MSMS 206 : Practical 03

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Question

(1) Calculate L and U such that $A = LU$ where L is a lower triangular matrix and U is an upper triangular matrix for given

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 3 \\ 2 & 3 & 1 \end{bmatrix}.$$

(2) Solve the following system of linear equations using LU decomposition method.

$$\begin{aligned} x_1 + x_2 - x_3 &= 4 \\ x_1 - 2x_2 + 3x_3 &= -6 \\ 2x_1 + 3x_2 + x_3 &= 7 \end{aligned}$$

Standard LU decomposition is possible only possible for square matrices with all leading principal minors being non-zero.

```
check_LPM <- function(A){

  if(dim(A)[1] != dim(A)[2]) stop("Input must be a square matrix.")

  # leading_principal_minors
  m <- c()

  for (i in 1:dim(A)[1]) {
    m[i] <- det(as.matrix(A[1:i, 1:i]))
  }

  if(all((m != 0) == TRUE)){
    return(TRUE)
  }else{
    return(FALSE)
  }
}
```

```r
LU_decomposer <- function(A){

  if(!check_LPM(A)) stop("All the leading principal minors must be non-zero.")

  I <- matrix(data = 0, nrow = nrow(A), ncol = ncol(A))

  for (i in 1:nrow(I)) {
    I[i, i] <- I[i, i] + 1
  }

  r <- dim(A)[1]
  c <- dim(A)[2]

  i <- 1; j <- 1

  while(j <= c) {

    while(i <= r) {

      if(i != r){
        a1 <- as.matrix(A[(i+1):r, j] / A[i, j])

        a2 <- t(as.matrix(A[i, ]))

        A[(i+1):r, ] <- A[(i+1):r, ] - a1 %*% a2

        I[(i+1):r, j] <- as.vector(a1)

        break
      }
      i <- i + 1
    }
    j <- j + 1

    i <- j
  }

  return(list(I, A))
}
```

LU_decomposer() returns a list containing L and U respectively.

```r
A <- matrix(data = c(1, 1, -1,
                     1, -2, 3,
                     2, 3, 1),
            nrow = 3, ncol = 3, byrow = TRUE)
```

```r
LU_decomposer(A)

## [[1]]
```

```
##      [,1]       [,2] [,3]
## [1,]    1  0.0000000    0
## [2,]    1  1.0000000    0
## [3,]    2 -0.3333333    1
##
## [[2]]
##      [,1] [,2]      [,3]
## [1,]    1    1 -1.000000
## [2,]    0   -3  4.000000
## [3,]    0    0  4.333333
```

```
L <- LU_decomposer(A)[[1]]
U <- LU_decomposer(A)[[2]]
```

➜

```
b <- matrix(data = c(4, -6, 7), nrow = 3, ncol = 1, byrow = TRUE)
```

-

$$
\begin{aligned}
A\underset{\sim}{x} &= \underset{\sim}{b} \\
\Rightarrow LU\underset{\sim}{x} &= \underset{\sim}{b} \\
\Rightarrow \underset{\sim}{x} &= U^{-1}L^{-1}\underset{\sim}{b}
\end{aligned}
$$

```
solve(U) %*% solve(L) %*% b

##      [,1]
## [1,]    1
## [2,]    2
## [3,]   -1
```

✎ The solution of the given system of equations is

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.
$$

More LU Decompositions :

☞ Schaums Outline of Linear Algebra : Example 3.22

$$A = \begin{bmatrix} 1 & 2 & -3 \\ -3 & -4 & 13 \\ 2 & 1 & -5 \end{bmatrix}.$$

```
LU_decomposer(A)

## [[1]]
##      [,1] [,2] [,3]
## [1,]    1  0.0    0
## [2,]   -3  1.0    0
## [3,]    2 -1.5    1
##
## [[2]]
##      [,1] [,2] [,3]
## [1,]    1    2   -3
## [2,]    0    2    4
## [3,]    0    0    7
```

☞ Schaums Outline of Linear Algebra : Solved Problems 3.39(a)

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -4 & 7 \\ -1 & -2 & 1 \end{bmatrix}.$$

```
LU_decomposer(A)

## [[1]]
##      [,1] [,2] [,3]
## [1,]    1  0.0    0
## [2,]    2  1.0    0
## [3,]   -1 -2.5    1
##
## [[2]]
##      [,1] [,2] [,3]
## [1,]    1   -3  5.0
## [2,]    0    2 -3.0
## [3,]    0    0 -1.5
```

☞ Schaums Outline of Linear Algebra : Solved Problems 3.39(b)

$$A = \begin{bmatrix} 1 & 4 & -3 \\ 2 & 8 & 1 \\ -5 & -9 & 7 \end{bmatrix}.$$

```
LU_decomposer(A)

## Error in LU_decomposer(A): All the leading principal minors must be non-zero.
```

☞ Schaums Outline of Linear Algebra : Solved Problems 3.41

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ -3 & -10 & 2 \end{bmatrix}.$$

```
LU_decomposer(A)

## [[1]]
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    2    1    0
## [3,]   -3    4    1
##
## [[2]]
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    0   -1    1
## [3,]    0    0    1
```

☞ Schaums Outline of Linear Algebra : Supplementary Problems 3.69(a)

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 3 & -4 & -2 \\ 2 & -3 & -2 \end{bmatrix}.$$

```
LU_decomposer(A)

## [[1]]
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    3    1    0
## [3,]    2    1    1
##
## [[2]]
##      [,1] [,2] [,3]
## [1,]    1   -1   -1
## [2,]    0   -1    1
## [3,]    0    0   -1
```

☞ Schaums Outline of Linear Algebra : Supplementary Problems 3.69(b)

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 5 & 1 \\ 3 & 4 & 2 \end{bmatrix}.$$

```
LU_decomposer(A)

## [[1]]
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    2    1    0
## [3,]    3    5    1
##
## [[2]]
##      [,1] [,2] [,3]
## [1,]    1    3   -1
## [2,]    0   -1    3
## [3,]    0    0  -10
```

☞ Schaums Outline of Linear Algebra : Supplementary Problems 3.69(c)

$$A = \begin{bmatrix} 2 & 3 & 6 \\ 4 & 7 & 9 \\ 3 & 5 & 4 \end{bmatrix}.$$

```
LU_decomposer(A)

## [[1]]
##      [,1] [,2] [,3]
## [1,]  1.0  0.0    0
## [2,]  2.0  1.0    0
## [3,]  1.5  0.5    1
##
## [[2]]
##      [,1] [,2] [,3]
## [1,]    2    3  6.0
## [2,]    0    1 -3.0
## [3,]    0    0 -3.5
```

☞ Schaums Outline of Linear Algebra : Supplementary Problems 3.69(d)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 7 & 10 \end{bmatrix}.$$

```
LU_decomposer(A)

## Error in LU_decomposer(A): All the leading principal minors must be non-zero.
```