

## QUESTIONS

1. A random sample of size 9 is drawn from the distribution with PDF

$$f_{\theta}(x) \propto \frac{x^2}{\theta^3}; -3\theta < x < \theta; \theta > 0 \text{ and} \\ f_{\theta}(x) = 0; \text{ otherwise}$$

and the observations are found to be 10, -30, 14, -45, -34, 7, 12, 11, -13.

Find the maximum likelihood estimate of  $\theta$ . Also find (with justification) the maximum likelihood estimate of the variance for the above distribution.

2. The time a client waits to be served by the mortgage specialist at a bank has probability density function

$$f(x) = \frac{1}{2\theta^3} x^2 e^{-x/\theta}; x > 0; \theta > 0.$$

The waiting times of 15 clients are found to be 6, 12, 15, 14, 12, 10, 8, 9, 10, 9, 8, 7, 10, 7 and 3 minutes. Calculate the values of the maximum likelihood estimate and the method of moments estimate of  $\theta$ .

3. Consider the life-time of an electric bulb which is exponentially distributed with mean  $3\theta$ . The life-time of 20 bulbs are found to be 0.11, 2.28, 6.33, 0.67, 3.68, 1.46, 4.17, 2.96, 4.93, 9.49, 1.02, 0.60, 3.85, 1.32, 0.24, 2.38, 0.41, 4.98, 6.91, 2.14 years. Obtain

- (i) MLE of  $\theta$ .
- (ii) unbiased estimate of  $\theta$ .
- (iii) MLE of  $P[X_5 > 6]$ .

4. A random sample of size 20 is drawn from the distribution with PDF

$$f(x) = \begin{cases} \theta e^{-\theta x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

and the observations are found to be 0.10, 0.34, 0.77, 0.57, 1.63, 0.06, 1.14, 0.40, 0.07, 1.17, 0.17, 0.17, 0.16, 0.01, 0.79, 0.06, 0.38, 0.28, 0.17, 1.30.

Obtain method of moments estimate of  $\theta$ .

5. A random sample of size 20 is drawn from the distribution with PDF

$$f(x) = \begin{cases} \frac{\alpha^\beta}{\Gamma(\beta)} e^{-\alpha x} x^{\beta-1}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

and the observations are found to be 2.13, 3.30, 3.78, 1.99, 2.91, 1.68, 1.18, 2.97, 3.89, 4.71, 2.05, 1.56, 0.48, 2.26, 3.27, 2.16, 2.71, 4.46, 1.78, 4.47.

Obtain method of moments estimates of  $\alpha$  and  $\beta$ .

6. An urn contains 6 marbels of which  $\theta$  are white and the others black. In order to test the null hypothesis  $H_0 : \theta = 3$  against the alternative  $H_1 : \theta = 4$ , two marbels are drawn at random (without replacement) and  $H_0$  is rejected if both the marbels are white; otherwise  $H_0$  is accepted. Find the probabilities of committing type  $I$  and type  $II$  errors.
7. A random sample of size 10 is drawn from the distribution with PDF

$$f(x) = \begin{cases} \frac{1}{2\sqrt{2\pi x}} \cdot \exp\{-\frac{1}{8}(\ln x - \mu)^2\}, & x > 0, -\infty < \mu < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Let the random sample be  $X_1, X_2, \dots, X_{10}$ . To test  $H_0 : \mu = 1$  against  $H_1 : \mu > 1$ , we reject  $H_0$  if  $\frac{1}{10} \sum_{i=1}^{10} \ln X_i > 1$ .

Find  $P(\text{type } I \text{ error})$ . Also find  $P(\text{type } II \text{ error})$  and power of the test when  $\mu = 2$ . Also draw a sketch of the power curve for  $\mu = -3, -2, -1, 0, 1, 2, 3$  and comment on your findings.