

QUESTIONS

1. Signatures to a petition were collected on 700 sheets. Each sheet was provided with space for 50 signatures, but the signatories put their signatures in erratic ways and the number of signatures per sheet was not definite. 12 sheets were spoiled in transit. Of the available sheets, a random sample of 50 was drawn and the numbers of signatures per sheet counted, which are shown as below :

Number of signatures (y_i)	52	51	46	42	40	37	32	29	27	15	14	10	8
Number of sheets (n_i)	1	2	21	8	7	2	2	1	1	2	1	1	1

Estimate the total number of signatures to the petition and calculate 95% confidence limits.

2. The data given below to pertain to one complete lactation of milk yield (in 10 kg) of 250 cows in an organised dairy farm.
 - (i) Select a simple random sample of size 25.
 - (ii) Estimate the mean with its standard error.
 - (iii) Construct a 95% confidence limit for the population mean.

230	293	163	290	200	173	194	322	169	230
297	151	248	271	259	214	167	207	240	286
184	248	327	338	165	177	270	177	202	155
155	293	190	172	150	319	151	118	213	114
186	167	129	185	231	199	265	306	173	276
291	231	205	220	246	239	186	299	233	208
265	204	300	195	239	173	237	282	221	218
197	215	213	290	146	232	305	184	149	267
188	219	171	99	329	199	180	225	257	202
189	207	792	327	201	300	206	199	299	153
175	287	277	230	258	137	174	301	260	282
211	212	284	214	283	139	223	212	207	224
207	111	272	192	127	303	221	187	309	263
203	176	233	239	176	218	193	243	236	275

288	198	241	219	167	193	234	179	126	173
279	178	275	260	191	174	235	338	242	238
211	187	184	189	305	221	253	225	327	203
195	158	156	185	170	271	160	188	165	218
312	143	267	298	196	139	205	298	238	217
145	201	313	230	185	166	147	223	271	133
155	230	287	329	265	150	286	271	268	198
214	231	163	335	198	270	187	174	163	201
192	247	247	297	178	240	290	234	170	227
230	353	170	159	236	181	230	240	212	242
151	158	253	179	263	158	250	226	246	301

3. 2000 cultivators' holdings in Uttar Pradesh (India) were stratified according to their sizes. The number of holdings (N_i), mean area under wheat per holdings (\bar{Y}_i) and s.d. of area under wheat per holdings (S_i) are given below for each stratum :

Stratum Number	Number of holdings (N_i)	Mean area under wheat per-holding (\bar{Y}_i)	s.d. of area under wheat per-holding (S_i)
1	394	5.4	8.3
2	461	16.3	13.3
3	381	24.3	15.1
4	334	34.5	19.8
5	169	42.1	24.5
6	113	50.1	26.0
7	148	63.8	35.2

For a sample of 200 farms, compute the sample size in each stratum under proportional and optimum allocations.

Calculate the sampling variance of the estimated area under wheat from the sample

- if the farms are selected under proportional allocation by with and without replacement methods
- if the farms are selected under Neyman's allocation by with and without replacement methods.

Also compute the gain in efficiency from these procedures as compared to simple random sampling.

4. The number of pepper standards for selected villages in each of the three strata of Trivandrum zone is as follows :

Stratum	Total number of villages in the stratum	Number of villages selected from the stratum	Number of pepper standars in each of the selected villages
1	441	11	41, 116, 19, 15, 144, 159, 212, 57, 28, 119, 76
2	405	12	39, 70, 38, 37, 161, 38, 27, 119, 36, 128, 30, 208
3	103	7	252, 385, 192, 296, 115, 159, 120

Estimate the total number of pepper standards along with its standard error in Trivandrum zone. Also, estimate the gain in precision due to stratification.

5. A random sample of $n = 2$ households was drawn from a small colony of 5 households (hypothetical population) having monthly income (in rupees) as follows :

Household	1	2	3	4	5
Income (in rupees)	156	149	166	164	155

- (i) Calculate population mean (\bar{Y}), variance (σ^2) and mean square (S^2).
- (ii) Enumerate all possible samples of size 2 by replacement method and show that
 - (a) the sample mean gives an unbiased estimate of the population mean and find its sampling variance;
 - (b) the sample variance (s^2) is an unbiased estimate of the population variance (σ^2); and
 - (c) $v(\bar{y}) = \frac{(y_1 - y_2)^2}{4}$ is an unbiased estimator of $V(\bar{Y})$, i.e. $E(v(\bar{y})) = V(\bar{Y}) = \frac{\sigma^2}{2}$.
- (iii) Enumerate all possible samples of size 2 by without replacement method and show that
 - (a) the sample mean gives an unbiased estimate of the population mean and find its sampling variance;
 - (b) the sample variance (s^2) is an unbiased estimate of the population mean square (S^2); and
 - (c) $v(\bar{y}) = \frac{3(y_1 - y_2)^2}{20}$ is an unbiased estimator of $V(\bar{Y})$, i.e.

$$E(v(\bar{y})) = V(\bar{Y}) = \left(\frac{1}{2} - \frac{1}{5}\right) S^2 = \frac{3}{10} S^2.$$

6. An investigator desires to take a stratified random sample with the following assumptions:

Stratum	S_i	N_i	C_i (in Rs.)
1	400	10	4
2	600	20	9

- Estimate the values of $\frac{n_1}{n}$ and $\frac{n_2}{n}$ which minimize the total field cost $C = c_1n_1 + c_2n_2$ for a given value of $V(\bar{y}_{st})$.
- Estimate the total sample size required, under the scheme of optimum allocation, to make $V(\bar{y}_{st}) = 1$, when fpc is ignored.
- Also estimate the cost of the survey.

7. Given below are the daily milk yield (in liters) records of the first lactation of a specified cow belonging to the Tharparkar herd maintained at the Government Cattle Farm, Patna. The milk yields of the first five days were not recorded, being the colostrum period.

Day	1	2	3	4	5	6	7	8	9	10
Milk Yield	10	11	14	10	14	9	10	8	11	10
Day	11	12	13	14	15	16	17	18	19	20
Milk Yield	6	9	8	7	9	10	11	11	13	12
Day	21	22	23	24	25	26	27	28	29	30
Milk Yield	12	10	11	11	14	15	12	17	18	16
Day	31	32	33	34	35	36	37	38	39	40
Milk Yield	13	14	14	15	16	16	16	13	16	17
Day	41	42	43	44	45	46	47	48	49	50
Milk Yield	14	16	15	14	14	15	17	15	16	17
Day	51	52	53	54	55	56	57	58	59	60
Milk Yield	25	22	23	19	18	16	22	21	21	23
Day	61	62	63	64	65	66	67	68	69	70
Milk Yield	21	19	19	19	19	19	19	19	19	19
Day	71	72	73	74	75	76	77	78	79	80
Milk Yield	18	19	21	20	17	16	18	18	18	22
Day	81	82	83	84	85	86	87	88	89	90
Milk Yield	22	22	20	20	20	18	20	21	21	20

Day	91	92	93	94	95	96	97	98	99	100
Milk Yield	18	21	22	22	20	21	21	21	21	21
Day	101	102	103	104	105	106	107	108	109	110
Milk Yield	19	20	21	20	21	20	21	20	21	20
Day	111	112	113	114	115	116	117	118	119	120
Milk Yield	19	21	18	21	20	22	21	21	21	16
Day	121	122	123	124	125	126	127	128	129	130
Milk Yield	19	15	15	16	19	12	16	14	15	17
Day	131	132	133	134	135	136	137	138	139	140
Milk Yield	16	20	15	19	16	16	20	20	18	21
Day	141	142	143	144	145	146	147	148	149	150
Milk Yield	22	22	21	22	21	21	21	18	20	17
Day	151	152	153	154	155	156	157	158	159	160
Milk Yield	20	20	21	21	21	20	20	16	16	15
Day	161	162	163	164	165	166	167	168	169	170
Milk Yield	18	19	18	20	19	18	16	14	14	13
Day	171	172	173	174	175	176	177	178	179	180
Milk Yield	16	16	16	18	16	15	16	18	18	15
Day	181	182	183	184	185	186	187	188	189	190
Milk Yield	18	16	17	18	16	17	13	14	13	12
Day	191	192	193	194	195	196	197	198	199	200
Milk Yield	16	10	13	8	8	6	8	9	4	5
Day	201	202	203							
Milk Yield	6	6	4							

Find the efficiency of systematic sampling at 7 and 14 days' interval of recording, with respect to corresponding simple random sampling, in estimating the lactation yield of the cow.

8. In an experimental agricultural census carried out by I.A.S.R.I., New Delhi, in the Loni Block of Meerut District of U.P. (India) during 1967-68, two villages of the block were selected randomly. Out of 225 holding in two villages, namely Panch-lok and Agrola, 45 holdings were selected for systematic sampling (with 5 as the sampling interval). The total arable land (in kacha bigha)* for the 45 selected holdings are given below :

S.N.	1	2	3	4	5	6	7	8	9	10
Total arable land	60	50	14	10	1	0	0	0	0	0
S.N.	11	12	13	14	15	16	17	18	19	20
Total arable land	150	150	100	20	0	25	192	25	0	13
S.N.	21	22	23	24	25	26	27	28	29	30
Total arable land	0	0	50	0	10	0	0	0	85	30
S.N.	31	32	33	34	35	36	37	38	39	40
Total arable land	30	70	30	35	0	30	0	0	10	0
S.N.	41	42	43	44	45					
Total arable land	20	70	16	15	35					

Estimate the total arable land in the two villages and also the approximate standard error of the estimate.

9. For studying milk yield, feeding and management practices of milch animals in the year 1977-78, the whole Haryana state was divided into 4 zones according to agro-climatic conditions. The total number of milch animals in 17 randomly selected villages (in 1977-78) of zone A, along with their livestock census data in 1976, are as shown below :

Serial No. of Village	No. of Milch Animals in Survey (y)	No. of Milch Animals in Census (x)
1	1129	1141
2	1144	1144
3	1125	1127
4	1138	1153
5	1137	1117
6	1127	1140
7	1163	1153
8	1153	1146
9	1164	1189
10	1130	1137
11	1153	1170
12	1125	1115
13	1116	1130

14	1115	1118
15	1112	1122
16	1112	1113
17	1123	1166

Estimate the total number of milch animals in 117 villages of zone A

- (i) by ratio method and
- (ii) by simple mean per unit method

Also compare its precision, given the total number of milch animals in the census = 143968.

10. The number of labourers x (in thousands) and the quantity of raw materials y (in lakhs of bales) are given below for 20 jute mills. Draw a sample of 5 units by SRSWOR. Estimate the total amount of raw materials consumed by 20 mills by

- (i) sample mean estimator, and
- (ii) ratio estimator along with their variance estimators.

Also, compare the variance of these estimators.

Jute Mill	x	y	Jute Mill	x	y
1	368	31	11	512	31
2	384	33	12	503	29
3	361	37	13	472	38
4	347	39	14	429	41
5	403	43	15	387	40
6	529	61	16	376	38
7	703	68	17	412	42
8	396	42	18	345	45
9	473	41	19	297	32
10	509	49	20	633	50

11. The population in 1975 (x) and the population in 1985 (y) for 22 municipal town areas divided into two zones in Sweden are given in the following table.

Draw a simple random sample of size 4 from each of the two zones and obtain a separate ratio estimator and a combined ratio estimator of the total population in 1985 for the whole area taking (x) as an auxiliary variable. Obtain the variance estimators.

Zone A			Zone B		
Municipal Area	x	y	Municipal Area	x	y
1	27	33	1	29	32
2	15	19	2	14	20
3	20	26	3	40	53
4	15	19	4	27	28
5	52	56	5	43	48
6	15	16	6	671	653
7	62	70	7	78	79
8	54	66	8	54	59
9	12	12	9	28	27
10	50	60	10	35	49
			11	36	38
			12	6	6

12. The following data give the household sizes for 32 households in a village. Draw a simple random sample with replacement of 6 draws and hence obtain an estimate of the average household size along with its standard error. Obtain a 95% confidence interval for the average household size in the population.

5, 3, 7, 11, 4, 6, 10, 9, 8, 12, 11, 10, 10, 11, 8, 7, 6, 8, 9, 4, 1, 5, 7, 7, 12, 8, 9, 10, 9, 7, 6, 8.

13. The following data give the geographical area (in acres) under paddy for 58 villages. Draw an SRSWOR of 8 villages, find an estimate of average area per village under paddy, an estimate of its variance and its 95% confidence interval.

98, 270, 79, 273, 130, 158, 116, 194, 41, 33, 78, 56, 58, 19, 64, 81, 141, 58, 29, 46, 93, 127, 114, 88, 108, 58, 47, 69, 44, 56, 102, 102, 187, 161, 179, 76, 137, 179, 76, 137, 127, 104, 117, 170, 210, 101, 222, 223, 96, 114, 318, 272, 155, 292, 240, 201, 261, 189.

14. Using the following data, estimate the total number of milch animals in 117 villages of Zone A by the method of regression estimation. Also, compare its precision with the ratio estimate and mean per unit estimate.

Serial No. of Village	No. of Milch Animals in Survey (y)	No. of Milch Animals in Census (x)
1	1129	1141
2	1144	1144
3	1125	1127
4	1138	1153
5	1137	1117
6	1127	1140
7	1163	1153
8	1153	1146
9	1164	1189
10	1130	1137
11	1153	1170
12	1125	1115
13	1116	1130
14	1115	1118
15	1112	1122
16	1112	1113
17	1123	1166

15. Using the following data, estimate the total number of trees in the districts by the regression method of estimation and compare its precision.

Stratum Number	Total no. of villages (N_m)	Total area (in Hect.) under orchard (X_m)	No. of villages in sample (n_m)	Area under orchards in Hect. (x_m)	Total number of trees (y_m)
1	985	11253	6	10.63, 9.90, 1.45, 3.38, 5.17, 10.35	747, 719, 78, 201, 311, 448
2	2196	25115	8	14.66, 2.61, 4.35, 9.87, 2.42, 5.60, 4.70, 36.75	580, 103, 316, 739, 196, 235, 212, 1646
3	1020	18870	11	11.60, 5.29, 7.94, 7.29, 8.00, 1.20, 11.50, 7.96, 23.15, 1.70, 2.01	488, 227, 374, 491, 499, 50, 455, 47, 879, 115, 115

16. A pilot sample survey for study of cultivation practices and yield of guava was conducted by IASRI in Prayagraj district of Uttar Pradesh (India). From Umerpur-Neerna village, out of a total of 412 bearing trees, 15 clusters of size 4 trees each were selected and yields (in kg.) were recorded as given below :

Cluster	1st Tree	2nd Tree	3rd Tree	4th Tree
1	5.53	4.84	0.69	15.79
2	26.11	10.93	19.08	11.18
3	11.08	0.65	4.21	7.56
4	12.66	32.52	16.92	37.02
5	0.87	3.56	4.81	57.54
6	6.40	11.68	40.05	5.15
7	54.21	34.63	52.55	37.96
8	1.94	35.97	29.54	25.98
9	37.94	47.07	16.94	28.11
10	56.92	17.69	26.24	6.77
11	27.59	38.10	24.76	6.53
12	45.98	5.17	1.17	6.53
13	7.13	34.35	12.18	9.86
14	14.23	16.89	28.93	21.70
15	3.53	40.76	5.15	1.25

- Estimate the average yield (in kg.) per tree of guava in the Umerpur-Neerna village of Prayagraj along with its standard error.
- Estimate the intraclass correlation coefficient between trees within clusters and efficiency of cluster sampling as compared to simple random sampling.