





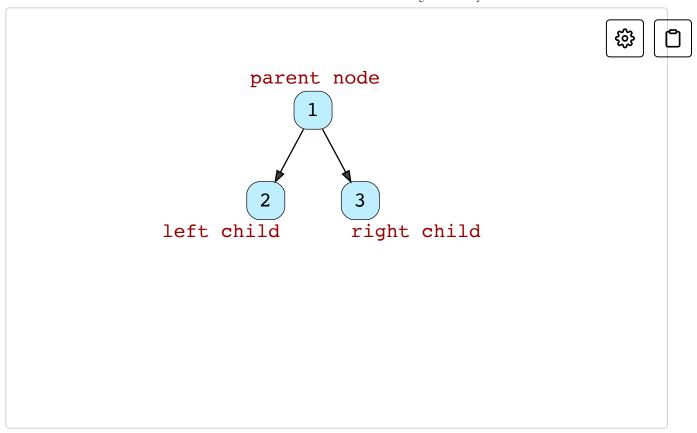
#### Introduction

In this lesson, you will be introduced to Binary Trees and their implementation in Python.

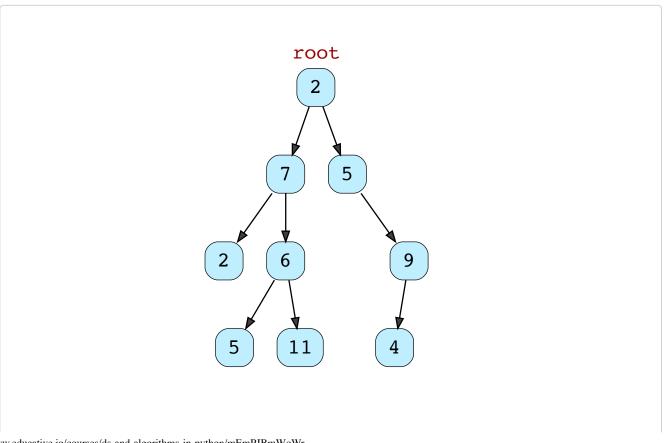
We'll cover the following ^

- Depth of a Node
- Height of a Tree
- Types of Binary Trees
  - Complete Binary Tree
  - Full Binary Tree
- Implementation

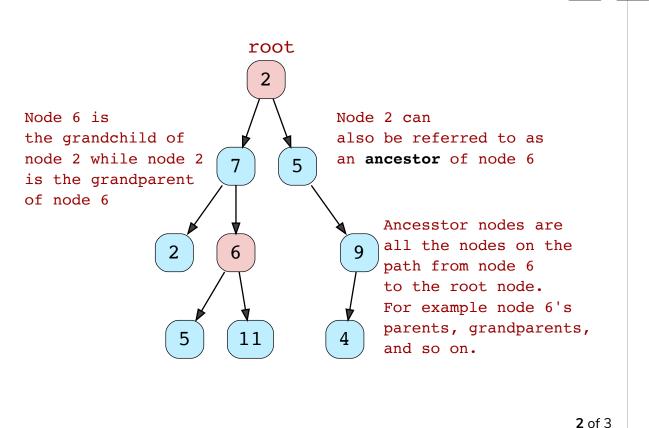
A **binary tree** is a tree data structure in which each node has at most two children, which are referred to as the *left child* and the *right child*. Have a look at an elementary example of a binary tree:

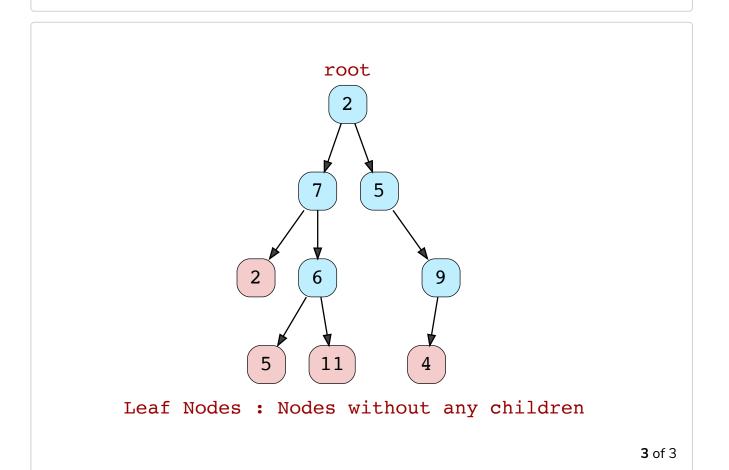


Here is another example of a binary tree that introduces us to other related terminologies:









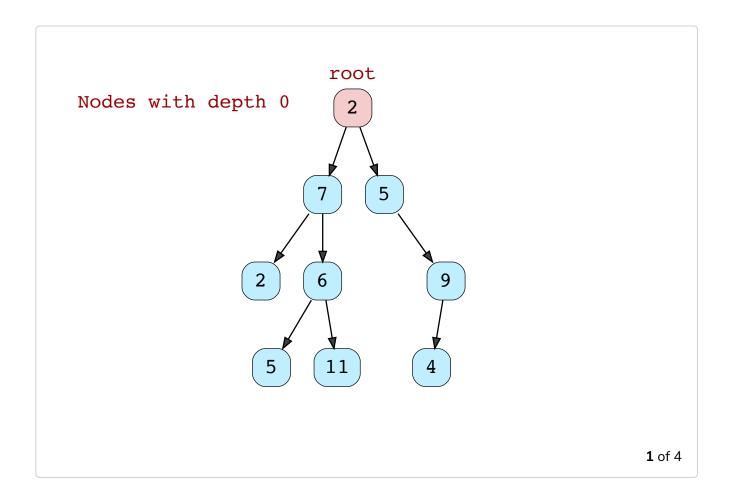


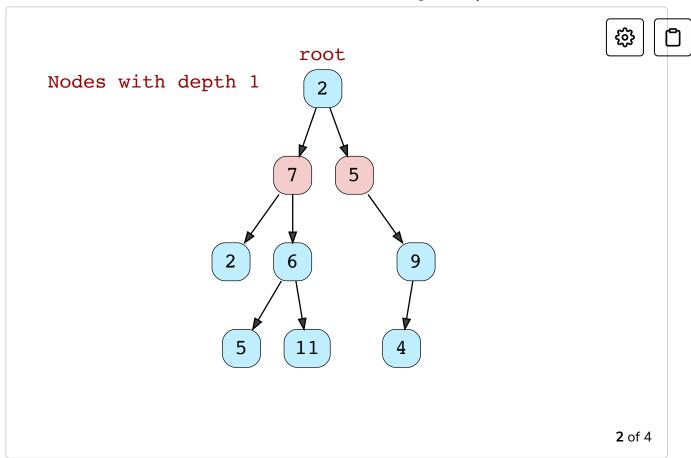


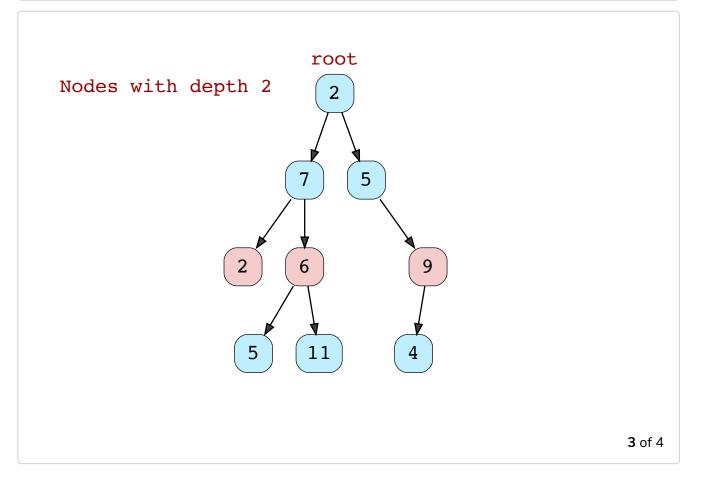


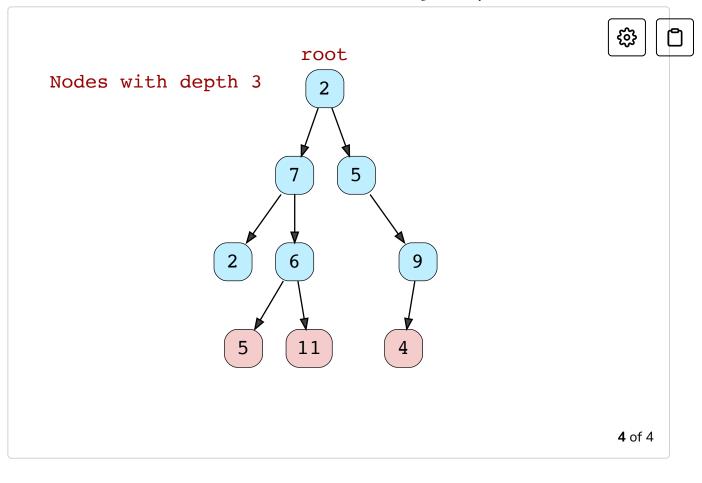
## Depth of a Node #

The length of the path from a node, n, to the root node. The depth of the root node is 0.



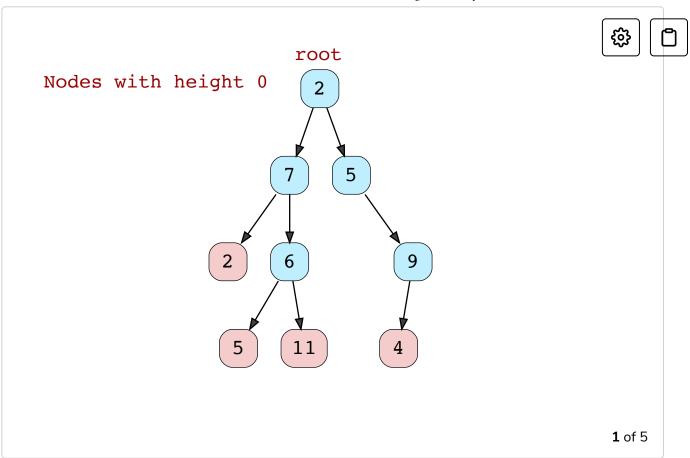


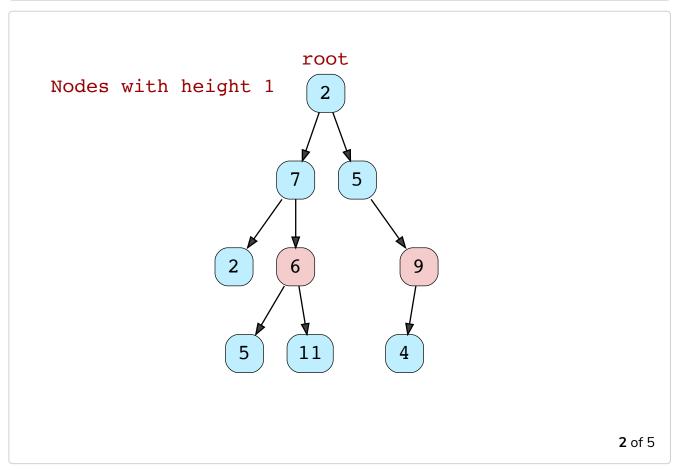


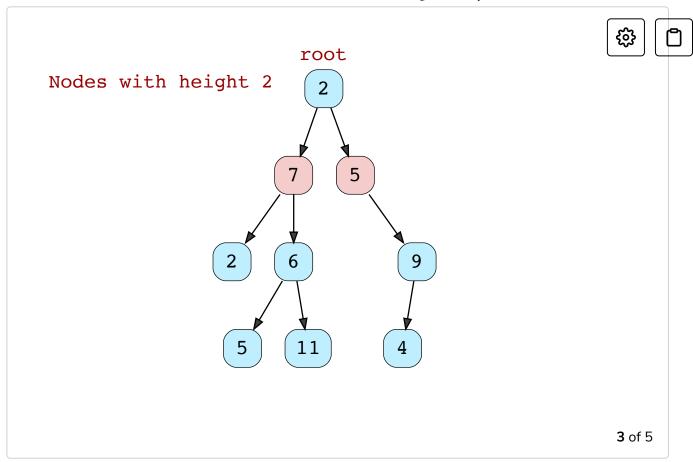


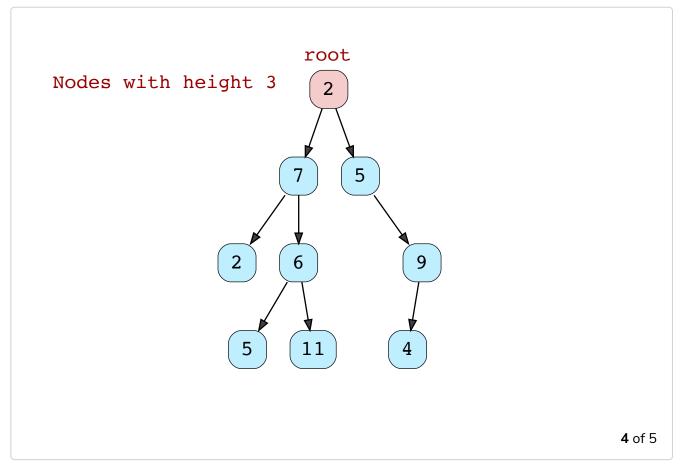
### Height of a Tree #

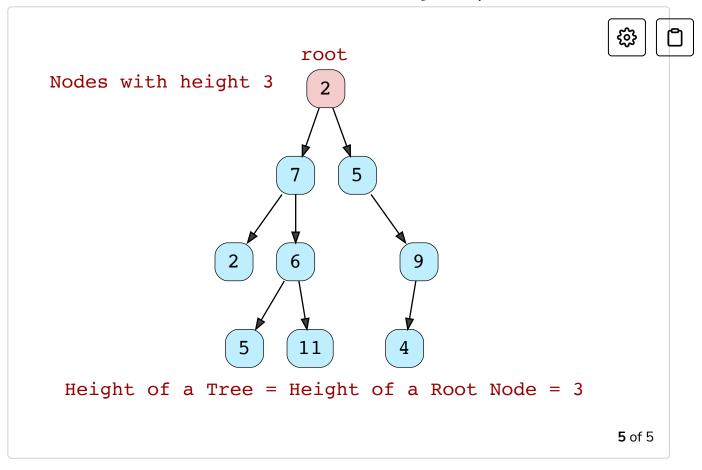
The length of the path from n to its deepest descendant. The height of the tree itself is the height of the root node, and the height of leaf nodes is always 0.









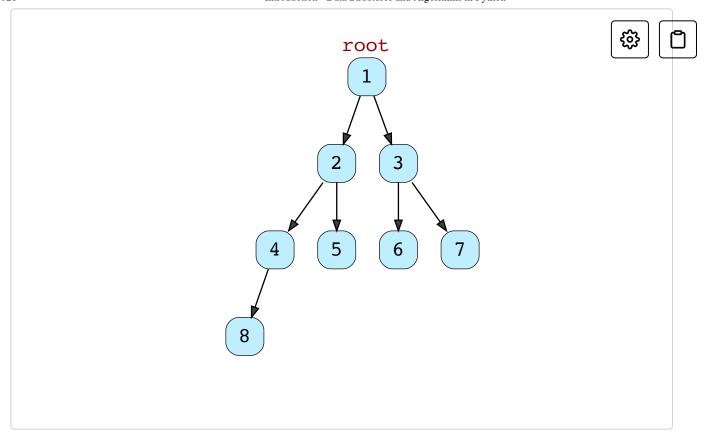


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# Types of Binary Trees #

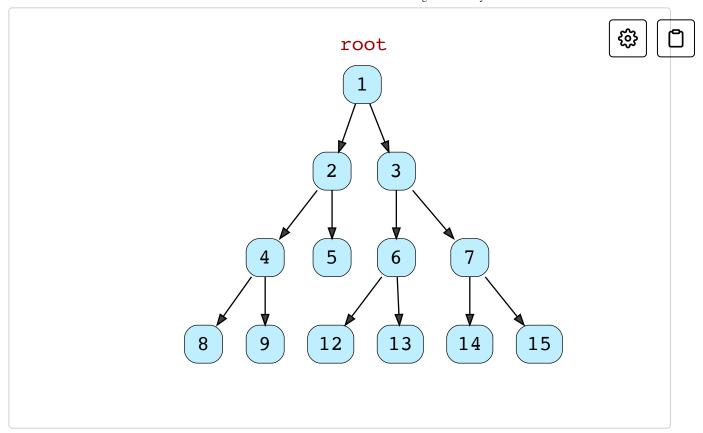
### **Complete Binary Tree #**

In a **complete** binary tree, every level *except possibly the last*, is completely filled and all nodes in the last level are as far left as possible.



### Full Binary Tree #

A **full** binary tree (sometimes referred to as a **proper** or **plane** binary tree) is a tree in which every node has either 0 or 2 children.



### Implementation #

To implement a binary tree in Python, we will first implement the Node class.

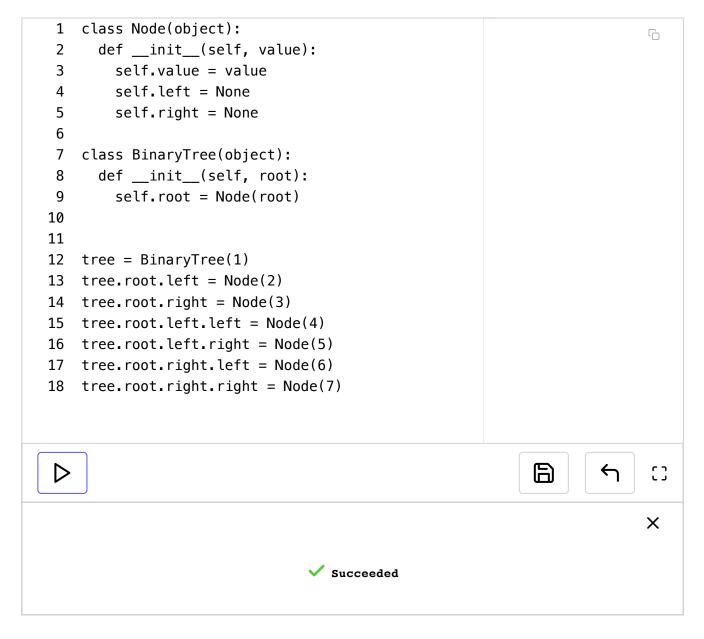
```
1 class Node(object):
2  def __init__(self, value):
3    self.value = value
4    self.left = None
5    self.right = None
```

In the code above, we have defined the Node class with a "new" style of defining classes in Python. The Node class has three attributes:

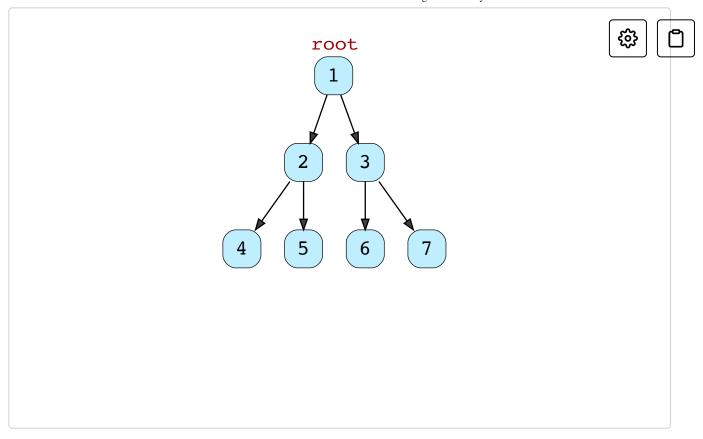
- self.value
- 2. self.left
- self.right

self.value will be equal to the value passed to the constructor while self.left and self.right will contain the nodes which will be the left and the right child of this node.

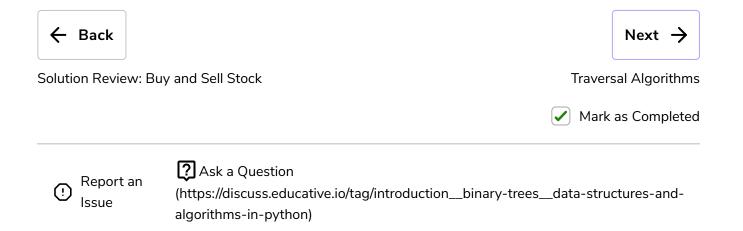
Let's go ahead and implement BinaryTree class:



The BinaryTree contains root, which is an object of the Node class and contains the value passed as root in the constructor of BinaryTree class. From **lines 12-18**, we construct an object from the BinaryTree class and populate the tree by creating nodes. The visual representation of that tree will be as follows:



The implementation of the Binary Tree was pretty straightforward. At this point, we need some way to traverse the tree and visualize through code in Python. In the next lesson, we will go over some of the tree traversal methods.













#### Traversal Algorithms

In this lesson, you will learn how to traverse binary trees using a depth-first search.

We'll cover the following

- Tree Traversal
- Pre-order Traversal
- In-order Traversal
- Post-order Traversal
- Helper Method

### Tree Traversal #

Tree Traversal is the process of visiting (checking or updating) each node in a tree data structure, exactly once. Unlike linked lists or one-dimensional arrays that are canonically traversed in linear order, trees may be traversed in multiple ways. They may be traversed in *depth-first* or *breadth-first* order.

There are three common ways to traverse a tree in depth-first order:

- 1. In-order
- 2. Pre-order
- 3. Post-order

Let's begin with the Pre-order Traversal.

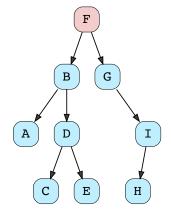
#### Pre-order Traversal #

#### Here is the algorithm for a pre-order traversal:



- 1. Check if the current node is empty/null.
- 2. Display the data part of the root (or current node).
- 3. Traverse the left subtree by recursively calling the pre-order method.
- 4. Traverse the right subtree by recursively calling the pre-order method.
  - 1. Check if the current node is empty/null.
  - 2. Display the data part of the root (or current node).
  - 3. Traverse the left subtree by recursively calling the pre-order method.
  - 4. Traverse the right subtree by recursively calling the pre-order method.

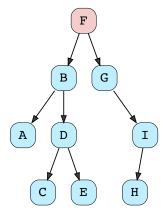
Start with the root node.



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- 1. Check if the current node is empty/null.
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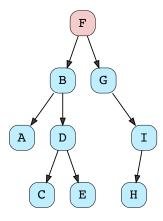






- 1. Check if the current node is empty/null.
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F



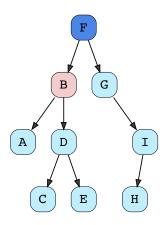
**3** of 37

- 1. Check if the current node is empty/null.
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 $\rightarrow$ 

- 3. Traverse the left subtree by recursively calling the pre-order method.
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F

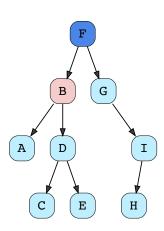


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F

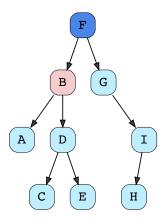






- 1. Check if the current node is empty/null.
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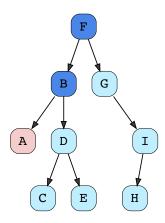
F, B



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- 1. Check if the current node is empty/null.
- 2. Display the data part of the root (or current node).
- 3.
- 3. Traverse the left subtree by recursively calling the pre-order method.
  - 4. Traverse the right subtree by recursively calling the pre-order method.

F, B

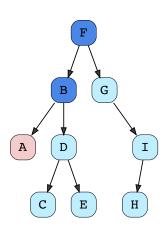


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- 1. Check if the current node is empty/null.
- 2. Display the data part of the root (or current node).
- 3. Traverse the left subtree by recursively calling the pre-order method.  $\,$
- 4. Traverse the right subtree by recursively calling the pre-order method.

F, B

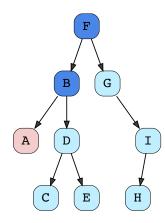






- 1. Check if the current node is empty/null.
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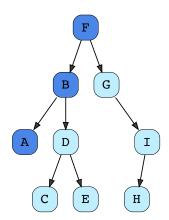
F, B, A



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- 1. Check if the current node is empty/null.
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F, B, A

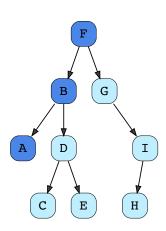


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- ▶ 1. Check if the current node is empty/null.
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F, B, A

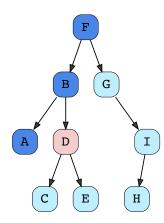






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F, B, A

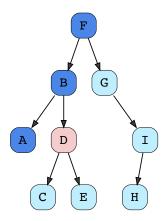


**12** of 37



- 1. Check if the current node is empty/null.
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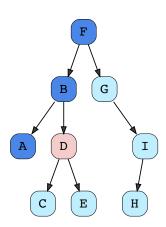
F, B, A



**13** of 37

- 1. Check if the current node is empty/null.
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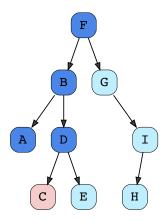
F, B, A, D







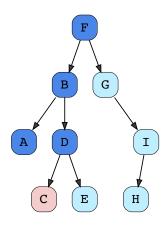
- 1. Check if the current node is empty/null.
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- 3
  - 3. Traverse the left subtree by recursively calling the pre-order method.
  - 4. Traverse the right subtree by recursively calling the pre-order method.
  - F, B, A, D



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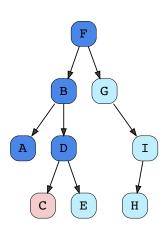
- · 1. Check if the current node is empty/null.
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- 4. Traverse the right subtree by recursively calling the pre-order method.
- F, B, A, D



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- 1. Check if the current node is empty/null.
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  - 4. Traverse the right subtree by recursively calling the pre-order method.
  - F, B, A, D, C

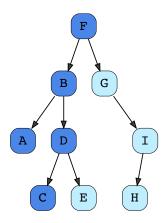






- 1. Check if the current node is empty/null.
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- $\rightarrow$
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- 4. Traverse the right subtree by recursively calling the pre-order method.

F, B, A, D, C

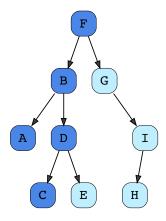


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- 1. Check if the current node is empty/null.
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F, B, A, D, C



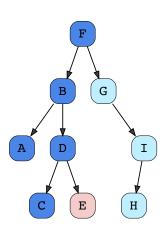
**19** of 37

- 1. Check if the current node is empty/null.
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4. Traverse the right subtree by recursively calling the pre-order method.

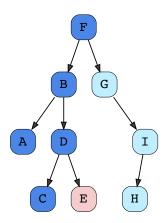
F, B, A, D, C





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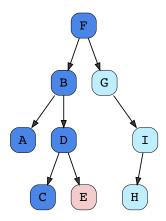
F, B, A, D, C



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F, B, A, D, C, E



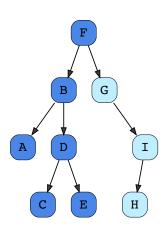
**22** of 37

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3. Traverse the left subtree by recursively calling the pre-order method.

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F, B, A, D, C, E

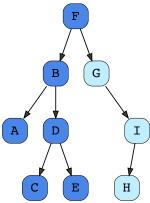






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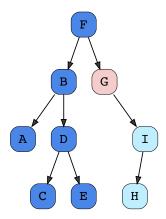
F, B, A, D, C, E



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F, B, A, D, C, E

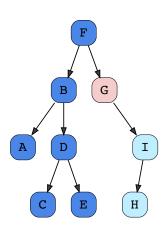


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- ▶ 1. Check if the current node is empty/null.
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F, B, A, D, C, E

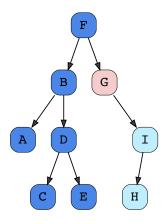


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1. Check if the current node is empty/null.



- 2. Display the data part of the root (or current node).
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- F, B, A, D, C, E, G

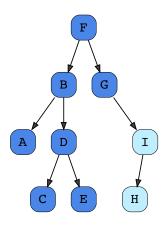


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- 1. Check if the current node is empty/null.
- 2. Display the data part of the root (or current node).



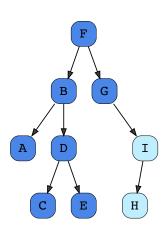
- 3. Traverse the left subtree by recursively calling the pre-order method.
- 4. Traverse the right subtree by recursively calling the pre-order method.
- F, B, A, D, C, E, G



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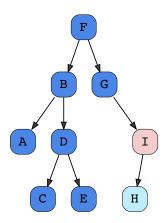


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- 4. Traverse the right subtree by recursively calling the pre-order method.
- F, B, A, D, C, E, G



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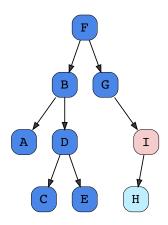
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  - F, B, A, D, C, E, G



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- 4. Traverse the right subtree by recursively calling the pre-order method.
- F, B, A, D, C, E, G



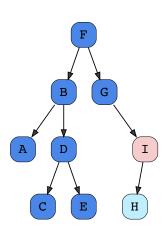
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1. Check if the current node is empty/null.

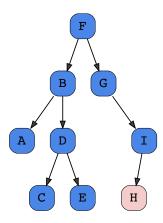


- 2. Display the data part of the root (or current node).
- 3. Traverse the left subtree by recursively calling the pre-order method.
- 4. Traverse the right subtree by recursively calling the pre-order method.
- F, B, A, D, C, E, G, I



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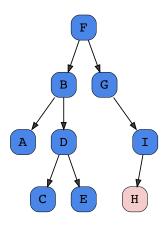
- 1. Check if the current node is empty/null.
- 2. Display the data part of the root (or current node).
- - 3. Traverse the left subtree by recursively calling the pre-order method.  $% \left( \frac{1}{2}\right) =\frac{1}{2}\left( \frac{1}{2}\right) ^{2}$
  - 4. Traverse the right subtree by recursively calling the pre-order method.
  - F, B, A, D, C, E, G, I



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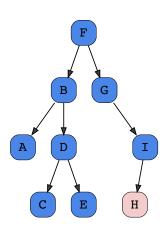
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- F, B, A, D, C, E, G, I

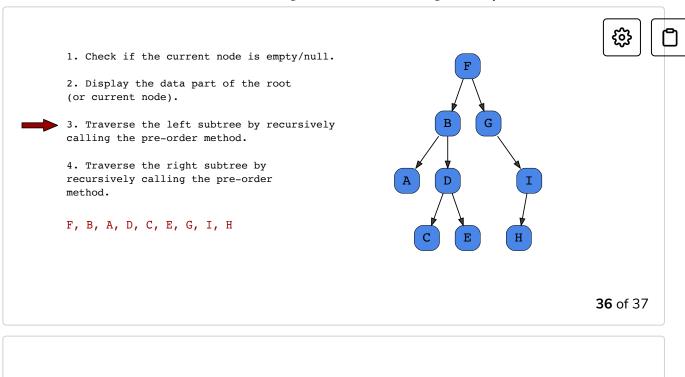


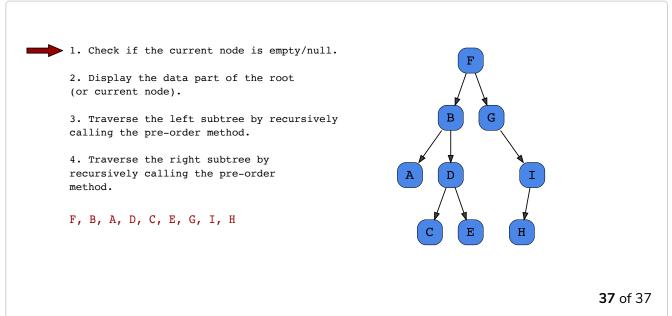
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- 1. Check if the current node is empty/null.
- 2. Display the data part of the root (or current node).
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  - 4. Traverse the right subtree by recursively calling the pre-order method.
  - F, B, A, D, C, E, G, I, H







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I hope the illustrations have made the algorithm pretty clear. Let's go over its implementation in Python:

```
1 def preorder_print(self, start, traversal):
2 """Root->Left->Right"""
3   if start:
4     traversal += (str(start.value) + "-")
5     traversal = self.preorder_print(start.left, to traversal = self.preorder_print(start.right, to traversal);
7   return traversal
```

#### preorder\_print(self, start, traversal)



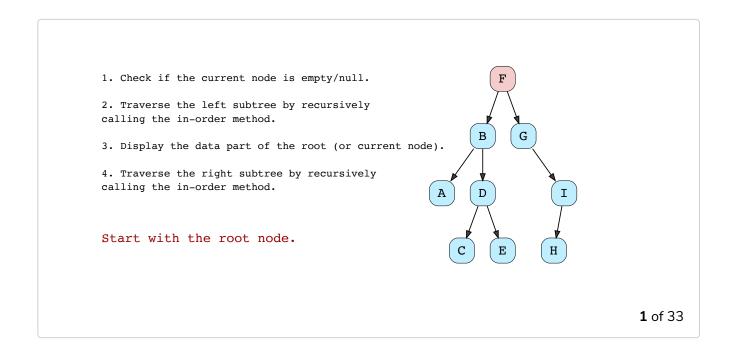


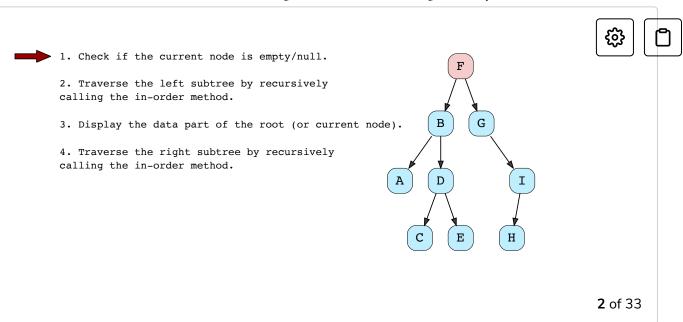
Just as specified in the algorithm, we check if start (i.e., the current node) is empty or not. If not, then we append start.value to the traversal string and recursively call preorder\_print on start.left and start.right which are the right and left child of the current node. Finally, we return traversal from the method after we have returned from all the recursive calls in case start is not None. traversal is just a string that will concatenate the value of nodes in an order that we visited them.

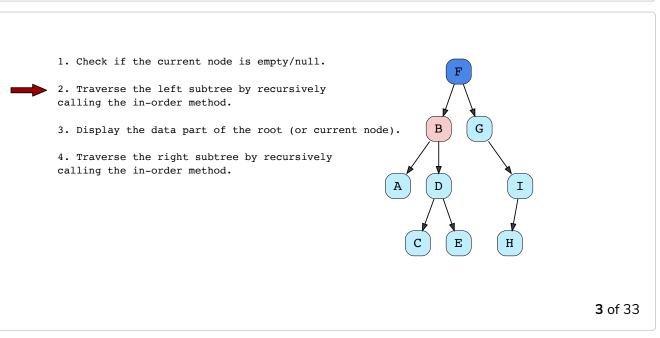
### In-order Traversal #

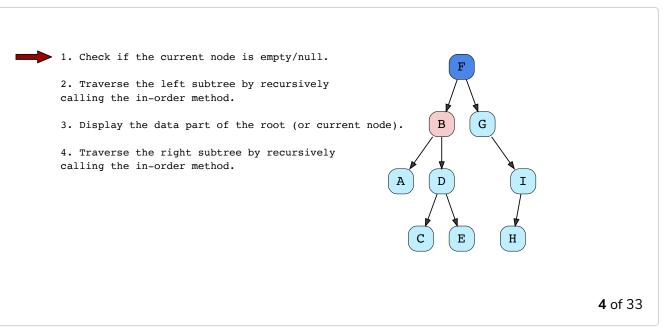
Here is the algorithm for an in-order traversal:

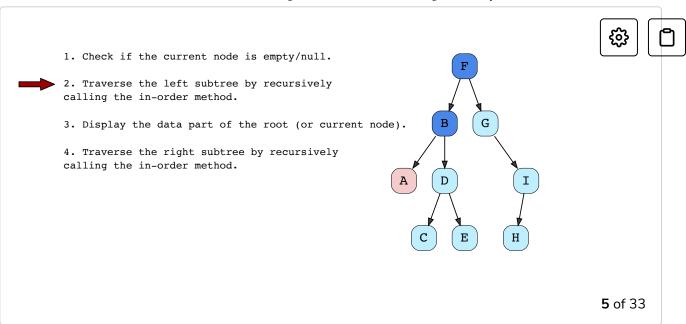
- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the in-order method.
- 3. Display the data part of the root (or current node).
- 4. Traverse the right subtree by recursively calling the in-order method.

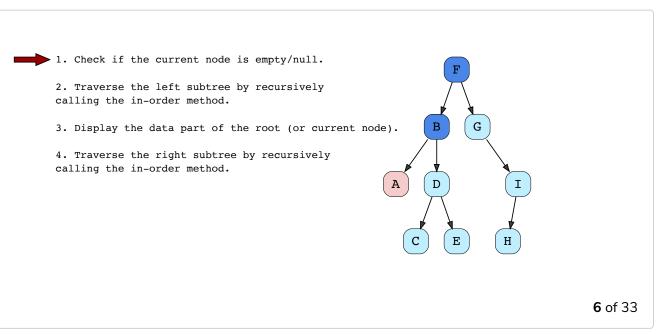


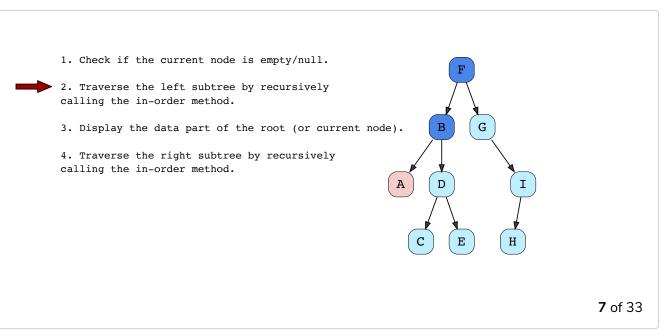


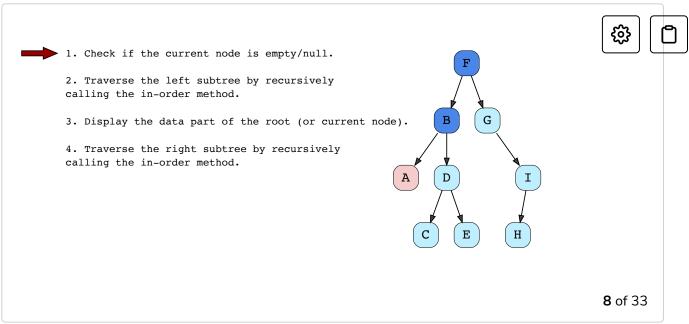


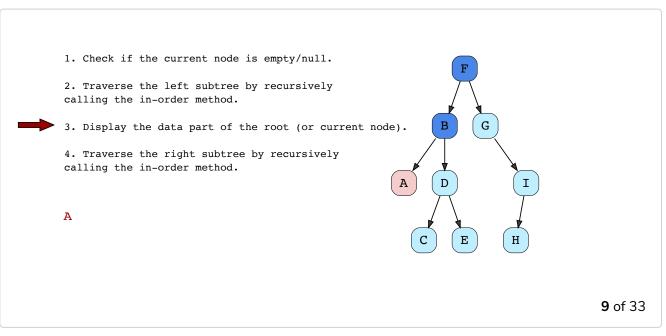


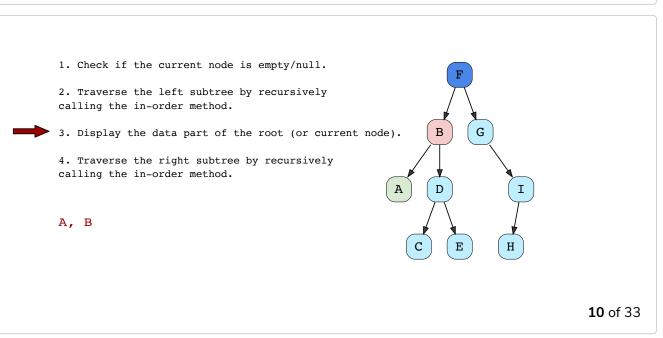


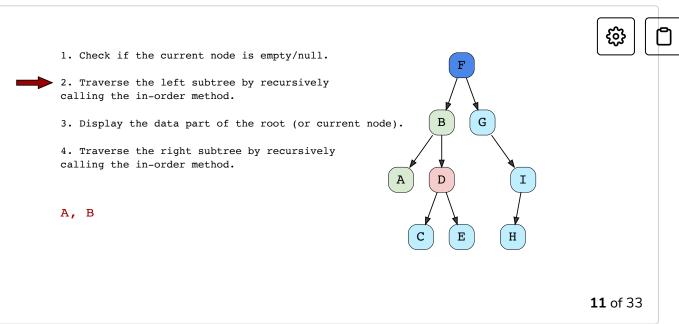


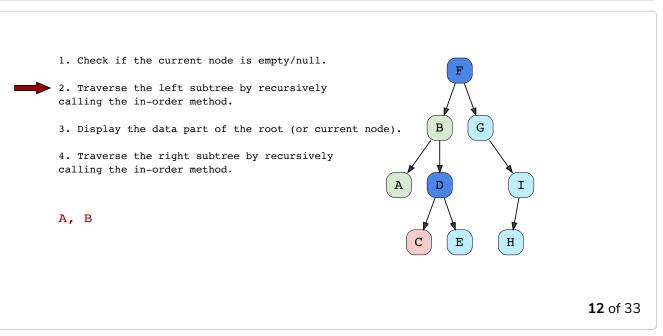


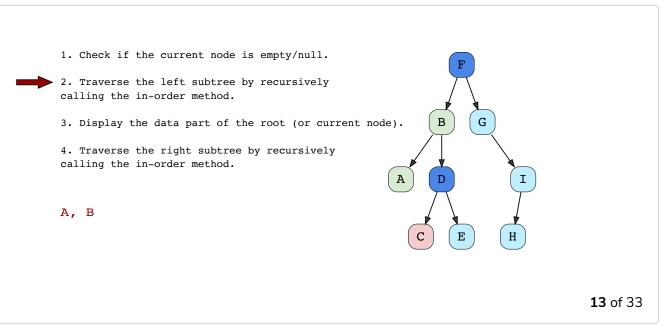


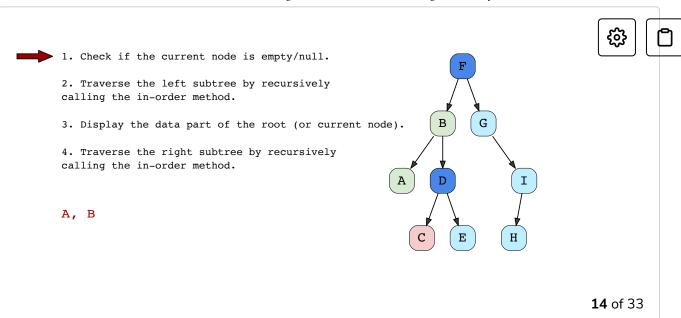












1. Check if the current node is empty/null.

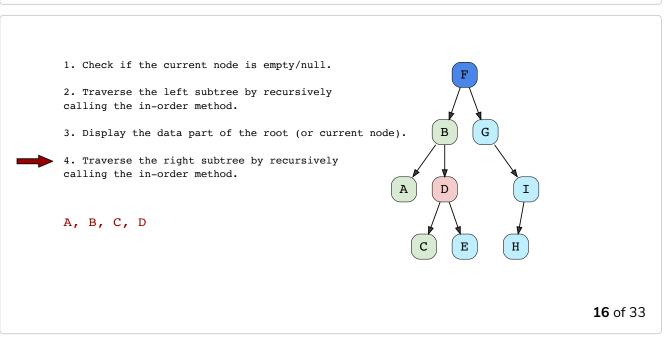
2. Traverse the left subtree by recursively calling the in-order method.

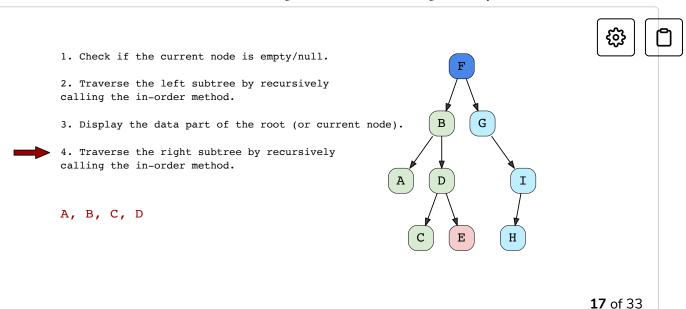
3. Display the data part of the root (or current node).

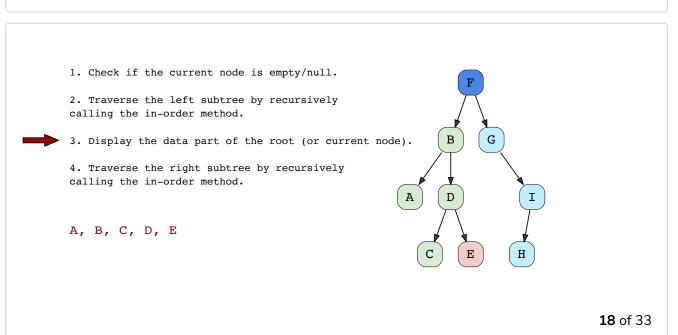
4. Traverse the right subtree by recursively calling the in-order method.

A, B, C

15 of 33







1. Check if the current node is empty/null.

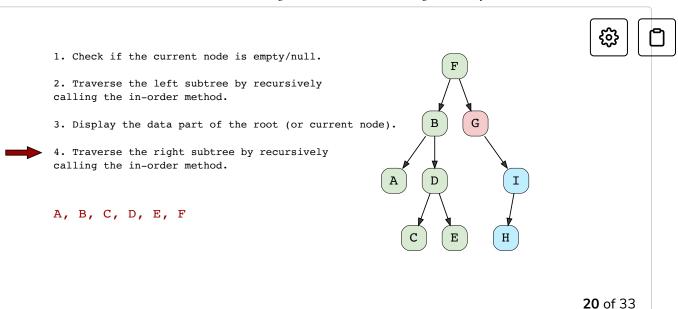
2. Traverse the left subtree by recursively calling the in-order method.

3. Display the data part of the root (or current node).

4. Traverse the right subtree by recursively calling the in-order method.

A, B, C, D, E, F

C E H



1. Check if the current node is empty/null.

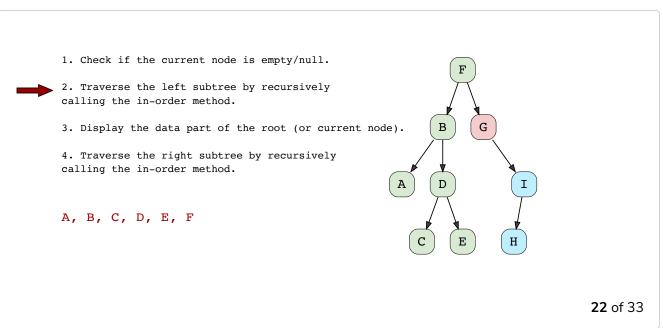
2. Traverse the left subtree by recursively calling the in-order method.

3. Display the data part of the root (or current node).

4. Traverse the right subtree by recursively calling the in-order method.

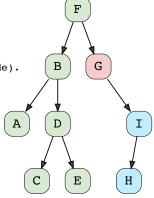
A, B, C, D, E, F

C E H



- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the in-order method.
- 3. Display the data part of the root (or current node).
  - 4. Traverse the right subtree by recursively calling the in-order method.

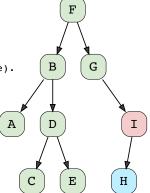
A, B, C, D, E, F, G



**23** of 33

- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the in-order method.
- 3. Display the data part of the root (or current node).
- 4. Traverse the right subtree by recursively calling the in-order method.

A, B, C, D, E, F, G

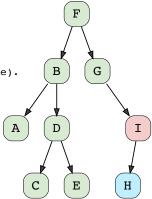


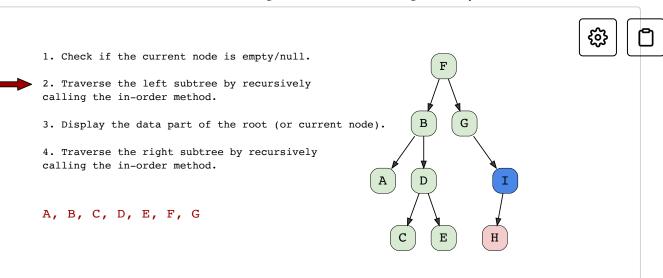
**24** of 33



- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the in-order method.
- 3. Display the data part of the root (or current node).
- 4. Traverse the right subtree by recursively calling the in-order method.

A, B, C, D, E, F, G





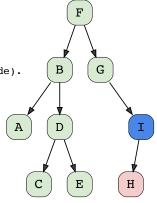
Check if the current node is empty/null.
 Traverse the left subtree by recursively

3. Display the data part of the root (or current node).

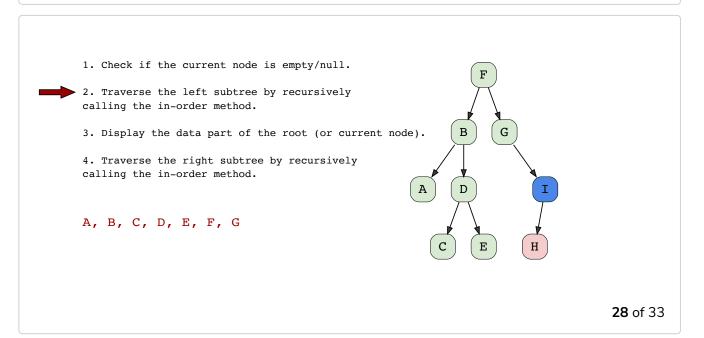
4. Traverse the right subtree by recursively calling the in-order method.

A, B, C, D, E, F, G

calling the in-order method.



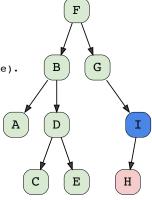
**27** of 33





- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the in-order method.
- 3. Display the data part of the root (or current node).
  - 4. Traverse the right subtree by recursively calling the in-order method.

A, B, C, D, E, F, G, H

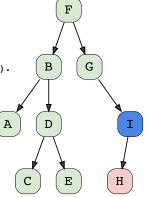


**29** of 33

- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the in-order method.
- 3. Display the data part of the root (or current node).

4. Traverse the right subtree by recursively calling the in-order method.

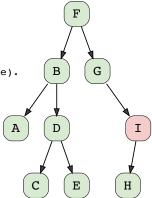
A, B, C, D, E, F, G, H

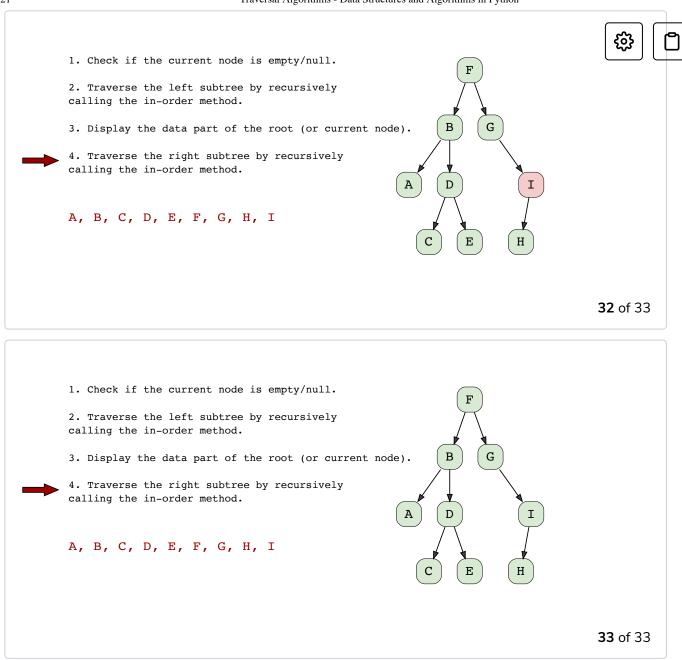


**30** of 33

- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the in-order method.
- 3. Display the data part of the root (or current node).
  - 4. Traverse the right subtree by recursively calling the in-order method.

A, B, C, D, E, F, G, H





— [ ·

Now that you are familiar with the algorithm, let's jump to the code in Python:

```
1 def inorder_print(self, start, traversal):
2    """Left->Root->Right"""
3    if start:
4         traversal = self.inorder_print(start.')
5         traversal += (str(start.value) + "-")
6         traversal = self.inorder_print(start.')
7    return traversal
```

inorder\_print(self, start, traversal)





The inorder\_print is pretty much the same as the preorder\_print except that the order *Root->Left->Right* from pre-order changes to *Left->Root->Right* in in-order traversal. In order to achieve this order, we just change the order of statements in the if-condition, i.e., we first make a recursive call on the left child and after we are done will all the subsequent calls from **line 4**, we concatenate the value of the current node with traversal on **line 5**. Then, we can make a recursive call to right subtree on **line 6**. This will help us keep the order required for the in-order traversal.

# Post-order Traversal #

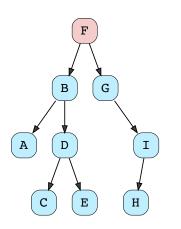
At this point, it will be very easy for you to guess the algorithm for post-order traversal. There you go:

- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).



- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).

Start with the root node.

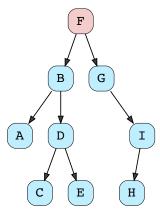






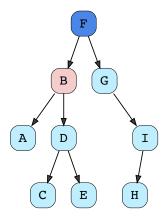


- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).





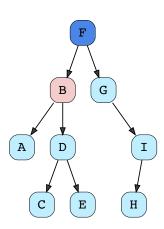
- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).



**3** of 21



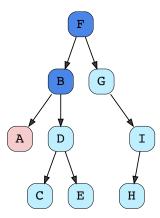
- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).







- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).

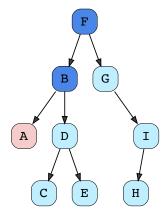


- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.



4. Display the data part of the root (or current node).

Α



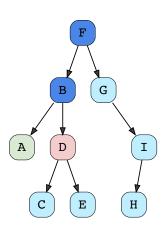
**6** of 21

- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.



- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).

Α



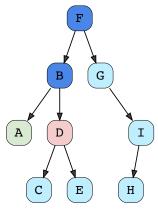






- ▶ 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).

Α



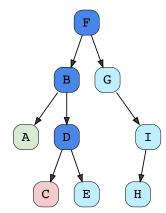
**8** of 21

1. Check if the current node is empty/null.



- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).

Α

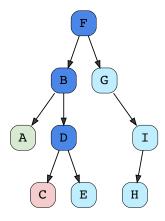






- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).

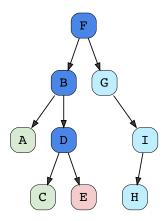
A, C



**10** of 21

- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
  - 4. Display the data part of the root (or current node).

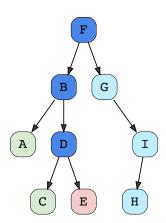
A, C



**11** of 21

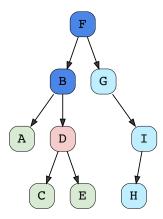
- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.  $\,$
- 4. Display the data part of the root (or current node).

A, C, E

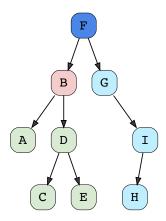


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- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).
  - A, C, E, D



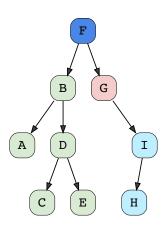
- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).
  - A, C, E, D, B



**14** of 21

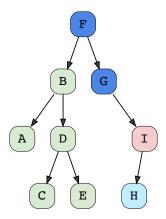
- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
  - 4. Display the data part of the root (or current node).

A, C, E, D, B



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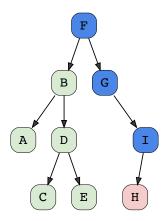
- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
  - 4. Display the data part of the root (or current node).
  - A, C, E, D, B



- **→**
- 2. Traverse the left subtree by recursively calling the post-order method.

1. Check if the current node is empty/null.

- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).
- A, C, E, D, B

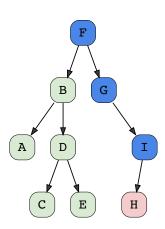


**17** of 21

- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.  $\,$

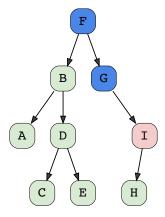


- 4. Display the data part of the root (or current node).
- A, C, E, D, B, H

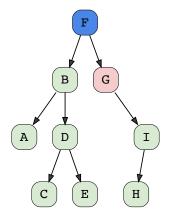


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- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.  $\,$
- 4. Display the data part of the root (or current node).
  - A, C, E, D, B, H, I



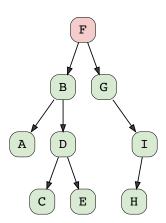
- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.
- 4. Display the data part of the root (or current node).
  - A, C, E, D, B, H, I, G



**20** of 21

- 1. Check if the current node is empty/null.
- 2. Traverse the left subtree by recursively calling the post-order method.
- 3. Traverse the right subtree by recursively calling the post-order method.  $\,$
- 4. Display the data part of the root (or current node).

A, C, E, D, B, H, I, G, F



- :





Here is the implementation of post-order traversal in Python:

```
1 def·postorder_print(self,·start,·traversal):
2 ··"""Left->Right->Root"""
3 ··if·start:
4 ····traversal·=·self.postorder_print(start.left,·*
5 ····traversal·=·self.postorder_print(start.right,
6 ····traversal·+=·(str(start.value)·+·"-")
7 ··return·traversal
```

postorder\_print(self, start, traversal)

Again, we just changed the order of statements. The recursive calls to the left and the right subtree have been placed before concatenating the value of the current node to traversal.

# Helper Method #

Below is the implementation of all the tree traversal methods within the Binary Tree class. Additionally, there is a helper method print\_tree(self, traversal\_type) which will invoke the specified method according to traversal\_type.

```
42
            if start:
43
                traversal = self.postorder_print(start
                traversal = self.postorder_print(start
44
                traversal += (str(start.value) + "-")
45
46
            return traversal
47
   # 1-2-4-5-3-6-7-
48
   # 4-2-5-1-6-3-7
   # 4-2-5-6-3-7-1
50
51
                     1
52
```

```
54
55
                             7
56
57 # Set up tree:
   tree = BinaryTree(1)
58
59 tree.root.left = Node(2)
60 tree.root.right = Node(3)
61 tree.root.left.left = Node(4)
62 tree.root.left.right = Node(5)
63 tree.root.right.left = Node(6)
64
    tree.root.right.right = Node(7)
65
66
    print(tree.print_tree("preorder"))
    print(tree.print_tree("inorder"))
67
    print(tree.print_tree("postorder"))
68
                                                          同*
 D
                                                                         X
Output
                                                                    0.19s
 1-2-4-5-3-6-7-
 4-2-5-1-6-3-7-
 4-5-2-6-7-3-1-
```

A suggestion is to take out a piece of paper and traverse a sample binary tree yourself according to all the traversal types. Once that is done, you can confirm your results by playing around with the implementation provided above.

Hope you find these depth-first tree traversals useful! See you in the next lesson for level-order traversal which is a kind of breadth-first tree traversal.



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	£	

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 $(https://discuss.educative.io/tag/traversal-algorithms\_binary-trees\_data-structures-and-algorithms-in-python)\\$ 







# Level-Order Traversal

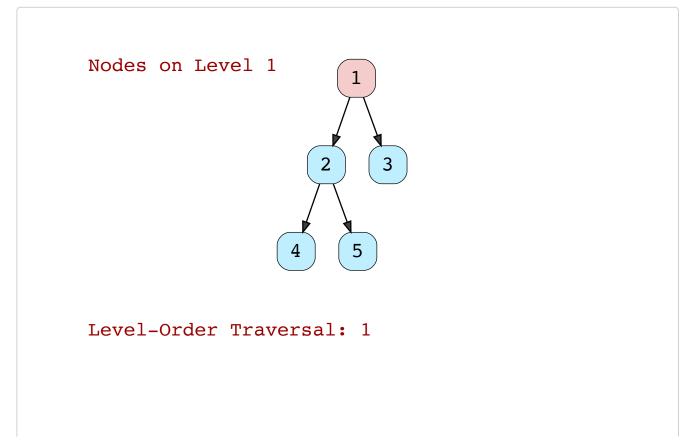
In this lesson, you will learn how to implement level-order traversal of a binary tree in Python.

We'll cover the following ^

- Algorithm
- Implementation

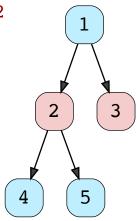
In this lesson, we go over how to perform a level-order traversal in a binary tree. We then code a solution in Python building upon our binary tree class.

Here is an example of a level-order traversal:





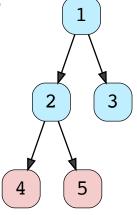
Nodes on Level 2



Level-Order Traversal: 1, 2, 3

**2** of 3

Nodes on Level 3



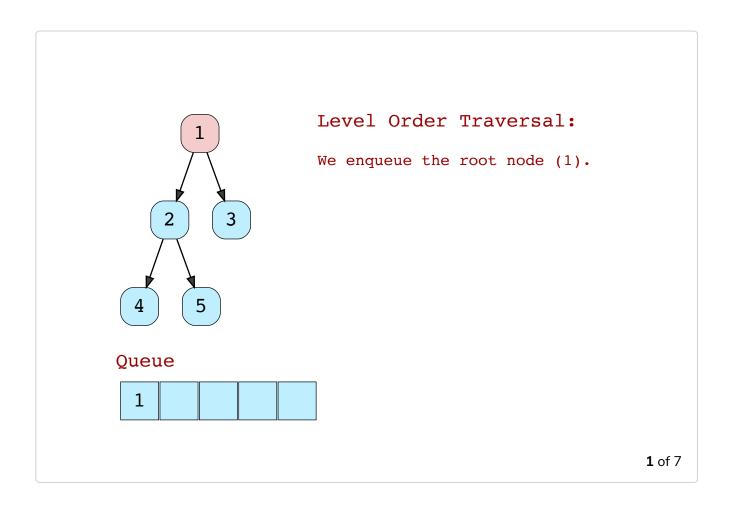
Level-Order Traversal: 1, 2, 3, 4, 5





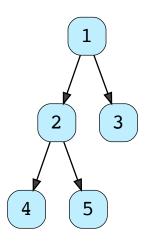
# Algorithm #

To do a level-order traversal of a binary tree, we require a queue. Have a look at the slides below for the algorithm:





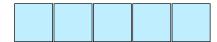




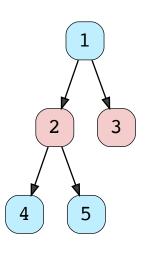
### Level Order Traversal: 1,

We denqueue from the queue and add it to the traversal.

### Queue



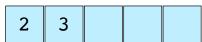
**2** of 7



### Level Order Traversal: 1,

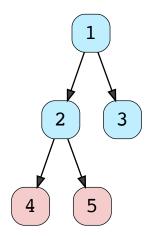
We enqueue the children of the node we dequeued.

### Queue









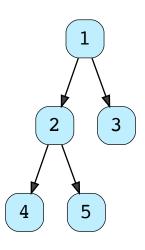
### Level Order Traversal: 1,2

We dequeue 2, add it to the traversal and enqueue its children.

### Queue



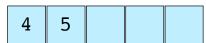
**4** of 7



## Level Order Traversal: 1,2,3

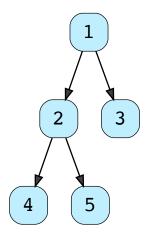
We dequeue 3, add it to the traversal and enqueue nothing as 3 has no children.

### Queue





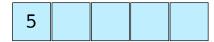




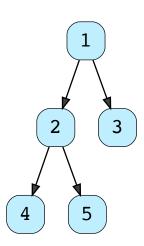
## Level Order Traversal: 1,2,3,4

We dequeue 4, add it to the traversal and enqueue nothing as 4 has no children.

### Queue



**6** of 7



## Level Order Traversal: 1,2,3,4,5

We dequeue 5, add it to the traversal and enqueue nothing as 5 has no children.

#### Queue



# Implementation #





Now that you are familiar with the algorithm, let's jump to the implementation in Python. First, we'll need to implement Queue so that we can use its object in our solution of level-order traversal.

```
class Queue(object):
 1
                                                                           6
 2
      def __init__(self):
 3
        self.items = []
 4
 5
      def enqueue(self, item):
 6
        self.items.insert(0, item)
 7
 8
      def dequeue(self):
 9
        if not self.is_empty():
          return self.items.pop()
10
11
12
      def is_empty(self):
13
        return len(self.items) == 0
14
15
      def peek(self):
16
        if not self.is_empty():
17
          return self.items[-1].value
18
      def __len__(self):
19
20
        return self.size()
21
22
      def size(self):
23
        return len(self.items)
```

class Queue

The constructor of the Queue class initializes self.items to an empty list on **line 3**. This list will store all the elements in the queue. We assume the last element to be the *front* of the queue and the first element to be the *back* of the queue.

To perform the enqueue operation, in the enqueue method, we make use of the insert method of Python list which will insert item on the 0 th index in self.items as specified on **line 6**. On the other hand, in the dequeue method, we use the pop method of Python list to pop out the last element as the queue follows the *First-In*, *First-Out* property. The method also ensures that the pop method is only called if the queue is not empty. To see if a queue is empty or not, the is\_empty method comes in handy which checks for the length of self.items and compares it with 0. If the length of self.items is 0, True is returned, otherwise, False is returned.

The peek method will return the value of the last element in self.items which we assume to be the front of our queue. We have also overridden the len method on **line 19** which calls the size method on **line 22**. The size method returns the length of self.items.

Now that we have successfully implemented the Queue class, let's go ahead and implement level-order traversal:

```
def levelorder_print(self, start):
 2
      if start is None:
 3
        return
 4
 5
      queue = Queue()
      queue.enqueue(start)
 6
 7
      traversal = ""
 8
 9
      while len(queue) > 0:
10
        traversal += str(queue.peek()) + "-"
        node = queue.dequeue()
11
12
13
        if node.left:
14
          queue.enqueue(node.left)
15
        if node.right:
16
          queue.enqueue(node.right)
17
18
      return traversal
```



In the code above, first of all, we handle an edge case on **line 2**, i.e., start (root node) is None or we have an empty tree. In such a case, we return from the levelorder\_print method.

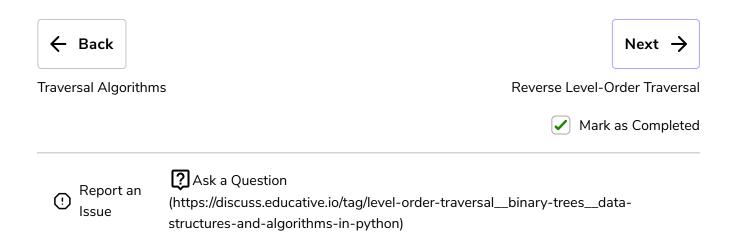
On **line 5**, we initialize a Queue object from the class we just implemented and name it as queue to which we enqueue start on **line 6** as described in the algorithm. traversal is initialized to an empty string on **line 8**. Next, we set up a while loop on **line 9** which runs until the length of the queue is greater than 0. Just as depicted in the algorithm, we append an element using the peek method to traversal and also concatenate a – so that the traversal appears in a format where the visited nodes will be divided by –. Once traversal is updated to register the node we visit, we dequeue that node and save it in the variable node on **line 11**. From **lines 13-16**, we check for the left and the right children of node and enqueue them to queue if they exist.

Finally, we return traversal on **line 18** which will have all the nodes we visited according to level-order.

In the code widget below, we have added levelorder\_print to BinaryTree class and have also added "levelorder" as a traversal\_type to print\_tree method.

```
75
        def levelorder_print(self, start):
76
             if start is None:
77
                 return
78
79
            queue = Queue()
80
            queue.enqueue(start)
81
            traversal = ""
82
83
            while len(queue) > 0:
                 traversal += str(queue.peek()) + "-"
84
85
                 node = queue.dequeue()
86
                 if node.left:
```

I hope level-order traversal is clear to you! In the next lesson, we will cover reverse level-order traversal. Stay tuned!













## Reverse Level-Order Traversal

In this lesson, you will learn how to implement reverse level-order traversal of a binary tree in Python.



- Algorithm
- Implementation

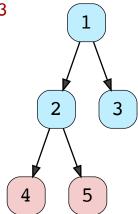
This lesson will be an extension of the previous lesson. In this lesson, we will go over how to perform a reverse level-order traversal in a binary tree. We then code a solution in Python building on our binary tree class.

Below is an example of reverse level-order traversal of a binary tree:





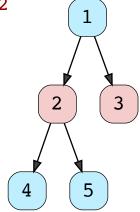
Nodes on Level 3



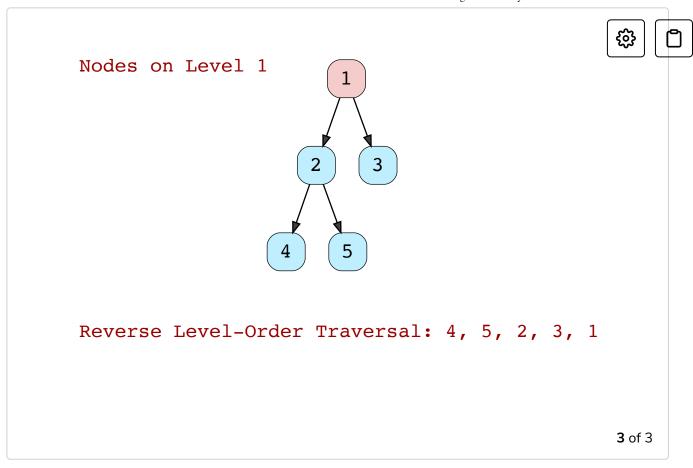
Reverse Level-Order Traversal: 4, 5

**1** of 3

### Nodes on Level 2



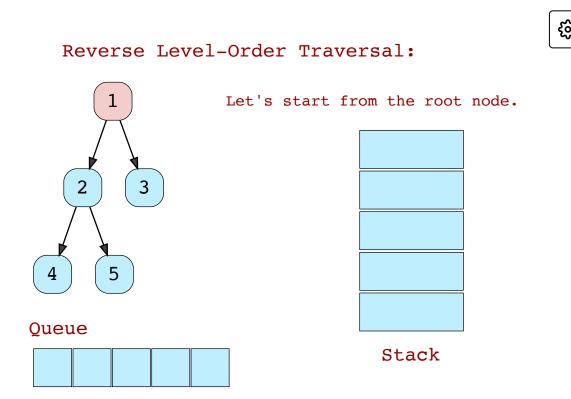
Reverse Level-Order Traversal: 4, 5, 2, 3

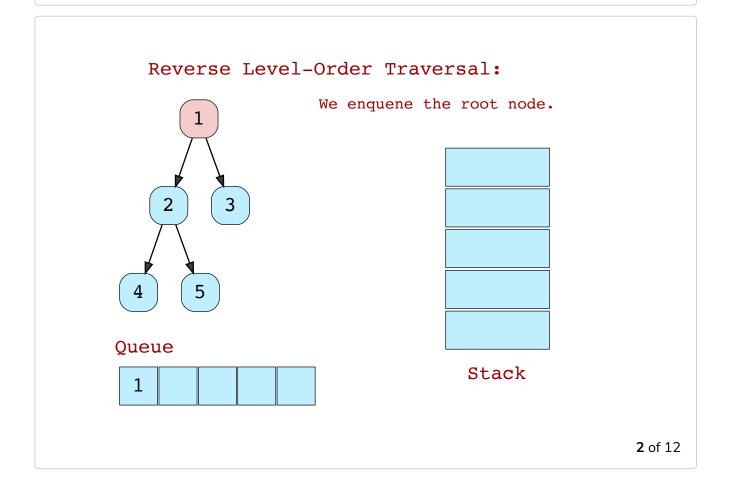


**–** []

# Algorithm #

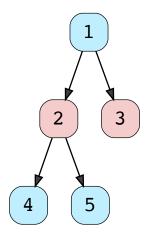
To solve this problem, we'll use a queue again just like we did with level-order traversal, but with a slight tweak; we'll enqueue the right child before the left child. Additionally, we will use a stack. The algorithm starts with enqueuing the root node. As we traverse the tree, we dequeue the nodes from the queue and push them to the stack. After we push a node on to the stack, we check for its children, and if they are present, we enqueue them. This process is repeated until the queue becomes empty. In the end, popping the element from the stack will give us the reverse-order traversal. Let's step through the algorithm using the illustrations below:



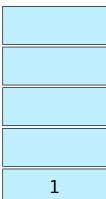




#### Reverse Level-Order Traversal:



We push 1 onto the stack and enqueue its children (from right to left).



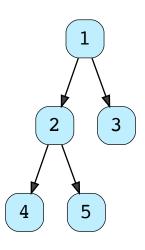
Queue

3 2

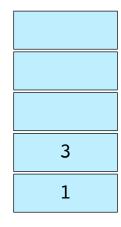
Stack

**3** of 12

### Reverse Level-Order Traversal:



We push 3 onto the stack.



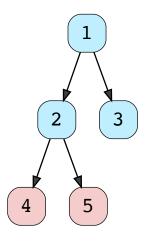
Queue



Stack



#### Reverse Level-Order Traversal:



We push 2 onto the stack and enqueue its children (from right to left).

2 3 1

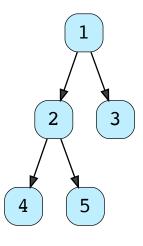
Queue

5 4

Stack

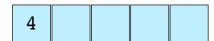
**5** of 12

### Reverse Level-Order Traversal:



We push 5 onto the stack.

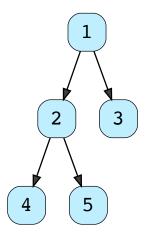
Queue



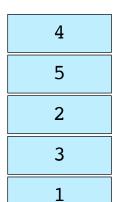
Stack



### Reverse Level-Order Traversal:

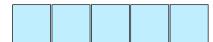


We push 4 onto the stack.



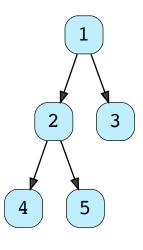
Stack

Queue



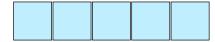
**7** of 12

## Reverse Level-Order Traversal: 4



Popping from the stack and appending it to the traversal

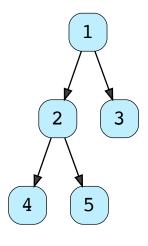
Queue



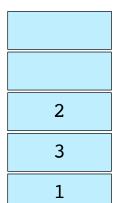
Stack



### Reverse Level-Order Traversal: 4,5,



Popping from the stack and appending it to the traversal

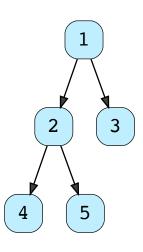


Queue

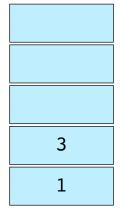
Stack

**9** of 12

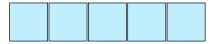
## Reverse Level-Order Traversal: 4,5,2



Popping from the stack and appending it to the traversal



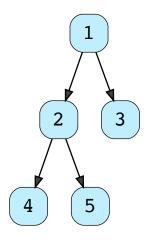
Queue



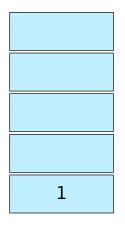
Stack



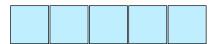
### Reverse Level-Order Traversal: 4,5,2,3



Popping from the stack and appending it to the traversal



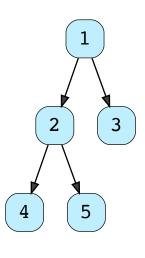
Queue



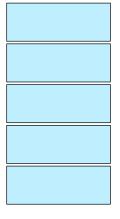
Stack

**11** of 12

## Reverse Level-Order Traversal: 4,5,2,3,1



Popping from the stack and appending it to the traversal



Queue



Stack

# Implementation #





Now that we have studied the algorithm, let's jump to the implementation in Python. First, we'll implement Stack class:

```
2 ····def·__init__(self):
                                                                 6
  ·····self.items·=·[]
4
5 ····def·__len__(self):
6 ····return·self.size()
7 .....
   ····def·size(self):
9
   ·····return·len(self.items)
10
11
   ····def·push(self, ·item):
12
   ·····self.items.append(item)
13
14
   ····def·pop(self):··
   ....if not self is empty():
16
   ·····return·self.items.pop()
17
   ····def·peek(self):
18
   ·····if·not·self.is empty():
20
   ·····return·self.items[-1]
21
22
   ····def·is_empty(self):
23
   ·····return·len(self.items)·==·0
24
25
  ····def· str (self):
  26
27
  ·····for·i·in·range(len(self.items)):
28
   ·····s·+=·str(self.items[i].value)·+·"-"
29
   ·····return·s
```

class Stack

In the constructor, we initialize <code>self.items</code> to an empty list just like we did with <code>Queue</code>. In the push method on <code>line 11</code>, the built-in append method is used to insert elements (<code>item</code>) to <code>self.items</code>. So whenever we push an element onto the stack, we append that element to <code>self.items</code>. The <code>pop</code> method on <code>line 14</code> first checks whether the stack is empty or not using the

is\_empty method implemented on **line 22**. In the pop method, we use pop method of Python list to pop out the last element as the stack follows the *First-In, Last-Out* property and the latest element we inserted is at the end of self.items. The is\_empty method on **line 22** checks for the length of self.items by comparing it with 0 and returns the boolean value accordingly. On **line 18**, peek method is implemented which may or may not be used in the solution of our lesson problem. If the stack is not empty, the last element of self.items is returned on **line 20**.

The size and len method have also been added in a way as to the Queue class. Also, we have an str method on **line 25** which iterates through self.items and concatenates them into a string which is returned from the method.

Now that we are done with the implementation of Stack class, let's discuss reverse\_levelorder\_print:

```
def reverse_levelorder_print(self, start):
 1
 2
      if start is None:
 3
        return
 4
      queue = Queue()
 5
      stack = Stack()
 6
 7
      queue.enqueue(start)
 8
 9
      traversal = ""
10
11
      while len(queue) > 0:
        node = queue.dequeue()
12
13
14
        stack.push(node)
15
        if node.right:
16
17
          queue.enqueue(node.right)
18
        if node.left:
19
          queue.enqueue(node.left)
20
21
      while len(stack) > 0:
22
        node = stack.pop()
        traversal += str(node.value) + "-"
23
```

2425 return traversal



reverse\_levelorder\_print(self, start)

In the code above, we handle an edge case on **line 2**, i.e., the start (root node) is None or we would have an empty tree. In such a case, we return from the reverse\_levelorder\_print method.

On **line 5** and **line 6**, we initialize a Queue object and a Stack object from the class we just implemented. In the next line, we enqueue start to queue as described in the algorithm. traversal is initialized to an empty string on **line 10**. Next, we set up a while loop on **line 11** which runs until the length of the queue is greater than 0. Just as depicted in the algorithm, we dequeue an element from the queue and push it on the stack on **line 12**. From **lines 16-19**, we check for the right and left children of the node and enqueue them to queue if they exist. At the end of the while loop, stack will contain all the nodes of the tree. On **line 21**, we are using a while loop to pop elements from the stack and concatenate them to traversal which is returned from the method on **line 25**.

In the code widget below, we have added reverse\_levelorder\_print to the BinaryTree class and have also added "reverse\_levelorder" as a traversal\_type to print\_tree method.

```
traversal = ""
135
136
             while len(queue) > 0:
137
                  node = queue.dequeue()
138
139
                  stack.push(node)
140
141
                  if node.right:
                      queue.enqueue(node.right)
142
                  if node.left:
143
                      queue.enqueue(node.left)
144
```

```
while len(stack) > 0:
146
147
                  node = stack.pop()
                  traversal += str(node.value) + "-"
148
149
150
              return traversal
151
152
153
154
    tree = BinaryTree(1)
155
     tree.root.left = Node(2)
     tree.root.right = Node(3)
156
     tree.root.left.left = Node(4)
157
158
     tree.root.left.right = Node(5)
159
     print(tree.print_tree("reverse_levelorder"))
160
161
 \triangleright
                                                              []
                                                                            X
                                                                        0.15s
Output
 4-5-2-3-1-
```

In the next lesson, we will learn how to calculate the height of a binary tree in Python.







#### Calculating the Height of a Binary Tree

In this lesson, you will learn how to calculate the height of a binary tree.

We'll cover the following ^

- · Height of Tree
  - Height of Node
- Algorithm
- Implementation

### Height of Tree #

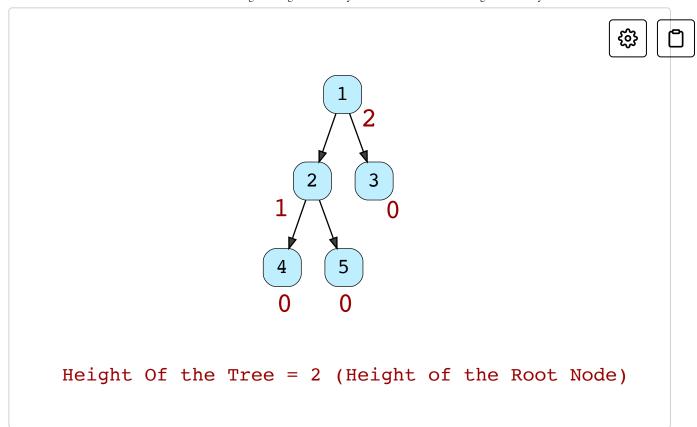
Let's start by defining the height of a tree. The height of a tree is the height of its root node. Now let's see what we mean by the height of a node:

#### Height of Node #

The height of a node is the number of edges on the longest path between that node and a leaf. The height of a leaf node is 0.

Recursively defined, the height of a node is one greater than the max of its right and left children's height.

Below is an example of a binary tree labeled with heights of individual nodes:



### Algorithm #

In this lesson, we will consider the recursive approach to calculate the height of a tree. The idea is to break down the problem using recursion and traverse through the left and right subtree of a node to calculate the height of that node. Once we get the height of the left and right subtree, we will consider the maximum of the two heights plus one to be the height of the tree.

### Implementation #

Let's move to the implementation in Python:

```
1 def height(self, node):
2   if node is None:
3    return -1
4   left_height = self.height(node.left)
5   right_height = self.height(node.right)
6
```

7 return 1 + max(left\_height, right\_height)

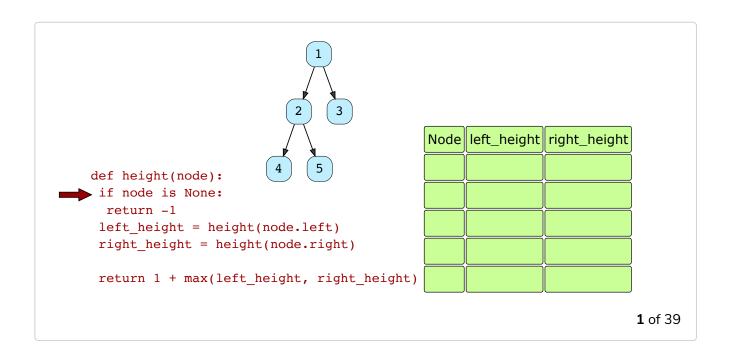


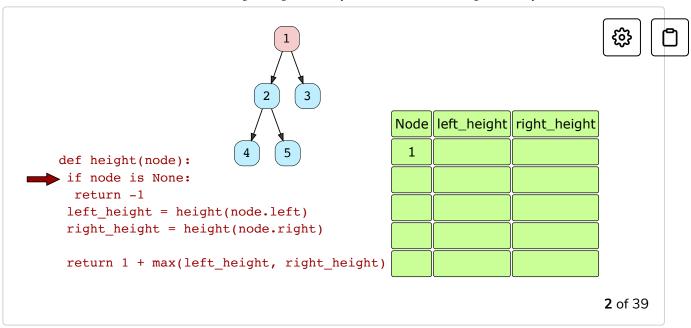
height(self, node)

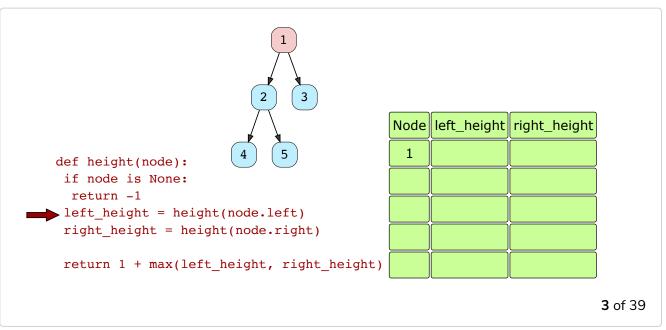
In the code above, our base case is when node equals None. If node equals None, we return -1 on **line 3** as we have gone past the leaf nodes. Once a leaf node discovers that its left and right children are reporting heights of -1 each, it will add 1 to -1 and return 0 as its height.

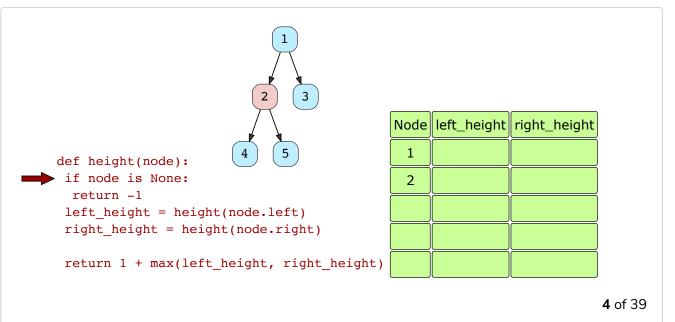
In **lines 4-5**, we recursively call the height method on the left child and the right child. The final height is calculated by adding 1 to the maximum height of the left and right subtree, as height is the longest path between that node and the leaf node. The final height is what is returned from the method.

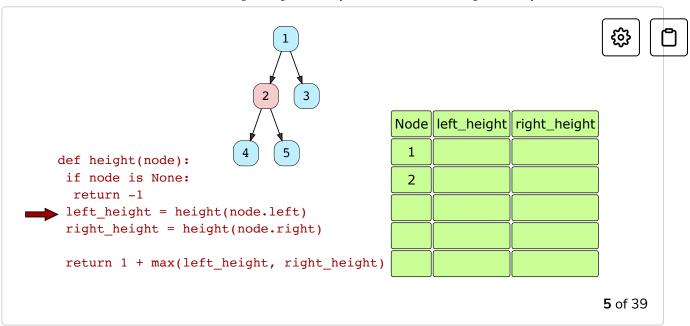
Let's run this code step by step in the illustration below:

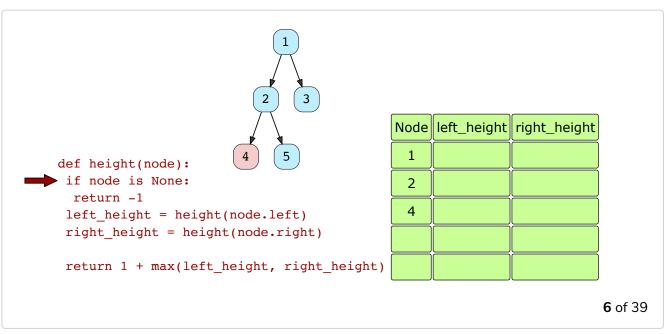


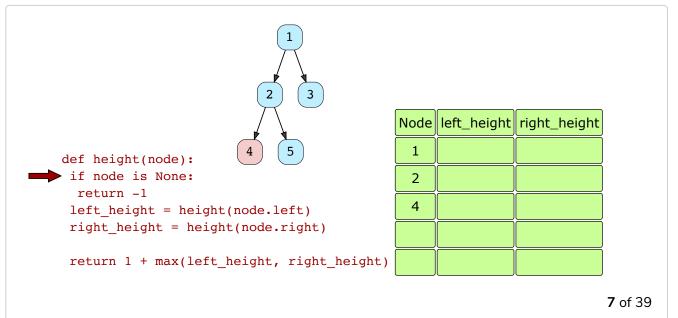


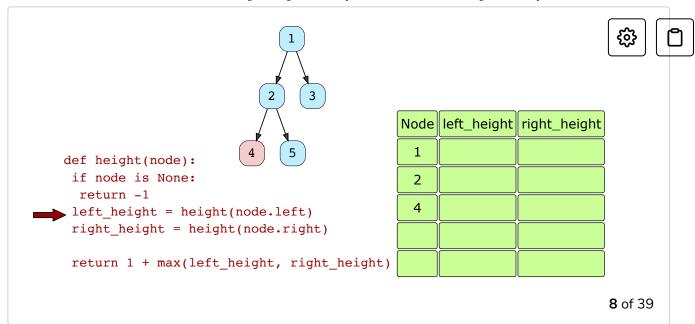


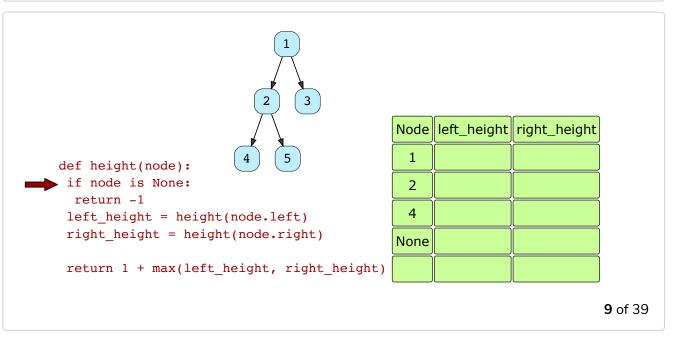


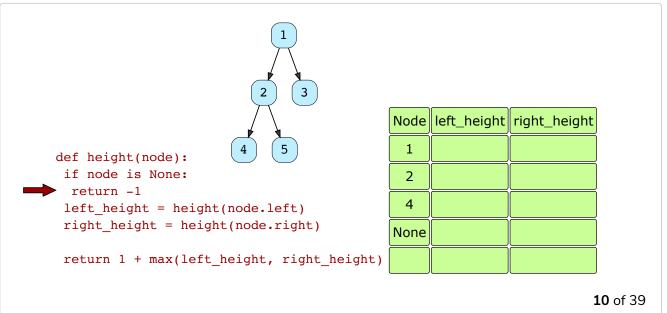


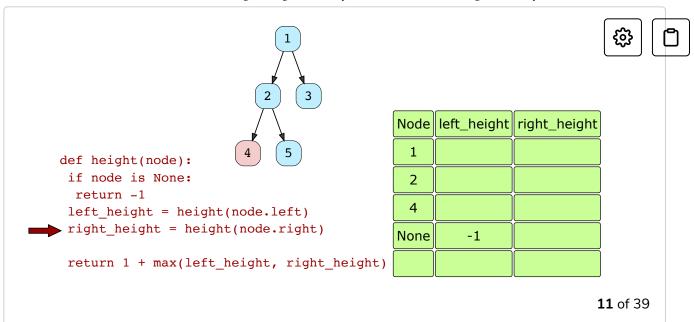


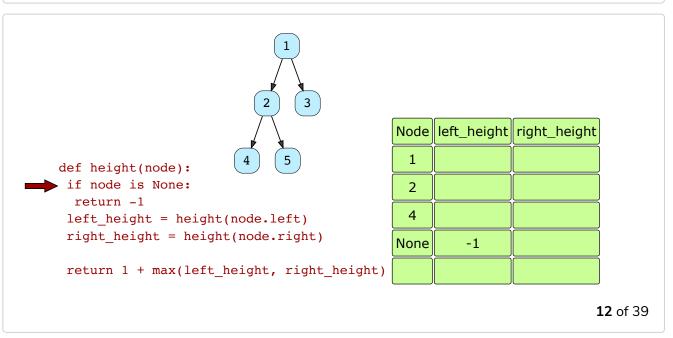


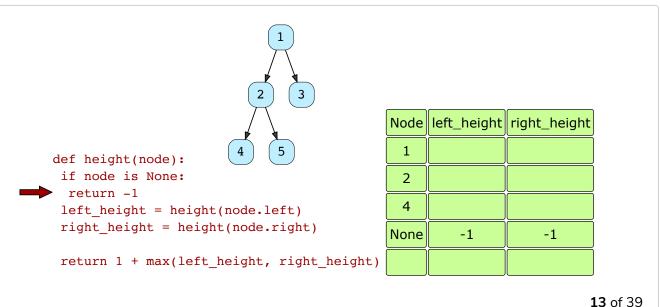


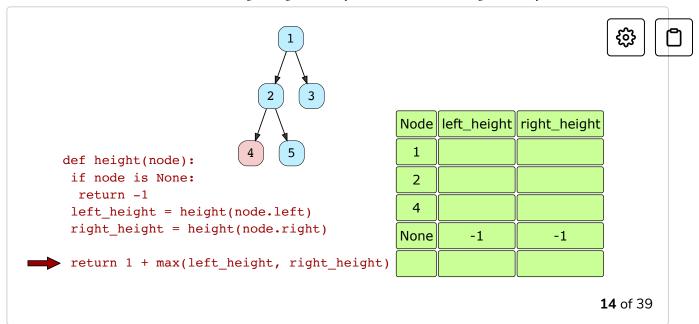


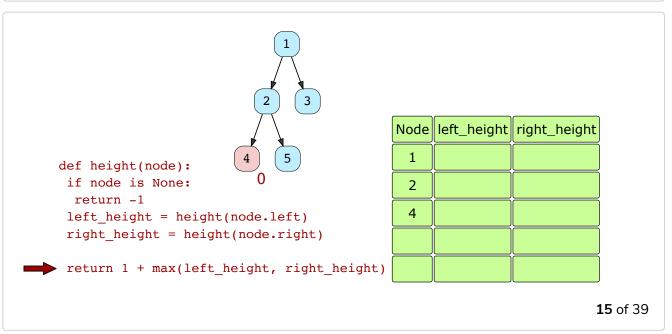


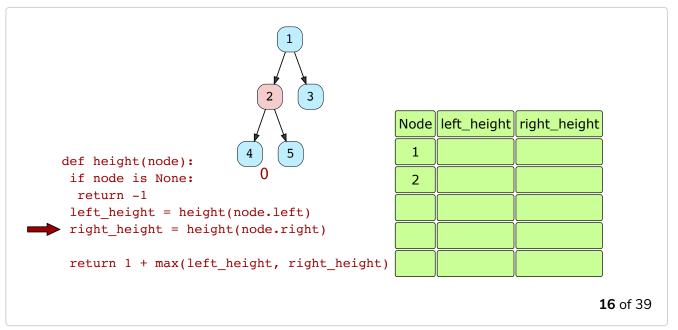


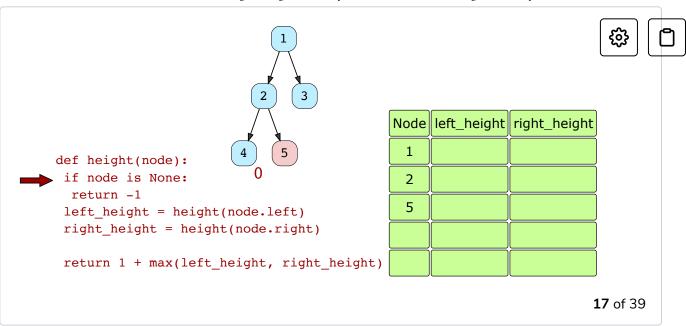


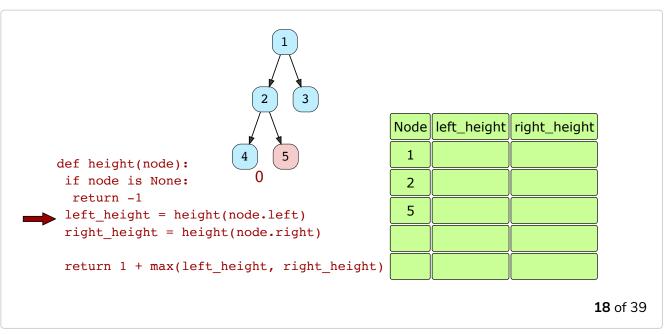


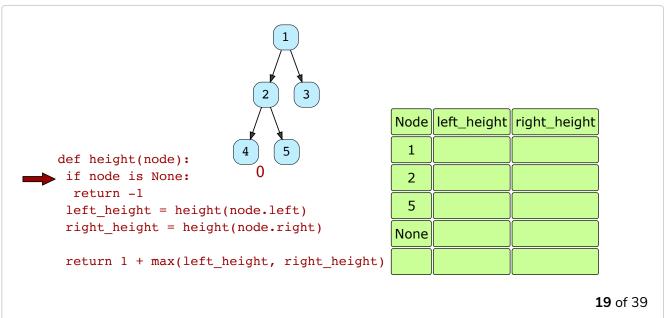


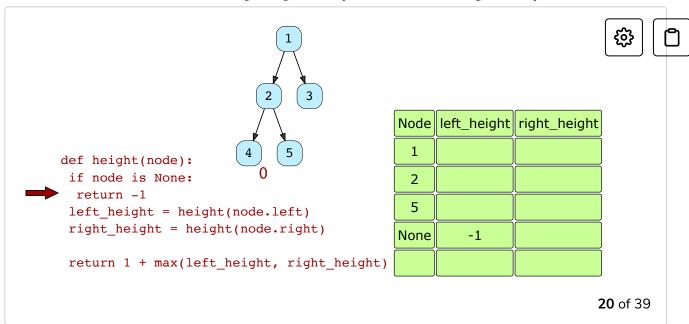


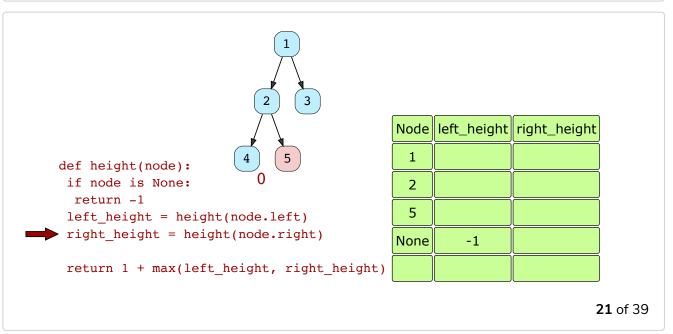


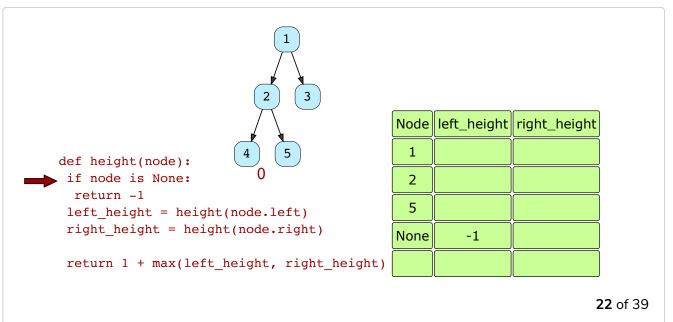


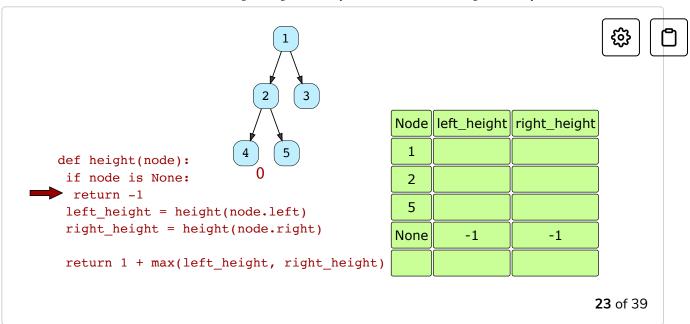


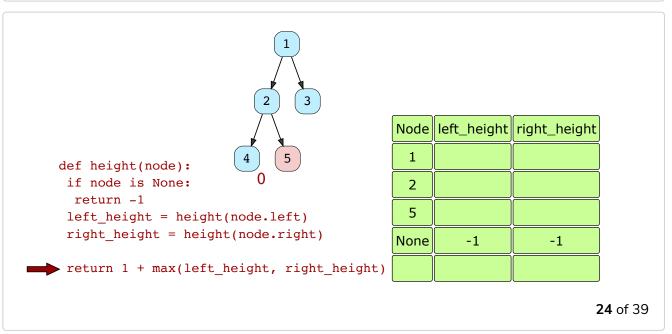


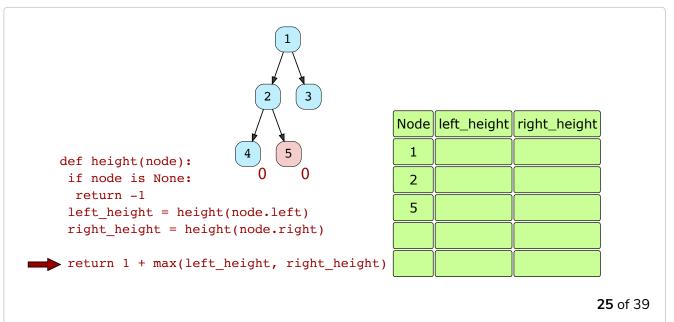


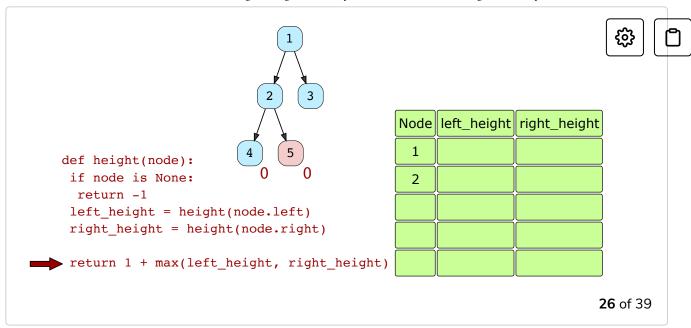


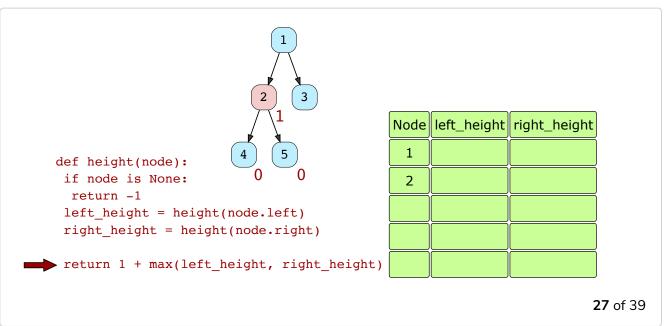


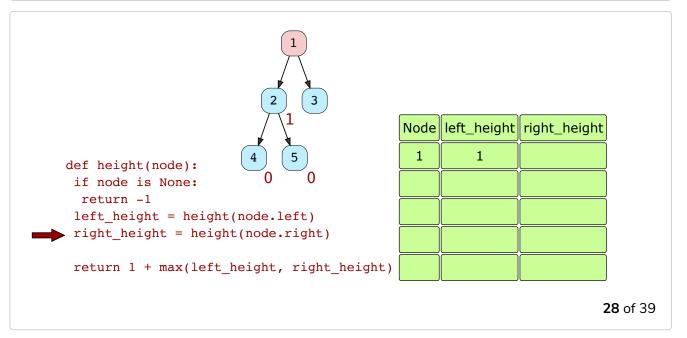


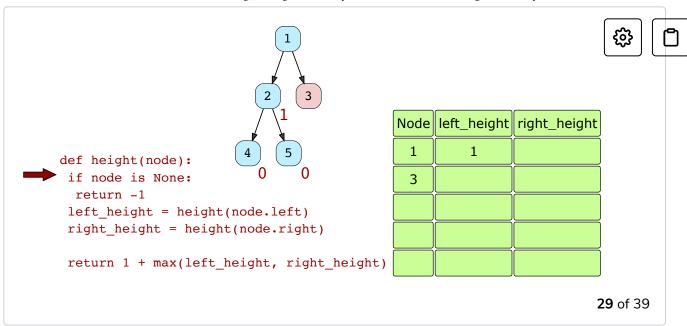


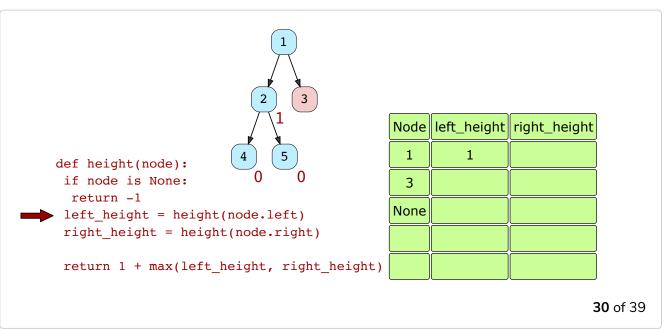


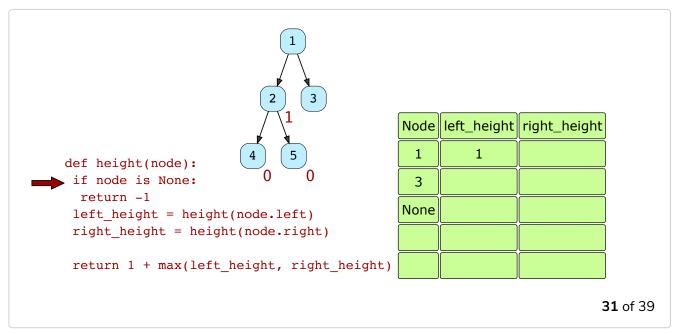


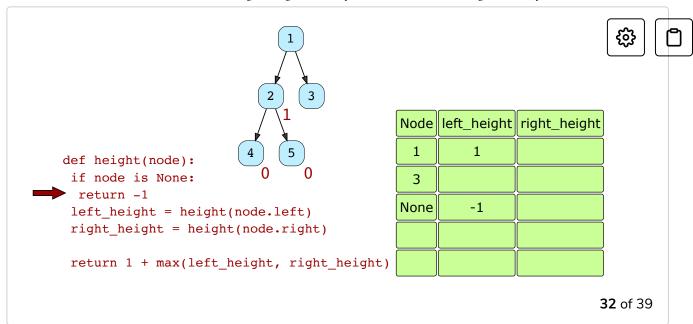


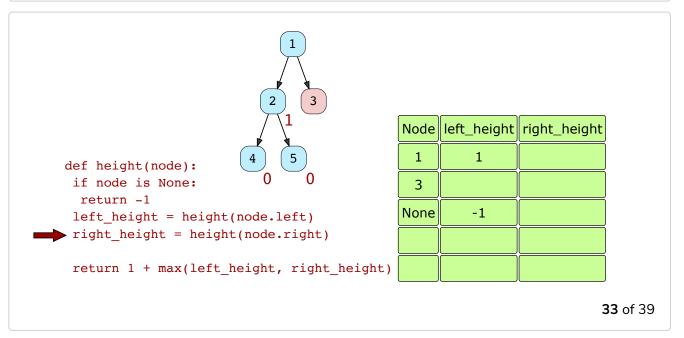


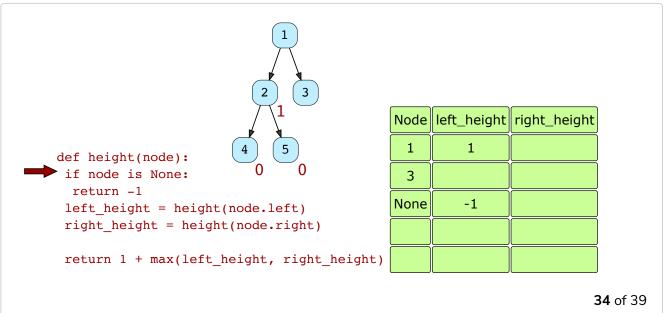


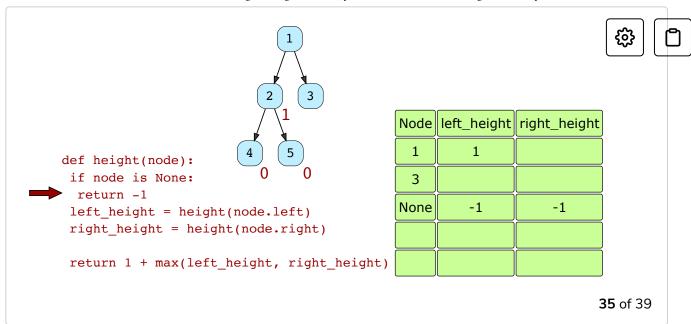


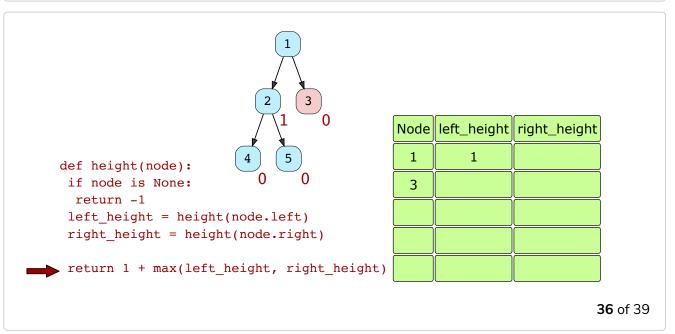


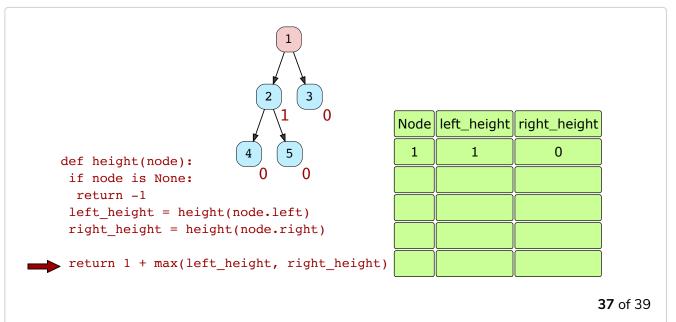


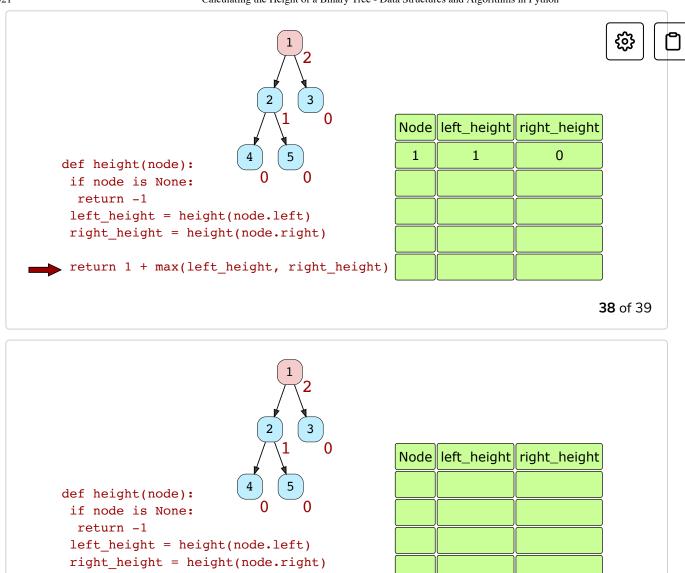












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I hope the visuals are helpful to understand the algorithm.

return 1 + max(left height, right height)

In the code widget, the height method is made part of the BinaryTree Class. Write your test cases to verify the height method. A sample test case has been given to you.

**39** of 39

```
150
             return traversal
151
         def height(self, node):
152
153
           if node is None:
154
             return -1
155
           left_height = self.height(node.left)
           right_height = self.height(node.right)
156
157
           return 1 + max(left_height, right_height)
158
159
160
    # Calculate height of binary tree:
161
162
          /\
         2 3
163 #
164
       / \
165
    #45
166
167 tree = BinaryTree(1)
168 tree.root.left = Node(2)
169 tree.root.right = Node(3)
170 tree.root.left.left = Node(4)
    tree.root.left.right = Node(5)
171
172
    print(tree.height(tree.root))
173
                                                            \triangleright
                                                                          X
                                                                      0.16s
Output
 2
```

This sums up our content on Binary Trees. Get ready as we have a challenge regarding binary trees waiting for you in the next lesson.



Reverse Level-Order Traversal

Exercise: Calculating the Size of a Tree

Mark as Completed

Report an Issue

? Ask a Question

(https://discuss.educative.io/tag/calculating-the-height-of-a-binary-tree\_\_binary-trees\_\_data-structures-and-algorithms-in-python)







#### Exercise: Calculating the Size of a Tree

Challenge yourself with an exercise in which you'll have to calculate the size of a binary tree!

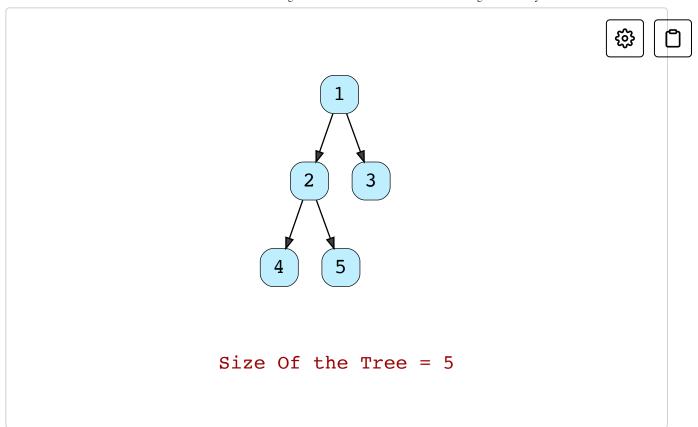
We'll cover the following ^

- Problem
- Coding Time!

#### Problem #

The size of the tree is the total number of nodes in a tree. You are required to return the size of a binary tree given the root node of the tree.

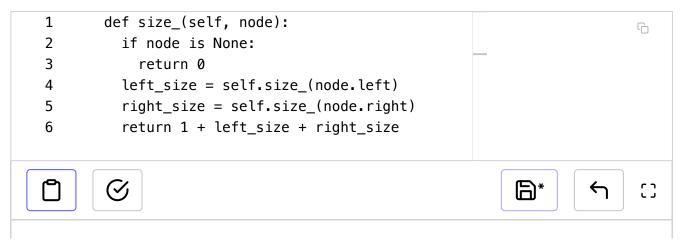
Below is an example illustrated for you:



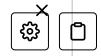
### Coding Time! #

In the code below, size\_ is a class method of the BinaryTree class. You cannot see the rest of the code as it is hidden. As size\_ is a class method, please make sure that you don't change the indentation of the code provided to you. You are required to write your solution under the method prototype and return the size of the tree from the method.

#### Good luck!



**Show Results** Show Console



0.23s

#### 3 of 3 Tests Passed

Result	Input	Expected Output	Actual Output	Reason
<b>~</b>	size(tree)	5	5	Succeeded
<b>~</b>	size(tree)	7	7	Succeeded
<b>✓</b>	size(tree)	1	1	Succeeded







#### Solution Review: Calculating the Size of a Tree

This lesson contains the solution review for the challenge of calculating the size of a binary tree.

# We'll cover the following ^

- Iterative Approach
- Recursive Approach

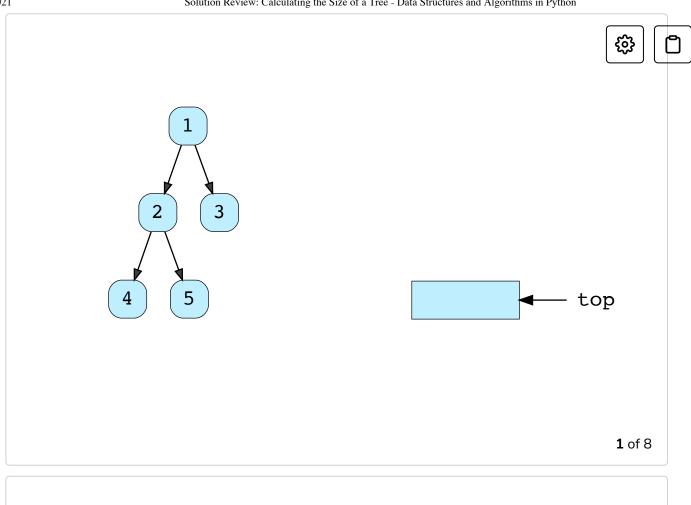
In this lesson, we will review a solution to the problem of determining the size of a binary tree.

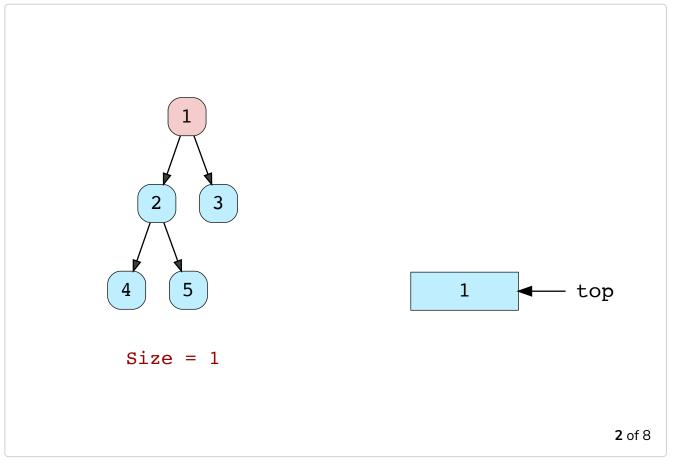
The "size" of a binary tree is the total number of nodes present in the binary tree. We will explicitly define this quantity in greater detail and cover a strategy for how one may calculate this quantity in the binary tree data structure we have been building in this chapter.

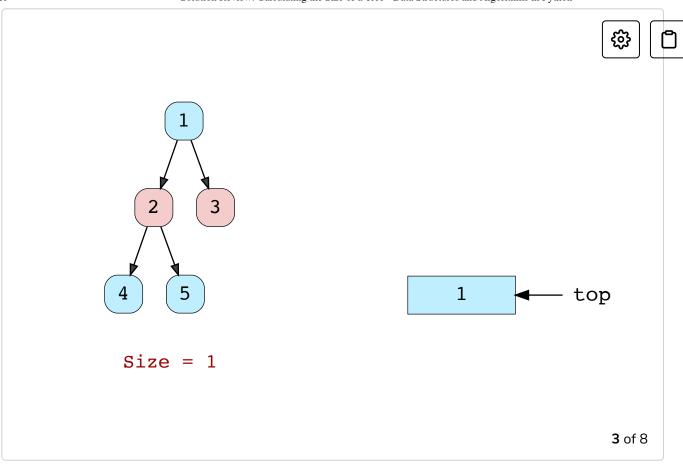
We will discuss an iterative and a recursive approach for solving this challenge.

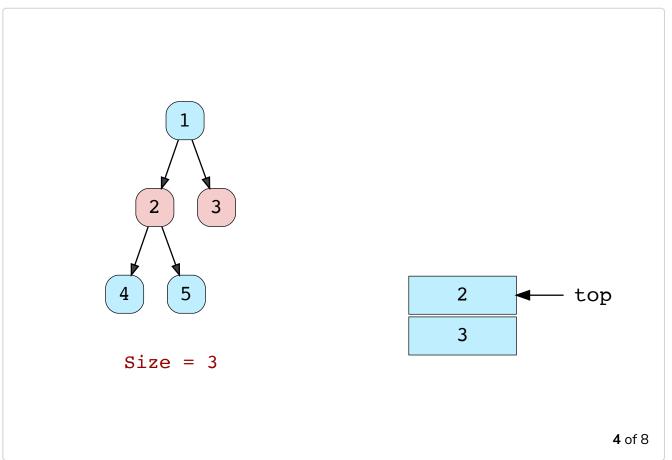
## Iterative Approach #

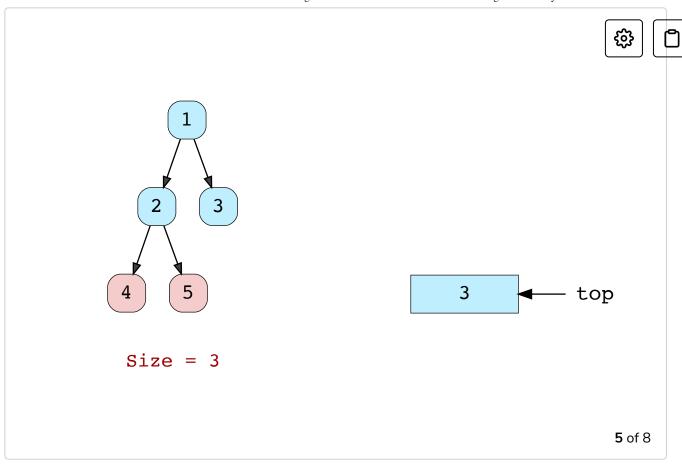
In the iterative solution, we'll make use of a stack on which we can push the starting node and increment size by 1. Next, we'll pop elements and push their children on to the stack if they have any. For every push, we'll increment size by 1. When the stack becomes empty, the count for the size will also be final. Have a look at the slides below to check out the algorithm:

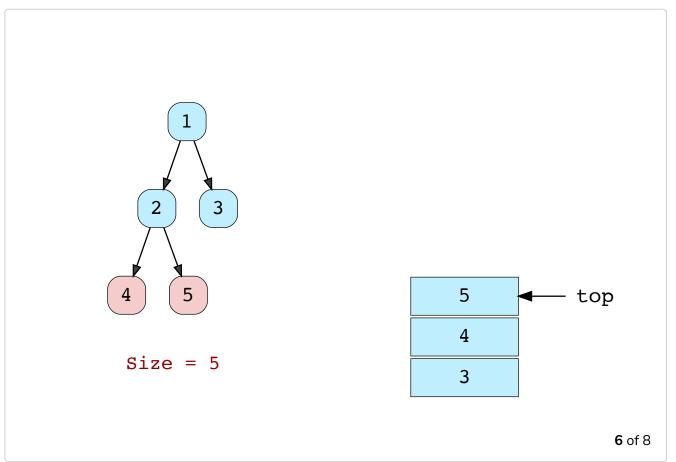


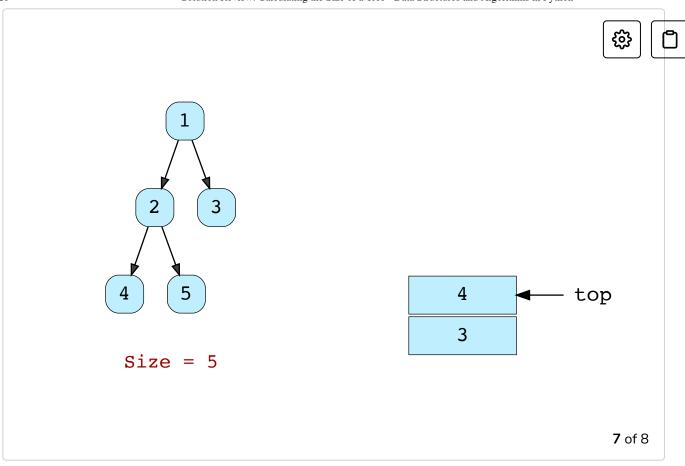


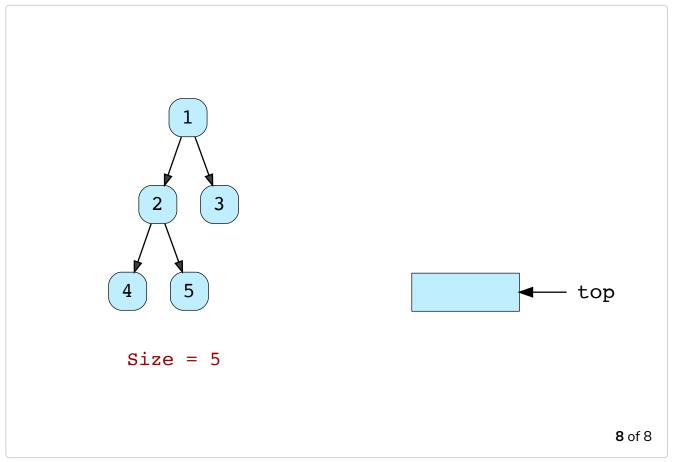












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Here is the implementation of the algorithm illustrated above in Python



**Lines 2-3** contain the edge case which checks for an empty tree and returns 0 in that case. On **line 5**, we declare stack to a Stack object and push the root node on to the stack on **line 6**. After the push, we initialize size to 1 as we have a node present in the stack.

Next, we have a while loop which runs as long as stack is not empty. On **line 9**, we pop from the stack and store the popped element in node. On **lines 10-15**, we check if node has a left or right child and push the child on to the stack while also incrementing the size by 1. Finally, when stack is empty, the while loop terminates, and size is returned on **line 16**.

#### Recursive Approach #

We will recursively traverse the nodes and keep track of the count of the nodes visited. Check out the implementation below:

```
1 def size_(self, node):
2  if node is None:
```

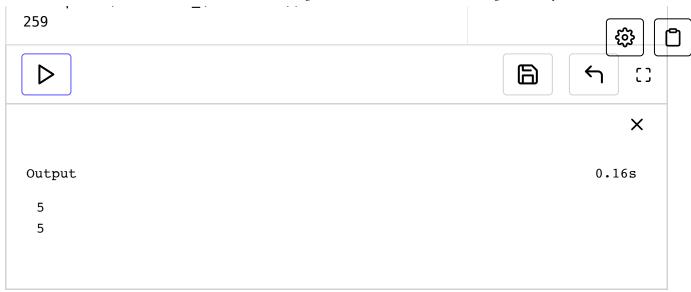
```
return 0
4 return 1 + self.size_(node.left) + self.size_(no
```

Recursively speaking, the size of the tree is the size of the left subtree of the root node + the size of the right subtree of the root node + 1 (for the root node).

The base case is that an empty binary tree has a size of 0 so when node becomes None, we return 0 as a count. Otherwise, we return 1 plus the count from the recursive call on the left and the right subtree.

Now, this was pretty straightforward. In the code widget below, you can play around with the entire implementation of BinaryTree that we have covered so far in this chapter.

```
233
             size = 1
234
             while stack:
235
                 node = stack.pop()
236
                  if node.left:
237
                      size += 1
238
                      stack.push(node.left)
239
                  if node.right:
240
                      size += 1
241
                      stack.push(node.right)
242
             return size
243
244
    # Calculate size of binary tree:
245
     #
           1
246
          /\
247
         2 3
248
249
     # 4 5
250
251
    tree = BinaryTree(1)
252
    tree.root.left = Node(2)
253
    tree.root.right = Node(3)
254
     tree.root.left.left = Node(4)
255
     tree.root.left.right = Node(5)
256
257
     print(tree.size())
258
     print(tree.size (tree.root))
```



I hope you had fun learning about Binary Trees. In the next chapter, we have a different type of binary tree, i.e., the binary search tree. Stay tuned to find out more!

