

MSMS 308 : Practical 05

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➔ Question

Generate a sample of lifetime data from a log-normal distribution, estimate its shape and scale parameters using Maximum Likelihood Estimation (MLE), and compute the Mean Squared Error (MSE) of the estimates through simulation. Using the estimated parameter values, plot the Probability Density Function (PDF), Survival Function, Hazard Function, and Cumulative Hazard Function of the log-normal model.

➔ Theory

Let T_1, T_2, \dots, T_n be a sample of size n from $\text{LN}(\mu, \sigma^2)$. Then $\forall i = 1(1)n, \ln T_i \stackrel{iid}{\sim} N(\mu, \sigma^2)$.

The Maximum Likelihood estimates of μ and σ^2 are given by

$$\hat{\mu}_{MLE} = \frac{1}{n} \sum_{i=1}^n \ln T_i \text{ and}$$

$$\widehat{\sigma^2}_{MLE} = \frac{1}{n-1} \sum_{i=1}^n (\ln T_i - \hat{\mu}_{MLE})^2$$

➔ R Program

```
times <- rlnorm(100)
log_times <- log(times)
```

```
mean(log_times)
```

```
## [1] 0.06885595
```

$\therefore \hat{\mu}_{MLE} = 0.0688559.$

```
sd(log_times); sd(log_times)^2
```

```
## [1] 0.9273728
```

```
## [1] 0.8600203
```

$\therefore \widehat{\sigma^2}_{MLE} = 0.8600203.$

```
true_mu <- 0; true_sigma <- 1
```

```
sample_size <- 100
```

Now we shall evaluate the bias and MSE of the estimates.

```
mu_estimates = sigma_square_estimates = c()

for (i in 1:100){

  times <- rlnorm(sample_size)

  mu_estimates[i] <- mean(log(times))

  sigma_square_estimates[i] <- sd(log(times))^2
}

mu_bias <- mean(mu_estimates) - true_mu

sigma_square_bias <- mean(sigma_square_estimates) - true_sigma^2

mu_MSE <- mean( (mu_estimates - true_mu)^2 )

sigma_square_MSE <- mean( (sigma_square_estimates - true_sigma^2)^2 )
```

```
mu_bias; sigma_square_bias; mu_MSE; sigma_square_MSE
```

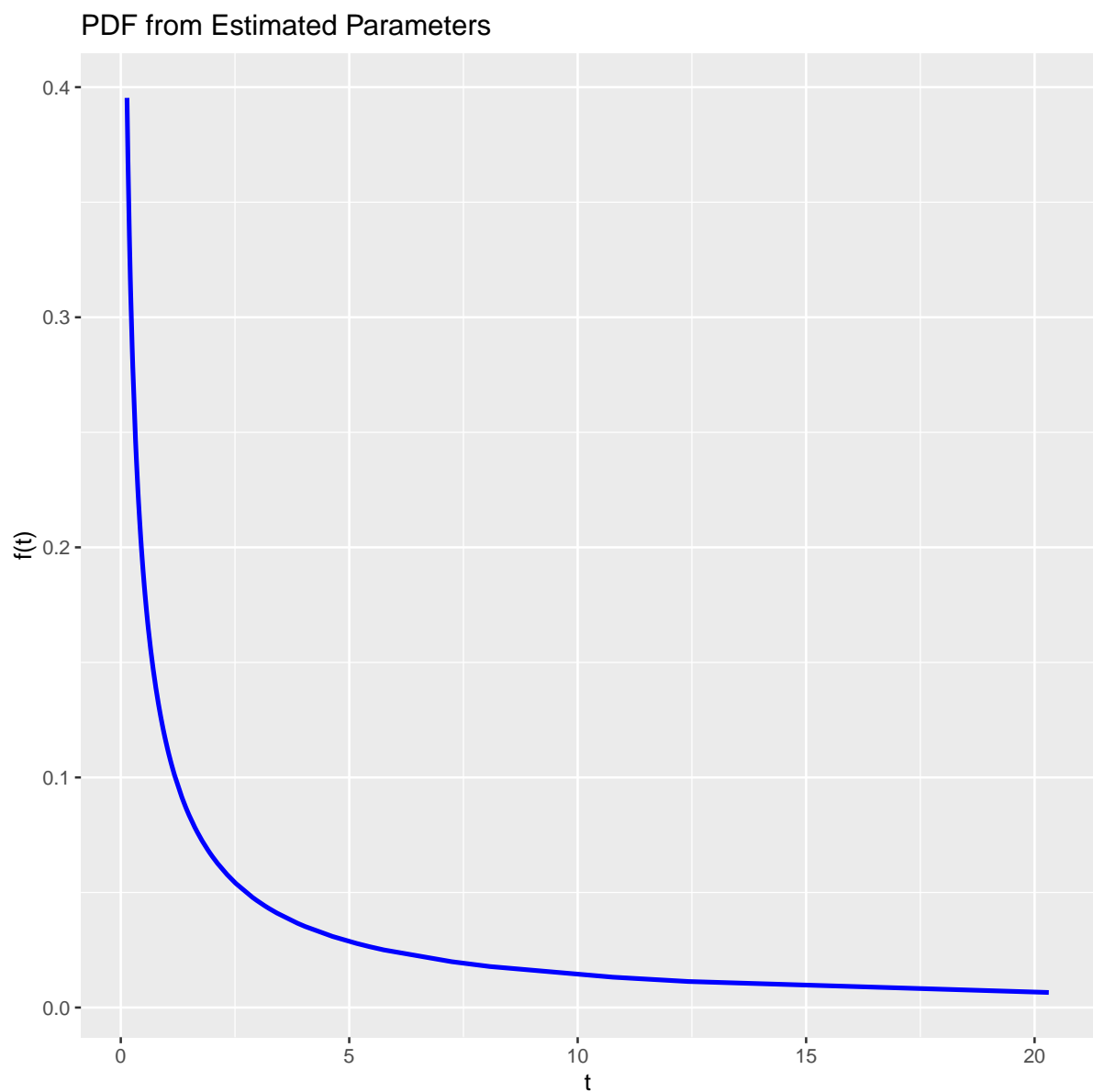
```
## [1] -0.0215415
## [1] 0.00972938
## [1] 0.008702948
## [1] 0.01729812
```

```
t_values <- rlnorm(sample_size)
```

```
mu_hat <- mean(t_values); sigma_hat <- sd(t_values)
```

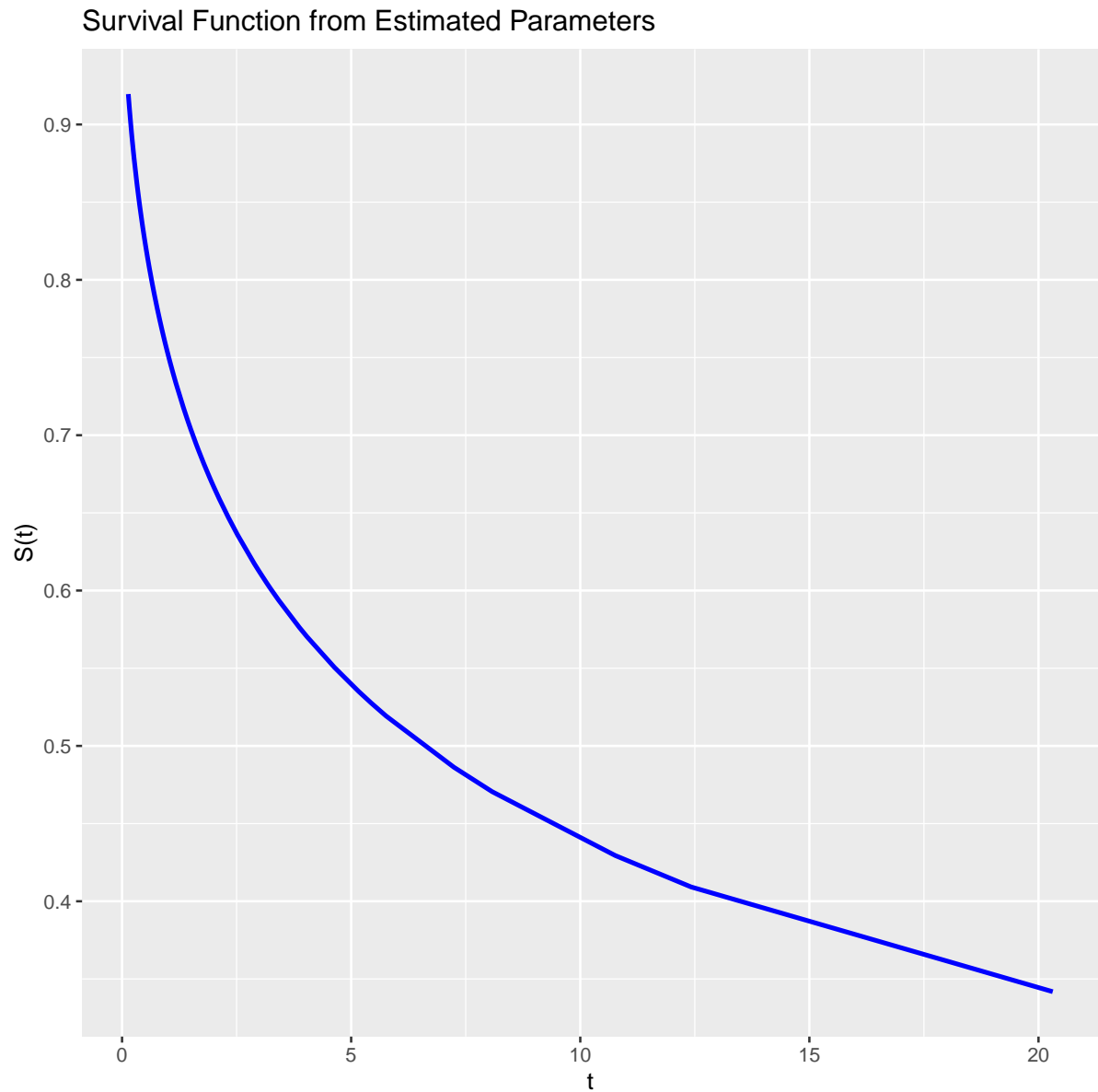
```
df1 <- data.frame(t = t_values,  
                  ft = dlnorm(t_values, meanlog = mu_hat, sdlog = sigma_hat))
```

```
df1 %>%  
  ggplot(aes(x = t, y = ft)) +  
  geom_line(col = 'blue', linewidth = 1) +  
  labs(x = "t", y = "f(t)",  
       title = "PDF from Estimated Parameters")
```



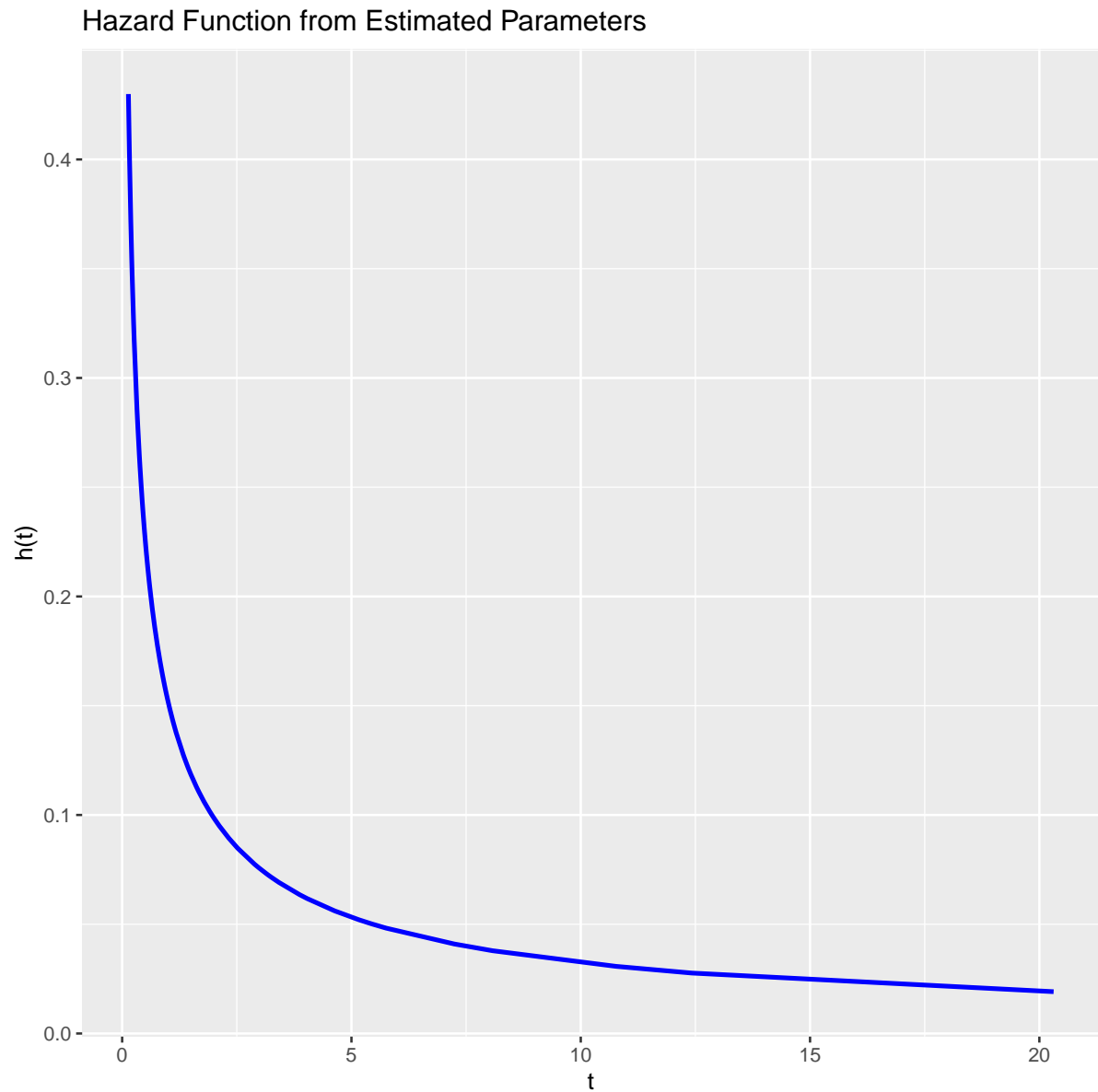
```
df2 <- data.frame(t = t_values,
                  St = 1 - plnorm(t_values, meanlog = mu_hat, sdlog = sigma_hat))

df2 %>%
  ggplot(aes(x = t, y = St)) +
  geom_line(col = 'blue', linewidth = 1) +
  labs(x = "t", y = "S(t)",
       title = "Survival Function from Estimated Parameters")
```



```
df3 <- data.frame(t = t_values,
                  ht = df1$ft / df2$St)

df3 %>%
  ggplot(aes(x = t, y = ht)) +
  geom_line(col = 'blue', linewidth = 1) +
  labs(x = "t", y = "h(t)",
       title = "Hazard Function from Estimated Parameters")
```



```
df4 <- data.frame(t = t_values,
                  Ht = -log(df2$St))

df4 %>%
  ggplot(aes(x = t, y = Ht)) +
  geom_line(col = 'blue', linewidth = 1) +
  labs(x = "t", y = "H(t)",
       title = "Cumulative Hazard Function from Estimated Parameters")
```

