Independent setting (exp + weibull)

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Example setting:

The survival time follows an exponential distribution $T \sim EXP(\theta)$, the censoring time follows a weibull distribution with parameter θ. And T and C are independent, random censoring.

$$P(T > x) = S_t(x) = e^{-\theta x}, \quad P(C > y) = S_c(y) = e^{-(\theta y)^k}$$

Then

- $f_T(x) = \frac{1-S_T(x)}{x} = \theta e^{-\theta x}$
- $f_C(x) = \frac{1-S_C(x)}{x} = k\theta(\theta y)^{k-1} e^{-(\theta x)^k}$
- $S_Z(x) = P(T > x, C > x) = e^{-\theta x - (\theta x)^k}$, $f_Z(x) = (\theta + k\theta(\theta x)^{k-1})e^{-\theta x - (\theta x)^k}$

Therefore the $m()$ function is

$$m(x) = \frac{f_T(x)/S_T(x)}{f_Z(X)/S_Z(x)} = \frac{\theta e^{-\theta x}/e^{-\theta x}}{(\theta + k\theta(\theta x)^{k-1})e^{-\theta x-(\theta x)^k}/e^{-\theta x-(\theta x)^k}} = \frac{1}{1 + k(\theta x)^{k-1}}$$

We could also transform $m()$ function as:

$$m(x) = \frac{1}{1 + \exp(\log(k(\theta x)^{k-1}))} = \frac{1}{1 + \exp(\log(k) + (k-1)\log(\theta) + (k-1)\log(x))}$$

The m() function can be estimated by a logistic regression

$$E(\delta|z) = \beta_0 + \beta_1 \log(z)$$

$$\hat{\beta}_1 = -(k-1), \hat{\beta}_0 = -\log(k) - (k-1)\log(\theta)$$

$$\rightarrow k = 1 - \hat{\beta}_1,$$

$$\rightarrow \theta = \frac{1}{k-1}(-\log(k) - \hat{\beta}_0) = \frac{1}{\hat{\beta}_1}(\log(1-\hat{\beta}_1) + \hat{\beta}_0)$$

For the simulation, 500 subjects were generated. Their survival time T and censoring time C were randomly generated from the above exponential distribution and extreme distribution. The observed time Z and status δ are then calculated

$$Z = T \wedge C, \delta = I(T < C)$$

500 iterations were taken.

The estimation of $90\%, 75\%, 50\%, 25\%, 10\%$ quantiles were reported.

Results

The mean value of the estimations of the quantiles of the KM, exponential m(), Dikta method 1 and Dikta method 2 were calculated.

The cells in the table are the mean value of the estimations over 500 iterations minus the true quantiles value i.e (0.9, 0.75, 0.5, 0.25, 0.1)

Table 1: Mean absolute difference between estimated and true S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 0.2								
t0.1	0.01078	0.01037	0.01038	0.01038	0.01078	0.01051	0.01053	0.01053
t0.25	0.01533	0.01350	0.01354	0.01354	0.01533	0.01485	0.01487	0.01488
t0.5	0.01988	0.01504	0.01510	0.01513	0.01988	0.01859	0.01866	0.01868
t0.75	0.02618	0.01442	0.01471	0.01502	0.02618	0.02210	0.02232	0.02254
t0.9	0.06109	0.02193	0.08636	0.08421	0.06109	0.02831	0.08669	0.08507
theta = 0.8								
t0.1	0.01078	0.01037	0.01038	0.01038	0.01078	0.01051	0.01053	0.01053
t0.25	0.01533	0.01350	0.01354	0.01354	0.01533	0.01485	0.01487	0.01488
t0.5	0.01988	0.01504	0.01510	0.01513	0.01988	0.01859	0.01866	0.01868
t0.75	0.02618	0.01443	0.01471	0.01502	0.02618	0.02211	0.02232	0.02254
t0.9	0.06109	0.02189	0.08636	0.08421	0.06109	0.02830	0.08669	0.08507
theta = 1								
t0.1	0.01078	0.01037	0.01038	0.01038	0.01078	0.01051	0.01053	0.01053
t0.25	0.01533	0.01350	0.01354	0.01354	0.01533	0.01485	0.01487	0.01488
t0.5	0.01988	0.01504	0.01510	0.01513	0.01988	0.01859	0.01866	0.01868
t0.75	0.02618	0.01443	0.01471	0.01502	0.02618	0.02211	0.02232	0.02254
t0.9	0.06109	0.02188	0.08636	0.08421	0.06109	0.02829	0.08669	0.08507
theta = 2								
t0.1	0.01076	0.01038	0.01039	0.01039	0.01076	0.01053	0.01054	0.01054
t0.25	0.01534	0.01352	0.01355	0.01355	0.01534	0.01485	0.01487	0.01487
t0.5	0.01992	0.01507	0.01513	0.01516	0.01992	0.01862	0.01869	0.01871
t0.75	0.02621	0.01450	0.01475	0.01506	0.02621	0.02211	0.02230	0.02251
t0.9	0.06139	0.02188	0.08635	0.08419	0.06139	0.02831	0.08671	0.08508
theta = 5								
t0.1	0.01076	0.01039	0.01039	0.01039	0.01076	0.01053	0.01054	0.01054
t0.25	0.01534	0.01352	0.01355	0.01355	0.01534	0.01486	0.01487	0.01487
t0.5	0.01992	0.01508	0.01513	0.01516	0.01992	0.01863	0.01869	0.01871
t0.75	0.02621	0.01452	0.01475	0.01506	0.02621	0.02214	0.02230	0.02251
t0.9	0.06139	0.02172	0.08635	0.08419	0.06139	0.02825	0.08671	0.08508

To make the table easy to look at, I used the column 2,3,4,5 to divide the column 1, column 7,8,9,10 to divide column 6.

The values that are less than 1 are showing that the methods have less bias than the KM.

Table 2: Mean absolute difference between estimated and true S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 0.2								
t0.1	1	0.96200	0.96322	0.96333	1	0.97547	0.97671	0.97681
t0.25	1	0.88090	0.88296	0.88349	1	0.96842	0.97027	0.97058
t0.5	1	0.75660	0.75959	0.76118	1	0.93521	0.93841	0.93972
t0.75	1	0.55076	0.56182	0.57377	1	0.84423	0.85271	0.86099
t0.9	1	0.35895	1.41373	1.37849	1	0.46344	1.41913	1.39258
theta = 0.8								
t0.1	1	0.96200	0.96322	0.96333	1	0.97547	0.97671	0.97681
t0.25	1	0.88089	0.88296	0.88349	1	0.96869	0.97027	0.97058

t0.5	1	0.75660	0.75959	0.76118	1	0.93521	0.93841	0.93972
t0.75	1	0.55126	0.56182	0.57377	1	0.84473	0.85271	0.86099
t0.9	1	0.35828	1.41373	1.37849	1	0.46323	1.41913	1.39258
theta = 1								
t0.1	1	0.96200	0.96322	0.96333	1	0.97547	0.97671	0.97681
t0.25	1	0.88089	0.88296	0.88349	1	0.96869	0.97027	0.97058
t0.5	1	0.75650	0.75959	0.76118	1	0.93511	0.93841	0.93972
t0.75	1	0.55108	0.56182	0.57377	1	0.84455	0.85271	0.86099
t0.9	1	0.35823	1.41373	1.37849	1	0.46318	1.41913	1.39258
theta = 2								
t0.1	1	0.96451	0.96545	0.96556	1	0.97810	0.97908	0.97917
t0.25	1	0.88125	0.88297	0.88348	1	0.96809	0.96927	0.96956
t0.5	1	0.75667	0.75966	0.76121	1	0.93498	0.93818	0.93943
t0.75	1	0.55296	0.56269	0.57445	1	0.84339	0.85058	0.85872
t0.9	1	0.35638	1.40646	1.37138	1	0.46106	1.41233	1.38575
theta = 5								
t0.1	1	0.96519	0.96545	0.96556	1	0.97879	0.97908	0.97917
t0.25	1	0.88142	0.88297	0.88348	1	0.96850	0.96927	0.96956
t0.5	1	0.75695	0.75966	0.76121	1	0.93527	0.93818	0.93944
t0.75	1	0.55398	0.56269	0.57445	1	0.84440	0.85058	0.85872
t0.9	1	0.35373	1.40646	1.37138	1	0.46013	1.41232	1.38575

The standard deviation of the estimations of each quantiles are reported in the following table.

Table 3: Standard deviations of the estimated S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 0.2								
t0.1	0.01346	0.01289	0.01290	0.01290	0.01346	0.01312	0.01313	0.01313
t0.25	0.01917	0.01682	0.01684	0.01684	0.01917	0.01857	0.01859	0.01859
t0.5	0.02426	0.01842	0.01845	0.01846	0.02426	0.02267	0.02270	0.02271
t0.75	0.03356	0.01824	0.01854	0.01877	0.03356	0.02832	0.02853	0.02865
t0.9	0.07041	0.01316	0.05408	0.04991	0.07041	0.02644	0.05455	0.05046
theta = 0.8								
t0.1	0.01346	0.01289	0.01290	0.01290	0.01346	0.01312	0.01313	0.01313
t0.25	0.01917	0.01682	0.01684	0.01684	0.01917	0.01857	0.01859	0.01859
t0.5	0.02426	0.01842	0.01845	0.01846	0.02426	0.02267	0.02270	0.02271
t0.75	0.03356	0.01825	0.01854	0.01877	0.03356	0.02833	0.02853	0.02865
t0.9	0.07041	0.01316	0.05408	0.04991	0.07041	0.02647	0.05455	0.05046
theta = 1								
t0.1	0.01346	0.01289	0.01290	0.01290	0.01346	0.01312	0.01313	0.01313
t0.25	0.01917	0.01682	0.01684	0.01684	0.01917	0.01857	0.01859	0.01859
t0.5	0.02426	0.01841	0.01845	0.01846	0.02426	0.02267	0.02270	0.02271
t0.75	0.03356	0.01823	0.01854	0.01877	0.03356	0.02832	0.02853	0.02865
t0.9	0.07041	0.01316	0.05408	0.04991	0.07041	0.02647	0.05455	0.05046
theta = 2								
t0.1	0.01344	0.01289	0.01290	0.01291	0.01344	0.01312	0.01313	0.01314
t0.25	0.01915	0.01682	0.01684	0.01684	0.01915	0.01855	0.01857	0.01857
t0.5	0.02431	0.01843	0.01847	0.01848	0.02431	0.02271	0.02274	0.02275
t0.75	0.03354	0.01830	0.01857	0.01880	0.03354	0.02830	0.02849	0.02861

t0.9	0.07073	0.01313	0.05405	0.04987	0.07073	0.02638	0.05457	0.05048
theta = 5								
t0.1	0.01344	0.01290	0.01290	0.01291	0.01344	0.01313	0.01313	0.01314
t0.25	0.01915	0.01683	0.01684	0.01684	0.01915	0.01856	0.01857	0.01857
t0.5	0.02431	0.01844	0.01847	0.01848	0.02431	0.02272	0.02274	0.02275
t0.75	0.03354	0.01833	0.01857	0.01880	0.03354	0.02833	0.02849	0.02861
t0.9	0.07073	0.01302	0.05405	0.04987	0.07073	0.02637	0.05457	0.05048

To make the table easy to look at, I used the column 2,3,4,5 to divide the column 1, column 7,8,9,10 to divide column 6.

The values that are less than 1 are showing that the methods have less standard deviation than the KM.

Table 4: Standard deviations of the estimated S()

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 0.2								
t0.1	1	0.95752	0.95849	0.95858	1	0.97502	0.97600	0.97606
t0.25	1	0.87778	0.87872	0.87894	1	0.96884	0.96985	0.96993
t0.5	1	0.75934	0.76062	0.76125	1	0.93465	0.93591	0.93620
t0.75	1	0.54340	0.55235	0.55920	1	0.84387	0.85016	0.85376
t0.9	1	0.18692	0.76813	0.70885	1	0.37559	0.77471	0.71672
theta = 0.8								
t0.1	1	0.95752	0.95849	0.95858	1	0.97502	0.97600	0.97606
t0.25	1	0.87780	0.87872	0.87894	1	0.96890	0.96985	0.96993
t0.5	1	0.75930	0.76062	0.76125	1	0.93468	0.93591	0.93620
t0.75	1	0.54366	0.55235	0.55920	1	0.84405	0.85016	0.85376
t0.9	1	0.18685	0.76813	0.70885	1	0.37593	0.77471	0.71672
theta = 1								
t0.1	1	0.95752	0.95849	0.95858	1	0.97502	0.97600	0.97606
t0.25	1	0.87780	0.87872	0.87894	1	0.96890	0.96985	0.96993
t0.5	1	0.75914	0.76062	0.76125	1	0.93456	0.93591	0.93620
t0.75	1	0.54334	0.55235	0.55920	1	0.84382	0.85016	0.85376
t0.9	1	0.18685	0.76813	0.70885	1	0.37592	0.77471	0.71672
theta = 2								
t0.1	1	0.95945	0.96036	0.96045	1	0.97657	0.97753	0.97759
t0.25	1	0.87850	0.87925	0.87946	1	0.96879	0.96967	0.96976
t0.5	1	0.75837	0.75986	0.76049	1	0.93421	0.93561	0.93590
t0.75	1	0.54547	0.55366	0.56055	1	0.84357	0.84925	0.85288
t0.9	1	0.18558	0.76407	0.70502	1	0.37288	0.77151	0.71368
theta = 5								
t0.1	1	0.96008	0.96036	0.96045	1	0.97729	0.97753	0.97759
t0.25	1	0.87868	0.87925	0.87946	1	0.96908	0.96967	0.96976
t0.5	1	0.75881	0.75986	0.76049	1	0.93483	0.93561	0.93590
t0.75	1	0.54648	0.55366	0.56055	1	0.84446	0.84925	0.85288
t0.9	1	0.18407	0.76407	0.70502	1	0.37282	0.77151	0.71368

The MSE of each estimation

Table 5: MSE

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 0.2								
t0.1	0.00018	0.00017	0.00017	0.00017	0.00018	0.00017	0.00017	0.00017
t0.25	0.00037	0.00028	0.00028	0.00028	0.00037	0.00035	0.00035	0.00035
t0.5	0.00059	0.00034	0.00034	0.00034	0.00059	0.00052	0.00052	0.00052
t0.75	0.00113	0.00033	0.00035	0.00036	0.00113	0.00080	0.00082	0.00083
t0.9	0.00520	0.00065	0.00820	0.00808	0.00520	0.00126	0.00828	0.00816
theta = 0.8								
t0.1	0.00018	0.00017	0.00017	0.00017	0.00018	0.00017	0.00017	0.00017
t0.25	0.00037	0.00028	0.00028	0.00028	0.00037	0.00035	0.00035	0.00035
t0.5	0.00059	0.00034	0.00034	0.00034	0.00059	0.00052	0.00052	0.00052
t0.75	0.00113	0.00033	0.00035	0.00036	0.00113	0.00080	0.00082	0.00083
t0.9	0.00520	0.00065	0.00820	0.00808	0.00520	0.00126	0.00828	0.00816
theta = 1								
t0.1	0.00018	0.00017	0.00017	0.00017	0.00018	0.00017	0.00017	0.00017
t0.25	0.00037	0.00028	0.00028	0.00028	0.00037	0.00035	0.00035	0.00035
t0.5	0.00059	0.00034	0.00034	0.00034	0.00059	0.00052	0.00052	0.00052
t0.75	0.00113	0.00033	0.00035	0.00036	0.00113	0.00080	0.00082	0.00083
t0.9	0.00520	0.00065	0.00820	0.00808	0.00520	0.00126	0.00828	0.00816
theta = 2								
t0.1	0.00018	0.00017	0.00017	0.00017	0.00018	0.00017	0.00017	0.00017
t0.25	0.00037	0.00028	0.00028	0.00028	0.00037	0.00034	0.00035	0.00035
t0.5	0.00060	0.00034	0.00034	0.00034	0.00060	0.00052	0.00052	0.00052
t0.75	0.00112	0.00033	0.00035	0.00037	0.00112	0.00080	0.00081	0.00083
t0.9	0.00524	0.00065	0.00820	0.00808	0.00524	0.00126	0.00828	0.00816
theta = 5								
t0.1	0.00018	0.00017	0.00017	0.00017	0.00018	0.00017	0.00017	0.00017
t0.25	0.00037	0.00028	0.00028	0.00028	0.00037	0.00034	0.00035	0.00035
t0.5	0.00060	0.00034	0.00034	0.00034	0.00060	0.00052	0.00052	0.00052
t0.75	0.00112	0.00034	0.00035	0.00037	0.00112	0.00080	0.00081	0.00083
t0.9	0.00524	0.00064	0.00820	0.00808	0.00524	0.00125	0.00828	0.00816

To make the table easy to look at, I used the column 2,3,4,5 to divide the column 1, column 7,8,9,10 to divide column 6.

The values that are less than 1 are showing that the methods have less MSE than the KM.

Table 6: MSE

	With true m()				With estimated m()			
Quantile	KM	Exp m()	Dikta 1	Dikta 2	KM	Exp m()	Dikta 1	Dikta 2
theta = 0.2								
t0.1	1	0.91603	0.91831	0.91853	1	0.94979	0.95214	0.95231
t0.25	1	0.76903	0.77160	0.77223	1	0.93872	0.94226	0.94282
t0.5	1	0.57091	0.57537	0.57778	1	0.86785	0.87437	0.87706
t0.75	1	0.29492	0.30877	0.32370	1	0.71131	0.72492	0.73755
t0.9	1	0.12560	1.57508	1.55277	1	0.24286	1.59069	1.56824
theta = 0.8								

t0.1	1	0.91603	0.91831	0.91853	1	0.94979	0.95214	0.95231
t0.25	1	0.76908	0.77160	0.77223	1	0.93886	0.94226	0.94282
t0.5	1	0.57086	0.57537	0.57778	1	0.86792	0.87437	0.87706
t0.75	1	0.29520	0.30877	0.32370	1	0.71161	0.72492	0.73755
t0.9	1	0.12523	1.57508	1.55277	1	0.24273	1.59069	1.56824
theta = 1								
t0.1	1	0.91603	0.91831	0.91853	1	0.94979	0.95214	0.95231
t0.25	1	0.76908	0.77160	0.77223	1	0.93886	0.94226	0.94282
t0.5	1	0.57064	0.57537	0.57778	1	0.86772	0.87437	0.87706
t0.75	1	0.29485	0.30877	0.32370	1	0.71122	0.72492	0.73755
t0.9	1	0.12521	1.57508	1.55277	1	0.24269	1.59069	1.56824
theta = 2								
t0.1	1	0.91985	0.92192	0.92214	1	0.95296	0.95516	0.95532
t0.25	1	0.77067	0.77272	0.77332	1	0.93881	0.94191	0.94243
t0.5	1	0.56977	0.57430	0.57661	1	0.86717	0.87359	0.87615
t0.75	1	0.29729	0.30997	0.32471	1	0.71117	0.72313	0.73538
t0.9	1	0.12408	1.56297	1.54087	1	0.24036	1.57872	1.55632
theta = 5								
t0.1	1	0.92109	0.92192	0.92214	1	0.95439	0.95516	0.95532
t0.25	1	0.77101	0.77272	0.77332	1	0.93943	0.94191	0.94243
t0.5	1	0.57050	0.57430	0.57662	1	0.86845	0.87359	0.87615
t0.75	1	0.29839	0.30997	0.32471	1	0.71262	0.72313	0.73538
t0.9	1	0.12220	1.56297	1.54087	1	0.23888	1.57872	1.55632

The bias of estimation of our $m()$ function from logistic regression, i.e. the absolute mean value of $\hat{m}(t)-$ true $m(t)$

Table 7: mean absolute difference between hat m() and true m()

0.2	0.8	1	2	5
0.0222325	0.0222325	0.0222325	0.0221058	0.0221058

The estimation of $\hat{\theta}$. The row name shows the true θ value

Table 8: estimated theta from logitic regression

0.2	0.8	1	2	5
0.1994043	0.8001827	1.00401	2.044344	5.305224

The estimation of $\hat{k}$. The true k value is 2.

Table 9: estimated theta from logitic regression

theta = 0.2	theta = 0.8	theta = 1	theta = 2	theta = 5
2.013633	2.013633	2.013633	2.01367	2.01367