Problem 1 (a)

In[1]:=
```
zero = {1, 0};
ρ0 = ArrayFlatten[Outer[Times, zero, zero]]; MatrixForm[ρ0]
```
Out[1]//MatrixForm=
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

In[2]:=
```
σx = {{0, 1}, {1, 0}}; MatrixForm[σx]
```
Out[2]//MatrixForm=
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

In[3]:=
```
hrabi = 1/2 * ħ * Ω * σx; MatrixForm[hrabi]
```
Out[3]//MatrixForm=
$$\begin{pmatrix} 0 & \frac{\Omega\,\hbar}{2} \\ \frac{\Omega\,\hbar}{2} & 0 \end{pmatrix}$$

In[4]:=
```
U = MatrixExp[-I * t/ħ * hrabi]; MatrixForm[U]
```
Out[4]//MatrixForm=
$$\begin{pmatrix} \text{Cos}\left[\frac{t\,\Omega}{2}\right] & -i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right] \\ -i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right] & \text{Cos}\left[\frac{t\,\Omega}{2}\right] \end{pmatrix}$$

In[5]:=
```
Udagger = MatrixExp[I * t/ħ * hrabi]; MatrixForm[Udagger]
```
Out[5]//MatrixForm=
$$\begin{pmatrix} \text{Cos}\left[\frac{t\,\Omega}{2}\right] & i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right] \\ i\,\text{Sin}\left[\frac{t\,\Omega}{2}\right] & \text{Cos}\left[\frac{t\,\Omega}{2}\right] \end{pmatrix}$$

In[6]:=
```
ρt = FullSimplify[U.ρ0.Udagger]; MatrixForm[ρt]
```
Out[6]//MatrixForm=
$$\begin{pmatrix} \text{Cos}\left[\frac{t\,\Omega}{2}\right]^2 & \frac{1}{2}\,i\,\text{Sin}[t\,\Omega] \\ -\frac{1}{2}\,i\,\text{Sin}[t\,\Omega] & \text{Sin}\left[\frac{t\,\Omega}{2}\right]^2 \end{pmatrix}$$

In[7]:=
```
σ0 = {{1, 0}, {0, 1}}; MatrixForm[σ0 ]
```
Out[7]//MatrixForm=
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[8]:=
```
σy = {{0, -I}, {I, 0}}; MatrixForm[σy]
```
Out[8]//MatrixForm=
$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

In[9]:=
```
σz = {{1, 0}, {0, -1}}; MatrixForm[σz ]
```
Out[9]//MatrixForm=
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

In[10]:=
```
pos = 1/2 * (σ0 + rx * σx + ry * σy + rz * σz); MatrixForm[pos]
```
Out[10]//MatrixForm=
$$\begin{pmatrix} \frac{1+rz}{2} & \frac{1}{2}(rx - i\, ry) \\ \frac{1}{2}(rx + i\, ry) & \frac{1-rz}{2} \end{pmatrix}$$

In[11]:=
```
FullSimplify[Solve[Cos[t Ω/2]^2 == (1 + rz)/2, {rz}]]
```
Out[11]=
```
{{rz → Cos[t Ω]}}
```

In[12]:=
```
FullSimplify[
  Solve[{1/2 (rx - i ry) == 1/2 i Sin[t Ω], 1/2 (rx + i ry) == -1/2 i Sin[t Ω]}, {rx, ry}]]
```
Out[12]=
```
{{rx → 0, ry → -Sin[t Ω]}}
```

Problem 1 (b)

In[13]:=
```
sigplus = {{ 0, 0}, { 1, 0}}; MatrixForm[sigplus]
sigminus = {{ 0, 1}, {0, 0}}; MatrixForm[sigminus]
```
Out[13]//MatrixForm=
$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Out[14]//MatrixForm=
$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

In[15]:=
```
zeromat = {{0, 0}, {0, 0}}; MatrixForm[zeromat]
```
Out[15]//MatrixForm=
$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In[16]:=
```
rhomat = {{ρ00, ρ01}, {ρ10, ρ11}}; MatrixForm[rhomat]
```

Out[16]//MatrixForm=

$$\begin{pmatrix} \rho 00 & \rho 01 \\ \rho 10 & \rho 11 \end{pmatrix}$$

In[17]:=
```
ρdot =
 FullSimplify[(-I)/ħ * (hrabi.rhomat - rhomat.hrabi) + γ1 * (sigminus.rhomat.sigplus -
      1/2 * (sigplus.sigminus.rhomat + rhomat.sigplus.sigminus))]; MatrixForm[ρdot]
```

Out[17]//MatrixForm=

$$\begin{pmatrix} \gamma 1\, \rho 11 + \frac{1}{2}\, i\, (\rho 01 - \rho 10)\, \Omega & \frac{1}{2}\, (-\gamma 1\, \rho 01 + i\, (\rho 00 - \rho 11)\, \Omega) \\ \frac{1}{2}\, (-\gamma 1\, \rho 10 - i\, (\rho 00 - \rho 11)\, \Omega) & -\gamma 1\, \rho 11 - \frac{1}{2}\, i\, (\rho 01 - \rho 10)\, \Omega \end{pmatrix}$$

Problem 1 (c)

In[18]:=
```
Solve[{ρdot == zeromat, ρ00 + ρ11 == 1}, {ρ00, ρ01, ρ10, ρ11}]
```

Out[18]=
$$\left\{\left\{\rho 00 \to -\frac{-\gamma 1^2 - \Omega^2}{\gamma 1^2 + 2\,\Omega^2},\ \rho 01 \to \frac{i\,\gamma 1\,\Omega}{\gamma 1^2 + 2\,\Omega^2},\ \rho 10 \to -\frac{i\,\gamma 1\,\Omega}{\gamma 1^2 + 2\,\Omega^2},\ \rho 11 \to \frac{\Omega^2}{\gamma 1^2 + 2\,\Omega^2}\right\}\right\}$$

In[19]:=
```
sssoln = FullSimplify[{{-(-γ1^2 - Ω^2)/(γ1^2 + 2 Ω^2), (i γ1 Ω)/(γ1^2 + 2 Ω^2)}, {-(i γ1 Ω)/(γ1^2 + 2 Ω^2), Ω^2/(γ1^2 + 2 Ω^2)}}];
MatrixForm[sssoln]
```

Out[19]//MatrixForm=

$$\begin{pmatrix} \frac{\gamma 1^2+\Omega^2}{\gamma 1^2+2\,\Omega^2} & \frac{i\,\gamma 1\,\Omega}{\gamma 1^2+2\,\Omega^2} \\ -\frac{i\,\gamma 1\,\Omega}{\gamma 1^2+2\,\Omega^2} & \frac{\Omega^2}{\gamma 1^2+2\,\Omega^2} \end{pmatrix}$$

Problem 1 (d)

In[20]:=
```
Solve[sssoln == pos, {rx, ry, rz}]
```

Out[20]=
$$\left\{\left\{rx \to 0,\ ry \to -\frac{2\,\gamma 1\,\Omega}{\gamma 1^2 + 2\,\Omega^2},\ rz \to \frac{\gamma 1^2}{\gamma 1^2 + 2\,\Omega^2}\right\}\right\}$$