

# Graphical Method of Solving L.P.P.

1. Solve the following L.P.P. graphically :

$$\begin{array}{ll}\text{Minimize} & z = 20x_1 + 10x_2 \\ \text{subject to} & x_1 + 2x_2 \leq 40, \\ & 3x_1 + x_2 \geq 30, \\ & 4x_1 + 3x_2 \geq 60, \\ & x_1, x_2 \geq 0.\end{array}$$

2. Solve the following L.P.P. graphically :

$$\begin{array}{ll}\text{Maximize} & z = 6x_1 + 4x_2 \\ \text{subject to} & 7x_1 + 5x_2 \leq 35, \\ & 5x_1 + 7x_2 \leq 35, \\ & 4x_1 + 3x_2 \geq 12, \\ & 3x_1 + x_2 \geq 3, \\ & x_1, x_2 \geq 0.\end{array}$$

3. Solve the following L.P.P. graphically :

$$\begin{array}{ll}\text{Maximize} & z = 3x_1 + 4x_2 \\ \text{subject to} & x_1 - x_2 \geq 0, \\ & -x_1 + 3x_2 \leq 3, \\ & x_1, x_2 \geq 0.\end{array}$$

4. Solve the following L.P.P. graphically :

$$\begin{array}{ll}\text{Maximize} & z = 6x_1 + 10x_2 \\ \text{subject to} & 3x_1 + 5x_2 \leq 10, \\ & 5x_1 + 3x_2 \leq 15, \\ & x_1, x_2 \geq 0.\end{array}$$

# Basic Solution of System of Equations

5. Find the basic feasible solutions of the following system of equations.

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8;$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3;$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

6. Find all the basic solutions of the following equations and identify the basic vectors and the basic variables in each case.

$$x_1 + x_2 + x_3 = 4;$$

$$2x_1 + 5x_2 - 2x_3 = 3.$$

# Simplex Method

7. Solve the following *L.P.P.* using *simplex method*.

$$\begin{array}{ll}\text{Maximize} & z = 4x_1 + 7x_2 \\ \text{subject to} & 2x_1 + x_2 \leq 1000, \\ & 10x_1 + 10x_2 \leq 6000, \\ & 2x_1 + 4x_2 \leq 2000, \\ & x_1, x_2 \geq 0.\end{array}$$

8. Solve the following *L.P.P.* using *simplex method*.

$$\begin{array}{ll}\text{Maximize} & z = 3x_1 + x_2 + 3x_3 \\ \text{subject to} & 2x_1 + x_2 + x_3 \leq 2, \\ & x_1 + 2x_2 + 3x_3 \leq 5, \\ & 2x_1 + 2x_2 + x_3 \leq 6, \\ & x_1, x_2, x_3 \geq 0.\end{array}$$

9. Solve the following *L.P.P.* using *penalty method*.

$$\begin{array}{ll}\text{Maximize} & z = 2x_1 + 3x_2 + x_3 \\ \text{subject to} & -3x_1 + 2x_2 + 3x_3 = 8, \\ & -3x_1 + 4x_2 + 2x_3 = 7, \\ & x_1, x_2, x_3 \geq 0.\end{array}$$

10. Solve the following *L.P.P.* using *penalty method*.

$$\begin{array}{ll}\text{Maximize} & z = -2x_1 + x_2 + 3x_3 \\ \text{subject to} & x_1 - 2x_2 + 3x_3 = 2, \\ & 3x_1 + 2x_2 + 4x_3 = 1, \\ & x_1, x_2, x_3 \geq 0.\end{array}$$

# Duality

11. Use duality to find the optimal solution, if any, of the following *L.P.P.*

$$\begin{array}{ll}\text{Maximize} & z = 3x_1 + 2x_2 \\ \text{subject to} & 2x_1 + x_2 \leq 5, \\ & x_1 + x_2 \leq 3, \\ & x_1, x_2 \geq 0.\end{array}$$

12. Use duality to find the optimal solution, if any, of the following *L.P.P.*

$$\begin{array}{ll}\text{Minimize} & z = 15x_1 + 10x_2 \\ \text{subject to} & 3x_1 + 5x_2 \geq 5, \\ & 5x_1 + 2x_2 \geq 3, \\ & x_1, x_2 \geq 0.\end{array}$$

# Mathematical Formulation of L.P.P.

13. At a cattle breeding firm it is prescribed that the food ration for one animal must contain at least 14, 22 and 1 units of nutrients  $A$ ,  $B$ , and  $C$  respectively. Two different kinds of fodder are available. Each unit weight of these two contains the following amounts of the three nutrients :

	Fodder 1	Fodder 2
Nutrient $A$	2	1
Nutrient $B$	2	3
Nutrient $C$	1	1

It is given that the costs of unit quantity of fodder 1 and 2 are 3 and 2 monetary units respectively. Pose a linear programming problem in terms of minimizing the cost of purchasing the fodders for the above cattle breeding firm.

14. Three products are processed through three different operations. The times (in minutes) required per unit of each product, the daily capacity of the operations (in minutes per day) and the profit per unit sold for each product (in rupees) are as follows :

Operation	Time per unit			Operation Capacity
	Product 1	Product 2	Product 3	
1	3	4	3	42
2	5	0	3	45
3	3	6	2	41
Profit	3	2	1	

The zero time indicates that the product does not require the given operation. The problem is to determine the optimum daily production for three products that maximizes the profit.

Formulate the above production planning problem as a linear programming problem assuming that all units produced are sold.

# Transportation Problem

15. Use *North-West Corner Rule* to find the basic feasible solution of the following transportation problem :

	$D_1$	$D_2$	$D_3$	$D_4$	$a_i$
$O_1$	19	20	50	10	7
$O_2$	70	30	40	60	9
$O_3$	40	8	70	20	18
$b_j$	5	8	7	14	

16. Use *Row-minima method* to find the basic feasible solution of the following transportation problem :

	$D_1$	$D_2$	$D_3$	$D_4$	$a_i$
$O_1$	21	16	25	13	11
$O_2$	17	18	14	23	13
$O_3$	32	27	18	41	19
$b_j$	6	10	12	15	

17. Use *Column-minima method* to find the basic feasible solution of the following transportation problem :

	$D_1$	$D_2$	$D_3$	$a_i$
$O_1$	2	7	4	5
$O_2$	3	3	1	8
$O_3$	5	4	7	7
$O_4$	1	6	2	14
$b_j$	7	9	18	

18. Use *Matrix-minima method* to find the basic feasible solution of the following transportation problem :

	$A$	$B$	$C$	$a_i$
$F_1$	10	9	8	8
$F_2$	10	7	10	7
$F_3$	11	9	7	9
$F_4$	12	14	10	4
$b_j$	10	10	8	

19. Obtain an optimal basic feasible solution of the following transportation problem :

	$D_1$	$D_2$	$D_3$	$D_4$	$a_i$
$O_1$	1	2	1	4	30
$O_2$	3	3	2	1	50
$O_3$	4	2	5	9	20
$b_j$	20	40	30	10	

20. Obtain an optimal basic feasible solution of the following transportation problem :

	$W_1$	$W_2$	$W_3$	$W_4$	$a_i$
$F_1$	19	30	50	10	7
$F_2$	70	30	40	60	9
$F_3$	40	8	70	20	18
$b_j$	5	8	7	14	

21. A steel company has three open hearth furnaces and five rolling mills. Transportation costs (rupees per quintal) for transporting steel from furnaces to rolling mills are shown in the following table :

	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	$a_i$
$F_1$	4	2	3	2	6	8
$F_2$	5	4	5	2	1	12
$F_3$	6	5	4	7	7	14
$b_j$	4	4	6	8	8	

What is the optimal transportation schedule ?

# Assignment Problem

22. Find the optimal assignment to obtain the minimum cost for the assignment problem with the following cost matrix.

	$M_1$	$M_2$	$M_3$	$M_4$
$J_1$	10	24	30	15
$J_2$	16	22	28	12
$J_3$	12	20	32	10
$J_4$	9	26	34	16

23. Find the optimal assignment to obtain the minimum cost for the assignment problem with the following cost matrix.

	a	b	c	d	e
1	2	9	2	7	1
2	6	8	7	6	1
3	4	6	5	3	1
4	4	2	7	3	1
5	5	3	9	5	1



# Game Theory

24. In a rectangular game the payoff matrix is given by

10	5	5	20	4
11	15	10	17	25
7	12	8	9	8
5	13	9	10	5

State, giving reasons, whether the players will use pure or mixed strategies. What is the value of the game ?

25. Use dominance to reduce the following payoff matrix and solve the game.

		B			
A	8	15	−4	−2	
	19	15	17	16	
	10	20	15	5	

26. Use dominance to reduce the following payoff matrix and solve the game.

		B		
A	8	5	8	
	8	6	5	
	7	4	5	
	6	5	6	

27. Reduce the following matrix game into a  $2 \times 2$  matrix game.

2	2	1	-2	-3
4	3	4	-2	0
5	1	2	5	6

28. Reduce the following matrix game into a  $2 \times 2$  matrix game.

4	2	0	2	1	1
4	3	1	3	2	2
4	3	7	-5	1	2
4	3	4	-1	2	2
4	3	3	-2	2	2

29. Solve graphically the game whose payoff matrix is given as follows :

		B	
		$B_1$	$B_2$
A	$A_1$	1	-3
	$A_2$	3	5
	$A_3$	-1	6
	$A_4$	4	1
	$A_5$	2	2
	$A_6$	-5	0