Random Sample Generation from Multivariate Normal Distribution

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While there exists function to generate random sample from a multivariate normal distribution, but here we discuss a very simple technique that accomplishes the job. Suppose we have to generate a sample of size n from $N_p\left(\underset{\sim}{\mu}, \Sigma\right)$. The steps are as follows :

Step I. Generate a sample of $n \times p$ from $N(0,1)$. Club each p of them to get a sample of size n from $N_p\left(\underset{\sim}{0}, I_p\right)$.

Step II. Consider eigenvalue decomposition of Σ *i.e.* $\Sigma = P\Lambda P'$ where P is a $p \times p$ matrix with columns as eigenvectors of Σ and Λ is a diagonal matrix of order p with eigenvalues of Σ in its diagonal entries. Σ is a symmetric matrix, so P is an orthogonal matrix. Now,

$$\begin{aligned} \underset{\sim}{X} &\sim N_p\left(\underset{\sim}{0}, I_p\right) \\ \Rightarrow P\sqrt{\Lambda} \cdot \underset{\sim}{X} &\sim N_p\left(\underset{\sim}{0}, P\sqrt{\Lambda} \cdot I_p \cdot \left(P\sqrt{\Lambda}\right)'\right) \equiv N_p\left(\underset{\sim}{0}, P\Lambda P'\right) \\ \Rightarrow \underset{\sim}{\mu} + P\sqrt{\Lambda} \cdot \underset{\sim}{X} &\sim N_p\left(\underset{\sim}{\mu}, \Sigma\right). \end{aligned}$$

Thus, $\underset{\sim}{\mu} + P\sqrt{\Lambda} \cdot \underset{\sim}{X}$ becomes a sample from $N_p\left(\underset{\sim}{\mu}, \Sigma\right)$.

R Program

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n <- 100; p <- 3
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a <- matrix(rnorm(n, mean = 0, sd = 1), nrow = p, ncol = n, byrow = TRUE)
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