Comparison of purity and likelihood criteria in GEM and non GEM models

2020-06-29

Simulation

The data sets are generated following the below parameter setting:

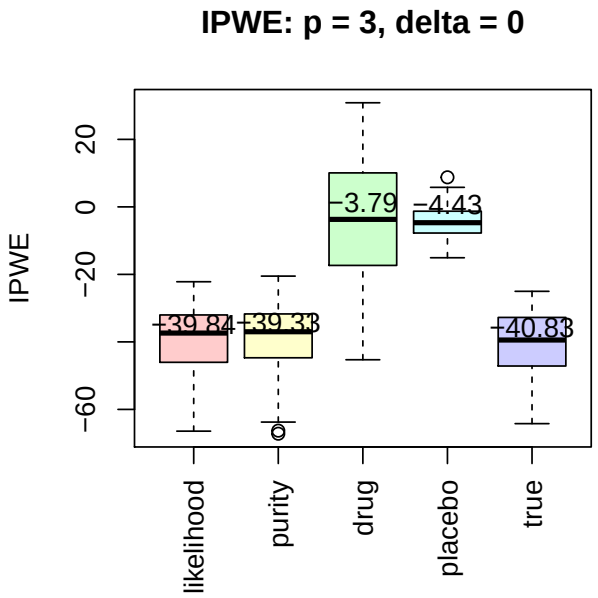
$$\mathbf{y_{ki}} = \mathbf{X_i}(\beta_\mathbf{k} + \mathbf{b_{ki}} + \mathbf{\Gamma_k}(\alpha' \mathbf{x})) + \epsilon_\mathbf{ki} \tag{1}$$

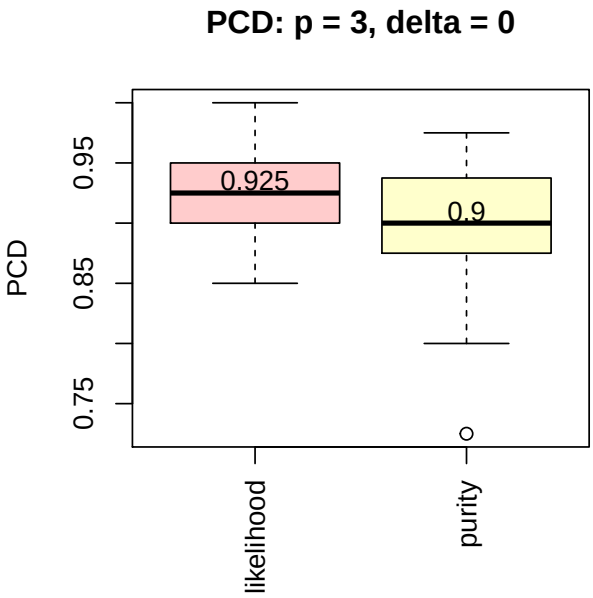
where

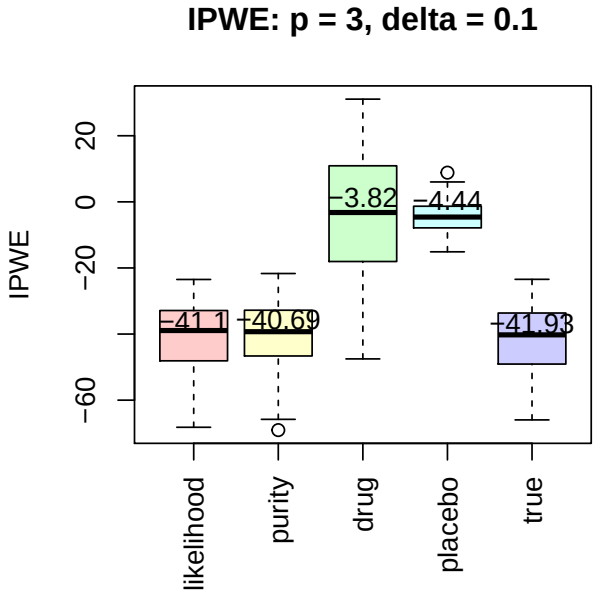
- $\alpha = \alpha_\mathbf{1} + \delta\alpha_\mathbf{2}$
  - $\alpha_1 = 1 : p$;
  - $\alpha_2 = (\alpha_{21}, ..., \alpha_{2p}) \sim Exp(4)$
  - $||\alpha||_2^2 = 1$
  - when $\delta = 0$, equation (1) is a GEM model while $\delta \neq$, equation (1) is a non GEM model
- Suppose we have $k = \{1, 2\}$ treatments. $k = 1$ represents the placebo group while $k = 2$ represents the drug group.
- $\mathbf{X_i} = [1, t, t^2]$, and $t = [0, 1, 2, 3, 4, 6, 8]$, which is the design matrix for fixed effect and random effect
- $\beta_\mathbf{1} = \beta_\mathbf{2} = [0, -0.1, -0.05]$
- $\mathbf{\Gamma_1} = [0, 1, 0]$, $\mathbf{\Gamma_1} = [0, \cos(\pi/3), sin(\pi/3)]$,
- $\mathbf{b_{1i}} \sim N(0, \mathbf{D_1}), \mathbf{b_{2i}} \sim N(0, \mathbf{D_2})$, $\mathbf{D_1} = \mathbf{D_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0.05 \\ 0 & 0.05 & 0.05 \end{pmatrix}$
- $\mathbf{x_i} \sim MVN(\mu_x, \Sigma_x)$, $\mu_x = \mathbf{0}_p$, Σ_x has diagonal equals to 1 and 0.5 everywhere else.
- $\epsilon_\mathbf{1}, \epsilon_\mathbf{2} \sim N(0, 1^2)$
- $p = \{3, 10\}$, which is the dimension of the predictors x_i.

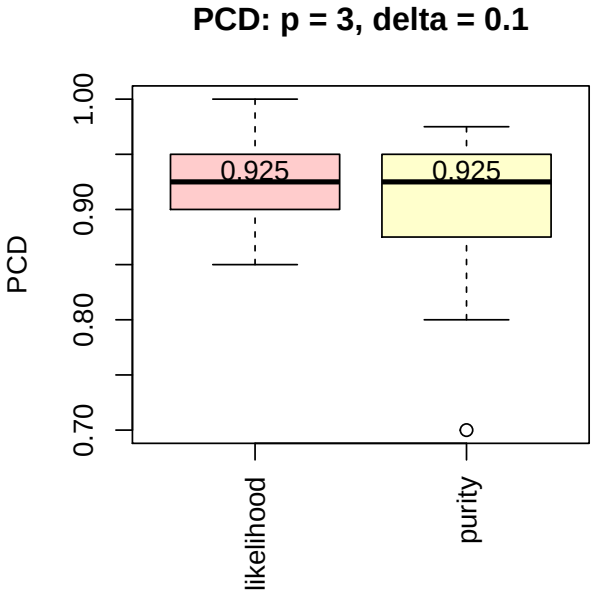
The IPWE are calculated with the estimated treatment assignment by using the purity criterion and by using the log-likelihood criterion. The proportions of correct assignment are also calculated. The whole procedures are repeated for 100 times.

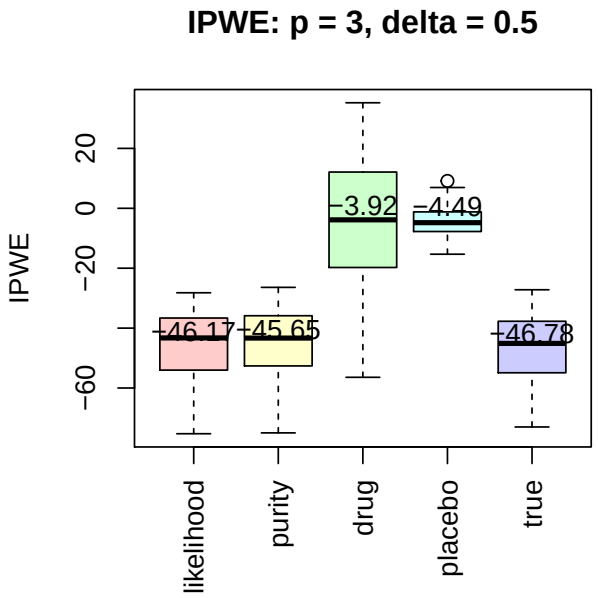
p = 3

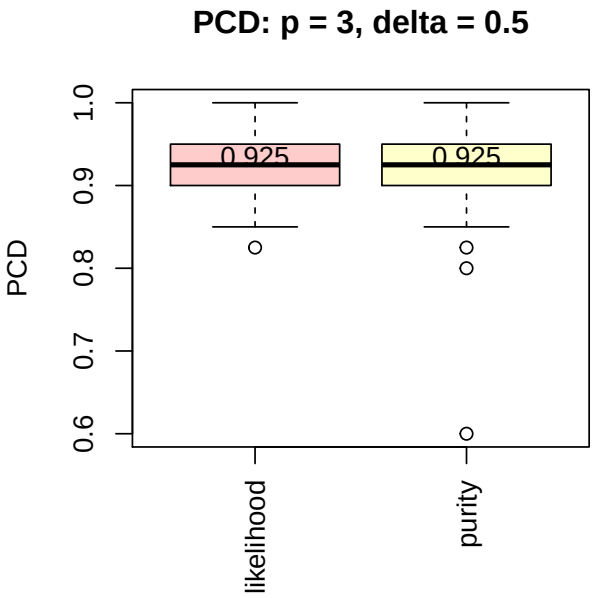

IPWE: p = 3, delta = 0
IPWE
20
0
−20
−60
−39.84
−39.33
−3.79
−4.43
−40.83
likelihood
purity
drug
placebo
true

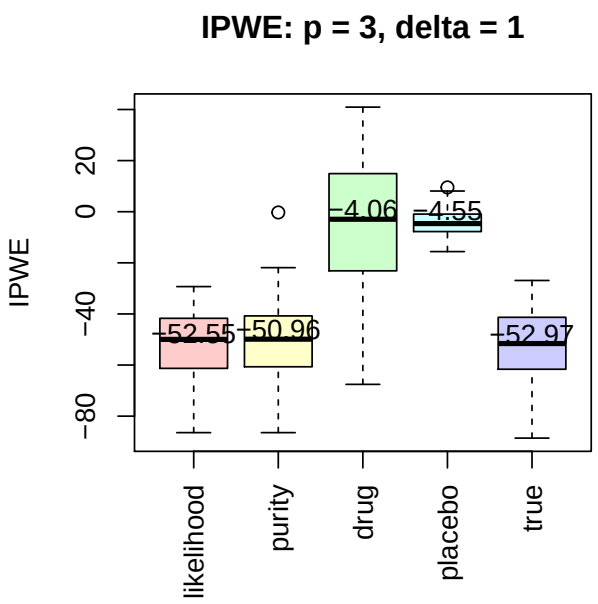

PCD: p = 3, delta = 0
PCD
0.95
0.85
0.75
0.925
0.9
likelihood
purity

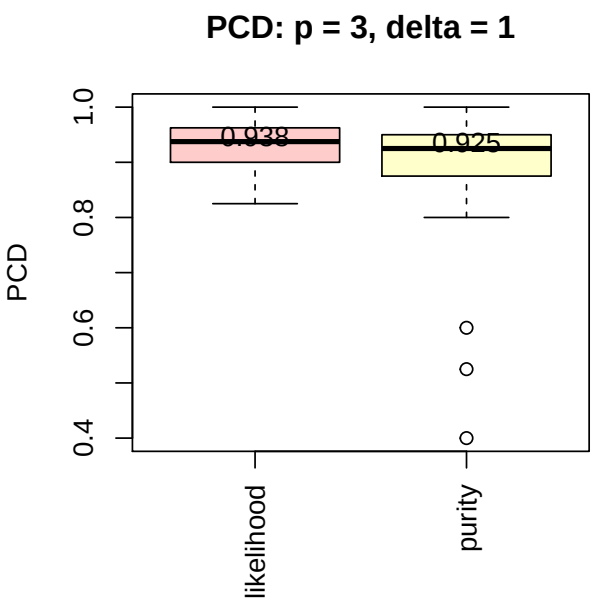

IPWE: p = 3, delta = 0.1
IPWE
20
0
−20
−60
−41.1
−40.69
−3.82
−4.44
−41.93
likelihood
purity
drug
placebo
true

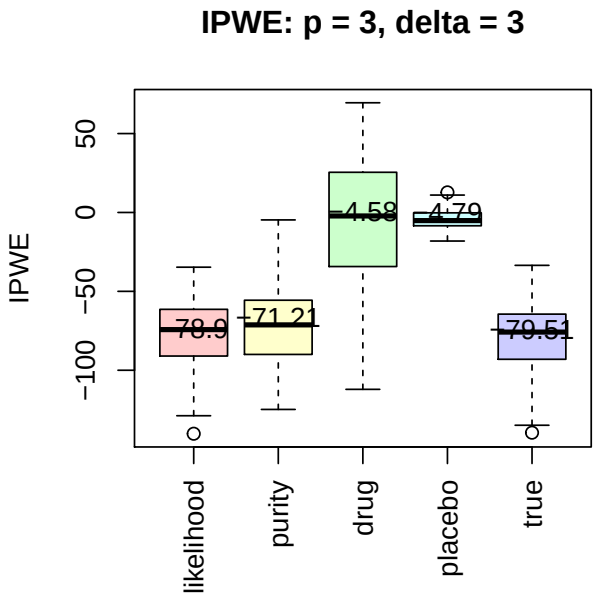

PCD: p = 3, delta = 0.1
PCD
1.00
0.90
0.80
0.70
0.925
0.925
likelihood
purity

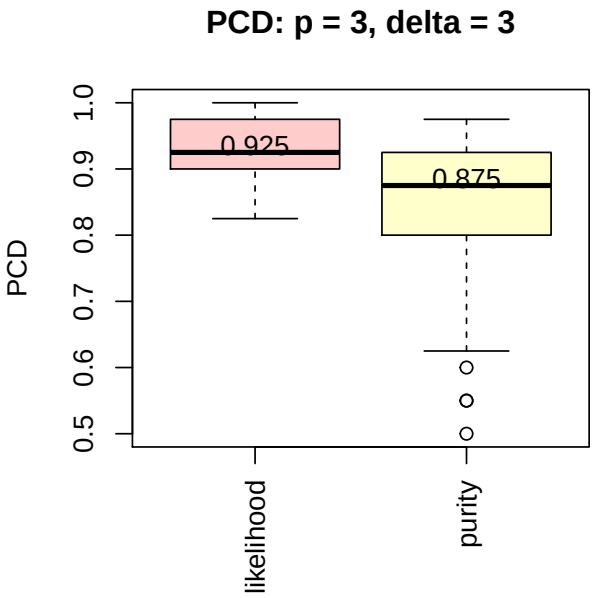

IPWE: p = 3, delta = 0.5
IPWE
20
0
−20
−60
−46.17
−45.65
−3.92
−4.49
−46.78
likelihood
purity
drug
placebo
true


PCD: p = 3, delta = 0.5
PCD
1.0
0.9
0.8
0.7
0.6
0.925
0.925
likelihood
purity

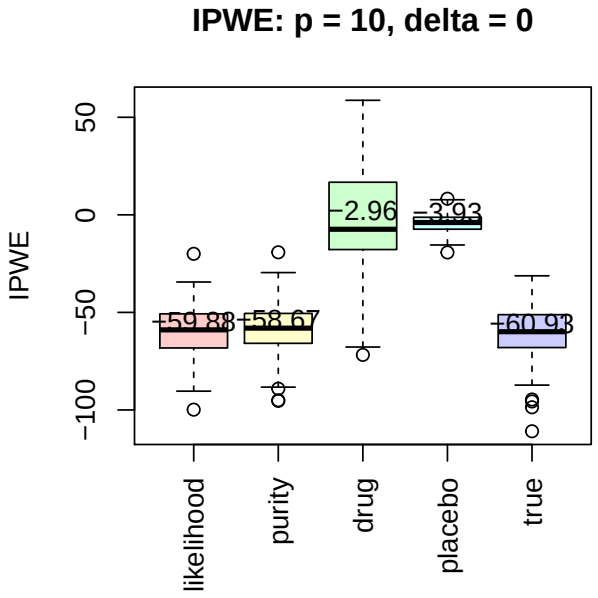

IPWE: p = 3, delta = 1
IPWE
20
0
−40
−80
−52.55
−50.96
−4.06
−4.55
−52.97
likelihood
purity
drug
placebo
true

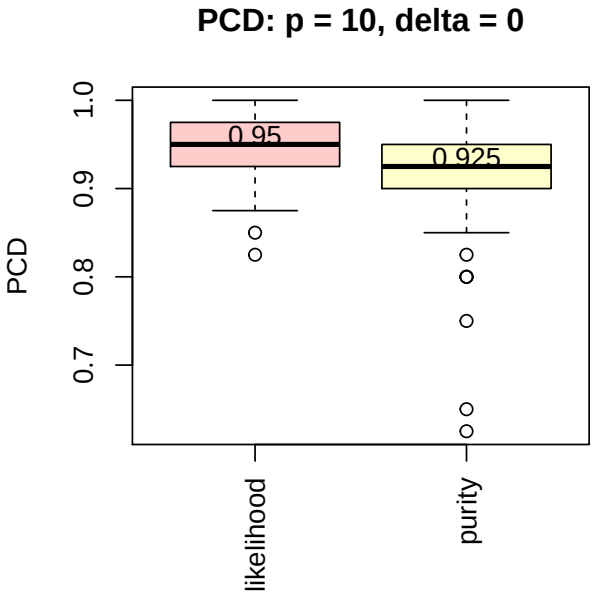

PCD: p = 3, delta = 1
PCD
1.0
0.8
0.6
0.4
0.938
0.925
likelihood
purity

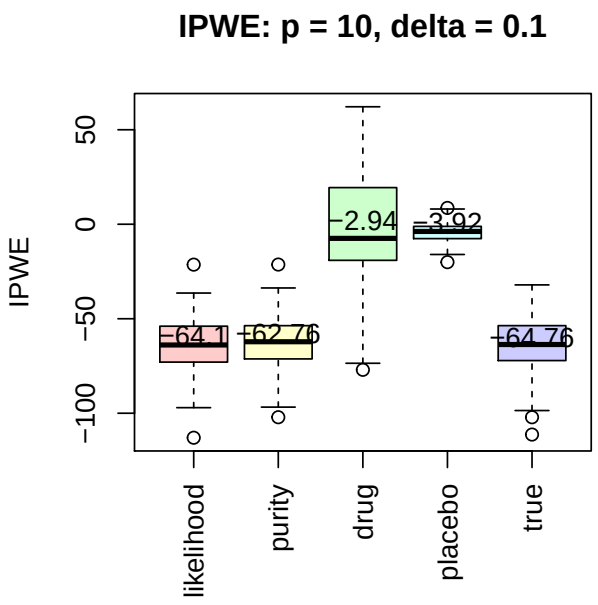

IPWE: p = 3, delta = 3
IPWE
50
0
−50
−100
−79.9
−71.21
−4.58
−4.79
−79.51
likelihood
purity
drug
placebo
true

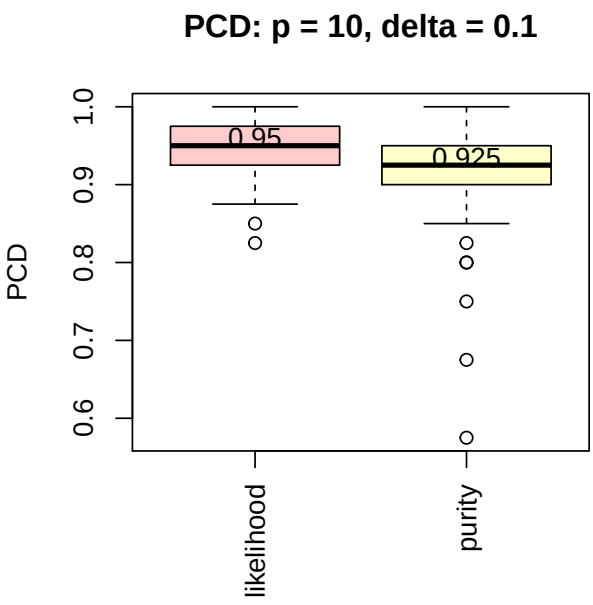

PCD: p = 3, delta = 3
PCD
1.0
0.9
0.8
0.7
0.6
0.5
0.925
0.875
likelihood
purity

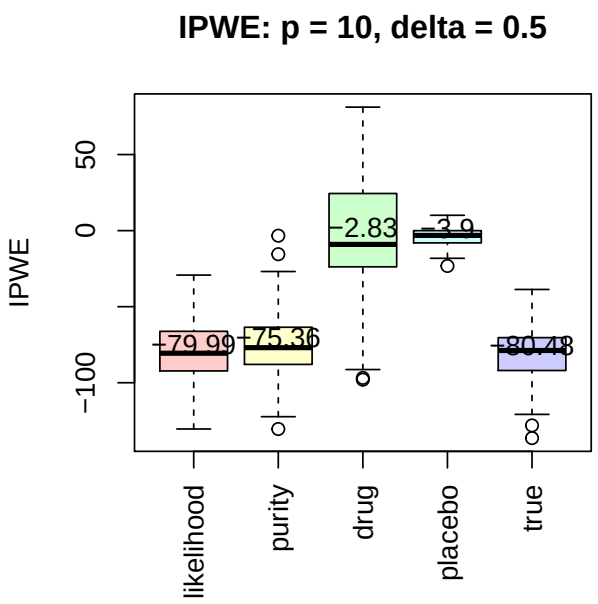
p = 10

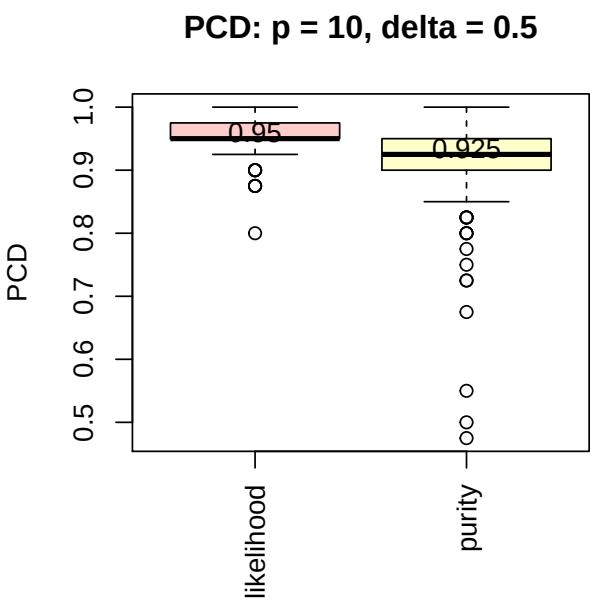

IPWE: p = 10, delta = 0
IPWE
50
0
−50
−100
−59.88
−58.67
−2.96
−3.93
−60.93
likelihood
purity
drug
placebo
true

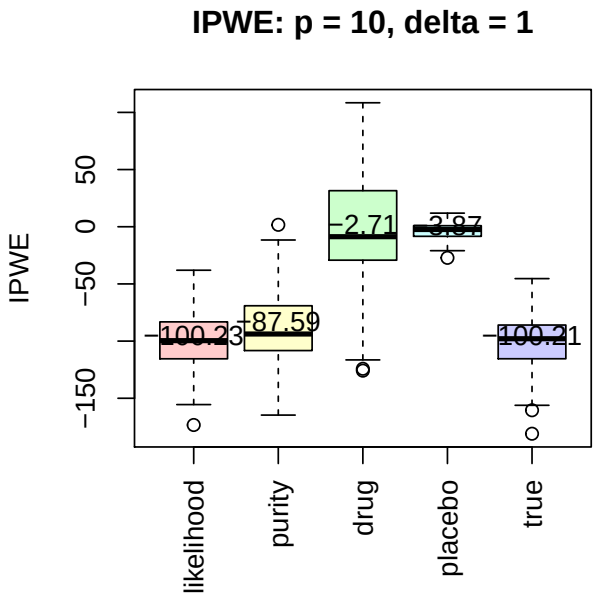

PCD: p = 10, delta = 0
PCD
1.0
0.9
0.8
0.7
0.95
0.925
likelihood
purity

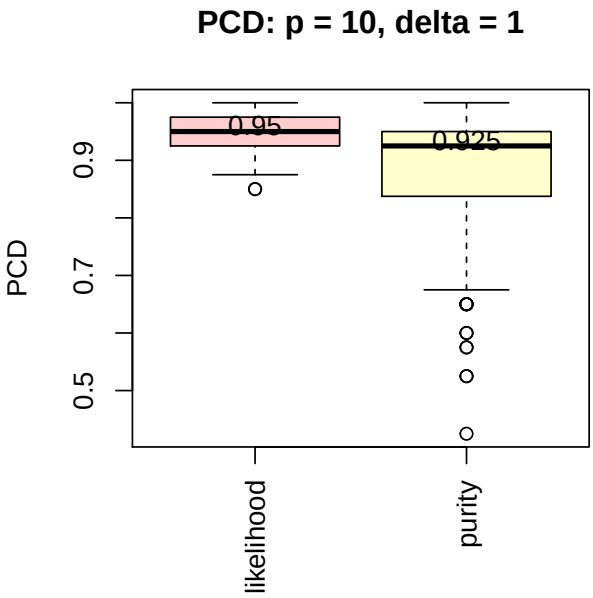

IPWE: p = 10, delta = 0.1
IPWE
50
0
−50
−100
−64.1
−62.76
−2.94
−3.92
−64.76
likelihood
purity
drug
placebo
true

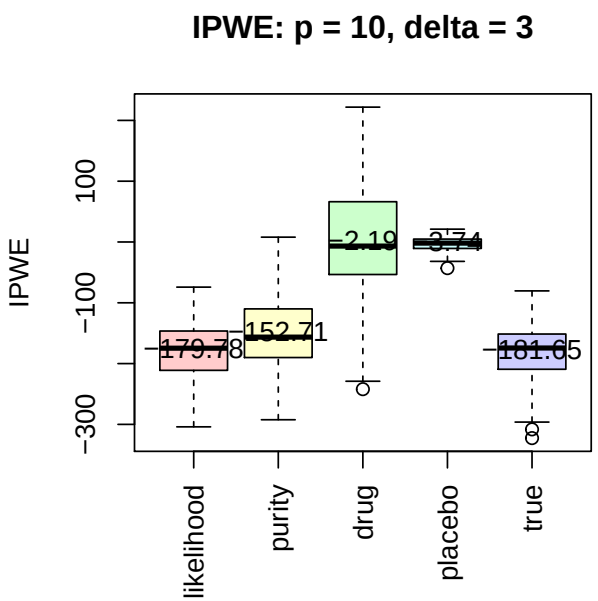

PCD: p = 10, delta = 0.1
PCD
1.0
0.9
0.8
0.7
0.6
0.95
0.925
likelihood
purity

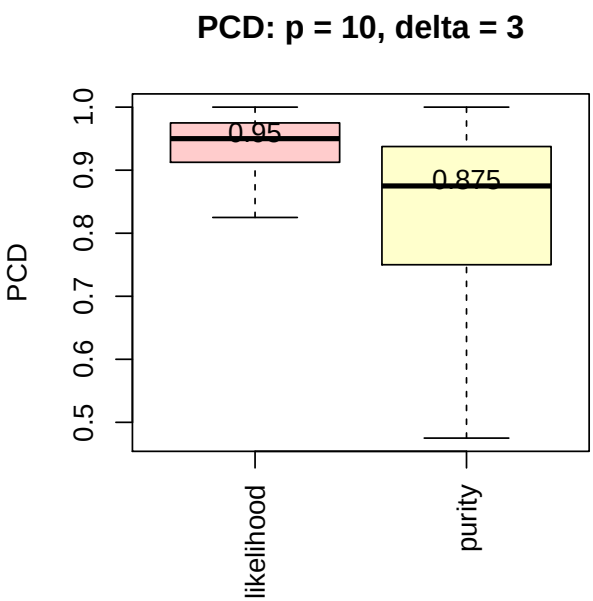

IPWE: p = 10, delta = 0.5
IPWE
50
0
−100
−79.99
−75.36
−2.83
−3.9
−80.48
likelihood
purity
drug
placebo
true

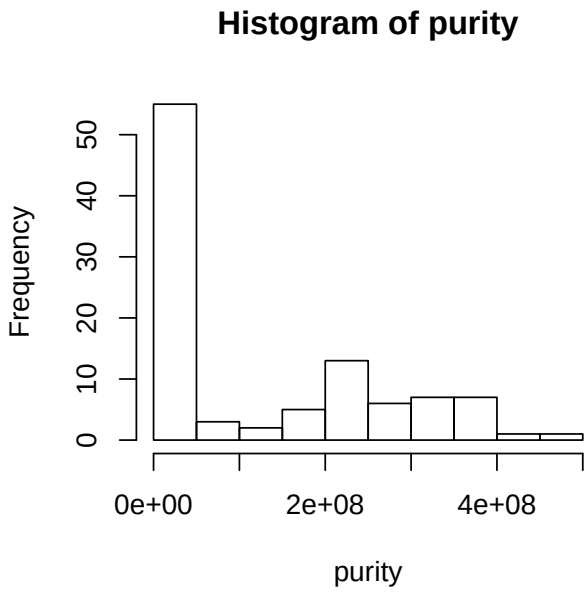

PCD: p = 10, delta = 0.5
PCD
1.0
0.9
0.8
0.7
0.6
0.5
0.95
0.925
likelihood
purity

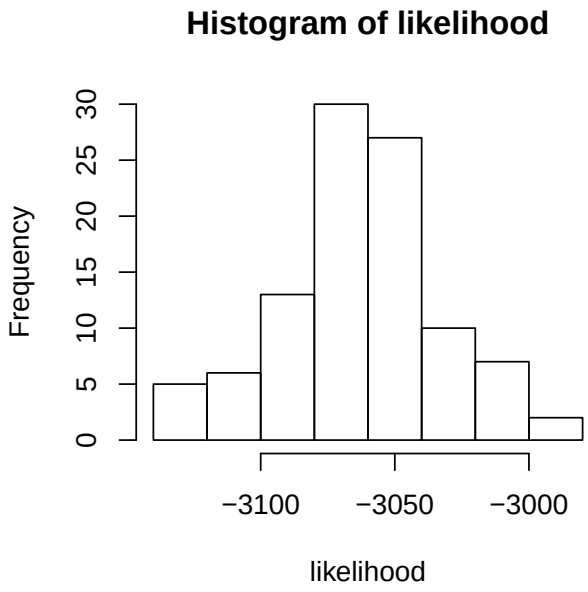

IPWE: p = 10, delta = 1
IPWE
50
0
−50
−150
−100.23
−87.59
−2.71
−3.87
−100.21
likelihood
purity
drug
placebo
true


PCD: p = 10, delta = 1
PCD
0.9
0.7
0.5
0.95
0.925
likelihood
purity


IPWE: p = 10, delta = 3
IPWE
100
−100
−300
−179.78
−152.71
−2.19
−3.74
−181.65
likelihood
purity
drug
placebo
true


PCD: p = 10, delta = 3
PCD
1.0
0.9
0.8
0.7
0.6
0.5
0.95
0.875
likelihood
purity


Histogram of purity
Frequency
0
10
20
30
40
50
0e+00
2e+08
4e+08
purity


Histogram of likelihood
Frequency
0
5
10
15
20
25
30
−3100
−3050
−3000
likelihood