MSMS - 105

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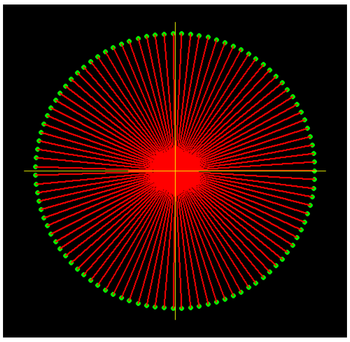
Assignment 04

❖ **Objective :** To create animated plots to get visual illustrations of different aspects of **Matrix Multiplication**.

➲ ***Theory*** : Essence of matrix multiplication is best understood when it is seen as a linear transformation. The geometric interpretation of matrix multiplication provides insights into how matrices transform vectors of a vector space.

In the following we shall see different aspects of matrix multiplication. For best visualization experience, we have considered $\mathbb{R}^2$ as our vector space. In each of the illustrations, we have taken 100 vectors originated at $(0, 0)$ and their tips together form a circle. We shall see how matrix multiplication changes the vectors and consequently the circular shape.

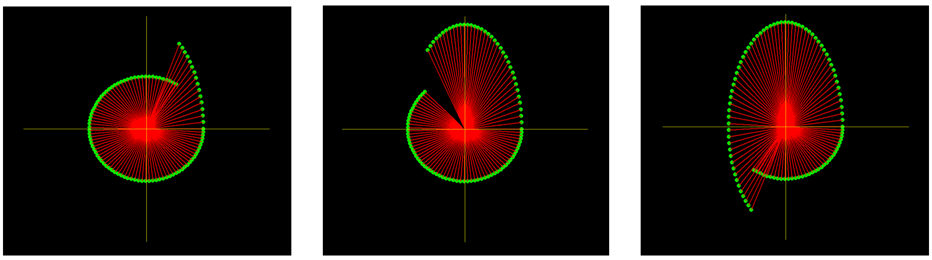


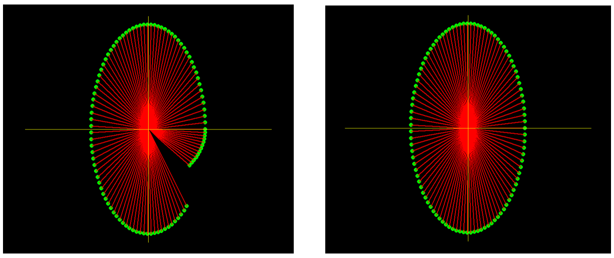
Program to create the above plot is here.

✎ ***Scaling*** : Pre-multiplying any vector $\begin{bmatrix} a \\ b \end{bmatrix} \in \mathbb{R}^2$ by $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$ scales the vector by a factor of s_x along $x-$axis and by a factor of s_y along $y-$axis.

Here we pre-multiply $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ with initial 100 vectors.

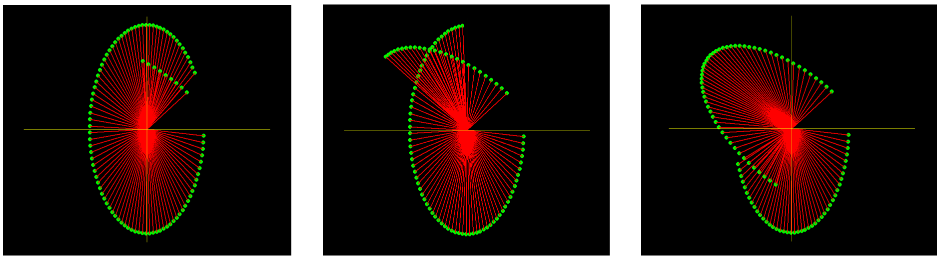
➲ ***Visualization*** : **Program to create the following animation is here.**

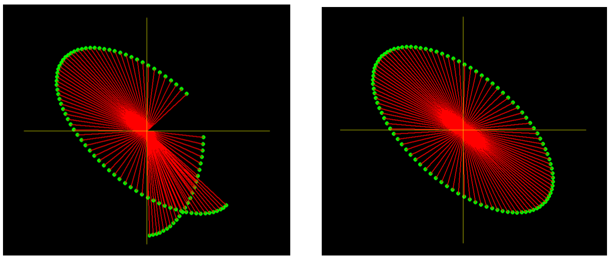




Rotating : Pre-multiplying any vector $\begin{bmatrix} a \\ b \end{bmatrix} \in \mathbb{R}^2$ by $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ rotates the vector by an angle θ anti-clockwise. Here we take $\theta = \dfrac{\pi}{4}$ and rotate the above vectors by 45° anti-clockwise.

Visualization : **Program to create the following animation is here.**

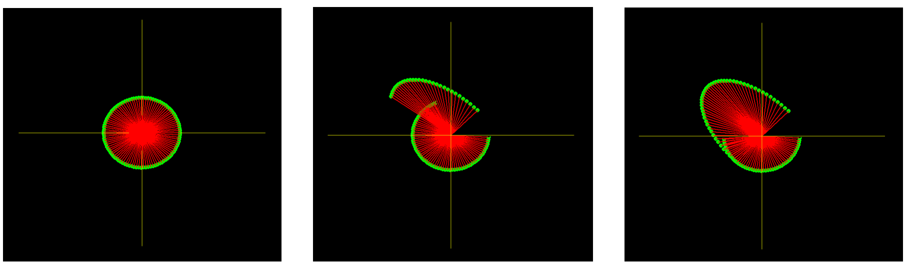


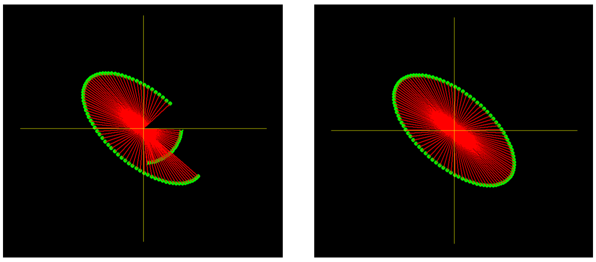


Composition of Transformation : Multiplication of two matrices results in a matrix that represents the combination of their transformations. Above we first stretched the 100 vectors two times along $y-$axis and then rotated them 45° anti-clockwise. We can achieve the same by just pre-multiplying all the vectors by

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}_{\theta=\frac{\pi}{4}} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0.7071068 & -1.414214 \\ 0.7071068 & 1.414214 \end{bmatrix}$$

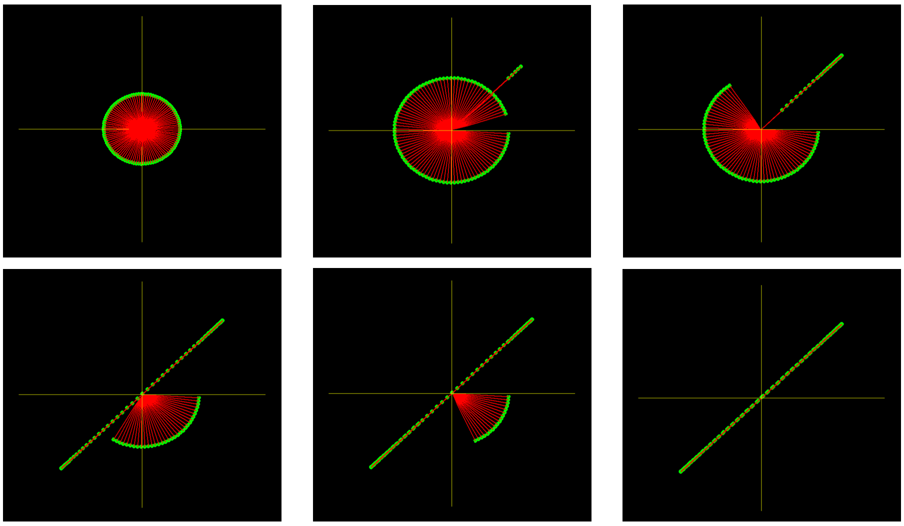
Visualization : **Program to create the following animation is here.**





Vector Spaces and Dimension : Multiplying all the vectors of a vector space by a matrix of rank r creates a new vector space of dimension r. Here we pre-multiply all the 100 vectors by $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ which has rank 1 and see how $\mathbb{R}^2$ reduces to a straight line only.

Visualization : **Program to create the following animation is here.**



Eigenvectors and Eigenvalues : Eigenvectors are those special vectors that, under the linear transformation defined by a matrix, remain within their own span, being scaled by a value, positive or negative depending on change of direction, called corresponding eigenvalues.

In the ***Scaling*** example, observe that the vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ get transformed to $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ respectively but remain in their corresponding spans only. This makes $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ eigenvectors of $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ with corresponding eigenvalues 1 and 2.

Visualization :