summary of examples

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Example 1 (Copula example):

We have the joint distribution

- $S_{T,C}(t,s) = (1-t)(1-s)(1+\frac{C_0}{8}ts(t-s)(t+s-1))$, $(t,s) \in [0,1] \times [0,1], C_0 \in [-4,4]$
- $f_{T,C}(t,s) = 1 + C_0(t-\frac{1}{2})(s-\frac{1}{2})(t+s-1)(t-s)$

The marginal distributions

- $$\begin{aligned} f_T(t) = &\int_0^1 f_{T,C}(t,s)dy \\ =&\big\{s - \frac{C_0}{4}(t-\frac{1}{2})(s^4 - 2s^3 + (-2t^2+2t+1)s^2 + (2t^2-2t)s)\big\}\Big|_0^1 = 1 \end{aligned}$$
- $f_T(t) = I_{[0,1]}(t)$
- $S_T(t) = t$
- $\psi(t) = \displaystyle\int_t^1 f(t,s)ds = \frac{1}{8}\big((1-t)\big(C_0(t-1)t^2(2t-1)+8\big)\big)$
- $S_H(t) = P(Z > t) = (1-t)^2$, $\Lambda_H(t) = -2\log(1-t), \lambda_H(t) = \frac{2}{1-t}$

The $m(t)$ and $\rho(t)$

- $m(t) = \frac{\lambda_F(t)}{\lambda_H(t)} = 0.5$
- $\rho(t) = \dfrac{f(t)/\psi(t) - 1}{S(t)/S_x(t) - 1} = \dfrac{C_0(2t-1)(t-1)^2t + 8}{C_0t^2(t-1)(2t-1) + 8}$

Example 2 (Slud example):

The joint distribution:

- $f(t,s) = \begin{cases} \exp(-t-s) & (t \le s) \\ \rho\exp(-\rho t + (\rho-2)s) & (t > s) \end{cases}$

The marginal distribution

- $f_T(t) = \frac{2\rho-2}{\rho-2}\exp(-2t) - \frac{\rho}{\rho-2}\exp(-\rho t)$
- $S_T(t) = \frac{\rho-1}{\rho-2}exp(-2t) - \frac{1}{\rho-2}\exp(-\rho t)$
- $\psi(t) = \exp(-2t)$
- $S_H(x) =$

$$\begin{aligned} S_H(x) =& \int_x^\infty \int_x^t f(t,s)dsdt + \int_x^\infty \int_x^s f(t,s)dtds \\ =& \int_x^\infty \int_x^t \rho\exp(-\rho t + (\rho-2)s)dsdt + \int_x^\infty \int_x^t \exp(-t-s)dtds \\ =& \int_x^\infty \rho\big(\frac{\exp(-2t)}{\rho-2} - \frac{\exp(\rho x - 2x - \rho t)}{\rho-2}\big)dt + \int_x^\infty \exp -x - s - \exp(-2s)ds \\ =& \frac{\rho}{\rho-2}\frac{\rho-2}{2\rho}\exp(-2x) + \frac{\exp(-2x)}{2} \\ =& \exp(-2x) \end{aligned}$$

- $\lambda_H(t) = 2$

The $m(t)$ and $\rho(t)$

- $m(t)$

$$m(t) = \frac{\lambda_F(t)}{\lambda_H(t)} = \frac{\frac{\frac{2\rho-2}{\rho-2}\exp(-2t) - \frac{\rho}{\rho-2}\exp(-\rho t)}{\frac{\rho-1}{\rho-2}\exp(-2t) - \frac{1}{\rho-2}\exp(-\rho t)}}{2} = \frac{1}{2}\frac{(2\rho-2)\exp(-2t) - \rho\exp(-\rho t)}{(\rho-1)\exp(-2t) - \exp(-\rho t)}$$

- $\rho(t) = \rho$

Example 3 (exponential + extreme distribution):

The joint distribution

- $S_{T,C}(t,s) = \begin{cases} e^{-\theta t} e^{-(e^{\theta s}-1)\left((\theta t - \theta s)^2+1\right)} & t \geq s \\ e^{-\theta t} e^{-(e^{\theta s}-1)} & t < s \end{cases}$

The marginal distribution

- $S_T(t) = P(T > t) = P(T > t, C > 0) = e^{-\theta t} e^{-(e^{\theta 0}-1)\left((t-0)^2+1\right)} = e^{-\theta t}$
- $f_T(t) = \frac{\partial}{\partial t}(1 - S_T(t)) = \frac{\partial}{\partial t}(1 - e^{-\theta t}) = \theta e^{-\theta t}$
- $S_Z(t) = P(T > t, C > t) = e^{-\theta t} e^{-(e^{\theta t}-1)} = e^{-e^{\theta t} - \theta t + 1}$
- $f_Z(t) = \frac{\partial}{\partial t}(1 - S_Z(t)) = 1 - e^{-e^{\theta t} - \theta t + 1} = \theta(1 + e^{\theta t}) e^{-e^{\theta t} - \theta t + 1}$
- $\psi(t) = \int_t^\infty f(t,c)dc = \int_t^\infty \theta^2 e^{-e^{\theta c} + \theta c - \theta t + 1} dc = \theta e^{-e^{\theta t} - \theta t + 1}$

The $m(t)$ and $\rho(t)$ function:

- $m(t) = \frac{\lambda_F(t)}{\lambda_H(t)} = \frac{f_T(t)}{S_T(t)} / \frac{f_Z(t)}{S_Z(t)} = \frac{\theta e^{-\theta t}}{e^{-\theta t}} / \frac{\theta(1+e^{\theta t}) e^{-e^{\theta t} - \theta t + 1}}{e^{-e^{\theta t} - \theta t + 1}} = \frac{1}{1 + e^{\theta t}}$
- $\rho = \frac{f(t)/\psi(t) - 1}{S(t)/S_Z(t) - 1} = \frac{\theta e^{-\theta t} / (\theta e^{-e^{\theta t} - \theta t + 1}) - 1}{e^{-\theta t} / e^{-e^{\theta t} - \theta t + 1} - 1} = 1$

Example 4 (exponential + weibull distribution):

The joint distribution

- $S_{T,C}(x,y) = \begin{cases} e^{-\theta x} e^{-(\theta y)^k\left((\theta x - \theta y)^2+1\right)} & x \geq y \\ e^{-\theta x} e^{-(\theta y)^k} & x < y \end{cases}$

The marginal distribution

- $S_T(x) = P(T \geq x, C \geq 0) = S(x, 0) = e^{-\theta x}$
- $f_T(x) = \frac{1 - S_T(x)}{x} = \theta e^{-\theta x}$
- $S_C(x) = P(T \geq 0, C \geq x) = S(0, x) = e^{-\theta 0} e^{-(\theta x)^k} = e^{-(\theta x)^k}$
- $f_C(x) = \frac{1 - S_C(x)}{x} = k\theta(\theta y)^{k-1} e^{-(\theta x)^k}$

The death time is from an exponential distribution with paramter θ, the censor time is from a Weibull distribution with shape parameter k and scale parameter $1/\theta$.

- $S_Z(x) = P(T > x, C > x) = e^{-\theta x - (\theta x)^k}$
- $f_Z(x) = (\theta + k\theta(\theta x)^{k-1}) e^{-\theta x - (\theta x)^k}$
- $\psi(x) = \int_x^\infty f_{T,C}(x,y)dy = \int_x^\infty k\theta^{k+1} y^{k-1} \exp(-\theta x - (\theta y)^k) dy = \theta \exp(-\theta x - (\theta x)^k)$

The $m(t)$ and $\rho(t)$ function

- $m(x) = \frac{f_T(x)/S_T(x)}{f_Z(X)/S_Z(x)} = \frac{\theta e^{-\theta x} / e^{-\theta x}}{(\theta + k\theta(\theta x)^{k-1}) e^{-\theta x - (\theta x)^k} / e^{-\theta x - (\theta x)^k}} = \frac{1}{1 + k(\theta x)^{k-1}}$

We could also transform $m()$ function as:

- $m(x) = \frac{1}{1+\exp(\log(k(\theta x)^{k-1}))} = \frac{1}{1+\exp(\log(k)+(k-1)\log(\theta)+(k-1)\log(x))}$

We can then estimate the k and θ by fitting logistic regression.

- $\rho(x) = \frac{f_T(x)/\psi(x)-1}{S(x)/S_Z(x)-1} = \frac{\theta\exp(-\theta x)/(\theta\exp(-\theta x-(\theta x)^k))-1}{\exp(-\theta x)/exp(-\theta x-(\theta x)^k)-1} = 1$

Example 5 (exponential + weibull distribution):

The joint distribution

- $S_{T,C}(x,y) = \begin{cases} e^{-\theta x}e^{-(e^{\theta y}-1)\big((x-y)+1\big)} & x \geq y \\ e^{-\theta x}e^{-(e^{\theta y}-1)} & x < y \end{cases}$

The marginal distribution

- $S_T(t) = P(T > t) = P(T > t, C > 0) = e^{-\theta t}e^{-(e^{\theta 0}-1)\big((t-0)+1\big)} = e^{-\theta t}$
- $f_T(t) = \frac{\partial}{\partial t}(1 - S_T(t)) = \frac{\partial}{\partial t}(1 - e^{-\theta t}) = \theta e^{-\theta t}$
- $S_Z(t) = P(T > t, C > t) = e^{-\theta t}e^{-(e^{\theta t}-1)} = e^{-e^{\theta t}-\theta t+1}$
- $f_Z(t) = \frac{\partial}{\partial t}(1 - S_Z(t)) = 1 - e^{-e^{\theta t}-\theta t+1} = \theta(1+e^{\theta t})e^{-e^{\theta t}-\theta t+1}$

$\psi(t) = \int_t^\infty f(t,c)dc = \int_t^\infty \theta^2 e^{-e^{\theta c}+\theta c-\theta t+1}dc = \theta e^{-e^{\theta t}-\theta t+1}$

The $m(t)$ and $\rho(t)$ function

- $m(t) = \frac{\lambda_F(t)}{\lambda_H(t)} = \frac{f_T(t)}{S_T(t)} / \frac{f_Z(t)}{S_Z(t)} = \frac{\theta e^{-\theta t}}{e^{-\theta t}} / \frac{\theta(1+e^{\theta t})e^{-e^{\theta t}-\theta t+1}}{e^{-e^{\theta t}-\theta t+1}} = \frac{1}{1+e^{\theta t}}$
- $\rho = \frac{f(t)/\psi(t)-1}{S(t)/S_x(t)-1} = \frac{\theta e^{-\theta t}/(\theta e^{-e^{\theta t}-\theta t+1})-1}{e^{-\theta t}/e^{-e^{\theta t}-\theta t+1}-1} = 1$

Simulation results tables

Sample size: 500; Repetitions: 500

Example 1

Table 1: Example 1: Mean absolute difference between estimated and true S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
C0 = 1								
t0.1	0.01110	0.00727	0.00728	0.00728	0.01110	0.00807	0.00805	0.00805
t0.25	0.01711	0.01280	0.01280	0.01281	0.01711	0.01560	0.01558	0.01557
t0.5	0.02307	0.01565	0.01560	0.01561	0.02307	0.02146	0.02135	0.02132
t0.75	0.02667	0.01835	0.01826	0.01831	0.02667	0.02330	0.02297	0.02288
t0.9	0.02603	0.01772	0.01847	0.01953	0.02603	0.02021	0.02059	0.02104
C0 = 2								
t0.1	0.01101	0.00722	0.00721	0.00721	0.01101	0.00815	0.00813	0.00812
t0.25	0.01687	0.01271	0.01271	0.01272	0.01687	0.01567	0.01565	0.01565
t0.5	0.02279	0.01572	0.01570	0.01569	0.02279	0.02135	0.02127	0.02124
t0.75	0.02652	0.01792	0.01798	0.01806	0.02652	0.02306	0.02285	0.02277
t0.9	0.02623	0.01789	0.01850	0.01955	0.02623	0.02050	0.02080	0.02121
C0 = 4								
t0.1	0.01133	0.00733	0.00735	0.00735	0.01133	0.00814	0.00814	0.00814
t0.25	0.01697	0.01282	0.01283	0.01284	0.01697	0.01565	0.01562	0.01561
t0.5	0.02314	0.01594	0.01587	0.01586	0.02314	0.02174	0.02161	0.02156
t0.75	0.02685	0.01819	0.01823	0.01830	0.02685	0.02351	0.02335	0.02327
t0.9	0.02718	0.01777	0.01851	0.01956	0.02718	0.02052	0.02080	0.02123

Table 2: Example 1: Standard deviations of the estimated S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
C0 = 1								
t0.1	0.01381	0.00915	0.00917	0.00917	0.01381	0.00997	0.00998	0.00998
t0.25	0.02102	0.01583	0.01584	0.01585	0.02102	0.01925	0.01925	0.01925
t0.5	0.02798	0.01958	0.01957	0.01959	0.02798	0.02629	0.02627	0.02628
t0.75	0.03271	0.02201	0.02214	0.02223	0.03271	0.02847	0.02852	0.02856
t0.9	0.03328	0.02174	0.02337	0.02393	0.03328	0.02453	0.02587	0.02631
C0 = 2								
t0.1	0.01387	0.00923	0.00923	0.00923	0.01387	0.01008	0.01008	0.01008
t0.25	0.02098	0.01593	0.01596	0.01597	0.02098	0.01931	0.01932	0.01933
t0.5	0.02795	0.01971	0.01976	0.01979	0.02795	0.02632	0.02633	0.02635
t0.75	0.03257	0.02172	0.02193	0.02203	0.03257	0.02824	0.02833	0.02838
t0.9	0.03323	0.02159	0.02327	0.02383	0.03323	0.02454	0.02586	0.02629
C0 = 4								
t0.1	0.01388	0.00919	0.00921	0.00922	0.01388	0.01001	0.01002	0.01002
t0.25	0.02106	0.01596	0.01597	0.01598	0.02106	0.01918	0.01918	0.01919
t0.5	0.02805	0.01984	0.01986	0.01989	0.02805	0.02661	0.02661	0.02662
t0.75	0.03279	0.02193	0.02211	0.02221	0.03279	0.02871	0.02879	0.02883
t0.9	0.03381	0.02157	0.02326	0.02382	0.03381	0.02456	0.02592	0.02635

Table 3: Example 1: MSE of the estimated S()

Quantile	With true m() KM	Exp m()	Dikta 1	Dikta 2	With estimated m() KM	Exp m()	Dikta 1	Dikta 2
C0 = 1								
t0.1	0.00019	0.00008	0.00008	0.00008	0.00019	0.00010	0.00010	0.00010
t0.25	0.00045	0.00025	0.00025	0.00025	0.00045	0.00038	0.00037	0.00037
t0.5	0.00080	0.00039	0.00039	0.00039	0.00080	0.00073	0.00072	0.00072
t0.75	0.00111	0.00049	0.00049	0.00049	0.00111	0.00086	0.00084	0.00083
t0.9	0.00111	0.00047	0.00056	0.00061	0.00111	0.00062	0.00067	0.00070
C0 = 2								
t0.1	0.00019	0.00008	0.00008	0.00008	0.00019	0.00010	0.00010	0.00010
t0.25	0.00044	0.00025	0.00025	0.00025	0.00044	0.00038	0.00038	0.00038
t0.5	0.00079	0.00040	0.00039	0.00039	0.00079	0.00073	0.00072	0.00072
t0.75	0.00110	0.00048	0.00048	0.00048	0.00110	0.00084	0.00082	0.00082
t0.9	0.00112	0.00047	0.00055	0.00061	0.00112	0.00062	0.00067	0.00070
C0 = 4								
t0.1	0.00019	0.00008	0.00008	0.00008	0.00019	0.00010	0.00010	0.00010
t0.25	0.00044	0.00025	0.00025	0.00025	0.00044	0.00037	0.00037	0.00037
t0.5	0.00079	0.00040	0.00040	0.00040	0.00079	0.00075	0.00074	0.00074
t0.75	0.00113	0.00049	0.00049	0.00049	0.00113	0.00087	0.00086	0.00085
t0.9	0.00118	0.00046	0.00055	0.00061	0.00118	0.00062	0.00067	0.00071

Example 3

Table 4: Example 2: Mean absolute difference between estimated and true S()

Quantile	With true m() KM	Exp m()	Dikta 1	Dikta 2	With estimated m() KM	Exp m()	Dikta 1	Dikta 2
theta = 0.3								
t0.1	0.01071	0.00956	0.00962	0.00965	0.01071	0.00976	0.00982	0.00985
t0.25	0.01460	0.02100	0.02128	0.02143	0.01460	0.02346	0.02376	0.02392
t0.5	0.04125	0.06398	0.06511	0.06580	0.04125	0.07632	0.07746	0.07815
t0.75	0.10133	0.04082	0.05102	0.05665	0.10133	0.06283	0.07307	0.07850
t0.9	0.05235	0.05139	0.09990	0.09986	0.05235	0.02789	0.10000	0.09996
theta = 0.8								
t0.1	0.01001	0.00846	0.00850	0.00852	0.01001	0.00853	0.00857	0.00859
t0.25	0.01416	0.01863	0.01888	0.01901	0.01416	0.02015	0.02041	0.02055
t0.5	0.03326	0.05150	0.05255	0.05321	0.03326	0.06035	0.06142	0.06207
t0.75	0.09109	0.04377	0.05576	0.06180	0.09109	0.06079	0.07236	0.07823
t0.9	0.05912	0.05143	0.09992	0.09989	0.05912	0.03319	0.09989	0.09985
theta = 1								
t0.1	0.00947	0.00826	0.00831	0.00834	0.00947	0.00839	0.00844	0.00847
t0.25	0.01380	0.01903	0.01928	0.01942	0.01380	0.02075	0.02102	0.02116
t0.5	0.03366	0.05310	0.05417	0.05484	0.03366	0.06186	0.06294	0.06359
t0.75	0.09109	0.04340	0.05253	0.05865	0.09109	0.05979	0.06886	0.07481
t0.9	0.05637	0.05244	0.10000	0.10000	0.05637	0.03446	0.10000	0.10000
theta = 1.5								
t0.1	0.00986	0.00811	0.00813	0.00814	0.00986	0.00813	0.00816	0.00817
t0.25	0.01415	0.01674	0.01695	0.01707	0.01415	0.01795	0.01818	0.01831
t0.5	0.03170	0.04993	0.05097	0.05163	0.03170	0.05853	0.05958	0.06023
t0.75	0.06719	0.01941	0.02279	0.02549	0.06719	0.03227	0.03648	0.03962
t0.9	0.05985	0.05800	0.09995	0.09991	0.05985	0.03937	0.09981	0.09978
theta = 2								
t0.1	0.01000	0.00772	0.00774	0.00775	0.01000	0.00776	0.00779	0.00780

t0.25	0.01425	0.01652	0.01674	0.01686	0.01425	0.01779	0.01803	0.01816
t0.5	0.03329	0.05195	0.05302	0.05367	0.03329	0.06001	0.06110	0.06176
t0.75	0.05248	0.01777	0.01798	0.01860	0.05248	0.02427	0.02601	0.02751
t0.9	0.07273	0.08153	0.10000	0.09997	0.07273	0.06150	0.09987	0.09983
theta = 5								
t0.1	0.01083	0.01598	0.01608	0.01613	0.01083	0.01687	0.01697	0.01703
t0.25	0.01591	0.03690	0.03722	0.03741	0.01591	0.04328	0.04361	0.04380
t0.5	0.07241	0.10069	0.10195	0.10278	0.07241	0.12810	0.12941	0.13020
t0.75	0.18745	0.06923	0.08175	0.08904	0.18745	0.11240	0.12385	0.13007
t0.9	0.06338	0.06325	0.06141	0.05180	0.06338	0.02267	0.02741	0.02676

Table 5: Example 3: Standard deviations of the estimated S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 0.3								
t0.1	0.01318	0.01040	0.01042	0.01042	0.01318	0.01040	0.01042	0.01042
t0.25	0.01768	0.01430	0.01432	0.01433	0.01768	0.01431	0.01433	0.01435
t0.5	0.02559	0.02005	0.02015	0.02021	0.02559	0.02166	0.02175	0.02180
t0.75	0.04435	0.02678	0.04130	0.04231	0.04435	0.02961	0.04100	0.04172
t0.9	0.06024	0.00853	0.00844	0.00776	0.06024	0.02219	0.01039	0.00969
theta = 0.8								
t0.1	0.01256	0.00968	0.00969	0.00970	0.01256	0.00969	0.00970	0.00970
t0.25	0.01760	0.01540	0.01543	0.01544	0.01760	0.01541	0.01544	0.01545
t0.5	0.02290	0.01909	0.01917	0.01923	0.02290	0.01978	0.01986	0.01990
t0.75	0.05050	0.02613	0.04558	0.04598	0.05050	0.03048	0.04555	0.04580
t0.9	0.06710	0.00800	0.00885	0.00819	0.06710	0.02131	0.00818	0.00754
theta = 1								
t0.1	0.01178	0.00934	0.00935	0.00935	0.01178	0.00935	0.00936	0.00936
t0.25	0.01710	0.01493	0.01495	0.01496	0.01710	0.01507	0.01509	0.01510
t0.5	0.02282	0.01927	0.01936	0.01942	0.02282	0.02049	0.02057	0.02062
t0.75	0.05134	0.02637	0.03997	0.04102	0.05134	0.03117	0.04155	0.04230
t0.9	0.06793	0.00856	0.00000	0.00000	0.06793	0.02315	0.00000	0.00000
theta = 1.5								
t0.1	0.01245	0.00989	0.00990	0.00990	0.01245	0.00988	0.00989	0.00990
t0.25	0.01750	0.01474	0.01475	0.01476	0.01750	0.01460	0.01461	0.01463
t0.5	0.02505	0.01981	0.01990	0.01995	0.02505	0.02039	0.02047	0.02052
t0.75	0.04198	0.02202	0.02550	0.02639	0.04198	0.02685	0.02949	0.03010
t0.9	0.07040	0.00714	0.00922	0.00852	0.07040	0.02248	0.00653	0.00591
theta = 2								
t0.1	0.01227	0.00935	0.00936	0.00937	0.01227	0.00935	0.00936	0.00937
t0.25	0.01760	0.01468	0.01470	0.01472	0.01760	0.01467	0.01469	0.01471
t0.5	0.02349	0.01913	0.01922	0.01927	0.02349	0.01987	0.01995	0.02000
t0.75	0.03840	0.02193	0.02295	0.02379	0.03840	0.02811	0.02892	0.02952
t0.9	0.04559	0.01157	0.01027	0.00965	0.04559	0.02698	0.00763	0.00701
theta = 5								
t0.1	0.01357	0.01039	0.01040	0.01040	0.01357	0.01047	0.01048	0.01048
t0.25	0.01796	0.01552	0.01554	0.01555	0.01796	0.01592	0.01593	0.01595
t0.5	0.02835	0.02001	0.02014	0.02022	0.02835	0.02056	0.02066	0.02071
t0.75	0.02984	0.02156	0.03704	0.03758	0.02984	0.02336	0.03257	0.03285
t0.9	0.02260	0.01651	0.05320	0.05011	0.02260	0.02168	0.04103	0.03849

Table 6: Example 3: MSE

	With true m()				With estimated m()			

Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 0.3								
t0.1	0.00017	0.00015	0.00015	0.00015	0.00017	0.00015	0.00015	0.00016
t0.25	0.00032	0.00061	0.00062	0.00063	0.00032	0.00073	0.00075	0.00075
t0.5	0.00221	0.00449	0.00464	0.00474	0.00221	0.00629	0.00647	0.00658
t0.75	0.01220	0.00232	0.00426	0.00496	0.01220	0.00480	0.00700	0.00789
t0.9	0.00404	0.00271	0.00998	0.00998	0.00404	0.00117	0.01000	0.00999
theta = 0.8								
t0.1	0.00016	0.00011	0.00011	0.00011	0.00016	0.00011	0.00011	0.00011
t0.25	0.00031	0.00050	0.00051	0.00051	0.00031	0.00057	0.00058	0.00059
t0.5	0.00151	0.00301	0.00312	0.00319	0.00151	0.00403	0.00416	0.00424
t0.75	0.01065	0.00255	0.00515	0.00591	0.01065	0.00457	0.00726	0.00817
t0.9	0.00496	0.00271	0.00999	0.00998	0.00496	0.00152	0.00998	0.00998
theta = 1								
t0.1	0.00014	0.00011	0.00011	0.00011	0.00014	0.00011	0.00011	0.00011
t0.25	0.00030	0.00053	0.00054	0.00055	0.00030	0.00061	0.00062	0.00063
t0.5	0.00158	0.00319	0.00331	0.00338	0.00158	0.00424	0.00438	0.00446
t0.75	0.01069	0.00252	0.00431	0.00508	0.01069	0.00451	0.00644	0.00736
t0.9	0.00469	0.00282	0.01000	0.01000	0.00469	0.00168	0.01000	0.01000
theta = 1.5								
t0.1	0.00015	0.00010	0.00010	0.00010	0.00015	0.00010	0.00011	0.00011
t0.25	0.00031	0.00041	0.00042	0.00042	0.00031	0.00046	0.00047	0.00048
t0.5	0.00150	0.00288	0.00299	0.00306	0.00150	0.00384	0.00396	0.00404
t0.75	0.00610	0.00062	0.00093	0.00113	0.00610	0.00149	0.00196	0.00226
t0.9	0.00507	0.00342	0.00999	0.00999	0.00507	0.00203	0.00998	0.00997
theta = 2								
t0.1	0.00015	0.00009	0.00009	0.00009	0.00015	0.00009	0.00009	0.00009
t0.25	0.00032	0.00040	0.00041	0.00041	0.00032	0.00045	0.00046	0.00047
t0.5	0.00152	0.00306	0.00317	0.00325	0.00152	0.00399	0.00413	0.00421
t0.75	0.00382	0.00050	0.00053	0.00057	0.00382	0.00089	0.00104	0.00117
t0.9	0.00681	0.00678	0.01000	0.00999	0.00681	0.00451	0.00998	0.00998
theta = 5								
t0.1	0.00019	0.00034	0.00034	0.00034	0.00019	0.00037	0.00037	0.00038
t0.25	0.00040	0.00160	0.00162	0.00163	0.00040	0.00212	0.00215	0.00217
t0.5	0.00604	0.01054	0.01080	0.01097	0.00604	0.01683	0.01717	0.01738
t0.75	0.03603	0.00526	0.00805	0.00934	0.03603	0.01318	0.01640	0.01799
t0.9	0.00450	0.00427	0.00413	0.00321	0.00450	0.00079	0.00169	0.00162

Example 4

Table 7: Example 4: Mean absolute difference between estimated and true S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 1								
t0.1	0.01433	0.01276	0.01281	0.01282	0.01433	0.01234	0.01239	0.01240
t0.25	0.02735	0.02253	0.02279	0.02284	0.02735	0.02368	0.02394	0.02399
t0.5	0.05885	0.04872	0.04944	0.04977	0.05885	0.06795	0.06868	0.06901
t0.75	0.18087	0.09472	0.10128	0.10543	0.18087	0.12850	0.13453	0.13802
t0.9	0.09934	0.00619	0.09649	0.09619	0.09934	0.03384	0.09689	0.09719
theta = 2								
t0.1	0.01085	0.01061	0.01059	0.01059	0.01085	0.01059	0.01059	0.01059
t0.25	0.01295	0.01280	0.01274	0.01273	0.01295	0.01238	0.01241	0.01242
t0.5	0.01333	0.01345	0.01340	0.01339	0.01333	0.01156	0.01169	0.01178
t0.75	0.02056	0.01573	0.01545	0.01547	0.02056	0.01769	0.01728	0.01706

t0.9	0.05968	0.01864	0.09742	0.09697	0.05968	0.03254	0.09762	0.09724

Table 8: Example 4: Standard deviations of the estimated S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 1								
t0.1	0.01475	0.01409	0.01411	0.01411	0.01475	0.01417	0.01418	0.01419
t0.25	0.01825	0.01694	0.01697	0.01697	0.01825	0.01741	0.01743	0.01744
t0.5	0.02041	0.01986	0.01993	0.01995	0.02041	0.01822	0.01829	0.01831
t0.75	0.02947	0.02235	0.02852	0.02945	0.02947	0.01954	0.02412	0.02473
t0.9	0.00341	0.00733	0.02207	0.02012	0.00341	0.01146	0.01541	0.01390
theta = 2								
t0.1	0.01341	0.01263	0.01265	0.01265	0.01341	0.01306	0.01307	0.01307
t0.25	0.01620	0.01537	0.01538	0.01539	0.01620	0.01569	0.01570	0.01571
t0.5	0.01650	0.01671	0.01669	0.01671	0.01650	0.01345	0.01343	0.01344
t0.75	0.02582	0.01904	0.01906	0.01929	0.02582	0.01984	0.01982	0.02002
t0.9	0.06723	0.00757	0.02660	0.02443	0.06723	0.01955	0.02779	0.02567

Table 9: Example 4: MSE

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 1								
t0.1	0.00031	0.00025	0.00025	0.00025	0.00031	0.00023	0.00024	0.00024
t0.25	0.00102	0.00072	0.00074	0.00074	0.00102	0.00079	0.00080	0.00081
t0.5	0.00388	0.00276	0.00284	0.00287	0.00388	0.00495	0.00505	0.00510
t0.75	0.03358	0.00947	0.01107	0.01198	0.03358	0.01689	0.01868	0.01966
t0.9	0.00988	0.00007	0.00961	0.00960	0.00988	0.00127	0.00962	0.00964
theta = 2								
t0.1	0.00018	0.00018	0.00017	0.00017	0.00018	0.00017	0.00017	0.00017
t0.25	0.00026	0.00026	0.00026	0.00026	0.00026	0.00025	0.00025	0.00025
t0.5	0.00028	0.00029	0.00028	0.00028	0.00028	0.00020	0.00021	0.00021
t0.75	0.00067	0.00038	0.00037	0.00037	0.00067	0.00047	0.00044	0.00043
t0.9	0.00509	0.00040	0.00965	0.00963	0.00509	0.00140	0.00968	0.00965

Example 5

Table 10: Example 5: Mean absolute difference between estimated and true S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 1								
t0.1	0.00992	0.01044	0.01051	0.01055	0.00992	0.01058	0.01066	0.01070
t0.25	0.01399	0.02018	0.02046	0.02062	0.01399	0.02162	0.02190	0.02206
t0.5	0.02562	0.02891	0.02977	0.03030	0.02562	0.03509	0.03599	0.03654
t0.75	0.05239	0.02606	0.03253	0.03672	0.05239	0.03863	0.04548	0.04991
t0.9	0.07029	0.06051	0.09993	0.09988	0.07029	0.04441	0.09981	0.09976
theta = 1.5								
t0.1	0.01018	0.01666	0.01678	0.01683	0.01018	0.01721	0.01732	0.01738
t0.25	0.01610	0.03935	0.03969	0.03988	0.01610	0.04305	0.04340	0.04359
t0.5	0.04131	0.04206	0.04303	0.04364	0.04131	0.05601	0.05701	0.05762

t0.75	0.06403	0.02911	0.03489	0.03966	0.06403	0.05629	0.06278	0.06762
t0.9	0.06371	0.05706	0.10000	0.10000	0.06371	0.02589	0.10000	0.10000
theta = 2								
t0.1	0.01054	0.01571	0.01582	0.01588	0.01054	0.01618	0.01629	0.01635
t0.25	0.01562	0.03483	0.03517	0.03536	0.01562	0.03809	0.03843	0.03862
t0.5	0.04020	0.03610	0.03701	0.03758	0.04020	0.04830	0.04926	0.04984
t0.75	0.04648	0.02224	0.02651	0.02989	0.04648	0.04340	0.04898	0.05300
t0.9	0.07095	0.06153	0.09996	0.09993	0.07095	0.03195	0.09990	0.09988
theta = 5								
t0.1	0.01021	0.02920	0.02933	0.02940	0.01021	0.03205	0.03217	0.03224
t0.25	0.02689	0.09228	0.09269	0.09293	0.02689	0.11146	0.11190	0.11213
t0.5	0.22492	0.19632	0.19827	0.19956	0.22492	0.25869	0.26066	0.26176
t0.75	0.22701	0.08810	0.09980	0.10726	0.22701	0.16783	0.17647	0.18087
t0.9	0.08807	0.04907	0.04312	0.03318	0.08807	0.03332	0.04463	0.05010

Table 11: Example 5: Standard deviations of the estimated S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 1								
t0.1	0.01245	0.01020	0.01021	0.01022	0.01245	0.01023	0.01024	0.01025
t0.25	0.01697	0.01421	0.01423	0.01424	0.01697	0.01425	0.01428	0.01429
t0.5	0.02247	0.01848	0.01855	0.01860	0.02247	0.01939	0.01945	0.01949
t0.75	0.04784	0.02601	0.03581	0.03709	0.04784	0.03144	0.03867	0.03963
t0.9	0.08103	0.00883	0.01146	0.01066	0.08103	0.02328	0.00962	0.00884
theta = 1.5								
t0.1	0.01305	0.01033	0.01034	0.01035	0.01305	0.01039	0.01040	0.01040
t0.25	0.01760	0.01516	0.01518	0.01520	0.01760	0.01543	0.01545	0.01546
t0.5	0.02376	0.01829	0.01836	0.01841	0.02376	0.01972	0.01979	0.01983
t0.75	0.04315	0.02450	0.02828	0.03036	0.04315	0.02930	0.03209	0.03355
t0.9	0.07412	0.00798	0.00000	0.00000	0.07412	0.02122	0.00000	0.00000
theta = 2								
t0.1	0.01324	0.01015	0.01016	0.01017	0.01324	0.01019	0.01020	0.01021
t0.25	0.01743	0.01464	0.01465	0.01467	0.01743	0.01464	0.01465	0.01467
t0.5	0.02475	0.01994	0.02001	0.02006	0.02475	0.02064	0.02071	0.02075
t0.75	0.04287	0.02400	0.02892	0.03023	0.04287	0.02989	0.03311	0.03402
t0.9	0.07804	0.00872	0.00805	0.00747	0.07804	0.02281	0.00680	0.00623
theta = 5								
t0.1	0.01282	0.01051	0.01051	0.01052	0.01282	0.01075	0.01075	0.01076
t0.25	0.01867	0.01593	0.01597	0.01599	0.01867	0.01679	0.01683	0.01684
t0.5	0.03767	0.01977	0.02005	0.02020	0.03767	0.01631	0.01652	0.01664
t0.75	0.01188	0.01906	0.03150	0.03228	0.01188	0.01795	0.02145	0.02177
t0.9	0.00518	0.01321	0.03445	0.03264	0.00518	0.01472	0.01843	0.01748

Table 12: Example 5: MSE

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 1								
t0.1	0.00015	0.00017	0.00017	0.00018	0.00015	0.00018	0.00018	0.00018
t0.25	0.00030	0.00056	0.00057	0.00058	0.00030	0.00062	0.00064	0.00065
t0.5	0.00099	0.00112	0.00118	0.00121	0.00099	0.00157	0.00164	0.00169
t0.75	0.00429	0.00111	0.00211	0.00252	0.00429	0.00222	0.00332	0.00384
t0.9	0.00686	0.00374	0.00999	0.00998	0.00686	0.00250	0.00997	0.00997
theta = 1.5								

t0.1	0.00017	0.00037	0.00037	0.00038	0.00017	0.00039	0.00039	0.00040
t0.25	0.00040	0.00177	0.00180	0.00181	0.00040	0.00208	0.00212	0.00213
t0.5	0.00222	0.00209	0.00218	0.00223	0.00222	0.00352	0.00364	0.00371
t0.75	0.00562	0.00128	0.00187	0.00237	0.00562	0.00395	0.00492	0.00566
t0.9	0.00563	0.00332	0.01000	0.01000	0.00563	0.00101	0.01000	0.01000
theta = 2								
t0.1	0.00017	0.00034	0.00034	0.00034	0.00017	0.00035	0.00036	0.00036
t0.25	0.00038	0.00143	0.00145	0.00146	0.00038	0.00166	0.00169	0.00171
t0.5	0.00216	0.00168	0.00175	0.00180	0.00216	0.00275	0.00285	0.00291
t0.75	0.00341	0.00080	0.00128	0.00157	0.00341	0.00263	0.00338	0.00387
t0.9	0.00684	0.00386	0.00999	0.00999	0.00684	0.00145	0.00999	0.00998
theta = 5								
t0.1	0.00017	0.00096	0.00097	0.00097	0.00017	0.00114	0.00115	0.00116
t0.25	0.00100	0.00877	0.00885	0.00889	0.00100	0.01271	0.01280	0.01286
t0.5	0.05201	0.03893	0.03971	0.04023	0.05201	0.06719	0.06822	0.06879
t0.75	0.05167	0.00812	0.01095	0.01255	0.05167	0.02849	0.03160	0.03319
t0.9	0.00778	0.00258	0.00218	0.00153	0.00778	0.00131	0.00233	0.00281