Purity estimation

2020-01-18

Simulation: Calculate the purity with true β value or true Γ value or true D value to check which parameter dominate the variance of the purity estimation.

Conclusion:

From the below results, the variance of purity estimation is mostly affected by the D estimation, and then the estimation of Γ. The estimation of β is the most stable.

Table 1: purity calculated with estimated β, estimated Γ and estimated D

truep	purity	purity_sd	sd_ratio	coverage	cossim	theta_est
1.195	1.505	0.533	0.446	0.910	0.965	0.800
241.835	261.166	51.682	0.214	0.925	0.999	0.785
187.044	201.892	41.082	0.220	0.930	0.999	0.785
20.170	21.870	3.551	0.176	0.920	0.997	0.787

The columns of the table show:

- truep: true purity value.
- purity: the mean value of the estimated purity with 200 repetition.
- purity_sd: the standard derivation of the estimated purity with 200 repetition.
- sd_ratio: the ratio of the standard derivation of the estimated purity and the true purity
- coverage: the coverage of the estimated purity, i.e. how many true purity value is covered in the estimated confidence interval.
- cossim: the cosine similarity
- theta_est: the estimated angle, θ of $\alpha = [\cos(\theta), \sin(theta)]$; the true value is $\frac{\pi}{4} \approx 0.7854$

Table 2: purity calculated with estimated β, estimated Γ and true value of D

truep	purity	purity_sd	sd_ratio	coverage	cossim	theta_est
1.195	1.648	0.458	0.383	0.865	0.489	1.179
241.835	239.426	25.075	0.104	0.955	0.986	0.806
187.044	185.270	19.534	0.104	0.950	0.986	0.806
20.170	19.985	2.221	0.110	0.960	0.997	0.786

The columns of the table show:

- truep: true purity value.
- purity: the mean value of the estimated purity with 200 repetition.
- purity_sd: the standard derivation of the estimated purity with 200 repetition.
- sd_ratio: the ratio of the standard derivation of the estimated purity and the true purity
- coverage: the coverage of the estimated purity, i.e. how many true purity value is covered in the estimated confidence interval.
- cossim: the cosine similarity
- theta_est: the estimated angle, θ of $\alpha = [\cos(\theta), \sin(theta)]$; the true value is $\frac{\pi}{4} \approx 0.7854$

Table 3: purity calculated with true value of β, estimated Γ and estimated D

truep	purity	purity_sd	sd_ratio	coverage	cossim	theta_est
1.195	1.425	0.518	0.433	0.930	0.971	0.790
241.835	259.539	51.525	0.213	0.930	0.999	0.785
187.044	200.620	40.968	0.219	0.935	0.999	0.785
20.170	21.585	3.399	0.169	0.930	0.997	0.787

The columns of the table show:

- truep: true purity value.
- purity: the mean value of the estimated purity with 200 repetition.
- purity_sd: the standard derivation of the estimated purity with 200 repetition.
- sd_ratio: the ratio of the standard derivation of the estimated purity and the true purity
- coverage: the coverage of the estimated purity, i.e. how many true purity value is covered in the estimated confidence interval.
- cossim: the cosine similarity
- theta_est: the estimated angle, θ of $\alpha = [\cos(\theta), \sin(theta)]$; the true value is $\frac{\pi}{4} \approx 0.7854$

Table 4: purity calculated with estimated of β, true value of Γ and estimated D

truep	purity	purity_sd	sd_ratio	coverage	cossim	theta_est
1.195	1.436	0.524	0.439	0.920	0.970	0.780
241.835	266.102	46.713	0.193	0.890	1.000	0.784
187.044	205.650	37.253	0.199	0.890	1.000	0.784
20.170	22.115	2.781	0.138	0.895	0.999	0.786

The columns of the table show:

- truep: true purity value.
- purity: the mean value of the estimated purity with 200 repetition.
- purity_sd: the standard derivation of the estimated purity with 200 repetition.
- sd_ratio: the ratio of the standard derivation of the estimated purity and the true purity
- coverage: the coverage of the estimated purity, i.e. how many true purity value is covered in the estimated confidence interval.
- cossim: the cosine similarity
- theta_est: the estimated angle, θ of $\alpha = [\cos(\theta), \sin(theta)]$; the true value is $\frac{\pi}{4} \approx 0.7854$

Table 5: purity calculated with true value of β, true value of Γ and estimated D

truep	purity	purity_sd	sd_ratio	coverage	cossim	theta_est
1.195	1.355	0.510	0.427	0.940	0.973	0.788
241.835	264.480	46.551	0.192	0.895	1.000	0.784
187.044	204.381	37.133	0.199	0.895	1.000	0.784
20.170	21.905	2.718	0.135	0.895	0.999	0.787

The columns of the table show:

- truep: true purity value.
- purity: the mean value of the estimated purity with 200 repetition.

- purity_sd: the standard derivation of the estimated purity with 200 repetition.
- sd_ratio: the ratio of the standard derivation of the estimated purity and the true purity
- coverage: the coverage of the estimated purity, i.e. how many true purity value is covered in the estimated confidence interval.
- cossim: the cosine similarity
- theta_est: the estimated angle, θ of $\alpha = [\cos(\theta), \sin(theta)]$; the true value is $\frac{\pi}{4} \approx 0.7854$

Table 6: purity calculated with true value of β, estimated of Γ and true value of D

truep	purity	purity_sd	sd_ratio	coverage	cossim	theta_est
1.195	1.399	0.235	0.197	0.900	0.611	1.004
241.835	237.873	24.846	0.103	0.945	0.987	0.806
187.044	184.050	19.354	0.103	0.945	0.986	0.805
20.170	19.791	2.202	0.109	0.960	0.997	0.787

The columns of the table show:

- truep: true purity value.
- purity: the mean value of the estimated purity with 200 repetition.
- purity_sd: the standard derivation of the estimated purity with 200 repetition.
- sd_ratio: the ratio of the standard derivation of the estimated purity and the true purity
- coverage: the coverage of the estimated purity, i.e. how many true purity value is covered in the estimated confidence interval.
- cossim: the cosine similarity
- theta_est: the estimated angle, θ of $\alpha = [\cos(\theta), \sin(theta)]$; the true value is $\frac{\pi}{4} \approx 0.7854$

Table 7: purity calculated with estimated of β, true value of Γ and true value of D

truep	purity	purity_sd	sd_ratio	coverage	cossim	theta_est
1.195	1.501	0.356	0.298	0.855	0.486	1.204
241.835	242.621	9.011	0.037	0.960	0.999	0.783
187.044	187.670	6.967	0.037	0.960	0.999	0.784
20.170	20.304	0.798	0.040	0.971	0.999	0.786

The columns of the table show:

- truep: true purity value.
- purity: the mean value of the estimated purity with 200 repetition.
- purity_sd: the standard derivation of the estimated purity with 200 repetition.
- sd_ratio: the ratio of the standard derivation of the estimated purity and the true purity
- coverage: the coverage of the estimated purity, i.e. how many true purity value is covered in the estimated confidence interval.
- cossim: the cosine similarity
- theta_est: the estimated angle, θ of $\alpha = [\cos(\theta), \sin(theta)]$; the true value is $\frac{\pi}{4} \approx 0.7854$