

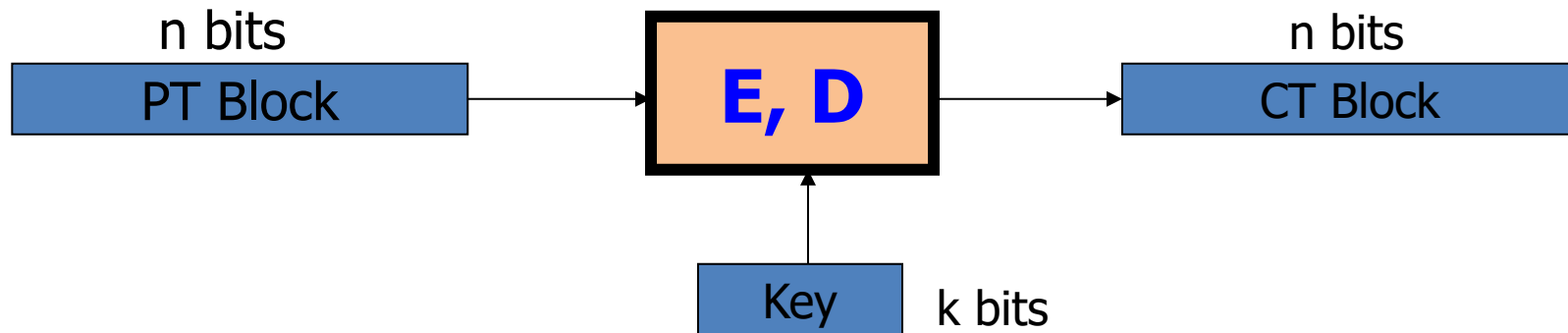


# Using block ciphers

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Review: PRPs and PRFs

# Block ciphers: crypto work horse



Canonical examples:

1. 3DES:  $n = 64$  bits,  $k = 168$  bits
2. AES:  $n = 128$  bits,  $k = 128, 192, 256$  bits

# Abstractly: PRPs and PRFs

- Pseudo Random Function (**PRF**) defined over  $(K, X, Y)$ :

$$F: K \times X \rightarrow Y$$

such that exists “efficient” algorithm to evaluate  $F(k, x)$

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- Pseudo Random Permutation (**PRP**) defined over  $(K, X)$ :

$$E: K \times X \rightarrow X$$

such that:

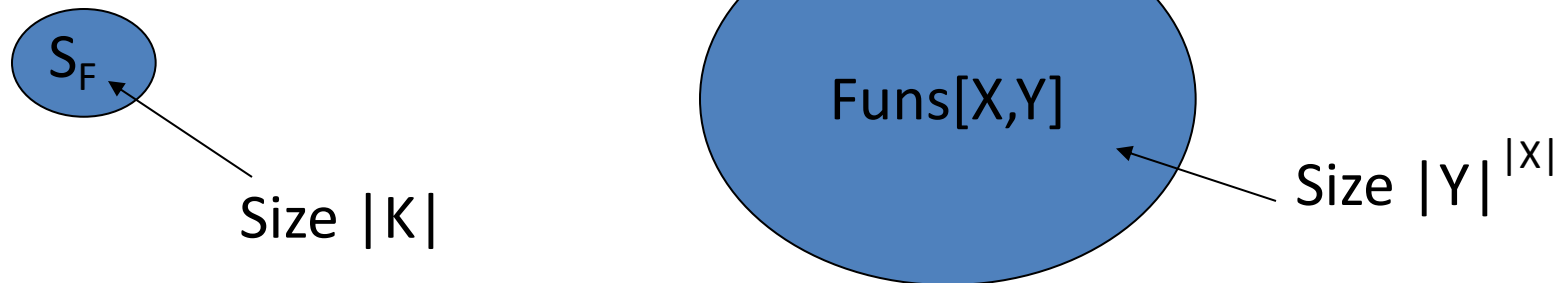
1. Exists “efficient” deterministic algorithm to evaluate  $E(k, x)$
2. The function  $E(k, \cdot)$  is one-to-one
3. Exists “efficient” inversion algorithm  $D(k, x)$

# Secure PRFs

- Let  $F: K \times X \rightarrow Y$  be a PRF

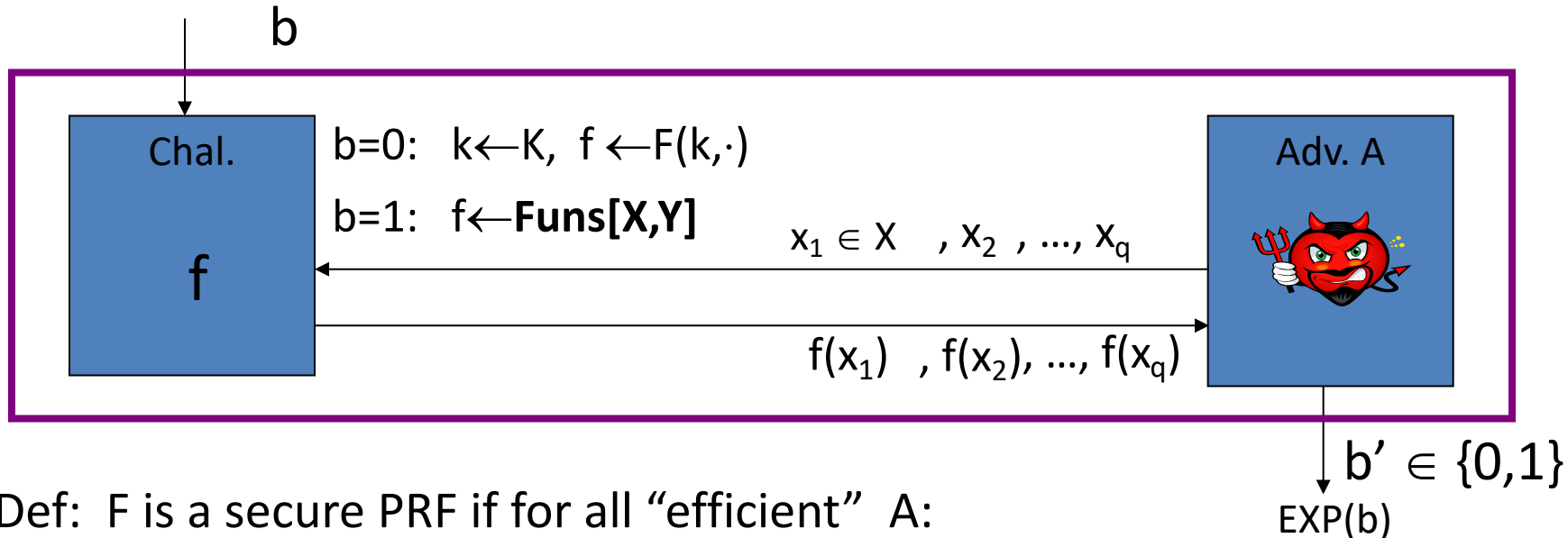
$$\left\{ \begin{array}{l} \text{Funs}[X,Y]: \text{ the set of } \underline{\text{all}} \text{ functions from } X \text{ to } Y \\ S_F = \{ F(k, \cdot) \text{ s.t. } k \in K \} \subseteq \text{Funs}[X,Y] \end{array} \right.$$

- Intuition: a PRF is **secure** if  
a random function in  $\text{Funs}[X,Y]$  is indistinguishable from  
a random function in  $S_F$



# Secure PRF: definition

- For  $b=0,1$  define experiment  $\text{EXP}(b)$  as:



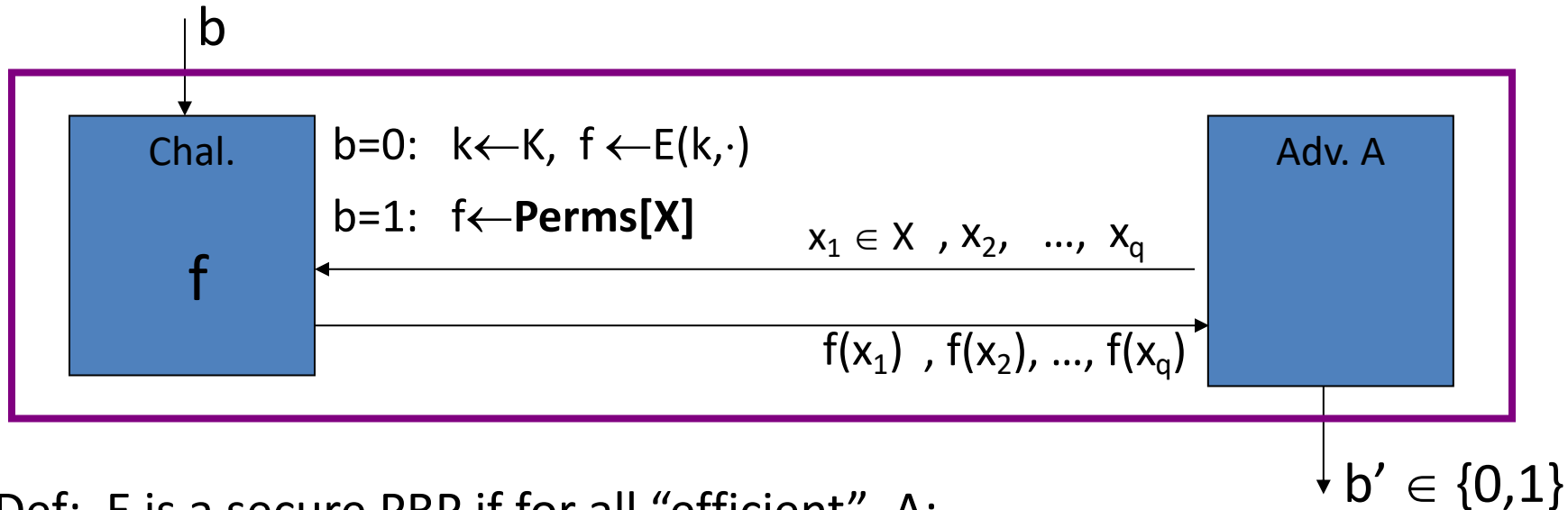
- Def:  $F$  is a secure PRF if for all “efficient”  $A$ :

$$\text{Adv}_{\text{PRF}}[A, F] := \left| \Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1] \right|$$

is “negligible.”

# Secure PRP (secure block cipher)

- For  $b=0,1$  define experiment  $\text{EXP}(b)$  as:



- Def:  $E$  is a secure PRP if for all “efficient”  $A$ :

$$\text{Adv}_{\text{PRP}}[A, E] = \left| \Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1] \right|$$

is “negligible.”

Let  $X = \{0,1\}$ .  $\text{Perms}[X]$  contains two functions

Consider the following PRP:

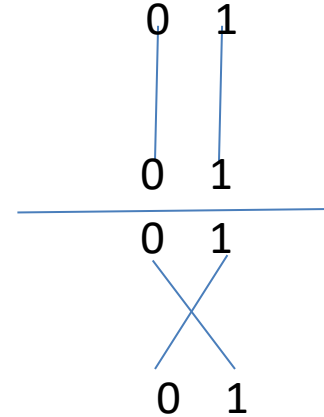
key space  $K = \{0,1\}$ , input space  $X = \{0,1\}$ ,

PRP defined as:

$$E(k,x) = x \oplus k$$

Is this a secure PRP?

- ✓ ☐ Yes
- ☐ No
- ☐ It depends
- ☐



# Example secure PRPs

- PRPs believed to be secure: 3DES, AES, ...

AES-128:  $K \times X \rightarrow X$  where  $K = X = \{0,1\}^{128}$

- An example concrete assumption about AES:

All  $2^{80}$ -time algs. A have  $\text{Adv}_{\text{PRP}}[A, \text{AES}] < 2^{-40}$



Consider the 1-bit PRP from the previous question:  $E(k,x) = x \oplus k$

Is it a secure PRF?



Note that  $\text{Funs}[X,X]$  contains four functions



- ☐ Yes
- ✓ ☐ No
- ☐ It depends
- ☐

Attacker A:

(1) query  $f(\cdot)$  at  $x=0$  and  $x=1$

(2) if  $f(0) = f(1)$  output “1”, else “0”

$$\text{Adv}_{\text{PRF}}[A,E] = |0 - \frac{1}{2}| = \frac{1}{2}$$

# PRF Switching Lemma

Any secure PRP is also a secure PRF, if  $|X|$  is sufficiently large.

Lemma: Let  $E$  be a PRP over  $(K, X)$

Then for any  $q$ -query adversary  $A$ :

$$\left| \underbrace{\text{Adv}_{\text{PRF}}[A, E] - \text{Adv}_{\text{PRP}}[A, E]}_{\text{neg.}} \right| < \underbrace{q^2 / 2|X|}_{\text{neg.}}$$

$\Rightarrow$  Suppose  $|X|$  is large so that  $q^2 / 2|X|$  is “negligible”

Then  $\text{Adv}_{\text{PRP}}[A, E]$  “negligible”  $\Rightarrow \text{Adv}_{\text{PRF}}[A, E]$  “negligible”

# Final note

- Suggestion:
  - don't think about the inner-workings of AES and 3DES.
- We assume both are secure PRPs and will see how to use them

End of Segment



## Using block ciphers

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Modes of operation:  
one time key

example: encrypted email, new key for every message.

# Using PRPs and PRFs

Goal: build “secure” encryption from a secure PRP (e.g. AES).

This segment: **one-time keys**

1. Adversary’s power:

Adv sees only one ciphertext (one-time key)

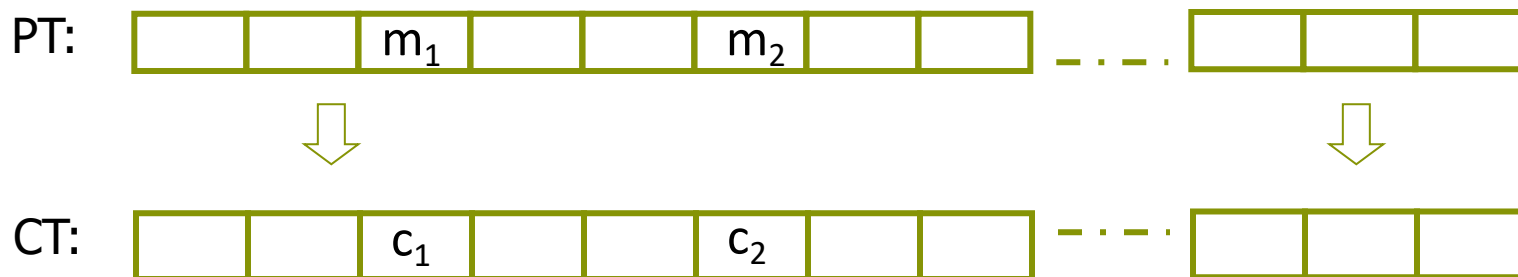
2. Adversary’s goal:

Learn info about PT from CT (semantic security)

Next segment: many-time keys (a.k.a chosen-plaintext security)

# Incorrect use of a PRP

Electronic Code Book (ECB):



Problem:

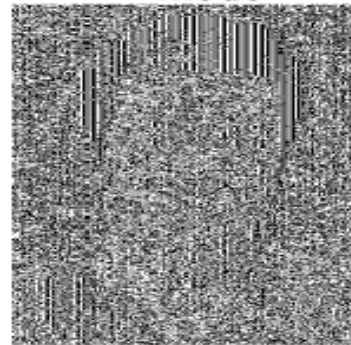
– if  $m_1 = m_2$  then  $c_1 = c_2$

# In pictures

An example plaintext



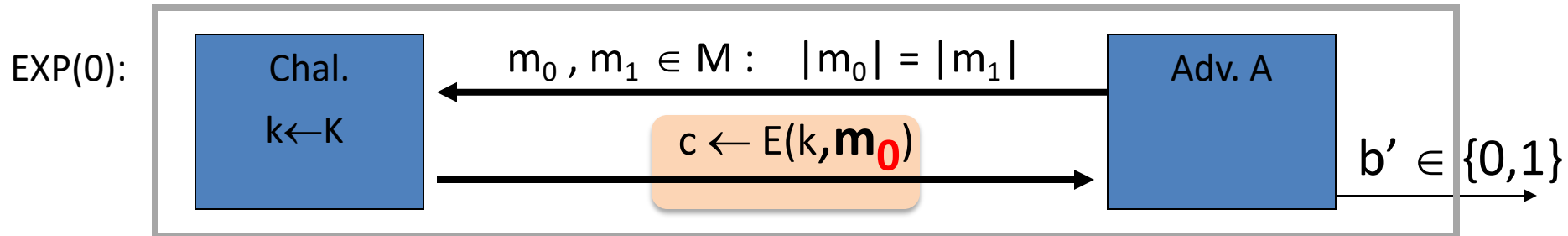
Encrypted with AES in ECB mode



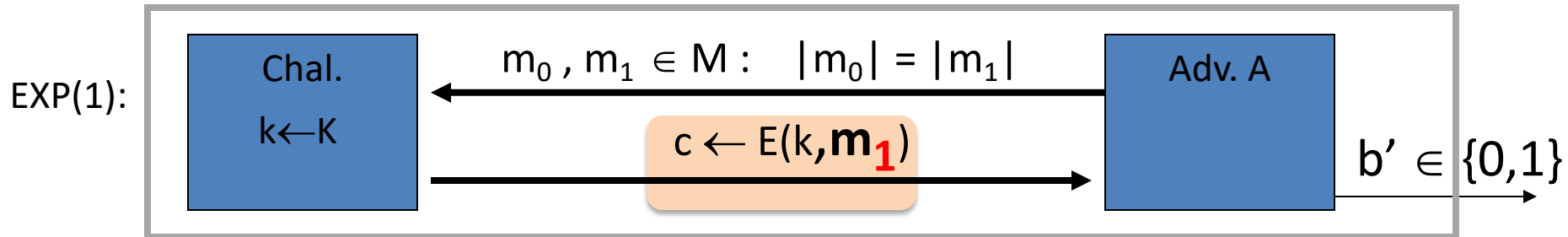
(courtesy B. Preneel)



# Semantic Security (one-time key)



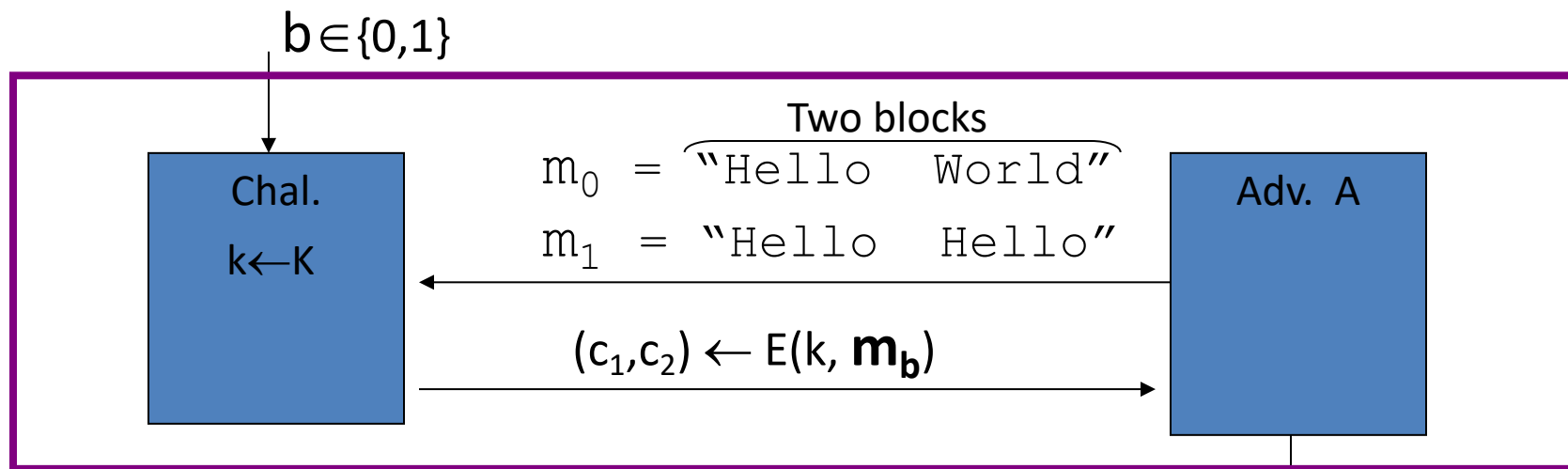
one time key  $\Rightarrow$  adversary sees only one ciphertext



$$\text{Adv}_{\text{SS}}[A, \text{OTP}] = \left| \Pr[\mathbf{EXP(0)}=1] - \Pr[\mathbf{EXP(1)}=1] \right| \text{ should be "neg."}$$

# ECB is not Semantically Secure

ECB is not semantically secure for messages that contain more than one block.



Then  $\text{Adv}_{\text{SS}} [A, \text{ECB}] =$

If  $c_1 = c_2$  output 0, else output 1

# Secure Construction I

Deterministic counter mode from a PRF  $F$  :

$k \times \{0,1\}^n \rightarrow \{0,1\}^n$  (e.g.  $n = 128$ )

•  $E_{\text{DETCTR}}(k, m) =$

$m[0]$	$m[1]$	...	$m[L]$
--------	--------	-----	--------

  
 $\oplus$   

$F(k,0)$	$F(k,1)$	...	$F(k,L)$
----------	----------	-----	----------

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$c[0]$	$c[1]$	...	$c[L]$
--------	--------	-----	--------

$\Rightarrow$  Stream cipher built from a PRF (e.g. AES, 3DES)

# Det. counter-mode security

Theorem: For any  $L > 0$ ,

If  $F$  is a secure PRF over  $(K, X, X)$  then

$E_{\text{DETCTR}}$  is sem. sec. cipher over  $(K, X^L, X^L)$ .

In particular, for any eff. adversary  $A$  attacking  $E_{\text{DETCTR}}$   
there exists a n eff. PRF adversary  $B$  s.t.:

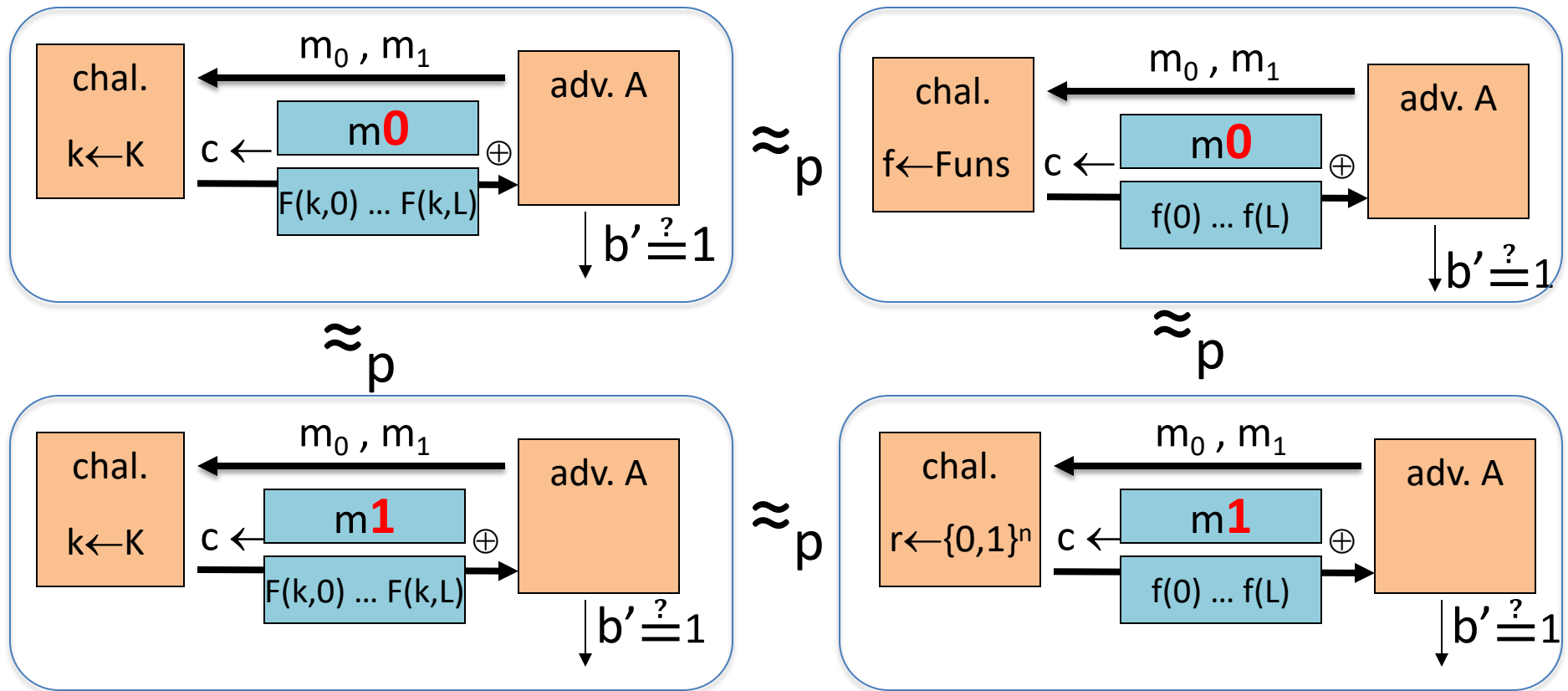
$$\text{Adv}_{\text{SS}}[A, E_{\text{DETCTR}}] = 2 \cdot \text{Adv}_{\text{PRF}}[B, F]$$

---

$\text{Adv}_{\text{PRF}}[B, F]$  is negligible (since  $F$  is a secure PRF)

Hence,  $\text{Adv}_{\text{SS}}[A, E_{\text{DETCTR}}]$  must be negligible.

# Proof



End of Segment



## Using block ciphers

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### Security for many-time key

#### Example applications:

1. File systems: Same AES key used to encrypt many files.
2. IPsec: Same AES key used to encrypt many packets.

# Semantic Security for many-time key

Key used more than once  $\Rightarrow$  adv. sees many CTs with same key

**Adversary's power:** chosen-plaintext attack (CPA)

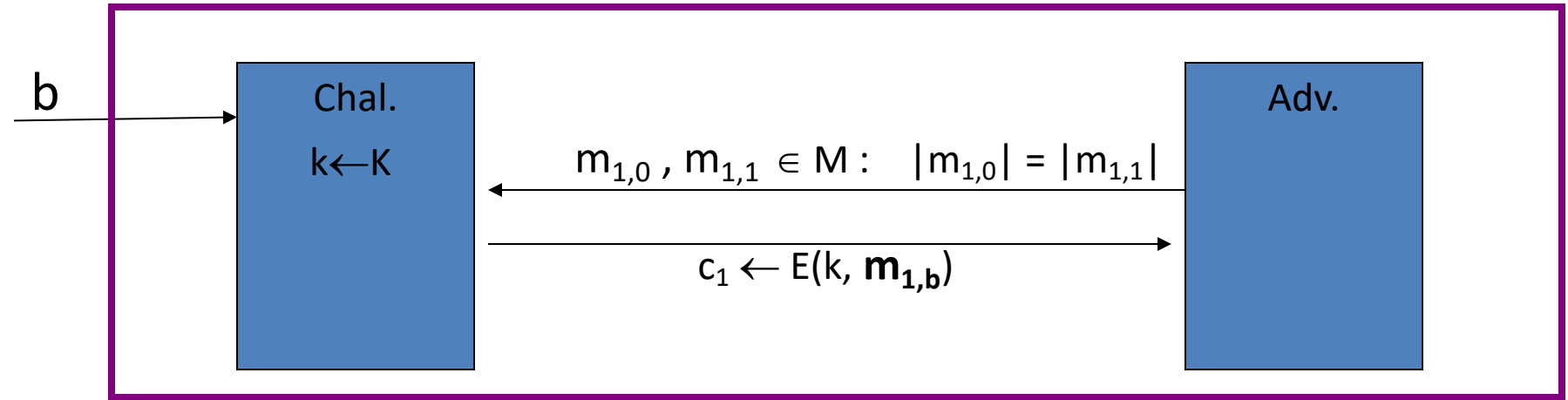
- Can obtain the encryption of arbitrary messages of his choice  
(conservative modeling of real life)

**Adversary's goal:** Break semantic security



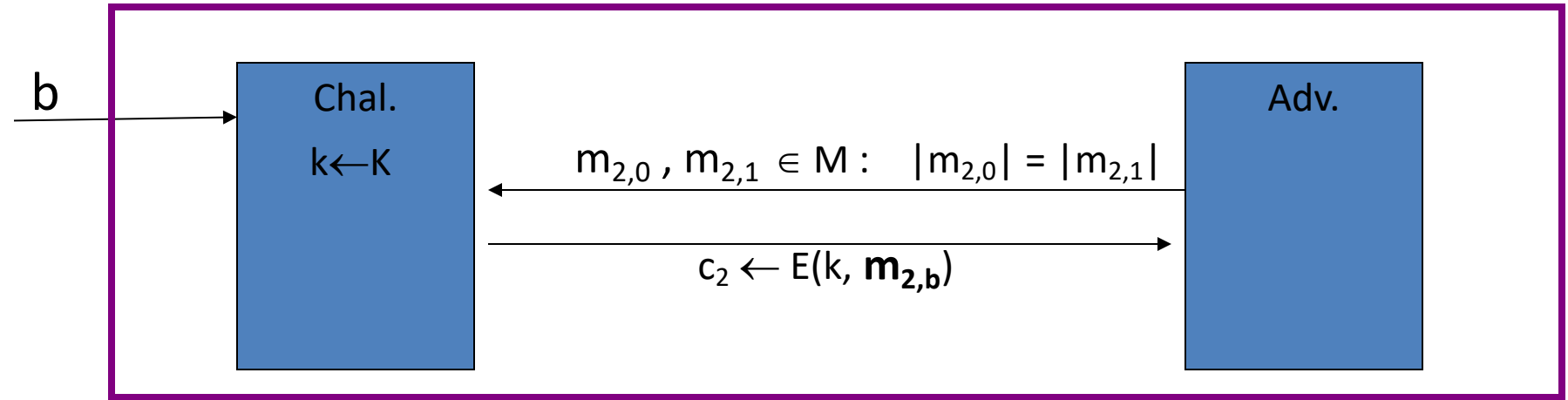
# Semantic Security for many-time key

$\mathbb{E} = (E, D)$  a cipher defined over  $(K, M, C)$ . For  $b=0,1$  define  $\text{EXP}(b)$  as:



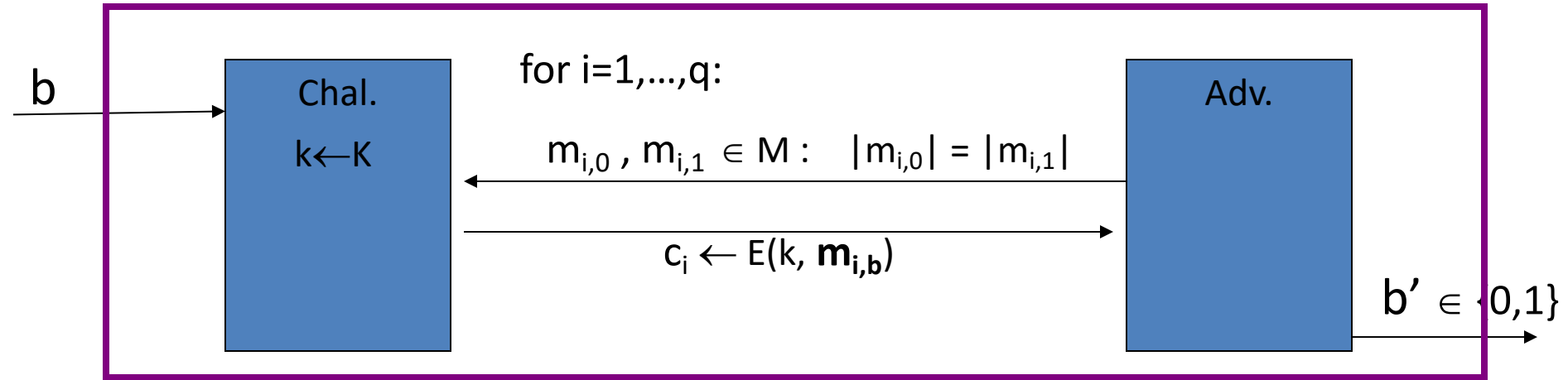
# Semantic Security for many-time key

$\mathbb{E} = (E, D)$  a cipher defined over  $(K, M, C)$ . For  $b=0,1$  define  $\text{EXP}(b)$  as:



# Semantic Security for many-time key (CPA security)

$\mathbb{E} = (E, D)$  a cipher defined over  $(K, M, C)$ . For  $b=0,1$  define  $\text{EXP}(b)$  as:



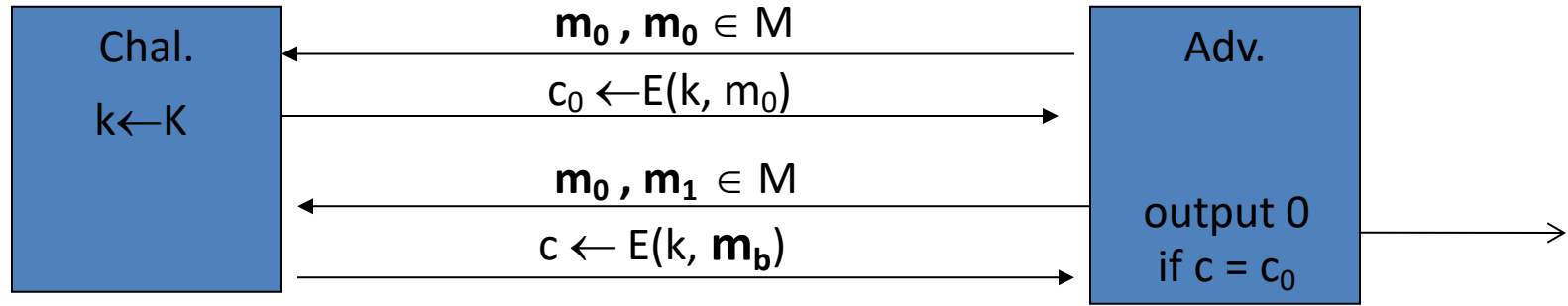
if adv. wants  $c = E(k, m)$  it queries with  $m_{j,0} = m_{j,1} = m$

Def:  $\mathbb{E}$  is sem. sec. under CPA if for all "efficient"  $A$ :

$$\text{Adv}_{\text{CPA}}[A, \mathbb{E}] = \left| \Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1] \right| \text{ is "negligible."}$$

# Ciphers insecure under CPA

Suppose  $E(k,m)$  always outputs same ciphertext for msg  $m$ . Then:

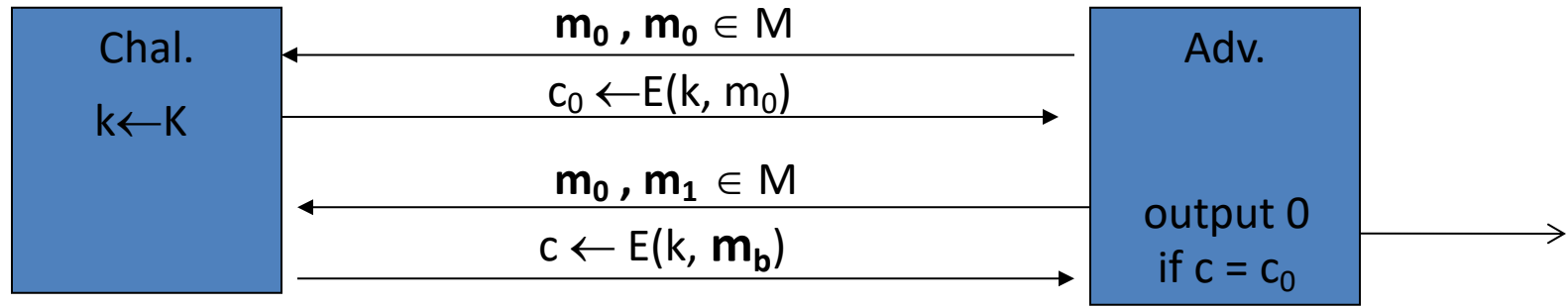


So what?      an attacker can learn that two encrypted files are the same, two encrypted packets are the same, etc.

- Leads to significant attacks when message space  $M$  is small

# Ciphers insecure under CPA

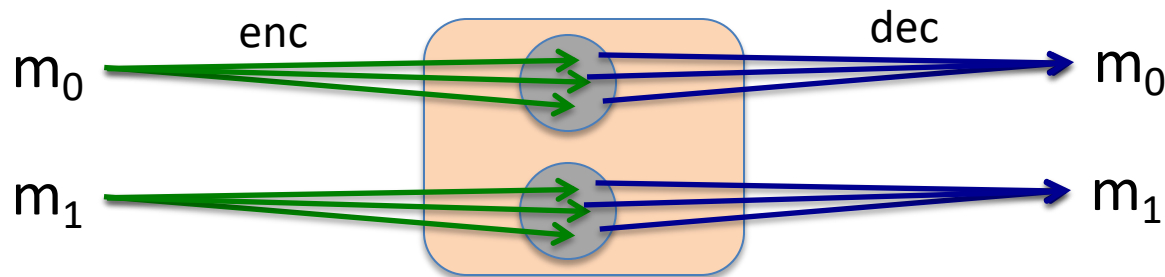
Suppose  $E(k,m)$  always outputs same ciphertext for msg  $m$ . Then:



If secret key is to be used multiple times  $\Rightarrow$   
given the same plaintext message twice,  
encryption must produce different outputs.

# Solution 1: randomized encryption

- $E(k,m)$  is a randomized algorithm:



⇒ encrypting same msg twice gives different ciphertexts

⇒ ciphertext must be longer than plaintext

Roughly speaking: CT-size = PT-size + “# random bits”

Let  $F: K \times R \rightarrow M$  be a secure PRF.

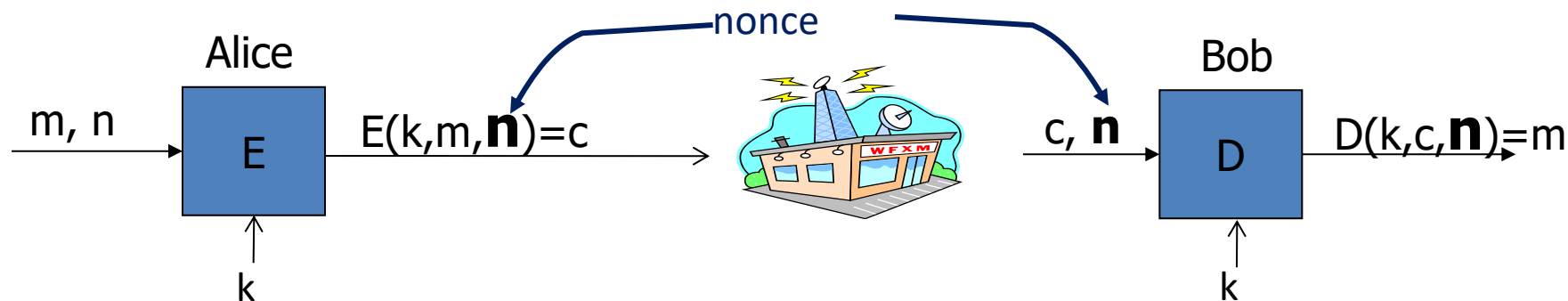
For  $m \in M$  define  $E(k, m) = [ r \xleftarrow{R} R, \text{ output } (r, F(k, r) \oplus m) ]$

$\swarrow \approx_p (r, f(r) \oplus m)$

Is  $E$  semantically secure under CPA?

- ☐ Yes, whenever  $F$  is a secure PRF
- ☐ No, there is always a CPA attack on this system
- ✓ ☐ Yes, but only if  $R$  is large enough so  $r$  never repeats (w.h.p)
- ☐ It depends on what  $F$  is used

# Solution 2: nonce-based Encryption

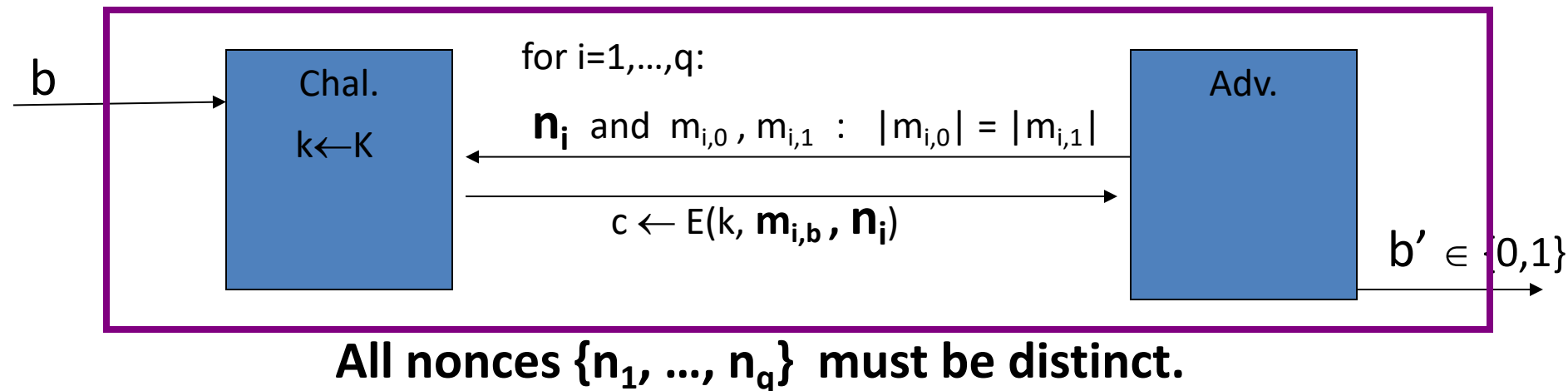


- nonce  $n$ : a value that changes from msg to msg.  
 $(k, n)$  pair never used more than once
- method 1: nonce is a **counter** (e.g. packet counter)
  - used when encryptor keeps state from msg to msg
  - if decryptor has same state, need not send nonce with CT
- method 2: encryptor chooses a **random nonce**,  $n \leftarrow \mathcal{N}$



# CPA security for nonce-based encryption

System should be secure when nonces are chosen adversarially.



Def: nonce-based  $\mathbb{E}$  is sem. sec. under CPA if for all “efficient”  $A$ :

$$\text{Adv}_{\text{nCPA}}[A, \mathbb{E}] = \left| \Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1] \right| \text{ is “negligible.”}$$

Let  $F: K \times R \rightarrow M$  be a secure PRF. Let  $r = 0$  initially.

For  $m \in M$  define  $E(k, m) = [ r++, \text{ output } (r, F(k, r) \oplus m) ]$   
 $\approx_p (r, f(r) \oplus m)$

Is  $E$  CPA secure nonce-based encryption?

- ☐ Yes, whenever  $F$  is a secure PRF
- ☐ No, there is always a nonce-based CPA attack on this system
- ☐ Yes, but only if  $R$  is large enough so  $r$  never repeats
- ☐ It depends on what  $F$  is used

End of Segment



## Using block ciphers

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Modes of operation:  
many time key (CBC)

### Example applications:

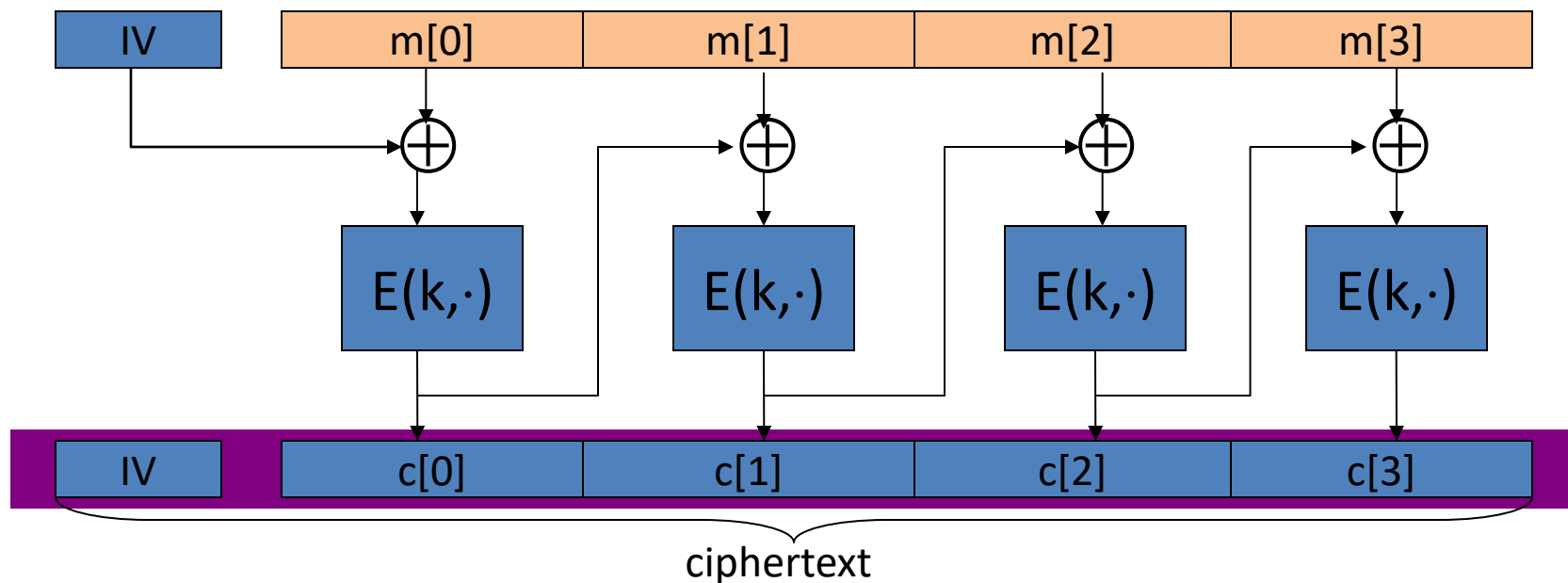
1. File systems: Same AES key used to encrypt many files.
2. IPsec: Same AES key used to encrypt many packets.

# Construction 1: CBC with random IV

Let  $(E,D)$  be a PRP.  $E_{\text{CBC}}(k,m)$ : choose random  $IV \in X$  and do:

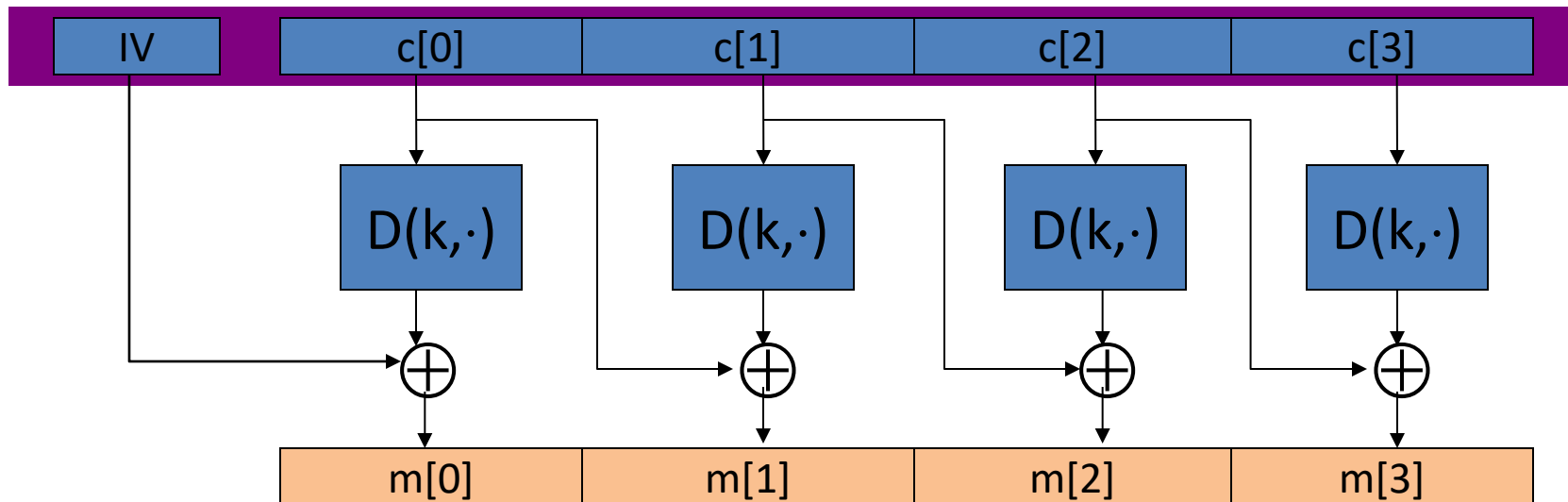
$$E: k \times \{0,1\}^n \times \{0,1\}^n$$

$$IV \in \{0,1\}^n$$



# Decryption circuit

In symbols:  $c[0] = E(k, IV \oplus m[0]) \Rightarrow m[0] =$



# CBC: CPA Analysis

CBC Theorem: For any  $L > 0$ ,

If  $E$  is a secure PRP over  $(K, X)$  then

$E_{\text{CBC}}$  is a sem. sec. under CPA over  $(K, X^L, X^{L+1})$ .

In particular, for a  $q$ -query adversary  $A$  attacking  $E_{\text{CBC}}$  there exists a PRP adversary  $B$  s.t.:

$$\text{Adv}_{\text{CPA}}[A, E_{\text{CBC}}] \leq 2 \cdot \text{Adv}_{\text{PRP}}[B, E] + 2q^2 L^2 / |X|$$

Note: CBC is only secure as long as  $q^2 L^2 \ll |X|$

# An example

$$\text{Adv}_{\text{CPA}} [A, E_{\text{CBC}}] \leq 2 \cdot \text{PRP Adv}[B, E] + 2 q^2 L^2 / |X|$$

$q$  = # messages encrypted with  $k$  ,  $L$  = length of max message

Suppose we want  $\text{Adv}_{\text{CPA}} [A, E_{\text{CBC}}] \leq 1/2^{32} \iff q^2 L^2 / |X| < 1/2^{32}$

- AES:  $|X| = 2^{128} \Rightarrow q L < 2^{48}$

So, after  $2^{48}$  AES blocks, must change key

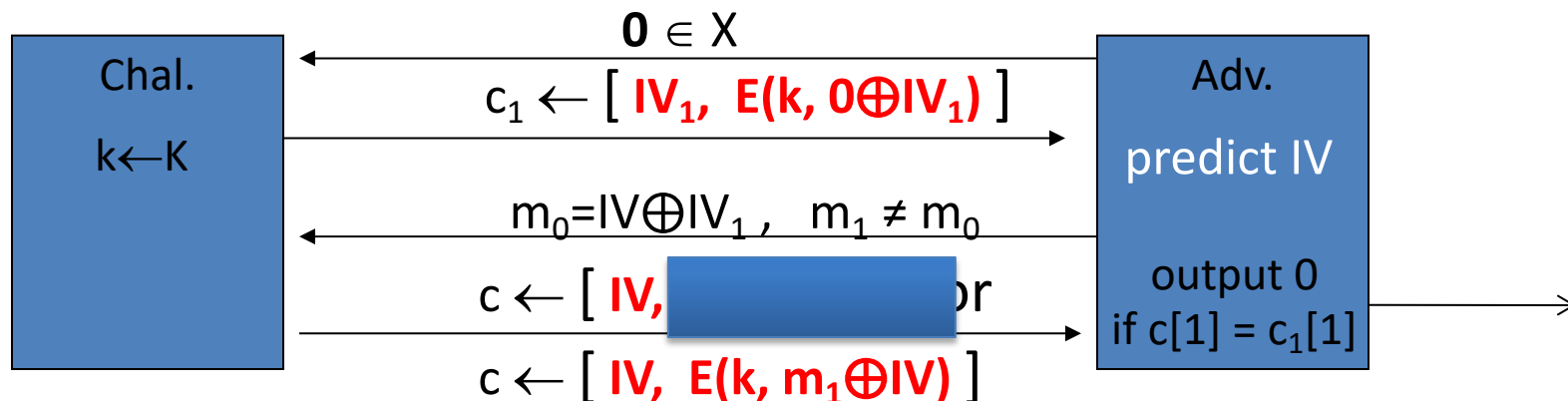
- 3DES:  $|X| = 2^{64} \Rightarrow q L < 2^{16}$



# Warning: an attack on CBC with rand. IV

CBC where attacker can predict the IV is not CPA-secure !!

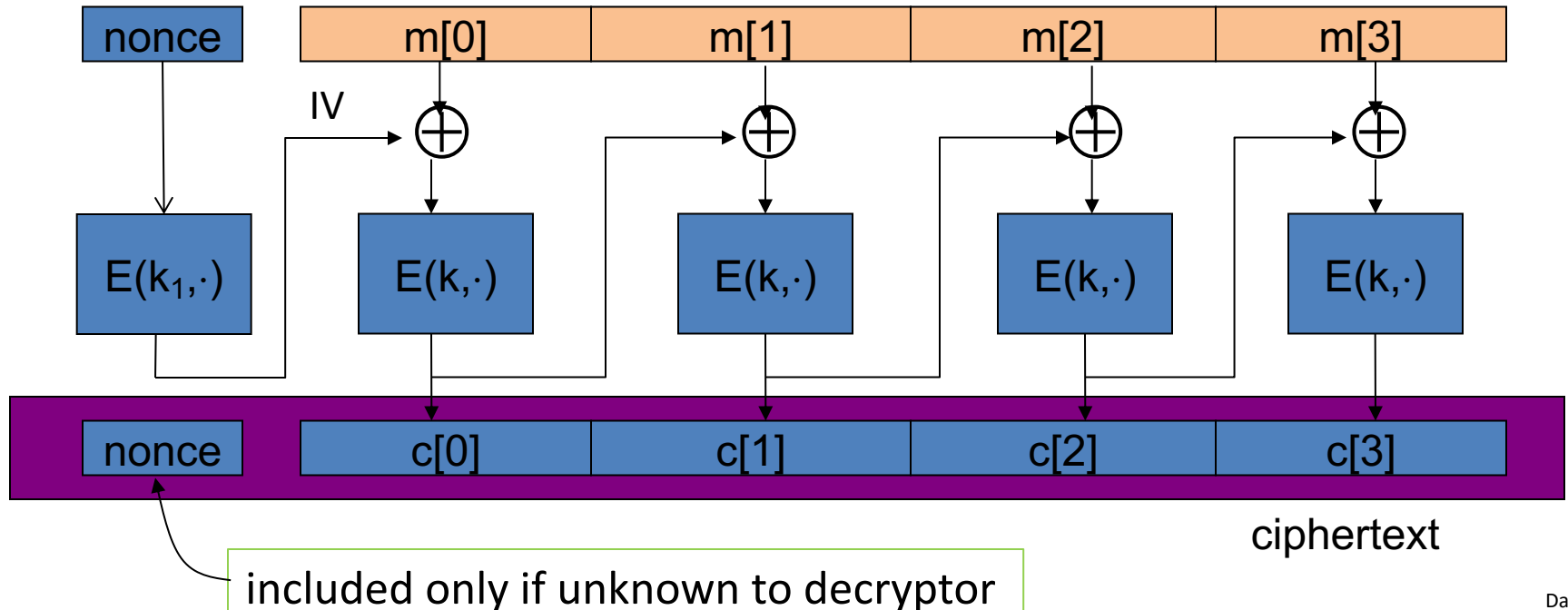
Suppose given  $c \leftarrow E_{\text{CBC}}(k, m)$  can predict IV for next message



Bug in SSL/TLS 1.0: IV for record #i is last CT block of record #(i-1)

# Construction 1': nonce-based CBC

- Cipher block chaining with unique nonce:  $\text{key} = (k, k_1)$   
unique nonce means:  $(\text{key}, n)$  pair is used for only one message

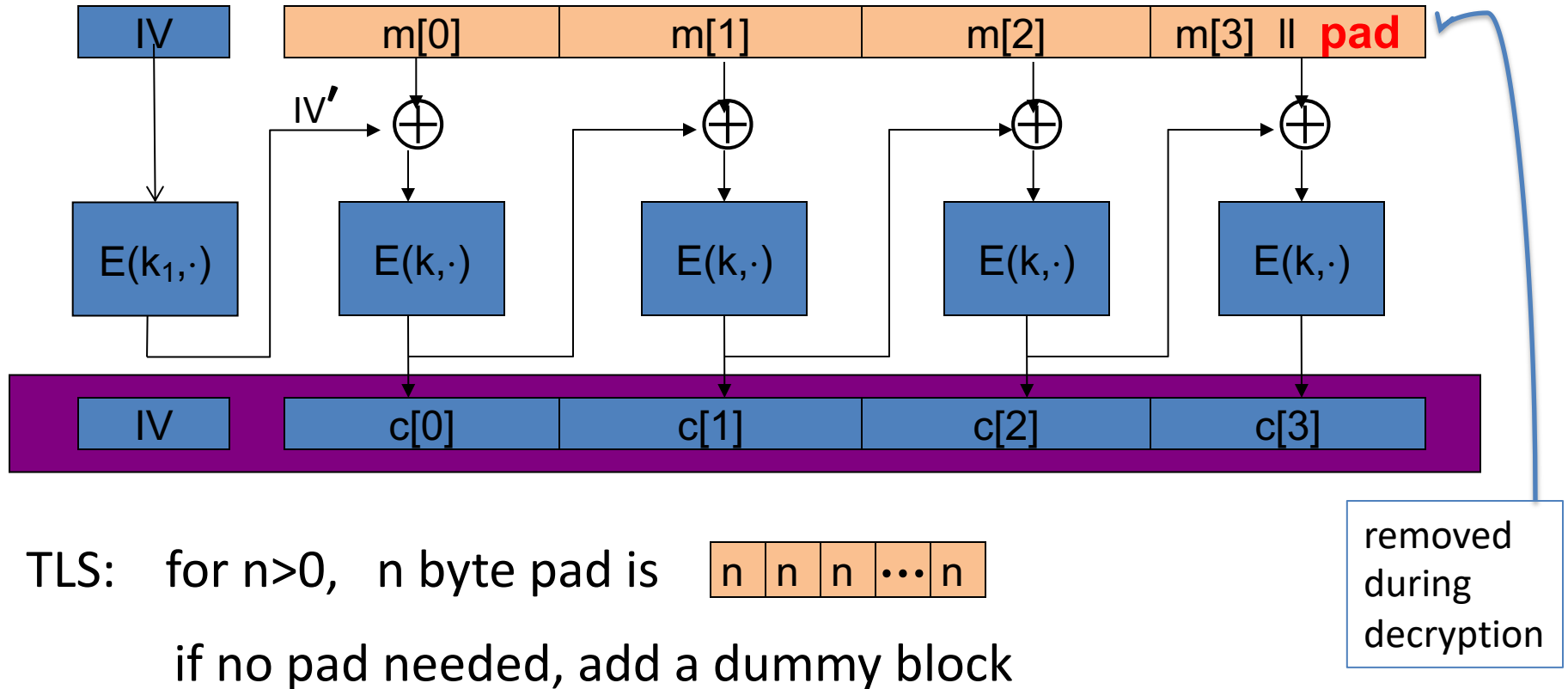


# An example Crypto API (OpenSSL)

```
void AES_cbc_encrypt(  
    const unsigned char *in,  
    unsigned char *out,  
    size_t length,  
    const AES_KEY *key,  
    unsigned char *ivec,           ← user supplies IV  
    AES_ENCRYPT or AES_DECRYPT);
```

When nonce is non random need to encrypt it before use  
Otherwise no CPA security

# A CBC technicality: padding



End of Segment



## Using block ciphers

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Modes of operation:  
many time key (CTR)

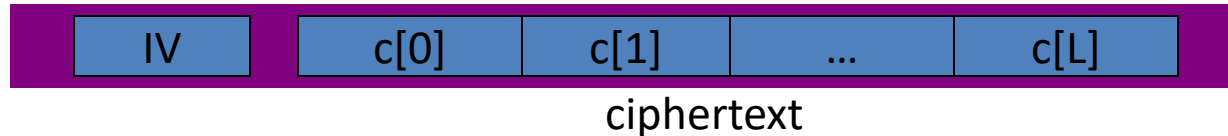
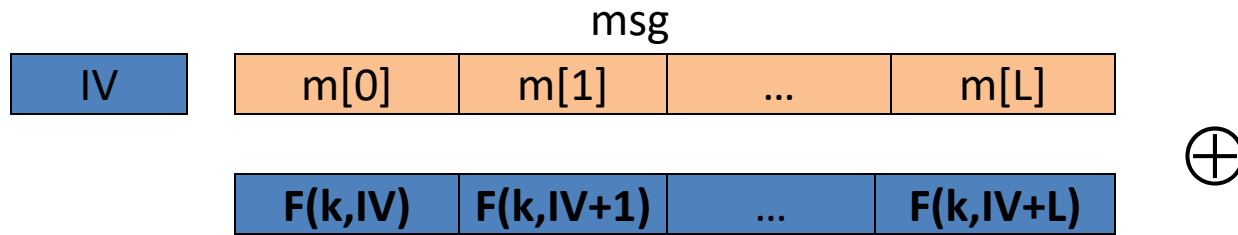
### Example applications:

1. File systems: Same AES key used to encrypt many files.
2. IPsec: Same AES key used to encrypt many packets.

# Construction 2: rand ctr-mode

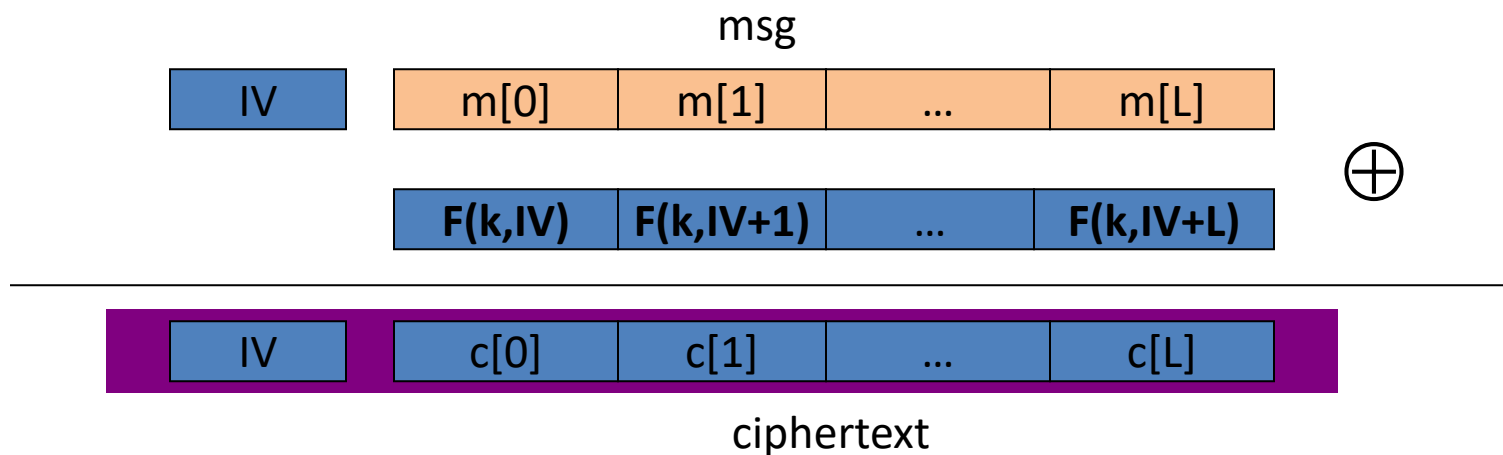
Let  $F: K \times \{0,1\}^n \rightarrow \{0,1\}^n$  be a secure PRF.

$E(k,m)$ : choose a random  $IV \in \{0,1\}^n$  and do:

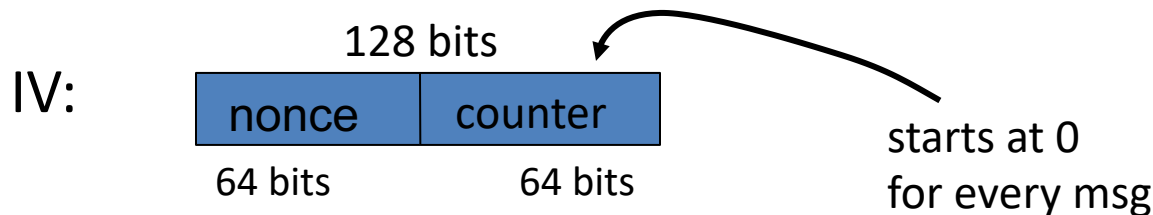


note: parallelizable (unlike CBC)

# Construction 2': nonce ctr-mode



To ensure  $F(k, x)$  is never used more than once, choose IV as:





# rand ctr-mode (rand. IV): CPA analysis

- Counter-mode Theorem: For any  $L > 0$ ,

If  $F$  is a secure PRF over  $(K, X, X)$  then

$E_{\text{CTR}}$  is a sem. sec. under CPA over  $(K, X^L, X^{L+1})$ .

In particular, for a  $q$ -query adversary  $A$  attacking  $E_{\text{CTR}}$  there exists a PRF adversary  $B$  s.t.:

$$\text{Adv}_{\text{CPA}}[A, E_{\text{CTR}}] \leq 2 \cdot \text{Adv}_{\text{PRF}}[B, F] + 2q^2 L / |X|$$

Note: ctr-mode only secure as long as  $q^2 L \ll |X|$ . Better than CBC !

# An example

$$\text{Adv}_{\text{CPA}} [A, E_{\text{CTR}}] \leq 2 \cdot \text{Adv}_{\text{PRF}} [B, E] + 2 q^2 L / |X|$$

$q$  = # messages encrypted with  $k$  ,  $L$  = length of max message

Suppose we want  $\text{Adv}_{\text{CPA}} [A, E_{\text{CTR}}] \leq 1/2^{32} \quad \Leftarrow \quad q^2 L / |X| < 1/2^{32}$

- AES:  $|X| = 2^{128} \Rightarrow q L^{1/2} < 2^{48}$

So, after  $2^{32}$  CTs each of len  $2^{32}$  , must change key

(total of  $2^{64}$  AES blocks)

# Comparison: ctr vs. CBC

	CBC	ctr mode
uses	PRP	PRF
parallel processing	No	Yes
Security of rand. enc.	$q^2 L^2 \ll  X $	$q^2 L \ll  X $
dummy padding block	Yes	No
1 byte msgs (nonce-based)	16x expansion	no expansion

(for CBC, dummy padding block can be solved using ciphertext stealing)

# Summary

- PRPs and PRFs: a useful abstraction of block ciphers.
- We examined two security notions: (security against eavesdropping)
  1. Semantic security against one-time CPA.
  2. Semantic security against many-time CPA.

Note: neither mode ensures data integrity.

- Stated security results summarized in the following table:

Power Goal	one-time key	Many-time key (CPA)	CPA and integrity
Sem. Sec.	steam-ciphers det. ctr-mode	rand CBC rand ctr-mode	later

# Further reading

- A concrete security treatment of symmetric encryption:  
Analysis of the DES modes of operation,  
M. Bellare, A. Desai, E. Jokipii and P. Rogaway, FOCS 1997
- Nonce-Based Symmetric Encryption, P. Rogaway, FSE 2004

End of Segment