MSMS 106

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Practical 02

✎ **Implement bisection method for solution of numerical equations.**

➜ In bisection method, first a sufficiently small interval (a, b) containing at least one root of the equation $f(x) = 0$ is taken. We must have $f(a) \cdot f(b) < 0$.

Let x_1 be the mid-point of the interval, i.e. $x_1 = \dfrac{a+b}{2}$. Then a real root of the equation must lie either in the interval $(a, x_1]$ or in the interval $[x_1, b)$.

If $f(x_1) = 0$, then x_1 is a root of the equation. Otherwise if $f(a) \cdot f(x_1) < 0$, our interval of interest becomes (a, x_1), or if $f(x_1) \cdot f(b) < 0$, our interval of interest becomes (x_1, b).

After repeating this procedure n times, the mid-point of the last interval is taken as an approximate solution of the given equation $f(x) = 0$.

The larger the value of n, the better will be the accuracy of the root.

```r
bisection_method <- function(func, a, b, iterations){

  if(func(a) * func(b) >= 0) stop("Incorrect a or b or both.")

  i <- 1

  while(i <= iterations){

    midpoint <- (a + b) / 2

    if(func(midpoint) == 0){
      break
    } else if(func(a) * func(midpoint) < 0){
      a <- a; b <- midpoint
    } else if(func(midpoint) * func(b) < 0){
      a <- midpoint; b <- b
    }

    i <- i + 1
  }
  return((a + b) / 2)
}
```

- **Example 1** : $f(x) = x^3 - 4x - 9$; $a = 2$; $b = 3$

```
f1 <- function(x) x^3 - 4*x - 9
sol1 <- bisection_method(func = f1, a = 2, b = 3, iterations = 5)
sol1

## [1] 2.703125
```

```
f1(sol1)

## [1] -0.06107712
```

- **Example 2** : $f(x) = x^4 + 2x^2 - x - 1$; $a = 0$; $b = 1$

```
f2 <- function(x) x^4 + 2*(x^2) - x - 1
sol2 <- bisection_method(func = f2, a = 0, b = 1, iterations = 7)
sol2

## [1] 0.8242188
```

```
f2(sol2)

## [1] -0.004047509
```

- **Example 3** : $f(x) = x^3 - x - 1$; $a = 1.25$; $b = 1.5$

```
f3 <- function(x) x^3 - x - 1
sol3 <- bisection_method(func = f3, a = 1.25, b = 1.5, iterations = 8)
sol3

## [1] 1.324707
```

```
f3(sol3)

## [1] -4.659488e-05
```