Correct the sampling method

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The distribtuion

In the piecewise example in Slud's paper,

To generate examples with the ρ_i value we want, we may use the piecewise example in Slud's paper, whose joint distribution is:

$$f(t,s) = \begin{cases} f_1(t)f_C(s) & (t \leq s) \\ f_C(s)\dfrac{S_1(s)}{S_2(s)}f_2(t) & (t > s) \end{cases}$$

Let

- $f_1(t) = exp(-t)$, $S_1(s) = exp(-x)$
- $f_C(s) = exp(-s)$, $S_C(s) = exp(-s)$
- $f_2(t) = \rho exp(-\rho t)$, $S_2(s) = exp(-\rho t)$
- $\rho(t) = \frac{h_2(t)}{h_1(t)} = \rho$, which is a constant.

Then

$$f(t,s) = \begin{cases} exp(-t-s) & (t \leq s) \\ \rho exp(-\rho t + (\rho - 2)s) & (t > s) \end{cases}$$

And

$$f(t) = \frac{2\rho - 2}{\rho - 2}exp(-2t) - \frac{\rho}{\rho - 2}exp(-\rho t)$$

$$S(t) = \frac{\rho - 1}{\rho - 2}exp(-2t) - \frac{1}{\rho - 2}exp(-\rho t)$$

$$\psi(t) = exp(-2t),\ S_x(t) = exp(-2t)$$

The sampling method

For the direction: $s \geq t$**:**

$$f(s,t) = exp(-s-t), (s \geq t)$$

. Let's make it as a pdf, then:

$$f(s,t) = 2exp(-s-t), (s \geq t)$$

. The marginal pdf of t is:

$f_t(t) = \int f(s,t)ds = \int_t^\infty 2exp(-s-t)ds = 2exp(-t)[-exp(-s)|_t^\infty] = 2exp(-t)exp(-t) = 2exp(-2t)$.

Therefore, the marginal of t comes from an exponential distribution with $\lambda = 2$.

We may generate n t from the EXP(2) distribution. Based on each t, we can generate $f(s|t)$.

- PDF: $f(s|t) = \frac{f(s,t)}{f_t(t)} = \frac{2exp(-s-t)}{2exp(-2t)} = exp(-s+t), (s \geq t)$
- CDF: $F(s|t) = \int_t^s f(s|t) = \int_t^s exp(-s+t) = 1 - exp(-s+t), (s \geq t)$

Follow the inverse probability sampling method, the s is generated from

$s = t - ln(1-x), x \sim UNI(0,1)$, for each t.

For the direction: $s < t$**:**

$$f(s,t) = \rho exp(-\rho t + (\rho - 2)s), (s < t)$$

. Let's make it as a pdf, then:

$$f(s,t) = 2\rho exp(-\rho t + (\rho - 2)s), (s < t)$$

.

We may sample s first. The marginal distribution for s is

$f_s(s) = \int f(s,t)dt = \int_s^\infty 2\rho exp(-\rho t + (\rho-2)s)dt = 2exp(\rho s - 2s)exp(-\rho s) = 2exp(-2s)$

Therefore, the marginal of s comes from an exponential distribution with $\lambda = 2$.

We may generate n s from the EXP(2) distribution. Based on each s, we can generate $f(t|s)$.

- PDF: $f(t|s) = \frac{f(s,t)}{f_s(s)} = \frac{2\rho exp(-\rho t + (\rho-2)s)}{2exp(-2s)} = \rho exp(-\rho(t-s)), (s < t)$
- CDF: $F(t|s) = \int_t^s f(t|s) = \int_t^s \rho exp(-\rho(t-s)) = 1 - exp(-\rho(t-s)), (s < t)$

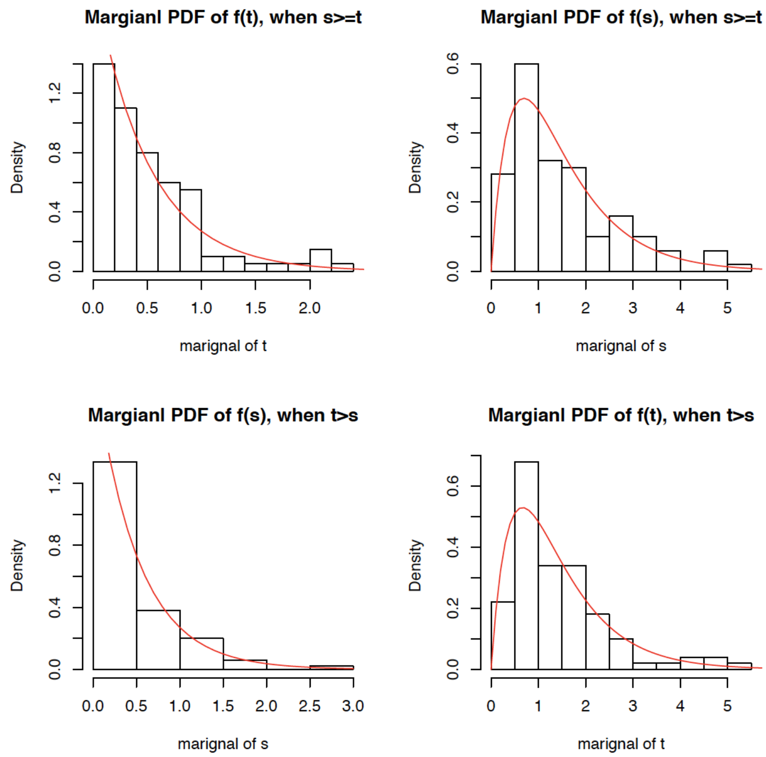
Follow the inverse probability sampling method, the s is generated from

$t = s - \frac{1}{\rho} ln(1-x), x \sim UNI(0,1)$, for each s.

The summary table:

	$s \geq t$	$s < t$
$f(t,s)$	$2exp(-s-t)$	$2\rho exp(-\rho t + (\rho-2)s)$
The first sample element	t	s
The marginal pdf of the first sample element	$f_t(t) = 2exp(-2t)$	$f_s(s) = 2exp(-2s)$
The sampling of first element	$t \sim EXP(2)$	$s \sim EXP(2)$
The conditional pdf	$f(s/t) = exp(-(s-t))$	$f(t/s) = \rho exp(-\rho(t-s))$
The conditional CDF	$F(s/t) = 1 - exp(-(s-t))$	$F(t/s) = 1 - exp(-\rho(t-s))$
The inverse function	$y = t - ln(1-x)$	$y = s - \frac{1}{\rho} ln(1-x)$
The second element's sampling	$x \sim UNI(0,1)$, get $s = t - ln(1-x)$	$x \sim UNI(0,1)$, get $t = s - \frac{1}{\rho} ln(1-x)$

The marginal plots:



Try MLE

Suppose we know the true $f(.), S(.)$ functions

What should the true MLE looks like?

ML expression	True $\rho = 1$	True $\rho = 0.1$	True $\rho = 0.5$	True $\rho = 3$	True $\rho = 10$
$\prod_{i=1}^{n} f(x_i)^{\delta_i} S(x_i)^{1-\delta_i}$	1	0.9	0.8	1	1
$\prod_{i=1}^{n} f(x_i)^{\delta_i}$	10	10	10	10	10
$\prod_{i=1}^{n} f(x_i)$	10	10	10	10	10
$\prod_{i=1}^{n} S(x_i)^{1-\delta_i}$	0.1	0.1	0.1	0.1	0.1
$\prod_{i=1}^{n} S(x_i)$	0.1	0.1	0.1	0.1	0.1