

1. $G'(k_1, k_2) = G(k_1) || G(k_2)$ is a **secure** PRG, because the $G(k)$ is a secure PRG and k_1, k_2 are different keys, the concatenation of $G(k_1)$ and $G(k_2)$ is still "random" and its output is still "indistinguishable" from a true random.

$G'(k) = G(0)$ is a **not secure** PRG, because the output of $G(0)$ is not "random".

$G'(k) = G(k)$ is a **secure** PRG, because the $G(k)$ is a secure PRG.

$G'(k) = G(k) || 0$ is a **not secure** PRG, because the last bit of $G'(k)$ is zero. Its output is not "random".

$G'(k) = G(k \oplus 1^S)$ is a **secure** PRG. Let $k_1 = k \oplus 1^S$, we can easily know $k_1 \xleftarrow{R} \{0,1\}^S$, and $G'(k) = G(k_1)$. $G(k_1)$ is a secure PRG, so $G'(k)$ is.

$G'(k) = \text{reserver}(G(k))$ is a **secure** PRG. After performing the reserver() operation, its output is still "random" and is "indistinguishable" from a true random.

2. First give the advantage formula:

$$\begin{aligned} Adv_{PRG}[A, G'] &= \left| Pr_{k_1, k_2 \xleftarrow{R} K} [A(G'(k_1, k_2)) = 1] - Pr_{r \xleftarrow{R} \{0,1\}^n} [A(r) = 1] \right| \\ &= \left| Pr_{k_1, k_2 \xleftarrow{R} K} [A(G(k_1) \wedge G(k_2)) = 1] - Pr_{r \xleftarrow{R} \{0,1\}^n} [A(r) = 1] \right| \end{aligned}$$

For a random string r in $\{0,1\}^n$, we have

$$\Pr[A(r) = 1] = \frac{1}{2}$$

For a secure PRG, $G: K \rightarrow \{0, 1\}^n$, we also have

$$\Pr[A(G(k)) = 1] = \frac{1}{2}$$

Only when the two corresponding binary bits are both 1, the result bit is 1, thus

$$Pr[A(G(k_1) \wedge G(k_2)) = 1] = \frac{1}{2} * \frac{1}{2} = \frac{1}{4}$$

Finally, we get

$$Adv_{PRG}[A, G'] = \left| \frac{1}{4} - \frac{1}{2} \right| = \frac{1}{4} = 0.25$$

3. $E'((k, k'), m) = E(k, m) || E(k', m)$ is **semantically secure**. Because (E, D) is a one-time semantically secure cipher and k, k' are different random key in K , for every $m_0, m_1 \in M$, the semantics security advantage of all efficient A against this E' is still negligible.

$E'(k, m) = E(0^n, m)$ is **not semantically secure**. Let $m_0 = 0^n$, $m_1 = 1^n$, an adversary A can ask for the encryption of m_0 and m_1 , and because the key is 0^n , A can easily distinguish $EXP(0)$ from $EXP(1)$.

$E'(k,m) = E(k,m)||k$ is **not semantically secure**. Because every CT contains the secret key k . For every $m_0, m_1 \in M$, adversary A can get the secret key k from the CT and use this key to decrypt the CT.

$E'(k,m) = E(k,m)||LSB(m)$ is **not semantically secure**. Let the LSB of m_0 be 0, the LSB of m_1 be 1, and the rest are the same. An adversary A can ask for the encryption of m_0, m_1 and can easily distinguish $EXP(0)$ from $EXP(1)$.

4. The ASCII of "attack at dawn": '61747461636b206174206461776e', let it be PT_1 .
The ASCII of "attack at dusk": '61747461636b206174206475736b', let it be PT_2 .

We have the CT_1 of PT_1

$$CT_1 = '6c73d5240a948c86981bc294814d'$$

Thus,

$$\begin{aligned} CT_2 &= OTP(k, PT_2) = OTP(CT_1 \oplus PT_1, PT_2) = CT_1 \oplus PT_1 \oplus PT_2 \\ &= '6c73d5240a948c86981bc2808548' \end{aligned}$$