

B. Math UGA 2010

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8. The roots of the equation $x^4 + x^2 = 1$ are

- (A) all real and positive;
- (B) never real;
- (C) 2 positive and 2 negative;
- (D) 1 positive, 1 negative and 2 non-real.

Answer :

Clearly, for $x \leq -1$ and $x \geq 1$ the given equation will not have any solution.

The equation will have solutions only when $-1 < x < 1$.

Let $x^2 = t$ where $t \in (0, 1)$

$$\text{Then, } t^2 + t - 1 = 0 \quad \therefore t = \frac{-1 \pm \sqrt{5}}{2}$$

When $x^2 = \frac{-1 - \sqrt{5}}{2} < 0$, we shall have 2 non-real roots of the given equation.

$$\text{Now, } \sqrt{5} > 2. \text{ So, } -1 + \sqrt{5} > 0 \quad \therefore \frac{-1 + \sqrt{5}}{2} > 0$$

$$\text{When } x^2 = \frac{-1 + \sqrt{5}}{2}, \text{ we shall have } x = \pm \sqrt{\frac{-1 + \sqrt{5}}{2}}$$

(D) 1 positive, 1 negative and 2 non-real