New likelihood function

Likelihood:

$$L(\rho; x, \delta) = \prod_{i=1}^{n} f(x_i, \delta_i; \rho) = \prod_{i=1}^{n} f(x_i, \delta_i = 1; \rho)^{\delta_i} f(x_i, \delta_i = 0; \rho)^{1-\delta_i}$$

And

- $f(x, \delta = 1; \rho) = \lim_{h \to 0} \frac{P(x<X<x+h, \delta=1)}{h}$
- $f(x, \delta = 0; \rho) = \lim_{h \to 0} \frac{P(x<X<x+h, \delta=0)}{h}$

For $P(x < X < x + h, \delta = 1)$

$$\begin{aligned} P(x < X < x + h, \delta = 1) =& P(x < T < x + h, T < C) \\ \approx& P(x < T < x + h, C > x) \\ =& P(C > x | x < T < x + h) P(x < T < x + h) \end{aligned} \tag{1}$$

Since $\psi(t) = \int_t^\infty f(t, s) ds = \int_t^\infty f(t) f(s|t) ds = f(t) \int_t^\infty f(s|t) ds = f(t) P(C > t | T = t) = f(t) P(C > T | t < T < t + h)$, $P(C > x | x < T < x + h) = \psi(t)/f(t)$

Therefore, **Eq 1** $= \frac{\psi(t)}{f(t)} \times f(t) h = \psi(t) h$.

Therefroe, $P(x < X < x + h, \delta = 1) = \psi(t)$ (is that correct?)

Similarly, for $P(x < X < x + h, \delta = 0)$

$$\begin{aligned} P(x < X < x + h, \delta = 0) =& P(x < C < x + h, T > C) \\ \approx& P(x < C < x + h, T > x) \\ =& P(T > x | x < C < x + h) P(x < C < x + h) \end{aligned} \tag{2}$$