MSMS 106

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Practical 01

✎ **Solve the following non-linear system of equations by Newton-Raphson Method.**

$$
\begin{aligned}
3x^2 + y^2 - 4 &= 0; \\
x^2 + xy + y^2 - 3 &= 0.
\end{aligned}
$$

Take $(\mathbf{x_0}, \mathbf{y_0}) = (\mathbf{0.8}, \mathbf{0.8})$ as your initial approximation of the solution.

➔ Let

$$
\begin{aligned}
f(x, y) &= 3x^2 + y^2 - 4; \\
g(x, y) &= x^2 + xy + y^2 - 3.
\end{aligned}
$$

```
f <- function(x, y) 3*(x^2) + y^2 - 4

g <- function(x, y) x^2 + x*y + y^2 - 3
```

We create a data-frame that stores our improved approximations of the solution and values of f and g at those approximate solutions.

```
df1 <- data.frame(x = c(0.8), y = c(0.8), f = f(0.8, 0.8), g = g(0.8, 0.8))
df1

##     x   y     f     g
## 1 0.8 0.8 -1.44 -1.08
```

Approximate solution (x_{k+1}, y_{k+1}) after $(k + 1)$ iteration(s) is given by

$$
\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \begin{bmatrix} f_x(x_k, y_k) & f_y(x_k, y_k) \\ g_x(x_k, y_k) & g_y(x_k, y_k) \end{bmatrix}^{-1} \cdot \begin{bmatrix} f(x_k, y_k) \\ g(x_k, y_k) \end{bmatrix}, \quad k = 0, 1, 2, \ldots
$$

Notations have their usual meanings.

```
library(Deriv)
```

```
f_x <- Deriv(f, "x")
f_x

## function (x, y)
## 6 * x
```

```r
f_y <- Deriv(f, "y")
f_y

## function (x, y)
## 2 * y
```

```r
g_x <- Deriv(g, "x")
g_x

## function (x, y)
## 2 * x + y
```

```r
g_y <- Deriv(g, "y")
g_y

## function (x, y)
## 2 * y + x
```

Here we shall perform 4 iterations.

```r
for (i in 1:4) {

  x_i <- df1$x[i]; y_i <- df1$y[i]

  A_matrix <- matrix(c(x_i, y_i), nrow = 2, byrow = TRUE)

  J_matrix <- matrix(c(f_x(x_i, y_i), f_y(x_i, y_i), g_x(x_i, y_i), g_y(x_i, y_i)),
                     nrow = 2, ncol = 2, byrow = TRUE)

  J_inv <- solve(J_matrix)

  B_matrix <- matrix(c(f(x_i, y_i), g(x_i, y_i)), nrow = 2, byrow = TRUE)

  result <- A_matrix - J_inv %*% B_matrix

  newrow <- data.frame(x = result[1, 1], y = result[2, 1],
                       f = f(result[1, 1], result[2, 1]), g = g(result[1, 1], result[2, 1]))

  df1 <- rbind(df1, newrow)
}
```

```r
df1

##          x        y             f             g
## 1 0.800000 0.800000 -1.440000e+00 -1.080000e+00
## 2 1.025000 1.025000  2.025000e-01  1.518750e-01
## 3 1.000305 1.000305  2.439396e-03  1.829547e-03
## 4 1.000000 1.000000  3.716892e-07  2.787669e-07
## 5 1.000000 1.000000  8.881784e-15  6.661338e-15
```

After the last iteration, value of f is $8.88 \times 10^{-15} \approx 0$ and that of g is $6.66 \times 10^{-15} \approx 0$. So we consider $(x, y) = (0, 0)$ as our solution to the given system of non-linear equations.