# MSMS 106

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**Practical 05**

## 🎯 Polynomial Approximation

✎ Find a linear fit to the following data.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 1.2 | 2.3 | 2.9 | 4.1 | 5.2 |

➔ Let $f(x) \approx P_1(x) = a_0 + a_1 x$.

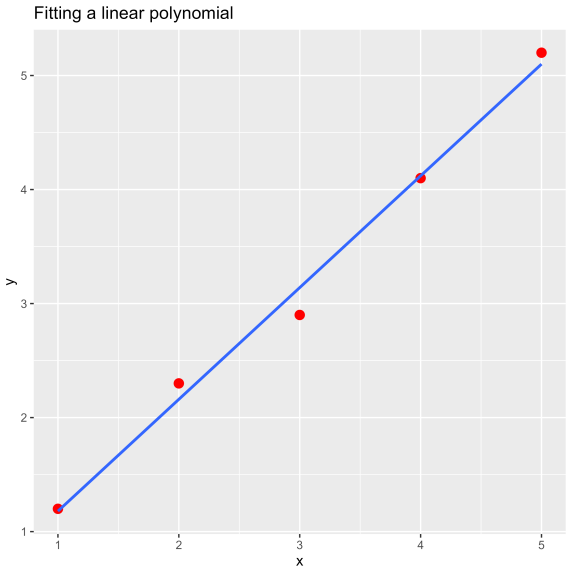
```
df1 <- data.frame(x = 1:5,
                  y = c(1.2, 2.3, 2.9, 4.1, 5.2))
```

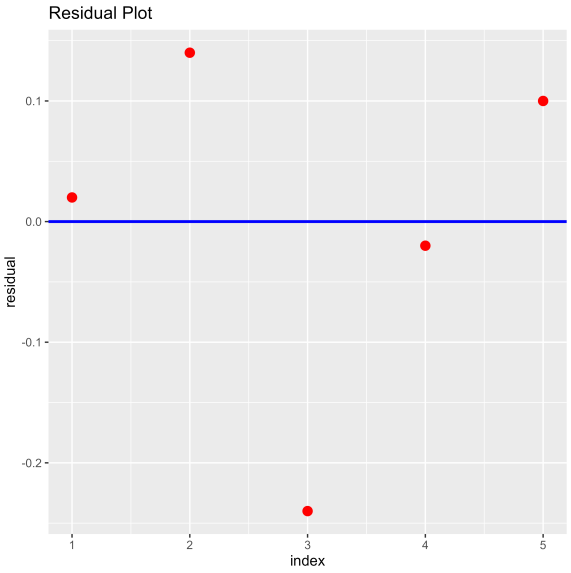
```
fit1 <- lm(y ~ x, data = df1)
fit1$coefficients

## (Intercept)           x
##        0.20        0.98
```

So, $P_1(x) = 0.2 + 0.98x$.

```
df1 %>%
  ggplot(aes(x = x, y = y)) +
  geom_point(col = "red", size = 3) +
  geom_smooth(method = "lm", formula = y ~ x, se = FALSE) +
  labs(title = "Fitting a linear polynomial")
```

```
fit1_residuals <- data.frame(index = 1:5, residual = fit1$residuals)

fit1_residuals %>%
  ggplot(aes(x = index, y = residual)) +
  geom_point(col = "red", size = 3) +
  geom_hline(yintercept = 0, col = "blue", linewidth = 1) +
  labs(title = "Residual Plot")
```




✎ Find a linear fit to the following data.

| $x$ | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
|---|---|---|---|---|---|
| $f(x)$ | 0.447 | 0.632 | 0.775 | 0.894 | 1 |

➔ Let $f(x) \approx P_1(x) = a_0 + a_1 x$.

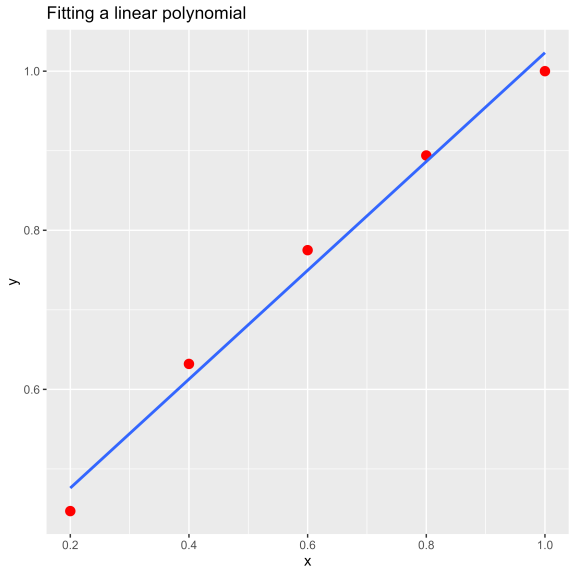
```
df2 <- data.frame(x = c(0.2, 0.4, 0.6, 0.8, 1),
                  y = c(0.447, 0.632, 0.775, 0.894, 1))
```

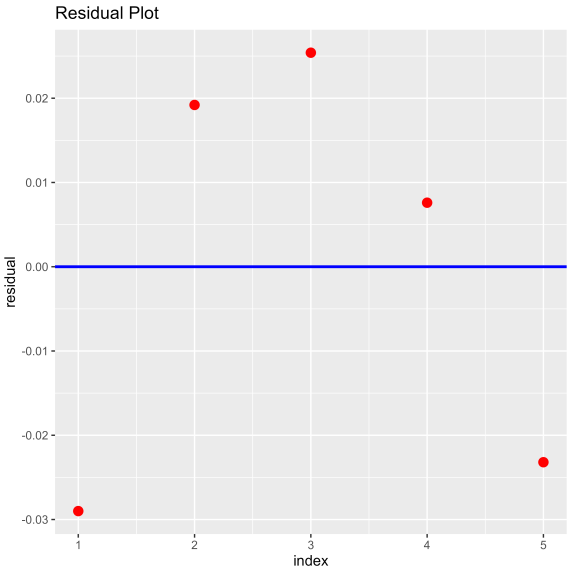
```
fit2 <- lm(y ~ x, data = df2)
fit2$coefficients

## (Intercept)           x
##      0.3392      0.6840
```

So, $P_1(x) = 0.3392 + 0.684x$.

```
df2 %>%
  ggplot(aes(x = x, y = y)) +
  geom_point(col = "red", size = 3) +
  geom_smooth(method = "lm", formula = y ~ x, se = FALSE) +
  labs(title = "Fitting a linear polynomial")
```

```
fit2_residuals <- data.frame(index = 1:5, residual = fit2$residuals)

fit2_residuals %>%
  ggplot(aes(x = index, y = residual)) +
  geom_point(col = "red", size = 3) +
  geom_hline(yintercept = 0, col = "blue", linewidth = 1) +
  labs(title = "Residual Plot")
```




✎ Fit a second-degree polynomial to the following data.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 2.2 | 4.8 | 8.5 | 14.1 | 20.2 |

➔ Let $f(x) \approx P_2(x) = a_0 + a_1 x + a_2 x^2$.

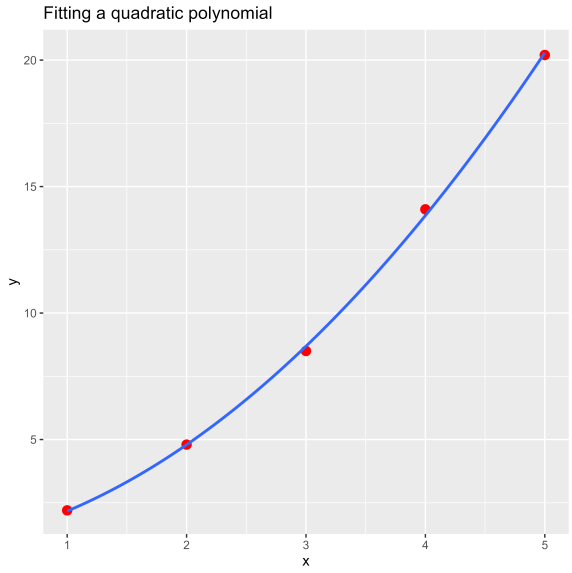
```
df3 <- data.frame(x = 1:5,
                  y = c(2.2, 4.8, 8.5, 14.1, 20.2))
```

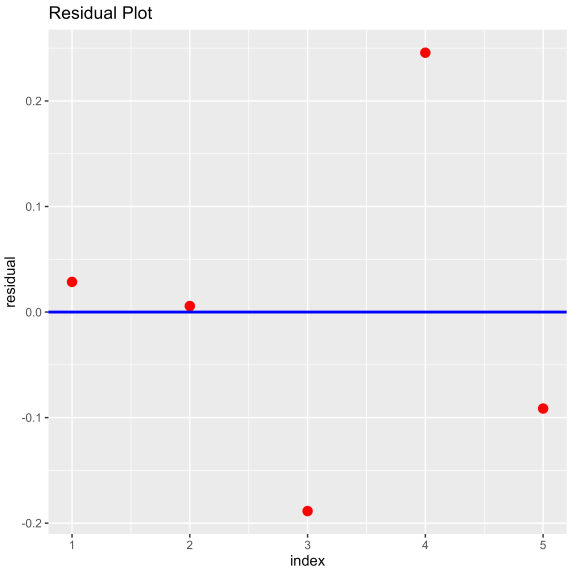
```
fit3 <- lm(y ~ x + I(x^2), data = df3)
fit3$coefficients

## (Intercept)           x      I(x^2)
##   0.8200000   0.7157143   0.6357143
```

So, $P_2(x) = 0.82 + 0.7157143x + 0.6357143x^2$.

```
df3 %>%
  ggplot(aes(x = x, y = y)) +
  geom_point(col = "red", size = 3) +
  geom_smooth(method = "lm", formula = y ~ x + I(x^2), se = FALSE) +
  labs(title = "Fitting a quadratic polynomial")
```

```
fit3_residuals <- data.frame(index = 1:5, residual = fit3$residuals)

fit3_residuals %>%
  ggplot(aes(x = index, y = residual)) +
  geom_point(col = "red", size = 3) +
  geom_hline(yintercept = 0, col = "blue", linewidth = 1) +
  labs(title = "Residual Plot")
```


Fitting a quadratic polynomial
20
15
10
5
y
1
2
3
4
5
x


Residual Plot
0.2
0.1
0.0
-0.1
-0.2
residual
1
2
3
4
5
index