MSMS 105 : Assignment 06

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Objective

Random number generation from an Exponential distribution by "Inverse Transformation Method".

Theory

Let X be a continuous random variable with CDF $F_X(x)$. We want random numbers from X.

We must have $F_X(X) = Y \sim U(0,1)$.

Thus, $X = F_X^{-1}(Y)$ yields a similar distribution as of X with Y being an Uniform $(0,1)$ variate, provided F_X^{-1} exists in a closed form.

So, at first we generate n many $U(0,1)$ random numbers, say y_i $\forall i = 1(1)n$.

Then we calculate $x_i = F_X^{-1}(y_i)$ $\forall i = 1(1)n$ to finally get random numbers from our desired distribution.

For an Exponential distribution with rate $\lambda > 0$,

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

and

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Thus,

$$F_X(x) = y$$

$$\implies 1 - e^{-\lambda x} = y$$

$$\implies e^{-\lambda x} = 1 - y$$

$$\implies -\lambda x = ln(1-y)$$

$$\implies x = -\frac{ln(1-y)}{\lambda}$$

➔ R Program

```r
random_numbers <- function(n, seed = NULL){
  a <- 1103515245
  b <- 12345
  m <- 2^31 - 1

  if(is.null(seed)){
    start_date <- as.POSIXct("2003-01-01 00:00:00", tz = "UTC")

    current_date <- Sys.time()

    seed <- as.numeric(difftime(current_date, start_date, units = "secs"))
  }

  x <- c(seed)

  for (i in 2:n) {
    x[i] <- (a * x[i-1] + b) %% m
  }

  return(x)
}
```

```r
uniform_random_numbers <- function(n, seed = NULL){
  return(random_numbers(n, seed) / (2^31 - 1))
}
```

```r
exponential_random_numbers <- function(n, lambda, seed = NULL){
  num <- -(log(1 - uniform_random_numbers(n, seed)) / lambda)
  return(num)
}
```

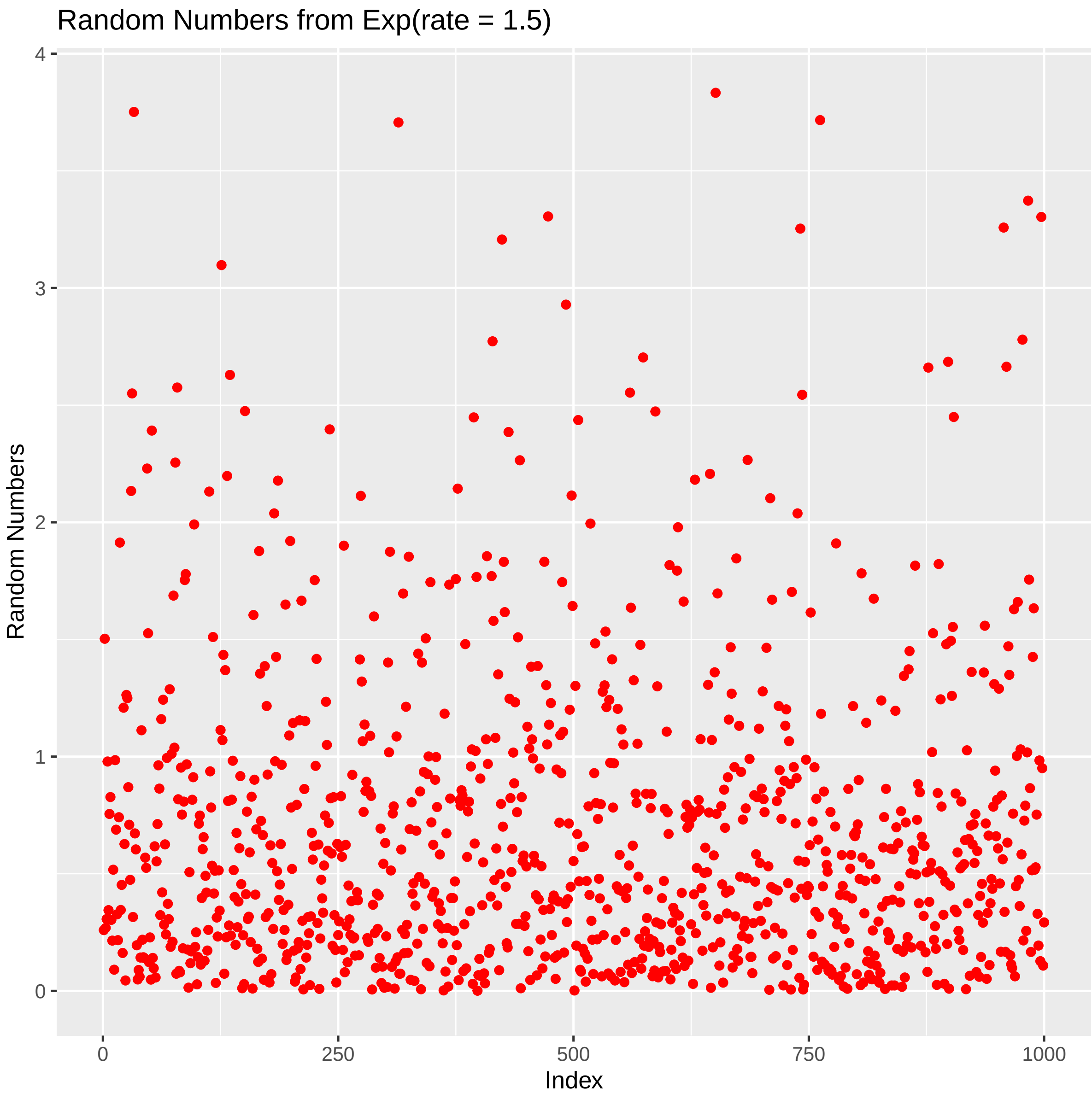
```r
exponential_random_numbers(10, 2)

##  [1] 0.19473564 2.88198404 0.09388916 0.03137359 0.32825849 0.14272313
##  [7] 0.59074292 0.04967599 0.22936684 0.28456345
```

➜ Visualization

```r
size = 1000
exp_numbers <- data.frame(n = 1:size,
                          num = exponential_random_numbers(size, 1.5))
```

```r
exp_numbers %>%
  ggplot(aes(x = n, y = num)) +
  geom_point(size = 1.5, col = "red") +
  labs(x = "Index", y = "Random Numbers",
       title = "Random Numbers from Exp(rate = 1.5)")
```



➜ Conclusion

✎ Scatteredness of the points mimics that of an Exponential Distribution.