# MSMS 206 : Practical 03

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## Question

(1) Calculate $L$ and $U$ such that $A = LU$ where $L$ is a lower triangular matrix and $U$ is an upper triangular matrix for given

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 3 \\ 2 & 3 & 1 \end{bmatrix}.$$

(2) Solve the following system of linear equations using $LU$ decomposition method.

$$\begin{aligned} x_1 + x_2 - x_3 &= 4 \\ x_1 - 2x_2 + 3x_3 &= -6 \\ 2x_1 + 3x_2 + x_3 &= 7 \end{aligned}$$

$LU$ decomposition is only possible for non-singular matrices. It is assumed that there is no need for row swapping in the Gaussian elimination.

```r
LU_decomposer <- function(A){

  if(det(A) == 0) stop("Matrix must be non-singular.")

  I <- matrix(data = 0, nrow = nrow(A), ncol = ncol(A))

  for (i in 1:nrow(I)) {
    I[i, i] <- I[i, i] + 1
  }

  r <- dim(A)[1]
  c <- dim(A)[2]

  i <- 1; j <- 1

  while(j <= c) {

    while(i <= r) {
```

```r
      if(i != r){
        a1 <- as.matrix(A[(i+1):r, j] / A[i, j])

        a2 <- t(as.matrix(A[i, ]))

        A[(i+1):r, ] <- A[(i+1):r, ] - a1 %*% a2

        I[(i+1):r, j] <- as.vector(a1)

        break
      }
      i <- i + 1
    }
    j <- j + 1

    i <- j
  }

  return(list(I, A))
}
```

***LU_decomposer()*** returns a list containing $L$ and $U$ respectively.

```r
A <- matrix(data = c(1, 1, -1,
                     1, -2, 3,
                     2, 3, 1),
            nrow = 3, ncol = 3, byrow = TRUE)
```

```r
LU_decomposer(A)

## [[1]]
##      [,1]       [,2] [,3]
## [1,]    1  0.0000000    0
## [2,]    1  1.0000000    0
## [3,]    2 -0.3333333    1
##
## [[2]]
##      [,1] [,2]      [,3]
## [1,]    1    1 -1.000000
## [2,]    0   -3  4.000000
## [3,]    0    0  4.333333
```

```r
L <- LU_decomposer(A)[[1]]
U <- LU_decomposer(A)[[2]]
```

➔

```r
b <- matrix(data = c(4, -6, 7), nrow = 3, ncol = 1, byrow = TRUE)
```

- $$\begin{aligned} A\underset{\sim}{x} &= \underset{\sim}{b} \\ \Rightarrow LU\underset{\sim}{x} &= \underset{\sim}{b} \\ \Rightarrow \underset{\sim}{x} &= U^{-1}L^{-1}\underset{\sim}{b} \end{aligned}$$

```r
solve(U) %*% solve(L) %*% b
```

```
##      [,1]
## [1,]    1
## [2,]    2
## [3,]   -1
```

✎ The solution of the given system of equations is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$