Tsiatis's copula

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The copula can be written as:

$$H(t,s) = C(H_1(t), H_2(s))H_1(t)H_2(s)$$

where

- $H_{t,s}(t,s)$ is the CDF of the joint distribution, H(t,s) = P(T > t, C>s)
- $H_1(t)$ is the CDF of the survival time H_1(t) = P(T > t), H_2(t) is the CDF of the censor time, H_2(t) = P(C>t)
- C(H_1(t), H_2(s)) is the copula

In Tsiatis's example,

$H_{t,s}(t,s) = exp(-\lambda t - \mu s - \theta ts), H_1(t) = exp(-\lambda t), H_2(t) = exp(-\mu t)$

Therefore,

$$\begin{aligned} C(H_1(t), H_2(s)) =& \frac{exp(-\lambda t - \mu s - \theta ts)}{exp(-\lambda t)exp(-\mu t)} = exp(-\theta ts) \\ =& exp(-\theta \frac{logU}{\lambda}\frac{logV}{\mu}) \end{aligned}$$

where $U = exp(-\lambda t), V = exp(-\mu s)$.

Therefore, the copula is

$$C(U,V) = exp(-\theta \frac{logU}{\lambda}\frac{logV}{\mu}),$$

where $U, V \sim U(0,1)$