Example 5

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Mimic Zhiliang Ying's paper and to make things easy, let's look at the Joint CDF which is:

$$S(T \geq x, C \geq y) = \begin{cases} e^{-\theta x} e^{-(e^{\theta y}-1)\big((x-y)+1\big)} & x \geq y \\ e^{-\theta x} e^{-(e^{\theta y}-1)} & x < y \end{cases}$$

For $x > y$ scenario

$$\frac{\partial}{\partial x} e^{-(e^{\theta y}-1)\big((x-y)+1\big)} = -(e^{\theta y} + \theta - 1) e^{-\theta x - (e^{\theta y}-1)\big((x-y)+1\big)}$$

$$-\frac{\partial}{\partial y}(e^{\theta y} + \theta - 1) e^{-\theta x - (e^{\theta y}-1)\big((x-y)+1\big)}$$

$$= \Big\{ \big[(\theta x - \theta y) + (\theta - 1)\big] e^{2\theta y} + \big[(\theta^2 - \theta)(x-y) + (\theta-1)(\theta-2)\big] e^{\theta y} + (\theta - 1) \Big\} e^{-\theta x - (e^{\theta y}-1)\big((x-y)+1\big)}$$

which is bigger than 0 for sure when $\theta > 2$

Therefore, the pdfs are

$$f_{T,C}(x,y) = \begin{cases} \Big\{ \big[(\theta x - \theta y) + (\theta - 1)\big] e^{2\theta y} + \big[(\theta^2 - \theta)(x-y) + (\theta-1)(\theta-2)\big] e^{\theta y} + (\theta - 1) \Big\} \\ \times e^{-\theta x - (e^{\theta y}-1)\big((x-y)+1\big)} & x \geq y \\ \\ \theta^2 e^{-e^{\theta y} + \theta y - \theta x + 1} & x < y \end{cases}$$

Besides, the functions $S(t), S_z(t), f(t), f_z(t)$ are the same, therefore, $m(t)$ is the same as Dr. Ying's example.

$$S_T(t) = P(T > t) = P(T > t, C > 0) = e^{-\theta t} e^{-(e^{\theta 0}-1)\big((t-0)+1\big)} = e^{-\theta t}$$

$$f_T(t) = \frac{\partial}{\partial t}(1 - S_T(t)) = \frac{\partial}{\partial t}(1 - e^{-\theta t}) = \theta e^{-\theta t}$$

$$S_Z(t) = P(T > t, C > t) = e^{-\theta t} e^{-(e^{\theta t}-1)} = e^{-e^{\theta t} - \theta t + 1}$$

$$f_Z(t) = \frac{\partial}{\partial t}(1 - S_Z(t)) = 1 - e^{-e^{\theta t} - \theta t + 1} = \theta(1 + e^{\theta t}) e^{-e^{\theta t} - \theta t + 1}$$

$$\psi(t) = \int_t^\infty f(t,c)dc = \int_t^\infty \theta^2 e^{-e^{\theta c} + \theta c - \theta t + 1} dc = \theta e^{-e^{\theta t} - \theta t + 1}$$

Therefore, the $m()$ function is:

$$m(t) = \frac{\lambda_F(t)}{\lambda_H(t)} = \frac{f_T(t)}{S_T(t)} / \frac{f_Z(t)}{S_Z(t)} = \frac{\theta e^{-\theta t}}{e^{-\theta t}} / \frac{\theta(1 + e^{\theta t}) e^{-e^{\theta t} - \theta t + 1}}{e^{-e^{\theta t} - \theta t + 1}} = \frac{1}{1 + e^{\theta t}}$$

And for the $\rho()$ function,

$$\begin{aligned} \rho = & \frac{f(t)/\psi(t) - 1}{S(t)/S_x(t) - 1} \\ = & \frac{\theta e^{-\theta t} / (\theta e^{-e^{\theta t} - \theta t + 1}) - 1}{e^{-\theta t} / e^{-e^{\theta t} - \theta t + 1} - 1} \\ = & 1 \end{aligned}$$

Results

Simulation settings

The survival time and the censor time were jointly simulated from the joint distribution above. The parameter θ is chosen as 1, 1.5, 2, and 5.

Sample size: 200

Iteration time: 500

Table 1: Mean absolute difference between estimated and true S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 1								
t0.1	0.01595	0.01442	0.01455	0.01463	0.01595	0.01450	0.01464	0.01471
t0.25	0.02215	0.02491	0.02550	0.02582	0.02215	0.02593	0.02654	0.02687
t0.5	0.03298	0.03292	0.03462	0.03571	0.03298	0.03800	0.03984	0.04099
t0.75	0.06624	0.03165	0.06358	0.07021	0.06624	0.04329	0.07444	0.08086
t0.9	0.09563	0.08600	0.10000	0.10000	0.09563	0.07227	0.10000	0.10000
theta = 1.5								
t0.1	0.01616	0.01768	0.01790	0.01802	0.01616	0.01806	0.01829	0.01841
t0.25	0.02455	0.03892	0.03973	0.04018	0.02455	0.04196	0.04281	0.04326
t0.5	0.04540	0.04209	0.04427	0.04563	0.04540	0.05362	0.05606	0.05751
t0.75	0.07266	0.03328	0.06842	0.07598	0.07266	0.05617	0.08921	0.09706
t0.9	0.08776	0.08306	0.10000	0.10000	0.08776	0.05522	0.10000	0.10000
theta = 2								
t0.1	0.01645	0.01820	0.01842	0.01854	0.01645	0.01858	0.01881	0.01893
t0.25	0.02299	0.03547	0.03625	0.03667	0.02299	0.03811	0.03891	0.03934
t0.5	0.04458	0.03860	0.04057	0.04179	0.04458	0.04804	0.05024	0.05156
t0.75	0.05850	0.02893	0.05303	0.05938	0.05850	0.04631	0.07086	0.07802
t0.9	0.09348	0.08716	0.09998	0.09995	0.09348	0.06080	0.09996	0.09992
theta = 5								
t0.1	0.01742	0.02929	0.02962	0.02979	0.01742	0.03188	0.03221	0.03239
t0.25	0.03140	0.09073	0.09182	0.09242	0.03140	0.10893	0.11006	0.11066
t0.5	0.21989	0.19104	0.19608	0.19932	0.21989	0.25029	0.25539	0.25816
t0.75	0.21982	0.06910	0.10944	0.11976	0.21982	0.14644	0.17083	0.17763
t0.9	0.08079	0.07243	0.07796	0.06781	0.08079	0.02170	0.04152	0.04632

Table 2: MSE

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 1								
t0.1	0.00039	0.00033	0.00034	0.00034	0.00039	0.00034	0.00034	0.00035
t0.25	0.00081	0.00095	0.00099	0.00101	0.00081	0.00101	0.00105	0.00107
t0.5	0.00178	0.00165	0.00181	0.00191	0.00178	0.00213	0.00232	0.00244
t0.75	0.00735	0.00163	0.01053	0.01124	0.00735	0.00296	0.01175	0.01260
t0.9	0.01240	0.00754	0.01000	0.01000	0.01240	0.00662	0.01000	0.01000
theta = 1.5								
t0.1	0.00040	0.00047	0.00048	0.00049	0.00040	0.00049	0.00050	0.00050

t0.25	0.00094	0.00199	0.00206	0.00210	0.00094	0.00227	0.00234	0.00239
t0.5	0.00308	0.00251	0.00274	0.00289	0.00308	0.00386	0.00416	0.00434
t0.75	0.00873	0.00175	0.01144	0.01227	0.00873	0.00450	0.01397	0.01518
t0.9	0.01050	0.00704	0.01000	0.01000	0.01050	0.00420	0.01000	0.01000
theta = 2								
t0.1	0.00043	0.00051	0.00052	0.00052	0.00043	0.00052	0.00053	0.00054
t0.25	0.00085	0.00174	0.00181	0.00184	0.00085	0.00197	0.00204	0.00208
t0.5	0.00291	0.00219	0.00239	0.00251	0.00291	0.00324	0.00350	0.00365
t0.75	0.00589	0.00140	0.00789	0.00856	0.00589	0.00331	0.00990	0.01092
t0.9	0.01188	0.00775	0.01000	0.00999	0.01188	0.00494	0.00999	0.00999
theta = 5								
t0.1	0.00047	0.00113	0.00115	0.00116	0.00047	0.00130	0.00133	0.00134
t0.25	0.00154	0.00890	0.00910	0.00922	0.00154	0.01259	0.01285	0.01298
t0.5	0.05182	0.03747	0.03949	0.04081	0.05182	0.06338	0.06600	0.06745
t0.75	0.04880	0.00551	0.01727	0.01904	0.04880	0.02227	0.03114	0.03328
t0.9	0.00673	0.00577	0.00641	0.00519	0.00673	0.00081	0.00331	0.00349

Table 3: Standard deviations of the estimated S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 1								
t0.1	0.01971	0.01628	0.01632	0.01635	0.01971	0.01627	0.01631	0.01634
t0.25	0.02793	0.02336	0.02344	0.02349	0.02793	0.02323	0.02331	0.02336
t0.5	0.03547	0.02986	0.03014	0.03033	0.03547	0.03171	0.03196	0.03212
t0.75	0.07583	0.03713	0.08773	0.08528	0.07583	0.04650	0.08736	0.08481
t0.9	0.10067	0.01212	0.00000	0.00000	0.10067	0.03769	0.00000	0.00000
theta = 1.5								
t0.1	0.02012	0.01588	0.01592	0.01594	0.02012	0.01596	0.01600	0.01603
t0.25	0.02912	0.02362	0.02371	0.02376	0.02912	0.02388	0.02397	0.02402
t0.5	0.03797	0.02999	0.03031	0.03052	0.03797	0.03264	0.03295	0.03312
t0.75	0.07358	0.03593	0.08791	0.08514	0.07358	0.04714	0.08395	0.08148
t0.9	0.09572	0.01178	0.00000	0.00000	0.09572	0.03583	0.00000	0.00000
theta = 2								
t0.1	0.02062	0.01632	0.01637	0.01639	0.02062	0.01638	0.01642	0.01645
t0.25	0.02760	0.02406	0.02416	0.02421	0.02760	0.02437	0.02447	0.02452
t0.5	0.03834	0.03154	0.03187	0.03208	0.03834	0.03374	0.03405	0.03422
t0.75	0.06814	0.03567	0.07854	0.07696	0.06814	0.04555	0.07658	0.07500
t0.9	0.09623	0.01224	0.00852	0.00777	0.09623	0.03643	0.00799	0.00726
theta = 5								
t0.1	0.02157	0.01694	0.01698	0.01701	0.02157	0.01723	0.01728	0.01730
t0.25	0.03122	0.02590	0.02600	0.02606	0.03122	0.02700	0.02711	0.02717
t0.5	0.05900	0.03130	0.03225	0.03291	0.05900	0.02715	0.02794	0.02841
t0.75	0.02192	0.02723	0.07286	0.06863	0.02192	0.02869	0.04430	0.04158
t0.9	0.01438	0.02286	0.07842	0.07183	0.01438	0.02739	0.04397	0.03962

The estimate of $m()$

Table 4: mean absolute difference between hat m() and true m()

1	1.5	2	5
0.0157802	0.0188633	0.0182303	0.0553309

The row name shows the θ value

Table 5: standard deviation of estimated m()

1	1.5	2	5
0.0119536	0.0137363	0.012792	0.0153594

The row name shows the θ value

Table 6: estimated theta from logitic regression

1	1.5	2	5
0.9169978	1.234852	1.680707	0.973398

The row name shows the true θ value

The survival function plots: