

MSMS - 105

Ananda Biswas

Assignment 03

❖ **Objective :** To visually illustrate the concept of **Statistical Regularity** through an example.

⊕ **Theory :** Statistics deals with random experiments. We may think of randomness as “irregularity”. However, in certain situations, even outcomes of random experiments show a regularity that is similar to deterministic behaviour. This is called Statistical Regularity.

In the following we shall consider a mathematical version of Ludo and see how a *deterministic behaviour* shapes out of many trials of a random experiment.

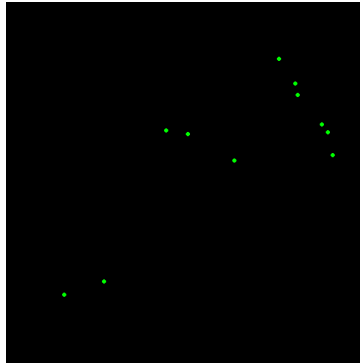
Assume \mathbb{R}^2 as our Ludo board and consider a die with 4 equally likely outcomes. We start our game with $(x, y) = (0, 0)$. We toss the die and update (x, y) as per the following rules.

Die result	New (x, y)
1	$(0.8x + 0.1, 0.8y + 0.04)$
2	$(0.5x + 0.25, 0.5y + 0.4)$
3	$(0.355(x - y) + 0.266, 0.355(x + y) + 0.078)$
4	$(0.355(x + y) + 0.378, 0.355(y - x) + 0.434)$

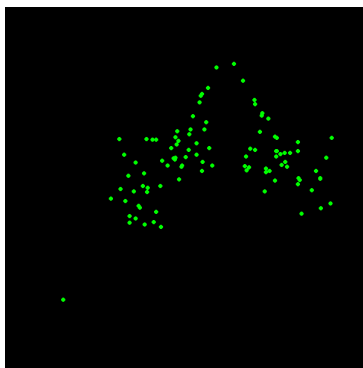
On getting new (x, y) , we plot it on \mathbb{R}^2 .

⊕ **Visualization :** [Program to create the following animation is here.](#)

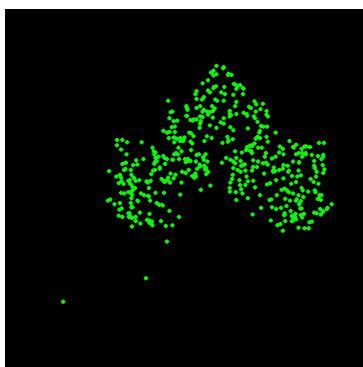
After playing the game for $n = 10$ times, we get the following scatter of points - just 10 random points with no visible pattern.



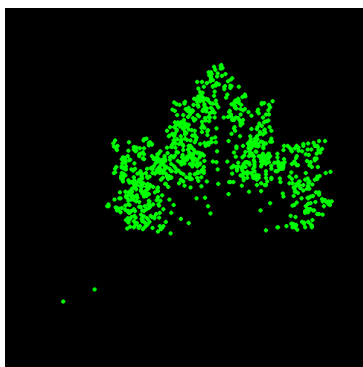
For $n = 100$, we get



For $n = 500$, we get

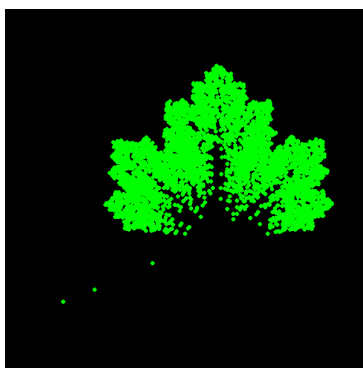


For $n = 1000$, we get



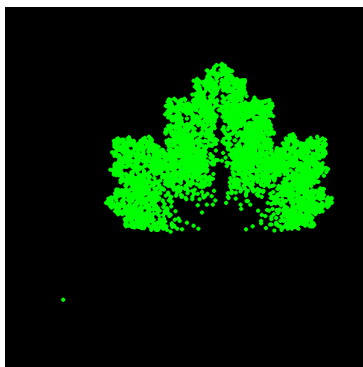
A certain cluster of points seems to be getting prominent gradually.

For $n = 5000$, we get



All these random points together create a leaf.

Notice that, all these 5000 points random points. But together they create a deterministic shape like a leaf. If we again play the game 5000 times, we shall get a new plot as below.



The new 5000 points are another set of random points, hardly similar with previous 5000 points. But they create the same scatter of points exactly identical as before. This phenomenon of getting a deterministic behaviour out of randomness is called **Statistical Regularity**.

Leaves of plants, fingerprints on hands are great examples how statistical regularity is part and parcel of nature.