MSMS 206 : Practical 10

Ananda Biswas

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🎯 **Question :** In a certain social mobility study, the population under consideration was divided into three income groups: upper, middle and lower. It has been found that 70% of sons of upper income group parents themselves become upper income, 20% middle income and 10% lower income. Of the sons of middle-income parents, 30% move to upper income group, 50% remain in middle income group and 20% become lower. Of the sons of lower income parents, 10% move to upper income group, 20% to middle income group and 70% remain lower. Draw up a matrix to represent these movements.

At a certain point of time, the population is found to have 10% men in upper income group, 50% in middle income group and 40% in lower. Assuming that each man has one son and one grandson, what will be the group composition of grandsons?

➊ Transition Probability Matrix

```
TPM <- matrix(c(0.7, 0.2, 0.1,
                0.3, 0.5, 0.2,
                0.1, 0.2, 0.7), nrow = 3, ncol = 3, byrow = TRUE)

states <- c("upper", "middle", "lower")

rownames(TPM) <- states
colnames(TPM) <- states
```

✎ The transition probability matrix is

```
TPM
```

```
##        upper middle lower
## upper    0.7    0.2   0.1
## middle   0.3    0.5   0.2
## lower    0.1    0.2   0.7
```

➋ Future Distribution from Initial Distribution

```
X_0 <- matrix(c(0.1, 0.5, 0.4), nrow = 1)
X_1 <- X_0 %*% TPM
X_2 <- X_1 %*% TPM
```

✎ The group composition of grandsons is

```
X_2
```

```
##      upper middle lower
## [1,] 0.326  0.305 0.369
```