lambda vs w plot

February 19, 2019

0.1 Try to plot the lambda function vs the baseline covariates

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```
In [2]: setwd('/Users/yaolanqiu/Desktop/NYU/rotation/Rotation2/Week3/from dr.tarpey')
        source("cvxcluster-0513.R")
```

```
In [12]: library('lme4')
```

```
In [13]: # read in data
         dat = read.table("hcaf.dat", header=T)
         d0 = dat[dat$trt == 0,]
         d1 = dat[dat$trt == 1,]
         head(dat)
```

subj	trt	y	age	BaselineCGI	t1	responder
2497	0	25	29	4	0	0
2497	0	18	29	4	1	0
2497	0	11	29	4	2	0
2497	0	9	29	4	3	0
2497	0	19	29	4	4	0
2497	0	15	29	4	5	0

```
In [14]: # create new covariates
         cov01 = rnorm(length(unique(d0$subj)),5,1)
         cov02 = rnorm(length(unique(d0$subj)),10,1)
         newcov0 = data.frame(subj = unique(d0$subj), newcov1 = cov01, newcov2 = cov02)
         d0 = merge(d0,newcov0, by = 'subj')
         head(d0)
```

subj	trt	y	age	BaselineCGI	t1	responder	newcov1	newcov2
2497	0	25	29	4	0	0	5.115883	10.0736
2497	0	18	29	4	1	0	5.115883	10.0736
2497	0	11	29	4	2	0	5.115883	10.0736
2497	0	9	29	4	3	0	5.115883	10.0736
2497	0	19	29	4	4	0	5.115883	10.0736
2497	0	15	29	4	5	0	5.115883	10.0736

In [15]:
```
# create new covariates
cov01 = rnorm(length(unique(d1$subj)),10,1)
cov02 = rnorm(length(unique(d1$subj)),5,1)
newcov1 = data.frame(subj = unique(d1$subj), newcov1 = cov01, newcov2 = cov02)
d1 = merge(d1,newcov1, by = 'subj')
head(d1)
```

subj	trt	y	age	BaselineCGI	t1	responder	newcov1	newcov2
2500	1	24	29	4	0	0	10.031118	4.965798
2500	1	22	29	4	1	0	10.031118	4.965798
2500	1	19	29	4	2	0	10.031118	4.965798
2500	1	25	29	4	3	0	10.031118	4.965798
2509	1	24	31	4	0	1	9.820069	7.081584
2509	1	10	31	4	1	1	9.820069	7.081584

In [16]:
```
# new covariate, which is the combination of the two new covariates
# let's make it a simple summation first
d0$newcov = d0$newcov1 + d0$newcov2
d1$newcov = d1$newcov1 + d1$newcov2
```

In [18]:
```
# Fit LME
fit_d0 = lmer(y ~ t1 + I(t1^2) + newcov + newcov * t1 + newcov * I(t1^2) + (t1+I(t1^2)
                data = d0, REML = FALSE)
fit_d1 = lmer(y ~ t1 + I(t1^2) + newcov + newcov * t1 + newcov * I(t1^2) + (t1+I(t1^2)
                data = d1, REML = FALSE)
```

```
singular fit
```

In [20]:
```
summary(fit_d0)
```

```
Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: y ~ t1 + I(t1^2) + newcov + newcov * t1 + newcov * I(t1^2) +
    (t1 + I(t1^2) | subj)
   Data: d0

     AIC      BIC   logLik deviance df.resid
  5970.0   6033.6  -2972.0   5944.0      965

Scaled residuals:
    Min      1Q  Median      3Q     Max
-4.2516 -0.5961 -0.0336  0.5374  2.9550

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 subj     (Intercept)  2.1349  1.4611
          t1           5.1128  2.2612    0.96
          I(t1^2)      0.1213  0.3482   -0.98 -0.88
 Residual             16.1672  4.0208
```

```
Number of obs: 978, groups:  subj, 162

Fixed effects:
               Estimate Std. Error t value
(Intercept)    29.31307    3.39179   8.642
t1             -9.08822    3.28602  -2.766
I(t1^2)         0.98201    0.55240   1.778
newcov         -0.41718    0.22569  -1.849
t1:newcov       0.32262    0.21865   1.476
I(t1^2):newcov -0.03216    0.03675  -0.875

Correlation of Fixed Effects:
            (Intr) t1     I(t1^2) newcov t1:nwc
t1          -0.344
I(t1^2)      0.242 -0.918
newcov      -0.996  0.343 -0.241
t1:newcov    0.343 -0.996  0.914  -0.344
I(t1^2):nwc -0.241  0.914 -0.996   0.242 -0.918
convergence code: 0
singular fit
```

```
In [21]: summary(fit_d1)

Linear mixed model fit by maximum likelihood  ['lmerMod']
Formula: y ~ t1 + I(t1^2) + newcov + newcov * t1 + newcov * I(t1^2) +
    (t1 + I(t1^2) | subj)
   Data: d1

     AIC      BIC   logLik deviance df.resid
  7281.4   7347.9  -3627.7   7255.4     1218

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.2540 -0.5487 -0.0448  0.5174  3.4531

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 subj     (Intercept)  8.0620  2.8394
          t1           7.5757  2.7524    0.07
          I(t1^2)      0.1552  0.3939   -0.22 -0.93
 Residual             11.7507  3.4279
Number of obs: 1231, groups:  subj, 196

Fixed effects:
               Estimate Std. Error t value
(Intercept)    27.49879    3.27545   8.395
```

```
t1              -9.88284    2.90459  -3.402
I(t1^2)          1.16388    0.44727   2.602
newcov          -0.25199    0.21587  -1.167
t1:newcov        0.36967    0.19139   1.932
I(t1^2):newcov -0.05253    0.02945  -1.784

Correlation of Fixed Effects:
            (Intr) t1     I(t1^2) newcov t1:nwc
t1          -0.338
I(t1^2)      0.210 -0.934
newcov      -0.996  0.337 -0.209
t1:newcov    0.337 -0.996  0.930  -0.338
I(t1^2):nwc -0.209  0.931 -0.996   0.210 -0.934
```

0.1.1 Plot the $\lambda(x)$ v.s. w figure

The λ here is just

$$\lambda(z_i|w_i) = \frac{f_2(z_i|w_i)}{f_1(z_i|w_i) + f_2(z_i|w_i)}$$

Steps:

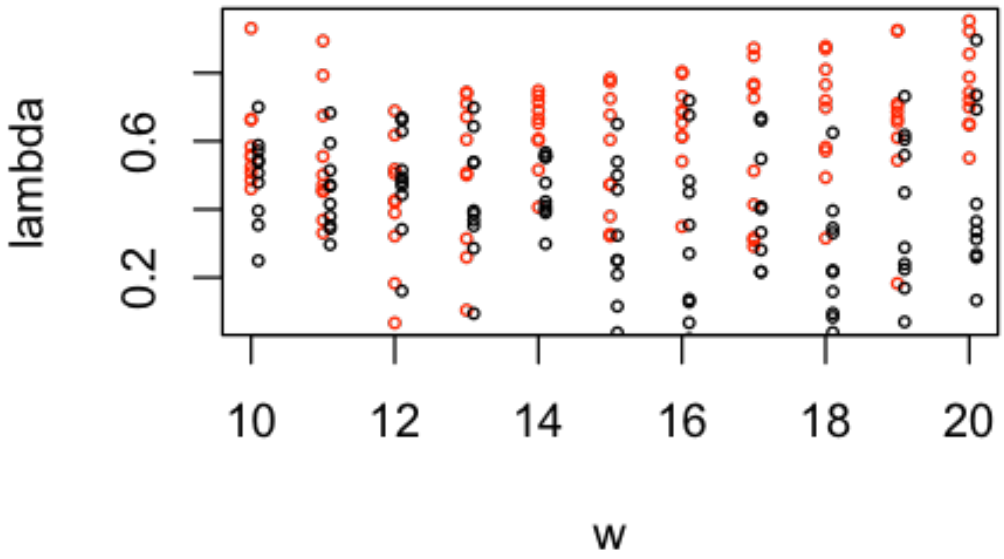
1. Fit LME, calculate the mean and variance of MVN for drug group and placebo group, separately.
2. Generate data from the above MVN by using the mean and variance calculated from the LME
3. Input values (the value from above generation) and selected w values into the λ function.
4. Draw the plot

```
In [23]: beta0 = as.matrix(fixef(fit_d0)[1:3])
         gamma0 = as.matrix(fixef(fit_d0)[4:6])
         beta1 = as.matrix(fixef(fit_d1)[1:3])
         gamma1 = as.matrix(fixef(fit_d1)[4:6])

In [26]: lambda2 = function(xx, mu0, mu1, sigma0, sigma1){

           quadratic0 = function(a,b) {
             X = matrix(c(a,b),nrow=2)
             Q = (-1/2)*t(X-mu0)%*%solve(sigma0)%*%(X-mu0)
           }
           quadratic1 = function(a,b) {
             X = matrix(c(a,b),nrow=2)
             Q = (-1/2)*t(X-mu1)%*%solve(sigma1)%*%(X-mu1)
           }

           f0 = (1/(2*pi))*(1/sqrt(det(sigma0)))*exp(quadratic0(xx[1],xx[2]))
           f1 = (1/(2*pi))*(1/sqrt(det(sigma1)))*exp(quadratic1(xx[1],xx[2]))
```

```
  return(f1/(f1 + f0))
}
```

In [30]:
```
drg_group = c()
pat_group = c()
Ww = c()
for(w in seq(10,20,1)){

  m0 = beta0 + gamma0 * w; m0 = m0[2:3]
  m1 = beta1 + gamma1 * w; m1 = m1[2:3]
  D0 = as.matrix(VarCorr(fit_d0)$subj)[2:3, 2:3]
  D1 = as.matrix(VarCorr(fit_d1)$subj)[2:3, 2:3]

  mu0 = matrix(m0, nrow=2)
  sigma0 = D0
  mu1 = matrix(m1, nrow=2)
  sigma1 = D1

  pat = mvrnorm(10, mu0, sigma0)
  drg = mvrnorm(10, mu1, sigma1)

  for(i in 1:10){
    pat_group = c(pat_group, lambda2(pat[i,], mu0, mu1, sigma0, sigma1))
  }

  for(i in 1:10){
    drg_group = c(drg_group, lambda2(drg[i,], mu0, mu1, sigma0, sigma1))
  }

  Ww = c(Ww, rep(w, 10))
}
```

In [35]:
```
options(repr.plot.width=4, repr.plot.height=3)
plot(Ww, drg_group, cex = 0.5, col = 'red', ylab = 'lambda', xlab= 'w')
points(Ww+0.1, pat_group, cex = 0.5)
```


lambda
0.2
0.6
10
12
14
16
18
20
w