MSMS 308 : Practical 03

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Question

Given that $\underset{\sim}{X} \sim N_3(\underset{\sim}{\mu}, \Sigma)$ with

$$\underset{\sim}{\mu} = \begin{bmatrix} 1 \\ 0.5 \\ 0.25 \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 1 & 0.8 & 0.4 \\ 0.8 & 1 & 0.56 \\ 0.4 & 0.56 & 1 \end{bmatrix}.$$

(1) Find the marginal distribution of $Y^{(1)}$ and $\underset{\sim}{Y}^{(2)}$ where

$$Y^{(1)} = X_1 - \Sigma_{12}\Sigma_{22}^{-1}\underset{\sim}{X}^{(2)}, \qquad \underset{\sim}{Y}^{(2)} = \underset{\sim}{X}^{(2)} = \begin{bmatrix} X_2 \\ X_3 \end{bmatrix}.$$

(2) Find the conditional distribution $X_1|X_2$.

(3) Find the multiple correlation coefficient $R_{1.23}$ and the partial correlation coefficient $r_{12.3}$.

R Program

- The marginal distribution of $Y^{(1)}$ is $N(\mu_1 - \Sigma_{12}\Sigma_{22}^{-1}\underset{\sim}{\mu}^{(2)}, \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$.

```r
mu <- matrix(c(1, 0.5, 0.25), nrow = 3, byrow = TRUE)

Sigma <- matrix(c(1, 0.8, 0.4,
                  0.8, 1, 0.56,
                  0.4, 0.56, 1), nrow = 3, ncol = 3, byrow = TRUE)
```

```r
a1 <- mu[1, 1]

a2 <- Sigma[1, 2:3] %*% solve(Sigma[-1, -1]) %*% mu[2:3]
```

```r
a1 - a2

##           [,1]
## [1,] 0.5979021
```

```r
a3 <- Sigma[1, 1] - Sigma[1, 2:3] %*% solve(Sigma[-1, -1]) %*% Sigma[2:3, 1]; a3

##           [,1]
## [1,] 0.3566434
```

$\therefore Y^{(1)} \sim N(0.5979021, 0.3566434)$.

The marginal distribution of $\underset{\sim}{Y}^{(2)}$ is $N_2(\mu^{(2)}, \Sigma_{22})$ where

$$\underset{\sim}{\mu}^{(2)} = \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}, \qquad \Sigma_{22} = \begin{bmatrix} 1 & 0.56 \\ 0.56 & 1 \end{bmatrix}.$$

⚁ The conditional distribution $X_1|X_2 = x_2$ is $N\left(\mu_1 - \rho\frac{\sigma_1}{\sigma_2}(x_2 - \mu_2), \sigma_1^2(1-\rho^2)\right)$.

```
x2 <- 1
b1 <- mu[1] - Sigma[1, 2] * (x2 - mu[2])

b2 <- (1 - Sigma[1, 2]^2)
```

```
b1; b2

## [1] 0.6
## [1] 0.36
```

$\therefore X_1|X_2 = 1 \sim N(0.6, 0.36)$.

In general, $X_1|X_2 = x_2 \sim N(1 - 0.8(x_2 - 0.5), 0.36)\ \forall x_2 \in \mathbb{R}$.

⚂ $R_{1.23} = \sqrt{\dfrac{\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}}{\Sigma_{11}}}$.

```
R_1.23 <- sqrt(Sigma[1, 2:3] %*% solve(Sigma[-1, -1]) %*% Sigma[2:3, 1])
R_1.23

##           [,1]
## [1,] 0.8020952
```

$\therefore R_{1.23} = 0.8020952$.

$r_{12.3} = -\dfrac{R_{12}}{\sqrt{R_{11}R_{22}}}$, here the variances are 1, so $R = \Sigma$.

```
cofactor <- function(mat, i, j) {
  minor <- mat[-i, -j]
  return ((-1)^(i + j) * det(minor))
}
```

```
r_1.23 <- - cofactor(Sigma, 1, 2) / sqrt(cofactor(Sigma, 1, 1) * cofactor(Sigma, 2, 2))
r_1.23

## [1] 0.7585674
```

$\therefore r_{1.23} = 0.7585674$.