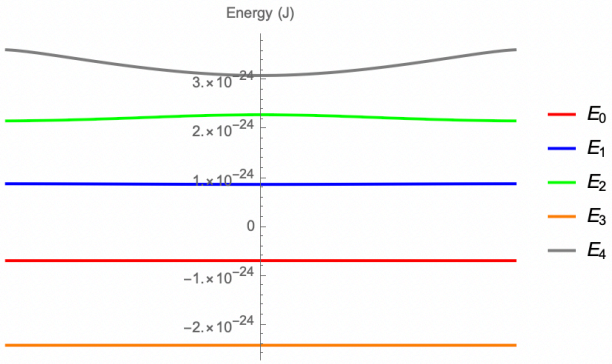


Problems 3(a) and 3(b)

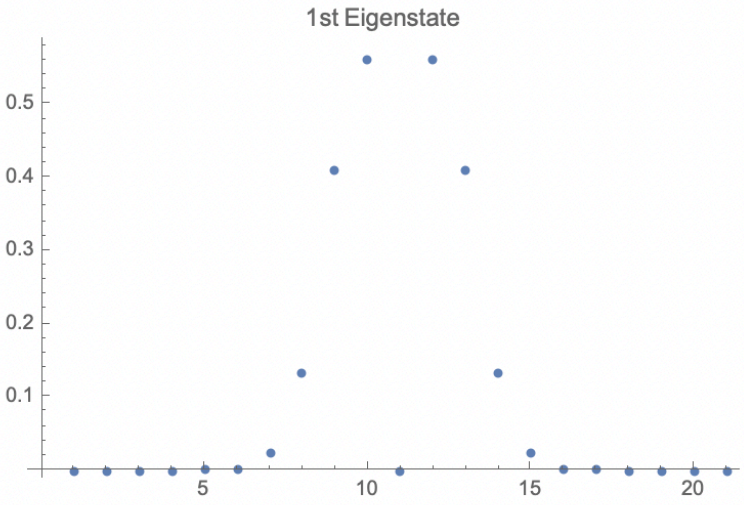
In[7]:=
```
num = 10;
size = (num * 2 + 1);
TruncHamil = IdentityMatrix[size];
For[i = -num, i < num, i++, {TruncHamil[[i + num + 1, i + num + 2]] = -eJ / 2}]
For[i = -num + 1, i < num + 1, i++, {TruncHamil[[i + num + 1, i + num]] = -eJ / 2}]
For[i = -num, i < num + 1, i++, {TruncHamil[[i + num + 1, i + num + 1]] = 4 * eC * ((i) - ng)^2}]
Hamil[ng_, eC_, eJ_] = TruncHamil;
MatrixForm[Hamil[ng, eC, eJ]]
```

$$\begin{pmatrix}
4\,eC\,(-10-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{eJ}{2} & 4\,eC\,(-9-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{eJ}{2} & 4\,eC\,(-8-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{eJ}{2} & 4\,eC\,(-7-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(-6-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(-5-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(-4-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(-3-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(-2-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(-1-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,ng^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(1-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(2-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(3-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(4-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(5-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(6-ng)^2 & -\frac{eJ}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(7-ng)^2 & -\frac{eJ}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(8-ng)^2 & -\frac{eJ}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(9-ng)^2 & -\frac{eJ}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{eJ}{2} & 4\,eC\,(10-ng)^2
\end{pmatrix}$$

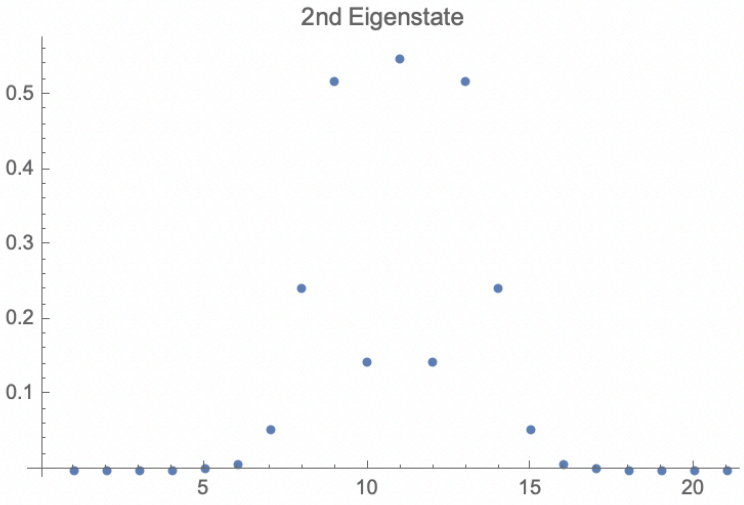
```
h = 6.63*10^-34; eC = 0.2*10^9*h; eJ = 5*10^9*h;
Plot[{Eigenvalues[Hamil[ng, eC, eJ]][[{size}]], Eigenvalues[Hamil[ng, eC, eJ]][[{size - 1}]], Eigenvalues[Hamil[ng, eC, eJ]][[{size - 2}]],
  Eigenvalues[Hamil[ng, eC, eJ]][[{size - 3}]], Eigenvalues[Hamil[ng, eC, eJ]][[{size - 4}]]}, {ng, -0.48, 0.48},
 PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}, {Orange, Thick}, {Gray, Thick}}, PlotStyle → Thick, AxesLabel → {"n_g", "Energy (J)"},
 PlotLegends → {"E_0", "E_1", "E_2", "E_3", "E_4"}, PlotRange → Full]
```



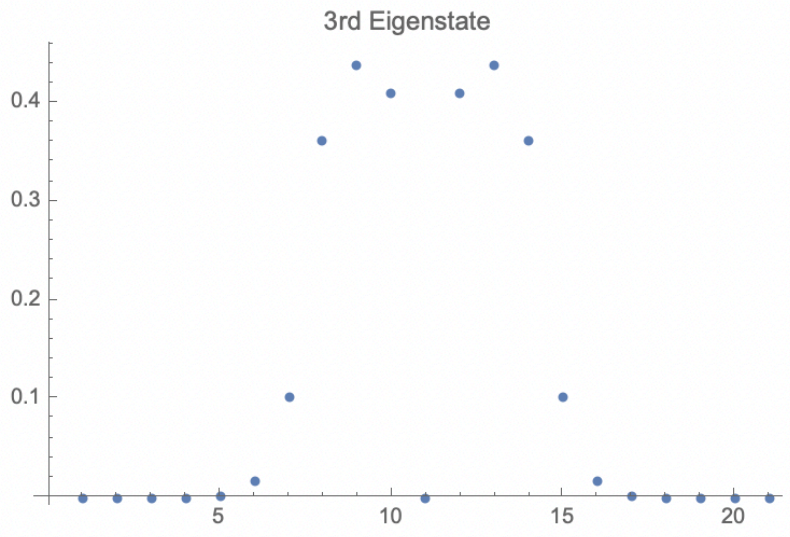

```
Show[ListPlot[(Abs[Eigenvectors[Hamil[0, eC, eJ]][[{size}]]])], PlotLabel → "1st Eigenstate"]
```



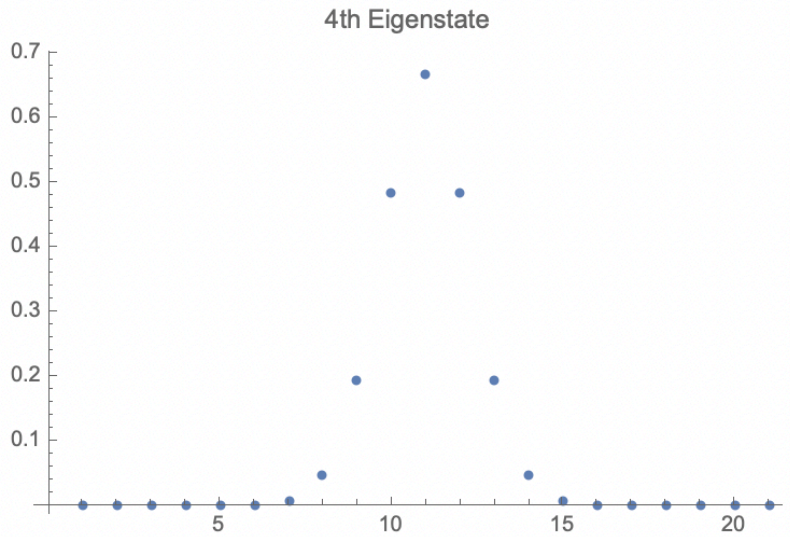

```
Show[ListPlot[(Abs[Eigenvectors[Hamil[0, eC, eJ]][[{size - 1}]]])], PlotLabel → "2nd Eigenstate"]
```



```
Show[ListPlot[(Abs[Eigenvectors[Hamil[0, eC, eJ]][[{size - 2}]]])], PlotLabel → "3rd Eigenstate"]
```




```
Show[ListPlot[(Abs[Eigenvectors[Hamil[0, eC, eJ]][[{size - 3}]]])], PlotLabel → "4th Eigenstate"]
```




```
Show[ListPlot[(Abs[Eigenvectors[Hamil[0, eC, eJ]][[{size - 4}]]])], PlotLabel → "5th Eigenstate"]
```