MSMS 308 : Practical 03

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Question

Generate survival times from Gamma distribution with shape $\alpha = 2$ and rate $\lambda = 0.5$ and estimate the parameters. Then plot

1. the Probability Density Function f
2. the Cumulative Distribution Function F
3. the Survival Function S
4. the Hazard Function h
5. the Cumulative Hazard Function H

R Program and Plot

```
Gamma_MLE <- function(gamma_sample, shape_initial, n_iteration){

  a <- c(shape_initial)

  n <- length(gamma_sample)

  f1 <- function(alpha){

    result <- - n * digamma(alpha) -
      n * log(mean(gamma_sample)) +
      n * log(alpha) +
      sum(log(gamma_sample))

    return(result)
  }

  f2 <- function(alpha){
    return(-n * trigamma(alpha) + n / alpha)
  }

  iterations <- n_iteration

  for (i in 2:iterations) {
    a[i] <- a[i-1] - f1(a[i-1]) / f2(a[i-1])
```

```
    if(abs(f1(a[length(a)])) < 0.001) break
  }

  alpha_hat <- a[length(a)]

  beta_hat <- mean(gamma_sample) / alpha_hat

  return(c(alpha_hat, beta_hat))
}
```

```
n <- 50; rate <- 0.5

set.seed(2)
our_sample <- rgamma(n, shape = 2, scale = 1/rate)
temp <- Gamma_MLE(our_sample, shape_initial = 1, n_iteration = 1000)
```

```
estimated_shape <- temp[1]; estimated_shape

## [1] 1.731768
```

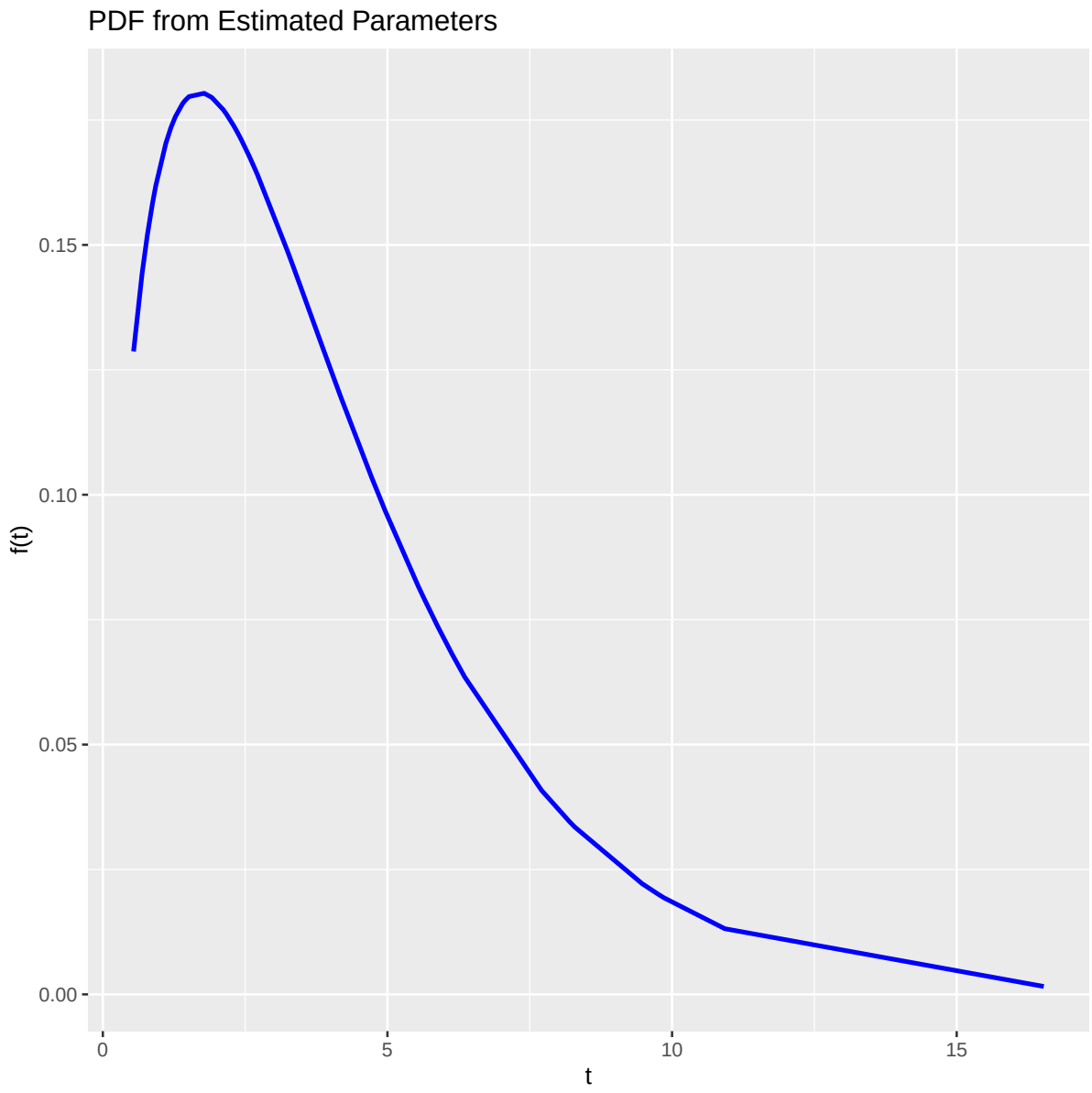
```
estimated_scale <- temp[2]; estimated_scale

## [1] 2.317431
```

```
t_values <- sort(our_sample)
```

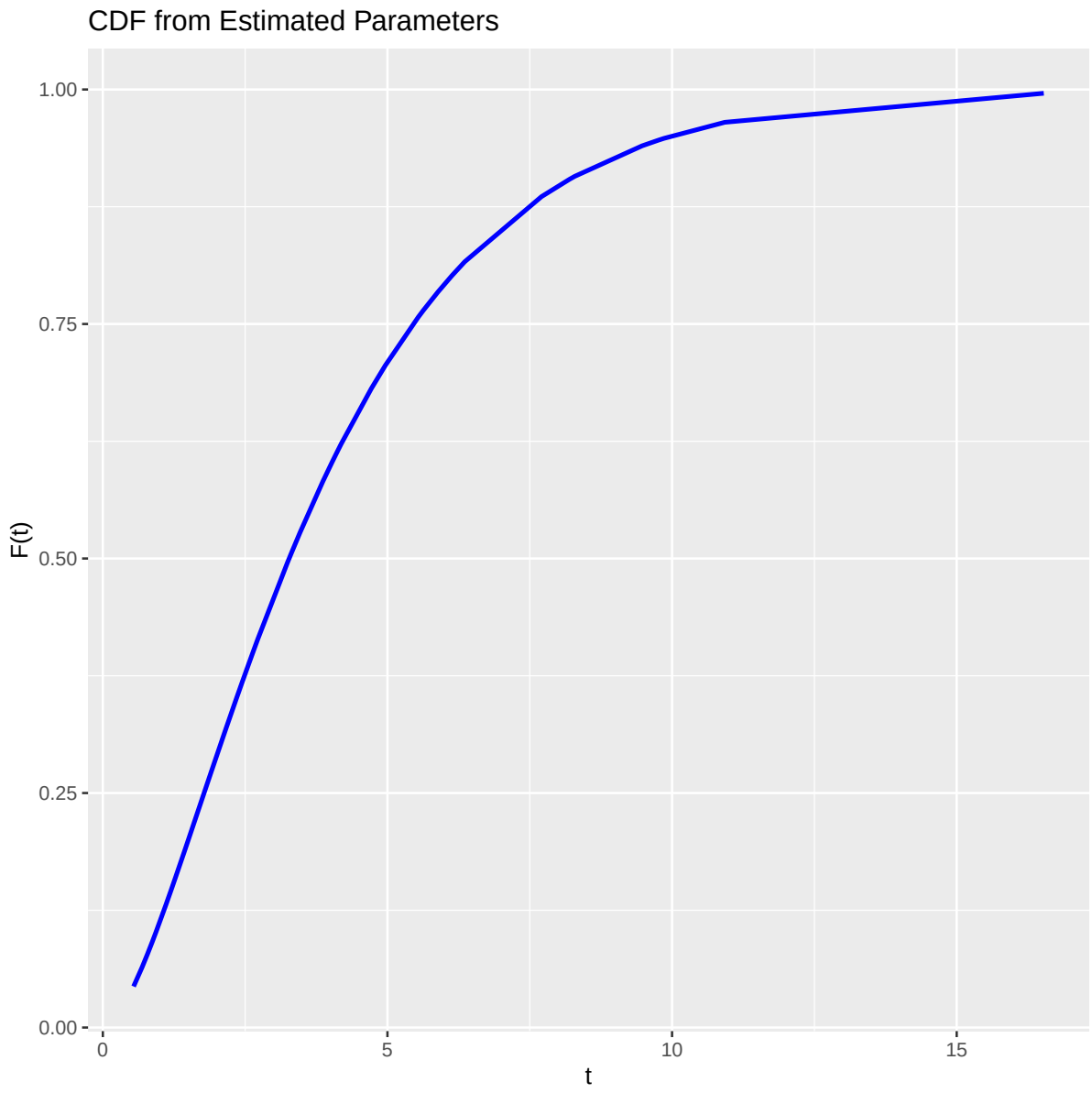
```r
df1 <- data.frame(t = t_values,
                  ft = dgamma(t_values, shape = estimated_shape,
                              scale = estimated_scale))

df1 %>%
  ggplot(aes(x = t, y = ft)) +
  geom_line(col = 'blue', linewidth = 1) +
  labs(x = "t", y = "f(t)", title = "PDF from Estimated Parameters")
```



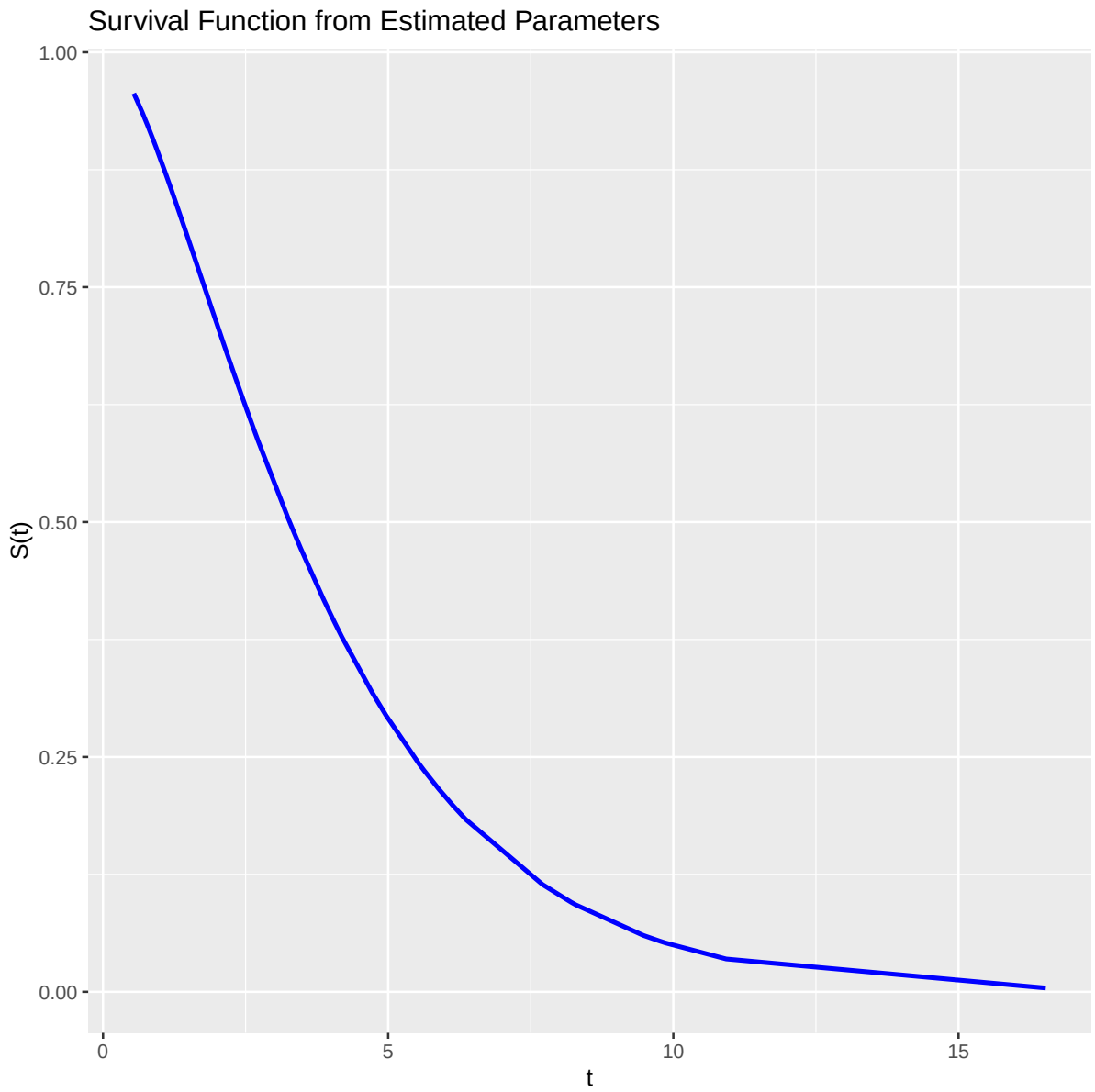
```r
df2 <- data.frame(t = t_values,
                  Ft = pgamma(t_values, shape = estimated_shape,
                              scale = estimated_scale))

df2 %>%
  ggplot(aes(x = t, y = Ft)) +
  geom_line(col = 'blue', linewidth = 1) +
  labs(x = "t", y = "F(t)", title = "CDF from Estimated Parameters")
```



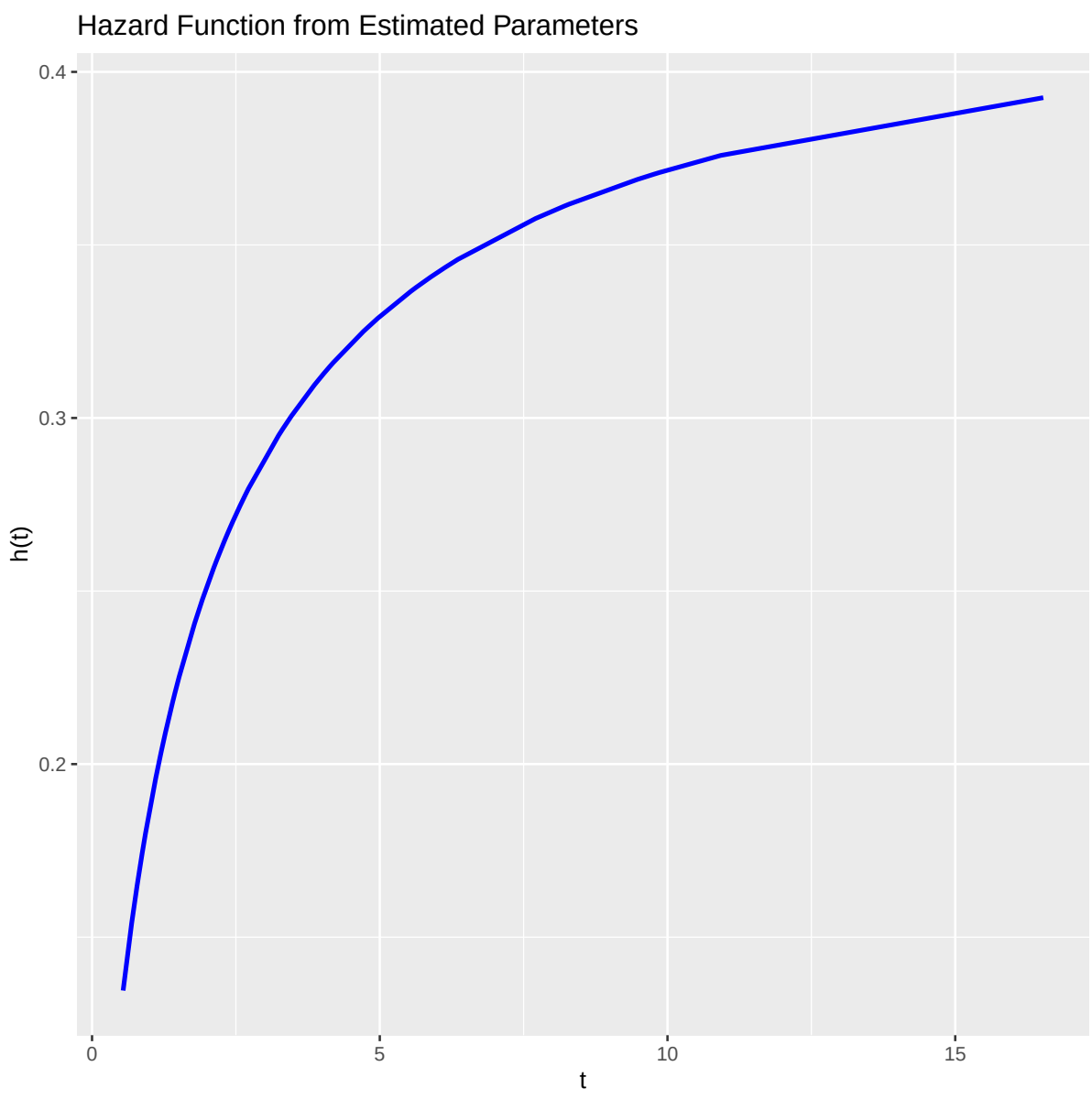
```r
df3 <- data.frame(t = t_values,
                  St = 1 - pgamma(t_values, shape = estimated_shape,
                                  scale = estimated_scale))

df3 %>%
  ggplot(aes(x = t, y = St)) +
  geom_line(col = 'blue', linewidth = 1) +
  labs(x = "t", y = "S(t)", title = "Survival Function from Estimated Parameters")
```



```r
df4 <- data.frame(t = t_values,
                  ht = df1$ft / df3$St)

df4 %>%
  ggplot(aes(x = t, y = ht)) +
  geom_line(col = 'blue', linewidth = 1) +
  labs(x = "t", y = "h(t)", title = "Hazard Function from Estimated Parameters")
```



✎ For shape parameter less than 1, the hazard function decreases monotonically.

```r
df5 <- data.frame(t = t_values,
                  Ht = -log(df3$St))

df5 %>%
  ggplot(aes(x = t, y = Ht)) +
  geom_line(col = 'blue', linewidth = 1) +
  labs(x = "t", y = "H(t)",
       title = "Cumulative Hazard Function from Estimated Parameters")
```