example 6 two weibull

2020-01-15

Two weibull distribution, mimic Dikta's example

The joint distribution

$$S_{T,C}(x,y) = \begin{cases} \exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})\exp\big(-(\frac{\theta_2}{\lambda_2}y)^{k_2}(x-y+1)\big) & x \geq y \\ \exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})\exp(-(\frac{\theta_2}{\lambda_2}y)^{k_2}) & x < y \end{cases}$$

Then the joint pdf when $x < y$ is

$$f_{T,C}(x,y) = k_1k_2(\frac{\theta_1}{\lambda_1})^{k_1}(\frac{\theta_2}{\lambda_2})^{k_2}x^{k_1-1}y^{k_2-1}\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1} - (\frac{\theta_2}{\lambda_2}y)^{k_2})$$

And then the survival function is

$$S_T(x) = P(T > x, C > 0) = \exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})$$

which is a weibull distribution with shape parameter k_1 and scale parameter λ_1. The θ_1 is the parameter associated with the covariates X.

The pdf of the survival function is

$$f_T(x) = k_1(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_1-1}\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})$$

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The CDF of the observed time is

$$S_H(x) = P(T > x, C > x) = \exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})\exp(-(\frac{\theta_2}{\lambda_2}x)^{k_2})$$

The pdf of the observed time is

$$f_H(x) = \big(k_1(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_1-1} + k_2(\frac{\theta_2}{\lambda_2})^{k_2}x^{k_2-1}\big)\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1} - (\frac{\theta_2}{\lambda_2}x)^{k_2})$$

The $\psi(x) = \int_x^\infty f_{T,C}(x,y)dy = k_1(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_1-1}\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})\exp(-(\frac{\theta_2}{\lambda_2}x)^{k_2})$

Therefore, the $\rho(x)$ function equals to 1.

$$\rho(x) = \frac{f_T(x)/\psi(x) - 1}{S_T(x)/S_H(x) - 1} = \frac{\frac{k_1(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_1-1}\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})}{k_1(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_1-1}\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})\exp(-(\frac{\theta_2}{\lambda_2}x)^{k_2})} - 1}{\frac{\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})}{\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})\exp(-(\frac{\theta_2}{\lambda_2}x)^{k_2})} - 1} = 1$$

The $m(x)$ function is

$$\begin{aligned} m(x) =& \frac{f_T(x)/S_T(x)}{f_H(x)/S_H(x)} = \frac{\frac{k_1(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_1-1}\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})}{\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})}}{\frac{\big(k_1(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_1-1}+k_2(\frac{\theta_2}{\lambda_2})^{k_2}x^{k_2-1}\big)\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1}-(\frac{\theta_2}{\lambda_2}x)^{k_2})}{\exp(-(\frac{\theta_1}{\lambda_1}x)^{k_1})\exp(-(\frac{\theta_2}{\lambda_2}x)^{k_2})}} \\ =& \frac{k_1(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_1-1}}{k_1(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_1-1} + k_2(\frac{\theta_2}{\lambda_2})^{k_2}x^{k_2-1}} \\ =& \frac{1}{1 + \frac{k_2}{k_1}(\frac{\theta_2}{\lambda_2})^{k_2}(\frac{\theta_1}{\lambda_1})^{k_1}x^{k_2-k_1}} \end{aligned}$$

The censoring percentage is

$$P(T < C) = \int_0^\infty \int_0^y k_1 k_2 (\frac{\theta_1}{\lambda_1})^{k_1} (\frac{\theta_2}{\lambda_2})^{k_2} x^{k_1 - 1} y^{k_2 - 1} \exp(-(\frac{\theta_1}{\lambda_1} x)^{k_1} - (\frac{\theta_2}{\lambda_2} y)^{k_2}) dx dy$$

To make it sample, if $k_1 = k_2 = 1$, the missing precentage is

$$\frac{\frac{\theta_1}{\lambda_1}}{\frac{\theta_1}{\lambda_1} + \frac{\theta_2}{\lambda_2}}$$

Let's then plug in numbers and make a simple example.

$$S_{T,C}(t, s|x) = \begin{cases} \exp(-(\beta^T x)t) \exp\big(-((\beta^T x)s)(t - s + 1)\big) & x \geq y \\ \exp(-(\beta^T x)t) \exp(-(\beta^T x)s) & x < y \end{cases}$$

The censoring percentage is 0.5.

The m function is

$$m(t|x) = \frac{1}{1 + (\beta^T x)^2}$$