MSMS 105 : Assignment 07

Ananda Biswas

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Objective

Random number generation from $N(\mu, \sigma^2)$.

Theory

Let U_1 and U_2 be two iid $U(0,1)$ variates.

Then,

$$X_1 = \sqrt{-2\ lnU_1}\ \cos(2\pi U_2) \text{ and}$$
$$X_2 = \sqrt{-2\ lnU_1}\ \sin(2\pi U_2)$$

are independently distributed $N(0,1)$ variates.

R Program

```
random_numbers <- function(n, seed = NULL){
  a <- 1103515245
  b <- 12345
  m <- 2^31 - 1

  if(is.null(seed)){
    start_date <- as.POSIXct("2003-01-01 00:00:00", tz = "UTC")

    current_date <- Sys.time()

    seed <- as.numeric(difftime(current_date, start_date, units = "secs"))
  }

  x <- c(seed)

  for (i in 2:n) {
    x[i] <- (a * x[i-1] + b) %% m
  }

  return(x)
}
```

```
uniform_random_numbers <- function(n, seed = NULL){
  return(random_numbers(n, seed) / (2^31 - 1))
}
```

```r
normal_random_numbers <- function(n, mu, sigma, seed = NULL){

  u1 <- uniform_random_numbers(n, seed)
  u2 <- uniform_random_numbers(n, seed)

  x <- sqrt(-2 * log(u1)) * cos(2 * pi * u2)

  x <- mu + sigma * x

  return(x)
}
```

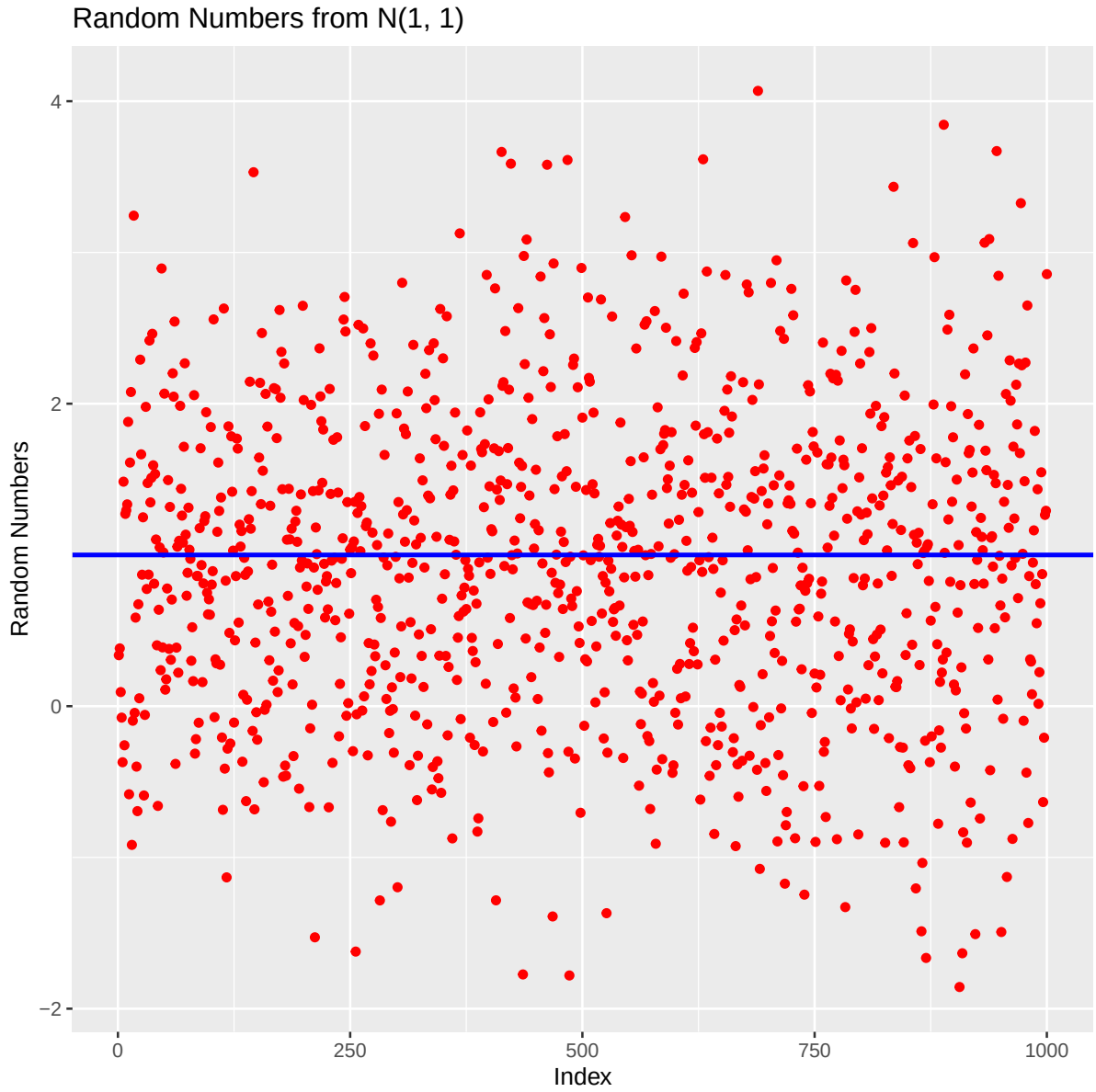
```r
size <- 10
normal_random_numbers(size, 1, 1)
```

```
##  [1] 0.3370971 1.3099430 2.7139209 1.5683085 1.4309843 3.1268405 1.5811996
##  [8] 0.9139678 2.9715475 1.6044944
```

Visualization

```
size <- 1000
normal_numbers <- data.frame(n = 1:size,
                             num = normal_random_numbers(size, 1, 1))
```

```
normal_numbers %>%
  ggplot(aes(x = n, y = num)) +
  geom_point(size = 1.5, col = "red") +
  geom_hline(yintercept = 1, col = "blue", linewidth = 1) +
  labs(x = "Index", y = "Random Numbers",
       title = "Random Numbers from N(1, 1)")
```



Conclusion

✎ Scatteredness of the points mimics that of a $N(1,1)$ Distribution.