Example of Independence (Slud piecewise)

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The pairwise example in Slud's paper

The joint distribution is:

$$f(t,s) = \begin{cases} f_1(t) f_C(s) & (t \le s) \\ f_C(s) \dfrac{S_1(s)}{S_2(s)} f_2(t) & (t > s) \end{cases}$$

Let

- $f_1(t) = \exp(-t)$, $S_1(s) = \exp(-x)$
- $f_C(s) = \exp(-s)$, $S_C(s) = \exp(-s)$
- $f_2(t) = \rho \exp(-\rho t)$, $S_2(s) =]exp(-\rho t)$
- $\rho(t) = \frac{h_2(t)}{h_1(t)} = \rho$, which is a constant.

Then

$$f(t,s) = \begin{cases} \exp(-t-s) & (t \le s) \\ \rho \exp(-\rho t + (\rho - 2)s) & (t > s) \end{cases}$$

And

$$f(t) = \frac{2\rho - 2}{\rho - 2} \exp(-2t) - \frac{\rho}{\rho - 2} \exp(-\rho t)$$

$$S(t) = \frac{\rho - 1}{\rho - 2} exp(-2t) - \frac{1}{\rho - 2} \exp(-\rho t)$$

$$\psi(t) = \exp(-2t)$$

$$S_x(x) = P(X = T \wedge C > x) = P(T > x, C > x) = P(T > C > x) + P(C > T > x)$$

$$\begin{aligned} S_x(x) =& P(X = T \wedge C > x) = P(T > x, C > x) = P(T > C > x) + P(C > T > x) \\ =& \int_x^\infty \int_x^t f(t,s) ds dt + \int_x^\infty \int_x^s f(t,s) dt ds \\ =& \int_x^\infty \int_x^t \rho \exp(-\rho t + (\rho - 2)s) ds dt + \int_x^\infty \int_x^t \exp(-t-s) dt ds \\ =& \int_x^\infty \rho \Big(\frac{\exp(-2t)}{\rho - 2} - \frac{\exp(\rho x - 2x - \rho t)}{\rho - 2} \Big) dt + \int_x^\infty \exp -x - s - \exp(-2s) ds \\ =& \frac{\rho}{\rho - 2} \frac{\rho - 2}{2\rho} \exp(-2x) + \frac{\exp(-2x)}{2} \\ =& \exp(-2x) \end{aligned}$$

Therefore,

$$S_H(t) = S_x(t) = \exp(-2t), \lambda_H(t) = 2 \text{ ,(consistent to previous notation))}$$

Then the $m()$ function is

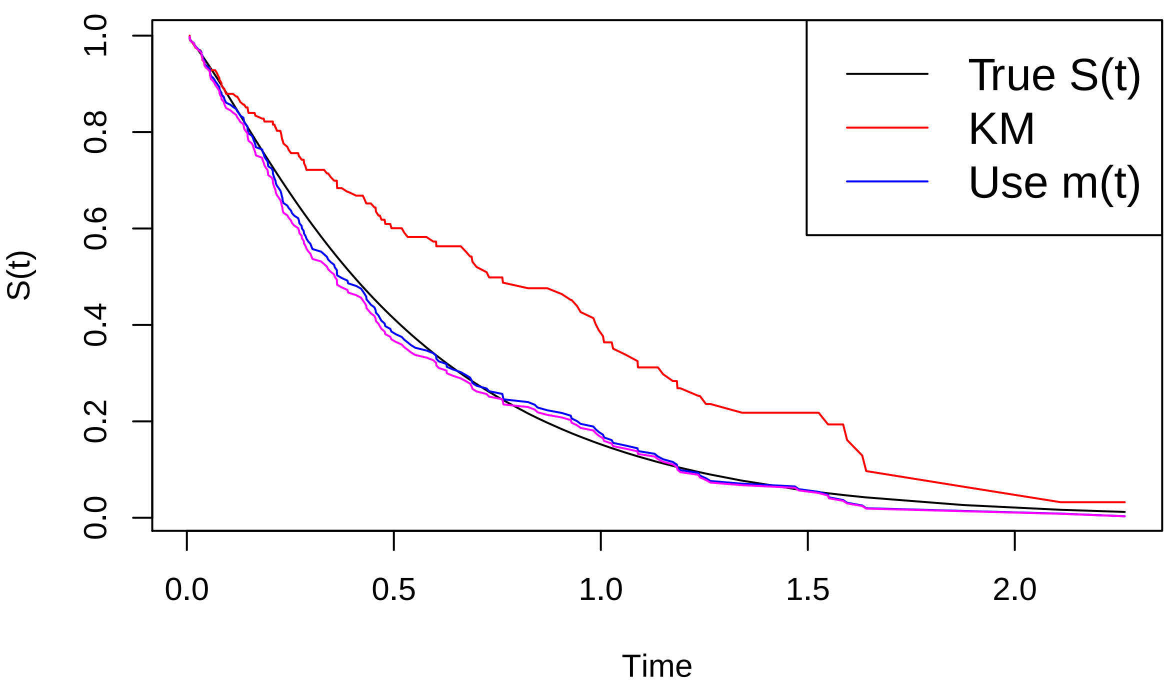
$$m(t) = \frac{\lambda_F(t)}{\lambda_H(t)} = \frac{\frac{\frac{2\rho-2}{\rho-2}\exp(-2t) - \frac{\rho}{\rho-2}\exp(-\rho t)}{\frac{\rho-1}{\rho-2}\exp(-2t) - \frac{1}{\rho-2}\exp(-\rho t)}}{2} = \frac{(2\rho-2)\exp(-2t) - \rho\exp(-\rho t)}{2(\rho-1)\exp(-2t) - 2\exp(-\rho t)}$$

And from the above formula, we can know that when $\rho = 1$, $m(t) = \frac{1}{2}$.

Let's an add parameter in the $m(t)$ function:

$$m_\theta(t) = \frac{\lambda_F(t)}{\lambda_H(t)} = \frac{(2\theta-2)\exp(-2t) - \theta\exp(-\theta t)}{2(\theta-1)\exp(-2t) - 2\exp(-\theta t)}$$

Then we could estimate the θ by calulating the MLE. Since it is hard to solve the $\hat{\theta}$, Newton Raphson method is applied.




```
mean(abs(fit_km$surv - S(fit_km$time)))
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```
## [1] 0.1183952
```

```
mean(abs(sest - S(fit_km$time)))
```

```
## [1] 0.0156356
```

```
mean(abs(sest_est - S(fit_km$time)))
```

```
## [1] 0.0253022
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