QUESTIONS

1. Prove that, the set $S = \{(1,0,1),(0,1,1),(1,1,0)\}$ is a basis of $\mathbb{R}^3$.
2. Find a basis of $\mathbb{R}^4$ that contains the vectors $\{(1,2,3,4),(2,3,0,1)\}$.
3. Let P_n be the space of all real polynomials of degree less than or equal to n. Determine if the following polynomials span P_2.

$$\begin{aligned} p_1 &= 1 + 2x - x^2 \\ p_2 &= 3 + x^2 \\ p_3 &= 5 + 4x - x^2 \\ p_4 &= -2 + 2x - 2x^2 \end{aligned}$$

4. **U** and **V** are the subspaces of the vector space $\mathbb{R}^4$ generated by the sets

$$\begin{aligned} B_1 &= \{(1,1,0,1),(1,2,3,0),(2,3,3,1)\}, \\ B_2 &= \{(1,2,2,2),(2,3,2,3),(1,3,4,3)\} \end{aligned}$$

respectively. Determine $dim(\mathbf{U} \cap \mathbf{V})$.

5. Determine the conditions for which the system of equations

$$\begin{aligned} x + y + z &= 1 \\ x + 2y - z &= b \\ 5x + 7y + az &= b^2 \end{aligned}$$

admits

(i) only one solution,
(ii) no solution,
(iii) many solutions.

6. For what values of a the following system of equations is consistent ? Solve completely in each case.

$$\begin{aligned} x - y + z &= 1 \\ x + 2y + 4z &= a \\ x + 4y + 6z &= a^2 \end{aligned}$$

7. For what values of k the system of equations

$$\begin{aligned} x+y+z &= kx \\ x+y+z &= ky \\ x+y+z &= kz \end{aligned}$$

has non-trivial solutions ?

8. Determine the conditions for which the system of equations

$$\begin{aligned} x+2y+z &= 1 \\ 2x+y+3z &= b \\ x+ay+3z &= b+1 \end{aligned}$$

has

(i) only one solution,
(ii) no solution,
(iii) many solutions.

9. Find the eigenvalues and the eigenvectors of the matrix

$$\begin{bmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix}.$$

10. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, find a matrix P such that $P^{-1}AP = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$.

11. Diagonalise the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$.

12. Use Cayley-Hamilton Theorem to find A^{100} where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

13. Reduce the following quadratic equation to the normal form and find the rank and signature.

$$2x^2 + 5y^2 + 10z^2 + 4xy + 12yz + 6zx$$

14. If the system of equations

$$\begin{aligned} ax+by+cz &= 0 \\ bx+cy+az &= 0 \\ cx+ay+bz &= 0 \end{aligned}$$

has a non-zero solution, prove that either $a+b+c=0$ or $a=b=c$.

15. Show that,

$$\begin{bmatrix} a^4 & a^2 & a & 1 \\ b^4 & b^2 & b & 1 \\ c^4 & c^2 & c & 1 \\ d^4 & d^2 & d & 1 \end{bmatrix} = (a+b+c+d) \begin{bmatrix} a^3 & a^2 & a & 1 \\ b^3 & b^2 & b & 1 \\ c^3 & c^2 & c & 1 \\ d^3 & d^2 & d & 1 \end{bmatrix}.$$

16. Find all real x so that the matrix

$$A = \begin{bmatrix} 2+x & 2 & 2 & 2 \\ 2 & 2+x & 2 & 2 \\ 2 & 2 & 2+x & 2 \\ 2 & 2 & 2 & 2+x \end{bmatrix}$$

is invertible. Assuming that A^{-1} exists find the sum of all the elements of A^{-1} without computing A^{-1}.

17. Find all real x for which the rank of the matrix $\begin{bmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{bmatrix}$ is less than 4.

18. Use elementary row operation on A to find A^{-1} where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$