QUESTIONS

1. A random sample of size 9 is drawn from the distribution with pdf

$$f_\theta(x) \propto \frac{x^2}{\theta^3};\ -3\theta < x < \theta;\ \theta > 0 \text{ and}$$
$$f_\theta(x) = 0;\ \text{otherwise}$$

   and the observations are found to be 10, –30, 14, –45, –34, 7, 12, 11, –13.

   Find the maximum likelihood estimate of θ. Also find (with justification) the maximum likelihood estimate of the variance for the above distribution.

2. The time a client waits to be served by the mortgage specialist at a bank has probability density function

$$f(x) = \frac{1}{2\theta^3} x^2 e^{-x/\theta};\ x > 0;\ \theta > 0.$$

   The waiting times of 15 clients are found to be 6, 12, 15, 14, 12, 10, 8, 9, 10, 9, 8, 7, 10, 7 and 3 minutes. Calculate the values of the maximum likelihood estimate and the method of moments estimate of θ.

3. Consider the life-time of an electric bulb which is exponentially distributed with mean 3θ. The life-time of 20 bulbs are found to be 0.11, 2.28, 6.33, 0.67, 3.68, 1.46, 4.17, 2.96, 4.93, 9.49, 1.02, 0.60, 3.85, 1.32, 0.24, 2.38, 0.41, 4.98, 6.91, 2.14 years. Obtain

   (i) MLE of θ.

   (ii) unbiased estimate of θ.

   (iii) MLE of $P[X_5 > 6]$.