Boostrapping

2019-11-04

Outline

- Model: Slud's piecewise example, with $\rho = 10$
- Simulated the origin dataset with 200 subjects
- Sample 200 subjects with replacement form the origin dataset; Repeat time: 1000
- The mean differences between true value and estimations are recorded, i.e. Kaplan Meier, Gerhard Dikta's m() function and our m() function.
- The estiamtes at time 1st, 30th, 50th, 100th, 150th (e.g. the 30th observed time in the data) were recorded and standard errors were calculated.

Model

The pairwise example in Slud's paper. The joint distribution is:

$$f(t,s) = \begin{cases} f_1(t) f_C(s) & (t \leq s) \\ f_C(s) \dfrac{S_1(s)}{S_2(s)} f_2(t) & (t > s) \end{cases}$$

Let

- $f_1(t) = \exp(-t)$, $S_1(s) = \exp(-x)$
- $f_C(s) = \exp(-s)$, $S_C(s) = \exp(-s)$
- $f_2(t) = \rho \exp(-\rho t)$, $S_2(s) =]exp(-\rho t)$
- $\rho(t) = \frac{h_2(t)}{h_1(t)} = \rho$, which is a constant.

Then

$$f(t,s) = \begin{cases} \exp(-t-s) & (t \leq s) \\ \rho \exp(-\rho t + (\rho - 2)s) & (t > s) \end{cases}$$

And

$$f(t) = \frac{2\rho - 2}{\rho - 2} \exp(-2t) - \frac{\rho}{\rho - 2} \exp(-\rho t)$$

$$S(t) = \frac{\rho - 1}{\rho - 2} exp(-2t) - \frac{1}{\rho - 2} \exp(-\rho t)$$

$$\psi(t) = \exp(-2t)$$

$$S_H(t) = S_x(t) = \exp(-2t), \lambda_H(t) = 2 \text{ ,(consistent to previous notation))}$$

Then the $m()$ function is

$$m(t) = \frac{\lambda_F(t)}{\lambda_H(t)} = \frac{\frac{\frac{2\rho-2}{\rho-2}\exp(-2t) - \frac{\rho}{\rho-2}\exp(-\rho t)}{\frac{\rho-1}{\rho-2}\exp(-2t) - \frac{1}{\rho-2}\exp(-\rho t)}}{2} = \frac{(2\rho-2)\exp(-2t) - \rho\exp(-\rho t)}{2(\rho-1)\exp(-2t) - 2\exp(-\rho t)}$$

Result

Mean absolute differences between the true $S(t)$

KM	Gerhard	New m()
0.1376875	0.0279516	0.0288048

Standard deviation

Just looked at 5 time points: the 1st observed time, and the 30th, 50th, 100th, 150th ovserved time.

1	30	50	100	150
0.0141611	0.0711328	0.1611304	0.3505427	0.736218

The standard deviation at those time points are:

	1	30	50	100	150
KM	0.0000000	0.0194751	0.0244678	0.0381762	0.0442257
Gerhard	0.0022889	0.0169957	0.0231279	0.0370569	0.0317589
New m()	0.0022640	0.0168599	0.0229375	0.0367498	0.0315048