MSMS 408 : Practical 01

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Question

Use Monte Carlo Integration to approximate the following integrals

(i) $\int_0^1 e^{e^x} dx$, (ii) $\int_{-2}^2 e^{x+x^2} dx$

and check how the estimates converge to the true value as the sample size increases.

Theory

Suppose we have to compute

$$I = \int_a^b f(x)dx. \tag{1}$$

We rewrite

$$I = (b-a)E[f(X)] \tag{2}$$

where $X \sim U(a,b)$.

To approximate I, we generate random numbers $X_1, X_2, \ldots, X_n$ from $U(a,b)$ and calculate

$$I \approx (b-a) \cdot \frac{1}{n} \sum_{i=1}^{n} f(X_i). \tag{3}$$

By law of large numbers, $\frac{1}{n} \sum_{i=1}^{n} f(X_i) \to E[f(X)]$ as $n \to \infty$.

R Program

```r
set.seed(22)
```

```r
Monte.Carlo.Integration <- function(integrand, lower.limit, upper.limit, sample.size){

  x <- runif(n = sample.size, min = lower.limit, max = upper.limit)

  I <- (upper.limit - lower.limit) * mean(integrand(x))

  return(I)
}
```

1 $\int_0^1 e^{e^x}\,dx$

```
func_1 <- function(x) exp(exp(x))
```

```
sample_sizes <- c(100, 500, 700, 1000, 3000, 5000)
```

```
results.1 <- c()

for (size in sample_sizes) {
  results.1 <- append(results.1,
                      Monte.Carlo.Integration(integrand = func_1, sample.size = size,
                                              lower.limit = 0, upper.limit = 1))
}
```

```
df1 <- data.frame(sample.size = sample_sizes,
                  approximate.integral.value = results.1)
```

```
stargazer(df1, summary = FALSE, rownames = FALSE, label = "Table 1")
```

Table 1:

sample.size	approximate.integral.value
100	6.006
500	6.015
700	6.322
1,000	6.410
3,000	6.246
5,000	6.316

2 $\int_{-2}^{2} e^{x+x^2} dx$

```
func_2 <- function(x) exp(x + x^2)
```

```
results.2 <- c()

for (size in sample_sizes) {
  results.2 <- append(results.2,
                      Monte.Carlo.Integration(integrand = func_2, sample.size = size,
                                              lower.limit = -2, upper.limit = 2))
}
```

```
df2 <- data.frame(sample.size = sample_sizes,
                  approximate.integral.value = results.2)
```

```
stargazer(df2, summary = FALSE, rownames = FALSE, label = "Table 2")
```

Table 2:

sample.size	approximate.integral.value
100	107.312
500	103.273
700	93.557
1,000	98.880
3,000	95.832
5,000	95.987

Conclusion

✎ As sample size increases, the approximates gradually converge to the true integral value.