Graphical Method of Solving L.P.P.

1. Solve the following L.P.P. graphically :

$$Minimize \quad z = 20x_1 + 10x_2$$
$$subject \ to \qquad x_1 + 2x_2 \leq 40,$$
$$3x_1 + x_2 \geq 30,$$
$$4x_1 + 3x_2 \geq 60,$$
$$x_1, x_2 \geq 0.$$

2. Solve the following L.P.P. graphically :

$$Maximize \quad z = 6x_1 + 4x_2$$
$$subject \ to \qquad 7x_1 + 5x_2 \leq 35,$$
$$5x_1 + 7x_2 \leq 35,$$
$$4x_1 + 3x_2 \geq 12,$$
$$3x_1 + x_2 \geq 3,$$
$$x_1, x_2 \geq 0.$$

3. Solve the following L.P.P. graphically :

$$Maximize \quad z = 3x_1 + 4x_2$$
$$subject \ to \qquad x_1 - x_2 \geq 0,$$
$$-x_1 + 3x_2 \leq 3,$$
$$x_1, x_2 \geq 0.$$

4. Solve the following L.P.P. graphically :

$$Maximize \quad z = 6x_1 + 10x_2$$
$$subject \ to \qquad 3x_1 + 5x_2 \leq 10,$$
$$5x_1 + 3x_2 \leq 15,$$
$$x_1, x_2 \geq 0.$$

Basic Solution of System of Equations

5. Find the basic feasible solutions of the following system of equations.

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8;$$
$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3;$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

6. Find all the basic solutions of the following equations and identify the basic vectors and the basic variables in each case.

$$x_1 + x_2 + x_3 = 4;$$
$$2x_1 + 5x_2 - 2x_3 = 3.$$

Simplex Method

7. Solve the following *L.P.P.* using *simplex method.*

$$\text{Maximize} \quad z = 4x_1 + 7x_2$$
$$\text{subject to} \quad 2x_1 + x_2 \leq 1000,$$
$$10x_1 + 10x_2 \leq 6000,$$
$$2x_1 + 4x_2 \leq 2000,$$
$$x_1, x_2 \geq 0.$$

8. Solve the following *L.P.P.* using *simplex method.*

$$\text{Maximize} \quad z = 3x_1 + x_2 + 3x_3$$
$$\text{subject to} \quad 2x_1 + x_2 + x_3 \leq 2,$$
$$x_1 + 2x_2 + 3x_3 \leq 5,$$
$$2x_1 + 2x_2 + x_3 \leq 6,$$
$$x_1, x_2, x_3 \geq 0.$$

9. Solve the following *L.P.P.* using *penalty method.*

$$\text{Maximize} \quad z = 2x_1 + 3x_2 + x_3$$
$$\text{subject to} \quad -3x_1 + 2x_2 + 3x_3 = 8,$$
$$-3x_1 + 4x_2 + 2x_3 = 7,$$
$$x_1, x_2, x_3 \geq 0.$$

10. Solve the following *L.P.P.* using *penalty method.*

$$\text{Maximize} \quad z = -2x_1 + x_2 + 3x_3$$
$$\text{subject to} \quad x_1 - 2x_2 + 3x_3 = 2,$$
$$3x_1 + 2x_2 + 4x_3 = 1,$$
$$x_1, x_2, x_3 \geq 0.$$

Duality

11. Use duality to find the optimal solution, if any, of the following $L.P.P.$

$$
\begin{aligned}
\textit{Maximize} \quad z &= 3x_1 + 2x_2 \\
\textit{subject to} \quad 2x_1 + x_2 &\leq 5, \\
x_1 + x_2 &\leq 3, \\
x_1, x_2 &\geq 0.
\end{aligned}
$$

12. Use duality to find the optimal solution, if any, of the following $L.P.P.$

$$
\begin{aligned}
\textit{Maximize} \quad z &= 15x_1 + 10x_2 \\
\textit{subject to} \quad 3x_1 + 5x_2 &\geq 5, \\
5x_1 + 2x_2 &\geq 3, \\
x_1, x_2 &\geq 0.
\end{aligned}
$$

Mathematical Formulation of L.P.P.

13. At a cattle breeding firm it is prescribed that the food ration for one animal must contain at least 14, 22 and 1 units of nutrients A, B, and C respectively. Two different kinds of fodder are available. Each unit weight of these two contains the following amounts of the three nutrients :

	Fodder 1	Fodder 2
Nutrient A	2	1
Nutrient B	2	3
Nutrient C	1	1

It is given that the costs of unit quantity of fodder 1 and 2 are 3 and 2 monetary units respectively. Pose a linear programming problem in terms of minimizing the cost of purchasing the fodders for the above cattle breeding firm.

14. Three products are processed through three different operations. The times (in minutes) required per unit of each product, the daily capacity of the operations (in minutes per day) and the profit per unit sold for each product (in rupees) are as follows :

Operation	Time per unit			Operation Capacity
	Product 1	Product 2	Product 3	
1	3	4	3	42
2	5	0	3	45
3	3	6	2	41
Profit	3	2	1	

The zero time indicates that the product does not require the given operation. The problem is to determine the optimum daily production for three products that maximizes the profit.

Formulate the above production planning problem as a linear programming problem assumig that all units produced are sold.

Transportation Problem

15. Use *North-West Corner Rule* to find the basic feasible solution of the following transportation problem :

	D_1	D_2	D_3	D_4	a_i
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

16. Use *Row-minima method* to find the basic feasible solution of the following transportation problem :

	D_1	D_2	D_3	D_4	a_i
O_1	21	16	25	13	11
O_2	17	18	14	23	13
O_3	32	27	18	41	19
b_j	6	10	12	15	

17. Use *Column-minima method* to find the basic feasible solution of the following transportation problem :

	D_1	D_2	D_3	a_i
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
b_j	7	9	18	

18. Use *Matrix-minima method* to find the basic feasible solution of the following transportation problem :

	A	B	C	a_i
F_1	10	9	8	8
F_2	10	7	10	7
F_3	11	9	7	9
F_4	12	14	10	4
b_j	10	10	8	

19. Obtain an optimal basic feasible solution of the following transportation problem :

	D_1	D_2	D_3	D_4	a_i
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
b_j	20	40	30	10	

20. Obtain an optimal basic feasible solution of the following transportation problem :

	W_1	W_2	W_3	W_4	a_i
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
b_j	5	8	7	14	

21. A steel company has three open hearth furnaces and five rolling mills. Transportation costs (rupees per quintal) for transporting steel from furnaces to rolling mills are shown in the following table :

	M_1	M_2	M_3	M_4	M_5	a_i
F_1	4	2	3	2	6	8
F_2	5	4	5	2	1	12
F_3	6	5	4	7	7	14
b_j	4	4	6	8	8	

What is the optimal transportation schedule ?

Assignment Problem

22. Find the optimal assignment to obtain the minimum cost for the assignment problem with the following cost matrix.

	M_1	M_2	M_3	M_4
J_1	10	24	30	15
J_2	16	22	28	12
J_3	12	20	32	10
J_4	9	26	34	16

23. Find the optimal assignment to obtain the minimum cost for the assignment problem with the following cost matrix.

	a	b	c	d	e
1	2	9	2	7	1
2	6	8	7	6	1
3	4	6	5	3	1
4	4	2	7	3	1
5	5	3	9	5	1

Game Theory

24. In a rectangular game the payoff matrix is given by

10	5	5	20	4
11	15	10	17	25
7	12	8	9	8
5	13	9	10	5

State, giving reasons, whether the players will use pure or mixed strategies. What is the value of the game ?

25. Use dominance to reduce the following payoff matrix and solve the game.

<div align="center">B</div>

	8	15	−4	−2
A	19	15	17	16
	10	20	15	5

26. Use dominance to reduce the following payoff matrix and solve the game.

<div align="center">B</div>

	8	5	8
	8	6	5
A	7	4	5
	6	5	6

27. Reduce the following matrix game into a 2×2 matrix game.

2	2	1	-2	-3
4	3	4	-2	0
5	1	2	5	6

28. Reduce the following matrix game into a 2×2 matrix game.

4	2	0	2	1	1
4	3	1	3	2	2
4	3	7	-5	1	2
4	3	4	-1	2	2
4	3	3	-2	2	2

29. Solve graphically the game whose payoff matrix is given as follows :

<table>
<tr><td></td><td></td><td colspan="2" align="center">B</td></tr>
<tr><td></td><td></td><td align="center">B_1</td><td align="center">B_2</td></tr>
<tr><td></td><td>A_1</td><td align="center">1</td><td align="center">-3</td></tr>
<tr><td></td><td>A_2</td><td align="center">3</td><td align="center">5</td></tr>
<tr><td rowspan="2">A</td><td>A_3</td><td align="center">-1</td><td align="center">6</td></tr>
<tr><td>A_4</td><td align="center">4</td><td align="center">1</td></tr>
<tr><td></td><td>A_5</td><td align="center">2</td><td align="center">2</td></tr>
<tr><td></td><td>A_6</td><td align="center">-5</td><td align="center">0</td></tr>
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