decision rule

2020-03-19

For the model:

$$Y_{ji} = S_{ji}(\beta_j + b_{ji} + \Gamma_j(\alpha' x_{ji})) + \epsilon_{ji},\ j \in \{1, 2\}$$

Parameter settings

I have tried different parameter settings.

Setting 1:

The fixed effect β:

	intercept	slope	concavity
Drug	18.6	-2.3	0.17
Placebo	18.6	-1.9	0.14

The fixed effect Γ:

	intercept	slope	concavity
Drug	0	1	0
Placebo	0	$\cos(\theta)$	$\sin(\theta)$

The random effect's covariance matrix:

	Drug				Placebo		
	intercept	slope	concavity		intercept	slope	concavity
intercept	8.869	2.766	-0.359	intercept	9.507	1.012	-0.093
slope	2.766	1.016	-0.104	slope	1.012	3.856	-0.395
concavity	-0.359	-0.104	1.5	concavity	-0.093	-0.395	1.5

Setting 2:

EMBARC's parameter

The random effect's covariance matrix:

	Drug				Placebo		
	intercept	slope	concavity		intercept	slope	concavity
intercept	8.869	2.766	-0.359	intercept	9.507	1.012	-0.093
slope	2.766	1.016	-0.104	slope	1.012	3.856	-0.395
concavity	-0.359	-0.104	0.015	concavity	-0.093	-0.395	0.045

Setting 3:

Make the intercept smaller

The fixed effect β:

	intercept	slope	concavity
Drug	1	-2.3	1.5
Placebo	1.1	-1.9	1

The fixed effect Γ:

	intercept	slope	concavity
Drug	0	1	0
Placebo	0	$\cos(\theta)$	$\sin(\theta)$

The random effect's covariance matrix:

	Drug				Placebo		
	intercept	slope	concavity		intercept	slope	concavity
intercept	4	0	0	intercept	4	1.732	1
slope	0	9	0	slope	1.732	1.75	0.433
concavity	0	0	1	concavity	1	0.433	1.25

Concordance proportion calculation

For the training dataset, we simulate $n = 100$ in each treatment group (200 subjects in total). The covariates are generated from $X \sim MVN(0, \Sigma)$, where

$$\Sigma_x = \begin{pmatrix} 1 & \rho & ... & \rho \\ \rho & 1 & ... & \rho \\ ... & ... & ... & ... \\ \rho & ... & \rho & 1 \end{pmatrix}_{p \times p}$$

The outcome Y is then generated from the model

$$Y_{ji} = S_{ji}(\beta_j + b_{ji} + \Gamma_j(\alpha' x_{ji})) + \epsilon_{ji}, \ j \in \{1, 2\}$$

For the testing dataset, $N = 1000$ subjects are generated. For each subjects, the outcome if he or she get treatment and the outcome if he or she get placebo are both generated.

For the testing dataset, since we have both outcomes, we know the treatment group assignment for each subject.

$$\text{group assignment} = \begin{cases} \text{Drug group} & \Delta Y_{pbo} > \Delta Y_{drg} \\ \text{Placebo group} & \Delta Y_{pbo} \leq \Delta Y_{drg} \end{cases}$$

To estimated the treatment assigment for the testing data, we tried two methods, longitudinal average slope methods and change score methods.

For the longitudinal average slope method, we used the ullback-Leibler divergence method to estimated $\hat{\alpha}$ and $\hat{\beta}_{drg}, \hat{\Gamma}_{drg}, \hat{\beta}_{pbo}, \hat{\Gamma}_{pbo}$. The the estimated fixed effects Z can be calculated as:

$$\hat{z}_{i,drg} = S[\hat{\beta}_{drg} + \hat{\Gamma}_{drg}\hat{\alpha}' x_i]$$

$$\hat{z}_{i,pbo} = S[\hat{\beta}_{pbo} + \hat{\Gamma}_{pbo}\hat{\alpha}' x_i]$$

Given that, the estimated group assignment can be calculated as

$$\text{estimated group assignment} = \begin{cases} \text{Drug group} & \Delta\hat{z}_{i,pbo} > \Delta\hat{z}_{i,drg} \\ \text{Placebo group} & \Delta\hat{z}_{i,pbo} \leq \Delta\hat{z}_{i,drg} \end{cases}$$

The concordance proportion is then calculated as

$$\frac{\sum_{i=1}^{N} I(\text{estimated group assignment} = \text{true group assignment})}{N}$$

p.s. previous I used $\hat{z}_{i,drg} = S[\beta_{drg} + \Gamma_{drg}\alpha' x_i]$ and $\Delta Y = \hat{z}_{7,drg} - \hat{z}_{1,drg}$ as the true group assignment rule for the longitudinal method, instead of the true ΔY in the testing dataset.

For the change score method, the change in outcome Y can be fitted with a linear regression with covariates X, i.e.

$$\Delta\hat{Y} = \hat{\gamma}_1 x_1 + ... + \hat{\gamma}_p x_p$$

where $\hat{\gamma}_i$ is estimated with the same model in training dataset. Then

$$\text{estimated group assignment} = \begin{cases} \text{Drug group} & \Delta\hat{Y}^{cs}_{pbo} > \Delta\hat{Y}^{cs}_{drg} \\ \text{Placebo group} & \Delta\hat{Y}^{cs}_{pbo} \leq \Delta\hat{Y}^{cs}_{drg} \end{cases}$$

The concordance proportion is then calculated as

$$\frac{\sum_{i=1}^{N} I(\text{estimated group assignment} = \text{true group assignment})}{N}$$

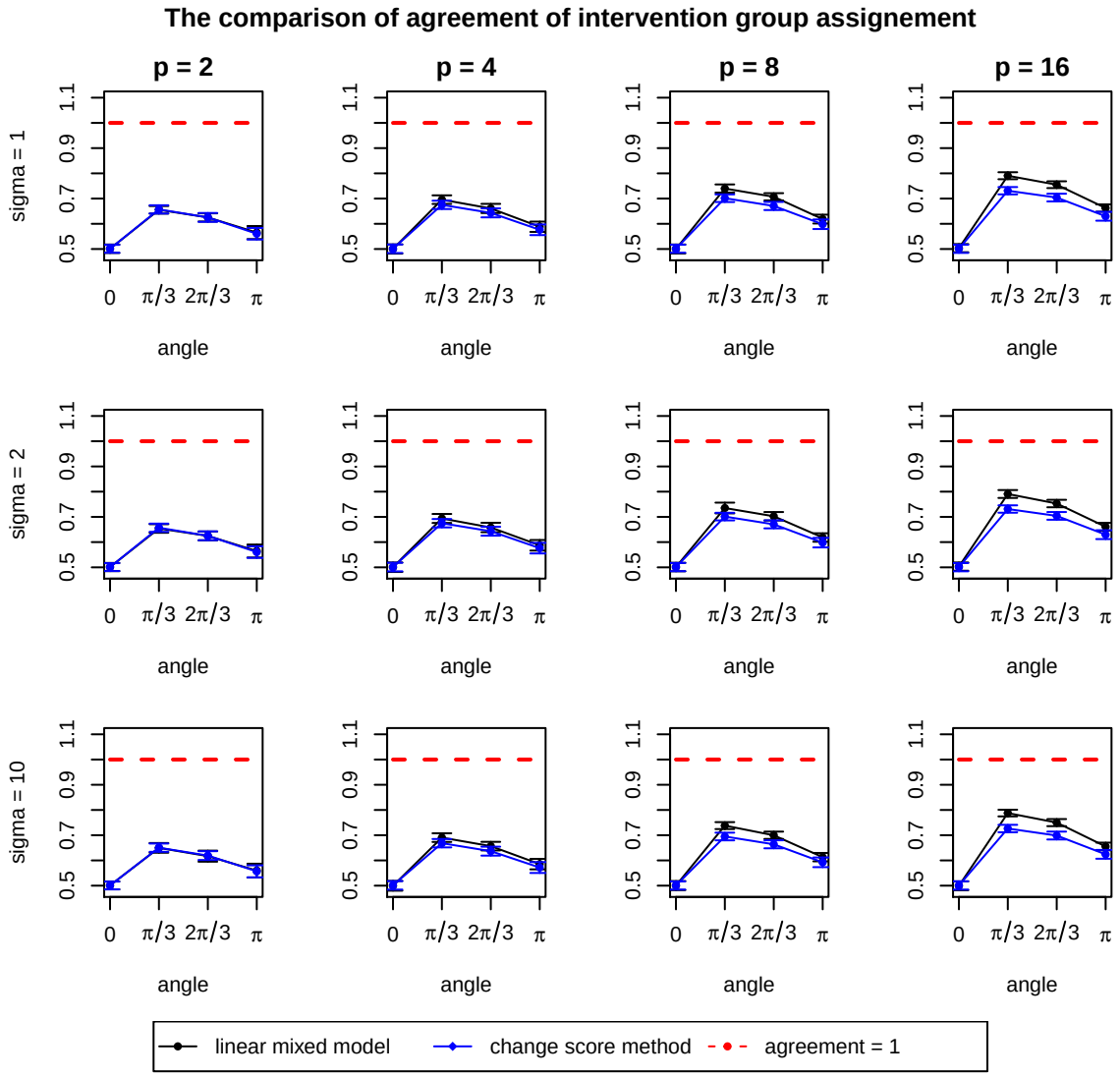
We would like to compare which method can achieve a higher concordance proportion.

Results

Setting 1

			Purity		Cosine similarity		Agreement	
Dimension	Sigma	Angle	Mean	SD	Mean	SD	Mean	SD
2	1	0	0.4941	0.2182	0.8520	0.2066	0.5011	0.0168
		60	1.5468	0.4369	0.9706	0.0465	0.6554	0.0150
		120	4.5609	1.0666	0.9857	0.0206	0.6253	0.0162
		180	7.4982	1.6440	0.9921	0.0120	0.5653	0.0259
	2	0	0.9088	1.0308	0.8660	0.1958	0.5012	0.0162
		60	2.5074	5.0316	0.9520	0.0889	0.6540	0.0170
		120	6.7709	12.5737	0.9744	0.0377	0.6246	0.0176
		180	10.6863	17.6086	0.9840	0.0228	0.5646	0.0257
	10	0	1020.8285	3945.5138	0.9550	0.1401	0.5010	0.0157
		60	1954.7484	9010.6063	0.9779	0.0839	0.6497	0.0192
		120	1817.5259	7761.0103	0.9652	0.1273	0.6170	0.0219
		180	3080.9066	11341.4179	0.9529	0.1168	0.5597	0.0272
4	1	0	0.5908	0.2283	0.8101	0.1722	0.5000	0.0184
		60	2.3377	0.5761	0.9303	0.0636	0.6957	0.0170
		120	7.5200	1.6407	0.9609	0.0323	0.6609	0.0182
		180	12.5537	2.5724	0.9766	0.0200	0.5883	0.0204
	2	0	1.5461	2.8809	0.8137	0.1863	0.5004	0.0188
		60	3.4636	3.0137	0.9041	0.1148	0.6938	0.0178
		120	12.0052	15.5890	0.9219	0.0907	0.6579	0.0187
		180	19.7302	28.1132	0.9546	0.0411	0.5873	0.0210
	10	0	545.7065	2666.8060	0.9899	0.0556	0.4994	0.0194
		60	1142.0264	7640.9531	0.9798	0.0733	0.6905	0.0172

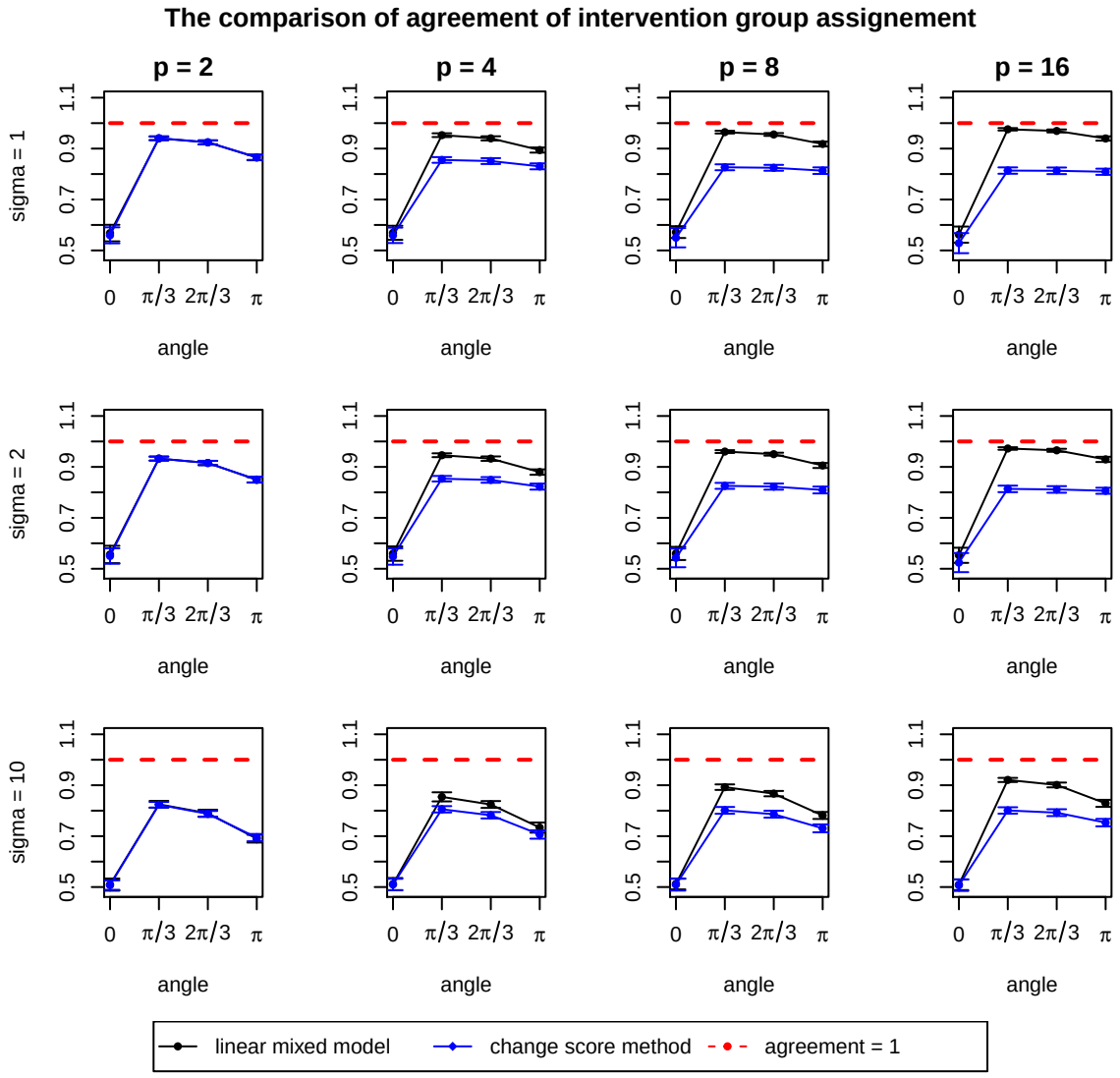
		120	2872.3701	11314.6790	0.9840	0.0663	0.6567	0.0171
		180	2046.5676	7963.4221	0.9824	0.0681	0.5853	0.0207
8	1	0	0.7183	0.3380	0.7003	0.1497	0.5002	0.0178
		60	3.6919	0.8491	0.8559	0.1040	0.7398	0.0162
		120	12.8547	3.1406	0.9146	0.0550	0.7070	0.0144
		180	22.4677	6.1550	0.9479	0.0287	0.6187	0.0175
	2	0	7.0189	51.6329	0.7575	0.1402	0.5012	0.0169
		60	10.3262	51.8376	0.8294	0.1548	0.7352	0.0213
		120	31.2540	130.1463	0.8571	0.1116	0.7025	0.0168
		180	421.0805	3841.9870	0.9085	0.0585	0.6183	0.0168
	10	0	1163.4518	5294.3756	0.9989	0.0055	0.5003	0.0179
		60	1369.4918	4853.7169	0.9930	0.0401	0.7381	0.0138
		120	4112.5607	13736.4404	0.9937	0.0405	0.6997	0.0145
		180	1732.4927	7971.6727	0.9959	0.0349	0.6130	0.0166
16	1	0	0.8735	0.3864	0.6842	0.1698	0.5034	0.0162
		60	6.9310	1.4942	0.8280	0.1028	0.7905	0.0139
		120	24.9727	5.2131	0.8904	0.0644	0.7550	0.0135
		180	44.4030	10.1076	0.9174	0.0457	0.6625	0.0145
	2	0	2.4095	3.9985	0.7778	0.1795	0.5029	0.0163
		60	8.8935	5.6064	0.8682	0.1402	0.7908	0.0156
		120	38.6863	33.9093	0.8640	0.1474	0.7530	0.0148
		180	77.4618	96.0817	0.8716	0.0911	0.6609	0.0154
	10	0	422.5353	1924.9107	0.9998	0.0002	0.4994	0.0176
		60	773.8957	3814.6546	0.9998	0.0002	0.7878	0.0132
		120	2098.7287	9566.4763	0.9998	0.0002	0.7499	0.0141
		180	1491.9371	8049.9292	0.9998	0.0002	0.6553	0.0152



Setting 2

			Purity		Cosine similarity		Agreement	
Dimension	Sigma	Angle	Mean	SD	Mean	SD	Mean	SD
2	1	0	1.5772	0.5836	0.9702	0.0426	0.5681	0.0325
		60	247.2797	50.7826	0.9995	0.0007	0.9401	0.0069
		120	191.9133	40.8291	0.9995	0.0007	0.9244	0.0076
		180	20.6900	3.2617	0.9976	0.0033	0.8650	0.0109
	2	0	879.4013	6369.8261	0.9638	0.0870	0.5560	0.0342
		60	518.4135	1293.5854	0.9994	0.0007	0.9325	0.0076
		120	616.8250	2544.6347	0.9993	0.0009	0.9154	0.0082
		180	60.8464	256.5212	0.9955	0.0084	0.8491	0.0108

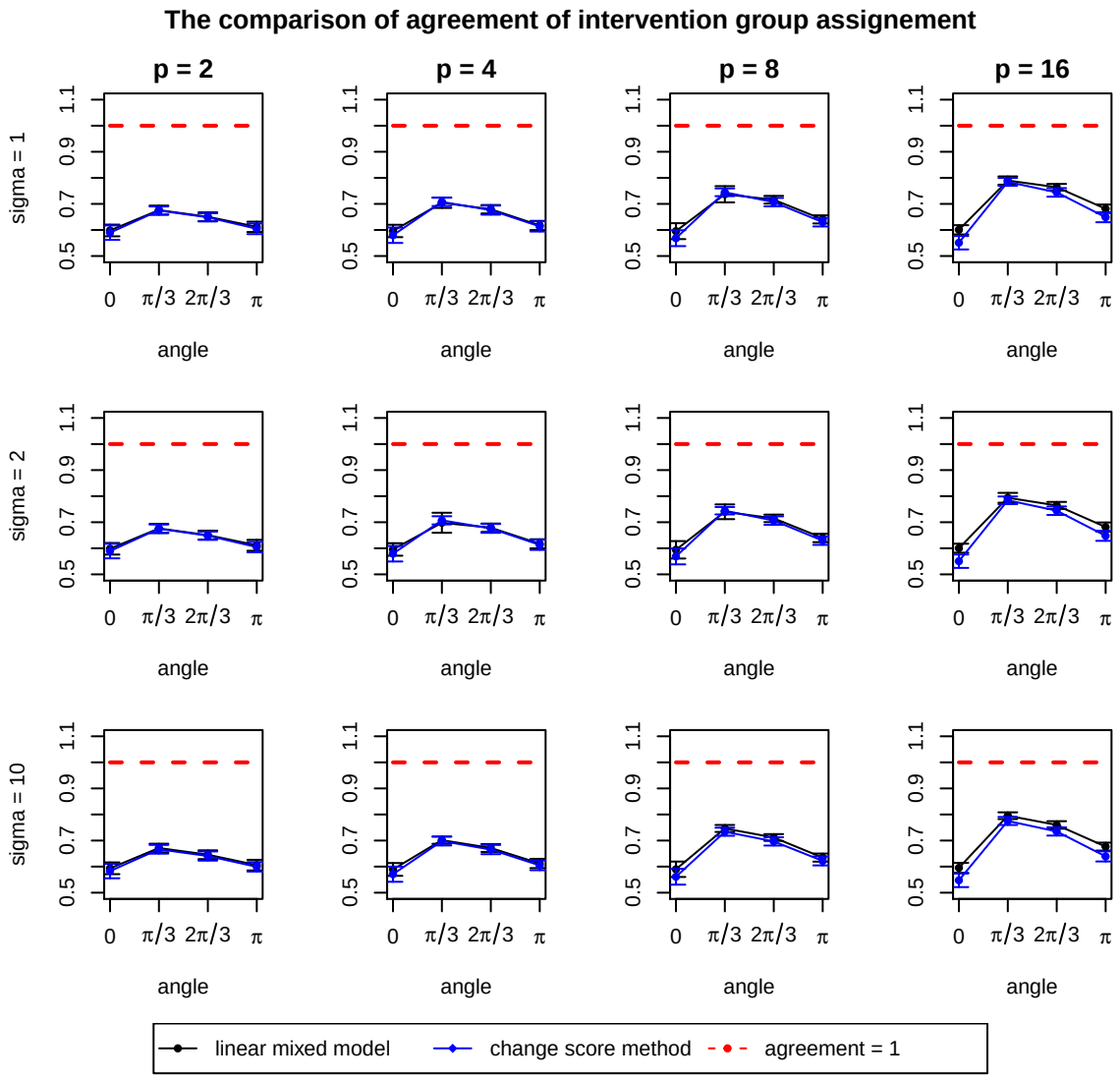
	10	0	4511.1242	15498.8996	0.9672	0.1184	0.5096	0.0238
		60	6811.5626	17095.2600	0.9917	0.0379	0.8251	0.0135
		120	4805.7298	14253.2210	0.9911	0.0382	0.7899	0.0139
		180	2397.9207	10464.8379	0.9843	0.0534	0.6900	0.0153
4	1	0	1.5910	0.5624	0.9243	0.1048	0.5692	0.0277
		60	405.4914	63.5055	0.9991	0.0006	0.9528	0.0073
		120	314.8149	50.2224	0.9990	0.0007	0.9403	0.0081
		180	33.5671	5.4529	0.9925	0.0067	0.8943	0.0101
	2	0	6.8261	29.4817	0.9153	0.1147	0.5594	0.0282
		60	1549.1616	5865.1312	0.9988	0.0010	0.9459	0.0075
		120	752.2896	2915.5866	0.9987	0.0011	0.9325	0.0087
		180	92.9996	321.0636	0.9822	0.0245	0.8797	0.0106
	10	0	7948.4350	20354.1703	0.9713	0.0896	0.5114	0.0242
		60	5621.4665	14339.6538	0.9799	0.0762	0.8543	0.0178
		120	7027.2184	15134.5024	0.9894	0.0250	0.8243	0.0134
		180	3836.7836	11960.5604	0.9657	0.0966	0.7336	0.0199
8	1	0	1.7436	0.6659	0.9046	0.1165	0.5720	0.0232
		60	780.5187	155.9886	0.9988	0.0007	0.9646	0.0053
		120	605.8159	125.4156	0.9987	0.0008	0.9554	0.0055
		180	62.6594	9.8145	0.9841	0.0097	0.9181	0.0095
	2	0	775.8581	7645.3834	0.9168	0.1123	0.5601	0.0258
		60	2206.8778	7694.7257	0.9984	0.0012	0.9603	0.0055
		120	1961.3569	5919.0847	0.9981	0.0013	0.9501	0.0057
		180	1076.1246	6614.0581	0.9670	0.0307	0.9056	0.0099
	10	0	7602.9954	20571.3544	0.9947	0.0344	0.5122	0.0214
		60	8424.6204	15303.8350	0.9913	0.0370	0.8924	0.0109
		120	3872.2562	6287.5508	0.9956	0.0134	0.8670	0.0105
		180	4967.3592	14827.8672	0.9877	0.0454	0.7812	0.0138
16	1	0	1.7547	0.6779	0.9443	0.0830	0.5620	0.0317
		60	1529.9761	297.9284	0.9989	0.0007	0.9758	0.0048
		120	1183.0168	224.7857	0.9986	0.0009	0.9694	0.0058
		180	122.8007	20.5727	0.9764	0.0122	0.9397	0.0085
	2	0	63.2273	528.4508	0.9453	0.0877	0.5528	0.0297
		60	4606.2175	12618.5773	0.9983	0.0013	0.9723	0.0050
		120	2379.9637	2809.5499	0.9977	0.0018	0.9652	0.0059
		180	670.2696	4592.1688	0.9525	0.0438	0.9295	0.0103
	10	0	7361.9942	19862.9779	0.9994	0.0040	0.5092	0.0211
		60	14852.4397	21270.1813	0.9976	0.0063	0.9208	0.0081
		120	11455.9901	19942.0523	0.9971	0.0081	0.9011	0.0094
		180	3835.6653	9336.8916	0.9950	0.0464	0.8288	0.0139



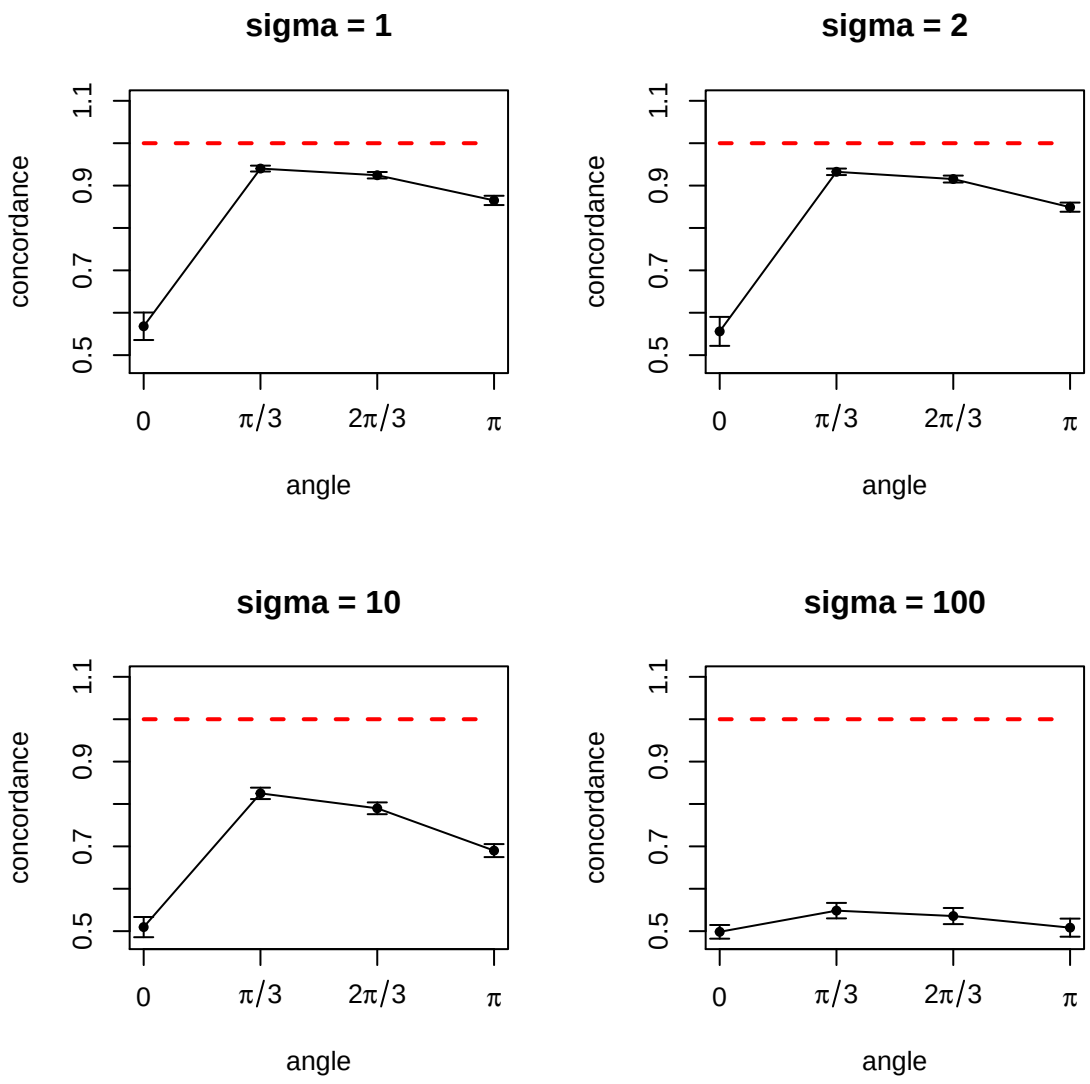
Setting 3

			Purity		Cosine similarity		Agreement	
Dimension	Sigma	Angle	Mean	SD	Mean	SD	Mean	SD
2	1	0	2.6131	0.9759	0.9629	0.0576	0.5981	0.0224
		60	4.1062	1.2375	0.9489	0.0856	0.6762	0.0173
		120	5.6561	1.6805	0.9849	0.0227	0.6506	0.0169
		180	5.0562	1.6149	0.9810	0.0316	0.6120	0.0198
	2	0	2.9668	2.1767	0.9409	0.0843	0.5982	0.0223
		60	4.4626	2.5513	0.9483	0.0859	0.6762	0.0175
		120	6.1751	3.3962	0.9781	0.0301	0.6506	0.0171
		180	5.5610	3.4237	0.9723	0.0412	0.6115	0.0210

	10	0	490.2517	3997.5421	0.9726	0.0861	0.5936	0.0226
		60	1511.9293	8407.6198	0.9832	0.0648	0.6713	0.0165
		120	1977.4778	10567.2261	0.9909	0.0244	0.6457	0.0168
		180	493.9024	3738.8173	0.9575	0.1220	0.6058	0.0203
4	1	0	3.0034	0.8487	0.9163	0.0706	0.5961	0.0237
		60	5.3294	1.1559	0.8700	0.1275	0.7044	0.0200
		120	8.0522	1.8684	0.9569	0.0331	0.6796	0.0163
		180	7.1907	1.8715	0.9530	0.0334	0.6173	0.0176
	2	0	3.5245	1.5636	0.8857	0.1013	0.5956	0.0240
		60	5.9353	2.1000	0.8506	0.1958	0.6979	0.0384
		120	9.2095	3.4897	0.9388	0.0549	0.6790	0.0160
		180	8.4028	3.7626	0.9300	0.0580	0.6173	0.0175
	10	0	1633.8166	9357.0301	0.9782	0.1078	0.5892	0.0245
		60	1740.0500	9491.5755	0.9909	0.0349	0.7015	0.0140
		120	2118.1459	6382.7453	0.9747	0.0880	0.6714	0.0151
		180	2960.4163	12064.4251	0.9759	0.0861	0.6120	0.0178
8	1	0	3.1261	0.9902	0.8018	0.0955	0.5954	0.0306
		60	7.1522	1.2583	0.7480	0.1743	0.7370	0.0311
		120	12.1406	2.5865	0.8922	0.0573	0.7159	0.0148
		180	10.5602	2.7459	0.8773	0.0596	0.6406	0.0156
	2	0	3.9619	2.1374	0.7835	0.1413	0.5942	0.0334
		60	7.9587	2.8342	0.7974	0.1879	0.7400	0.0283
		120	14.1737	5.3256	0.8792	0.0797	0.7148	0.0144
		180	12.3115	5.8318	0.8518	0.0939	0.6396	0.0161
	10	0	2260.8110	11880.1559	0.9993	0.0012	0.5898	0.0296
		60	4070.6805	15848.5434	0.9937	0.0327	0.7464	0.0134
		120	2489.8110	11910.5039	0.9956	0.0270	0.7117	0.0130
		180	2047.6900	10611.7847	0.9992	0.0033	0.6346	0.0155
16	1	0	3.4416	1.0994	0.7401	0.0690	0.6006	0.0178
		60	11.6418	2.5668	0.7071	0.1176	0.7896	0.0159
		120	21.6146	5.3181	0.8545	0.0519	0.7639	0.0127
		180	18.4528	4.8078	0.8267	0.0584	0.6814	0.0163
	2	0	4.2541	2.6433	0.8035	0.1604	0.6002	0.0173
		60	12.7104	4.4664	0.8410	0.1850	0.7938	0.0189
		120	25.6251	11.4249	0.8903	0.0957	0.7644	0.0136
		180	22.2075	11.7058	0.8465	0.1207	0.6816	0.0174
	10	0	2096.5607	9001.6770	0.9998	0.0001	0.5954	0.0191
		60	781.5405	3072.9190	0.9998	0.0002	0.7954	0.0129
		120	842.9563	2920.3134	0.9998	0.0002	0.7599	0.0144
		180	1179.7116	4456.8136	0.9998	0.0001	0.6768	0.0153



$\mathbf{p = 2,}\ \sigma \in (1, 2, 10, 100)$


sigma = 1
sigma = 2
sigma = 10
sigma = 100
concordance
angle
0.5
0.7
0.9
1.1
0
π/3
2π/3
π