MSMS 101
Statistical Inference : Point Estimation

Assignments

1. Show that the sample mean is a consistent estimator of θ in $N(\theta, 1)$ population.

2. Suppose we have a statistic T_n whose mean differs from θ by an order of $\dfrac{1}{n}$ and whose variance is of order $\dfrac{1}{n}$ and it tends to normality as $n \to \infty$. Then $\dfrac{T_n - \theta}{\sqrt{n}} \xrightarrow{P} 0$ and T_n is a consistent estimator of θ.

3. Show that the sample mean is not a consistent estimator of θ in Cauchy population, although the sample median is.

4. Show that the sample variance is a consistent estimator of σ^2 in $N(\mu, \sigma^2)$ population.

5. Consider a random samle of size n from $N(\mu, \sigma^2)$ population where σ^2 is known. Obtain the MVBUE of μ.

6. Suppose $X_1, X_2, \dots X_n$ is a random sample from $N(\mu, \sigma^2)$ where μ is known. Obtain the MVBUE of σ^2.

7. Consider a random sample of size n from Binomial(k, p) population where k is known. Obtain the MVBUE of p.

8. Suppose $X_1, X_2, \dots X_n$ is a random sample from Poisson(λ) population. Obtain the MVBUE of λ.

9. Suppose $X_1, X_2, \dots X_n$ is a random sample from Cauchy(μ, λ) population where λ is known. Obtain the MVBUE of μ.

10. Suppose X_1, X_2 be a random sample of size 2 from Poisson(λ). Show that $X_1 + X_2$ is a sufficient for λ but $X_1 + 2X_2$ is not.

11. Consider a random sample of size n from an Exponential distribution with mean θ. Obtain the UMVUE of $e^{-\frac{t}{\theta}}$, $t \in \mathbb{R}^+$.

12. Suppose $f(x; \theta_1, \theta_2) = \dfrac{1}{\sqrt{2\pi\theta_2}} \cdot \exp\left\{ -\dfrac{1}{2} \cdot \dfrac{(x - \theta_1)^2}{\theta_2} \right\}, \;\; x \in \mathbb{R}.$

    Consider a random sample of size n from $f(\cdot)$. Obtain the CRLBs for unbiased estimators of θ_1 and θ_2.

13. Suppose X be a random sample of size 1 from a population with PMF

    $$P(X = x) = p(1 - p)^x; \;\; x = 0, 1, 2, 3, \dots.$$

    Obtain lower bounds to the variance of an unbiased estimator of p.

14. Suppose $X_1, X_2, \ldots X_n$ is a random sample from $N(\mu, \sigma^2)$ where σ^2 is known. Show that the statistic $T(\underset{\sim}{X}) = \overline{X}$ is complete.

15. Suppose $X_1, X_2, \ldots X_n$ is a random sample from $N(\mu, \sigma^2)$ where μ is known. Show that the statistic $T(\underset{\sim}{X}) = \sum_{i=1}^{n}(X_i - \mu)^2$ is complete.

16. Consider a random sample of size n from Binomial(k, p) population where k is known. Obtain the MLE of p.

17. Suppose $X_1, X_2, \ldots X_n$ is a random sample from Poisson(λ) population. Obtain the MLE of λ.

18. Consider a random samle of size n from $N(\mu, \sigma^2)$ population where σ^2 is known. Obtain the MLE of μ.

19. Suppose $X_1, X_2, \ldots X_n$ is a random sample from $N(\mu, \sigma^2)$ where μ is known. Obtain the MLE of σ^2.

20. Suppose $X_1, X_2, \ldots X_n$ is a random sample from $N(\mu, \sigma^2)$ where both mean and variance are unknown. Obtain the MLE of the parameters μ and σ^2.

21. Let $f(x; \theta) = \dfrac{1}{\theta} e^{-\dfrac{x}{\theta}} I_{(0,\infty)}(x)$.

    $X_1, X_2, \ldots X_n$ be a random sample from $f(\cdot)$. Obtain the MLE of θ.