Parameter setting 1

The outcome is generated following

$$Y_k = S(\beta_k + b_k + \Gamma_k(\alpha_k' x)) + \epsilon_k, k = \{1, 2\} \text{ presents drug group and placebo group.} \quad (1)$$

The parameter settings are:

- dimension of the predictors $p = 3, 10, 20$
- $\beta_{drg} = \beta_{pbo} = (1, -0.05, -0.02)'$
- $\Gamma_{drg} = (0, 1, 0)$
- $\Gamma_{pbo} = (0, \cos(\frac{\pi}{3}, -\sin(\frac{\pi}{3}))$
- $S = [1, t, t^2]$, $t = [0, 1, 2, 3, 4, 6, 8]$ is the design matrix for fixed effect and random effect
- $x \sim MVN(\mu_x, \Sigma_x)$, $\mu_x = \mathbf{0}_p$, Σ_x has diagonal equals to 1 and 0.5 everywhere else.
- $D_{drg} = \begin{pmatrix} 1.45 & -0.11 & 0.2 \\ -0.11 & 0.17 & -0.08 \\ 0.2 & -0.08 & 0.23 \end{pmatrix}$, $D_{pbo} = \begin{pmatrix} 1.03 & -0.23 & -0.15 \\ -0.23 & 0.68 & 0.25 \\ -0.15 & 0.25 & 1.36 \end{pmatrix}$
- $\epsilon_{drg}, \epsilon_{pbo} \sim N(0, 1^2)$
- $\alpha = \alpha_1 + \delta\alpha_{2k}, k = 1, 2$, $\delta = 0, 1...$
  - $\alpha_1 = \text{rnorm(p)}$
  - drg: $\alpha_{21} = (1, ..., p)$
  - pbo: $\alpha_{22} = (-p, ..., -1)$

The Cosine similarity between α_{drg} and α_{pbo}

	$p = 3$	$p = 10$	$p = 20$
$\delta = 0$	1	1	1
$\delta = 1$	-0.14	-0.59	-0.54

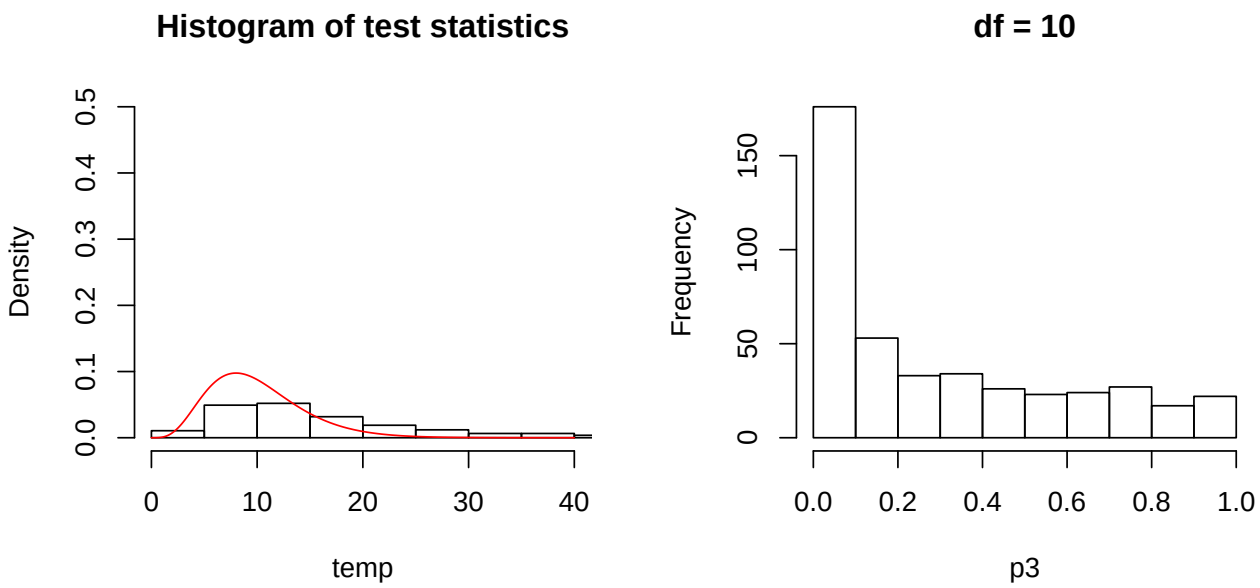
When $p = 3$, $\delta = 0$

When $\delta = 0$, it means that data is generated from GEM model, the GEM model is true. The hypothesis tests should have large p values

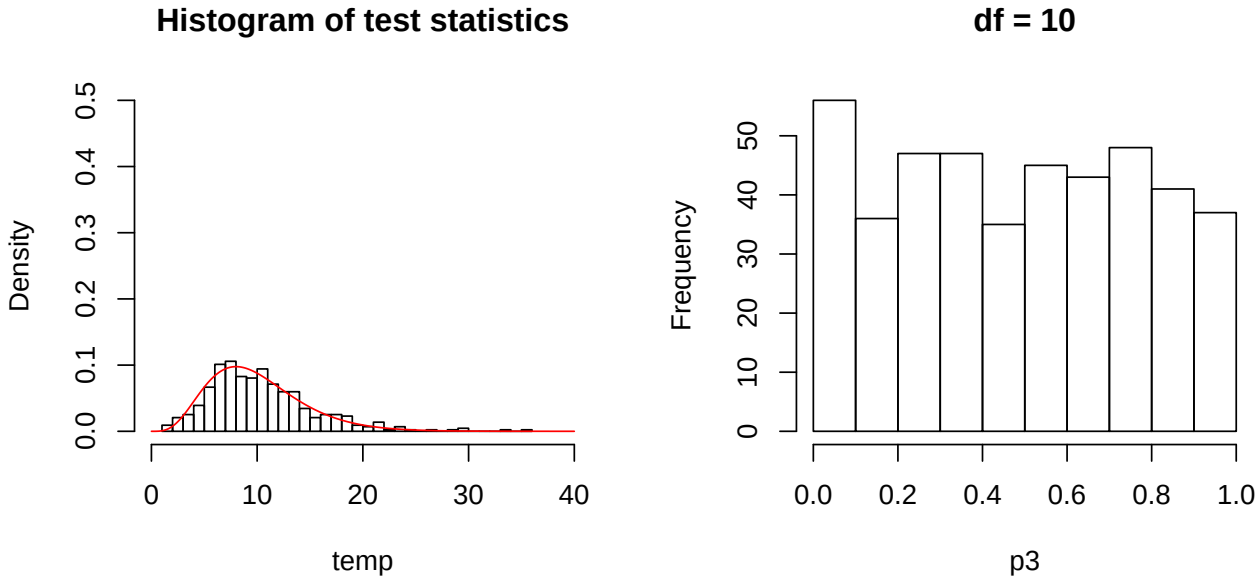
Test between GEM and unrestricted model

The histogram of test statistics and p-values

If the model is fited with α estimated by purity method:

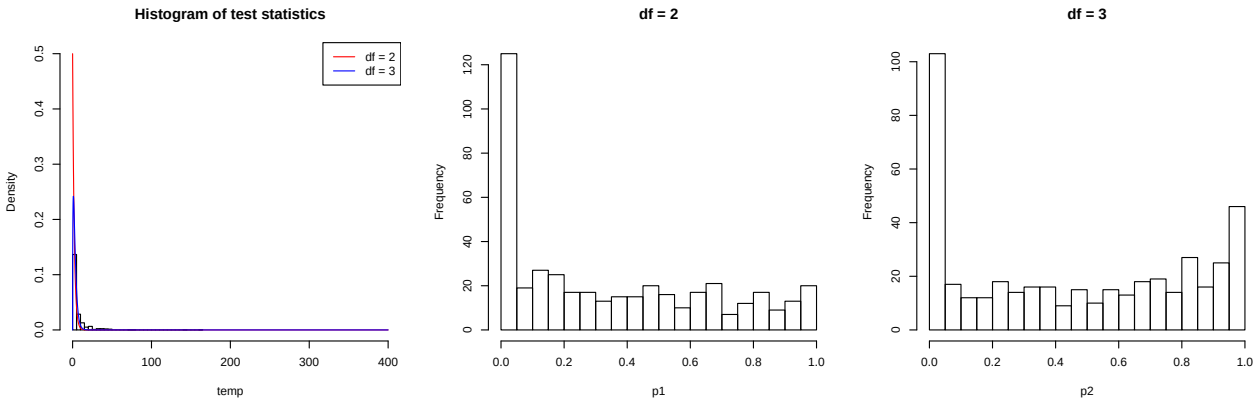


If the model is fited with α estimated by likelihood method:

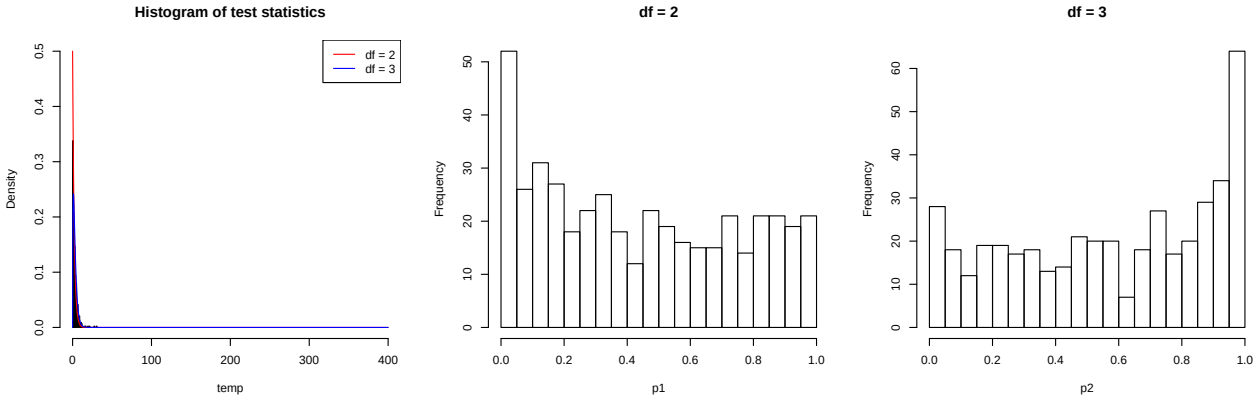


Test between GEM and multi-GEM model

If the model is fited with α estimated by purity method:



If the model is fited with α estimated by likelihood method:



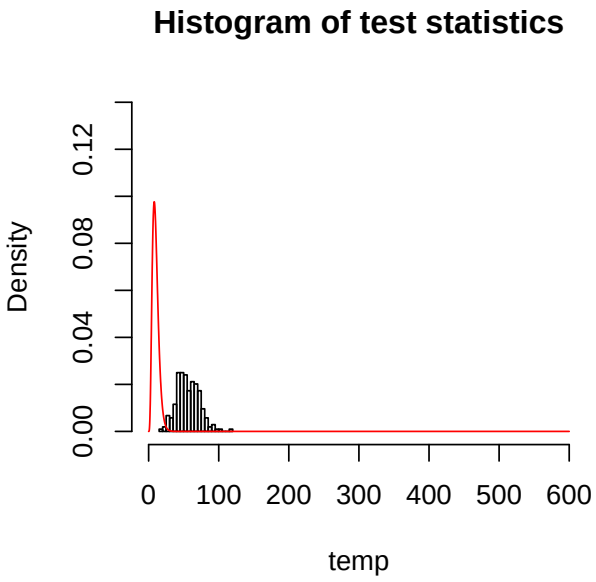
When $p = 3$, $\delta = 1$

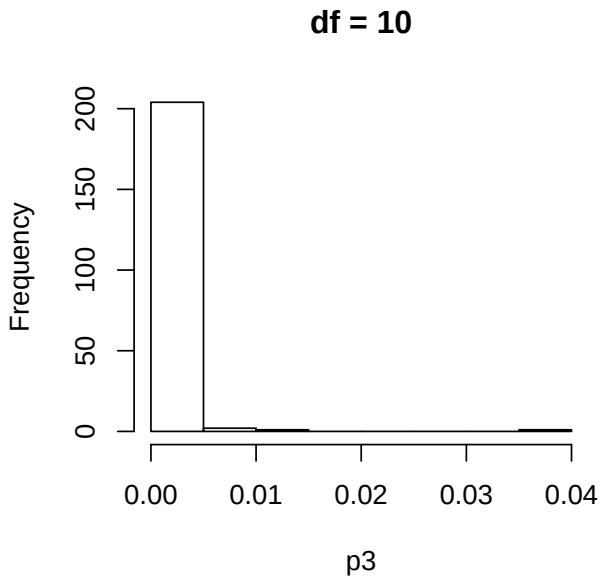
When $\delta = 1$, it means that data is generated from multi-GEM model, the GEM model is not true. The hypothesis tests should have small p values

Test between GEM and unrestricted model

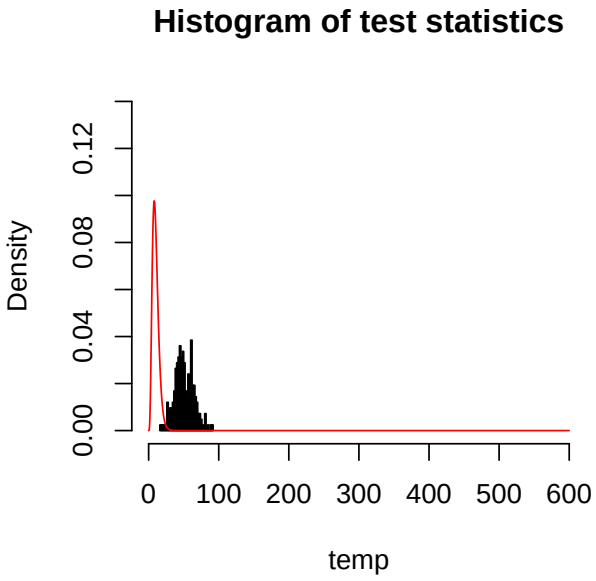
The histogram of test statistics and p-values

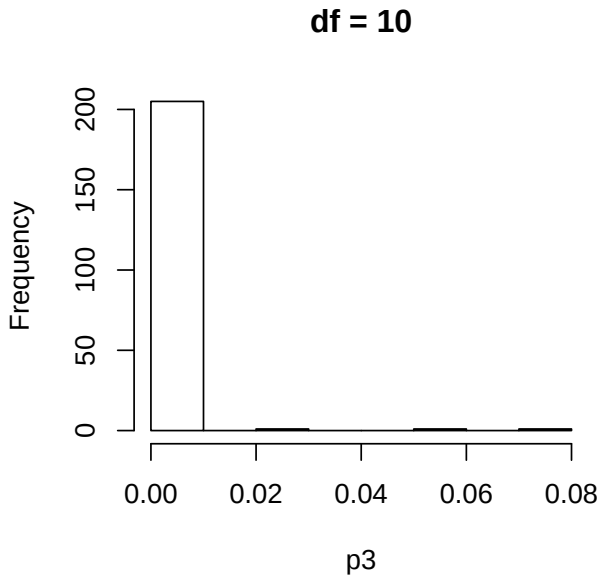
If the model is fited with α estimated by purity method:





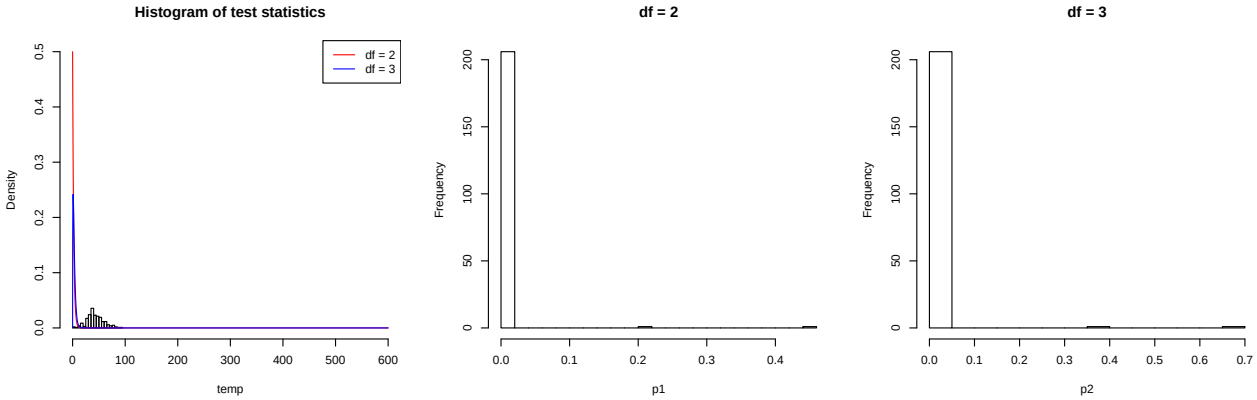
If the model is fited with α estimated by likelihood method:



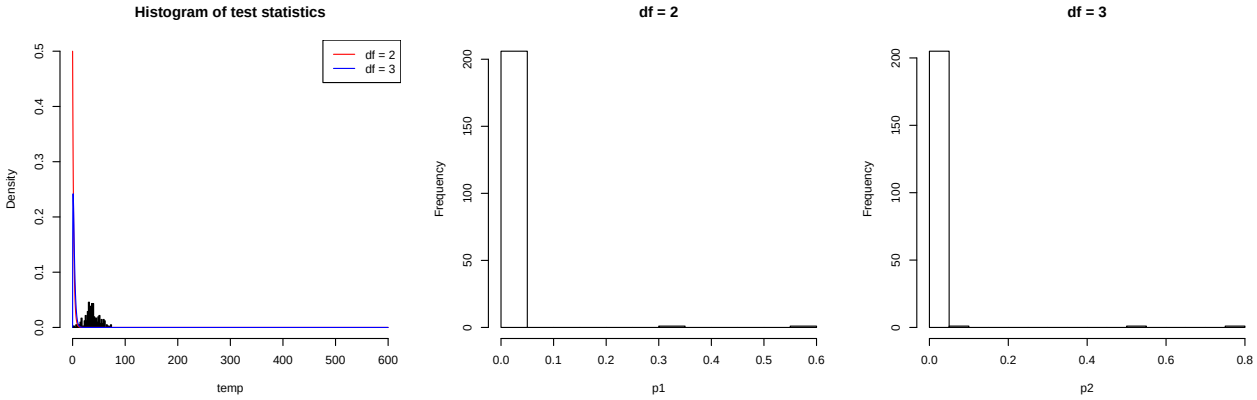


Test between GEM and multi-GEM model

If the model is fited with α estimated by purity method:



If the model is fited with α estimated by likelihood method:



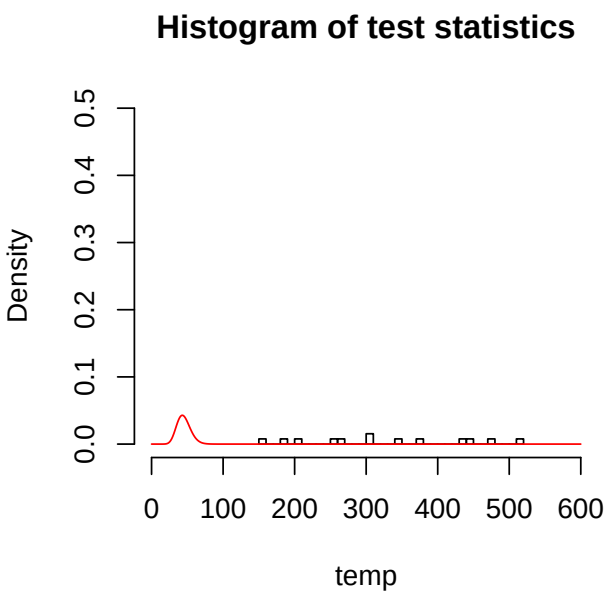
When $p = 10$, $\delta = 0$

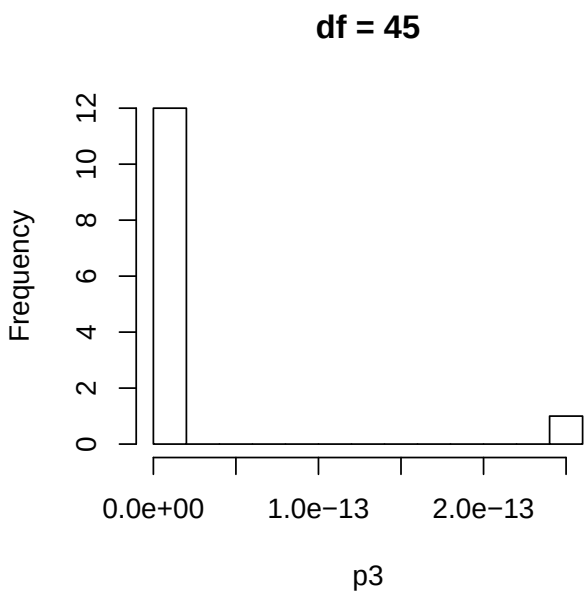
When $\delta = 0$, it means that data is generated from GEM model, the GEM model is true. The hypothesis tests should have large p values

Test between GEM and unrestricted model

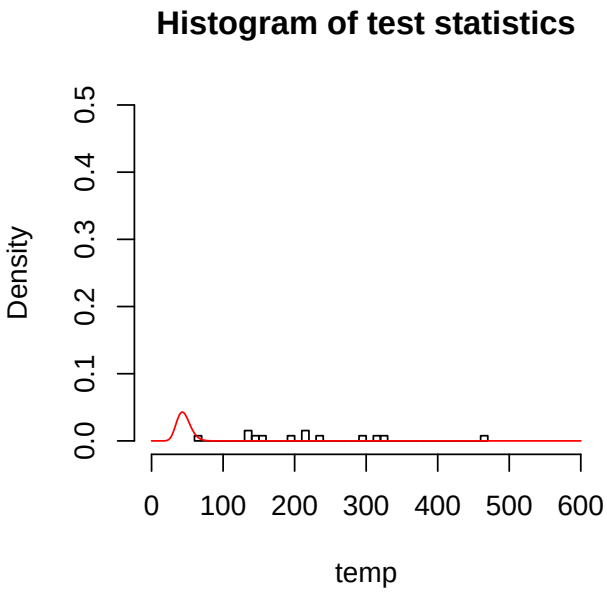
The histogram of test statistics and p-values

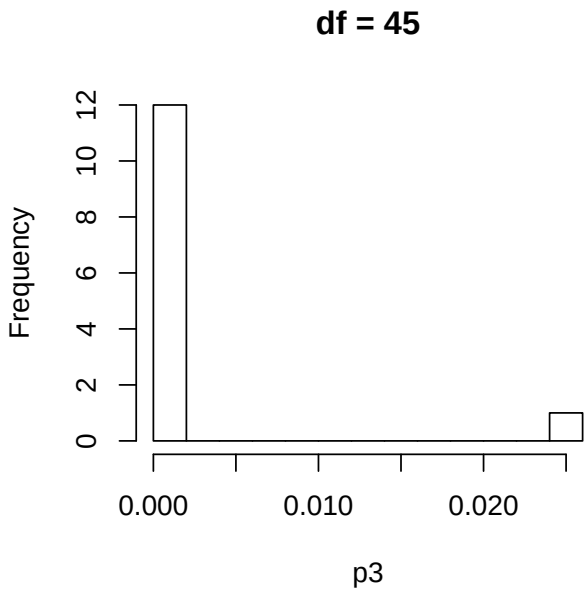
If the model is fited with α estimated by purity method:





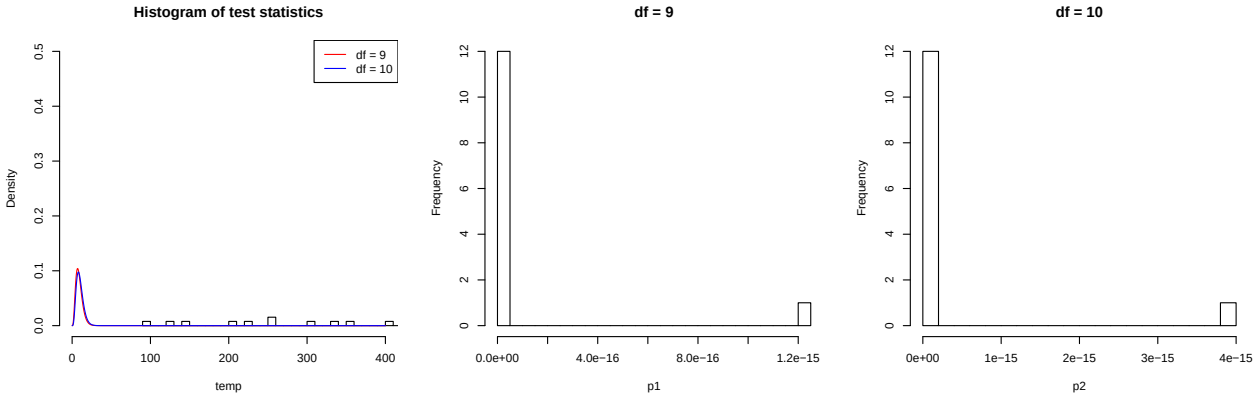
If the model is fited with α estimated by likelihood method:



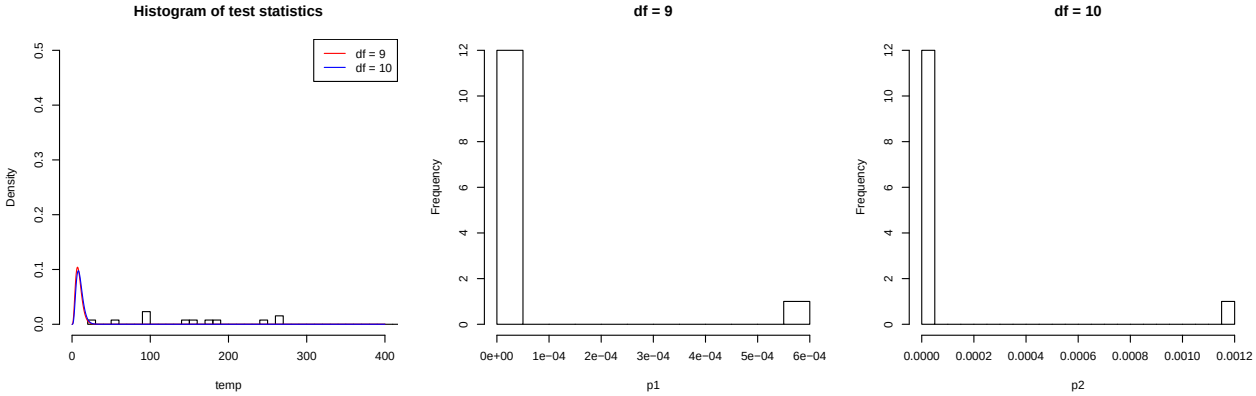


Test between GEM and multi-GEM model

If the model is fited with α estimated by purity method:



If the model is fited with α estimated by likelihood method:



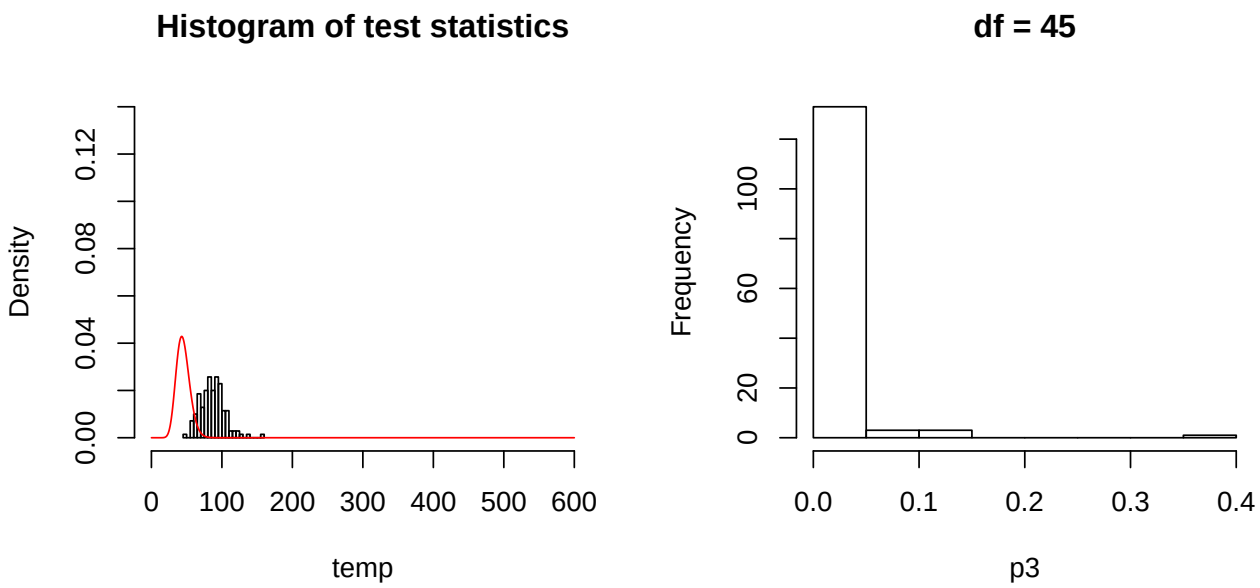
When $p = 10$, $\delta = 1$

When $\delta = 1$, it means that data is generated from multi-GEM model, the GEM model is not true. The hypothesis tests should have small p values

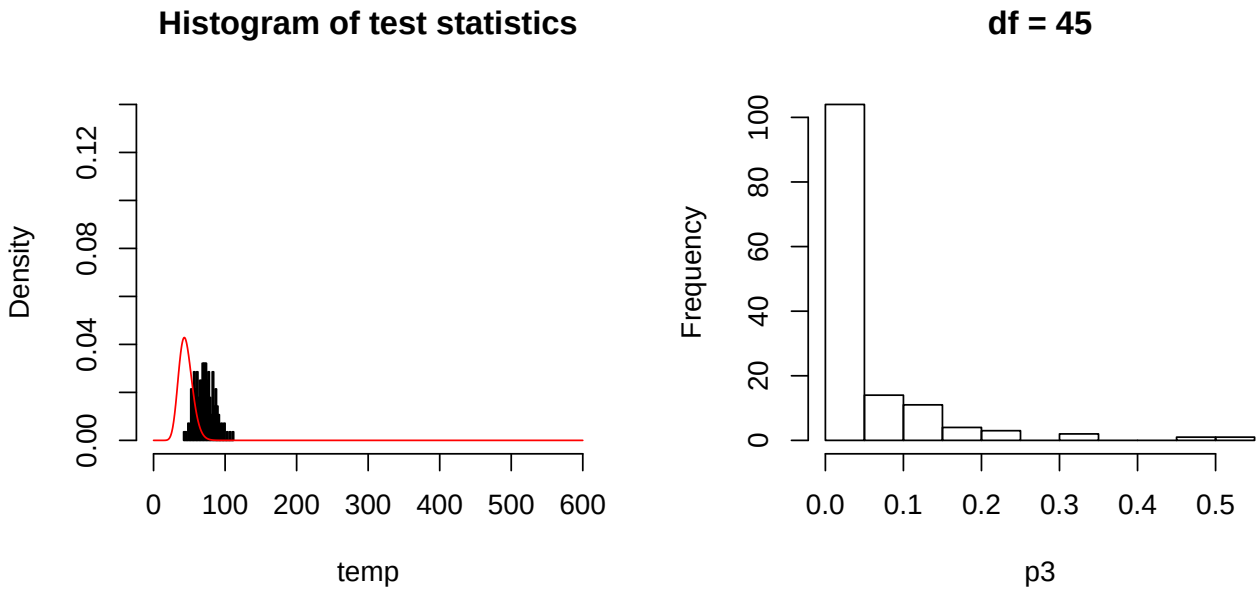
Test between GEM and unrestricted model

The histogram of test statistics and p-values

If the model is fited with α estimated by purity method:

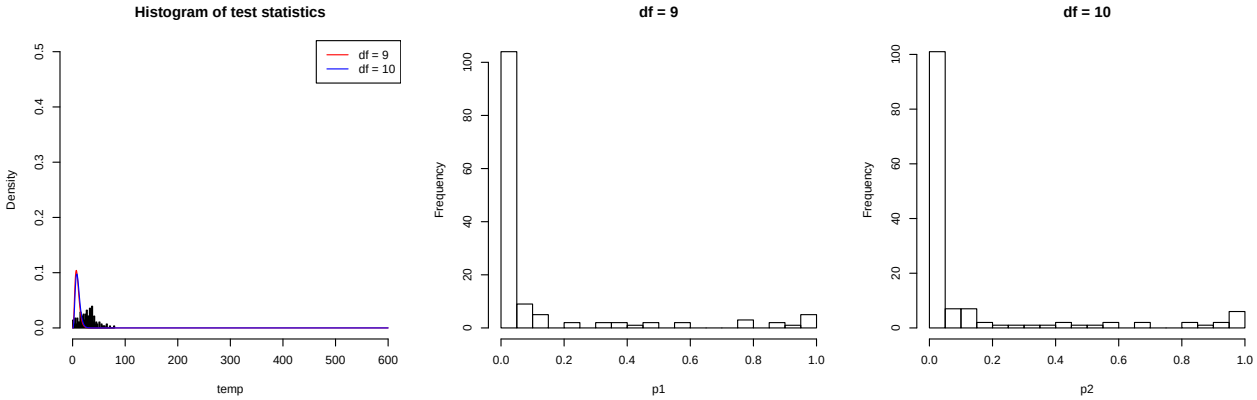


If the model is fited with α estimated by likelihood method:

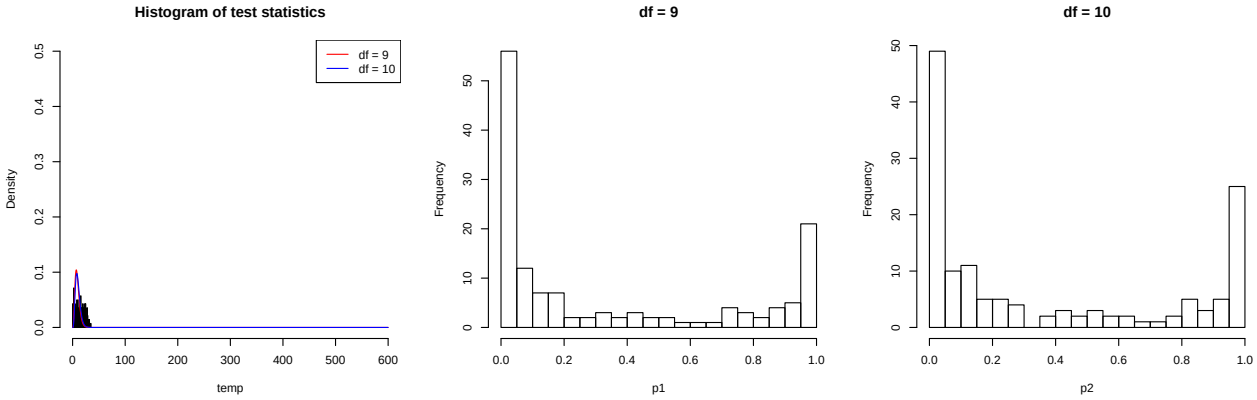


Test between GEM and multi-GEM model

If the model is fited with α estimated by purity method:



If the model is fited with α estimated by likelihood method:



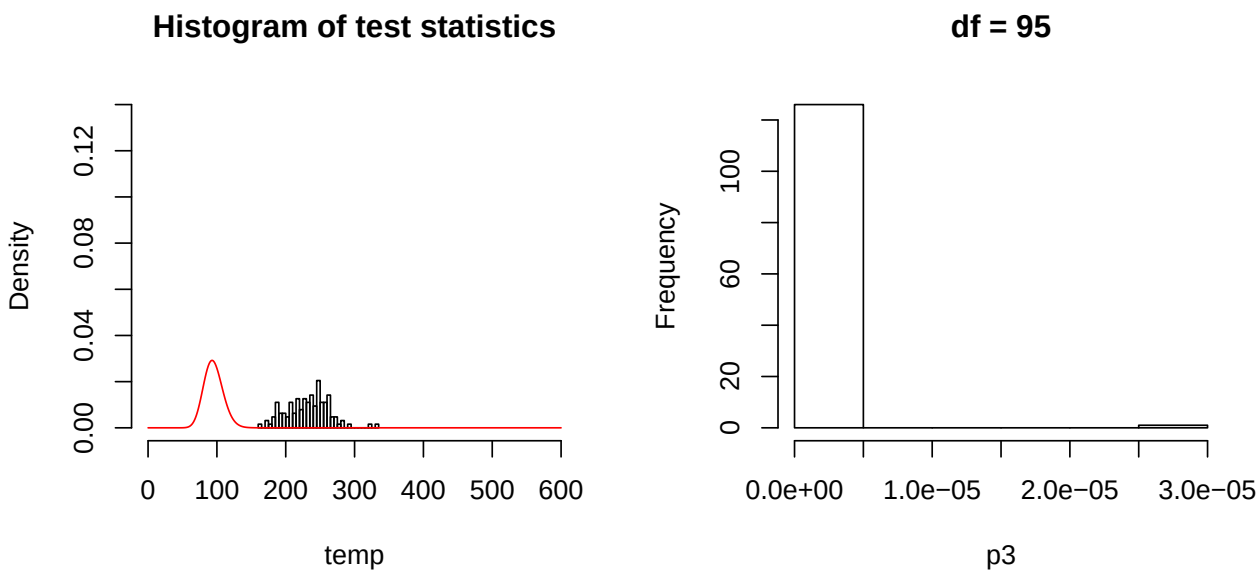
When $p = 20$, $\delta = 0$

When $\delta = 1$, it means that data is generated from multi-GEM model, the GEM model is not true. The hypothesis tests should have small p values

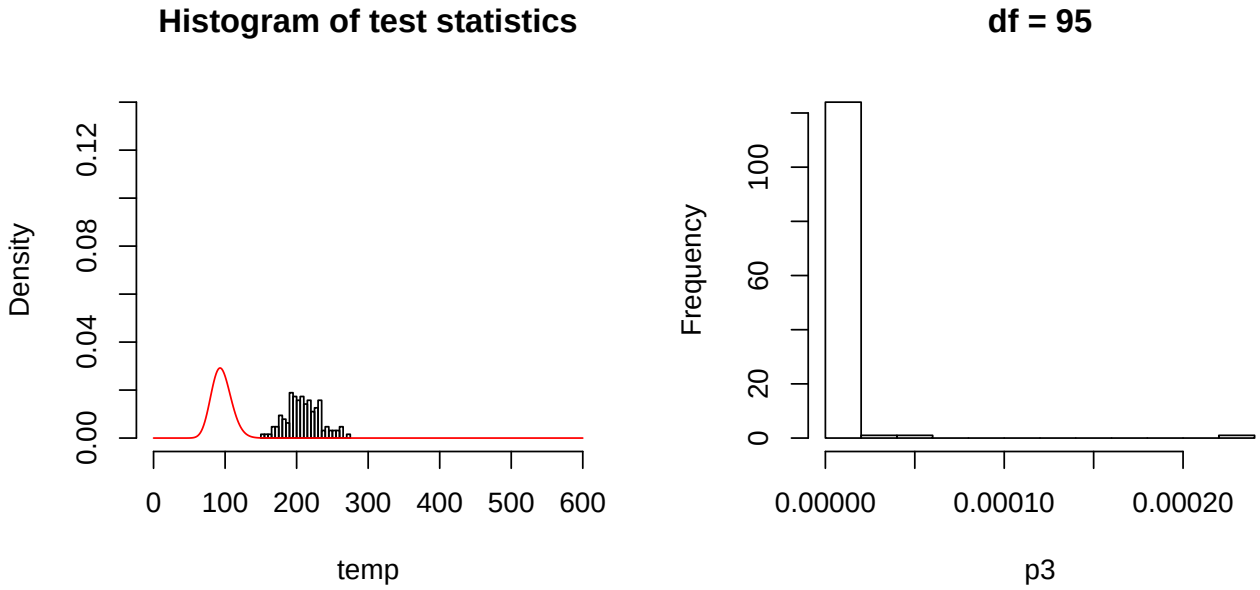
Test between GEM and unrestricted model

The histogram of test statistics and p-values

If the model is fited with α estimated by purity method:

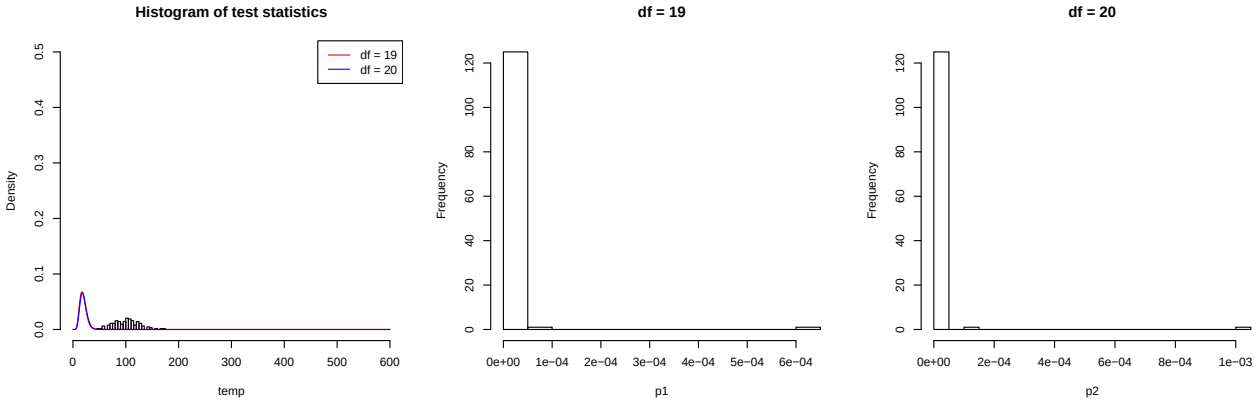


If the model is fited with α estimated by likelihood method:

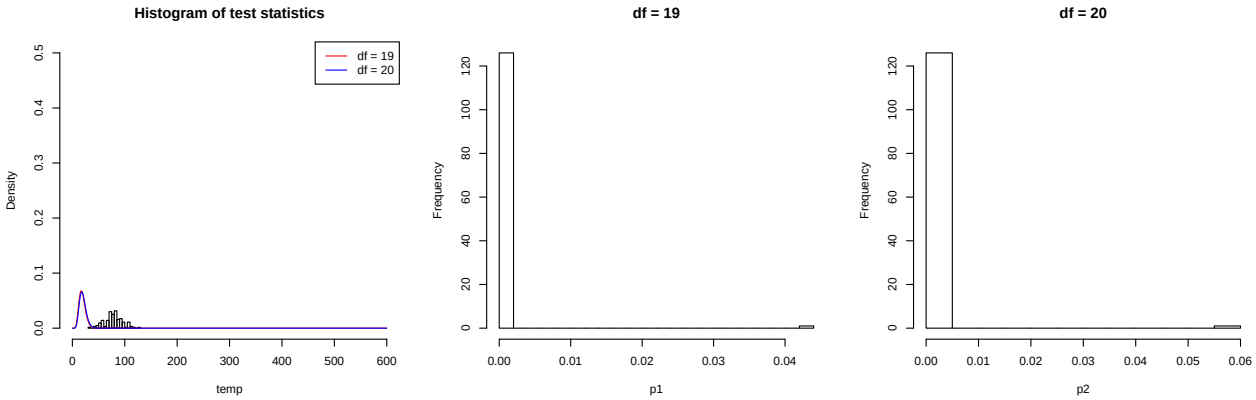


Test between GEM and multi-GEM model

If the model is fited with α estimated by purity method:



If the model is fited with α estimated by likelihood method:



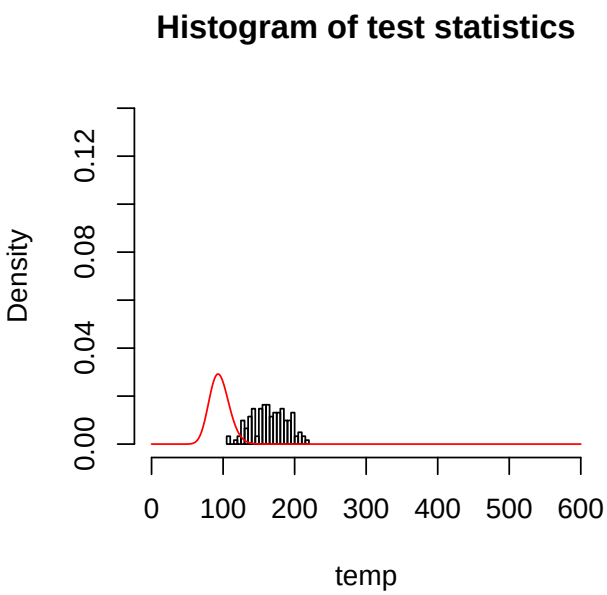
When $p = 20$, $\delta = 1$

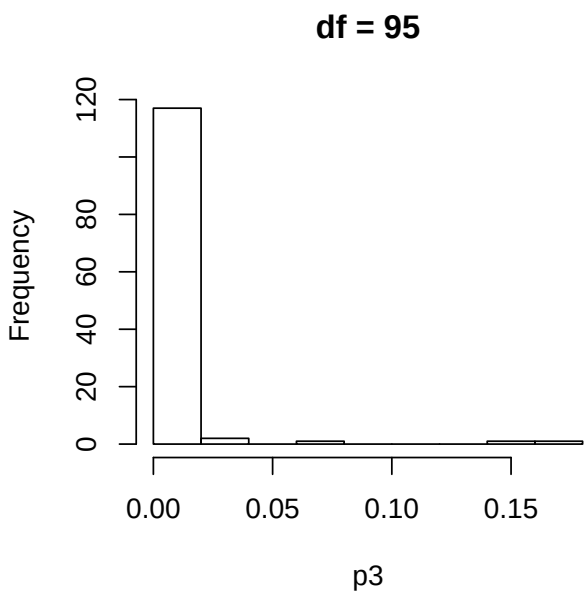
When $\delta = 1$, it means that data is generated from multi-GEM model, the GEM model is not true. The hypothesis tests should have small p values

Test between GEM and unrestricted model

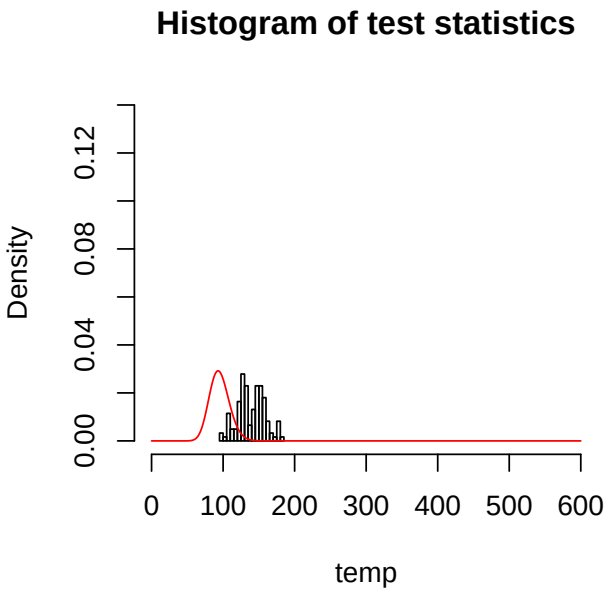
The histogram of test statistics and p-values

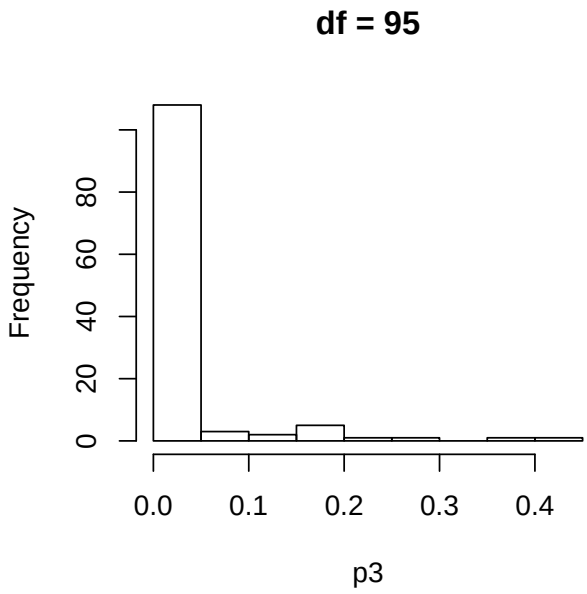
If the model is fited with α estimated by purity method:





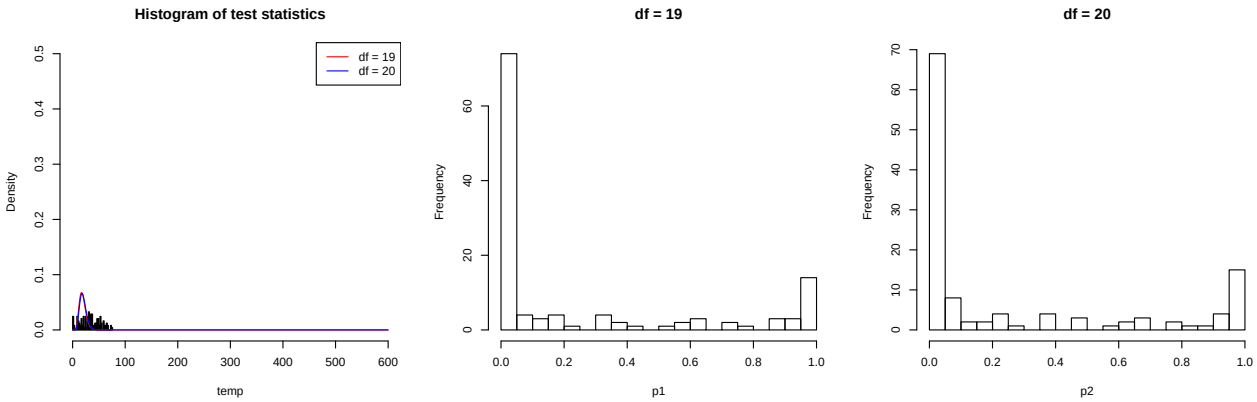
If the model is fited with α estimated by likelihood method:



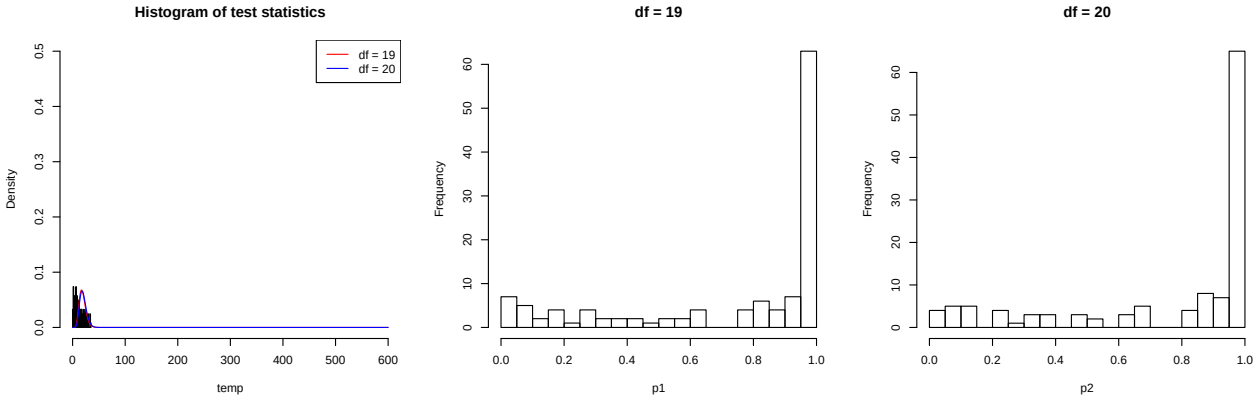


Test between GEM and multi-GEM model

If the model is fited with α estimated by purity method:

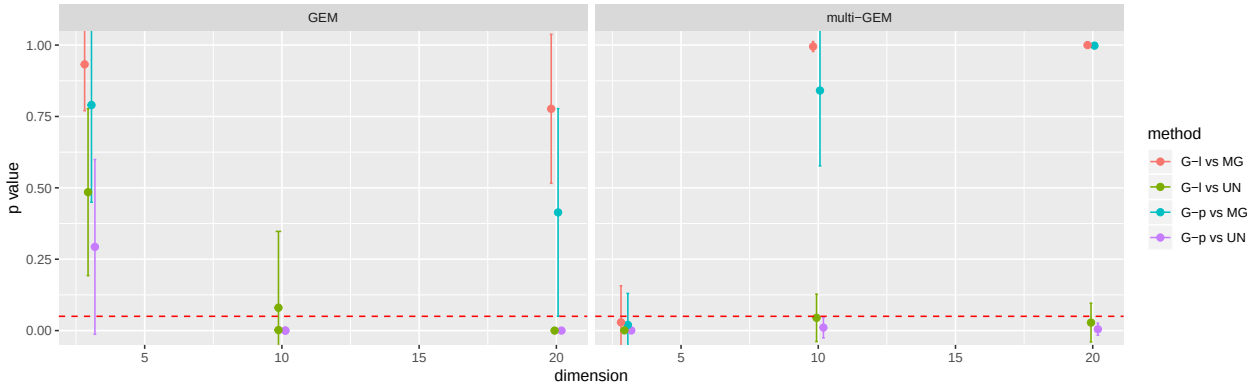


If the model is fited with α estimated by likelihood method:



Summary plots

p value over p = 3,10,20; $\delta = 0, 1$



- G-l vs MG: the GEM model fitted by using likelihood method, comparing with multi-GEM model
- G-L vs UN: the GEM model fitted by using likelihood method, comparing with unrestricted model
- G-p vs MG: the GEM model fitted by using purity method, comparing with multi-GEM model
- G-p vs UN: the GEM model fitted by using purity method, comparing with unrestricted model